

Black holes in Einstein-aether and Horava-Lifshitz gravity

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Why Lorentz-violating gravity?

Lorentz-violating effects are severely constrained in the matter sector.
However,

- ✂• Observational constraints are far weaker in the more weakly coupled gravitational sector
- ✂• A low energy effective theory of Lorentz-violating gravity is needed for such tests (e.g. Einstein-aether theory)

There might be an additional pay-off:

- ✂• Recently it has been claimed that some models of Lorentz-violating gravity are power-counting renormalizable (Hořava-Lifshitz gravity)

- ✧• Simpler to find than other solutions (e.g. stars)
- ✧• Potentially a probe for observable deviations from GR
- ✧• Intrinsically interesting in theories with Lorentz violations

What characterizes a black hole in this case?

- ✧• There can be different modes with different speeds, so there can be multiple horizons
- ✧• There can be modified dispersion relations, in which case short wavelength perturbations can travel with arbitrarily high speed

Tentative answer: The existence of a causal boundary for all modes

The action of the theory is

$$S_{\text{æ}} = \frac{1}{16\pi G_{\text{æ}}} \int d^4x \sqrt{-g} (-R - M^{\alpha\beta\mu\nu} \nabla_{\alpha} u_{\mu} \nabla_{\beta} u_{\nu})$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^{\alpha} u^{\beta} g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^{\mu} u_{\mu} = 1$$

- ✂• Most general theory with a unit timelike vector field which is second order in derivatives
- ✂• Extensively tested and still viable

Now assume

$$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu} \partial_\mu T \partial_\nu T}}$$

and choose T as the time coordinate

$$u_\alpha = \delta_{\alpha T} (g^{TT})^{-1/2} = N \delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S_{h.o. \infty} = \frac{1}{16\pi G_H} \int dT d^3x N \sqrt{h} (K_{ij} K^{ij} - \lambda K^2 + \xi^{(3)} R + \eta a_i a^i)$$

with $a_i = \partial_i \ln N$ and the parameter correspondence

$$\frac{G_H}{G_\infty} = \xi = \frac{1}{1 - c_{13}} \quad \lambda = \frac{1 + c_2}{1 - c_{13}} \quad \eta = \frac{c_{14}}{1 - c_{13}}$$

The action of the theory is

$$S_{HL} = \frac{1}{16\pi G_H} \int dT d^3x N \sqrt{h} \left(L_2 + \frac{1}{M_\star^2} L_4 + \frac{1}{M_\star^4} L_6 \right)$$

where

$$L_2 = K_{ij} K^{ij} - \lambda K^2 + \xi^{(3)} R + \eta a_i a^i$$

L_4 : contains all 4th order terms constructed with the induced metric h_{ij} and a_i

L_6 : contains all 6th order terms constructed in the same way

$$M_\star : 10^{10} \text{GeV} - 10^{16} \text{GeV}$$

- Higher order terms contain higher order spatial derivatives
- They modify the propagator and render the theory power-counting renormalizable
- All terms consistent with the symmetries will be generated by radiative corrections
- This version of the theory is viable so far
- “Low energy limit” is h.o. Einstein-aether theory!

Hypersurface orthogonality implies

$$\omega_\alpha = \epsilon_{\alpha\beta\mu\nu} u^\beta \nabla^\mu u^\nu = 0$$

Since the square of the twist is

$$\omega_\alpha \omega^\alpha = -(\nabla_\alpha u_\beta)(\nabla^\alpha u^\beta) + (\nabla_\alpha u_\beta)(\nabla^\beta u^\alpha) + (u^\beta \nabla_\beta u_\alpha)(u^\mu \nabla_\mu u^\alpha)$$

one can eliminate one of $c_{1,3,4}$ (but not c_2)

- ✂• The hypersurface orthogonal theory will have one less parameter

We are interested in vacuum black hole solutions which are

- spherically symmetric
- static
- asymptotically flat
- and can form from gravitational collapse

Finding such solutions analytically seems unfeasible, so we find them numerically

- Spherically symmetric vectors are hypersurface orthogonal!
- Consequence: Spherically symmetric solutions of ae-theory will be also solutions of low-energy HL-gravity
- The converse appears to be also true!

We use Eddington-Finkelstein(-like) coordinates

$$ds^2 = F(r)dv^2 - 2B(r)dvdr - r^2 d\Omega^2$$

$$u^\alpha \partial_\alpha = A(r) \partial_v - \frac{1 - F(r)A^2(r)}{2B(r)A(r)} \partial_r$$

In general there is a 3-parameter family of solutions. However

- asymptotic flatness
- regularity of the spin-0 horizon

will lead to a 1-parameter family.

- Fixing the horizon radius one has a unique solution

The field equations are

$$E^{\mu\nu} \equiv G^{\mu\nu} - T_{\text{æ}}^{\mu\nu} = 0 \qquad \text{\textit{Æ}}^\mu = 0$$

but the non-redundant or non-trivial components are

$$E^{vv} = E^{vr} = E^{rr} = E^{\theta\theta} = \text{\textit{Æ}}^v = 0$$

However, the combinations

$$C^\mu \equiv E^{r\mu} - u^r \text{\textit{Æ}}^\mu = 0$$

are actually constraint equations, so we will use instead the system

$$E^{vv} = E^{\theta\theta} = \text{\textit{Æ}}^v = 0, \quad C^v = C^r = 0$$

The first three equations can be recast in the form

$$F'' = F''(A, A', B, F, F')$$

$$A'' = A''(A, A', F, F')$$

$$B' = B'(A, A', B, F, F')$$

so we need 5 pieces of initial data. Additional, the constraints

- are automatically preserved if they are imposed initially
- relate initial data

As mentioned, one then generically has 3-parameter solutions.

In general there will be spin-2, spin-1 and spin-0 modes. For the spin-0

$$s_0^2 = \frac{c_{123}(2 - c_{14})}{c_{14}(1 - c_{13})(2 + c_{13} + 3c_2)}$$

The horizon for a mode is a null surface of the effective metric

$$g_{\alpha\beta}^{(i)} = g_{\alpha\beta} + (s_i^2 - 1)u_\alpha u_\beta$$

Together with the aether redefinition

$$u_{(i)}^\alpha = \frac{1}{s_i} u^\alpha$$

the metric redefinition leave the action formally invariant

- Perform the redefinition just described
- Define the common spin-0/metric horizon by $F(r_H) = 0$
- Require regularity of the horizon

$$B' = \frac{b_0}{F} + b_1 + b_2 F \quad \xrightarrow{\text{regularity}} \quad b_0(A, A', B, F')|_H = 0$$

- Impose the constraints: one is trivial on the horizon, the other is

$$C^v(A, A', B, F')|_H = 0$$

- Choose the time coordinate such that $B_H = 1$
- Parametrize the space of initial data by A_H
- Determine the value that leads to asymptotic flatness by a shooting method

We impose the following viability constraints

- ✂• Classical and quantum-mechanical stability
- ✂• Avoidance of vacuum Cherenkov radiation by matter.
- ✂• Exact agreement with Solar system experiments (vanishing preferred frame parameters)

The last constraint is more restrictive than actually required. However,

- ✂• It reduces the dimensions of the parameters space to 2
- ✂• It provided an important simplification with little given away

Exact agreement with Solar system test requires

$$c_2 = \frac{-2c_1^2 - c_1 c_3 + c_3^2}{3c_1} \qquad c_4 = -\frac{c_3^2}{c_1}$$

Then, defining

$$c_{\pm} = c_1 \pm c_3$$

the remaining constraint require

$$0 \leq c_+ \leq 1 \qquad 0 \leq c_- \leq \frac{c_+}{3(1 - c_+)}$$

We have explored the region that additionally satisfies

$$c_- \leq 1$$

It is more convenient to work in the parametrization

$$\xi = \frac{1}{1 - \beta} \quad \lambda = \frac{1 + \mu}{1 - \beta} \quad \eta = \frac{\alpha}{1 - \beta}$$

Then, exact agreement with Solar system test requires

$$\alpha = 2\beta$$

and the remaining constraint require

$$0 < \beta < 1/3 \quad \mu > \frac{\beta(\beta + 1)}{1 - 3\beta}$$

$$0 < \beta < 1/3 \quad \mu < -\frac{2 + \beta}{3}$$

$$1/3 < \beta < 1 \quad \frac{\beta(\beta + 1)}{1 - 3\beta} < \mu < -\frac{2 + \beta}{3}$$



Characteristic quantities

$\omega_{\text{ISCO}} r_g$: orbital frequency at the ISCO times gravitational radius.

Can be measured using X-ray spectra from accretion or gravitational waves from EMRIs

z_{max} : ($= \nu_{\text{emitted}}/\nu_{\text{measured}} - 1$) the maximum redshift for a photon emitted at the ISCO. Can be measured using iron-K α line

b_{ph}/r_g : impact parameter for circular photon orbit/grav. radius

Can be measured by gravitational lensing or in the future via black hole quasinormal modes

r_g/r_H : grav. radius/horizon radius

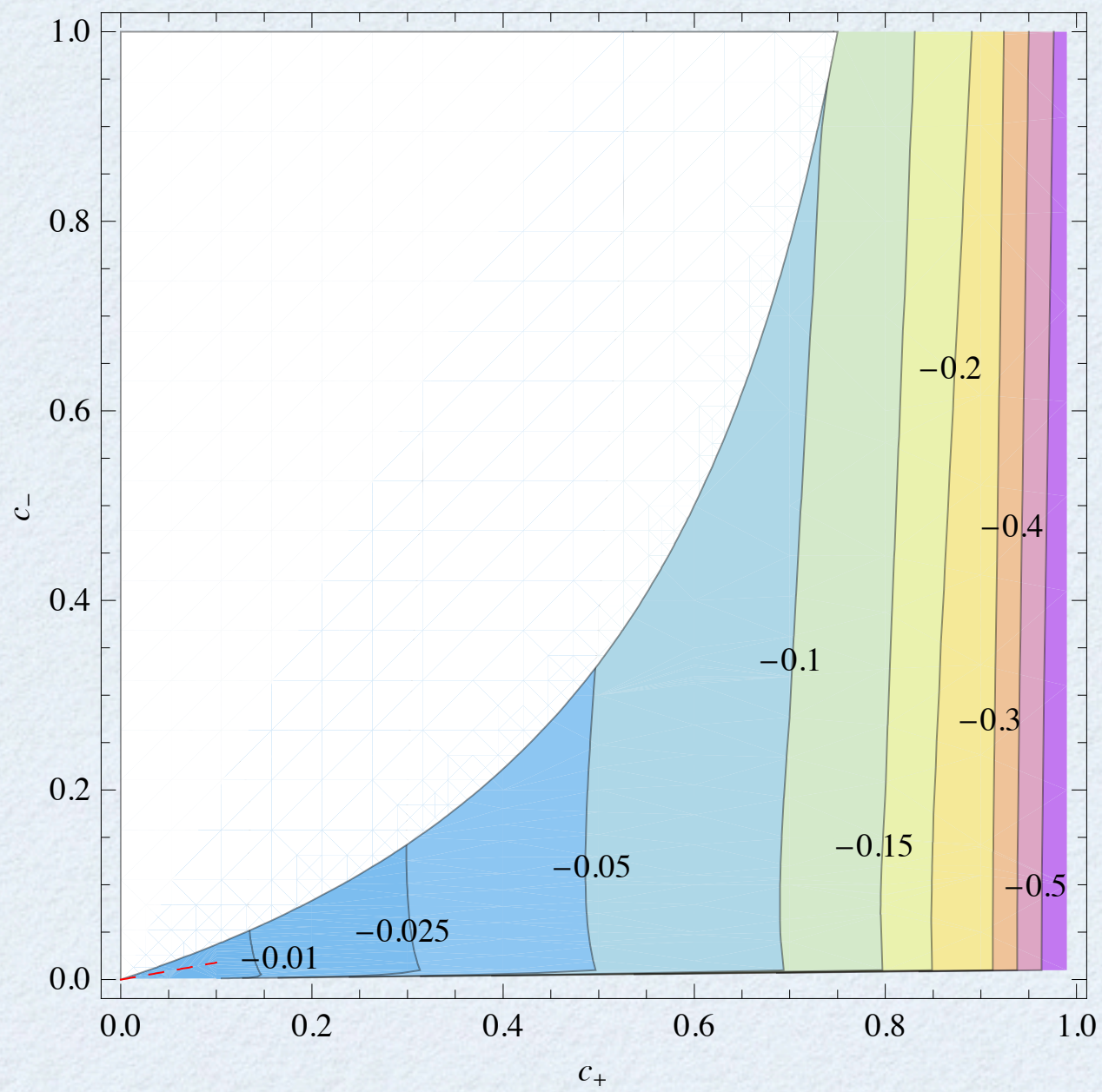
Not measurable but gives info about near horizon region

Eling and Jacobson had looked for black holes in ae-theory with

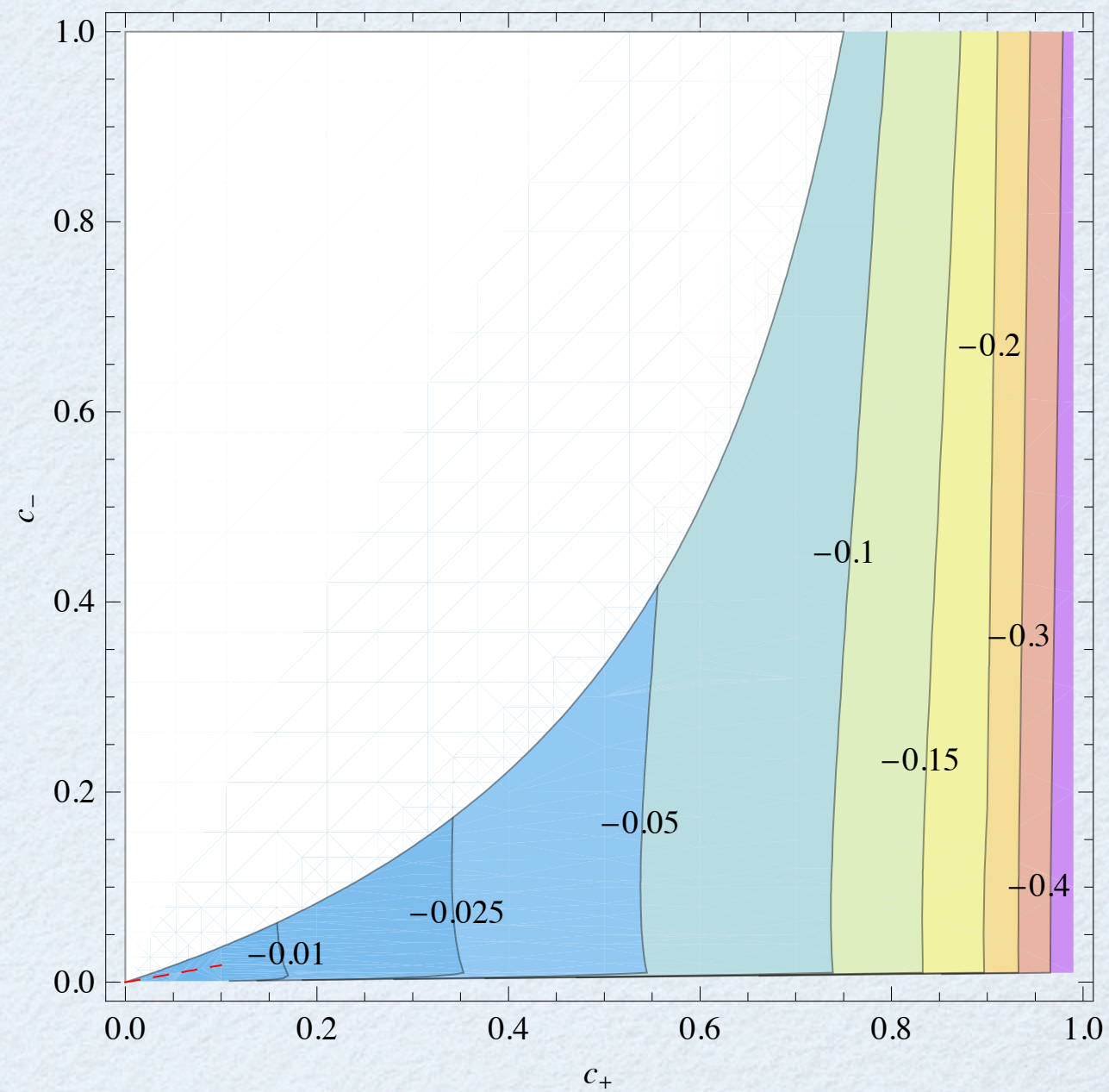
$$c_3 = c_4 = 0 \quad s_0 = 1 \quad \Rightarrow \quad c_2 = -c_1^3 / (3c_1^2 - 4c_1 + 2)$$

for the interval $0 < c_1 < 1$ and we have attempted to reproduce their results

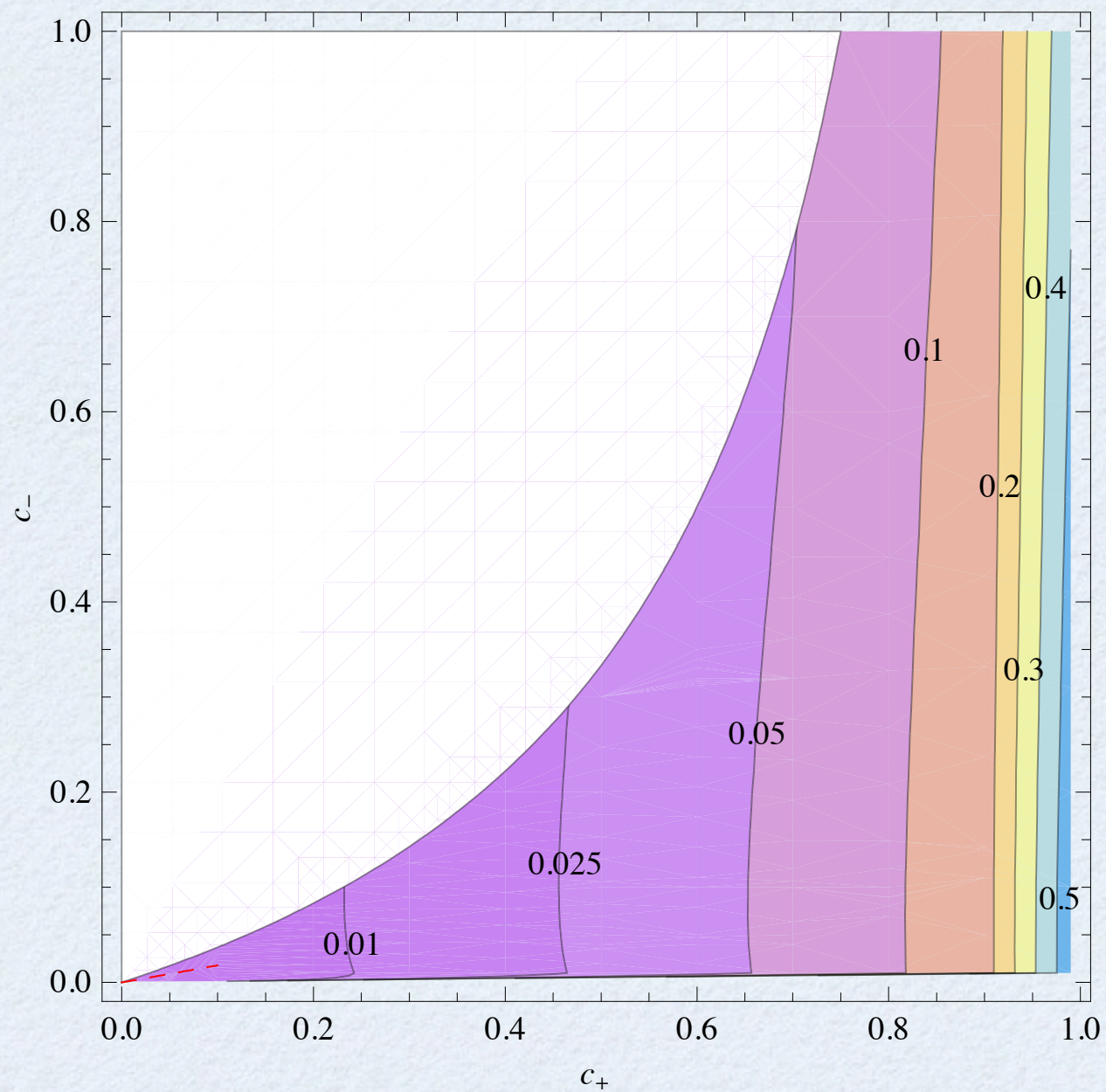
- ✂• We are successful in the range 0 to 0.7
- ✂• They did not find regular solutions in the range 0.7 to 1, whereas we did
- ✂• There is evidence that our code is simply more accurate than theirs. Example: our estimate of r_g/r_H agrees with theirs only up to the 3 decimal place for $c_1 > 0.3$ and the difference exceeds our estimated error (by orders of magnitude)



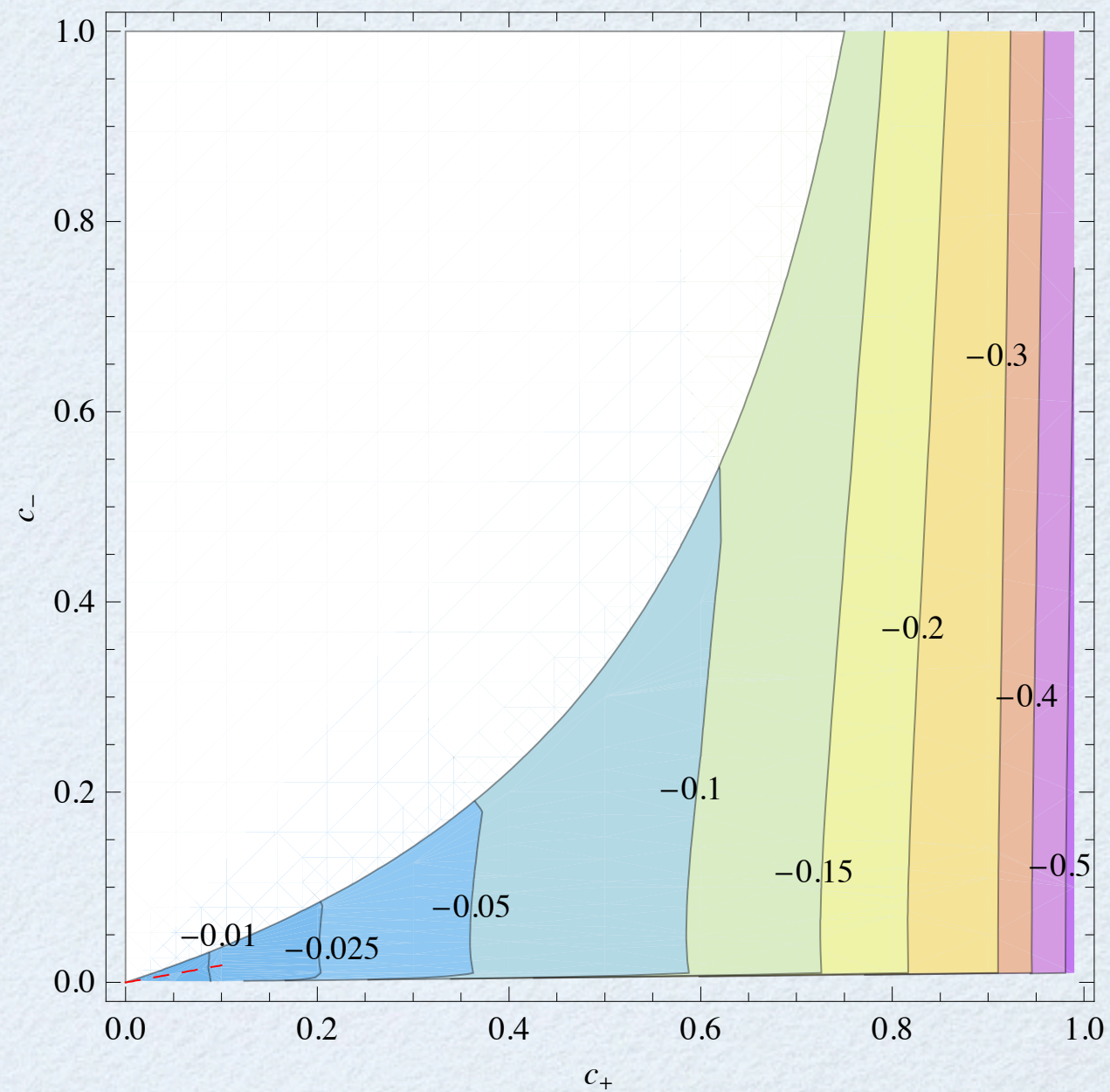
$\omega_{\text{ISCO}} r_g$



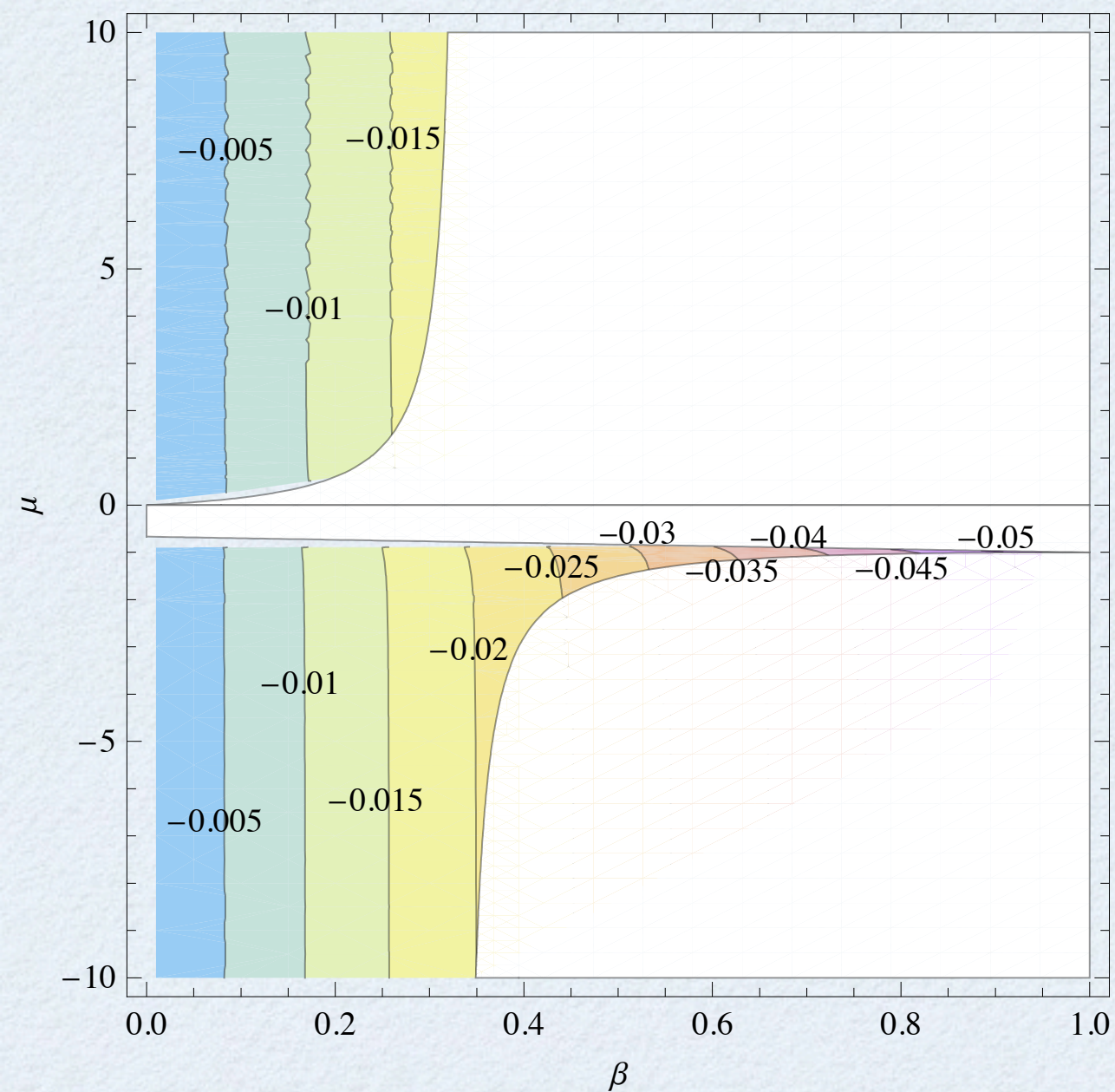
z_{max}



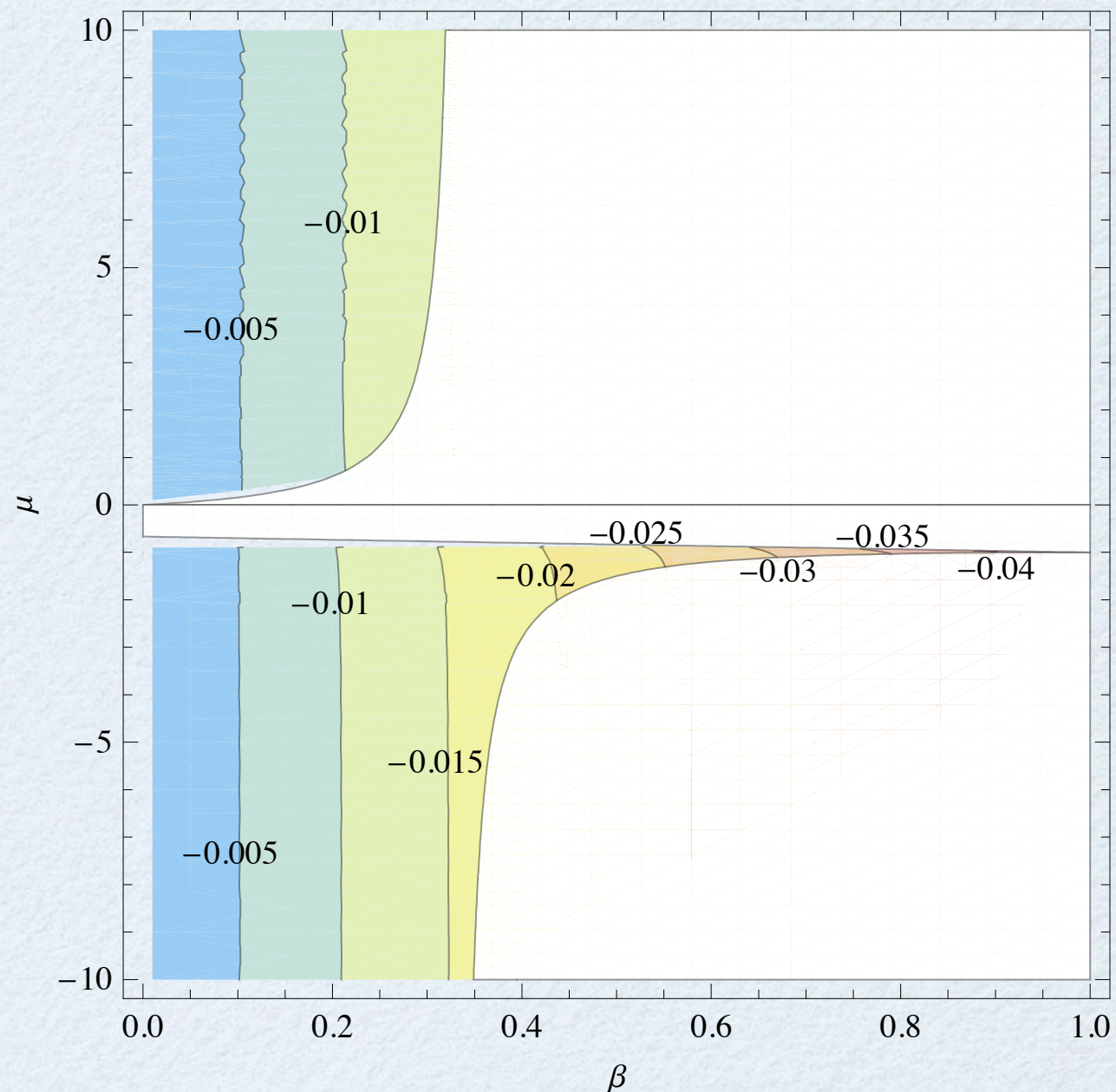
b_{ph}/r_g



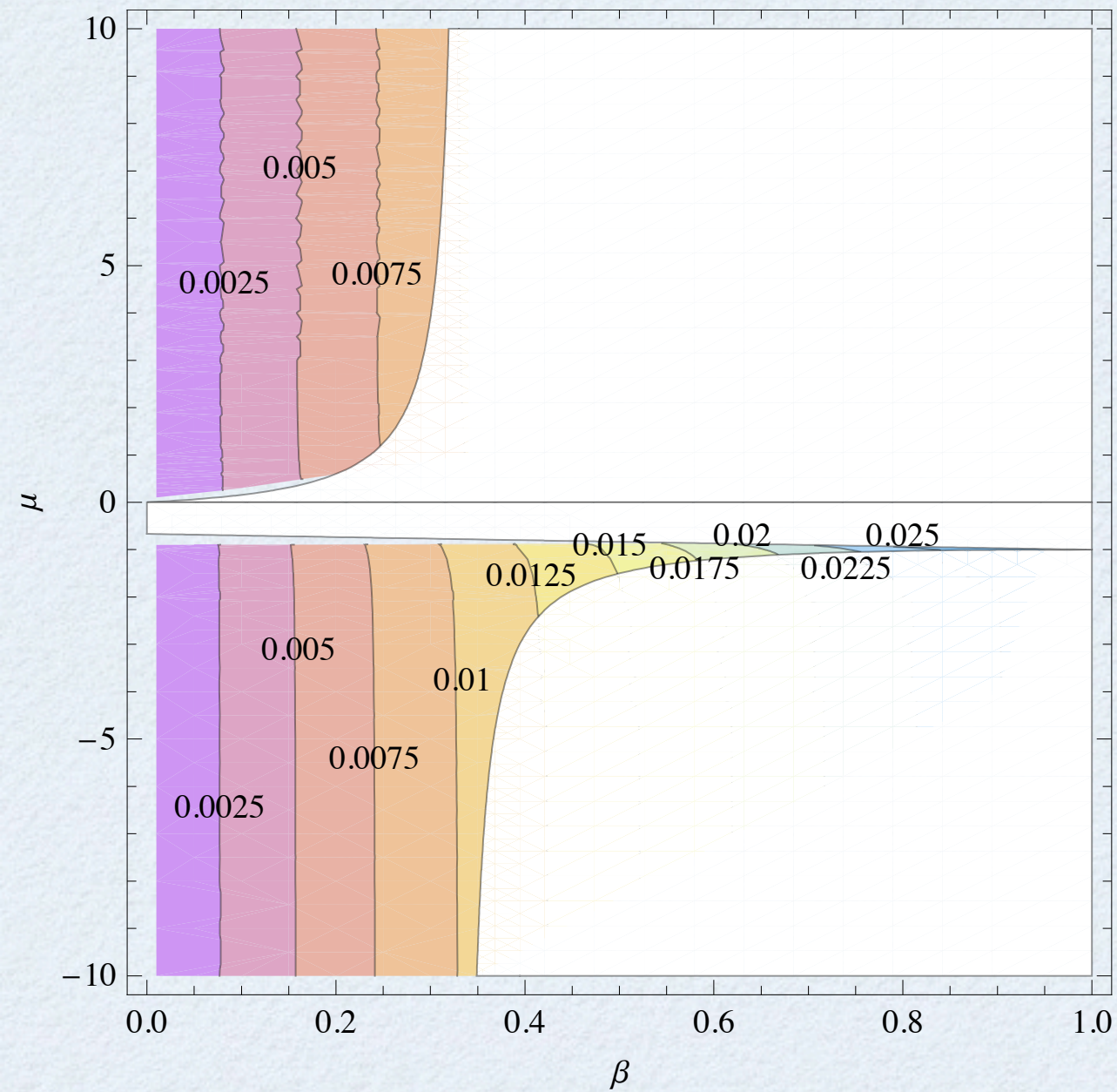
r_g/r_H



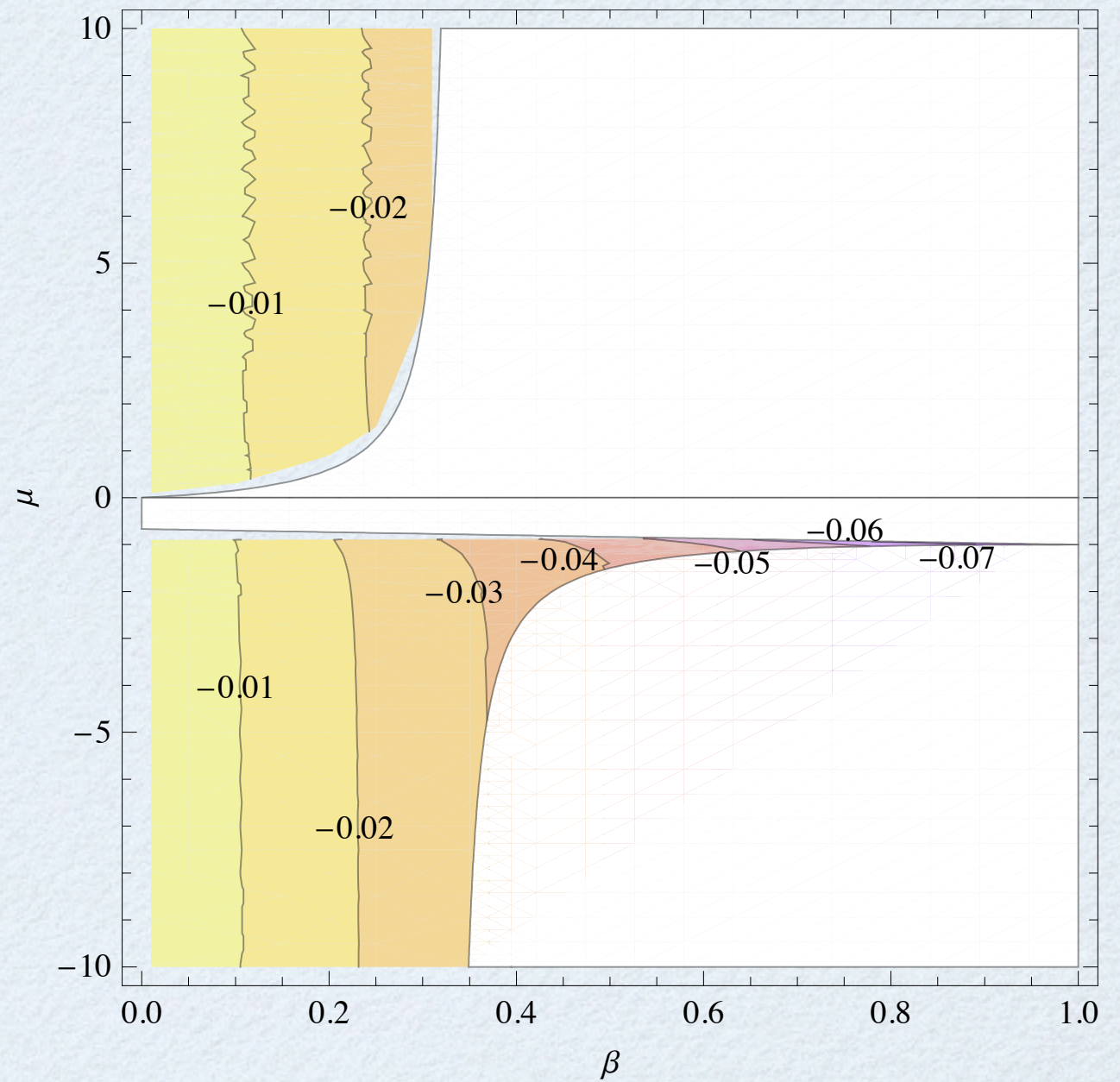
$\omega_{\text{ISCO}} r_g$



z_{max}



b_{ph}/r_g



r_g/r_H

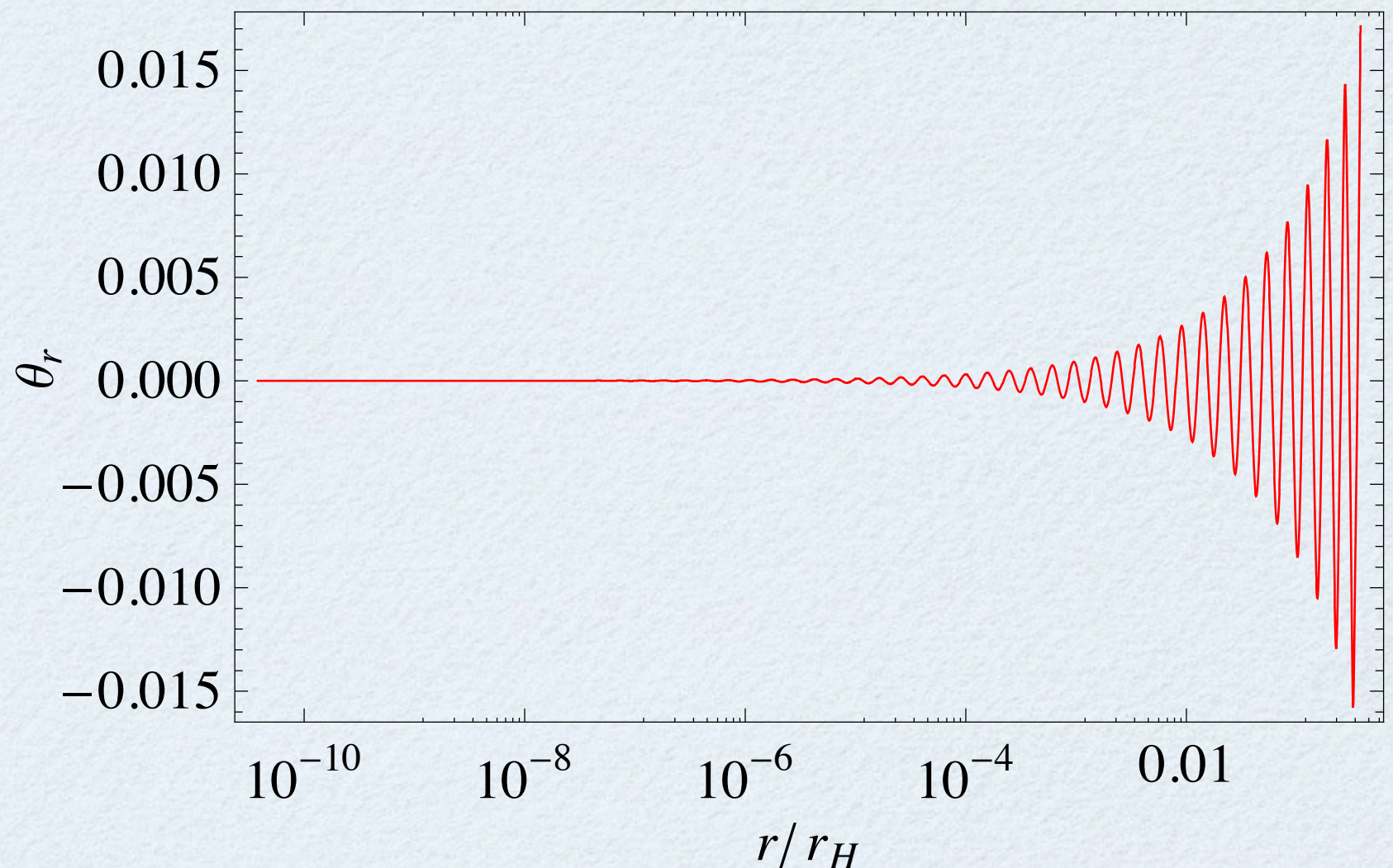
- Curvature singularity at the centre

We have also calculated the Lorentz factor of the aether as measured by the future directed observer orthogonal to $r = \text{const.}$ hypersurfaces

$$\gamma_r \equiv u_{\text{obs}}^\alpha u_\alpha$$

and the corresponding
boost angle

$$\theta_r = \text{arccosh} \gamma_r$$



- ⌘• Different modes have different horizons
- ⌘• When higher spatial derivatives are added short wavelength perturbations can travel at arbitrarily high speed

However,

- ⌘• Signals cannot travel backwards in time
- ⌘• Future and past direction are locally defined by the aether
- ⌘• The aether is orthogonal to constant time hypersurfaces in the preferred foliation
- ⌘• When the boost angle vanishes the aether is orthogonal to constant r hypersurfaces
- ⌘• Universal horizon!

- We have found a one parameter family of static, spherically symmetric, asymptotically flat black hole solutions with regular spin-0 horizons
- We have covered a significant portion of the allowed parameter space for both theories
- Deviations from general relativity are rather small. However, potentially observable with future observations
- A generic feature of these black holes seems to be the existence of an ultimate horizon (or a succession of them)
- What about stability and thermodynamics?