Towards a quantum treatment of leptogenesis

Mathias Garny (DESY)

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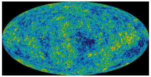
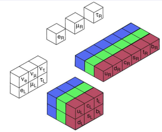
based on work done in collaboration with M. M. Müller; A. Hohenegger, A. Kartavtsev, M. Lindner

Physics beyond the Standard Model

Collider exp.

Baryon asymmetry

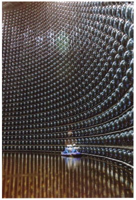




+ ?

⋮

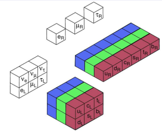
⋮



Neutrino exp.

Dark matter

Physics of the Early Universe



+ ?

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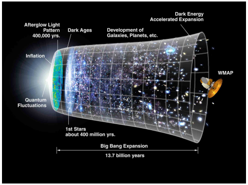

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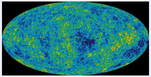
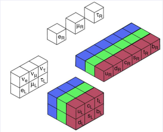


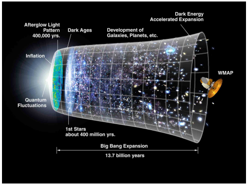
- Inflation, Reheating
- Baryogenesis
- Thermal relics (gravitino)
- Dark matter freeze-out
- ...

Physics of the Early Universe



$+$?

$\leftrightarrow$



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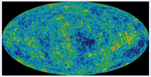

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Nonequilibrium dynamics at high energy

Outline

Towards a quantum treatment of leptogenesis

- Leptogenesis
- Quantum fields out of equilibrium
- Application to leptogenesis

Leptogenesis

Standard Model (SM) extended by three heavy singlet neutrino fields $N_i = N_i^c$, $i = 1, 2, 3$ with Majorana masses $\hat{M} = \text{diag}(M_i)$ in the mass eigenbasis

$$\mathcal{L} \quad = \quad \mathcal{L}_{SM} + \frac{1}{2}\bar{N}i\partial\!\!\!/ N - \frac{1}{2}\bar{N}\hat{M}N - \bar{\ell}\widetilde{\phi}hP_R N - \bar{N}P_L h^\dagger \widetilde{\phi}^\dagger \ell$$

Light neutrino masses via seesaw mechanism

$$m_\nu = -v_{EW}^2 h\hat{M}^{-1}h^T$$

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Baryogenesis via leptogenesis *Fukugita, Yanagida 86*

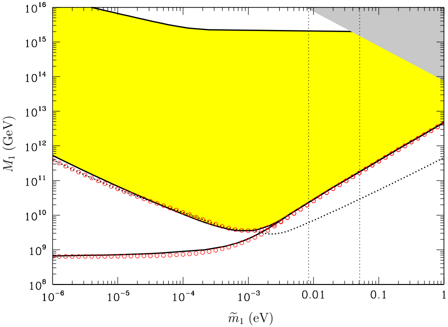
- B-violation via L-violating Majorana masses M_i
- CP-violation via Yukawa couplings $\mathrm{Im}[(h^\dagger h)_{ij}] \neq 0$
- Out-of-equilibrium (inverse) decay $N_i \leftrightarrow \ell\phi^\dagger$ and $N_i \leftrightarrow \ell^c\phi$

$$\begin{aligned}(\Gamma_i/H)|_{T=M_i} &\simeq \tilde{m}_i/\mathrm{meV} \quad \sim \mathcal{O}(1) \quad \text{where } \tilde{m}_i = v_{EW}^2 (h^\dagger h)_{ii}/M_i \\ (\Gamma_{SM}/H)|_{T=M_i} &\sim g^4 M_{pl}/M_i \gg 1 \qquad \text{for } M_i \ll 10^{16}\mathrm{GeV}\end{aligned}$$

Leptogenesis

Vanilla leptogenesis for hierarchical spectrum $M_1 \ll M_{2,3}$ requires large values of the reheating temperature $T_R \gtrsim \mathcal{O}(M_1) \gtrsim 10^9\,\text{GeV}$

Hamaguchi, Murayama, Yanagida; Davidson, Ibarra



Buchmüller, Di Bari, Plümacher

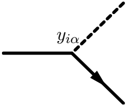
Gravitino production

$$\Omega_{3/2}^{th} h^2 \simeq 0.27 \left(\frac{T_R}{10^9\,\text{GeV}}\right)\left(\frac{10\,\text{GeV}}{m_{3/2}}\right)\left(\frac{m_{\widetilde{g}}}{1\,\text{TeV}}\right)^2$$

Moroi, Murayama, Yamaguchi, 93; Bolz, Brandenburg, Buchmüller, 01;Pradler, Steffen, 06; Rychkov, Strumia, 07

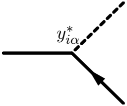
L-violating decay of heavy right-handed neutrino N_i

$$\mathcal{M}_{N_i \to \ell_\alpha h^\dagger} \quad = $$



$$+ \ldots$$

$$\mathcal{M}_{N_i \to \ell^c_\alpha h} \quad = $$



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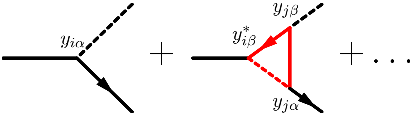

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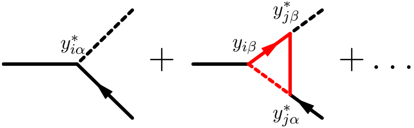
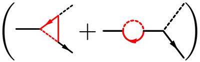


Matter-antimatter (CP) asymmetry

$\Leftrightarrow$ interference of tree and loop processes

$$\Gamma(N_i \to \ell_\alpha h^\dagger) - \Gamma(N_i \to \ell^c_\alpha h) \;\sim\; \mathrm{Im}(y_{i\alpha} y_{i\beta} y^*_{j\alpha} y^*_{j\beta}) \cdot \mathrm{Im}$$



Standard Boltzmann approach

$$N_L(t) = \int d^3x \sqrt{|g|} \int \frac{d^3p}{(2\pi)^3} \sum_{\ell} \left[f_\ell(t, \mathbf{x}, \mathbf{p}) - f_{\bar{\ell}}(t, \mathbf{x}, \mathbf{p}) \right]$$

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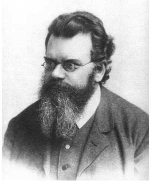

$$p^\mu \mathcal{D}_\mu f_\ell(t,\mathbf{x},\mathbf{p}) = \sum_i \int d\Pi_{N_i} d\Pi_h$$

$$\times (2\pi)^4 \delta(p_\ell + p_h - p_{N_i})$$

$$\times \Big[|\mathcal{M}|^2_{N_i \to \ell h^\dagger} f_{N_i}(1-f_\ell)(1+f_h) + \dots$$

$$- |\mathcal{M}|^2_{\ell h^\dagger \to N_i} f_\ell f_h (1 - f_{N_i}) + \dots \Big]$$

$f_\psi(t,\mathbf{x},\mathbf{p})$: distribution function of on-shell particles

$|\mathcal{M}|^2$: matrix elements computed in *vacuum*, off-shell effects

Corrections within Boltzmann framework

- Bose-enhancement, Pauli-Blocking; kinetic (non-)equilibrium
  - quantum statistical factors $1 \pm f_k$
  - non-integrated Boltzmann equations

Hannsestad, Basbøll 06; Garayoa, Pastor, Pinto, Rius, Vives 09; Hahn-Woernle, Plümacher, Wong 09

- Thermal corrections via thermal QFT
  - medium correction to CP-violating parameter $\epsilon = \epsilon^{vac} + \delta\epsilon^{th}$
  - thermal masses, decay width, ...

Covi, Rius, Roulet, Vissani 98; Giudice, Notari, Raidal, Riotto, Strumia 04; Besak, Bödeker 10

Kiessig, Thoma, Plümacher 10; Salvio Lodone Strumia 11...

- Flavour effects

Nardi, Nir, Roulet, Racker 06; Abada, Davidson, Ibarra, Josse-Micheaux, Losada, Riotto 06; Blanchet, diBari 06; ...

- Spectator processes, scatterings, N_2, ...

Double Counting Problem

Naive contribution from decay/inverse decay

$$|\mathcal{M}|^2_{N_i \to \ell h^\dagger} = |\mathcal{M}_0|^2(1+\epsilon_i) \qquad |\mathcal{M}|^2_{\ell h^\dagger \to N_i} = |\mathcal{M}_0|^2(1-\epsilon_i)$$

$$|\mathcal{M}|^2_{N_i \to \ell^c h} = |\mathcal{M}_0|^2(1-\epsilon_i) \qquad |\mathcal{M}|^2_{\ell^c h \to N_i} = |\mathcal{M}_0|^2(1+\epsilon_i)$$

$$\frac{dN_{B-L}}{dt} \propto (|\mathcal{M}|^2_{N_i \to \ell h^\dagger} - |\mathcal{M}|^2_{N_i \to \ell^c h}) N_{N_i} - (|\mathcal{M}|^2_{\ell h^\dagger \to N_i} - |\mathcal{M}|^2_{\ell^c h \to N_i}) N^{eq}_{N_i}$$

Double Counting Problem

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$$
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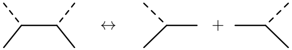
\end{aligned}
$$

$$
\begin{aligned}
\frac{dN_{B-L}}{dt} &\propto (|\mathcal{M}|^2_{N_i \to \ell h^\dagger} - |\mathcal{M}|^2_{N_i \to \ell^c h}) N_{N_i} \\
&\quad - (|\mathcal{M}|^2_{\ell h^\dagger \to N_i} - |\mathcal{M}|^2_{\ell^c h \to N_i}) N_{N_i}^{eq} \\
&\propto \epsilon_i (N_{N_i} + N_{N_i}^{eq})
\end{aligned}
$$

$\Rightarrow$ spurious generation of asymmetry even in equilibrium

Origin: Double Counting Problem 

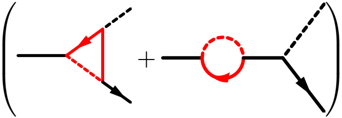
$\rightarrow$ need real intermediate state subtraction

... justification from first principles / generalization ?

Leptogenesis - resonant enhancement

Efficiency of leptogenesis depends on CP-violating parameter, which is one-loop suppressed

$$\epsilon_{N_i} = \frac{\Gamma(N_i \to \ell\phi^\dagger) - \Gamma(N_i \to \ell^c\phi)}{\Gamma(N_i \to \ell\phi^\dagger) + \Gamma(N_i \to \ell^c\phi)} \propto \mathrm{Im}$$



Self-energy (or ‘wave’) contribution to CP-violating parameter features a resonant enhancement for a quasi-degenerate spectrum $M_1 \simeq M_2 \ll M_3$

$$\epsilon_{N_i}^{wave} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \times \frac{M_1 M_2}{M_2^2 - M_1^2}$$

Flanz Paschos Sarkar 94/96; Covi Roulet Vissani 96;

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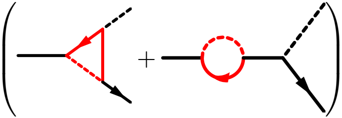

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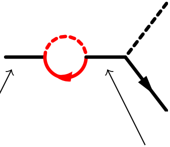


On-shell initial N_1: $p^2 = M_1^2$ Internal N_2: $\frac{i}{p^2 - M_2^2}$

Resonant leptogenesis

- *Flanz Paschos Sarkar Weiss 96;* effective Hamiltonian approach

$$\epsilon_{N_i} = -\frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{16\pi(h^\dagger h)_{ii}} \frac{M_1(M_2 - M_1)}{(M_2 - M_1)^2 + M_1^2(\mathrm{Re}(h^\dagger h)_{12}/(16\pi))^2}$$

- *Covi Roulet 96;* CP violating decay of mixing scalar fields described by effective mass matrix; formalism as in Liu Segre 93
- *Pilaftsis 97; Pilaftsis Underwood 03;* Pole mass expansion of resummed propagators

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + M_i^2\Gamma_j^2}$$

- *Buchmüller Plümacher 97;* Diagonalization of resummed propagators

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \frac{M_1 M_2(M_2^2 - M_1^2)}{(M_2^2 - M_1^2 - \frac{1}{\pi}\Gamma_i M_i \ln\frac{M_2^2}{M_1^2})^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2}$$

- *Rangarajan Mishra 99;* comparison of different approaches
- *Anisimov Broncano Plümacher 05;* Reconciliation of diagonalization approach with the pole mass expansion approach
- Invariant quantity $M_1 M_2(M_2^2 - M_1^2)\mathrm{Im}(h^\dagger h)_{12}^2$ related to CP violation appears in the enumerator

Resonant leptogenesis

The results can be summarized (neglecting log-corrections) as

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi (h^\dagger h)_{ii}} \times R, \qquad R \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

Different calculations correspond to different expressions for the 'regulator' A^2

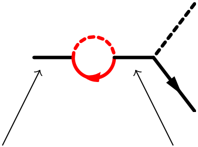
$$A^2 = \begin{cases} \frac{1}{4}(M_1+M_2)^4 \left(\frac{\mathrm{Re}(h^\dagger h)_{12}}{16\pi}\right)^2 & \textit{Flanz Paschos Sarkar Weiss 96} \\ M_i^2 \Gamma_j^2 & \textit{Pilaftsis 97; Pilaftsis Underwood 03} \\ (M_1\Gamma_1 - M_2\Gamma_2)^2 & \textit{Buchmüller Plümacher 97; Anisimov Broncano Plümacher 05; ...} \\ \dots & \end{cases}$$

The regulator is relevant for determining the maximal possible resonant enhancement, which occurs for $M_2^2 - M_1^2 = \pm A$, and is given by

$$R_{max} = \frac{M_1 M_2}{2|A|}$$

Resonant leptogenesis

The origin of the regulator is the finite width of N_1 and N_2



Off-shell initial N_1: $p^2 = M_1^2 + iM_1\Gamma_1$ Internal N_2: $\frac{i}{p^2 - M_2^2 - iM_2\Gamma_2}$

Resonant leptogenesis

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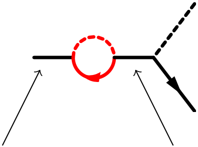
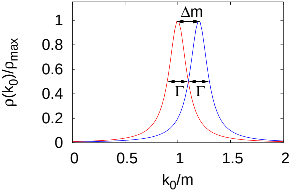


Off-shell initial N_1: $p^2 = M_1^2 + iM_1\Gamma_1$ Internal N_2: $\frac{i}{p^2 - M_2^2 - iM_2\Gamma_2}$

In the maximal resonant case $M_2 - M_1 = \mathcal{O}(\Gamma_i)$, the spectral functions overlap



$\Rightarrow$ Need to go beyond the quasi-particle approximation which underlies the conventional semi-classical Boltzmann approach.

Going beyond the Boltzmann picture

Statistical propagator	$G_F^{ij}(x,y)$	$=$	$\langle \Phi^i(x)\Phi^j(y) + \Phi^j(y)\Phi^i(x)\rangle/2$
Spectral function	$G_\rho^{ij}(x,y)$	$=$	$i\langle \Phi^i(x)\Phi^j(y) - \Phi^j(y)\Phi^i(x)\rangle$

Going beyond the Boltzmann picture

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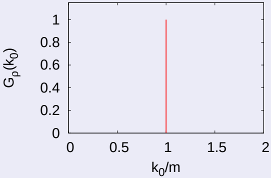
Boltzmann limit

- on-shell quasi-stable particles



$$G_\rho^{ij}(k) \sim \delta^{ij}\delta(k^2 - m_i^2)$$

- equilibrium-like fluctuation-dissipation relation

$$G_F^{ij}(t,k) = \left(f_k^i(t) + \frac{1}{2}\right) G_\rho^{ij}(k)$$

Going beyond the Boltzmann picture

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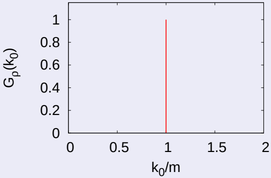

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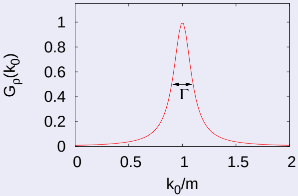
Propagation beyond Boltzmann

- spectrum with (thermal) width



$$G_\rho^{ij}(t,k) \sim \frac{\delta^{ij} 2k_0 \Gamma_i(t)}{(k^2 - m_{th,i}^2(t))^2 + k_0^2 \Gamma_i(t)^2} + \ldots$$

- on-/off-shell, cross-correlations

$$G_F^{ij}(t,k) = \begin{pmatrix} G_F^{11} & G_F^{12} \\ G_F^{21} & G_F^{22} \end{pmatrix}$$

Kadanoff-Baym equations

$$\left(\partial_{x^0}^2 - \nabla_x^2 + m_i^2(x)\right) G_F^{ij}(x,y) = \int_0^{y^0} d^4z\, \Pi_F^{ik}(x,z) G_\rho^{kj}(z,y) - \int_0^{x^0} d^4z\, \Pi_\rho^{ik}(x,z) G_F^{kj}(z,y)$$

$$\left(\partial_{x^0}^2 - \nabla_x^2 + m_i^2(x)\right) G_\rho^{ij}(x,y) = \int_{x_0}^{y^0} d^4z\, \Pi_\rho^{ik}(x,z) G_\rho^{kj}(z,y)$$

- Statistical propagator encodes time-evolution of the state
- Spectral function includes off-shell effects self-consistently
- Memory integrals

Kadanoff-Baym equations

- Obtained from stationarity condition of the 2PI effective action...

  Cornwall, Jackiw, Tomboulis (1974)

$$\frac{\delta\Gamma[\phi, G]}{\delta G} = 0 \quad \Leftrightarrow \quad G^{-1} = G_0^{-1} - \Pi[G]$$

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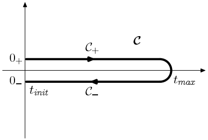
  ...evaluated on the closed Schwinger-Keldysh time contour



$$\begin{aligned} G(x,y) &= \langle T_{\mathcal{C}} \Phi(x)\Phi(y)\rangle \\ &= G_F(x,y) - \frac{i}{2}\,\mathrm{sign}_{\mathcal{C}}(x^0 - y^0)\,G_\rho(x,y) \end{aligned}$$

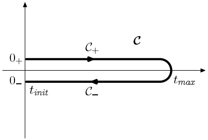
Kadanoff-Baym equations

- Obtained from stationarity condition of the 2PI effective action...

  Cornwall, Jackiw, Tomboulis (1974)

$$\frac{\delta\Gamma[\phi, G]}{\delta G} = 0 \quad \Leftrightarrow \quad G^{-1} = G_0^{-1} - \Pi[G]$$

...evaluated on the closed Schwinger-Keldysh time contour



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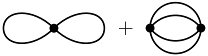
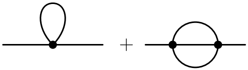
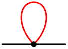
- Conserving approximation by truncation of the 2PI functional $\Gamma_2[\phi, G]$
- Example: Three-loop truncation in $\lambda\Phi^4$-theory (for $\langle\Phi\rangle = 0$)

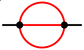
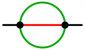
$\Gamma_2[G] =$ 

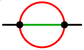
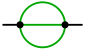
$\Pi[G] = \frac{2i\delta\Gamma_2}{\delta G} =$ 

Important: Π contains resummed propagator (self-consistent)

Kadanoff-Baym equations

$\left(\partial_{x^0}^2 + \mathbf{k}^2 + m^2 + \right.$  $\left. \right) G_F(x^0, y^0, \mathbf{k}) \quad =$

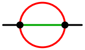
$\int_0^{y^0} dz^0 \left(\right.$  $+$  $\left. \right) G_\rho(z^0, y^0, \mathbf{k})$

$- \int_0^{x^0} dz^0 \left(\right.$  $+$  $\left. \right) G_F(z^0, y^0, \mathbf{k})$

Mixed two-time/momentum representation (spatially homogeneous)

$$G(x, y) = G(x^0, y^0, \mathbf{x} - \mathbf{y}) \quad \rightarrow \quad G(x^0, y^0, \mathbf{k}), \quad \mathbf{k} = (k_x, k_y, k_z)$$

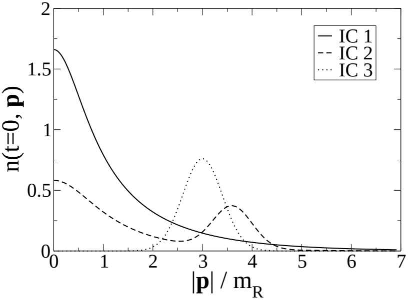
 $= \frac{\lambda}{2} \int \frac{d^3p}{(2\pi)^3} G_F(x^0, x^0, \mathbf{p})$

 $= -\frac{\lambda^2}{6} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} G_F(x^0, z^0, \mathbf{p}) G_\rho(x^0, z^0, \mathbf{q}) G_F(x^0, z^0, \mathbf{k} - \mathbf{p} - \mathbf{q})$

Numerical solution of Kadanoff-Baym equations

Initial condition (example): $\phi = \dot{\phi} = 0$,

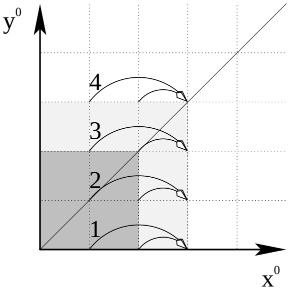
$$
\begin{aligned}
G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= \frac{n_{\mathbf{p}}(t_{init}) + 1/2}{\omega_{\mathbf{p}}(t_{init})} \\
(\partial_{x^0} + \partial_{y^0}) G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= 0 \\
\partial_{x^0}\partial_{y^0} G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= \omega_{\mathbf{p}}(t_{init})\,(n_{\mathbf{p}}(t_{init}) + 1/2)
\end{aligned}
$$



$$\omega_{\mathbf{p}}(t_{init}) = \sqrt{m_R^2 + \mathbf{k}^2}$$

Numerical solution of Kadanoff-Baym equations

- Time-stepping in the two-time plane for $G(x^0, y^0, \hat{\mathbf{k}})$



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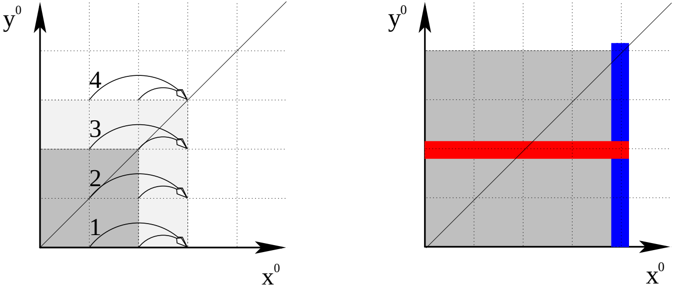


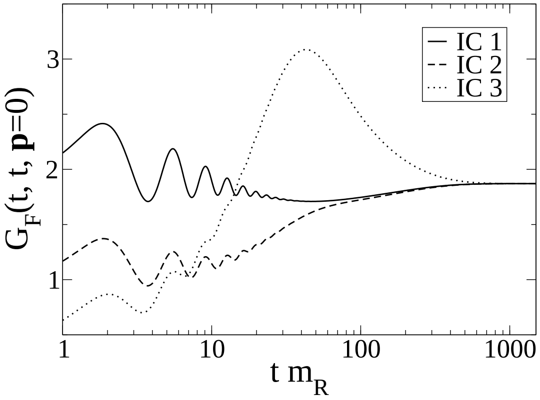
- History matrix for computing memory integrals

$$MEMINT\left(x^0, y^0, \hat{\mathbf{k}}\right) = \sum_{z^0} \Pi(x^0, z^0, \hat{\mathbf{k}})\, G(z^0, y^0, \hat{\mathbf{k}})$$

Danielewicz (1983); Köhler (1994); Berges, Cox (2001); Aarts, Berges (2002); Berges, Borsanyi, Serreau (2003); Juchem, Cassing, Greiner (2004); Arrizabalaga, Smit, Tranberg (2005); Lindner, Müller (2006); ...

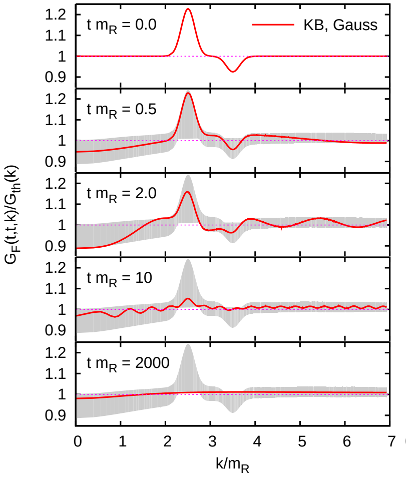
Numerical solution of Kadanoff-Baym equations

Quantum thermalization in Φ^4-theory *Lindner, Müller 2006*



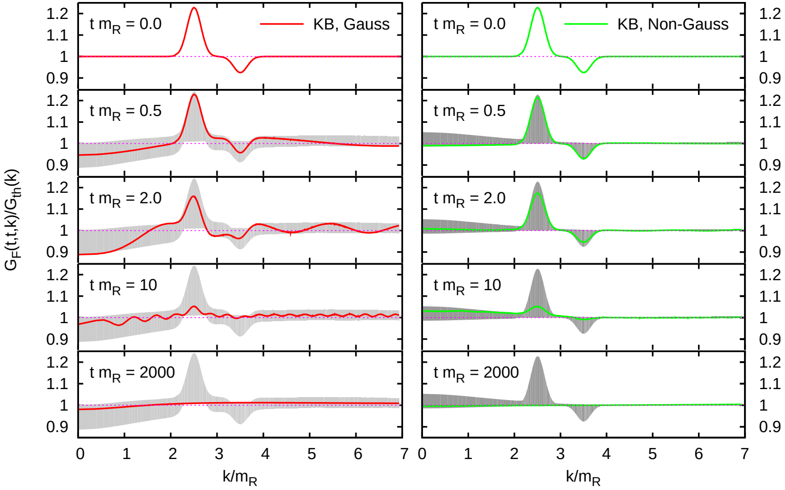
Numerical solution of Kadanoff-Baym equations

- Standard KBEs: interactions 'switched on' at $t = t_{init} \Rightarrow$ transients



Numerical solution of Kadanoff-Baym equations

- Standard KBEs: interactions 'switched on' at $t = t_{init}$ $\Rightarrow$ transients
- $\Rightarrow$ Extended KBEs with initial 4-point correlations *MG, M. Müller 2009*



CTP/Kadanoff-Baym approach to leptogenesis

Goal

- derivation of kinetic equations starting from first principles
- on-/off-shell treated in a unified way (avoid double-counting)
- thermal medium corrections, resonant leptogenesis, coherent flavor transitions

Buchmüller, Fredenhagen 00

De Simone, Riotto 07

Anisimov, Buchmüller, Drewes, Mendizabal 08,10

MG, Kartavtsev, Hohenegger, Lindner 09,10

Beneke, Fidler, Garbrecht, Herranen, Schwaller 10

CTP/Kadanoff-Baym approach to leptogenesis

Lepton current

$$j_L^\mu(x) = \left\langle \sum_\alpha \bar{\ell}_\alpha(x)\gamma^\mu \ell_\alpha(x) \right\rangle = -\text{tr}\left[\gamma^\mu {S_\ell}^{\alpha\beta}(x,x)\right]$$

CTP/Kadanoff-Baym approach to leptogenesis

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Lepton asymmetry

$$n_L(t) = \frac{1}{V}\int_V d^3x\, j_L^0(t,\mathbf{x})$$

CTP/Kadanoff-Baym approach to leptogenesis

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$$n_L(t) = \frac{1}{V}\int_V d^3x\, j_L^0(t,\mathbf{x})$$

Equation of motion

$$\frac{dn_L}{dt} = \frac{1}{V}\int_V d^3x\,\partial_\mu j_L^\mu(x) = -\frac{1}{V}\int_V d^3x\,\text{tr}\left[\gamma_\mu(\partial_x^\mu + \partial_y^\mu)S_\ell{}^{\alpha\beta}(x,y)\right]_{x=y}$$

CTP/Kadanoff-Baym approach to leptogenesis

Lepton current

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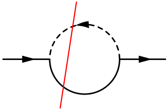
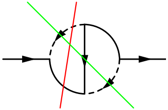
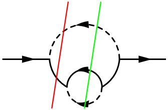
Use KB equations for leptons on the right-hand side $\Rightarrow$

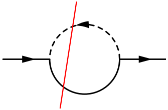
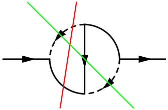
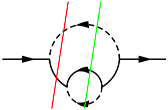
$$\begin{aligned}\frac{dn_L}{dt} \quad = \quad & i\int_0^t dt' \int \frac{d^3p}{(2\pi)^3}\,\mathrm{tr}\Big[\Sigma_{\ell\,\rho\,\mathbf{p}}^{\alpha\gamma}(t,t')S_{\ell\,F\,\mathbf{p}}^{\gamma\beta}(t',t) - \Sigma_{\ell\,F\,\mathbf{p}}^{\alpha\gamma}(t,t')S_{\ell\,\rho\,\mathbf{p}}^{\gamma\beta}(t',t) \\ & - S_{\ell\,\rho\,\mathbf{p}}^{\alpha\gamma}(t,t')\Sigma_{\ell\,F\,\mathbf{p}}^{\gamma\beta}(t',t) + S_{\ell\,F\,\mathbf{p}}^{\alpha\gamma}(t,t')\Sigma_{\ell\,\rho\,\mathbf{p}}^{\gamma\beta}(t',t)\Big]\end{aligned}$$

CTP/Kadanoff-Baym approach to leptogenesis

			
$N \leftrightarrow \ell\phi^\dagger$ $N \leftrightarrow \ell^c\phi$	$\|tree\|^2$	tree × vertex-corr.	tree × wave-corr.
$\ell\phi^\dagger \leftrightarrow \ell^c\phi$		s × t	s × s, t × t

- unified description of CP-violating decay, inverse decay, scattering

CTP/Kadanoff-Baym approach to leptogenesis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$\ell\phi^\dagger \leftrightarrow \ell^c\phi$		s × t	s × s, t × t

- unified description of CP-violating decay, inverse decay, scattering
- dn_L/dt vanishes in equilibrium due to KMS relations

$$S_F^{eq} = \frac{1}{2}\tanh\left(\frac{\beta k^0}{2}\right) S_\rho^{eq} \qquad \Sigma_F^{eq} = \frac{1}{2}\tanh\left(\frac{\beta k^0}{2}\right) \Sigma_\rho^{eq}$$

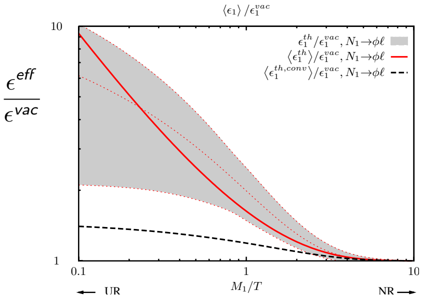
$\Rightarrow$ consistent equations free of double-counting problems

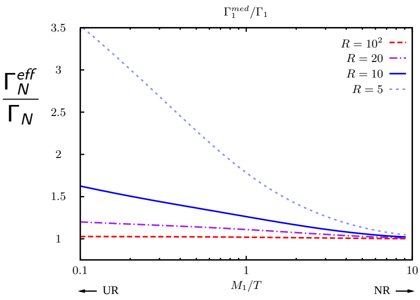
MG, Kartavtsev, Hohenegger, Lindner 09;

Beneke, Garbrecht, Herranen, Schwaller 10;

Quantum-corrected Boltzmann equations *MG, Kartavtsev, Hohenegger, Lindner 2010*

$$\epsilon^{vac} \mapsto \epsilon^{eff}(p, \ldots, T) \quad \Gamma_N \mapsto \Gamma_N^{eff}(p, \ldots, T)$$

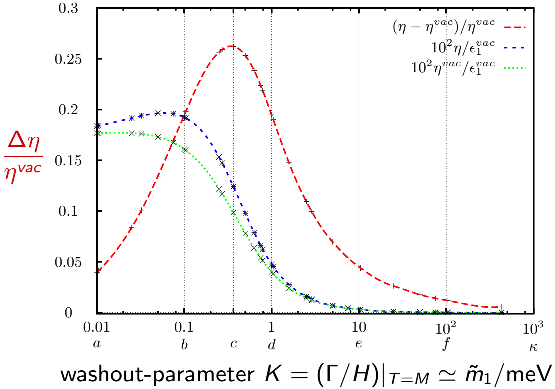




- Enhancement of the effective CP-violating parameter
- Applicable in hierarchical and mildly degenerate case $M_2 - M_1 \gg \Gamma_i$

CTP/Kadanoff-Baym approach to leptogenesis

Hierarchical case



thermal initial abundance

MG, Kartavtsev, Hohenegger, Lindner 10

Resonant enhancement

- Statistical propagator S_F and spectral function S_ρ are matrices in N_1, N_2, N_3 flavor space. We consider the sub-space N_1, N_2 of the quasi-degenerate states.

$$S^{ij}(x,y) = \langle T_{\mathcal{C}} N_i(x) \bar{N}_j(y) \rangle = \begin{pmatrix} S^{11} & S^{12} \\ S^{21} & S^{22} \end{pmatrix}$$

$\Rightarrow$ coherent N_1–N_2 transitions out-of-equilibrium

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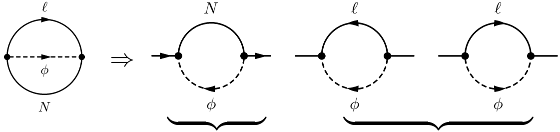
- Self-energies for leptons and for Majorana neutrinos

$\Gamma_2 =$ 

$${\Sigma_\ell}^{\alpha\beta}(x,y) = \frac{\partial i\Gamma_2}{\partial {S_\ell}^{\beta\alpha}(y,x)} \qquad {\Sigma_N}^{ij}(x,y) = \frac{\partial i\Gamma_2}{\partial S^{ji}(y,x)}$$

- Solve KBEs in Breit-Wigner approximation treating lepton and Higgs as a thermal bath *[hierarchical case: Anisimov, Buchmüller, Drewes, Mendizabal 08,10]*
- Important: lepton self-energy contains full Majorana propagator-matrix

Resonant enhancement

Kadanoff-Baym Equations

$$
\begin{aligned}
((i\partial\!\!\!/_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik})S_F^{kj}(x,y) &= \int_0^{x^0} dz^0 \int d^3z\, {\Sigma_N}_\rho^{ik}(x,z)S_F^{kj}(z,y) \\
&\quad - \int_0^{y^0} dz^0 \int d^3z\, {\Sigma_N}_F^{ik}(x,z)S_\rho^{kj}(z,y) \\
((i\partial\!\!\!/_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik})S_\rho^{kj}(x,y) &= \int_{y^0}^{x^0} dz^0 \int d^3z\, {\Sigma_N}_\rho^{ik}(x,z)S_\rho^{kj}(z,y)
\end{aligned}
$$

Decay of nearly-degenerate Majorana neutrinos

Goal: obtain analytical result by taking essential features into account
(width, coherent flavor-mixing, memory integrals)

Decay of nearly-degenerate Majorana neutrinos

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- First step: Non-equilibrium Majorana propagator

$$S_{F\,\mathbf{p}}^{ij}(t,t') \quad = \quad S_{F\,\mathbf{p}}^{ij\;th}(t-t') + \Delta S_{F\,\mathbf{p}}^{ij}(t,t')$$

Decay of nearly-degenerate Majorana neutrinos

Goal: obtain analytical result by taking essential features into account
(width, coherent flavor-mixing, memory integrals)

- First step: Non-equilibrium Majorana propagator

$$S^{ij}_{F\,\mathbf{p}}(t,t') = S^{ij\;th}_{F\,\mathbf{p}}(t-t') + \Delta S^{ij}_{F\,\mathbf{p}}(t,t')$$

- Second step: Lepton asymmetry

$$\begin{aligned} n_L(t) = & \; i(h^\dagger h)_{ji} \int_0^t dt' \int_0^t dt'' \int \frac{d^3p}{(2\pi)^3} \\ & \mathrm{tr}\Big[P_R \underbrace{\left(\Delta S^{ij}_{F\,\mathbf{p}}(t',t'') - \Delta \bar{S}^{ji}_{F\,\mathbf{p}}(t',t'') \right)}_{\propto \text{ Deviation from equilibrium, CP-violation}} P_L S_{\ell\phi\,\rho\,\mathbf{p}}(t''-t') \Big] \end{aligned}$$

$\Delta \bar{S}^{ji}_{F\,\mathbf{p}}(t',t'') = CP\Delta S^{ij}_{F\,\mathbf{p}}(t'',t')^T (CP)^{-1}$
lepton-Higgs loop $S_{\ell\phi} = S_\ell \Delta_\phi$

Majorana neutrino propagator

Retarded and advanced propagators

$$S_R(x,y) = \Theta(x^0 - y^0) S_\rho(x,y)$$
$$S_A(x,y) = -\Theta(y^0 - x^0) S_\rho(x,y)$$

$$(i\partial\!\!\!/_x - M - \delta\Sigma_N(x)) S_{R(A)}(x,y) = -\delta^{ij}\delta(x-y) + \int d^4z\, \Sigma_{N\,R(A)}(x,z) S_{R(A)}(z,y)$$

The Kadanoff-Baym equation for the statistical propagator can be written as

$$\int_0^\infty d^4z \left[\left(\left(i\partial\!\!\!/_x - M_i\right)\delta^{ik} - \delta\Sigma_N{}^{ik}(x)\right)\delta(x-z) - \Sigma_N{}_R^{ik}(x,z)\right] S_F^{kj}(z,y) = \int_0^\infty d^4z\, \Sigma_N{}_F^{ik}(x,z) S_A^{kj}(z,y)$$

Special solution of the inhomogeneous equation

$$S_F^{ij}(x,y)_{inhom} = -\int_0^\infty d^4u S_R^{ik}(x,u) \int_0^\infty d^4z\, \Sigma_N{}_F^{kl}(u,z) S_A^{lj}(z,y)$$

General solution of the homogeneous equation

$$S_F^{ij}(x,y)_{hom} = -\int d^3u\, S_R^{ik}(x,(0,\mathbf{u})) \int d^3v\ A^{kl}(\mathbf{u},\mathbf{v}) S_A^{lj}((0,\mathbf{v}),y)$$

Majorana neutrino propagator

Treating lepton/Higgs as a thermal bath, the Majorana neutrino propagator is given by

$$S^{ij}_{F\,\mathbf{p}}(t,t') \quad = \quad S^{ij\,th}_{F\,\mathbf{p}}(t-t') + \underbrace{S^{ik}_{R\,\mathbf{p}}(t)\gamma_0 \Delta S^{kl}_{F\,\mathbf{p}}(0,0)\gamma_0 S^{lj}_{A\,\mathbf{p}}(-t')}_{\equiv \Delta S^{ij}_{F\,\mathbf{p}}(t,t')}$$

The resummed retarded and advanced functions can be obtained by solving a SD equation (we use $\overline{MS}$ at $T=0$ and $\mu = (M_1 + M_2)/2$)

$$\left[\left(\not{p} - M_i\right)\delta^{ik} - \delta\Sigma_N{}^{ik}(p) - \Sigma_N{}^{ik}_{R(A)}(p)\right] S^{kj}_{R(A)}(p) = -\delta^{ij}$$

$$\Sigma_N{}^{ij}_{R(A)}(p) \quad = \quad -2\left[(h^\dagger h)_{ij} P_L + (h^\dagger h)_{ji} P_R\right] S_{\ell\phi\,R(A)}(p)$$

$$S_{\ell\phi}{}^{vac}_{R(A)}(p) = \frac{1}{32\pi^2}\left(\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 2 - \ln\left(\frac{|p^2|}{\mu^2}\right) \pm i\pi\Theta(p^2)\mathrm{sign}(p_0)\right)\not{p}$$

$$S_{\ell\phi}{}^{th}_{R(A)}(p) \simeq -T^2\not{p}/(12p^2) \pm 2i\not{p}/(e^{|p_0|/T}-1)\Theta(p^2)\mathrm{sign}(p_0)/(32\pi)$$

Majorana neutrino propagator

Solve SD equation for $S_{R(A)}(p)$ in Breit-Wigner approximation:

$$S^{ij}_{R(A)}(p) \simeq \frac{Z^{ij}_{1R(A)}}{p^2 - x_1} + \frac{Z^{ij}_{2R(A)}}{p^2 - x_2}$$

with residua $Z^{ij}_{IR(A)}$ and complex poles x_I (basis independent)

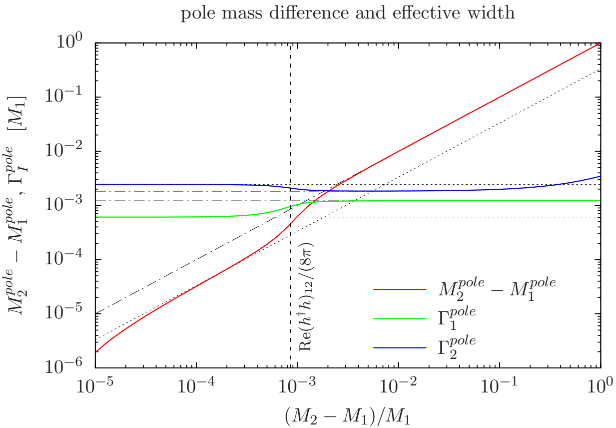
$$x_{1,2} = \frac{(V \pm W)^2}{4Q^2} \equiv \left(\omega_{pI} - i\frac{\Gamma_{pI}}{2}\right)^2 - \mathbf{p}^2$$

where

$$\begin{aligned} V &= \sqrt{(\eta_1 M_1(1+i\gamma_{22}) - \eta_2 M_2(1+i\gamma_{11}))^2 - 4\eta_1\eta_2 M_1 M_2 (\mathrm{Re}\gamma_{12})^2} \\ W &= \sqrt{(\eta_1 M_1(1+i\gamma_{22}) + \eta_2 M_2(1+i\gamma_{11}))^2 + 4\eta_1\eta_2 M_1 M_2 (\mathrm{Im}\gamma_{12})^2} \\ Q &= \det\Omega_{LR} = \det\Omega_{RL} = (1+i\gamma_{11})(1+i\gamma_{22}) + |\gamma_{12}|^2 \end{aligned}$$

$$\gamma_{ij} \simeq (h^\dagger h)_{ij}\left[\frac{\Theta(p^2)\mathrm{sign}(p_0)}{16\pi}\left(1 + \frac{2}{e^{|p_0|/T} - 1}\right) + i\left(\frac{\ln\frac{|p^2|}{\mu^2}}{16\pi^2} + \frac{T^2}{6p^2} - \frac{T^2}{6\mu^2}\right)\right]$$

Majorana neutrino propagator



Effective masses $M_I^{pole} \equiv \omega_{pI}|_{\mathbf{p}=0}$ and widths $\Gamma_I^{pole} \equiv \Gamma_{pI}|_{\mathbf{p}=0}$ of the sterile Majorana neutrinos extracted from complex poles of resummed ret/adv prop. for $(h^\dagger h)_{11} = 0.03$, $(h^\dagger h)_{22} = 0.045$, $(h^\dagger h)_{12} = 0.03 \cdot e^{i\pi/4}$ and $T = 0.25 M_1$.

Majorana neutrino propagator

- Regime $(M_2 - M_1)/M_1 \gtrsim \mathrm{Re}(h^\dagger h)_{12}/(8\pi)$

$$\begin{aligned} M_i^{pole} &\simeq M_i \, , \\ \Gamma_i^{pole} &\simeq \Gamma_i \equiv \frac{(h^\dagger h)_{ii}}{8\pi} M_i \left(1 + \frac{2}{e^{M_i/T} - 1}\right) \end{aligned}$$

- Regime $(M_2 - M_1)/M_1 \lesssim \mathrm{Re}(h^\dagger h)_{12}/(8\pi)$

$$\begin{aligned} M_i^{pole} &\simeq \frac{M_1 + M_2}{2} \pm \frac{(M_2 - M_1)((h^\dagger h)_{22} - (h^\dagger h)_{11})}{2\sqrt{((h^\dagger h)_{22} - (h^\dagger h)_{11})^2 + 4(\mathrm{Re}(h^\dagger h)_{12})^2}} \, , \\ \Gamma_i^{pole} &\simeq \frac{M_i}{16\pi}\left(1 + \frac{2}{e^{M_i/T} - 1}\right)\Big((h^\dagger h)_{11} + (h^\dagger h)_{22} \\ &\qquad \pm \sqrt{((h^\dagger h)_{22} - (h^\dagger h)_{11})^2 + 4(\mathrm{Re}(h^\dagger h)_{12})^2}\Big) \end{aligned}$$

The relation between the mass- and Yukawa coupling matrices at zero and finite temperature is

$$\begin{aligned} M(T) &= (P_L Z(T)^T + P_R Z(T)^\dagger) M(T=0) (P_L Z(T) + P_R Z(T)^*) \, , \\ (h^\dagger h)(T) &= Z(T)^T (h^\dagger h)(T=0) Z(T)^* \, , \end{aligned}$$

where $Z_{ij}(T) \equiv V_{ik}(T)(\delta_{kj} + (h^\dagger h)_{kj} T^2/(6\mu^2))$, $V(T)^\dagger V(T) = 1$

Result for the lepton asymmetry

$$n_L(t) = \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, 2q\, 2k} (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \sum_{I,J=1,2} \sum_{\epsilon_n=\pm 1} F_{JI}^{\epsilon_n} L_{IJ}^{\epsilon_n}(t)$$

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Coefficients F depend on Yukawa couplings, thermal distributions of lepton and Higgs, resummed ret/adv propagators and initial conditions

$$\begin{aligned} F^{\epsilon_n}_{JI} = & \sum_{ijkl=1,2} (h^\dagger h)_{ji} \left(\left(\tfrac{1}{2} + f_\phi(q)\right) + \epsilon_2\epsilon_3 \left(\tfrac{1}{2} - f_\ell(k)\right)\right) \mathrm{tr}\Big[P_L(|\mathbf{k}|\gamma_0 + \epsilon_2 \mathbf{k}\gamma) \\ & \times \left(S^{ik\epsilon_4}_{RI} \gamma_0 \Delta S^{kl}_{F\,\mathbf{p}}(0,0) \gamma_0 S^{lj\epsilon_1}_{AJ} - \bar{S}^{jk\epsilon_4}_{RI} \gamma_0 \Delta \bar{S}^{kl}_{F\,\mathbf{p}}(0,0) \gamma_0 \bar{S}^{li\epsilon_1}_{AJ} \right) \Big] \end{aligned}$$

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$$F_{JI}^{\epsilon_n} = \sum_{ijkl=1,2} (h^\dagger h)_{ji} \left(\left(\tfrac{1}{2} + f_\phi(q) \right) + \epsilon_2 \epsilon_3 \left(\tfrac{1}{2} - f_\ell(k) \right) \right) \mathrm{tr} \Big[P_L(|\mathbf{k}|\gamma_0 + \epsilon_2 \mathbf{k}\gamma)$$
$$\times \left(S_{RI}^{ik\epsilon_4} \gamma_0 \Delta S_{F\,\mathbf{p}}^{kl}(0,0) \gamma_0 S_{AJ}^{lj\epsilon_1} - \bar{S}_{RI}^{jk\epsilon_4} \gamma_0 \Delta \bar{S}_{F\,\mathbf{p}}^{kl}(0,0) \gamma_0 \bar{S}_{AJ}^{li\epsilon_1} \right) \Big]$$

Time-dependence: flavor diagonal and off-diagonal contributions:

$$L_{II}^{\pm}(t) = \frac{1 - e^{-\Gamma_{pI} t}}{\Gamma_{pI}} \mathrm{Re} \left(\frac{\Gamma_{\ell\phi}}{(\omega_{pI} - k - q + i\Gamma_{pI}/2)^2 + \Gamma_{\ell\phi}^2/4} \right)$$

$$L_{21}^{\pm}(t) = \frac{1 - e^{\mp i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} \pm 2i(\omega_{p1} - \omega_{p2})} \left(\frac{\Gamma_{\ell\phi}}{(\omega_{p1} - k - q \pm i\Gamma_{p1}/2)^2 + \Gamma_{\ell\phi}^2/4} \right.$$
$$\left. + \frac{\Gamma_{\ell\phi}}{(\omega_{p2} - k - q \mp i\Gamma_{p2}/2)^2 + \Gamma_{\ell\phi}^2/4} \right) = L_{12}^{\pm}(t)^*$$

Resonant enhancement

Comparison KB – Boltzmann: hierarchical limit

$$n_L(t) = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi}\frac{M_1}{M_2}\int \frac{d^3p\,d^3q\,d^3k}{(2\pi)^9\,\omega_{p1}\,2q\,2k}\,4k\cdot p_1$$
$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k}-\mathbf{q}) \times \mathrm{Re}\,\frac{\Gamma_{\ell\phi}}{(\omega_{p1}-k-q+i\Gamma_{p1}/2)^2+\Gamma_{\ell\phi}^2/4}$$
$$\times(1+f_\phi(q)-f_\ell(k))\,f_{FD}(\omega_{p1})\frac{1-e^{-\Gamma_{p1}t}}{\Gamma_{p1}}$$

$$n_L^{Boltzmann}(t) = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi}\frac{M_1}{M_2}\int \frac{d^3p\,d^3q\,d^3k}{(2\pi)^9\,\omega_{p1}\,2q\,2k}\,4k\cdot p_1$$
$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k}-\mathbf{q}) \times 2\pi\delta(\omega_{p1}-k-q)$$
$$\times(1+f_\phi(q)-f_\ell(k))\,f_{FD}(\omega_{p1})\frac{1-e^{-\Gamma_{p1}t}}{\Gamma_{p1}}\,.$$

The thermal width of lepton and Higgs $\Gamma_{\ell\phi}=\Gamma_\ell+\Gamma_\phi$ leads to a replacement of the on-shell delta function in the Boltzmann equations by a Breit-Wigner curve, in accordance with *Anisimov, Buchmüller, Drewes, Mendizabal 10*

The coherent contributions are suppressed with Γ_{p1}/ω_{p2}

Resonant enhancement

Comparison KB – Boltzmann: degenerate case

$$n_L(t) = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k} 4k\cdot p$$
$$\times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4} (1 + f_\phi - f_\ell) f_{FD}(\omega_p)$$
$$\times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} - 4\,\mathrm{Re}\left(\frac{1 - e^{-i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} + 2i(\omega_{p1} - \omega_{p2})} \right) \right]$$

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$$\times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q})\, 2\pi\delta(\omega_p - k - q)\, (1 + f_\phi - f_\ell) f_{FD}(\omega_p)$$
$$\times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} \right]$$

Resonant enhancement

Comparison KB – Boltzmann: degenerate case

$$n_L(t) = \frac{\mathrm{Im}[(h^\dagger h)^2_{12}]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k}\, 4k\cdot p$$
$$\times (2\pi)^3 \delta(\mathbf{p}-\mathbf{k}-\mathbf{q}) \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4} (1 + f_\phi - f_\ell) f_{FD}(\omega_p)$$
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- Regulator $M_1\Gamma_1 - M_2\Gamma_2$ is confirmed

Resonant enhancement

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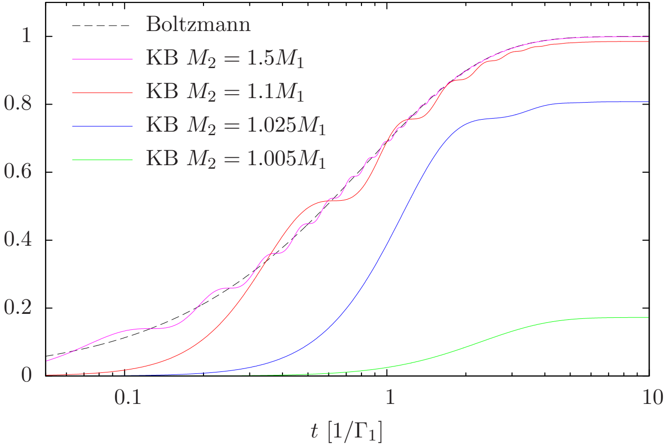
- Regulator $M_1\Gamma_1 - M_2\Gamma_2$ is confirmed
- Additional oscillating contribution due to coherent N_1—N_2 transitions

Resonant enhancement

MG, Kartavtsev, Hohenegger 1112.6428

$n_L(t, \mathbf{p}) / n_L^{Boltzmann}(t = \infty, \mathbf{p})$



$n_L(t) = \int \frac{d^3p}{(2\pi)^3} n_L(t, \mathbf{p}), \Gamma_1 = 0.01 M_1, \Gamma_2 = 0.015 M_1, \Gamma_{\ell\phi} \to 0$

Resonant enhancement

Resonant enhancement within the Boltzmann approach

$$R^{Boltzmann} \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

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Resonant enhancement within the Kadanoff-Baym approach, including coherent contributions ($|(h^\dagger h)_{12}| \ll (h^\dagger h)_{ii}$)

$$R^{KB}(t) = \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \times (1 - f_{coherent}(t))$$

Resonant enhancement

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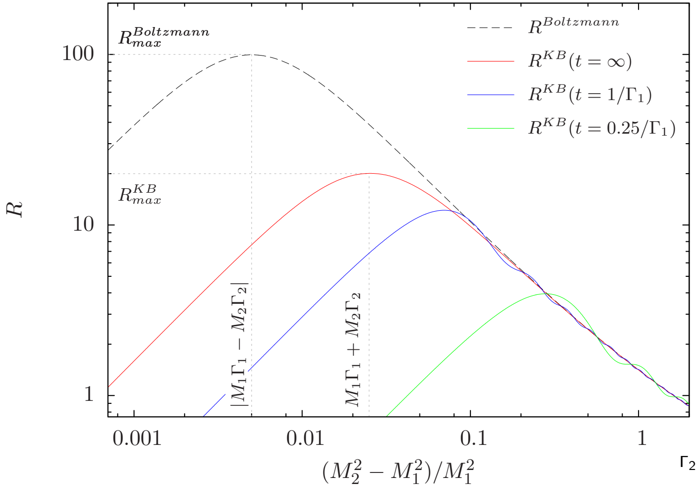
Partial cancellation of Boltzmann- and coherent contribution cuts off the enhancement in the doubly degenerate limit $M_1 \to M_2$ and $\Gamma_1 \to \Gamma_2$

$$R^{KB}(t)|_{t \gtrsim 1/\Gamma_{pl}} \quad \simeq \quad \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 + M_2\Gamma_2)^2}$$

MG, Kartavtsev, Hohenegger 1112.6428



$\Gamma_2/\Gamma_1 = 1.5$

$$R^{Boltzmann}_{max} = M_1 M_2/(2|\Gamma_1 M_1 - \Gamma_2 M_2|),\ R^{KB}_{max} = M_1 M_2/(2(\Gamma_1 M_1 + \Gamma_2 M_2))$$

Resonant enhancement

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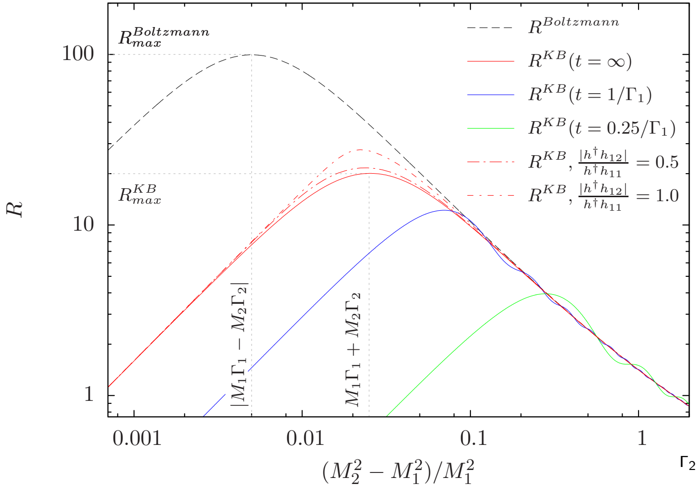

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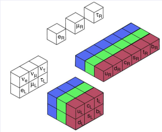
$R^{Boltzmann}_{max} = M_1 M_2/(2|\Gamma_1 M_1 - \Gamma_2 M_2|)$, $R^{KB}_{max} = M_1 M_2/(2(\Gamma_1 M_1 + \Gamma_2 M_2))$

Conclusions



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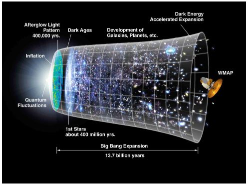

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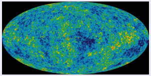


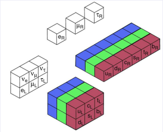
$\tilde{m}, \theta_{13}, \sigma v, \ldots$

microscopic description of out of equilibrium processes in the Early Universe

$\eta_b, \Omega_{dm}, n_s, \ldots$

- First-principles methods like Kadanoff-Baym equations are important to describe quantum effects and to scrutinize classical approximations
- Leptogenesis
  - resolve double counting issues
  - quantum-corrected Boltzmann equations
  - size of the resonant enhancement

Conclusions



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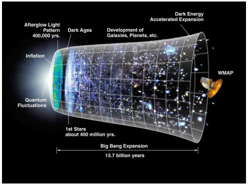

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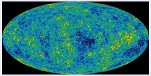

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thank you!