Super OPEs and Susy breaking mediation

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Outline

- Part 1: colloquium-type introduction
- Intro 2: why study (s)CFTs. QCD Analogy.
- Current OPEs in SCFTs.
- Applications to GGM.
- Example: apply to OGM.

Supersymmetry!

- Loophole in Coleman Mandula theorem forbidding extending Poincare' spacetime symmetry in an interacting theory. Conformal symmetry another exception. We'll consider combining them.
- Some exact results, insights into strongly interacting QFTs. For example, dualities.
- Might exist in Nature. Might help make the SM more natural.
- ....

Is Nature Natural?

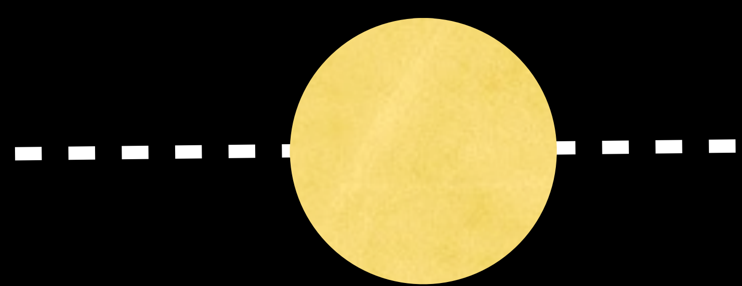
- Experimentally measured quantities, e.g. masses, or the CC, are complicated (generally divergent) sums of theory parameters. Apparent conspiracies of near-cancellations, highly sensitive to unknown UV physics. Unnatural.
- Fermions=naturally light thanks to chirality, but Higgs boson's mass is unnatural, e.g. top loops.
- ~ LHC energy scale susy helps somewhat restore Naturalness, by relating bosons to fermions.

Cornerstone of Standard Model

Higgs

Resting on a pencil found to be balancing on it's tip. Coincidence, or hidden supports?

Higgs mass corrections



Quantum loops of mass M fields give $\Delta m_H^2 \sim M^2 \gg m_H^2$ e.g. top loops. Susy partially cancels these, e.g. via stops.

Some valid concern that susy hasn't show itself yet at LHC. Experimental signatures highly dependent on susy breaking, enormous parameter space. Various slices now excluded. Many other parameter values remain viable.

SUSY BREAKING

Our matter ← * — Susy breaking

e.g. via

$Q_\alpha|0\rangle \neq 0$

* Many theory choices here. Susy breaking parameters and exp'l signatures highly dependent on choice. Here focus on "General gauge mediation" where breaking is communicated by gauge forces, e.g. $SU(3)_C$ $SU(2)_W$ $U(1)$. Susy breaking related to < J J > correlation functions (MSS).

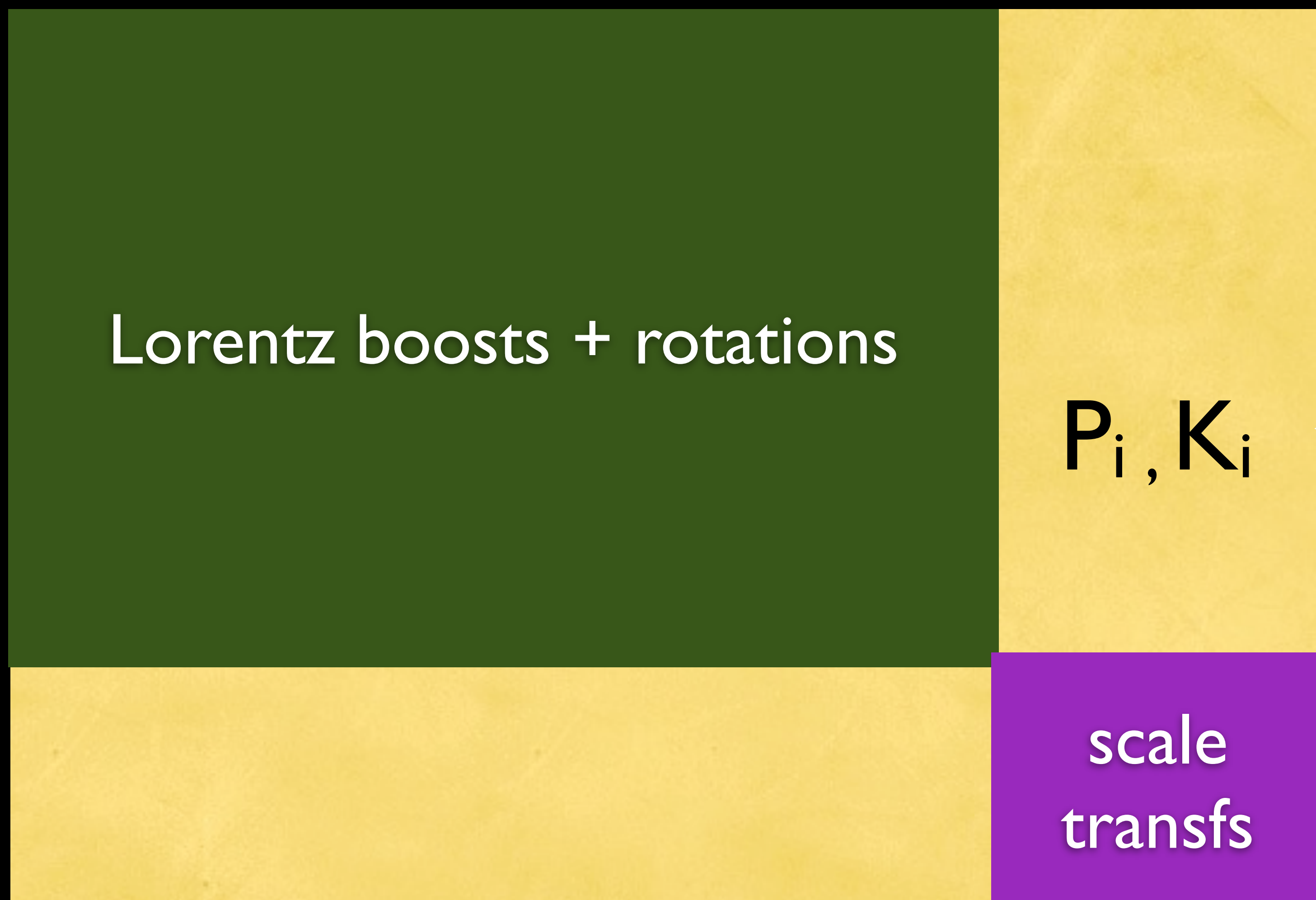
Conformal field theory

Extend Poincare to include scale (+ conformal) symmetry, e.g. $SO(4) \to SO(4,2)$. No S-matrix, rather just operator corelation functions.

$\langle \mathcal{O}_i \rangle = \delta_{i,0} \qquad \langle \mathcal{O}_i \mathcal{O}_j \rangle \sim c_{ij} \qquad \langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle \sim c_{ijk}$

suppressed operator position dependence here = fully determined by operator's spin and scaling dimension. Four and higher point functions = determined by above data, using the operator product expansion. (We'll mention some subtleties in the susy case.)

SO(4,2) ~ SU(2,2)



Lowest weight state (op) in irrep **"primary"**

$$\begin{cases} |\Delta, j_1, j_2\rangle \\ = \mathcal{O}_I(x)|0\rangle \end{cases}$$

Raising, lowering operators

susy version SU(2,2|1):

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^i_{\alpha\dot{\beta}} P_i$$

$$\{\bar{S}^{\dot{\alpha}}, S^\beta\} = 2\bar{\sigma}^{i\alpha\dot{\beta}} K_i$$

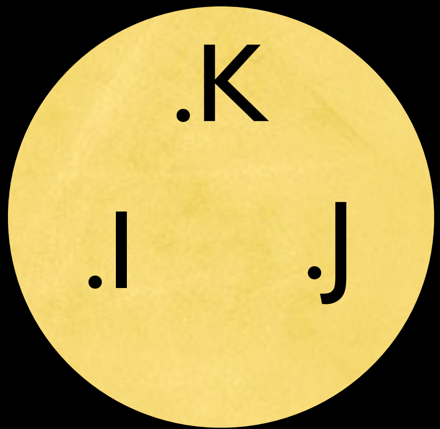
Add'l raising, lowering operators

OPE

$$\langle \mathcal{O}_K | \mathcal{O}_I(x) | \mathcal{O}_J \rangle = c_{IJ}^K$$

CFT data, x indep
primaries suffice

~Pants



=Basic building block
for all correlators
"conformal blocks"

vs Susy case

$$\langle \mathcal{O}_K | \mathcal{O}_I(x, \theta, \bar{\theta}) | \mathcal{O}_J \rangle \neq c_{IJ}^K$$

Because of nilpotent
3-point invariants
no universal superconf'l blks

Here

- Explore applications to this scenario of superconf'l symm e.g. $SU(2,2|1)$
- Note curiosity of SCFT representations: primaries generally don't suffice. Note conserved currents are better.
- Analyze SCFT conserved current operator product expansion constraints. Blocks.

End of part 1

Many 4d (s)CFTs

- SQCD in Seiberg conformal window.
- N=4.
- non-Lagrangian possibilities.
- Observables = spectrum of operators, their dimensions, OPE coefficients (+ non-local, Wilson loops.)

Applications of (s)CFTs (?)

- Help with model building challenges?
- E.g. O(1) anomalous dimensions could help suppress or enhance otherwise finely tuned quantities. Examples: sequestering, flavor hierarchy from anarchy, mu / Bmu in GM, conformal technicolor, etc.
- Or flowing near CFTs. E.g. walking technicolor, unparticles with mass gaps.

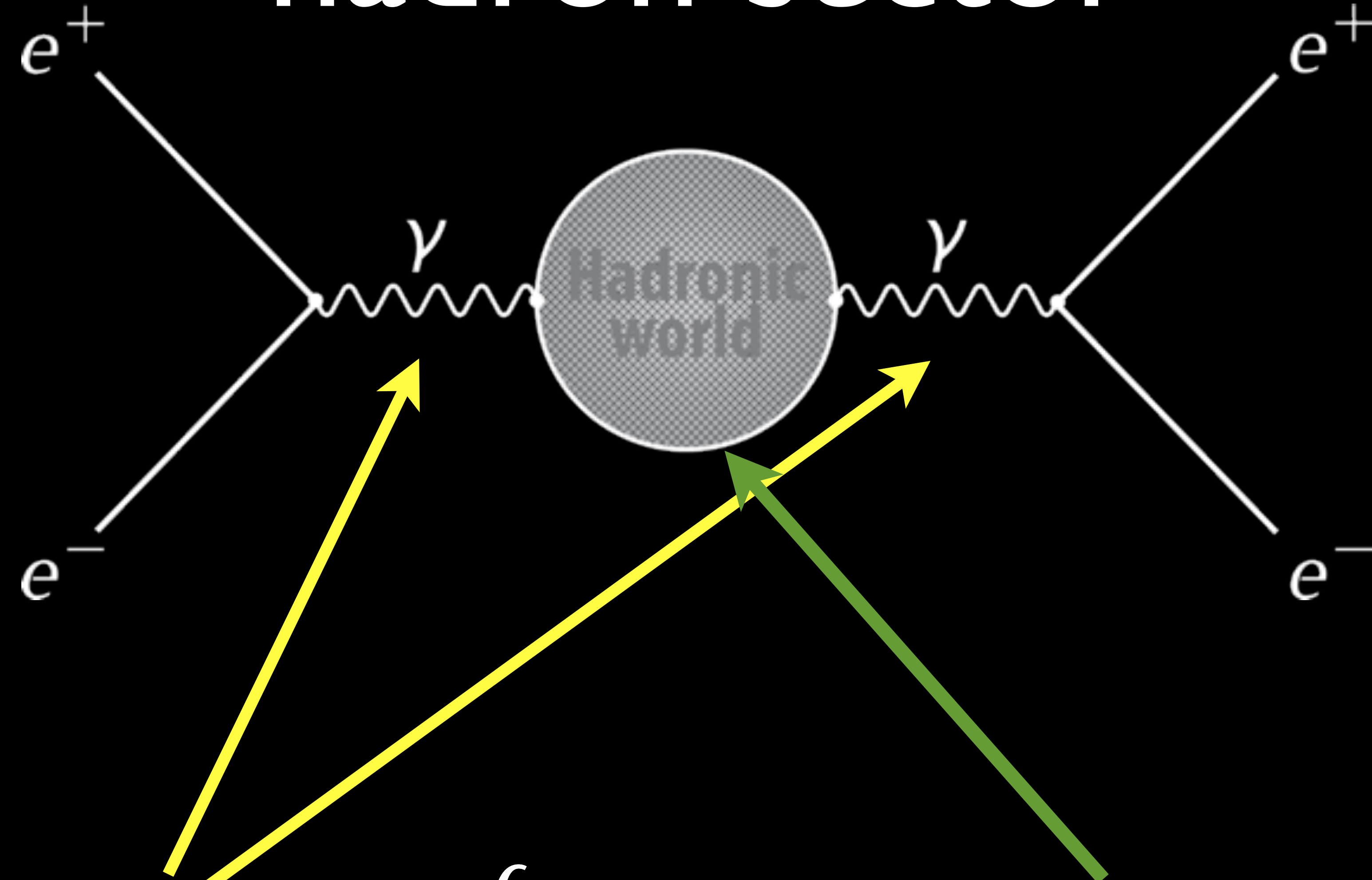
BSM?

- Direct observation doesn't look promising (at least not yet!). In any case, thinking about Naturalness challenges continues to lead to new insights into general aspects of quantum field theory.

Another kind of application of (s)CFTs

- Softly broken symmetries can be regarded as spontaneously broken, get selection rules. IR broken symmetry restored in UV.
- Example: QCD. Not conformal, RG flow. UV: asymptotically free CFT. IR: chiral symmetry breaking, confinement. Would like to relate IR to simpler to UV physics.

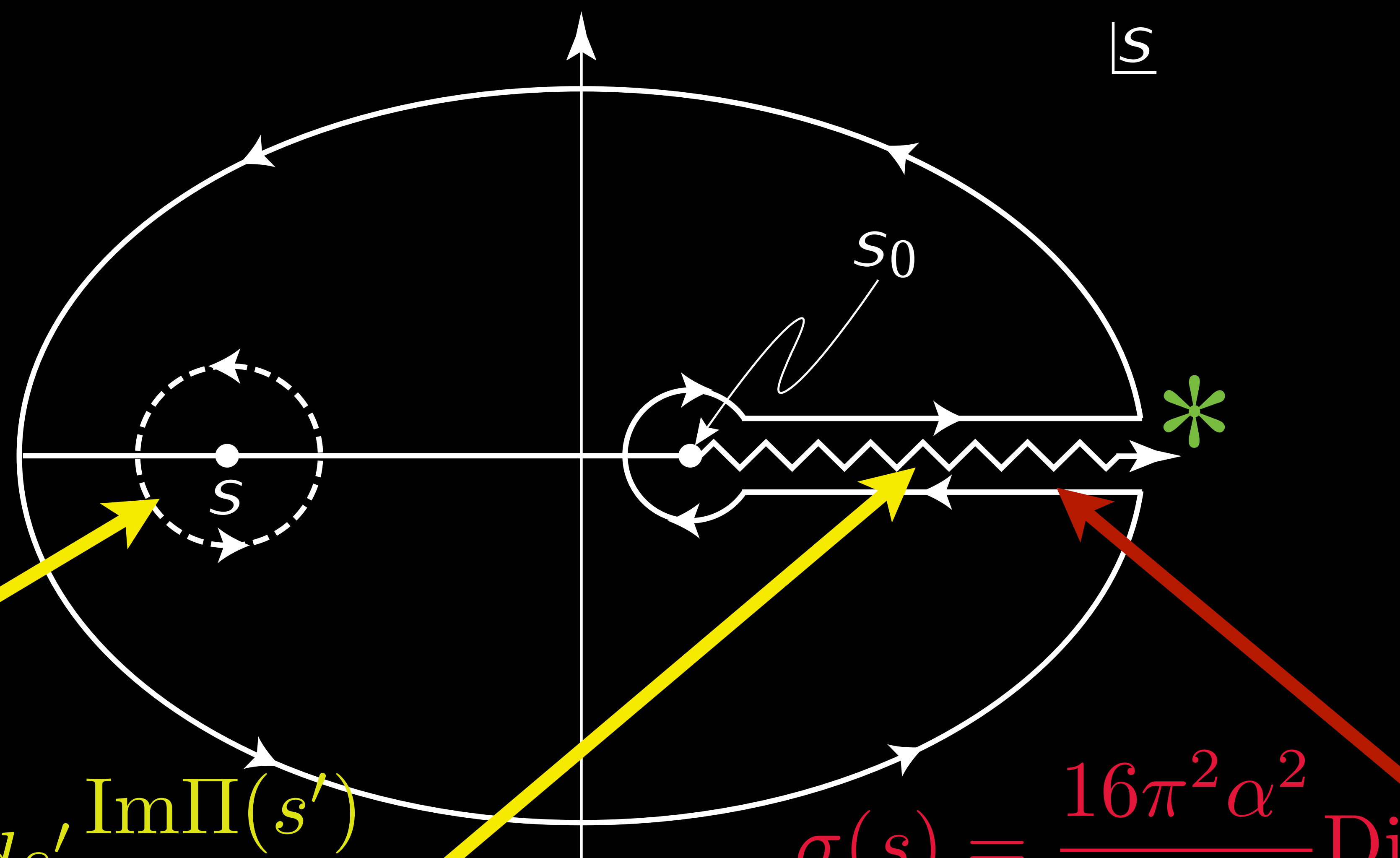
Example: leptons probing the hadron sector



$$i\Pi_h^{\mu\nu} = e^2 \int d^4x \, e^{-ip\cdot x} \langle J^\mu(x) J^\nu(0) \rangle$$

Tool: use OPE to separate UV and IR physics.

Analyticity, optical theorem, OPE



$$\Pi(s) = \frac{1}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}\Pi(s')}{s'-s}$$

$$\sigma(s) = \frac{16\pi^2\alpha^2}{s}\mathrm{Disc}\Pi(s)$$

* Use OPE @ large s. Use approx'ly for all s.

QCD Sum rules, etc. (Shifman, Vainshtein, Zakharov, Valentin.)

Change the subject to GGM

Visible sector soft susy breaking masses in GGM from hidden sector current current correlators:

$$M_{\text{gaugino}} \sim \alpha \int d^4x \langle Q^2(J(x)J(0))\rangle$$

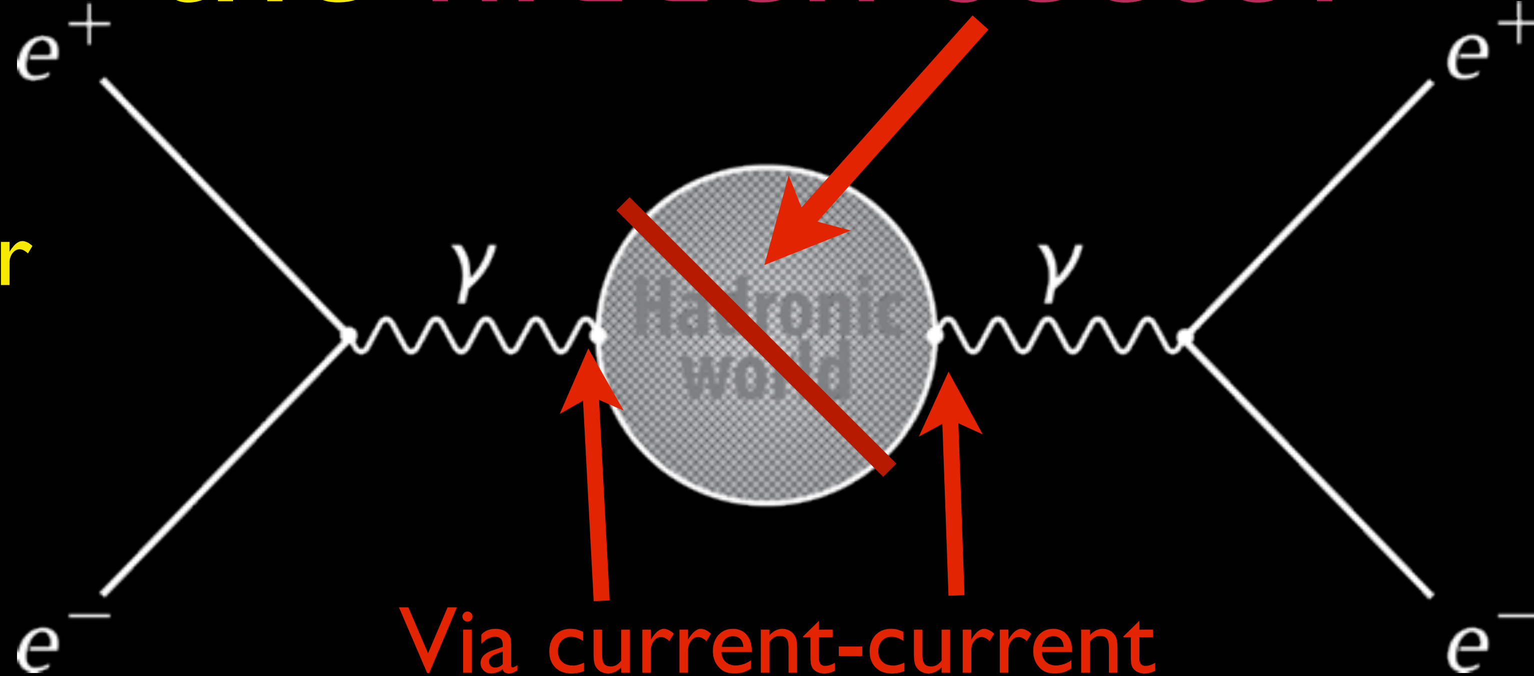
(Buican, Meade, Seiberg, Shih)

$$m^2_{\text{sfermion}} \sim \alpha^2 \int d^4x \ln(x^2M^2)\langle Q^4(J(x)J(0))\rangle$$

Bottom up: LHC might someday measure these few parameters, giving a tiny, indirect peek at the hidden sector. Indulge in some fantasy. Maybe someday humans will learn more about the hidden sector. Hidden sector production? Strongly coupled hidden sector? Dark matter = hidden sector baryons? Top down constraints?

QCD Analogy: visibles probing the hidden sector

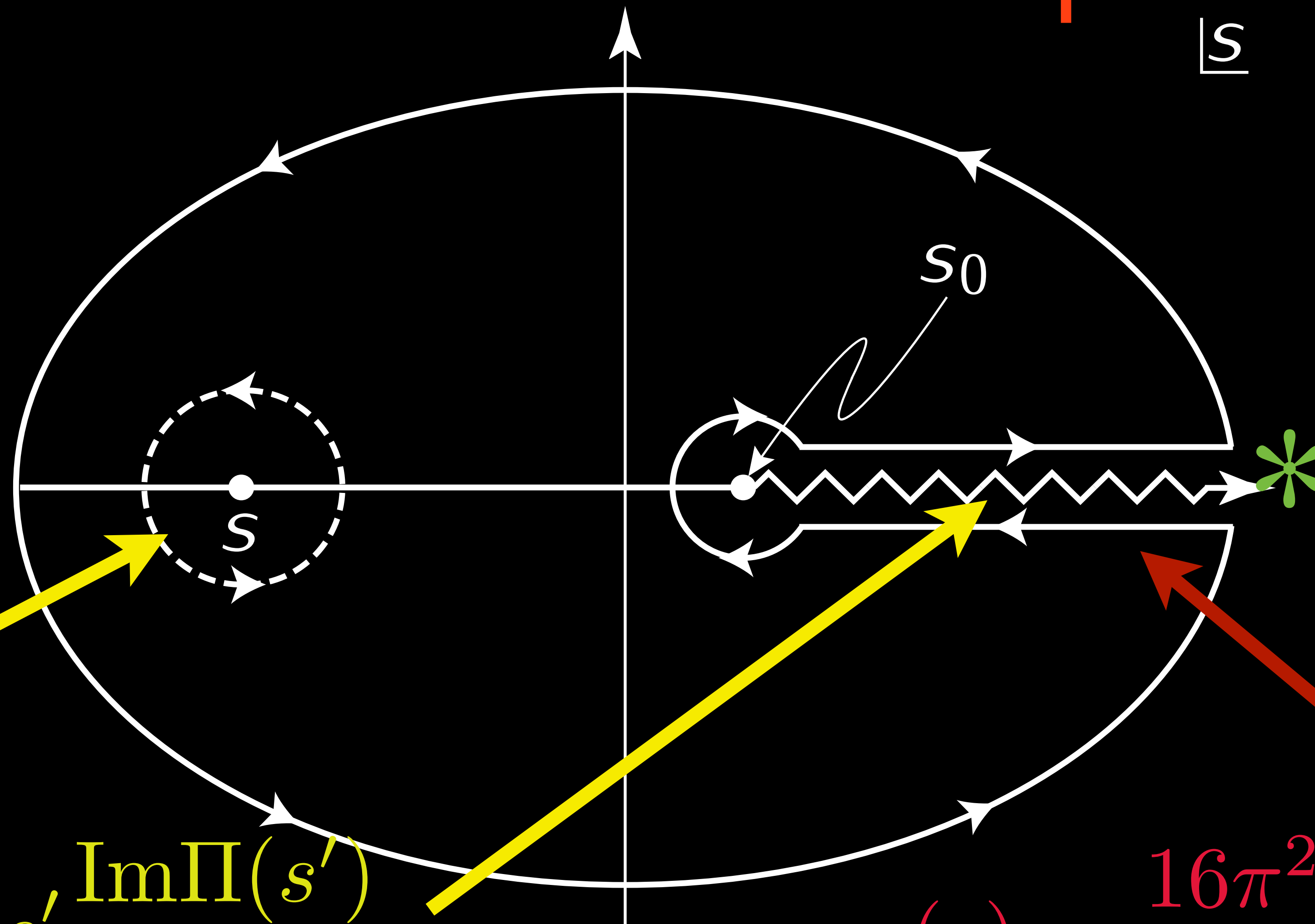
Visible sector matter



Via current-current 2-point function.

Visible sector soft masses in GGM is an example of this?
Other possible visible signatures of the hidden sector?
General constraints on current-current 2-point functions?

We use optical thm. etc. for hidden sector correlators + visible consequences



UV= “SCFT”

$$\Pi(s) = \frac{1}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}\Pi(s')}{s'-s}$$

gaugino or sfermion masses; hidden sec. bound states, a-la QCD sum rules?

$$\sigma(s) = \frac{16\pi^2\alpha^2}{s}\mathrm{Disc}\Pi(s)$$

Visibles to hiddens production x-section. Constrains Disc.'s

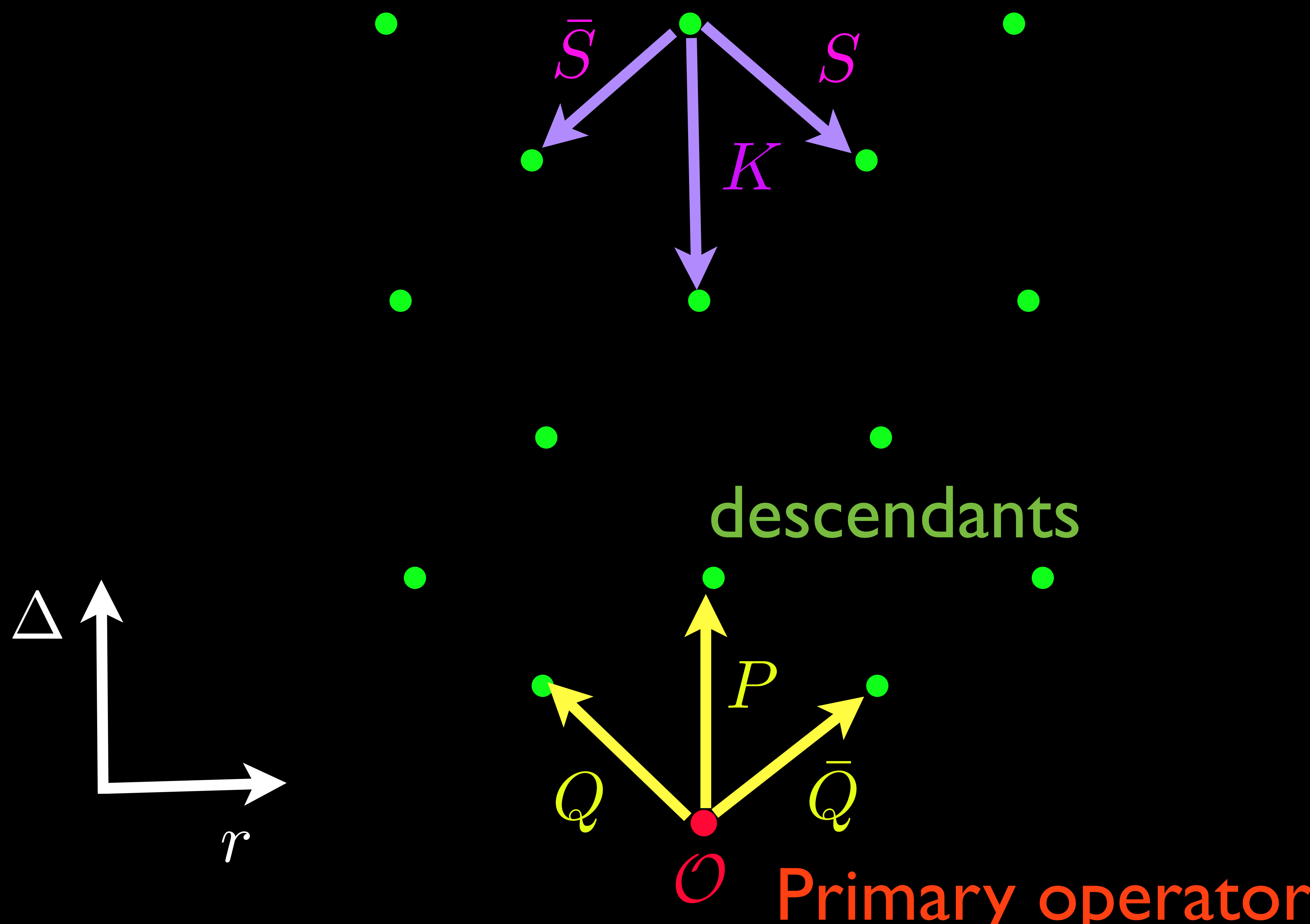
UV= "SCFT"

- Hidden sector: susy and conformal symm. is broken. But imagine they're restored in UV.
- Bigger UV symmetry implies UV relations. Some vestige can survive to IR.
- Broken symmetry in IR by operator or spurion vevs. $\langle \mathcal{O}_i \rangle \neq \delta_{i0}$ ➔ SC broken
- Apply the sOPE in the "sCFT", with $\langle \mathcal{O}_i \rangle \neq \delta_{i0}$ on the RHS.

sOPE in SCFTs

- Relate to 2-point and 3-point functions.
- Superconformal symmetry constrains their form (Osborn).
- Apply to our case of interest, correlators for conserved current OPEs.
- Relations among superconformal primaries and descendant sOPE coefficients.

Superconformal reps



Conserved current supermultiplets

$\mathcal{T}_\mu = j_R^\mu + \theta\sigma^\mu\bar{\theta} \;\; T_{\mu\nu} + \dots$ Ferrara -Zumino multiplet

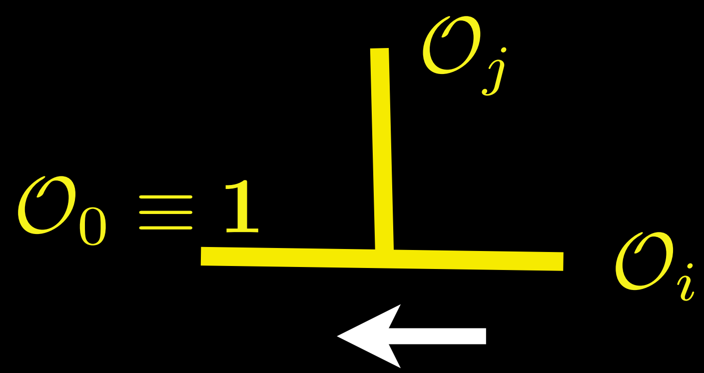
(Primary op. compt.)

$\mathcal{J} = J + i\theta j - i\bar{\theta}\bar{j} - \theta\sigma^\mu\bar{\theta} \;\; j^\mu + \dots$ Conserved current multiplet

$\bar{D}^{\dot{\alpha}}\mathcal{T}_{\alpha\dot{\alpha}} = 0$ "SCFT," $D^2\mathcal{J} = \bar{D}^2\mathcal{J} = 0$ Conserved

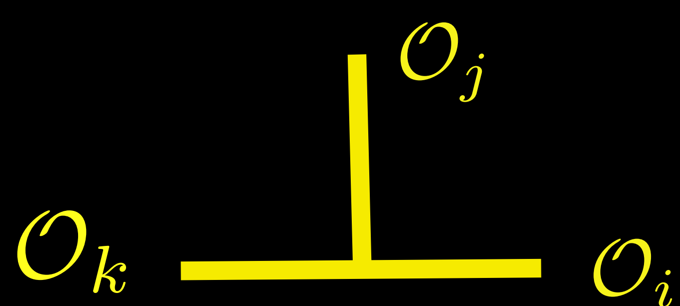
➔ Ward identities for their correlation fns.

2 and 3 point funs, OPE



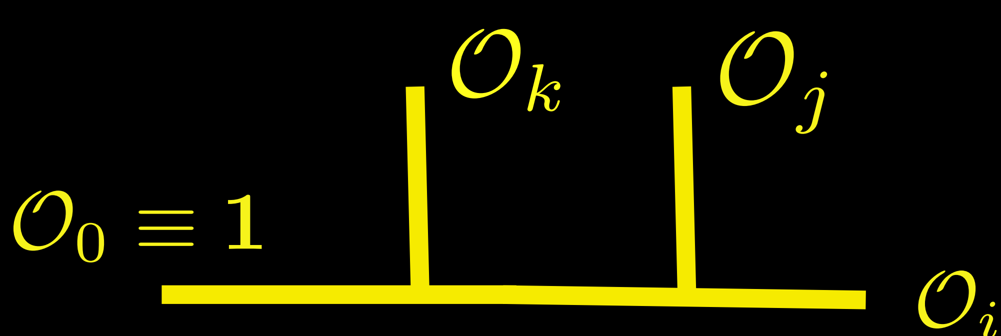
$\sim \quad c_{ij}^{0} = g_{ij}$

2-point fn coefficients
~ metric (Zamolodchikov)



$\sim \quad c_{ij}^{k}$

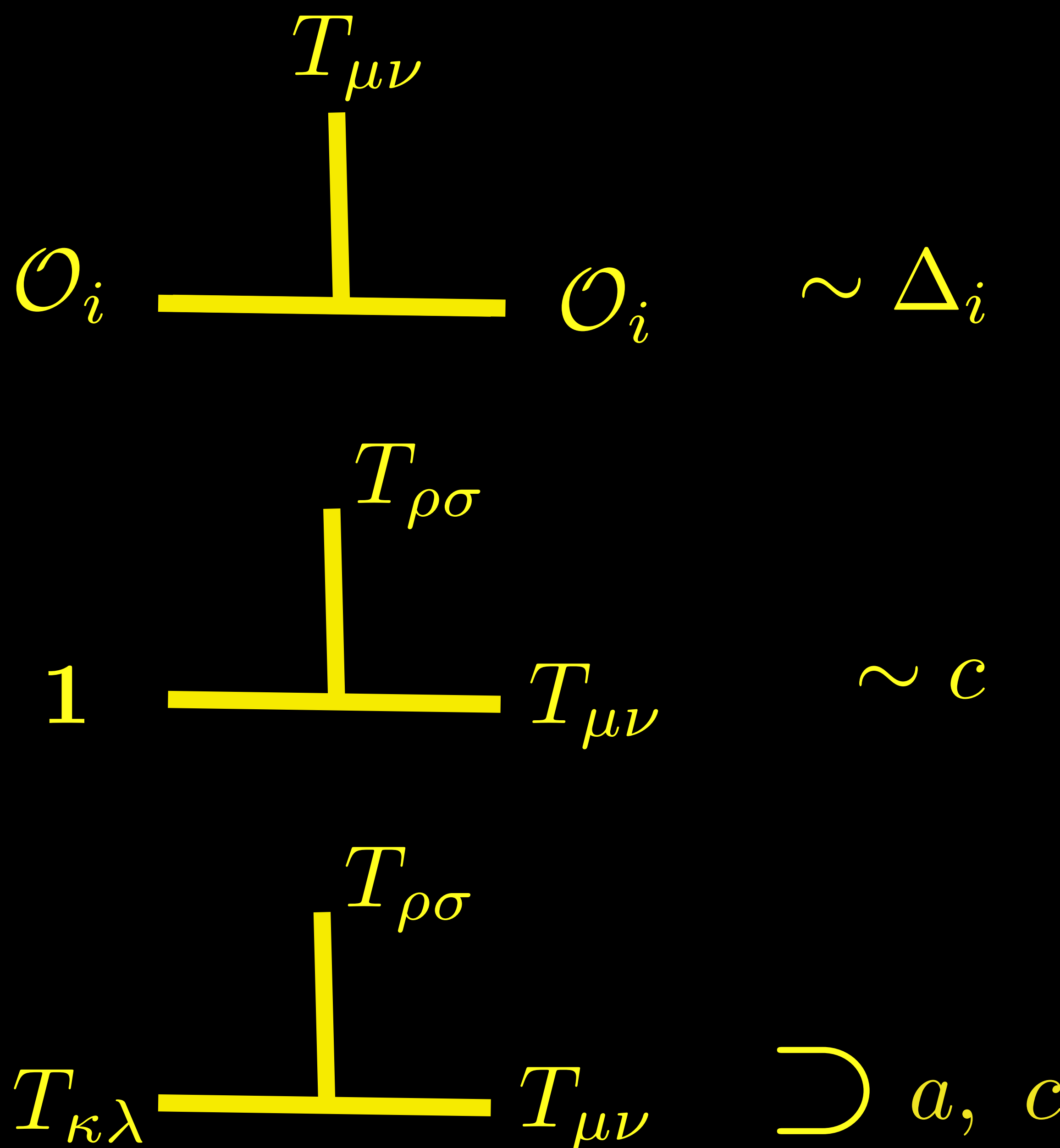
OPE coefficients



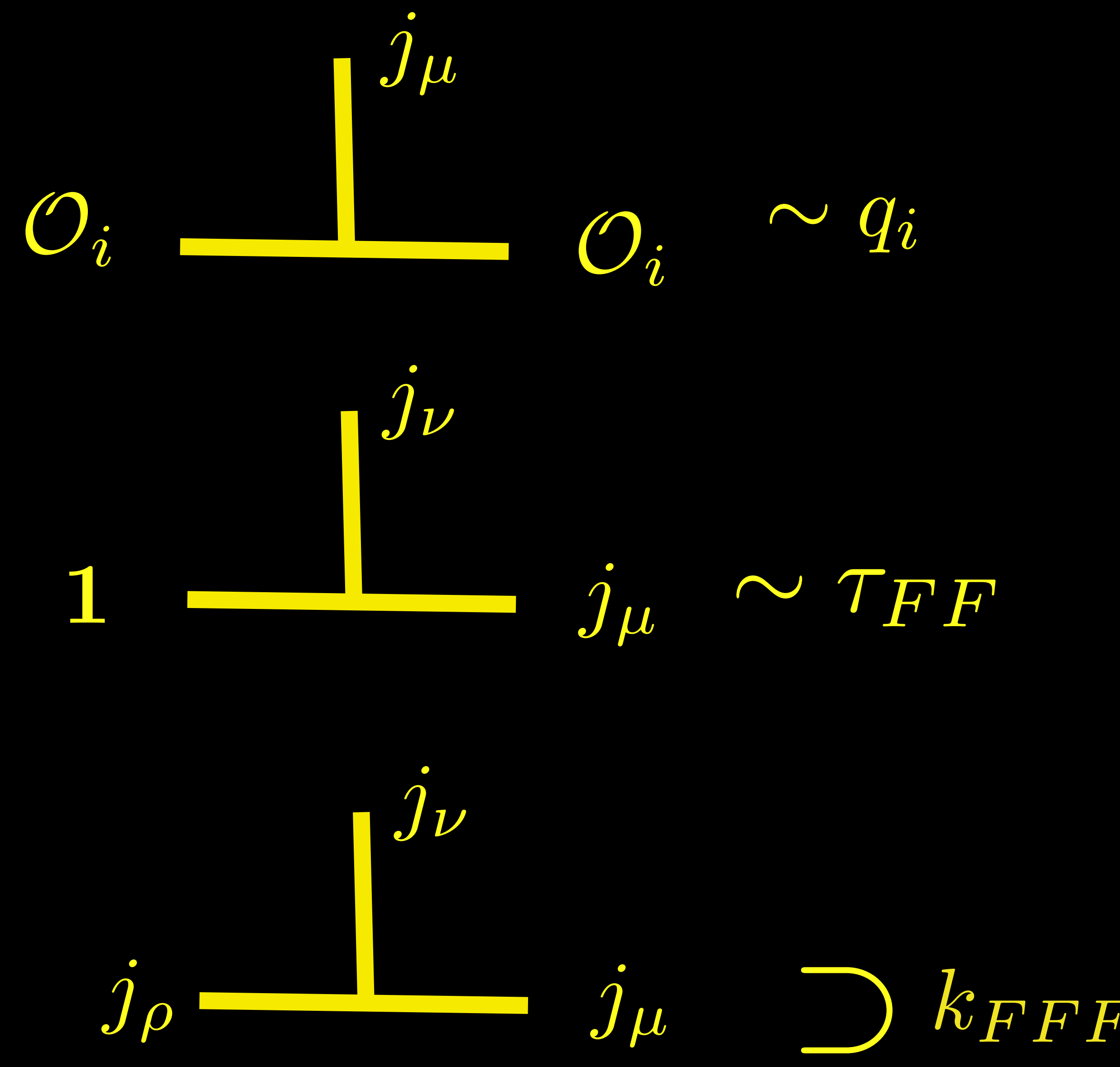
$\sim \quad c_{ij}^{\ell} g_{\ell k} = c_{ijk}$

3-point fn
coefficients

Examples (4d N=0, Osborn Petkou)

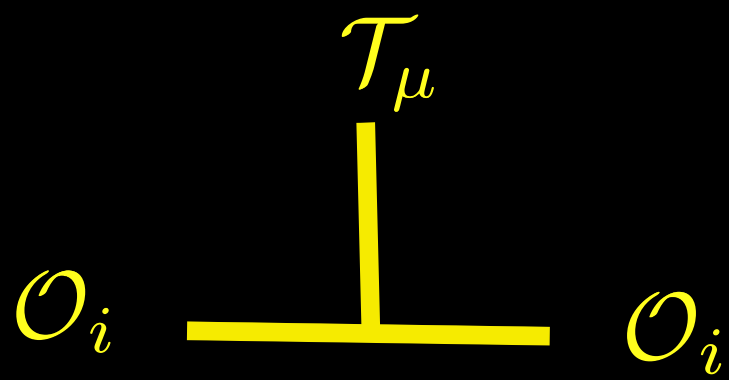


conf'l anomalies

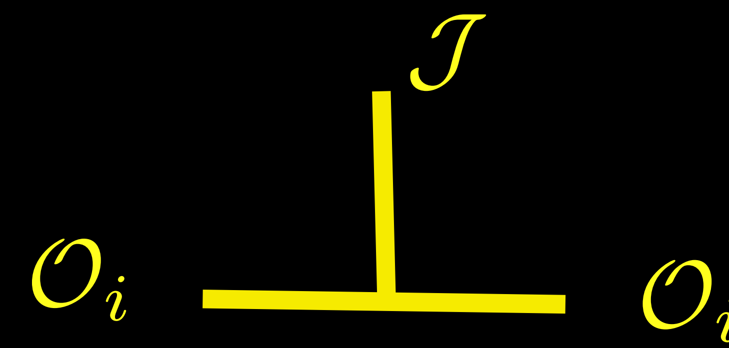


't Hooft anomalies

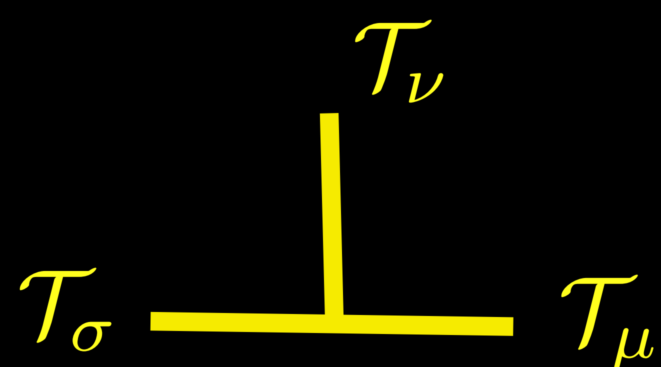
4d N=1 SCFT, e.g.



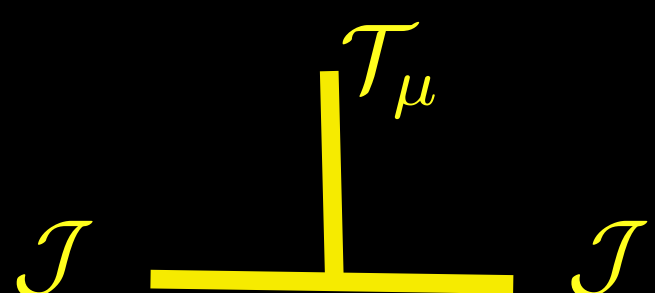
$\sim \Delta_i,\ r_i$



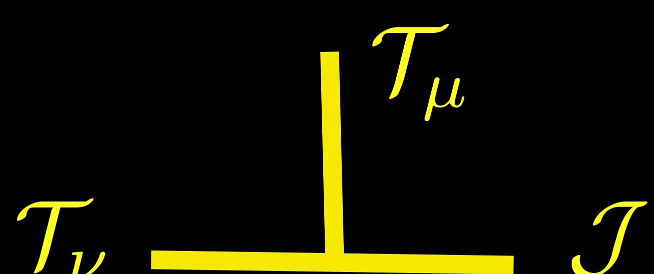
$\sim q_i$



Relates a, c to Tr R^3 and Tr R 't Hooft anomalies, Aneslmi et. al.; Osborn



Relates τ_{FF} to Tr RFF 't Hooft anomalies, Aneslmi et. al.; Osborn



Relates Tr RRF to Tr F anomalies: implies a-maximixation, KI+ Wecht.

Determining descendant OPEs from the primaries

Without susy, this was fully worked out in the '70s by Ferrara and collaborators. E.g. fixed coeffs

$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) = \sum_{[k]} \frac{c_{ij}^k}{r_{12}^{\Delta_i+\Delta_j-\Delta_k}} \left(1 - \frac{i}{2}\left(\frac{\Delta_k+\Delta_i-\Delta_j}{\Delta_k}\right) x\cdot P + \dots\right)\mathcal{O}_k(x_2)$$

You'd expect: (i) susy case is similar (ii) worked out long ago.

But both these are wrong. In fact, superconformal descendants are generally NOT fully determined from those of the primaries. In some special cases, it happens that the descendants are indeed fully determined by the primaries, but in generic OPEs they're not.

Superconformal 3-point functions and invariants

Implement the superconformal symmetry constraints using Osborn's superspace formalism and results. 3-point functions depend on $z_i = (x_i^\mu, \theta_i^\alpha, \bar{\theta}_i^{\dot{\alpha}}), \qquad i = 1, 2, 3.$

Make conformally nice coordinate $\qquad Z_3^\mu = \frac{x_{31}^\mu}{x_{31}^2} - \frac{x_{32}^\mu}{x_{32}^2}$

Complete it with corresponding theta terms to make ~ chiral or anti-chiral versions, called $X_3, \ \bar{X}_3$ (don't confuse with the chiral superfield X). These are coordinates, not fields.

There are corresponding nice combinations of coordinates that start with theta's, so they're nilpotent $\Theta_3, \ \bar{\Theta}_3$

General 3-point functions and the Theta invariant

Super operator 3-point functions

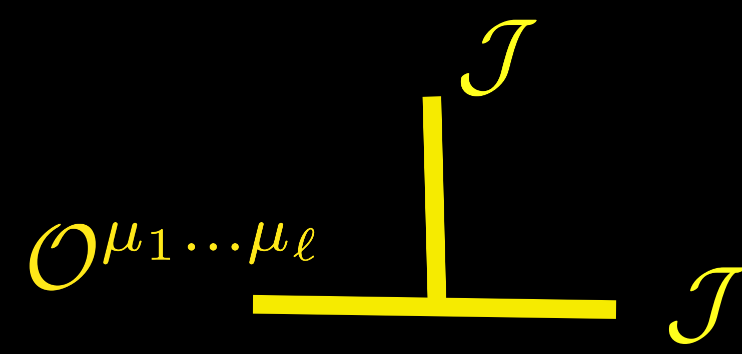
Determined dep on super-coords

$$\langle \mathcal{O}_i(z_1)\mathcal{O}_j(z_2)\mathcal{O}_k(z_3)\rangle = c_{ijk}\,(\ldots)\,f(X_3,\Theta_3,\bar{\Theta}_3)$$

Generally underdetermined function of Theta, means that super-descendant 3-point functions are generally not fully determined by the coefficient c_{ijk} of the three superconformal primaries. For shortened susy multiplets, the function f can be determined, giving descendants in terms of primaries.

Consider the J(x) J(0) OPE (Kl, **Fortin, Stergiou**)

Real, R=0, spin *l* primaries + descendants on RHS, only



$$\langle \mathcal{J}(z_1)\mathcal{J}(z_2)\mathcal{O}_i^{(\ell)(z_3)}\rangle = c_{JJ\mathcal{O}} f_{\Delta_\mathcal{O},\ell}(z_1,z_2,z_3)$$

$$f = \frac{t^{\mu_1\ldots\mu_\ell}(X_3,\Theta_3,\bar{\Theta}_3)}{x_{\bar{1}3}^2 x_{\bar{3}1}^2 x_{\bar{2}3}^2 x_{\bar{3}2}^2}$$

determined fn. of supercoordinates, via J conservation, e.g. for spin 0,

$$t = (X\cdot\bar{X})^{\frac{1}{2}\Delta_\mathcal{O}-2}\left(1-(\frac{\Delta_\mathcal{O}}{2}-2)(\frac{\Delta_\mathcal{O}}{2}-3)\frac{\Theta^2\bar{\Theta}^2}{X\cdot\bar{X}}\right)$$

$X,\Theta =$ combinations super coords (Osborn)

Implies relations in supermultipet of JJ OPEs, from superconformal symmetry + current conservation.

Relations, seen from algebra

$$j^\alpha(x) j_\alpha(0) = Q^2(J(x)J(0)) = \frac{1}{x^2} Q(x \cdot \bar{S})(J(x)J(0)) \quad \text{etc}$$

→ Relations among different OPEs on LHS

$$S^2(J(x)J(0)) = \bar{S}^2(J(x)J(0)) = 0 \quad \text{etc}$$

→ Relations among different terms on RHS of OPE.

Expand out superspace current-current OPEs, real operators on RHS:

$$\mathcal{O}^{\mu_1\cdots\mu_\ell}(x,\theta,\bar\theta) = A^{\mu_1\cdots\mu_\ell}(x) + \xi_\mu B^{\mu\mu_1\cdots\mu_\ell}(x) + \xi^2 D^{\mu_1\cdots\mu_\ell}(x) + \cdots$$

spin *l* spin *l* ± 1 spin *l*

$$\xi_\mu \equiv \theta\sigma_\mu\bar\theta$$

JJ OPE: only even spin

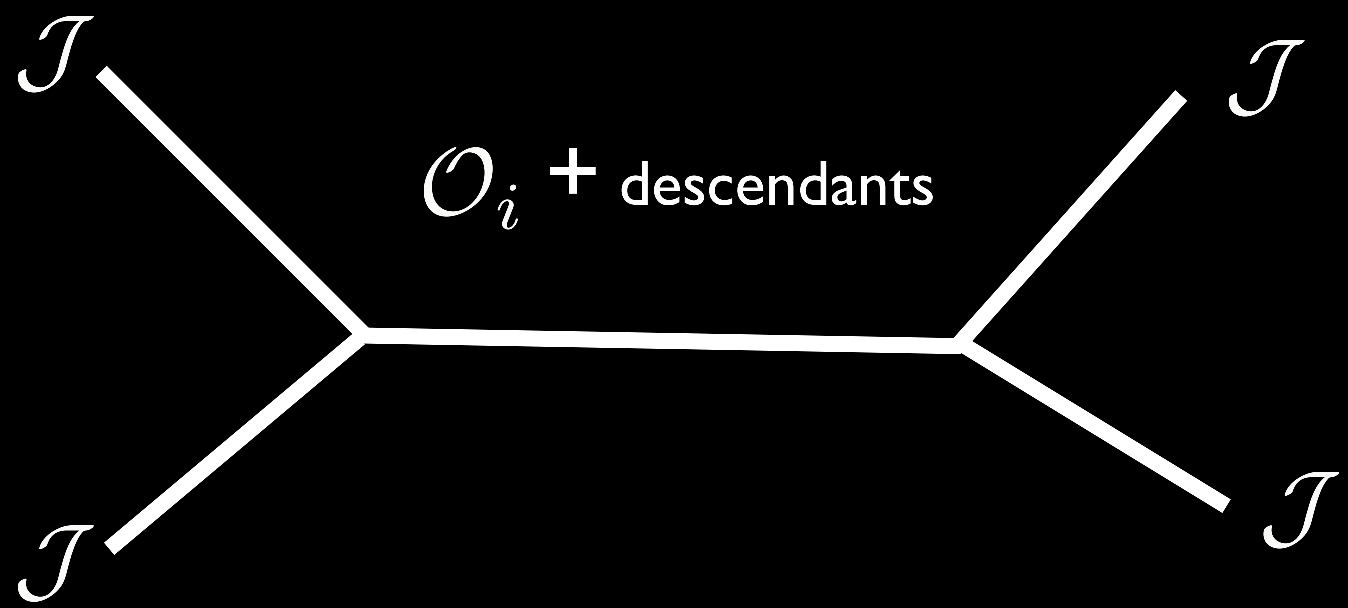
l even case
(odd case similar)

$$\langle JJA^{\mu_1\cdots\mu_\ell}\rangle = c_{JJ\mathcal{O}_\ell}\frac{Z^{\Delta-\ell}}{r_{12}^2}Z^{\mu_1}\cdots Z^{\mu_\ell}$$

$$\langle JJD^{\mu_1\ldots\mu_\ell}_{prim}\rangle = -c_{JJ\mathcal{O}_\ell}\frac{\Delta(\Delta+\ell)(\Delta-\ell-2)}{8(\Delta-1)}\frac{Z^{\Delta+2-\ell}}{r_{12}^2}Z^{\mu_1}\cdots Z^{\mu_\ell}$$

Results turn out to be similar to chiral-anti-chiral OPE of Poland & Simmons-Duffin.

Current 4-pt conformal blocks



$$u \equiv r_{12}r_{34}/r_{13}r_{24}$$

$$v \equiv r_{14}r_{23}/r_{13}r_{24}$$

$$\langle J(x_1)J(x_2)J(x_3)J(x_4)\rangle = \frac{1}{r_{12}^2 r_{34}^2} \sum_{\mathcal{O}_{\Delta,\ell}\in J\times J} \frac{(c_{JJA_\ell})^2}{c_{A_\ell A_\ell}} \mathcal{G}^{JJ;JJ}_{\Delta,\ell}(u,v),$$

$$\mathcal{G}^{JJ;JJ}_{\Delta,\,\ell\text{ even}} = g_{\Delta,\ell} + \frac{(\Delta+\ell)(\Delta-\ell-2)}{16(\Delta+\ell+1)(\Delta-\ell-1)} g_{\Delta+2,\ell}.$$

(No universal super-conformal blocks.)

Similar to $\mathcal{G}^{\phi\bar{\phi};\phi\bar{\phi}}_{\Delta,\,\ell}$

Poland & Simmons-Duffin.
For N=2, in SU(2) R-symmetry triplet multiplet.

Apply SCFT to GGM

e.g.

$$C_0(x) \equiv \langle J(x)J(0)\rangle = \sum_{\mathcal{O}} \frac{c_{JJ}^{\mathcal{O}}}{(x^2)^{\frac{1}{2}(4-\Delta_{\mathcal{O}})}} \left(\langle A_{\mathcal{O}} \rangle - \frac{x^2}{2\Delta_{\mathcal{O}}(\Delta_{\mathcal{O}}+1)} \langle D_{\mathcal{O};\mathrm{prim}} \rangle \right)$$

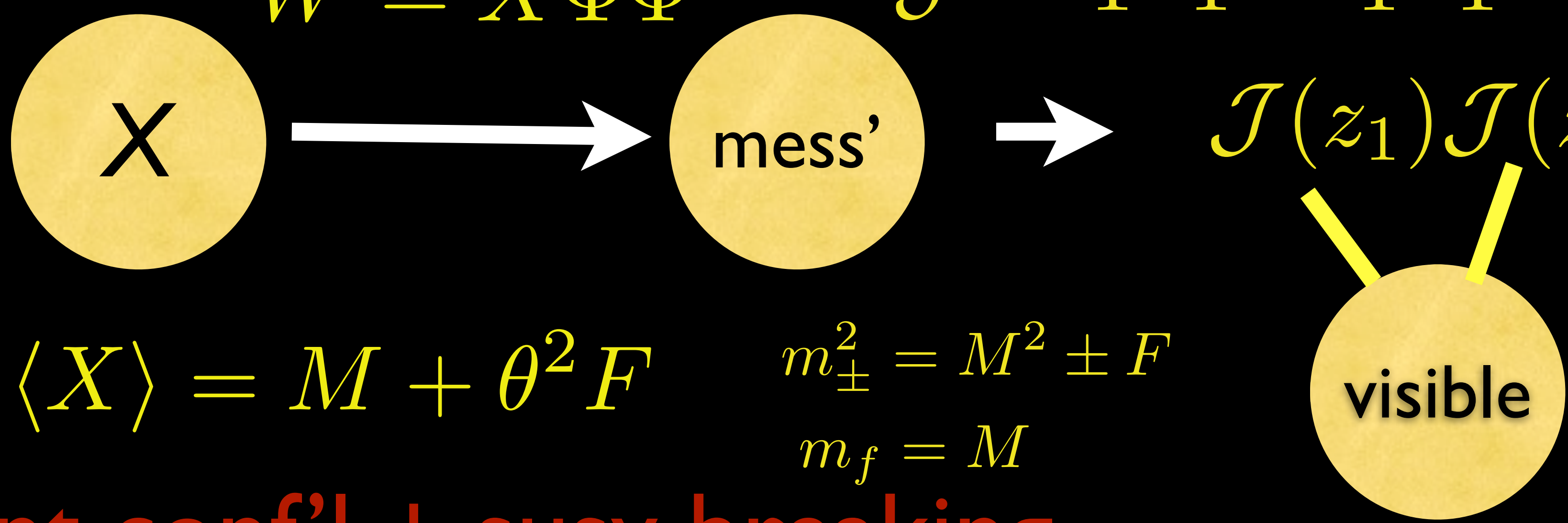
$$+ \sum_{\mathcal{O}^\mu} \frac{c_{JJ}^{N_{\mathcal{O}^\mu}}}{(x^2)^{\frac{1}{2}(3-\Delta_{\mathcal{O}^\mu})}} \langle N_{\mathcal{O}^\mu} \rangle$$

$$\langle j_\alpha(x)\bar{j}_{\dot{\alpha}}(0)\rangle = -i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu C_{1/2}(x) = -i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \sum_{\mathcal{O}} \frac{c_{JJ}^{\mathcal{O}}}{(x^2)^{\frac{1}{2}(4-\Delta_{\mathcal{O}})}} \left(\langle A_{\mathcal{O}} \rangle + \dots \right)$$

Simple Example: MGM as "SCFT"

Charged messenger pair + hidden goldstino field or spurion X

$W = X\Phi\widetilde{\Phi}$ $\qquad$ $\mathcal{J} = \Phi^\dagger\Phi - \widetilde{\Phi}^\dagger\widetilde{\Phi}$



$\mathcal{J}(z_1)\mathcal{J}(z_2) = \mathrm{OPE}$

IR: $\langle X\rangle = M + \theta^2 F$

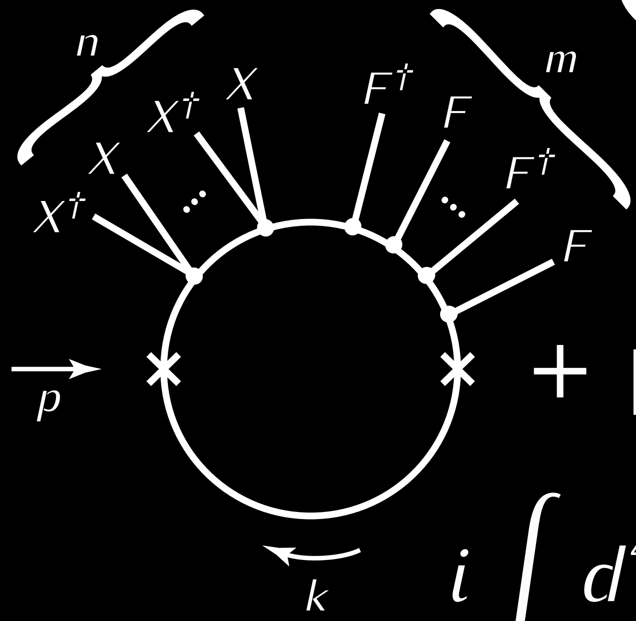
$m_\pm^2 = M^2 \pm F$

$m_f = M$

spont conf'l + susy breaking

Compute OPE coeffs in UV "SCFT", including X, F ops on RHS. Coeffs = UV data, unaffected by IR op vevs.

OPE Wilson Coeffs, Relns:



+ permutations

$$i\int d^4x\, e^{-ip\cdot x} J(x)J(0) \to \sum_{m,n=0}^{\infty} \tilde{c}_0(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n(0)$$

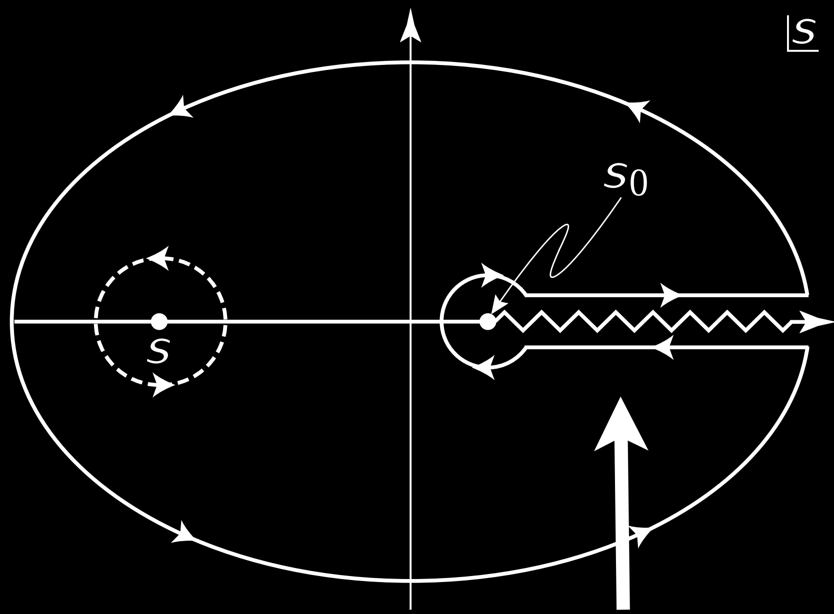
$$+\sum_{m,n=0}^{\infty} \tilde{d}_0(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n X^\dagger F^\dagger \chi^2(0) + \cdots$$

Use / check: $j^\alpha(x)j_\alpha(0) = Q^2(J(x)J(0)) = \frac{1}{x^2}Q(x\cdot\bar{S})(J(x)J(0))$

$$i\int d^4x\, e^{-ip\cdot x} j_\alpha(x)j_\beta(0) \to \epsilon_{\alpha\beta} F X^\dagger \sum_{m,n=0}^{\infty} \tilde{c}_{1/2}(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n$$

With: $\tilde{c}_{1/2}(m,n) = (n+1)\tilde{c}_0(m,n+1) + 2\tilde{d}_0(m-1,n)$ It works.

Only need JJ OPE



Cross sections

For GGM fns

$$\widetilde{C}_0(s) = \mathrm{F.T.}\langle J(x)J(0)\rangle$$
$$\widetilde{C}_{1/2}(s),\ \widetilde{C}_1(s)$$
$$\widetilde{B}_{1/2}(s) = \mathrm{F.T.}\langle j_\alpha(x)j^\alpha(0)\rangle$$

Cut: make on-shell hidden / messengers, disc = total cross sect.

$$\sigma_a(\mathrm{vis} \to \mathrm{hid}) = \frac{8\pi^2\alpha^2}{s}\mathrm{Disc}\widetilde{C}_a(s)$$ Also, e.g. $$\sigma_0(s) = \frac{\lambda^{1/2}(s, m_1, m_2)}{8\pi s^2}|\mathcal{M}|^2$$

$$\lambda = 4s|\vec{p}_{os}|^2 = (s-(m_1+m_2)^2)(s-(m_1-m_2)^2)$$ Textbook kinematic factor

Our example: $$\sigma_0(\mathrm{vis} \to \mathrm{hid}) = \frac{2\pi\alpha^2}{s^2}\lambda^{1/2}(s, m_+, m_-)$$

$$\sigma_0(s) = \frac{1}{s}\,\mathrm{Im}\sum_{m,n=0}^{\infty}\tilde{c}_0(m,n;s,\Lambda)(F^\dagger F)^m(X^\dagger X)^n$$

Compare with the Disc of our Wilson coefficients. It works. Use symms to find all other cross sects.

GGM soft masses from OPE

GGM:

$$M_r = g_r^2 M \widetilde{B}^{(r)}_{1/2}(0) \qquad m^2_{\tilde{f}} = \sum_{r=1}^{3} g_r^4 c_2(f;r) A_r$$

$$A_r = -\frac{M^2}{16\pi^2}\int dy \left[\widetilde{C}^{(r)}_0(y) - 4\widetilde{C}^{(r)}_{1/2}(y) + 3\widetilde{C}^{(r)}_1(y)\right]$$

original MSS expressions

Use the OPE + dispersion relations, e.g. $\widetilde{B}(s=0) = \int_{s_0}^{\infty} \frac{ds}{\pi}\frac{\mathrm{Im}(\widetilde{B}(s))}{s}$

IR :(for OPE UV :) for OPE.

$$M_{\mathrm{gaugino}} = \sum_k \frac{\alpha\, \mathrm{Im}[s^{d_k/2}\, \tilde{c}^k_{JJ}(s)]}{2^{d_k-1} d_k M^{d_k}} \langle Q^2(\mathcal{O}_k(0))\rangle$$

In our example, exactly yield the usual GM f(x), g(x), x=F/M^2

$$m^2_{\mathrm{sfermion}} = -\sum_k \frac{\alpha^2 c_2\, \mathrm{Im}[s^{d_k/2}\tilde{c}^k_{JJ}(s)]}{2^{d_k+1}\pi d_k^2 M^{d_k}} \langle \bar{Q}^2 Q^2(\mathcal{O}_k(0))\rangle$$

Get good approx. by keeping only few leading terms in OPE.

Approx. soft masses from OPE

$$J_a(x)J_b(0) = \tau\frac{\delta_{ab}\mathbb{1}}{16\pi^4 x^4} + \tau^{-1}kd^c_{ab}\frac{J_c(0)}{4\pi^2x^2} + w\frac{\delta_{ab}K(0)}{4\pi^2x^{2-\gamma_K}} + c^i_{ab}\frac{\mathcal{O}_i(0)}{x^{4-\Delta_i}} + \cdots$$

QQ(*)=0

"Konishi," lowest dim op with QQ(K) $\neq 0$

higher dim ops + descendants

We find:

$$M_{\text{gaugino}} \approx -\frac{\alpha\pi w\gamma_{Ki}}{8M^2}\langle Q^2(\mathcal{O}_i(0))\rangle$$

$$m^2_{\text{sfermion}} \approx \frac{\alpha^2 c_2 w\gamma_{Ki}}{64M^2}\langle \bar{Q}^2Q^2(\mathcal{O}_i(0))\rangle$$

Leading term from Konishi operator, and its mixing operators.

E.g. in our OGM example, standard soft mass functions f(x), g(x) ~ 1. The OPE approximation from including just a single term already accounts for ~ 1/2 of these functions. The higher dimension operators and descendants give just small corrections. Power of the OPE.

Summary

- Constrain current-current OPE in SCFTs, showed superconformal descendant coeffs are fully determined from primaries.
- Can apply to "SCFTs" with broken susy and conf'l symmetry. OPE coeffs =UV, so don't notice IR breaking by vevs.
- Can apply to GGM. Use OPE to find cross sections and approximations to soft masses.

Thank you.