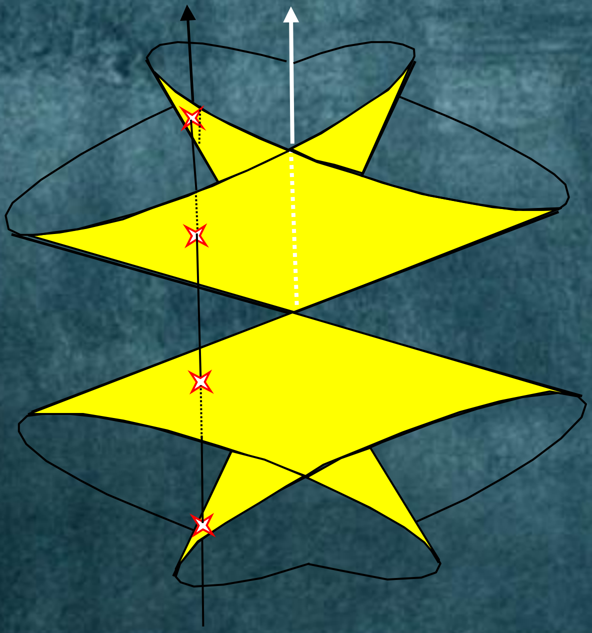
FREDERIC P. SCHULLER
Max Planck Institute for Gravitational Physics



Spacetimes beyond Einstein

Kavli Institute for the Physics and Mathematics of the Universe
Tokyo, Japan

SPACETIME

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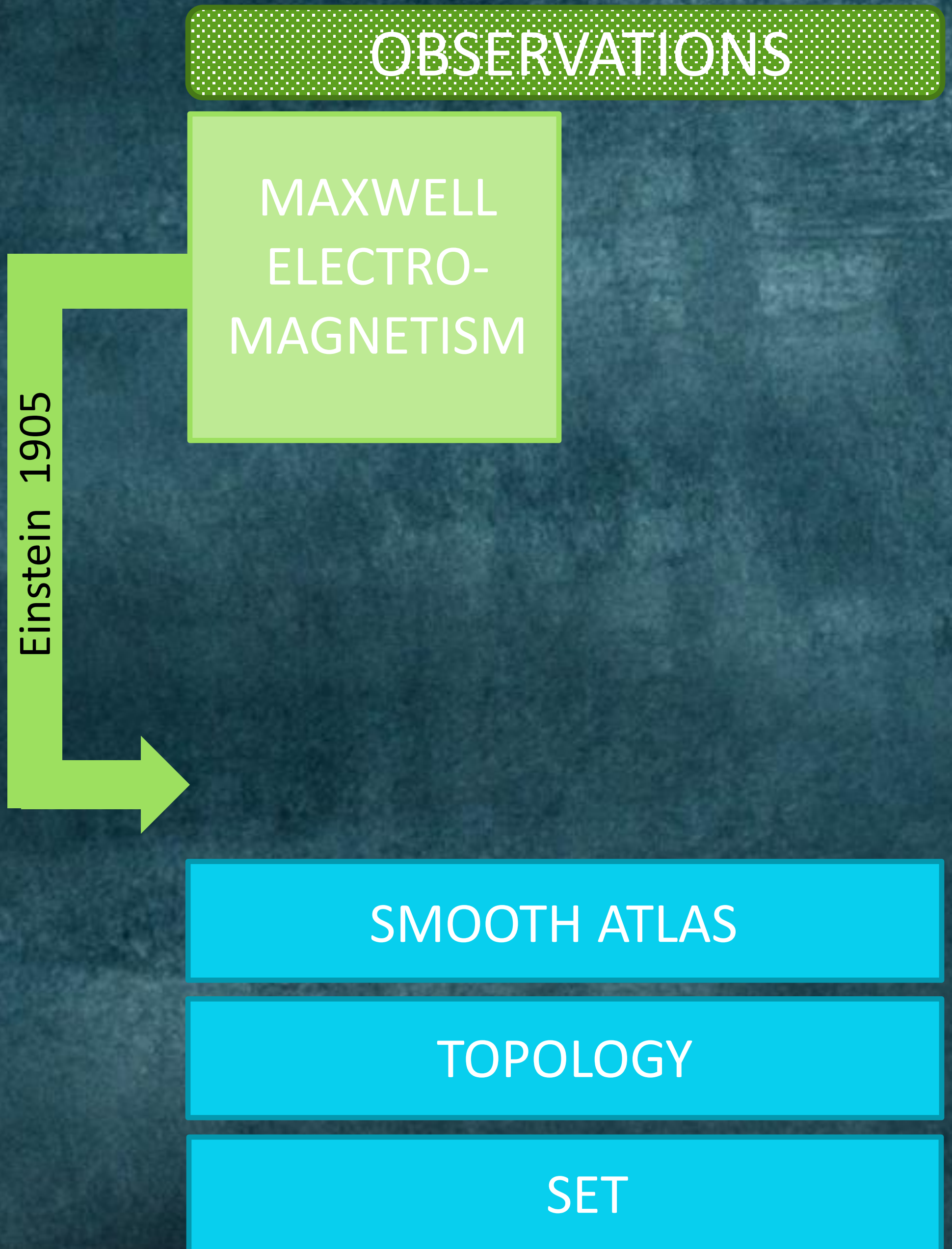
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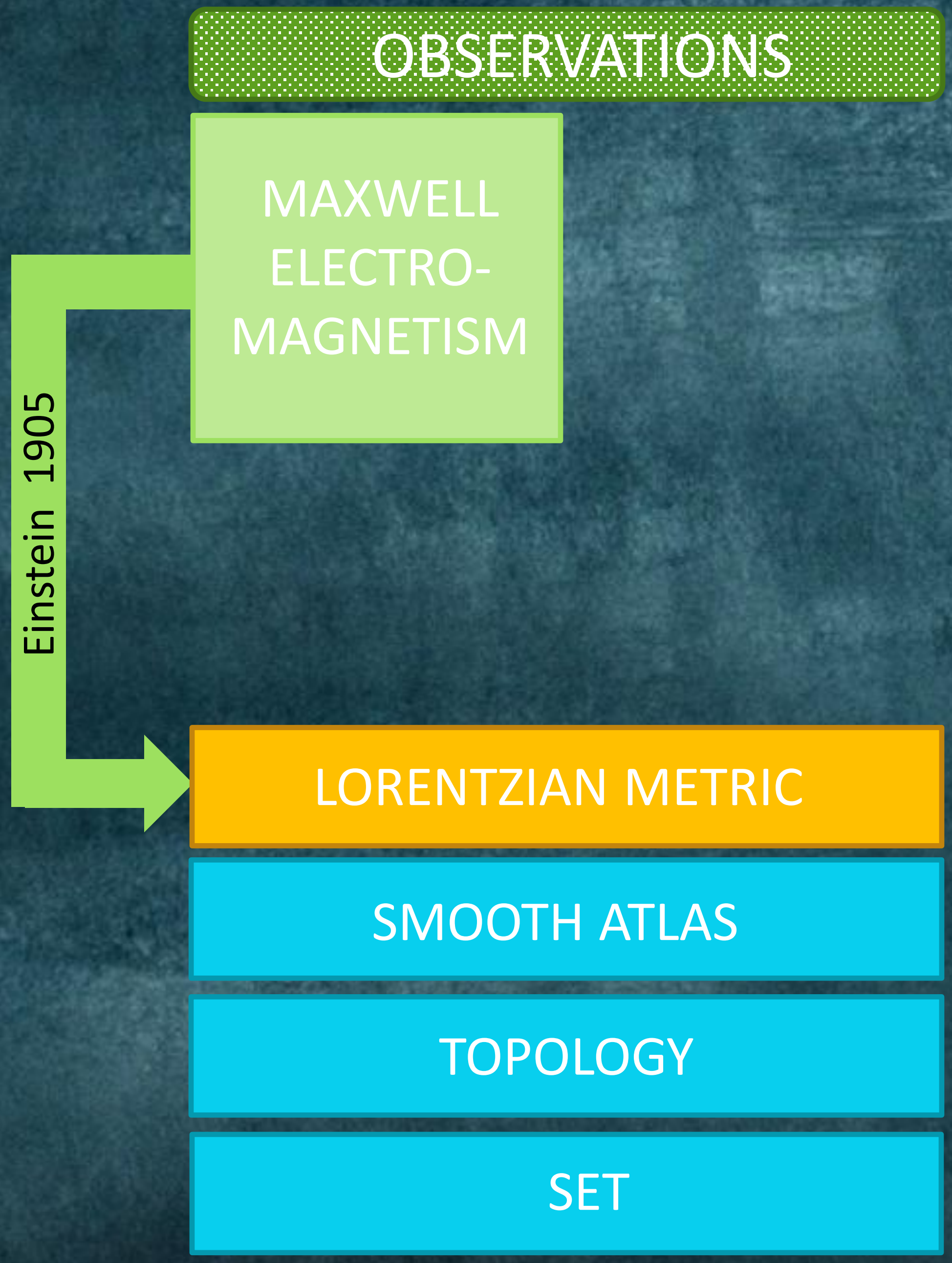
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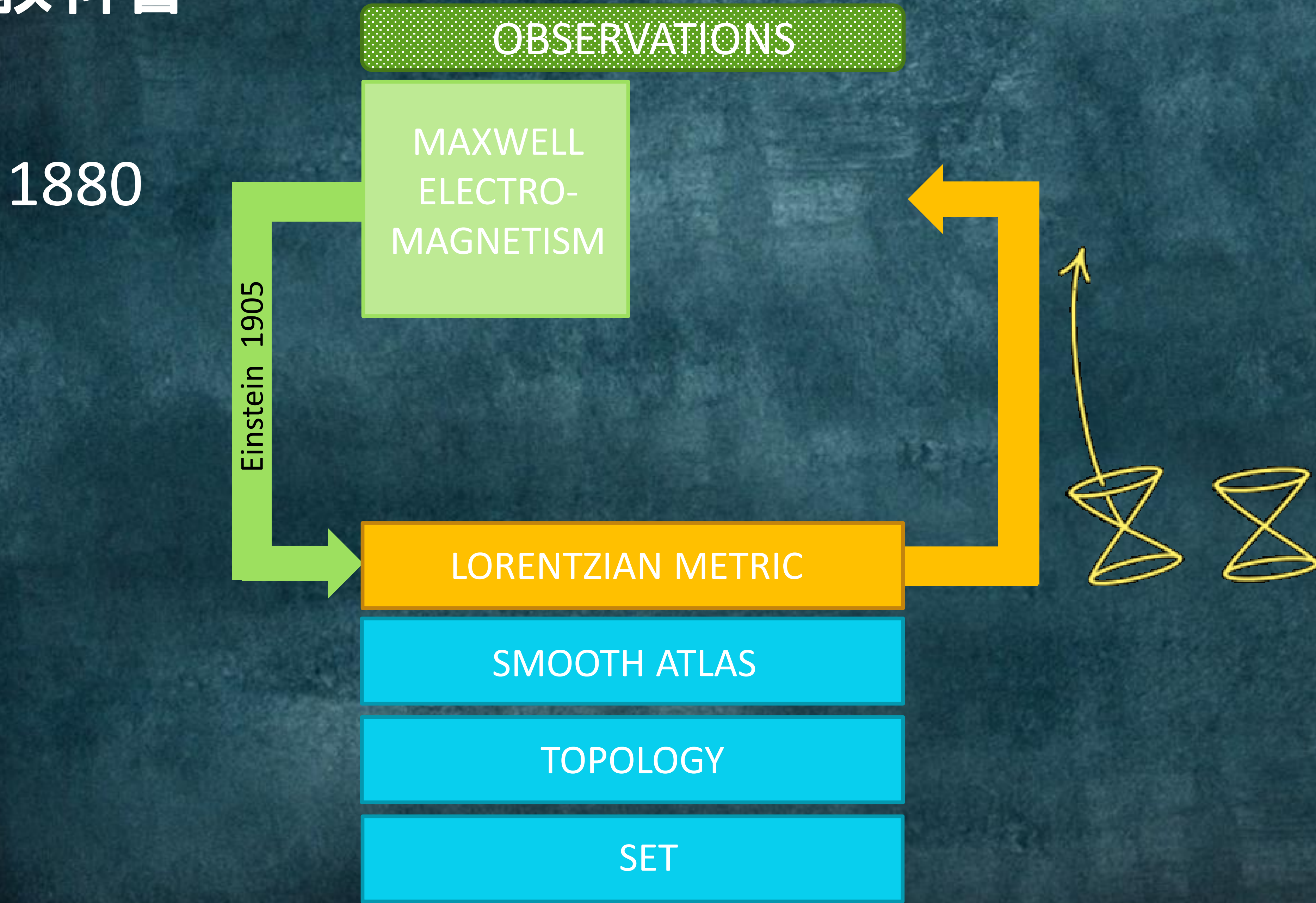

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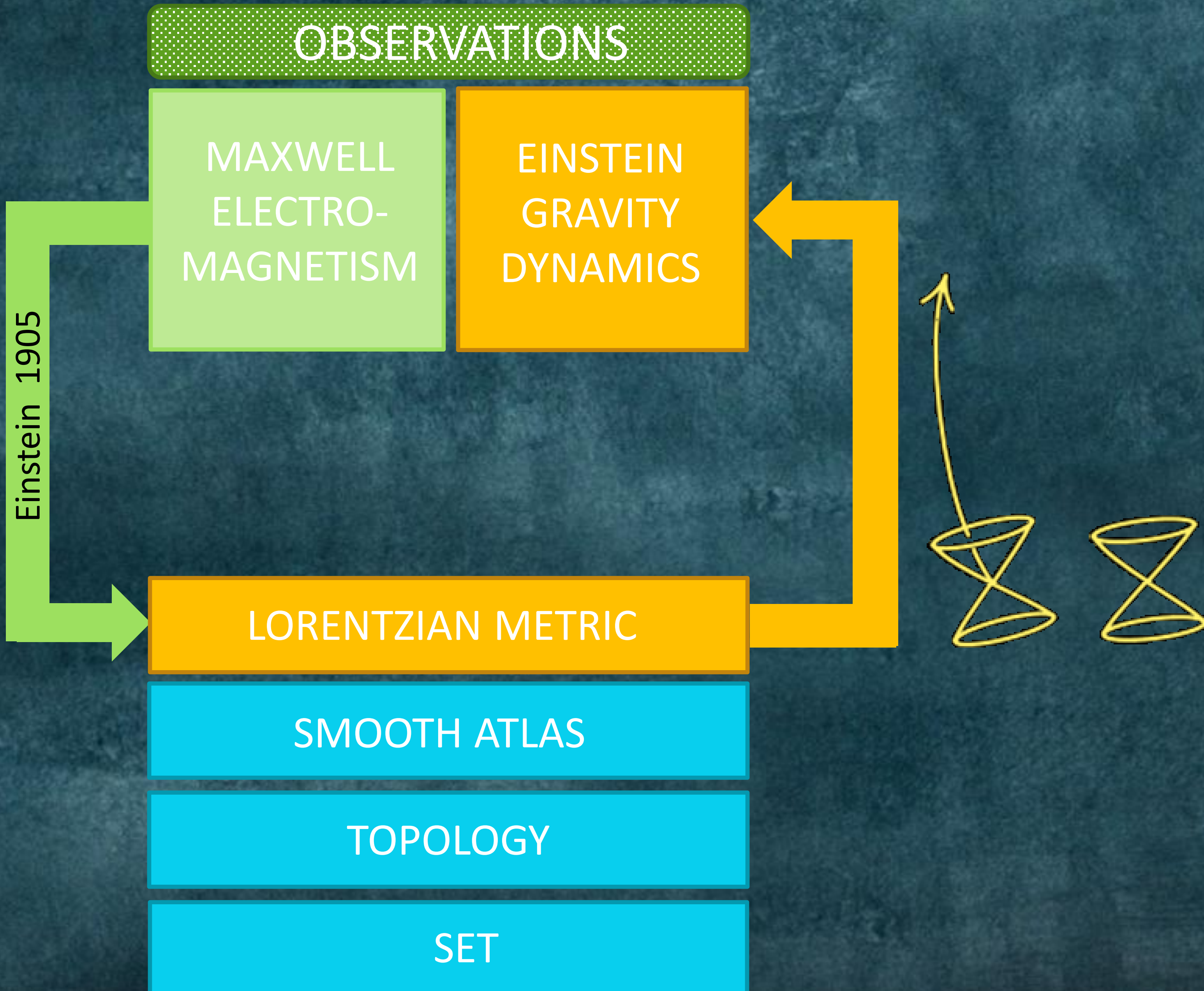

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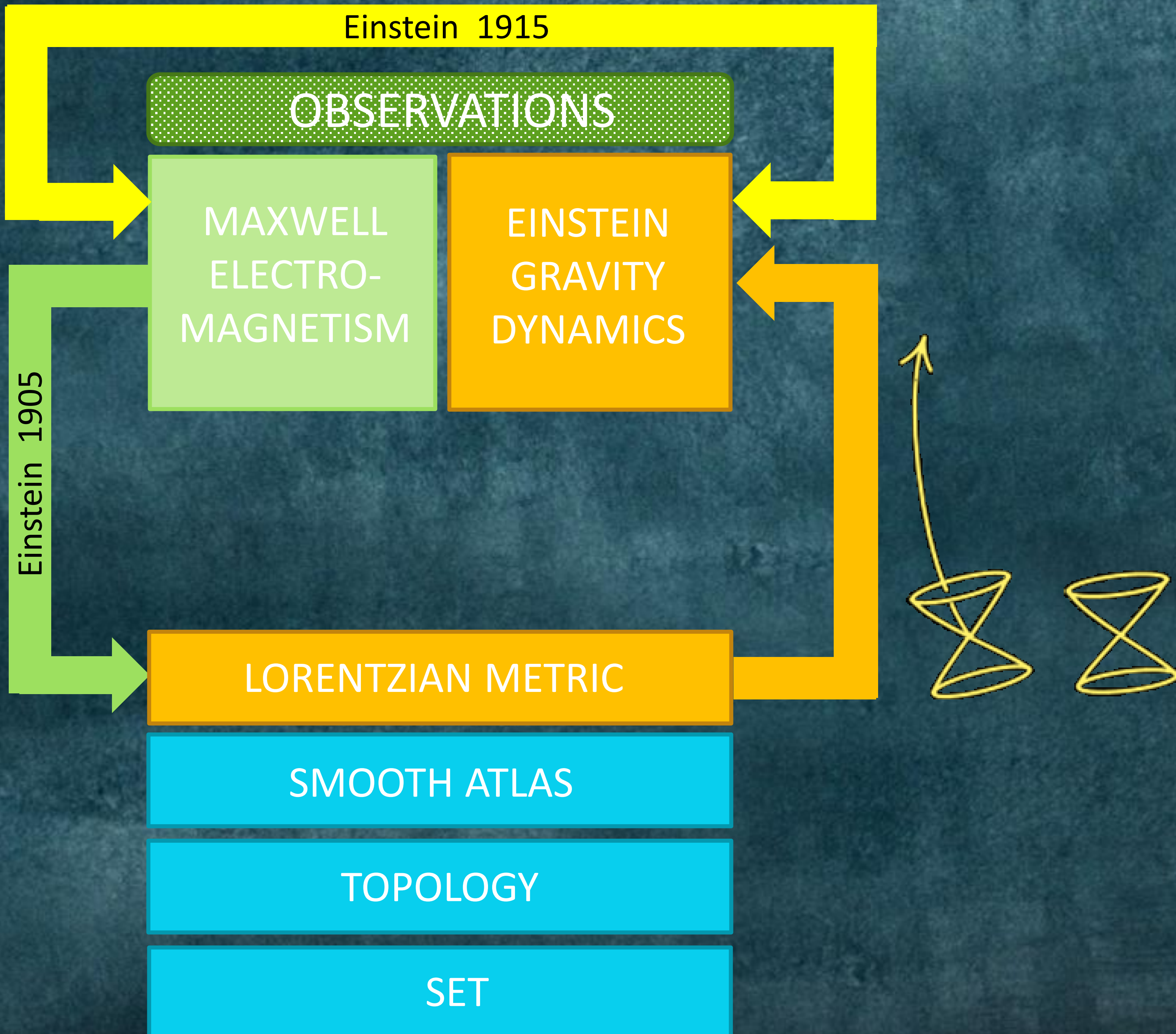
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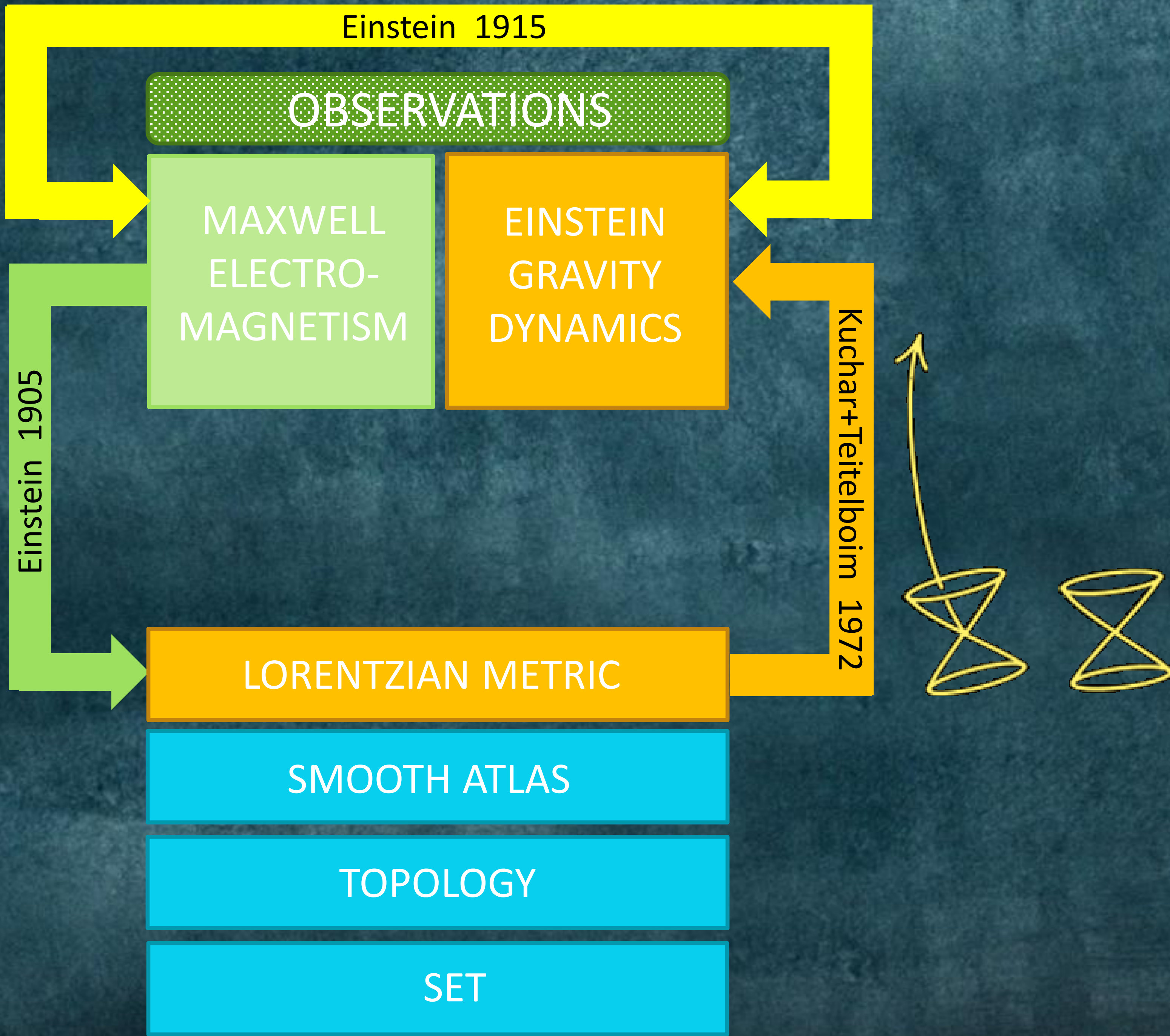

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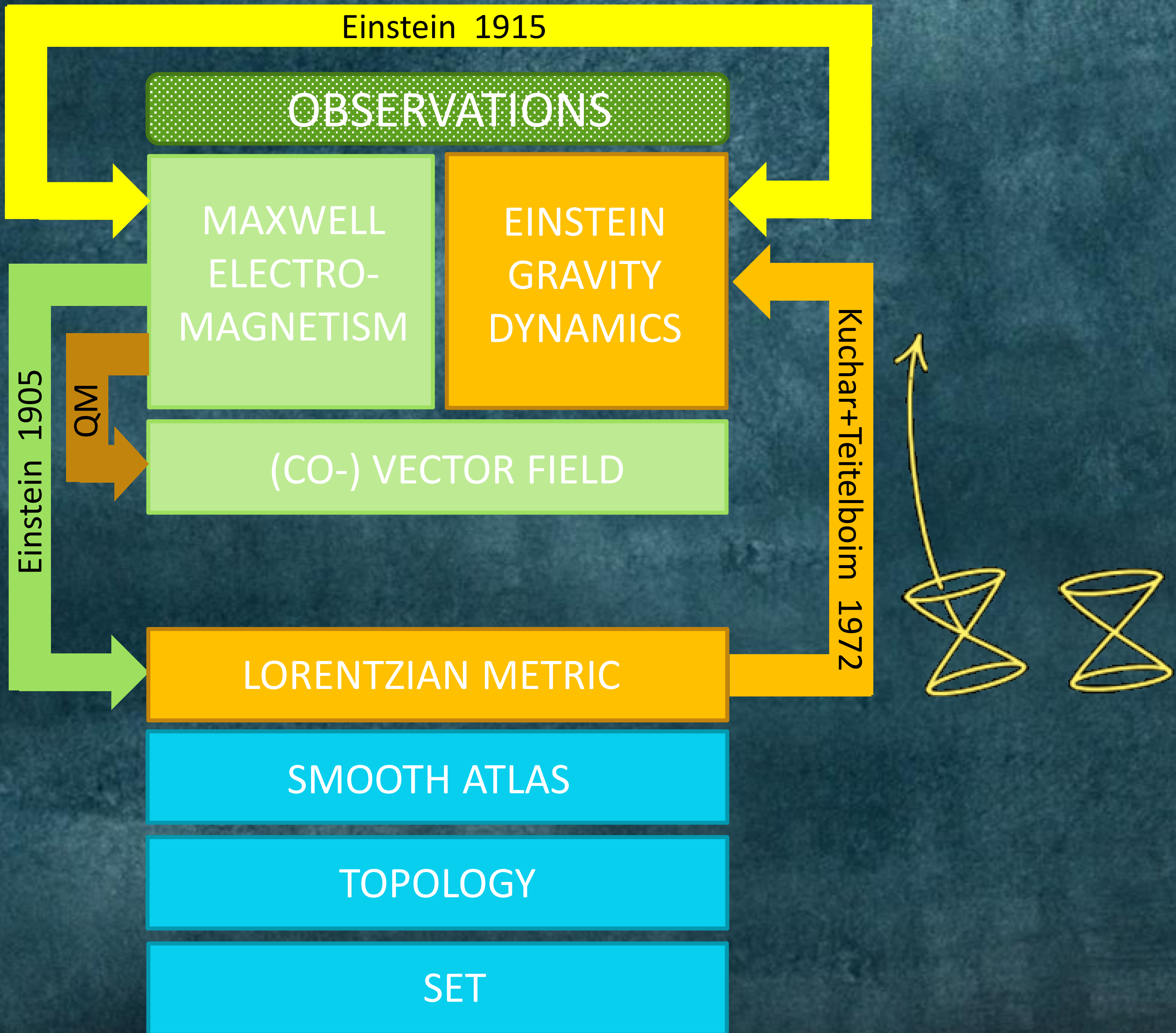

Einstein 1915
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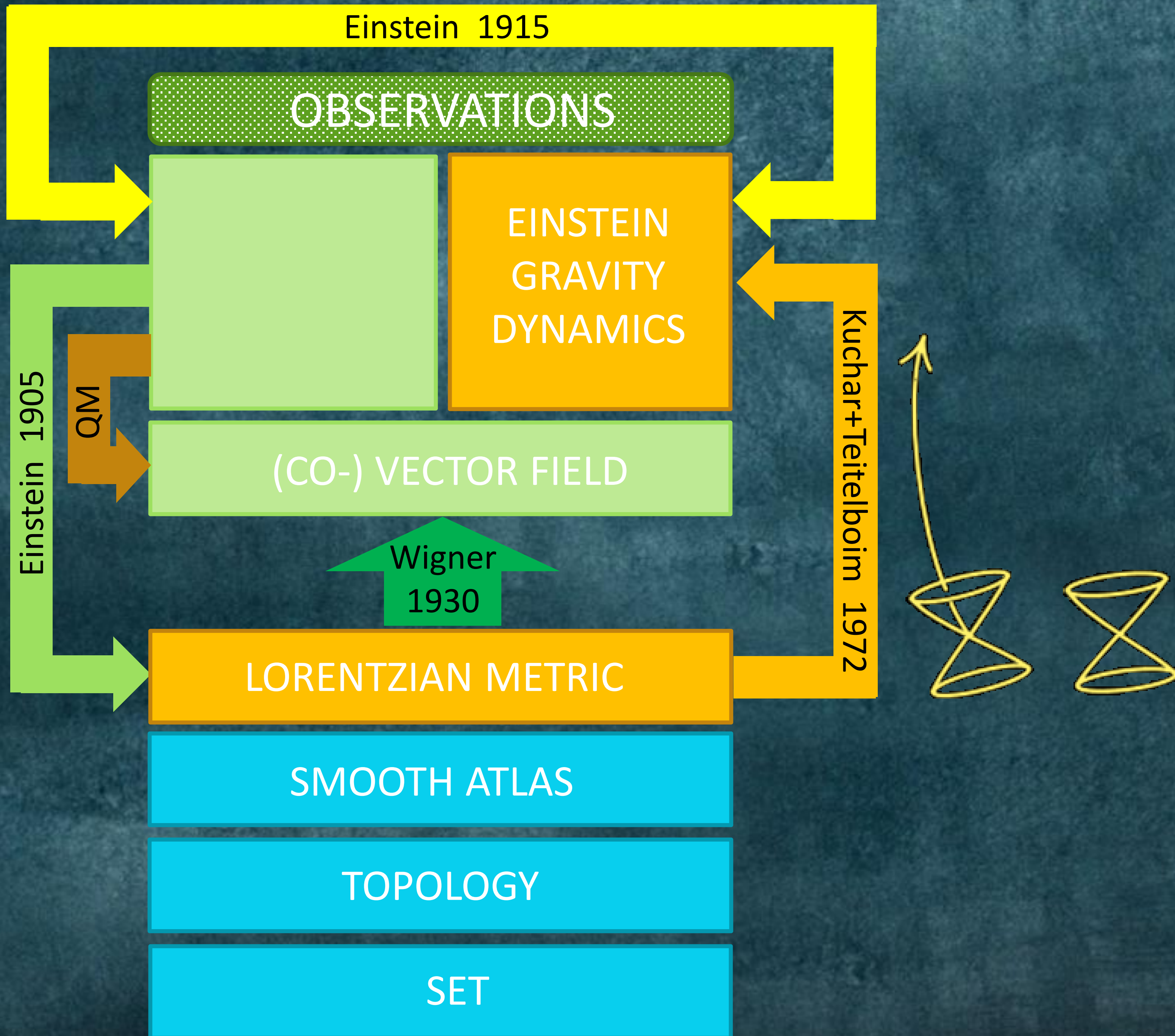

Einstein 1915
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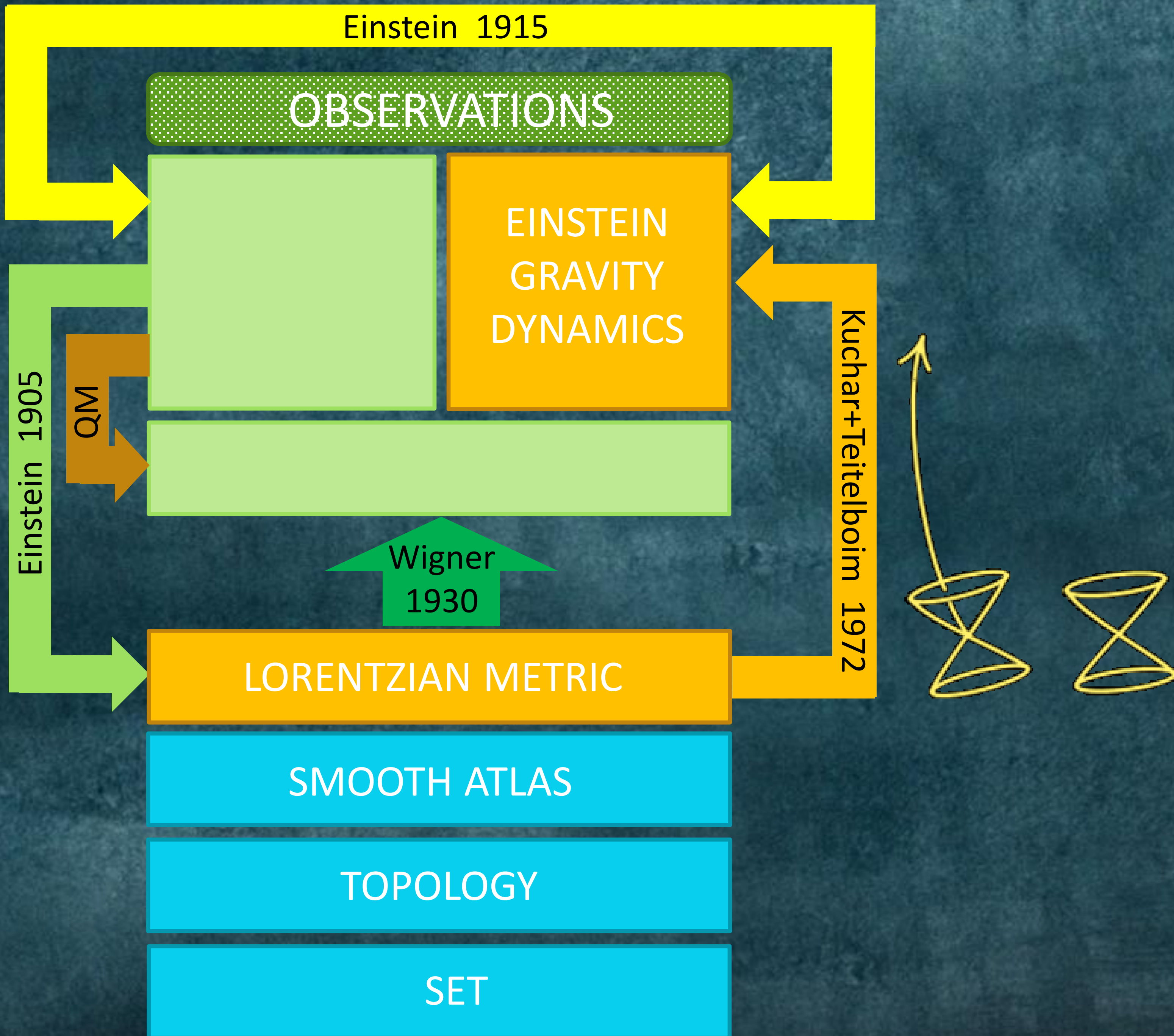
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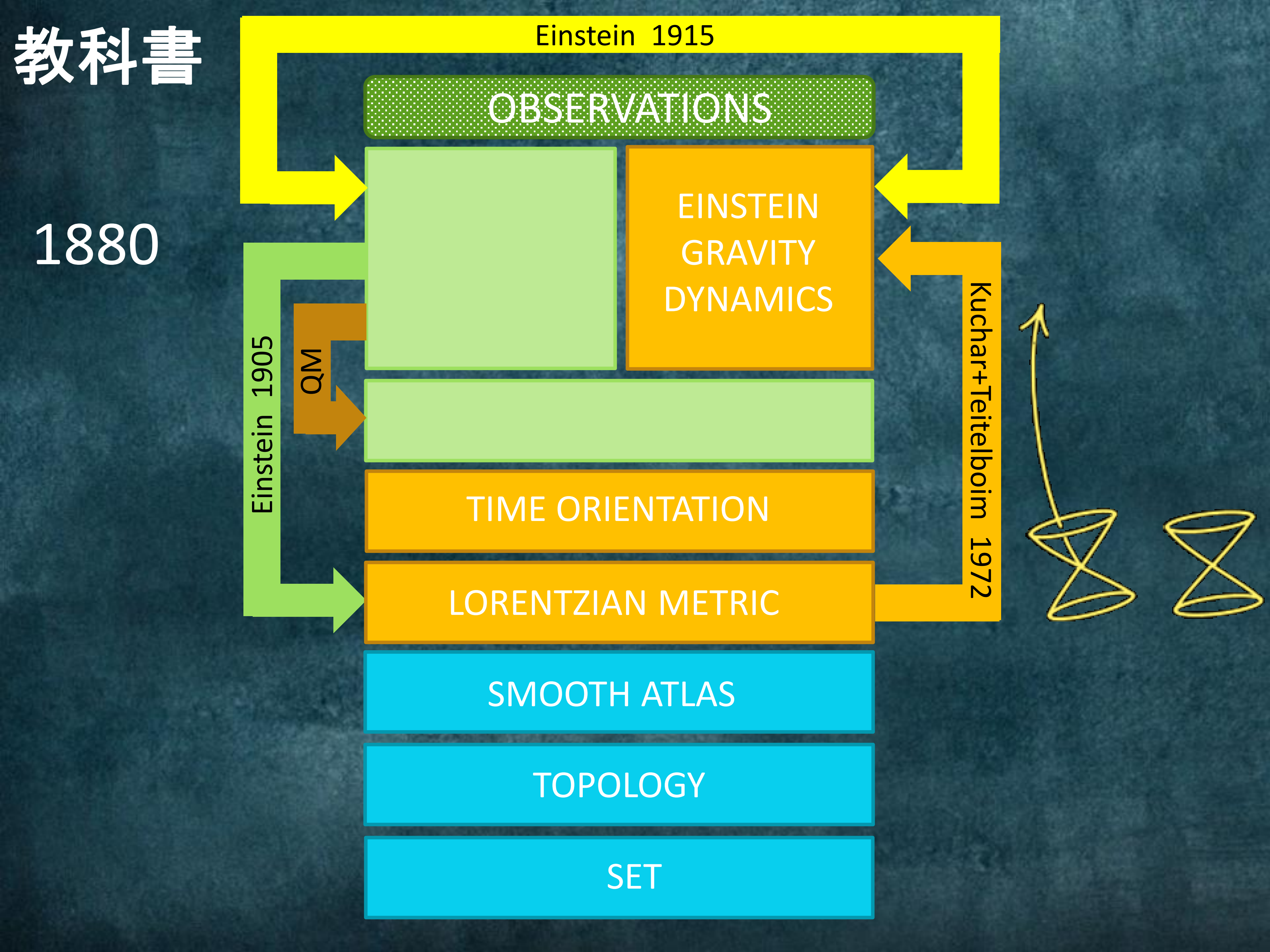

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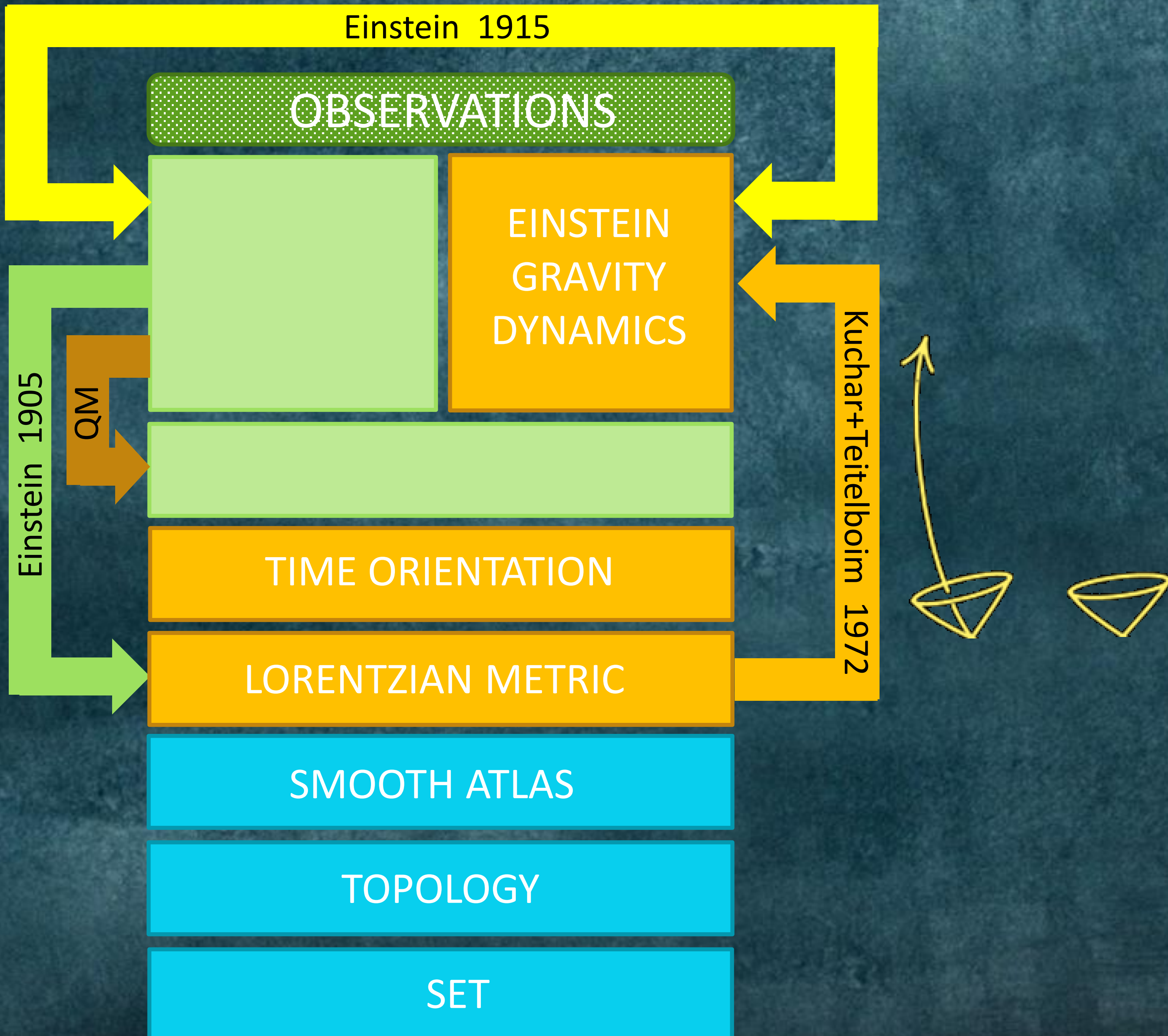
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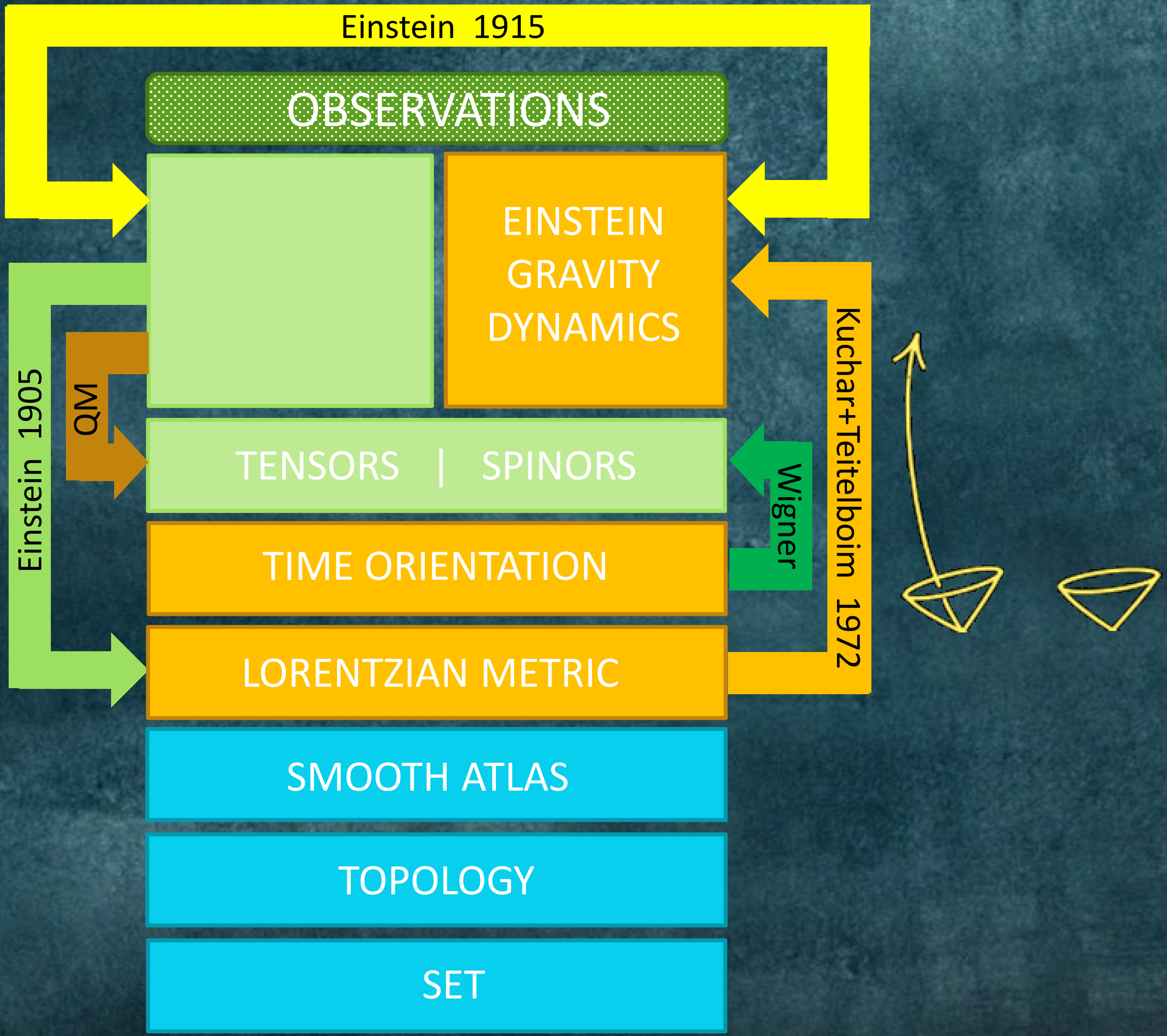
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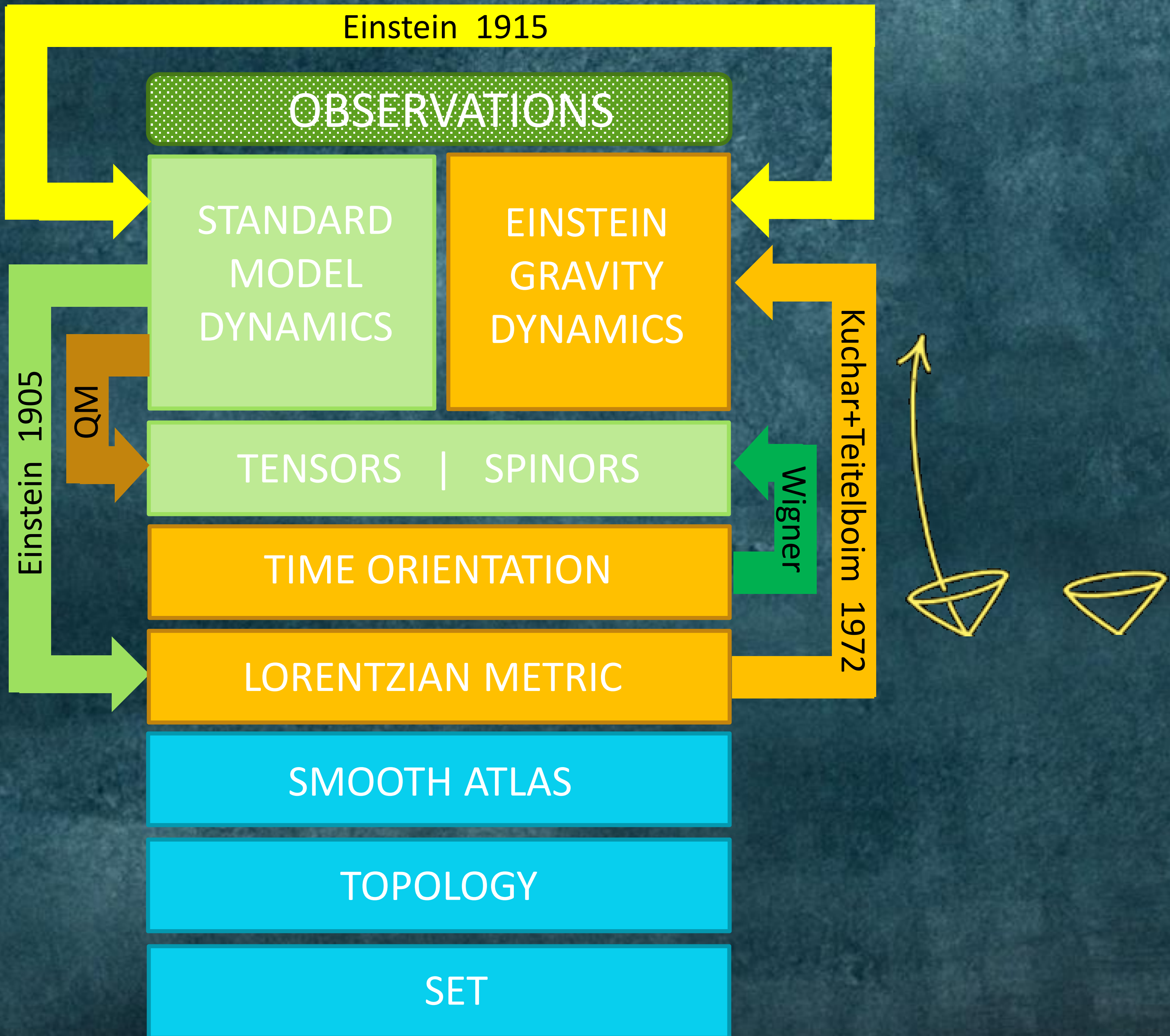
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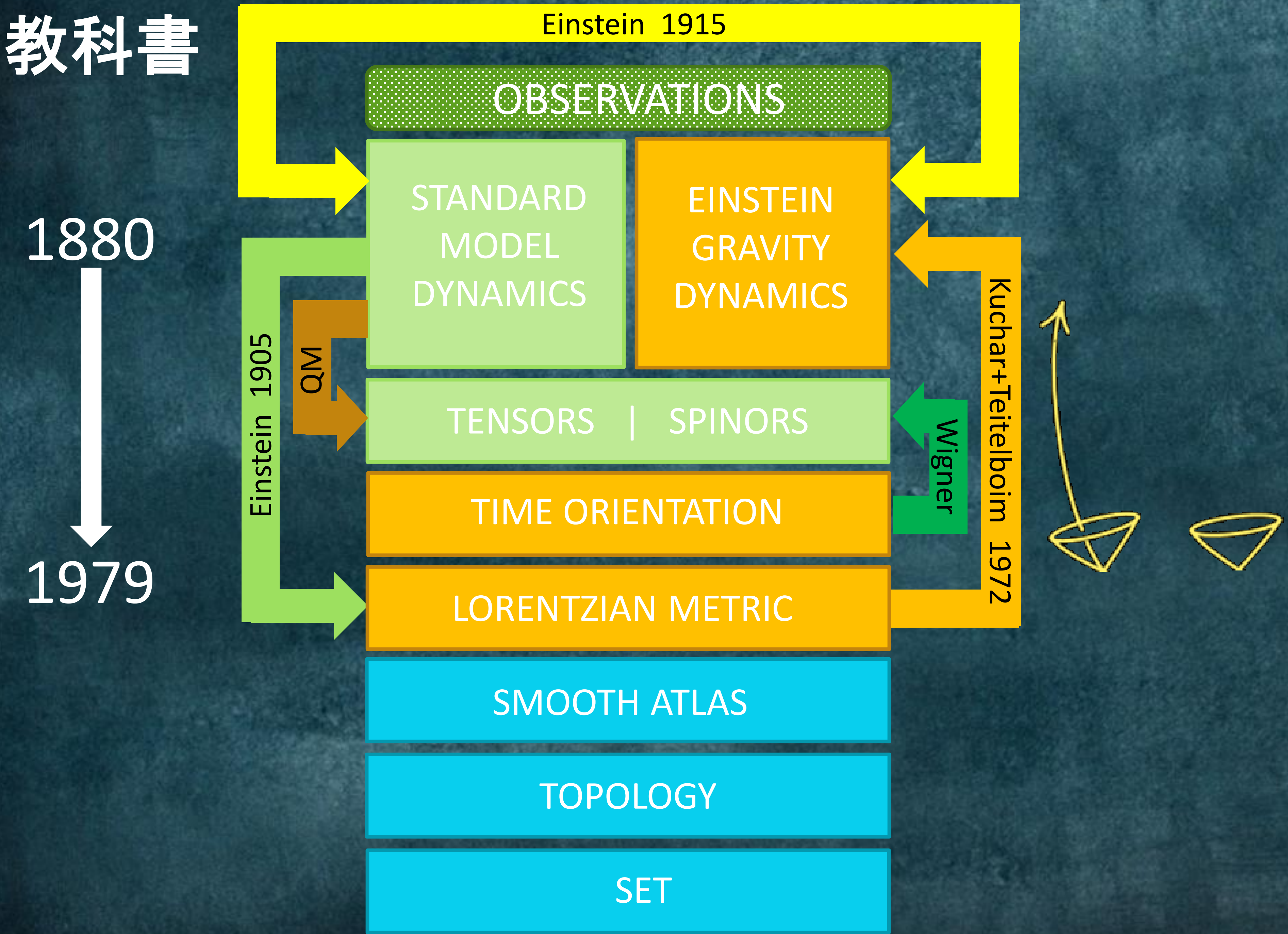
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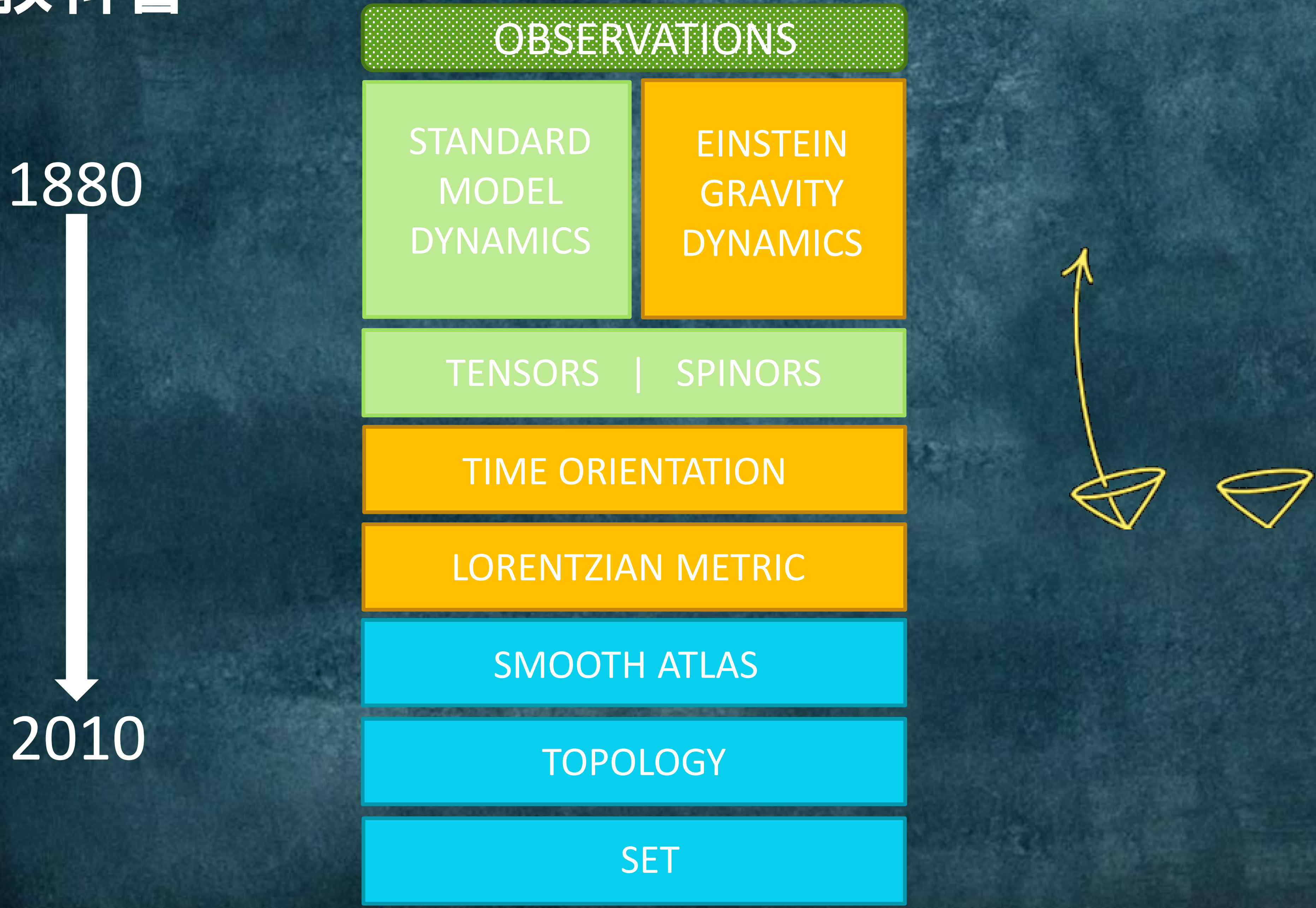
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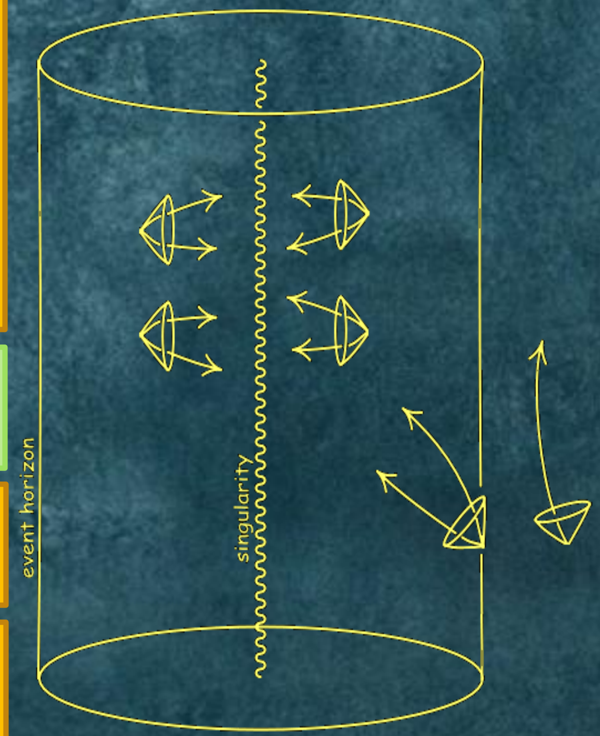

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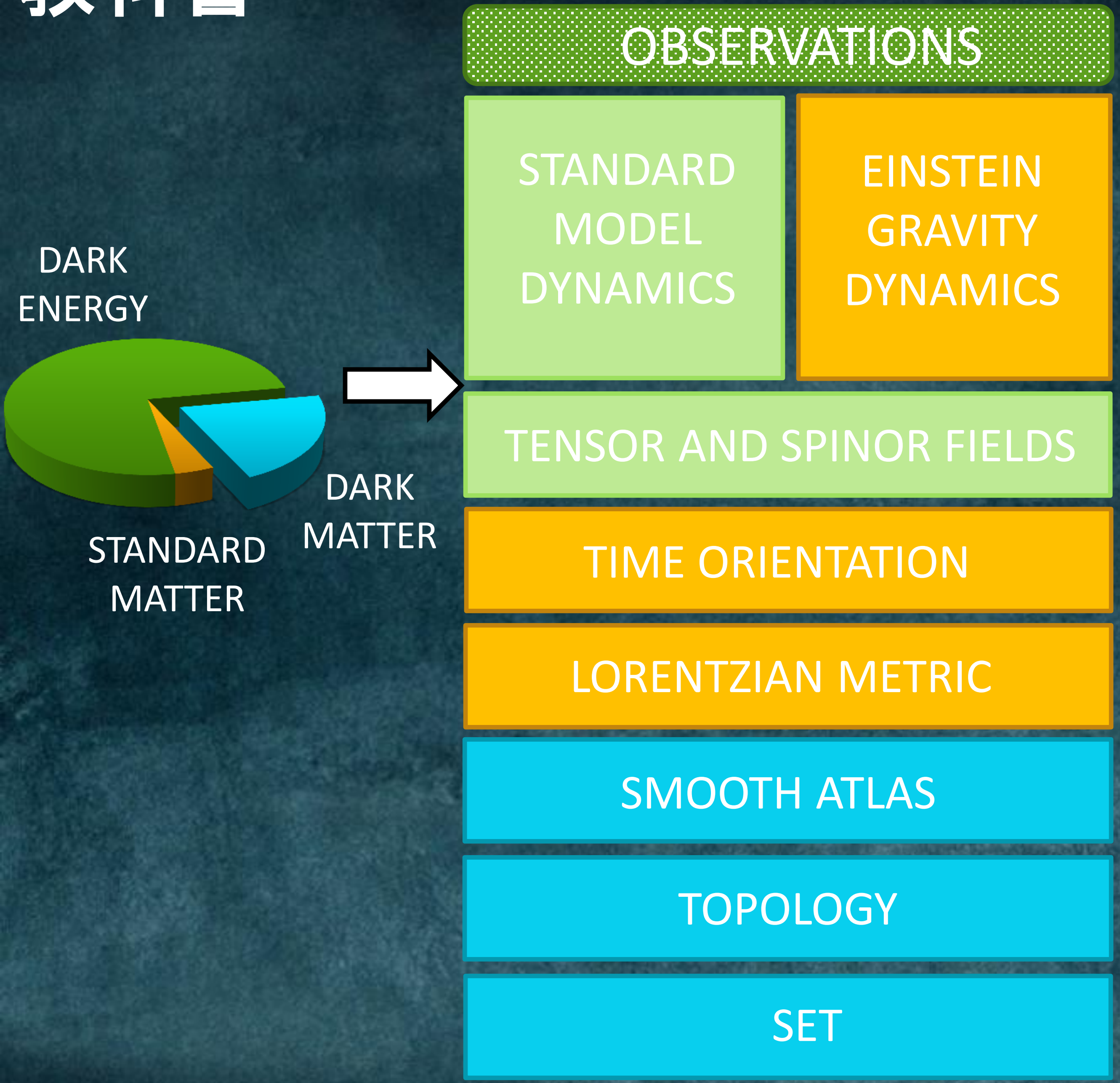
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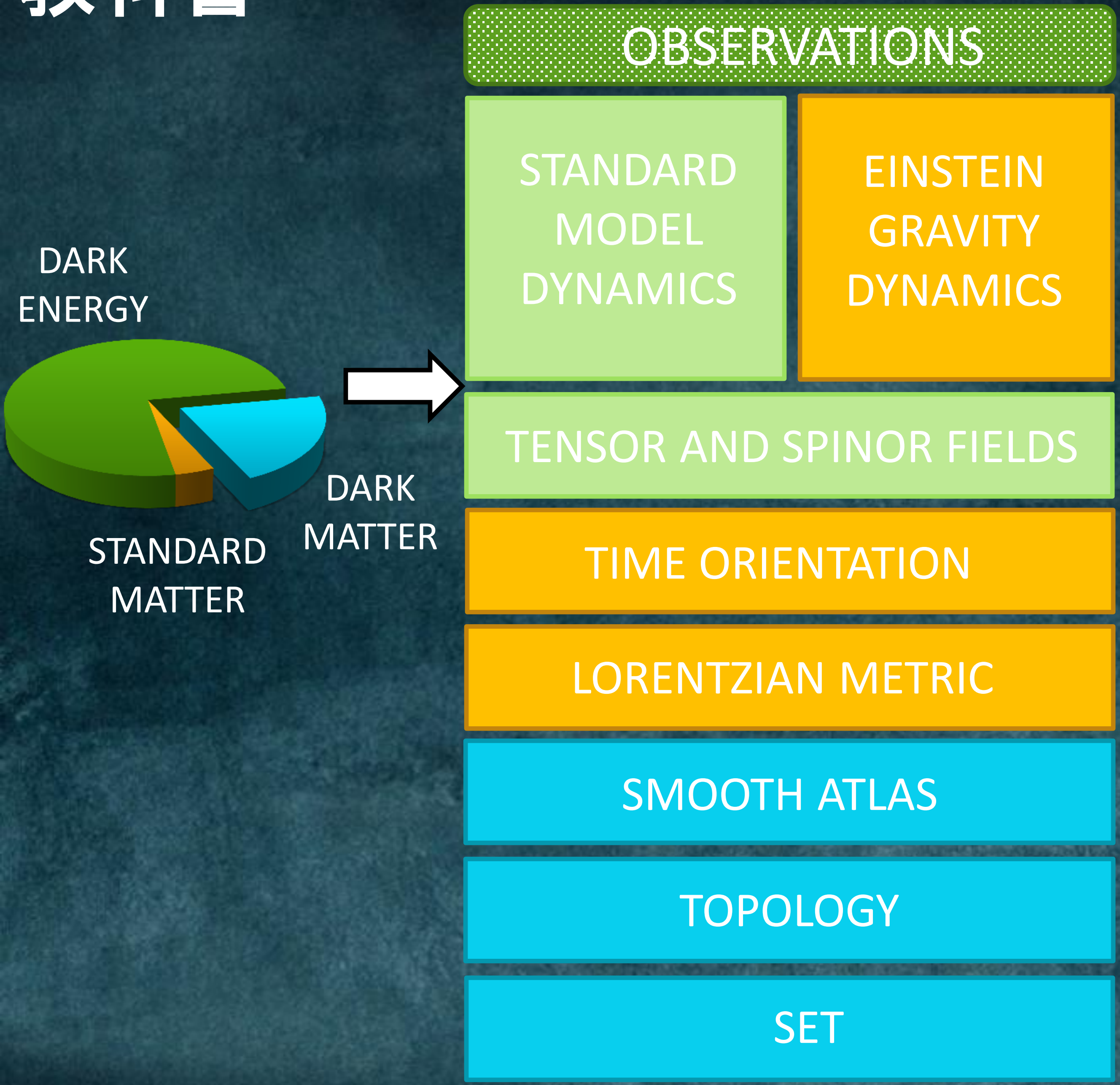

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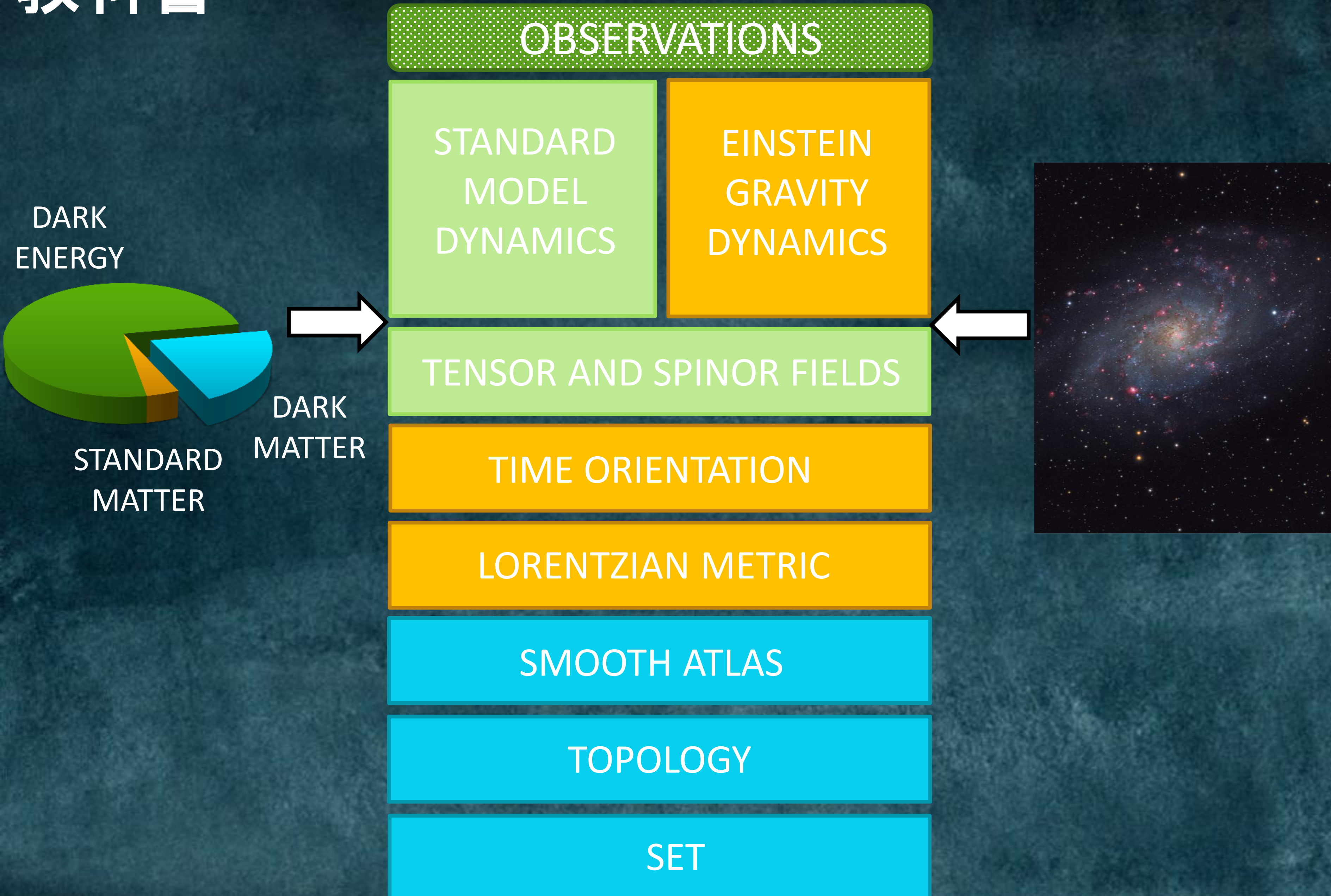

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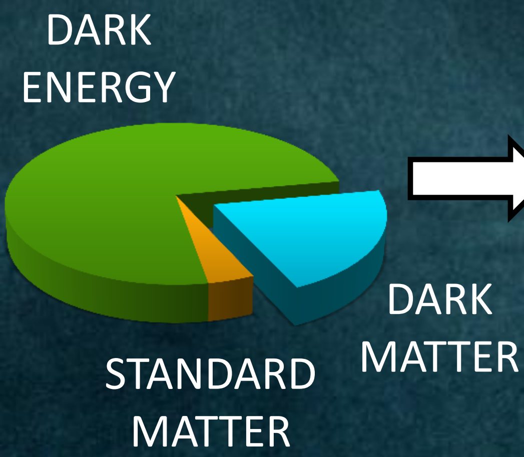
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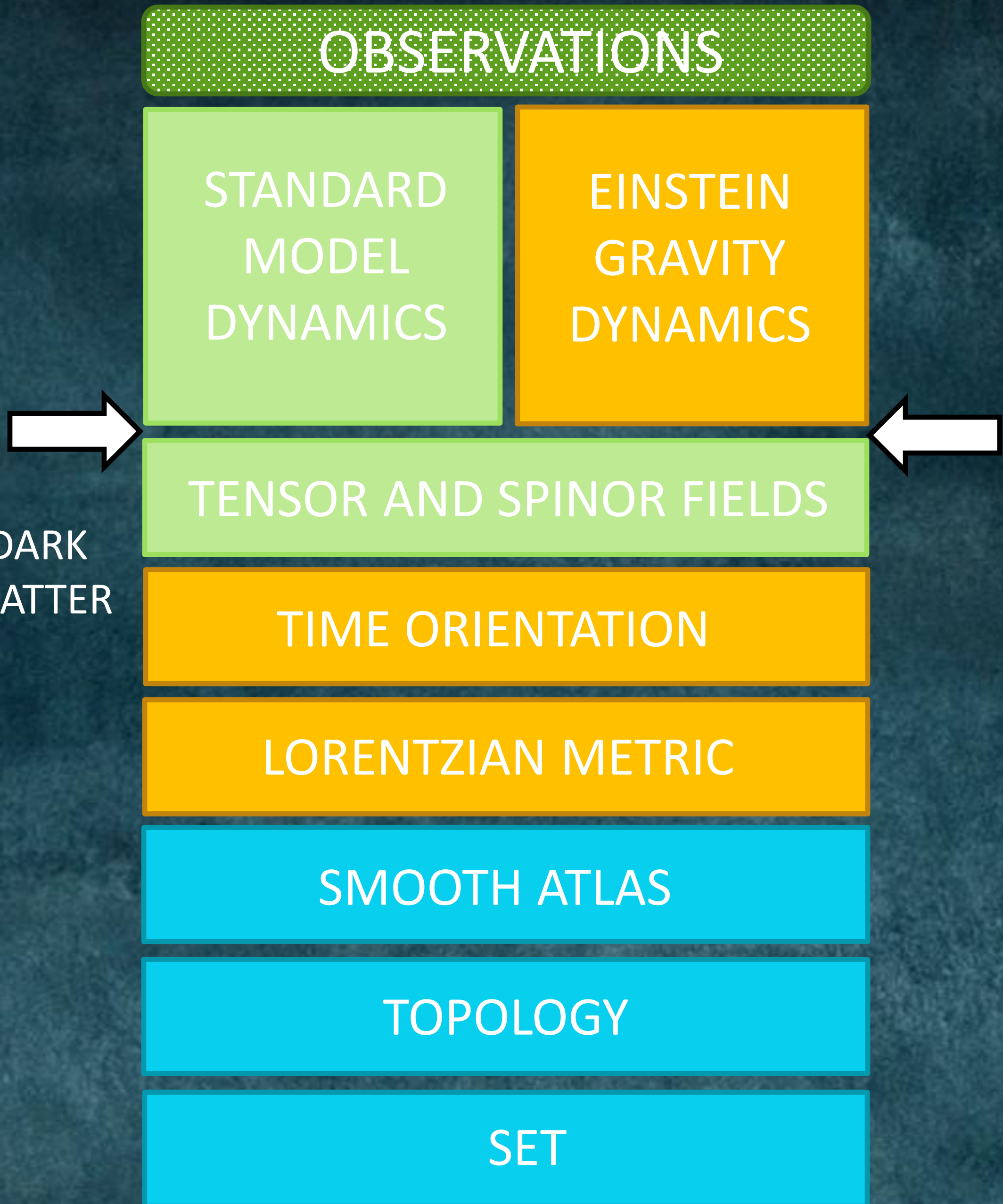
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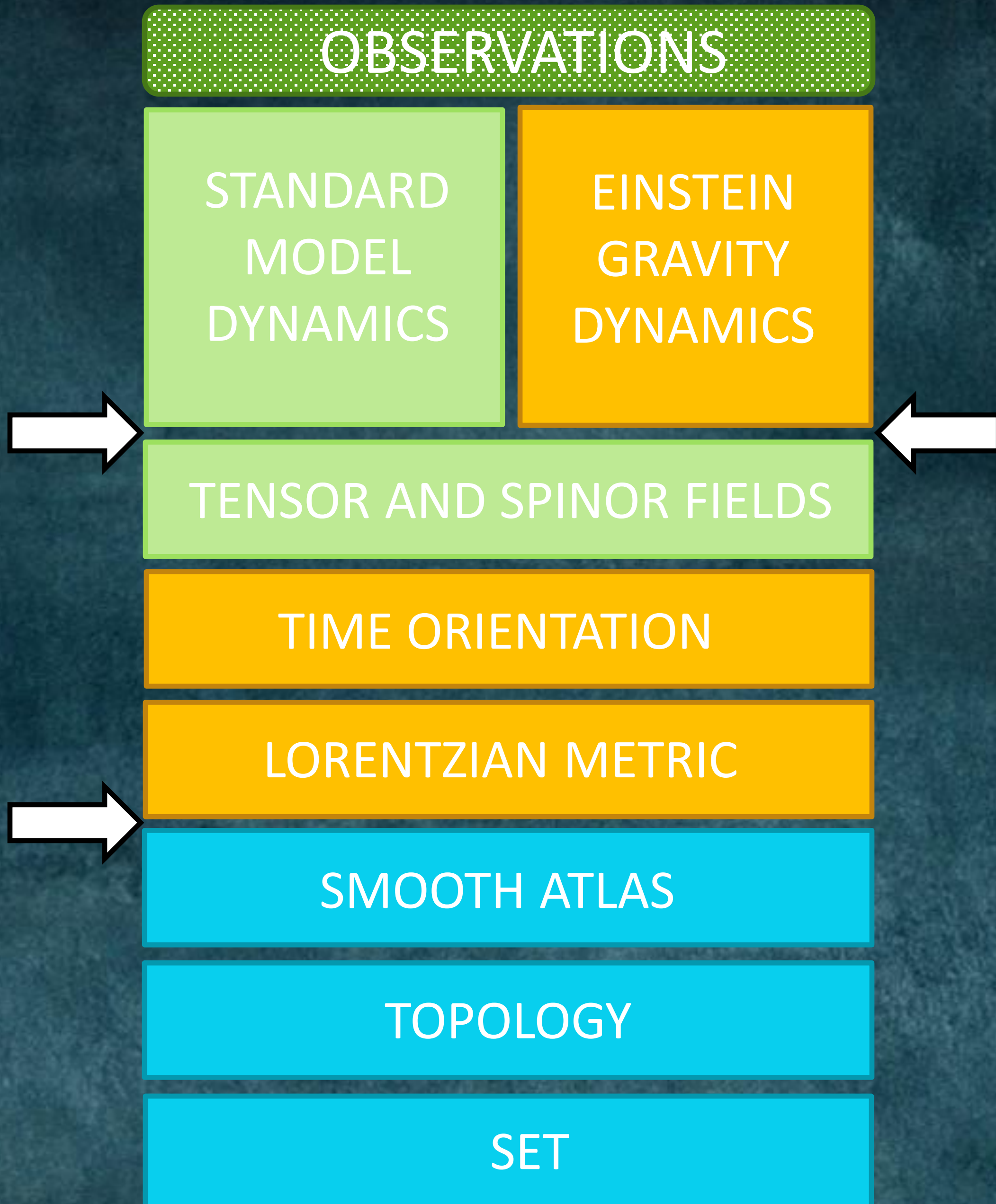
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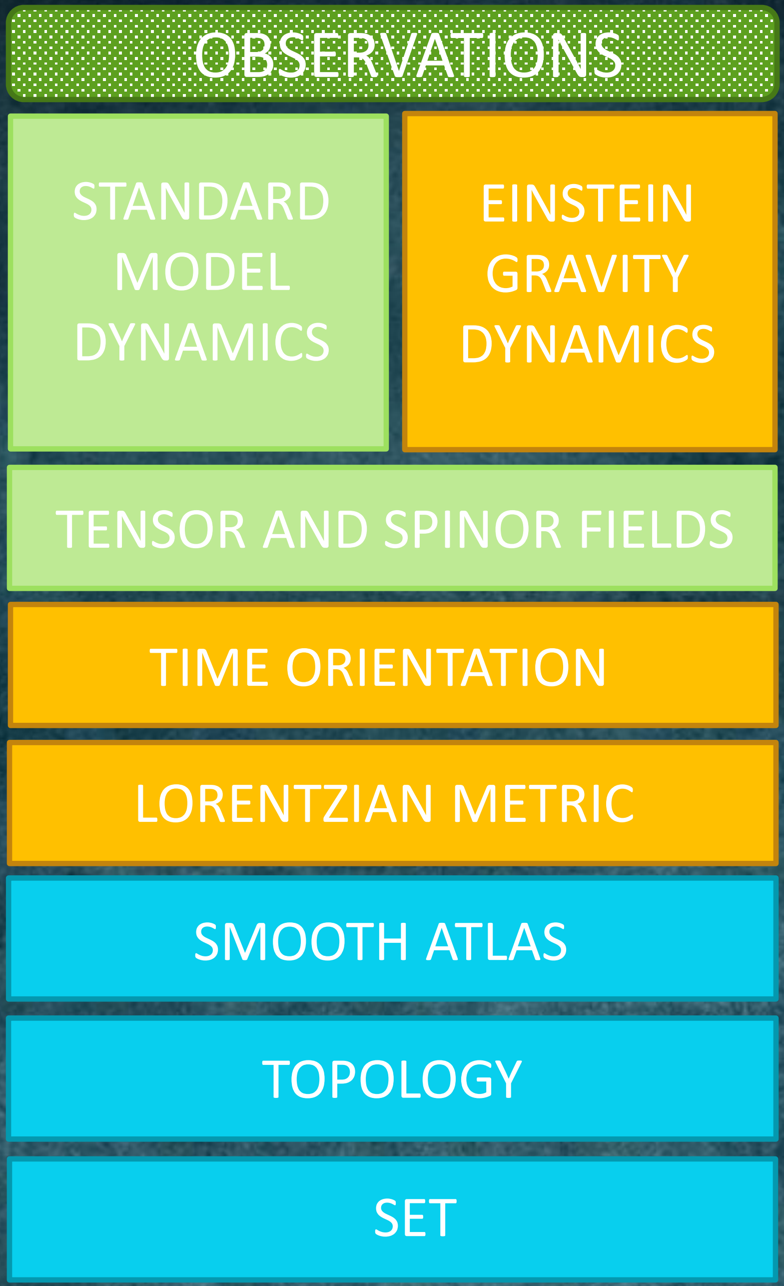
SPACETIME KINEMATICS

修正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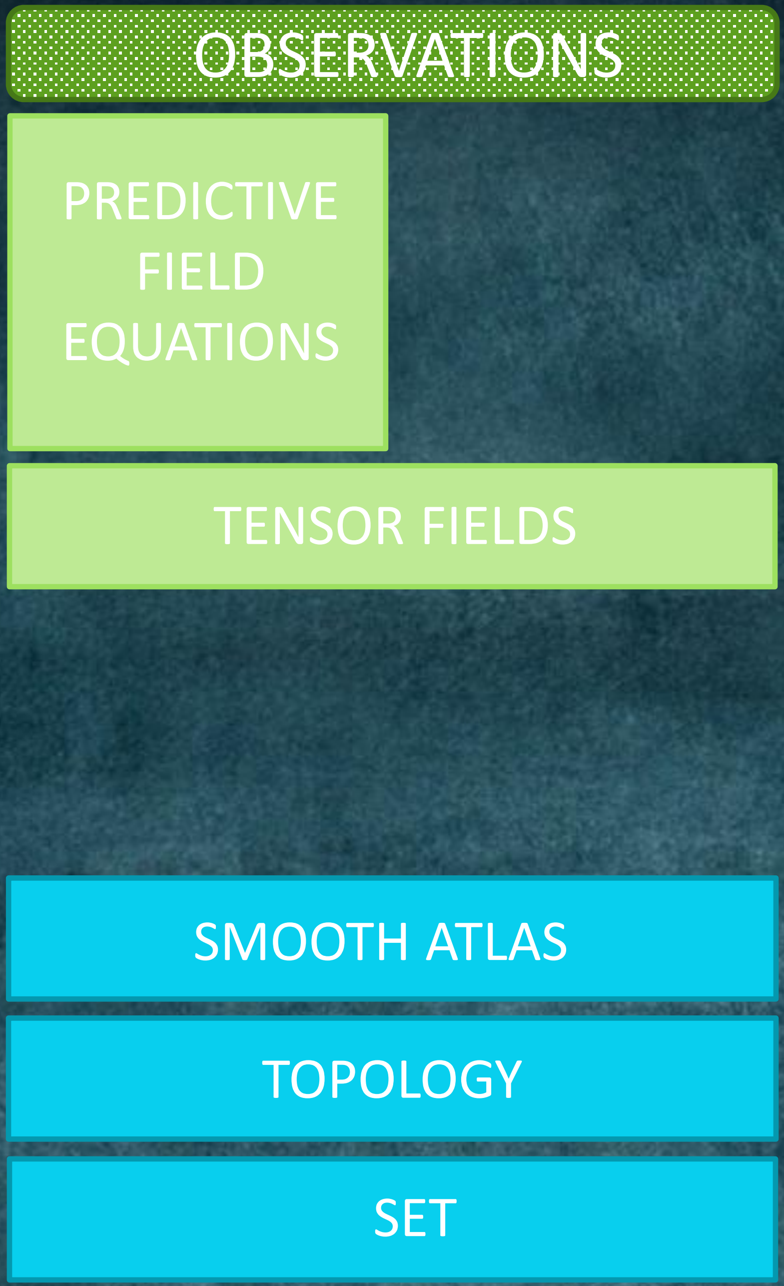
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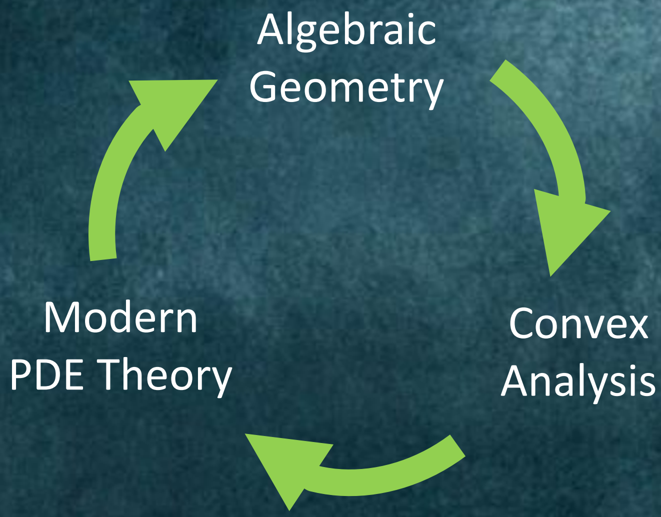

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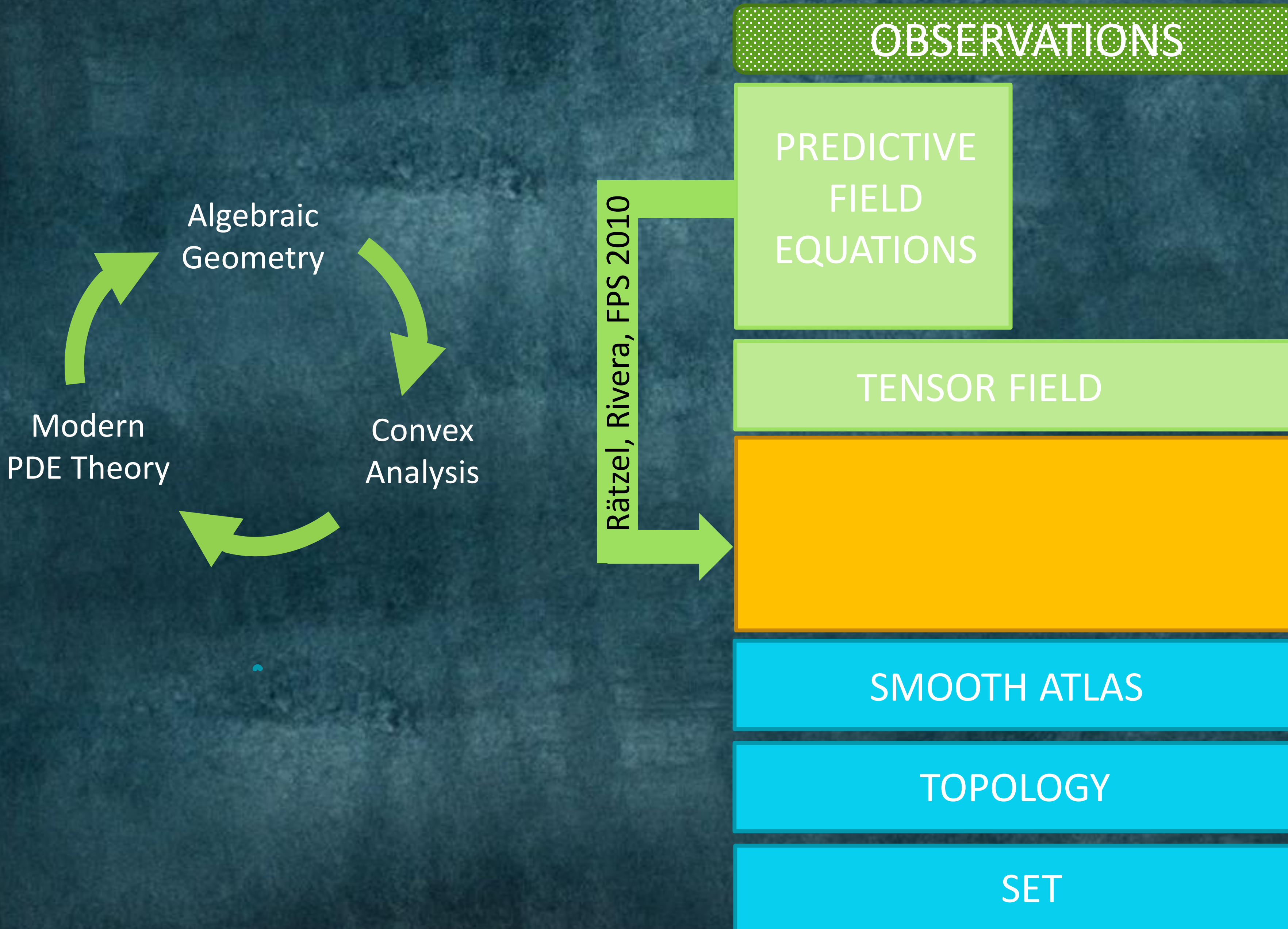
Algebraic Geometry
Convex Analysis
Modern PDE Theory
Rätzel, Rivera, FPS 2010
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$$\partial_{a_1} \ldots \partial_{a_n} \Phi = 0$$

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$$Q^{a_1 \dots a_n} \partial_{a_1} \dots \partial_{a_n} \Phi = 0$$

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$$\sum_{n=0}^{N} Q^{a_1 \ldots a_n} \partial_{a_1} \ldots \partial_{a_n} \Phi = 0$$

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$G^{\cdot\cdot}{}_{\dots}$

$$\sum_{n=0}^{N} Q^{a_1 \dots a_n} \partial_{a_1} \dots \partial_{a_n} \Phi = 0$$

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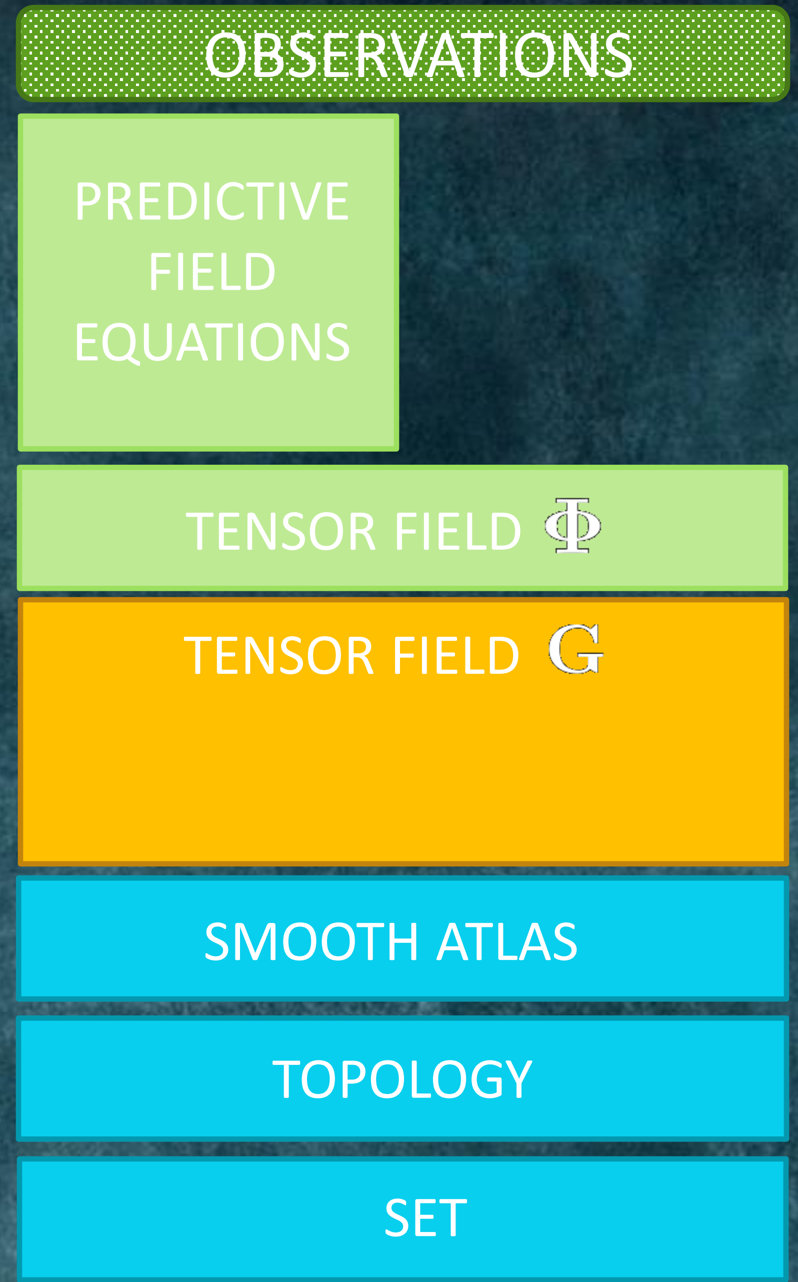
$$G^{\cdot\cdot}_{\ \ \cdots}$$

$$\sum_{n=0}^{N} Q^{a_1 \dots a_n} \partial_{a_1} \dots \partial_{a_n} \Phi = 0$$

$$Q^{a_1 \dots a_N}$$



$$G^{\cdot\cdot}{}_{\dots}$$

$$\sum_{n=0}^{N} Q^{a_1\dots a_n}\,\partial_{a_1}\dots\partial_{a_n}\,\Phi = 0$$

$$P(k) = Q^{a_1\dots a_N}\,k_{a_1}\dots k_{a_N}$$

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$$G^{\cdot\cdot}{}_{\dots}$$

$$\sum_{n=0}^{N} Q_{AB}^{a_1\dots a_n}\, \partial_{a_1}\dots\partial_{a_n}\Phi^B = 0$$

$$P(k) = Q^{a_1\dots a_N} k_{a_1}\dots k_{a_N}$$

$$G^{\cdot\cdot}{}_{\dots}$$

$$\sum_{n=0}^{N} Q_{AB}^{a_1\dots a_n}\,\partial_{a_1}\dots\partial_{a_n}\,\Phi^B = 0$$

$$P(k) = \det\left[Q^{a_1\dots a_N}k_{a_1}\dots k_{a_N}\right]$$

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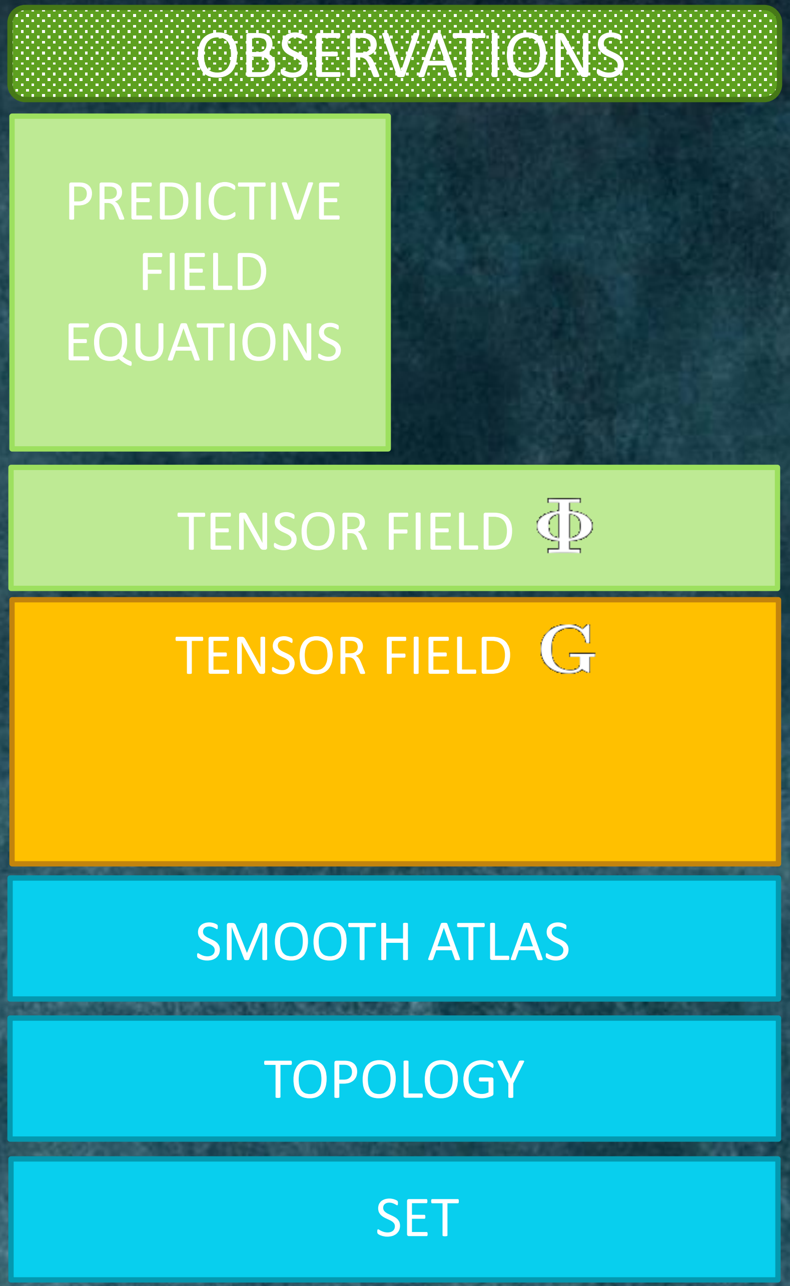
$$G^{\cdot\cdot}{}_{\dots}$$

$$\sum_{n=0}^{N} Q_{AB}^{a_1\dots a_n}\,\partial_{a_1}\dots\partial_{a_n}\,\Phi^B = 0$$

predictive

$$P(k) = \det\left[Q^{a_1\dots a_N} k_{a_1}\dots k_{a_N}\right]$$



$G^{\cdot\cdot}{}_{\dots}$

$$\sum_{n=0}^{N} Q_{AB}^{a_1 \dots a_n} \, \partial_{a_1} \dots \partial_{a_n} \Phi^B = 0$$

predictive

$\Downarrow$ [Hörmander]

$$P(k) = \det \left[Q^{a_1 \dots a_N} k_{a_1} \dots k_{a_N} \right]$$

hyperbolic polynomial

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$$\sum_{n=0}^{N} Q^{a_1\ldots a_n}_{AB}\, \partial_{a_1}\ldots\partial_{a_n}\Phi^B = 0$$

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hyperbolic polynomial

Example: Klein-Gordon

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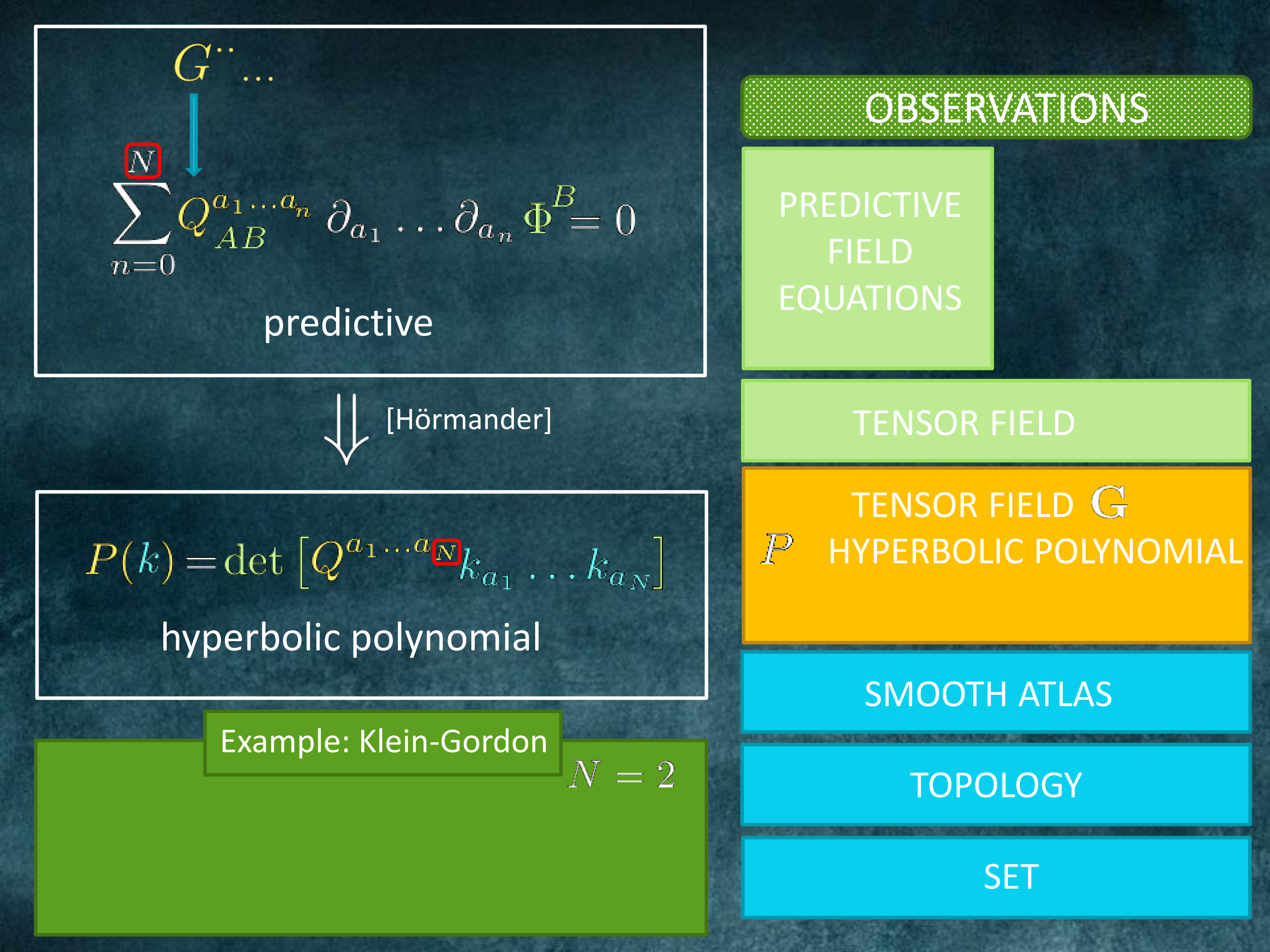
$G^{\cdot\cdot}{}_{\cdots}$
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predictive
[Hörmander]
$$P(k) = \det\left[Q^{a_1\dots a_N} k_{a_1}\dots k_{a_N}\right]$$
hyperbolic polynomial
Example: Klein-Gordon
$N = 2$
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$G^{\cdot\cdot}{}_{\dots}$

$$\sum_{n=0}^{N} Q^{a_1 \dots a_n}_{AB}\, \partial_{a_1} \dots \partial_{a_n} \Phi^B = 0$$

predictive

$\Downarrow$ [Hörmander]

$$P(k) = \det \left[Q^{a_1 \dots a_N} k_{a_1} \dots k_{a_N} \right]$$

hyperbolic polynomial

Example: Klein-Gordon

$$N = 2$$
$$G^{ab} = g^{ab}$$

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$$G^{\cdot\cdot}\ldots$$

$$\sum_{n=0}^{N} Q_{AB}^{a_1\ldots a_n}\,\partial_{a_1}\ldots\partial_{a_n}\Phi^B = 0$$

predictive

$\Downarrow$ [Hörmander]

$$P(k) = \det\left[Q^{a_1\ldots a_N} k_{a_1}\ldots k_{a_N}\right]$$

hyperbolic polynomial

Example: Klein-Gordon

$$N = 2$$
$$G^{ab} = g^{ab}$$

$$Q^{a_1 a_2} = g^{a_1 a_2}$$

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$$\sum_{n=0}^{N} Q_{AB}^{a_1\dots a_n}\,\partial_{a_1}\dots\partial_{a_n}\,\Phi^B = 0$$

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$\Downarrow$ [Hörmander]

$$P(k) = \det\left[Q^{a_1\dots a_N} k_{a_1}\dots k_{a_N}\right]$$

hyperbolic polynomial

Example: Klein-Gordon

$$N = 2$$

$$G^{ab} = g^{ab}$$

$$Q^{a_1} = \Gamma^{a_1}{}_{mn}\, g^{mn}$$

$$Q^{a_1 a_2} = g^{a_1 a_2}$$

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$$G^{\cdot\cdot}\dots$$

$$\sum_{n=0}^{N} Q_{AB}^{a_1 \dots a_n}\, \partial_{a_1} \dots \partial_{a_n} \Phi^B = 0$$

predictive

$\Downarrow$ [Hörmander]

$$P(k) = \det\left[Q^{a_1 \dots a_N} k_{a_1} \dots k_{a_N}\right]$$

hyperbolic polynomial

Example: Klein-Gordon

$Q = 0$

$Q^{a_1} = \Gamma^{a_1}{}_{mn} g^{mn}$

$Q^{a_1 a_2} = g^{a_1 a_2}$

$N = 2$

$G^{ab} = g^{ab}$

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SET

$$G^{\cdot\cdot}{}_{\dots}$$

$$\sum_{n=0}^{N} Q_{AB}^{a_1\dots a_n}\, \partial_{a_1}\dots\partial_{a_n}\Phi^B = 0$$

predictive

$\Downarrow$ [Hörmander]

$$P(k) = \det\left[Q^{a_1\dots a_N} k_{a_1}\dots k_{a_N}\right]$$

hyperbolic polynomial

Example: Klein-Gordon

$Q = 0$

$Q^{a_1} = \Gamma^{a_1}{}_{mn} g^{mn}$

$Q^{a_1 a_2} = g^{a_1 a_2} \Rightarrow P(k) = g^{a_1 a_2} k_{a_1} k_{a_2}$

$N = 2$

$G^{ab} = g^{ab}$

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hyperbolic polynomial

P-null surface in k-space

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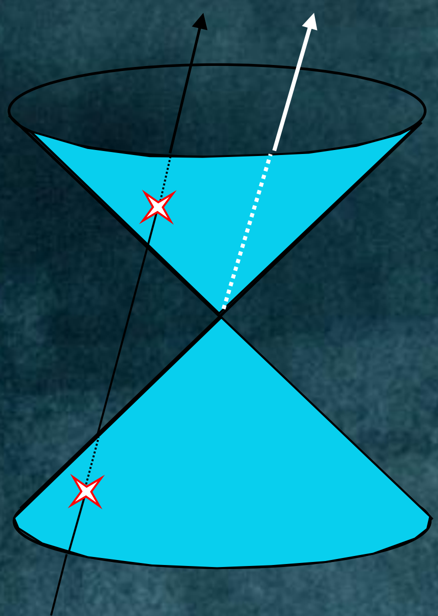

P-null surface in k-space

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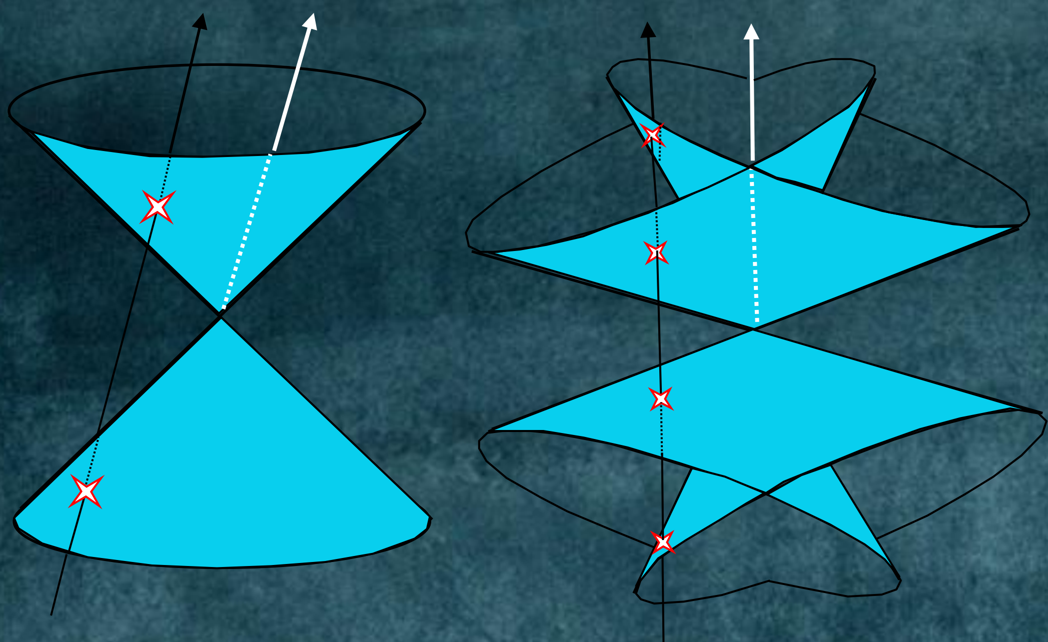

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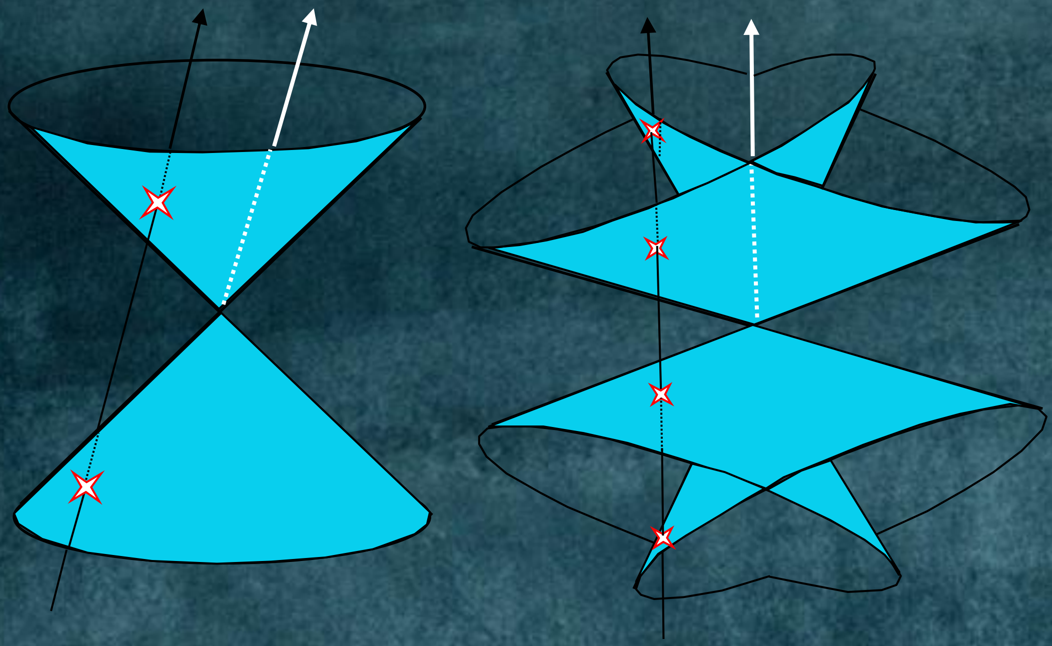
⇕

∃ piercing direction



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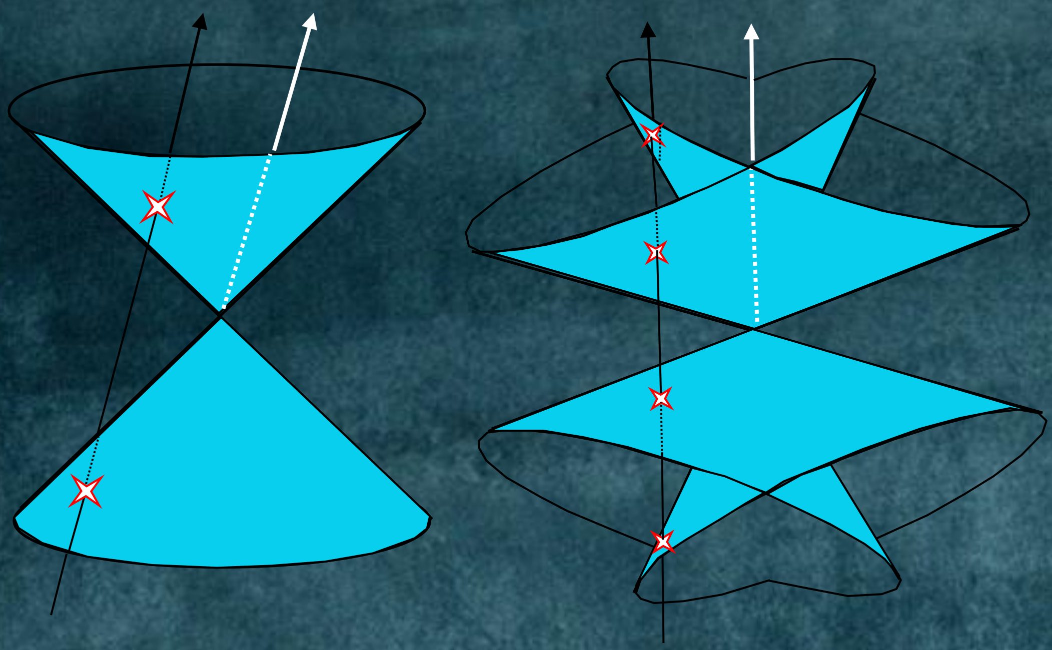
hyperbolic polynomial

$\Updownarrow$

∃ (deg P)–fold piercing direction



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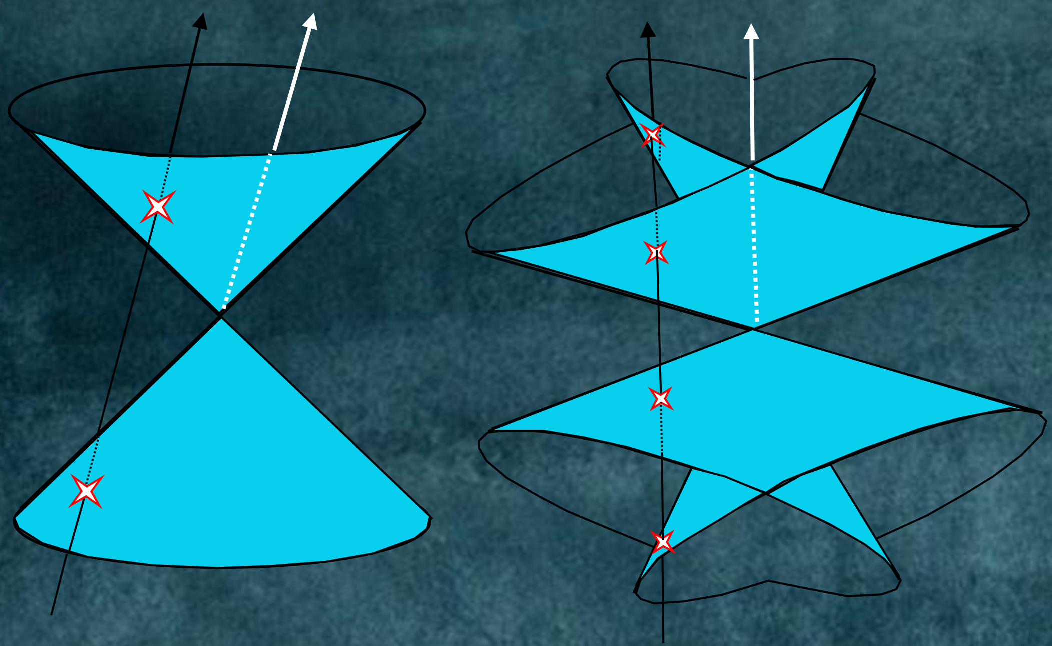
ALL (deg P)–fold piercing directions



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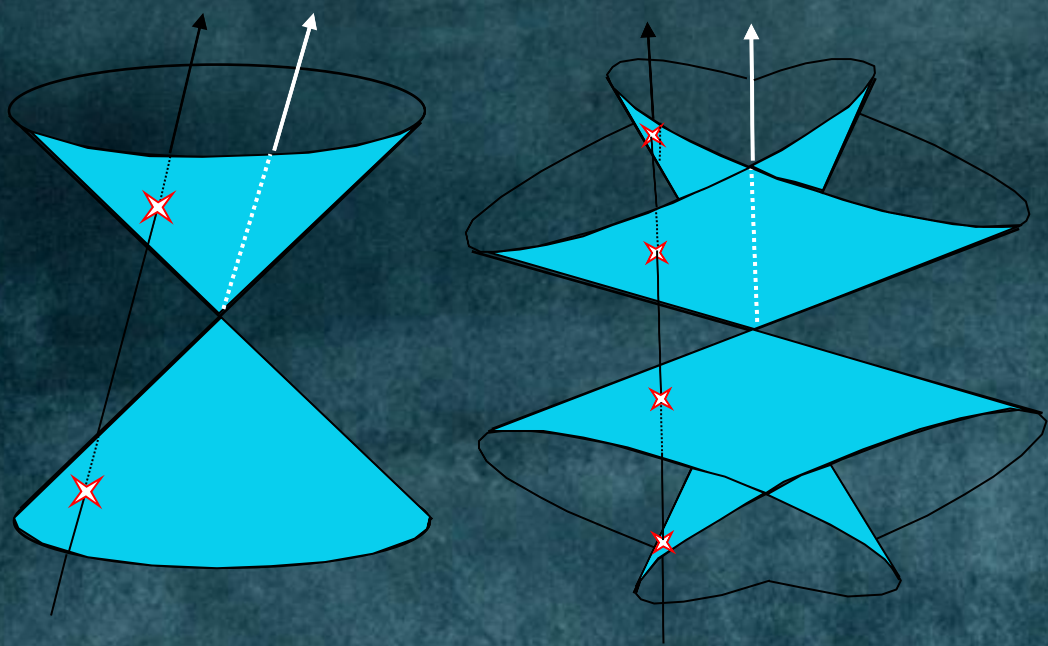

hyperbolicity cones

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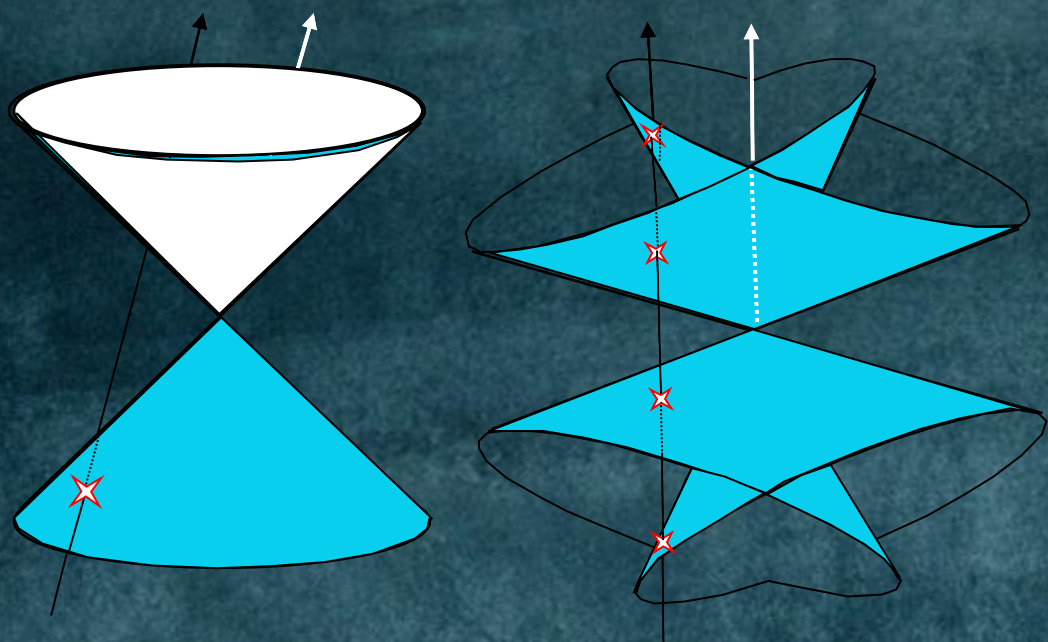

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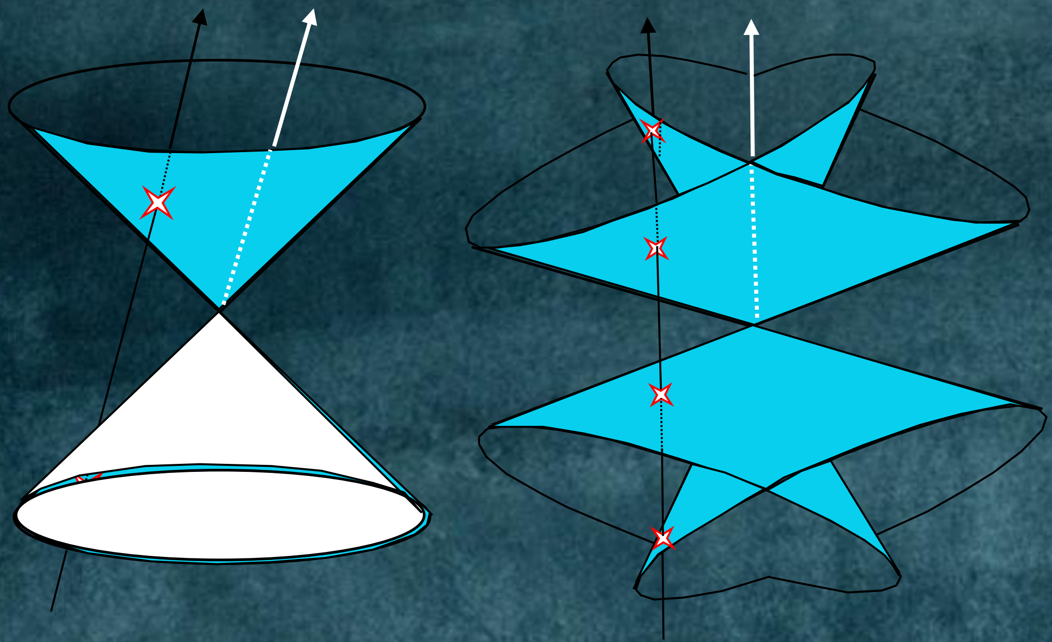

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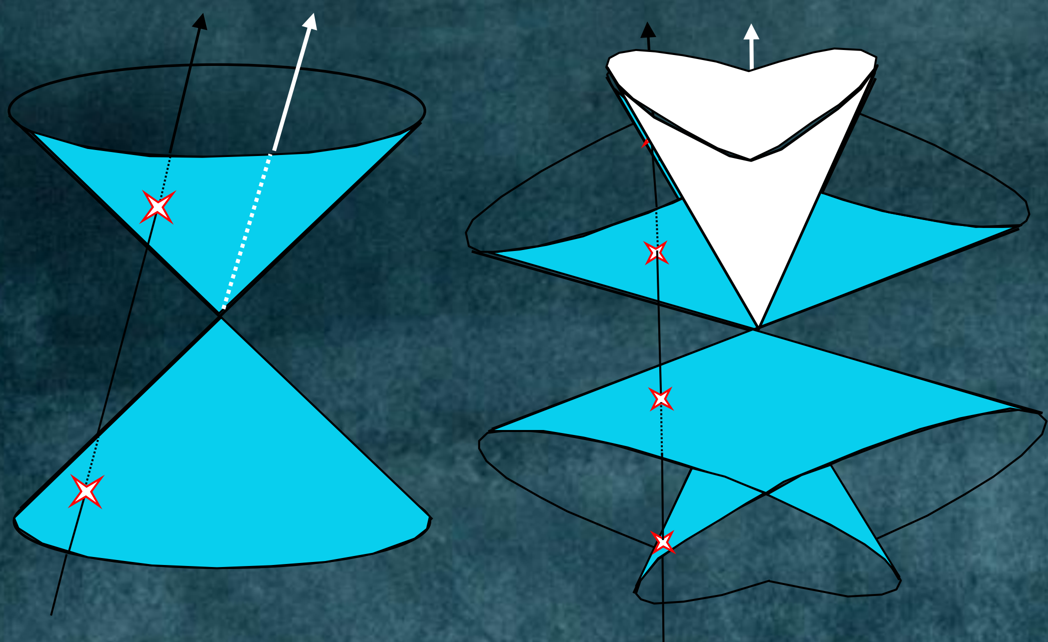

hyperbolicity cones

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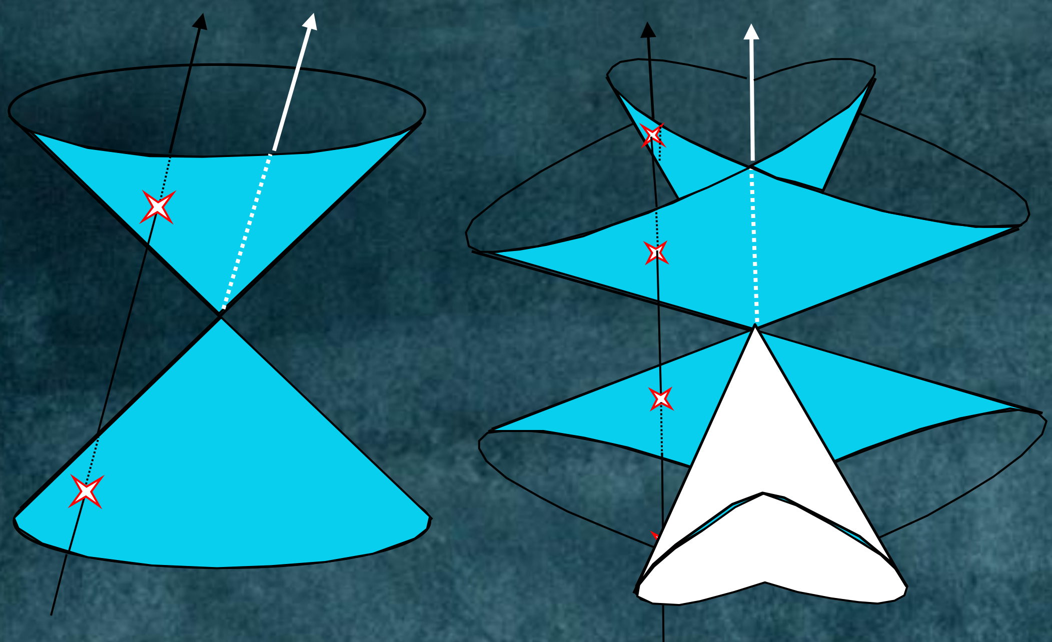

<u>hyperbolicity cones</u>

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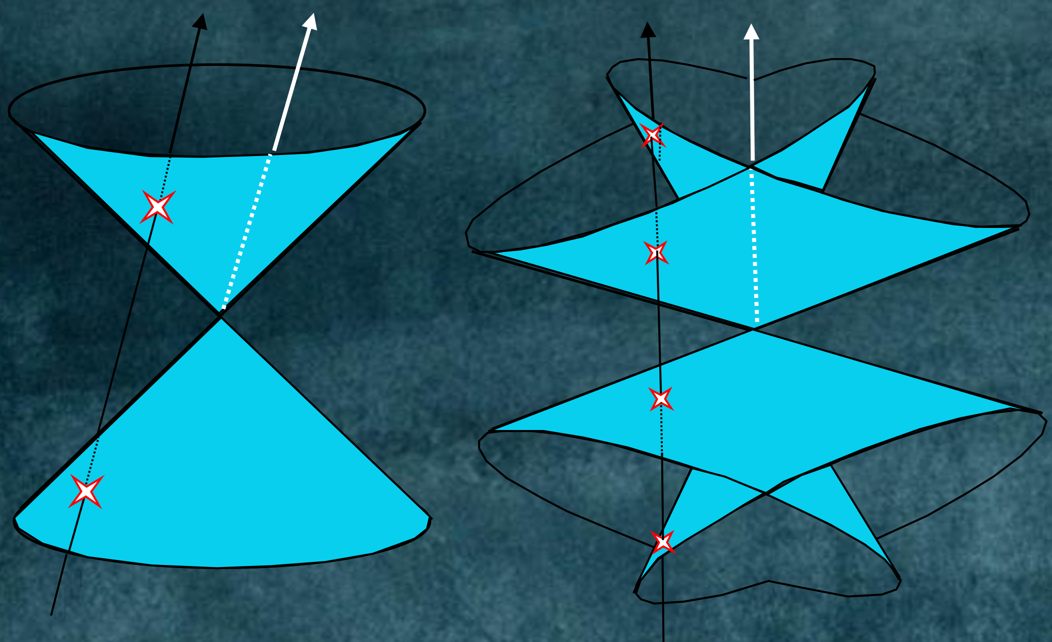

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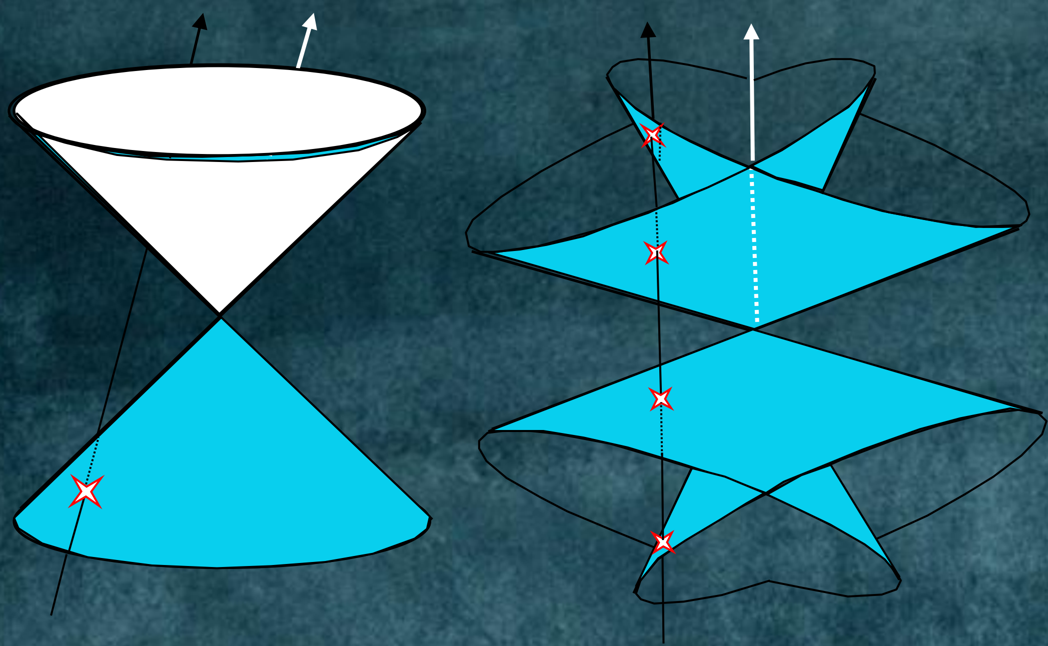

hyperbolicity cones

„co-normals to initial data surfaces“

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Theorem: massless dispersion

$$P(k) = 0$$

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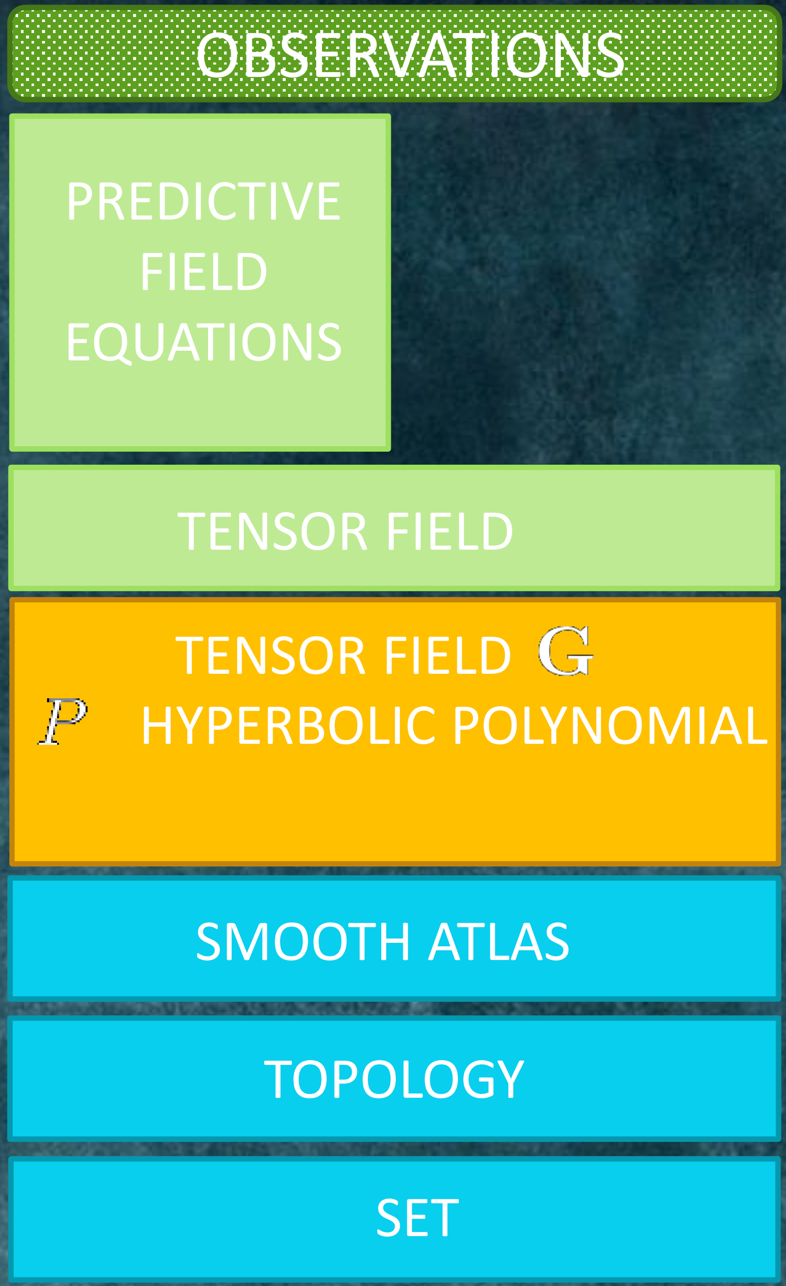
$$P(k) = 0$$

Corollary: massless particle action

$$S[x, k, \lambda] = \int d\tau \; [\dot{x}^a k_a - \lambda P(k)]$$



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eliminate k [real algebraic geometry]

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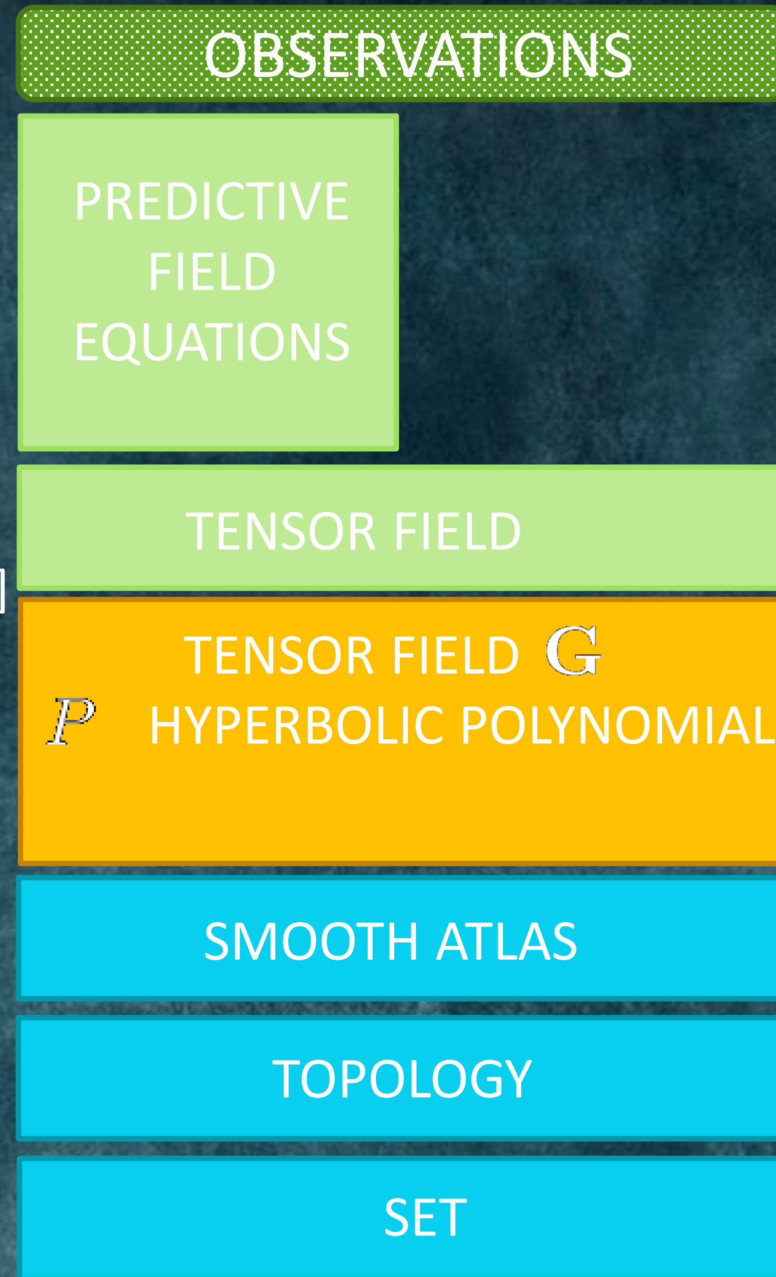
$$S[x, \cancel{k}, \lambda] = \int d\tau \, \lambda P^{\#}(\dot{x})$$



Theorem: massless dispersion

$$P(k) = 0$$

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↓ eliminate k [real algebraic geometry]

$$S[x, \not{k}, \lambda] = \int d\tau \, \lambda P^{\#}(\dot{x})$$

Theorem: P hyperbolic

$$\Downarrow$$

dual polynomial $P^{\#}$ exists / unique*

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Proof: Hyperbolicity saves the day over $\mathbb{R}$

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$$P^{\#}(\nabla P(k)) = 0 \quad [\text{where } P(k) = 0]$$

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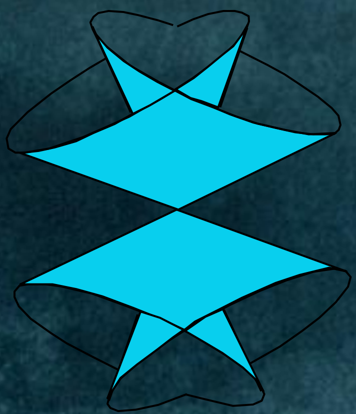

cotangent space

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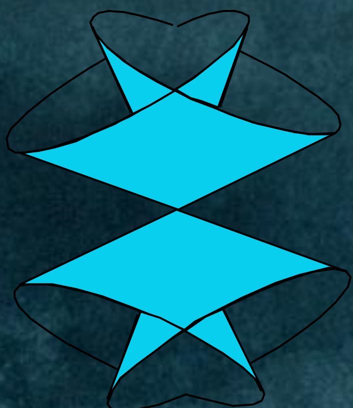
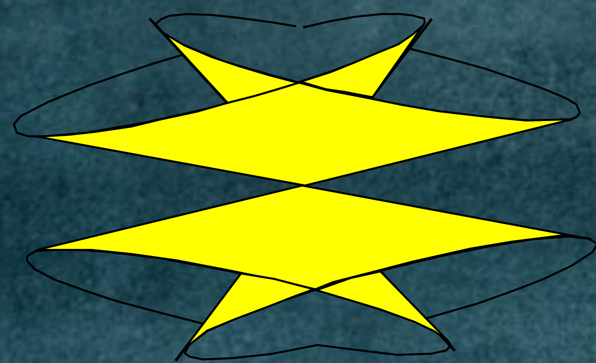


cotangent space

$P^{\#}(X) = 0$



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Theorem: P hyperbolic

$\Downarrow$

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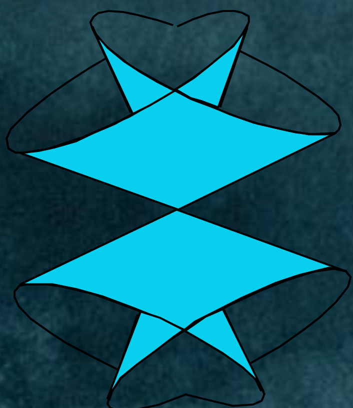
$P(k) = 0$



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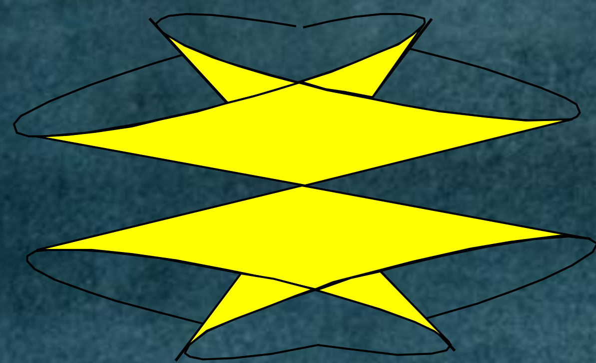

tangent space

massless momenta $\rightleftarrows$ velocities

∇P $\longrightarrow$ $\longleftarrow$ $\nabla P^{\#}$

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<u>Theorem</u>: P hyperbolic

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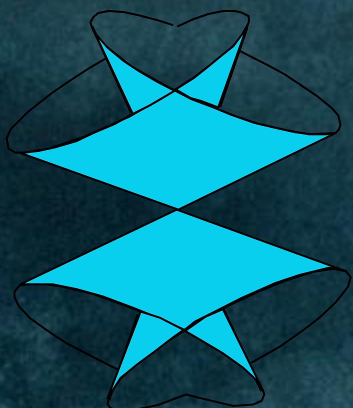

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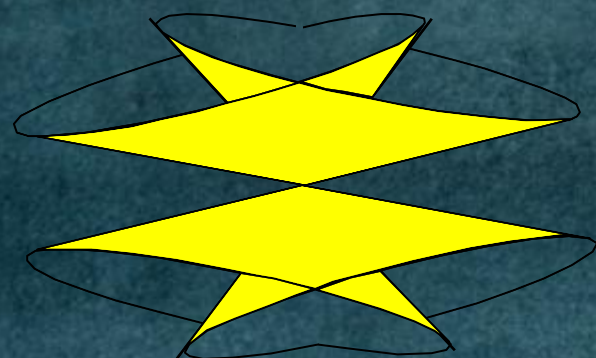

tangent space

massless momenta ⇄ velocities

Bi-hyperbolic Algebraic geometry

∇P → ← $\nabla P^{\#}$

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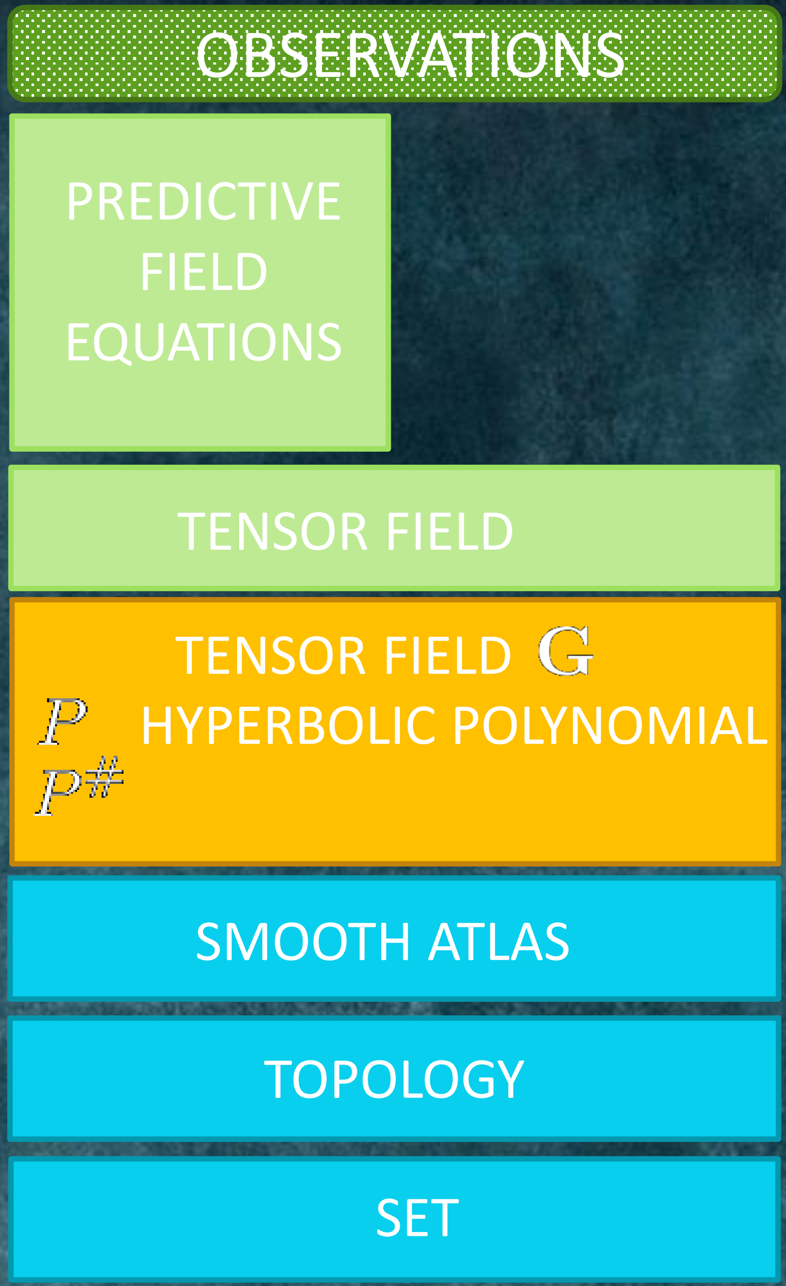
Postulate: massive dispersion

$$P(k) = m^{\deg \mathrm{P}}$$

only for

P hyperbolic AND $P^{\#}$ hyperbolic



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Theorem: massive particle action

$$S[x,k,\lambda] = \int d\tau \left[\dot{x}^a k_a - \lambda \ln P\left(\frac{k}{m}\right) \right]$$

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eliminate [convex analysis]

$$S[x, \cancel{k}, \cancel{\lambda}] = \int d\tau \, m \, P(L^{-1}(\dot{x}))^{-\frac{1}{\deg P}}$$

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massive momenta ⇄ velocities

$L \longrightarrow \quad \longleftarrow L^{-1}$

$$L(q) = \frac{\nabla P(q)}{P(q)}$$

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massive momenta ⇄ velocities

Bi-hyperbolic Convex analysis

L →   ← L^{-1}

$$L(q) = \frac{\nabla P(q)}{P(q)}$$

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massive momenta ⇄ velocities

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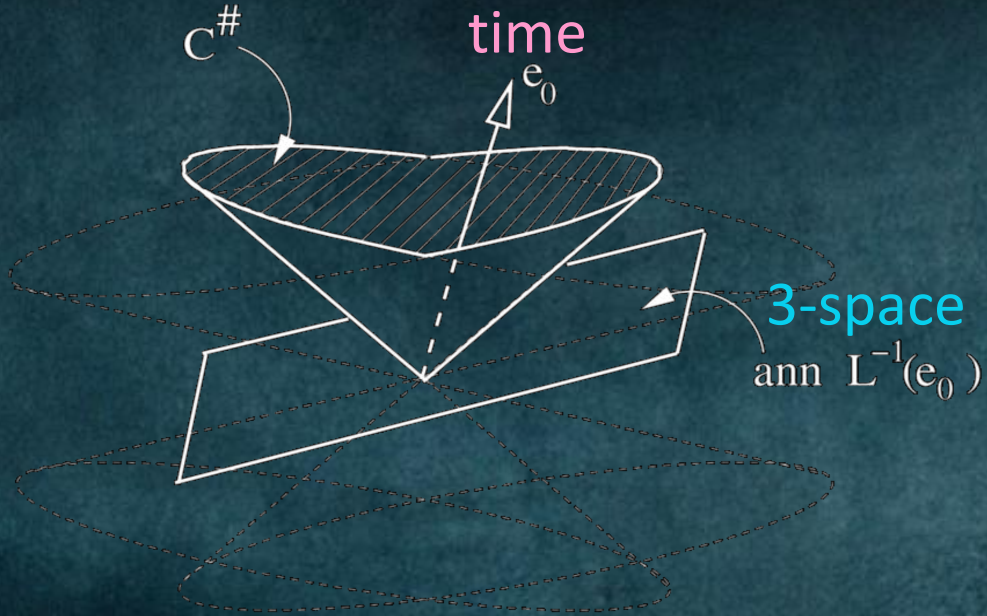
Application: 3+1 decomposition



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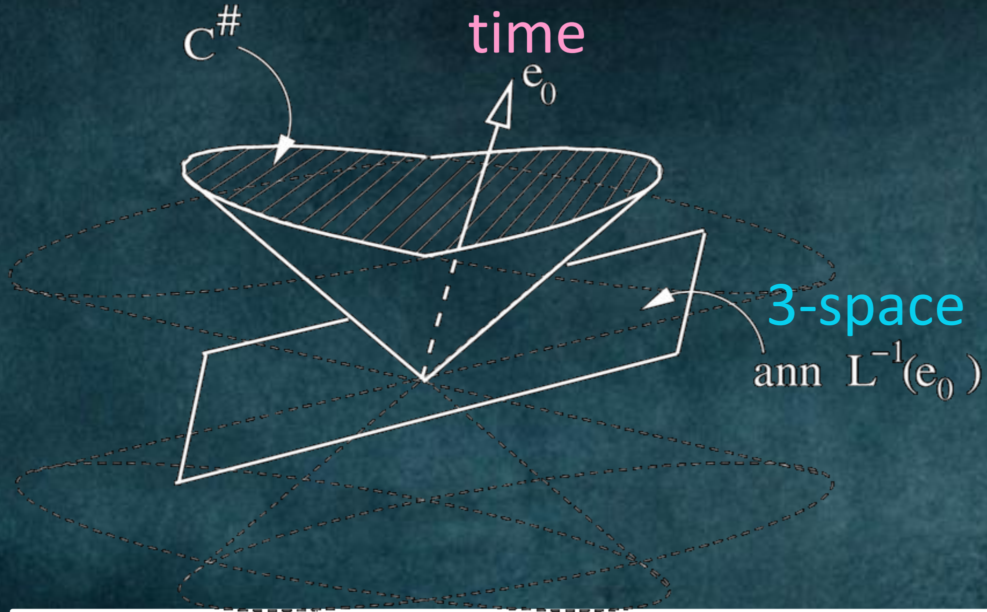

Corollary: modified dispersion E(p)

$$P(E\epsilon^0 + p_\alpha \epsilon^\alpha) = m^{\deg P}$$

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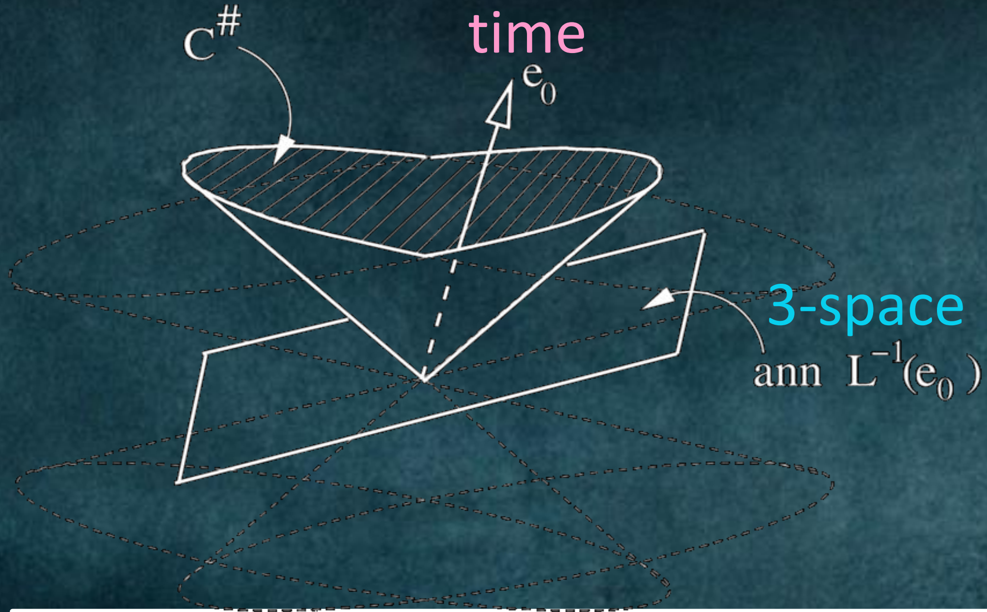

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$$P(E\epsilon^0 + p_\alpha \epsilon^\alpha) = m^{\deg\ P}$$

solve for E
[Galois]

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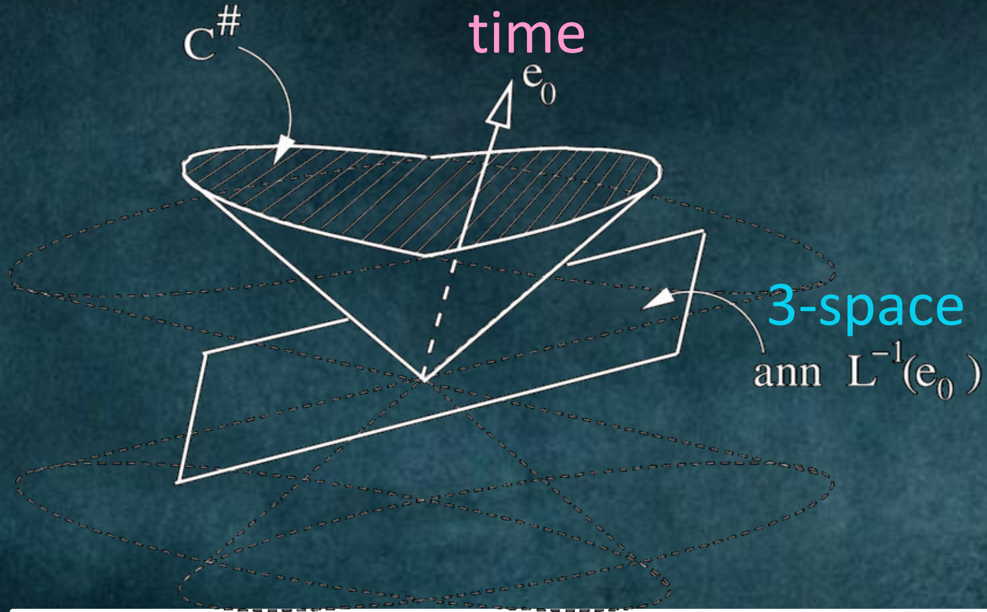
SET

Application: 3+1 decomposition



Corollary: modified dispersion E(p)

$$P(E\epsilon^0 + p_\alpha \epsilon^\alpha) = m^{\deg P}$$

solve for E
[Galois]

$$E = \sum_{n=0}^{\infty} C^{\alpha_1 \dots \alpha_n} \, p_{\alpha_1} \cdots p_{\alpha_n}$$

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

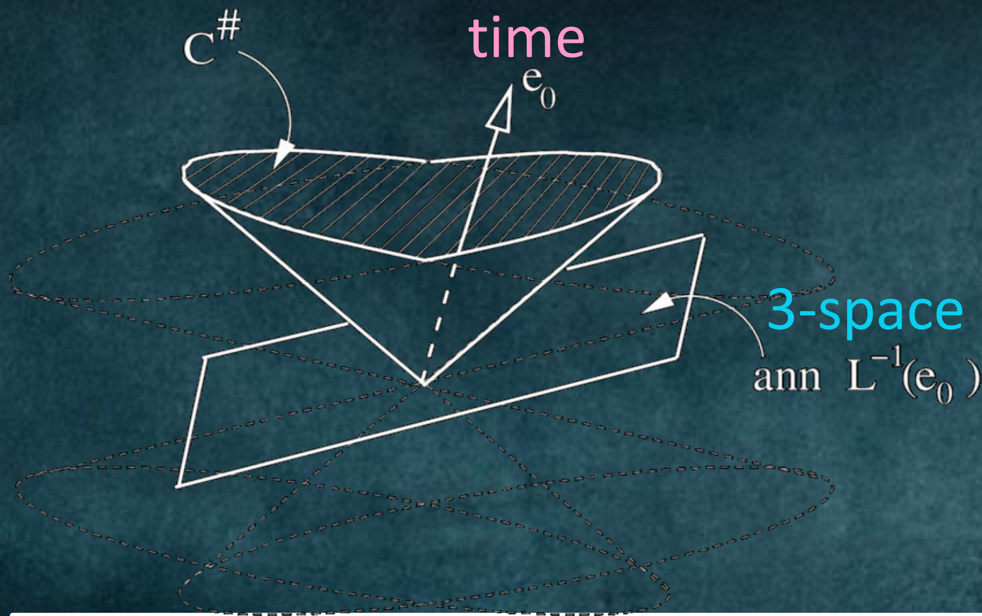
TENSOR FIELD G
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

SET

Application: 3+1 decomposition



OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

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TOPOLOGY

SET

Corollary: modified dispersion E(p)

$$P(E\epsilon^0 + p_\alpha \epsilon^\alpha) = m^{\deg P}$$

hard! ↑ ↓ solve for E [Galois]

$$E = \sum_{n=0}^{\infty} C^{\alpha_1 \dots \alpha_n} p_{\alpha_1} \cdots p_{\alpha_n}$$

Application: time↔energy orientation

OBSERVATIONS

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$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

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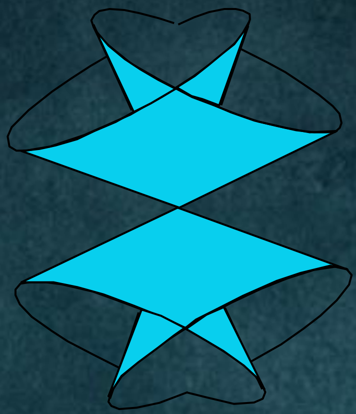
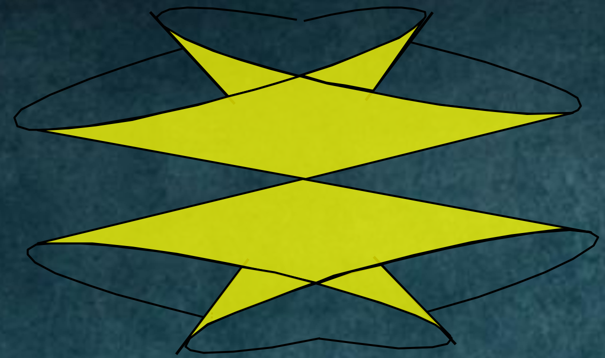
Application: time↔energy orientation



cotangent space



tangent space

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

TENSOR FIELD G

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

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TOPOLOGY

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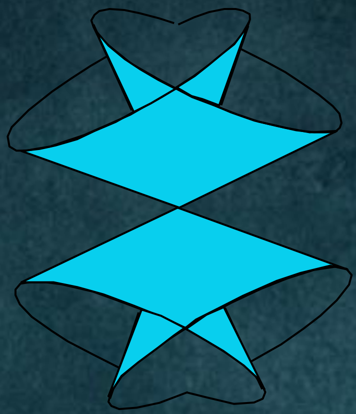
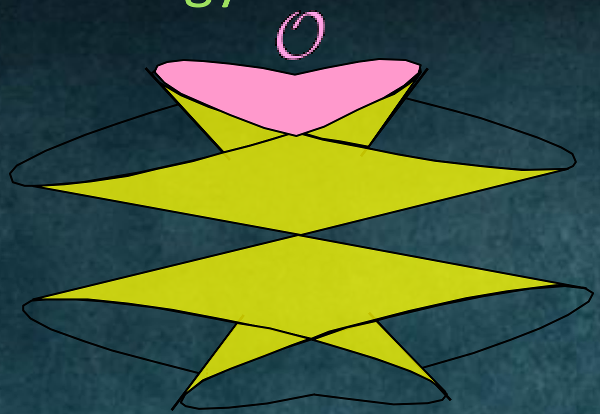
Application: time↔energy orientation



cotangent space



tangent space

OBSERVATIONS

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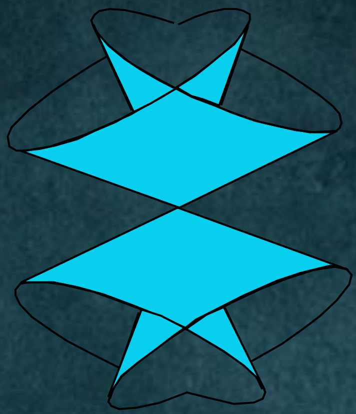

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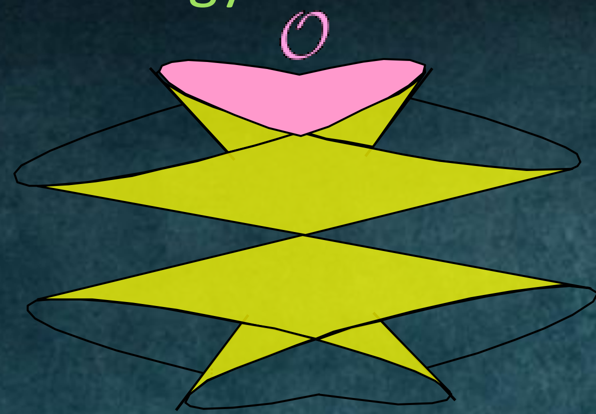

tangent space

pos/neg energy cones

$$\mathcal{O}^{\pm} = \{k \mid k(X) \gtrless 0 \ \text{ for all } X \in \mathcal{O}\}$$

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

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TENSOR FIELD G

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

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cotangent space

tangent space

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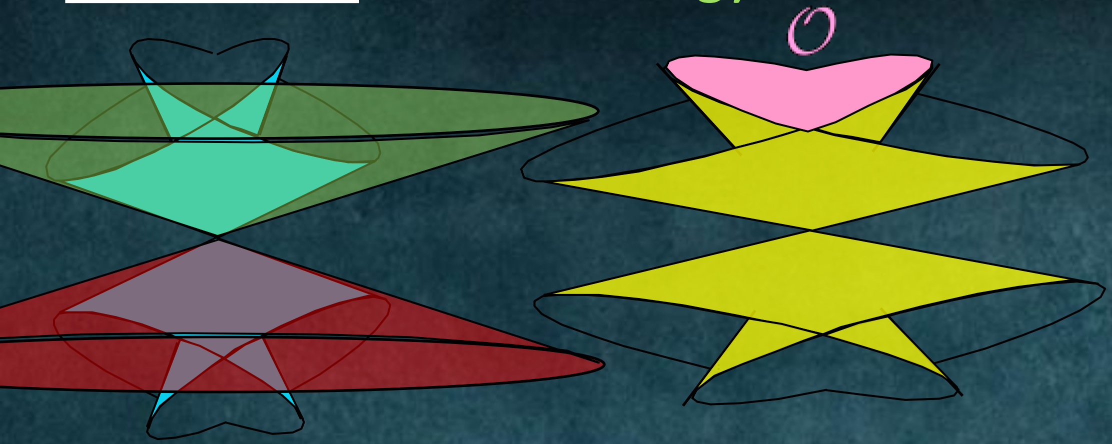
Application: time↔energy orientation



cotangent space

tangent space

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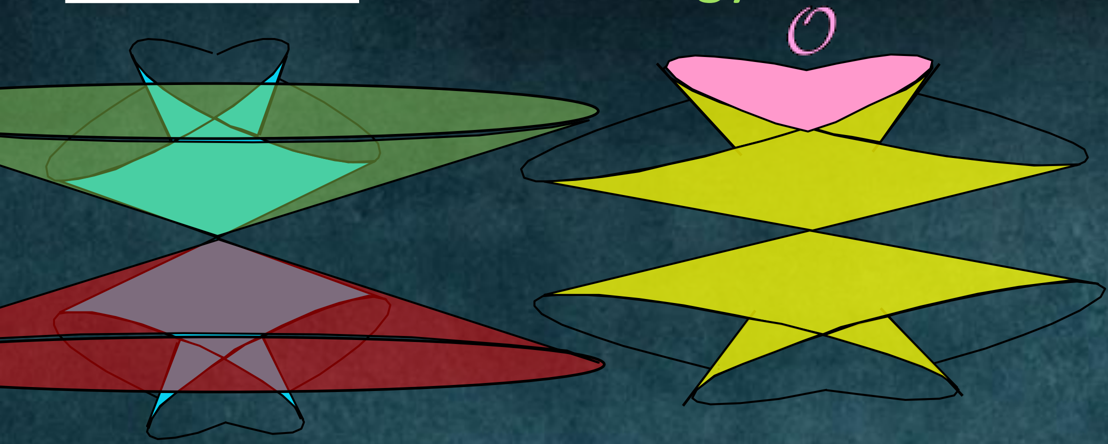
SET

Application: time↔energy orientation



cotangent space — tangent space

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

TENSOR FIELD $\mathbf{G}$
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

SET

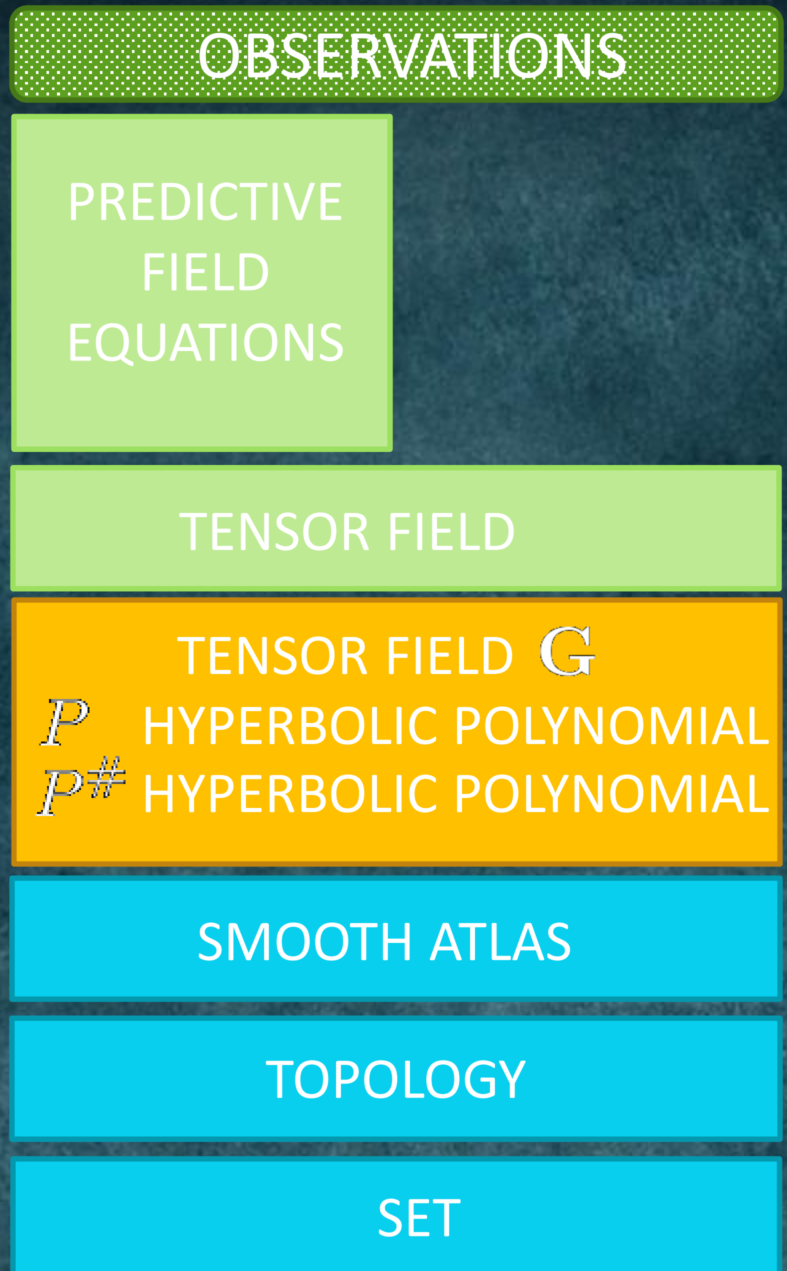
Theorem: pos/neg energy cones

$$\mathcal{O}^{\pm} = \{k \mid k(X) \gtrless 0 \text{ for all } X \in \mathcal{O}\}$$

disjointly cover massless momenta

$$\Uparrow$$

$\mathcal{O}$ hyperbolicity cone of $P^{\#}$


OBSERVATIONS
PREDICTIVE FIELD EQUATIONS
TENSOR FIELD
TENSOR FIELD G
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL
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P^{ab} (Einstein)

SMOOTH ATLAS

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$P^{\#}$ HYPERBOLIC POLYNOMIAL

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SET

P^{ab} (Einstein)

P^{abcd}

OBSERVATIONS

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P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

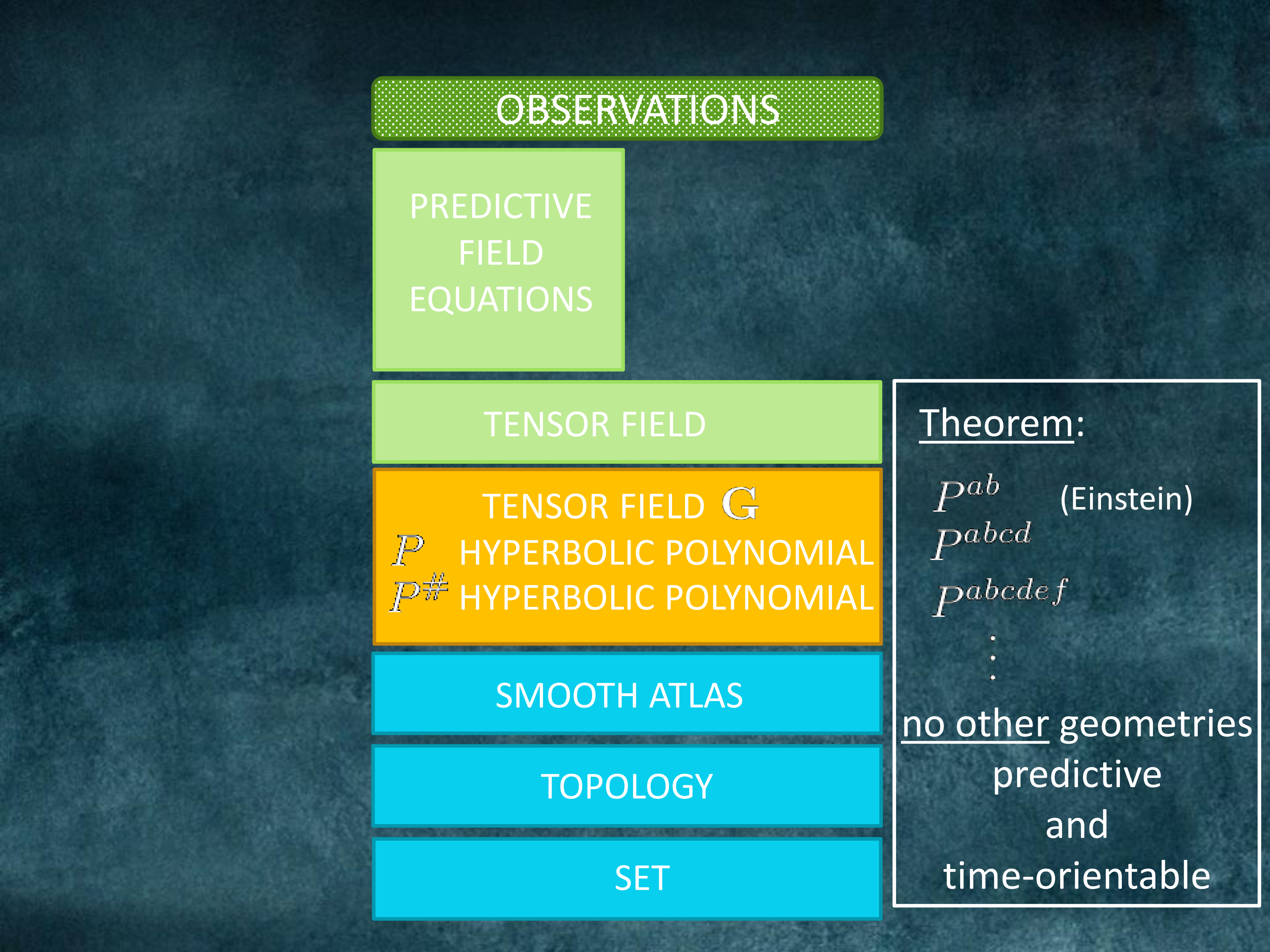
SET

P^{ab} (Einstein)

P^{abcd}

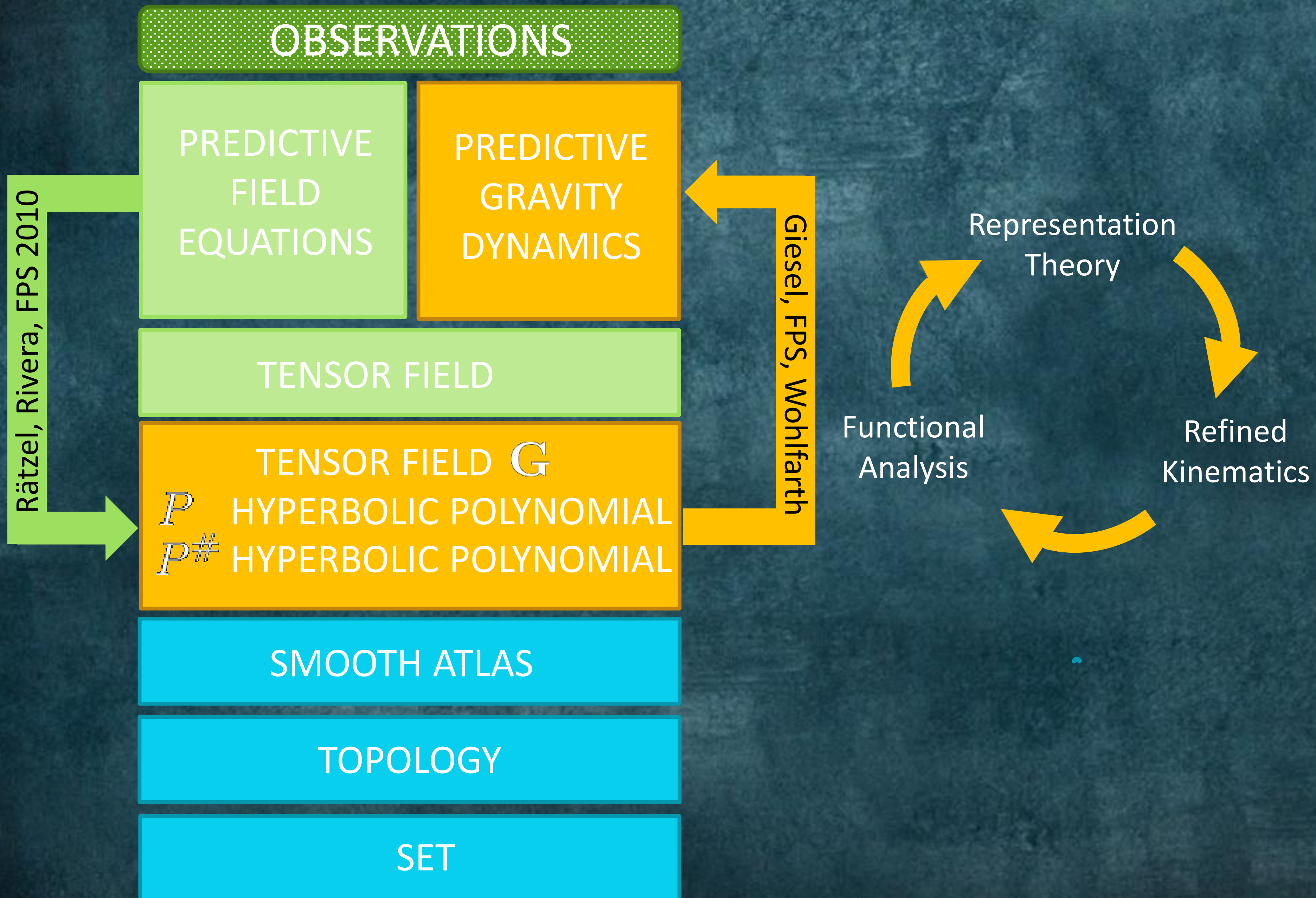
P^{abcdef}

$\vdots$


OBSERVATIONS
PREDICTIVE FIELD EQUATIONS
TENSOR FIELD
TENSOR FIELD $\mathbf{G}$
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL
SMOOTH ATLAS
TOPOLOGY
SET
Theorem:
P^{ab} (Einstein)
P^{abcd}
P^{abcdef}
$\vdots$
no other geometries predictive and time-orientable

SPACETIME DYNAMICS

修正


OBSERVATIONS
PREDICTIVE FIELD EQUATIONS
PREDICTIVE GRAVITY DYNAMICS
TENSOR FIELD
TENSOR FIELD G
P HYPERBOLIC POLYNOMIAL
P# HYPERBOLIC POLYNOMIAL
SMOOTH ATLAS
TOPOLOGY
SET
Rätzel, Rivera, FPS 2010
Giesel, FPS, Wohlfarth
Representation Theory
Refined Kinematics
Functional Analysis

OBSERVATIONS

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TENSOR FIELD

TENSOR FIELD G

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

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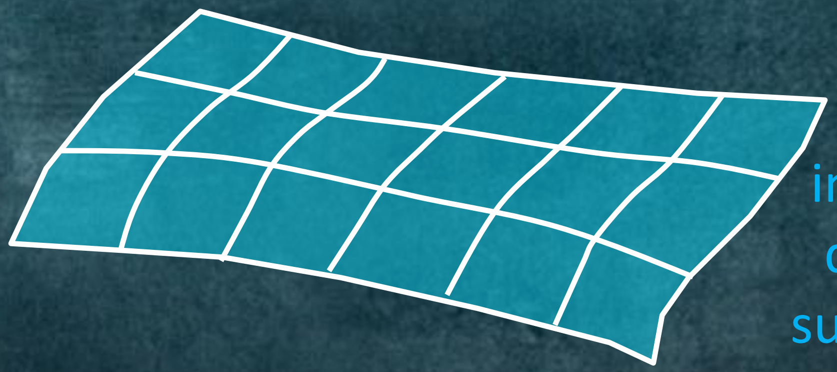
SET



OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

TENSOR FIELD G

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

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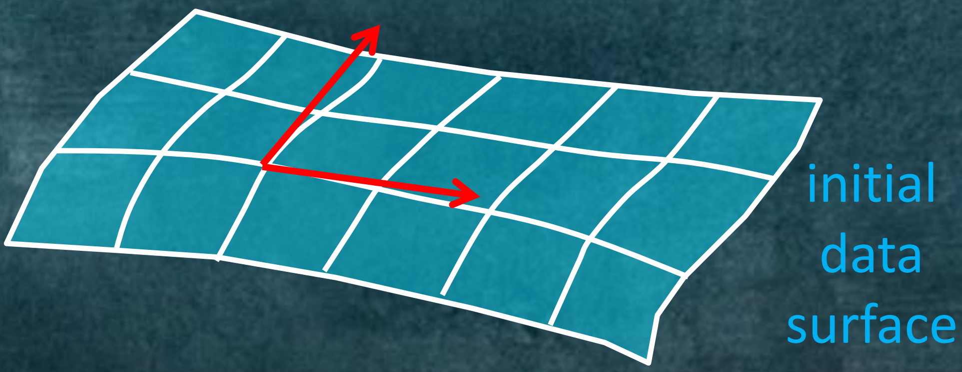

OBSERVATIONS

PREDICTIVE
FIELD
EQUATIONS

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TENSOR FIELD G

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$P^{\#}$ HYPERBOLIC POLYNOMIAL

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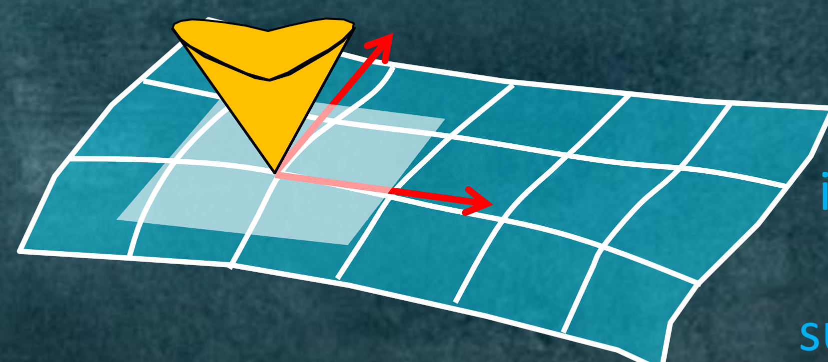

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

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TENSOR FIELD G

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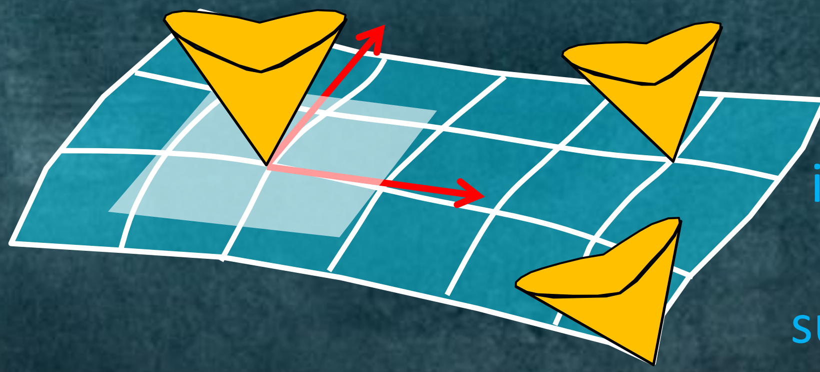

initial data surface

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

TENSOR FIELD G

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

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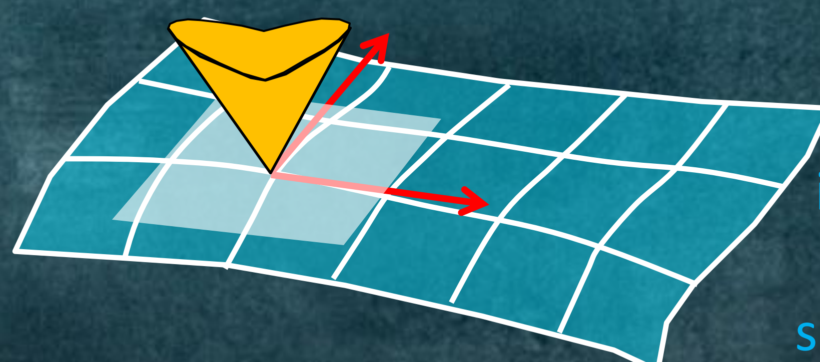
TOPOLOGY

SET



initial data surface

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

TENSOR FIELD $\mathbf{G}$

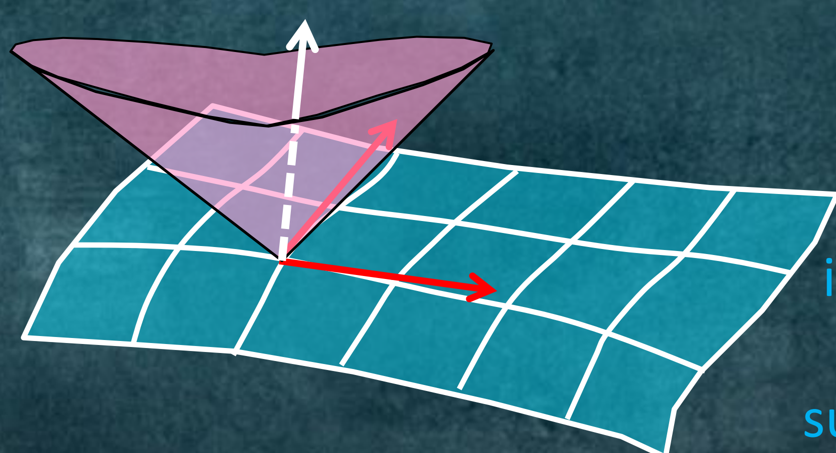
P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

SET



initial data surface

normal = Legendre(co-normal)

OBSERVATIONS

PREDICTIVE
FIELD
EQUATIONS

TENSOR FIELD

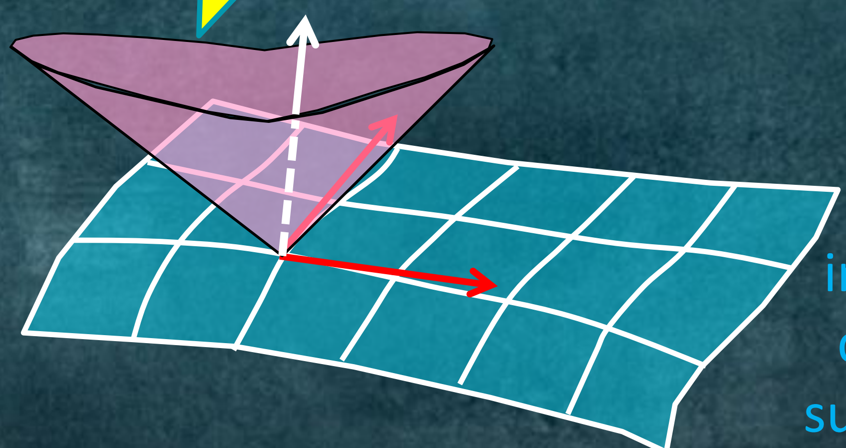
TENSOR FIELD $\mathbf{G}$
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

SET

vital injection from physics:
Legendre map tells
geometrodynamics
about the **matter** it must carry



initial
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OBSERVATIONS

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TENSOR FIELD

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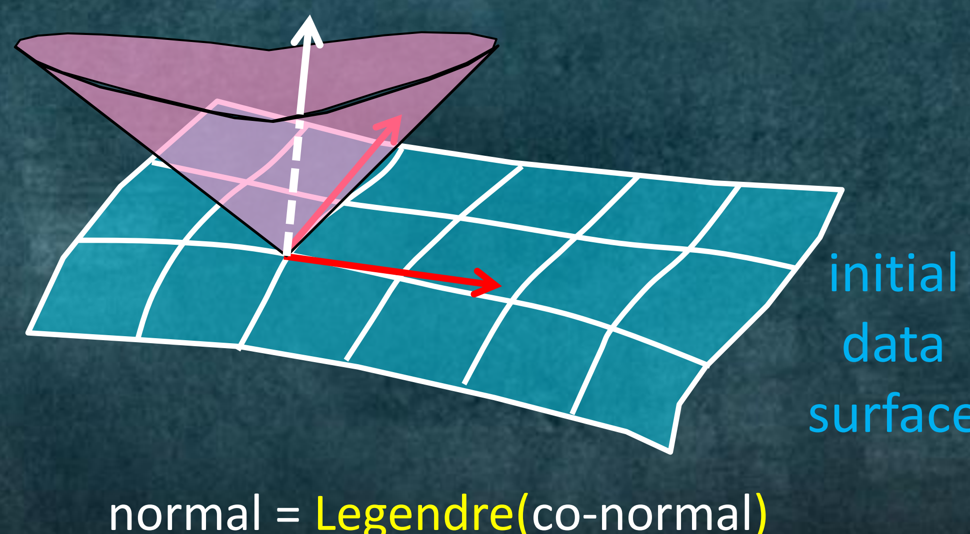

OBSERVATIONS

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$P^{\#}$ HYPERBOLIC POLYNOMIAL

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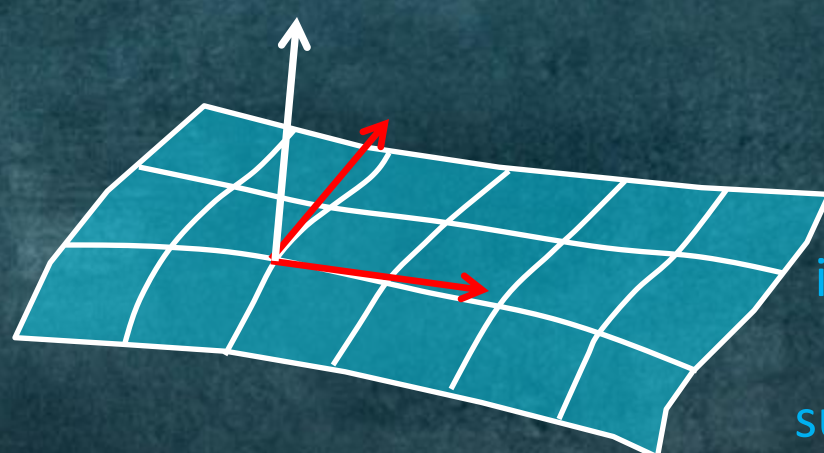

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OBSERVATIONS

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TENSOR FIELD G

P HYPERBOLIC POLYNOMIAL

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SMOOTH ATLAS

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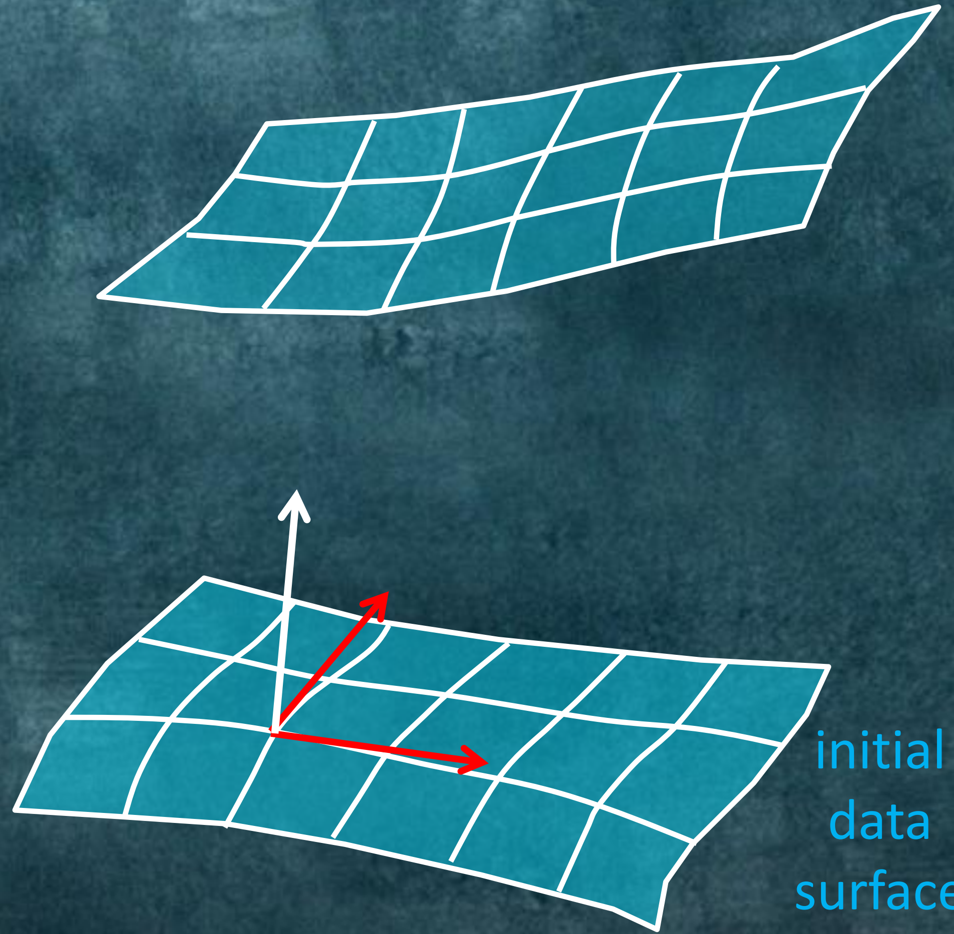

normal = Legendre(co-normal)

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

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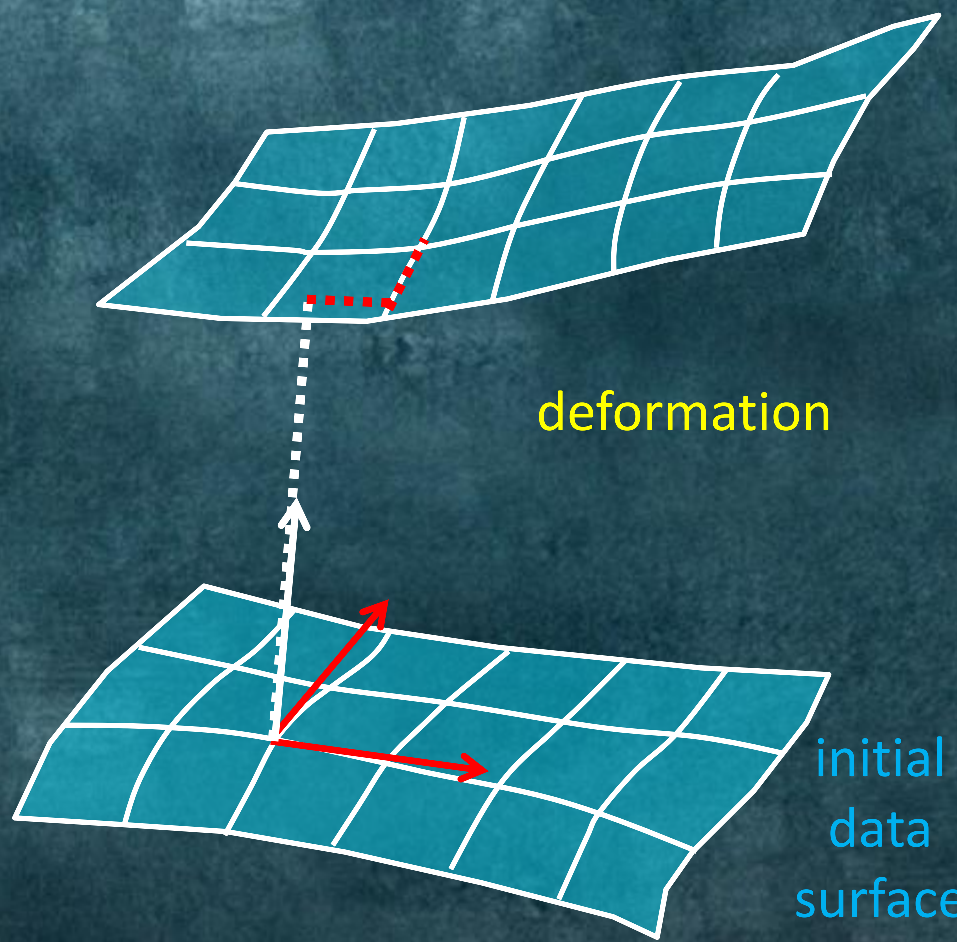
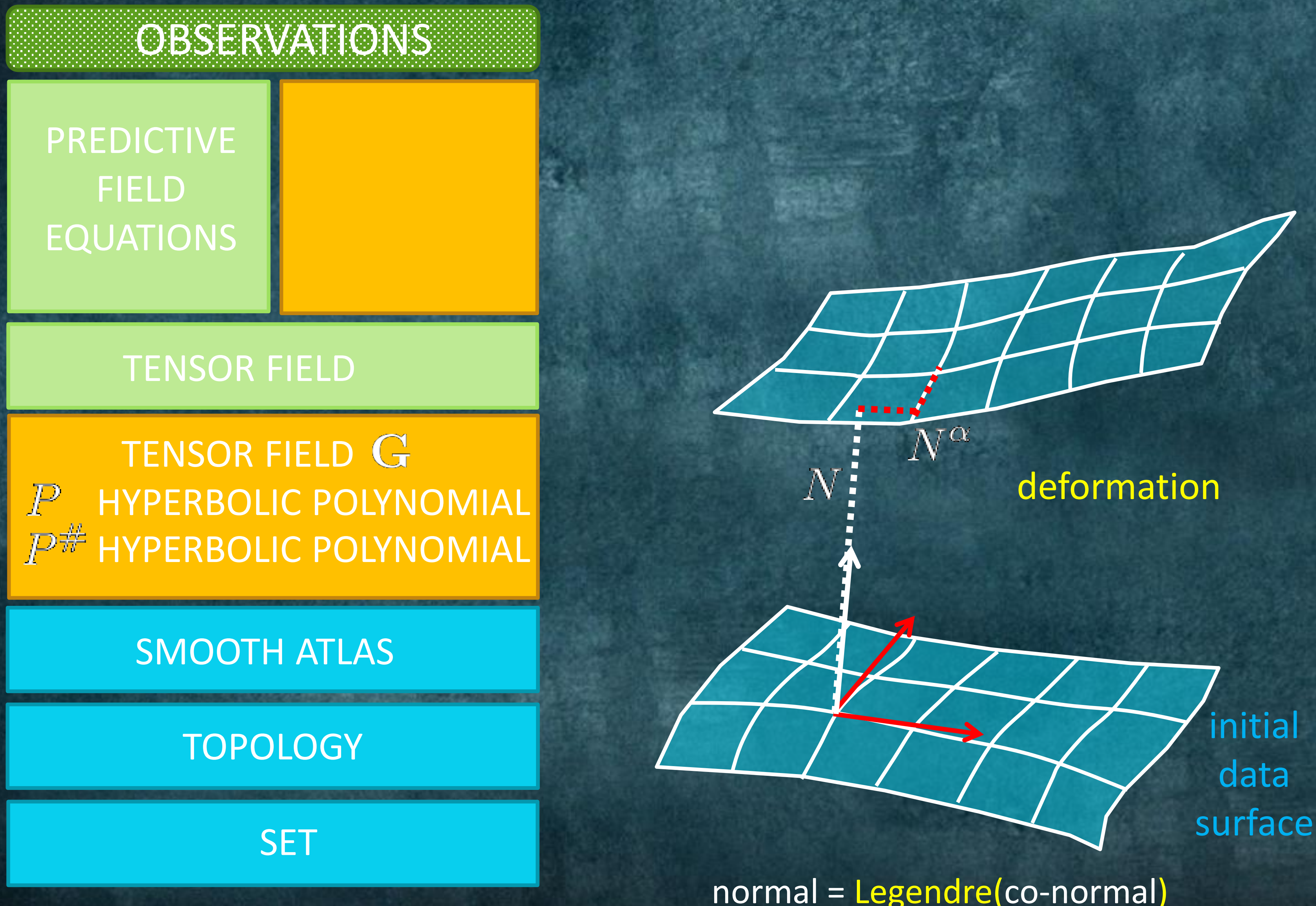
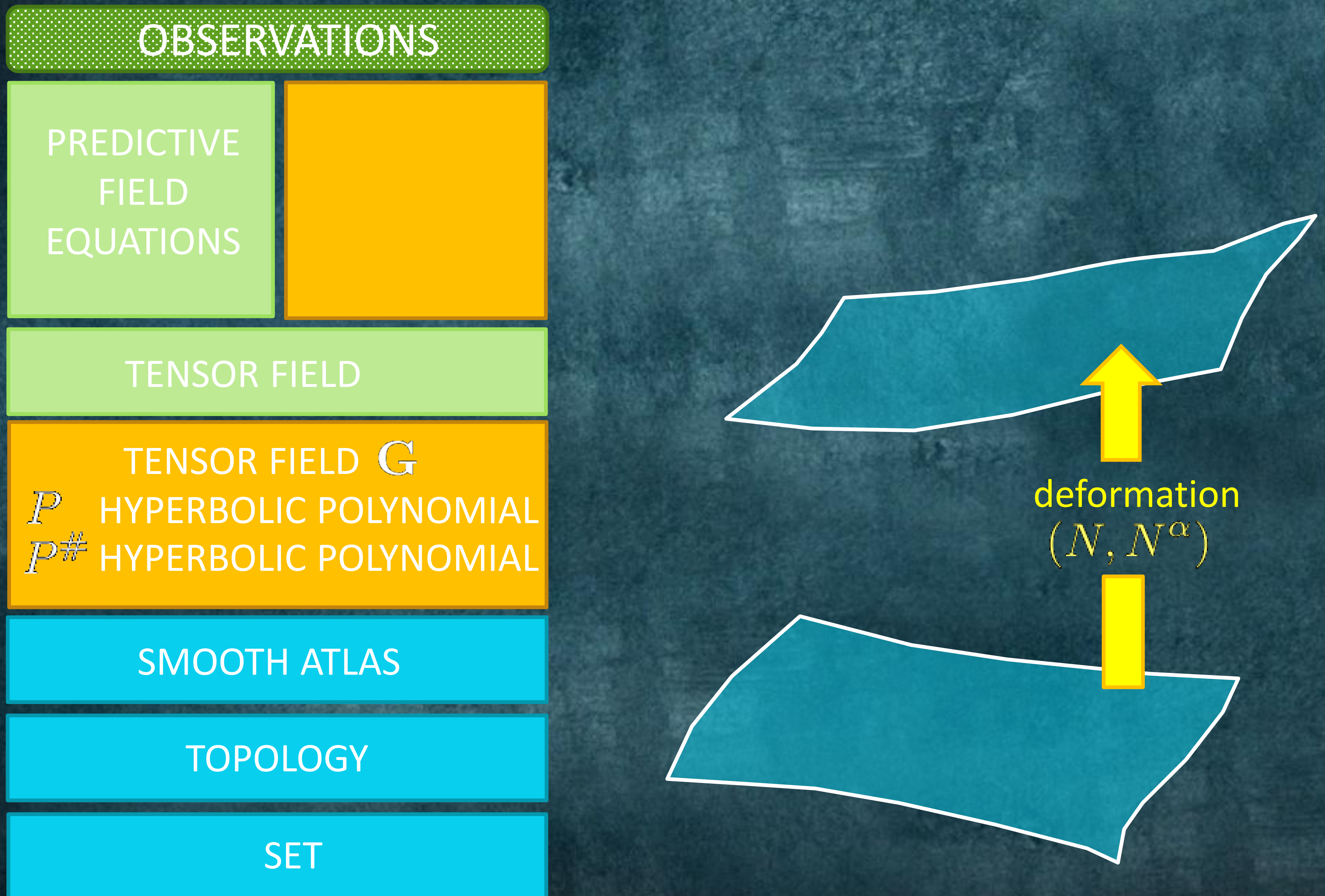


normal = Legendre(co-normal)


OBSERVATIONS
PREDICTIVE FIELD EQUATIONS
TENSOR FIELD
TENSOR FIELD G
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL
SMOOTH ATLAS
TOPOLOGY
SET
N^{α}
N
deformation
initial data surface
normal = Legendre(co-normal)


OBSERVATIONS
PREDICTIVE FIELD EQUATIONS
TENSOR FIELD
TENSOR FIELD G
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL
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TOPOLOGY
SET
deformation
(N, N^{α})

OBSERVATIONS

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P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

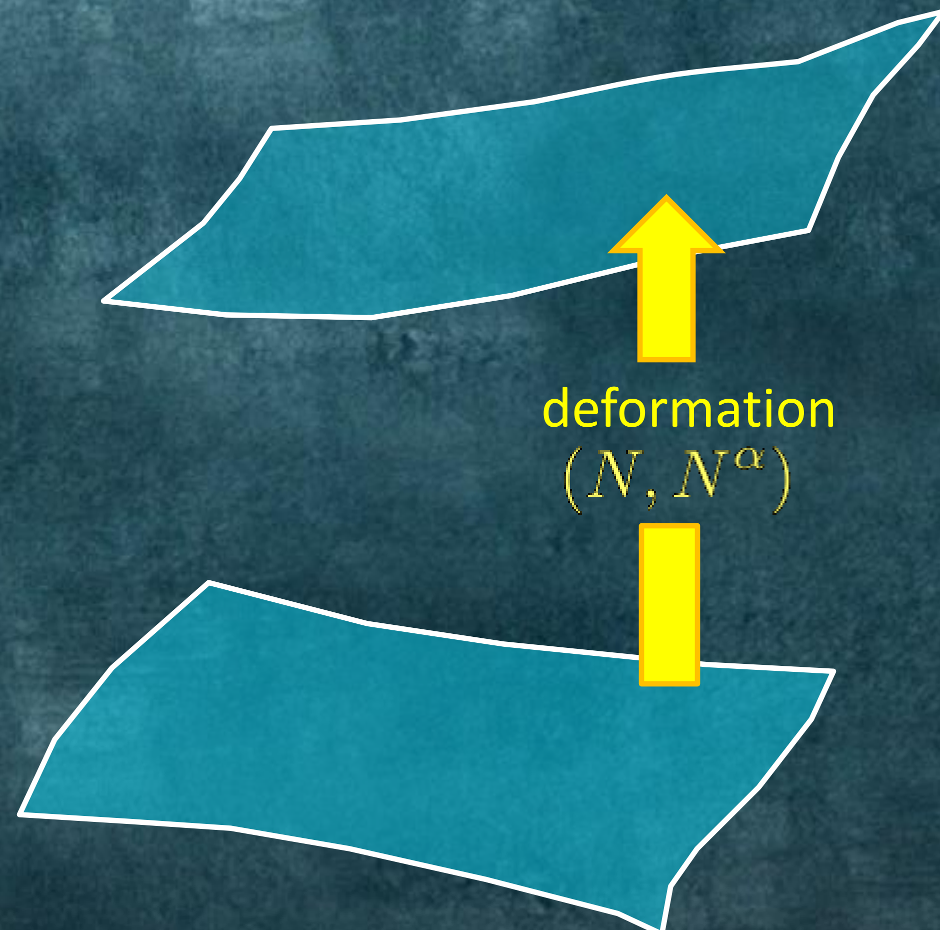
SMOOTH ATLAS

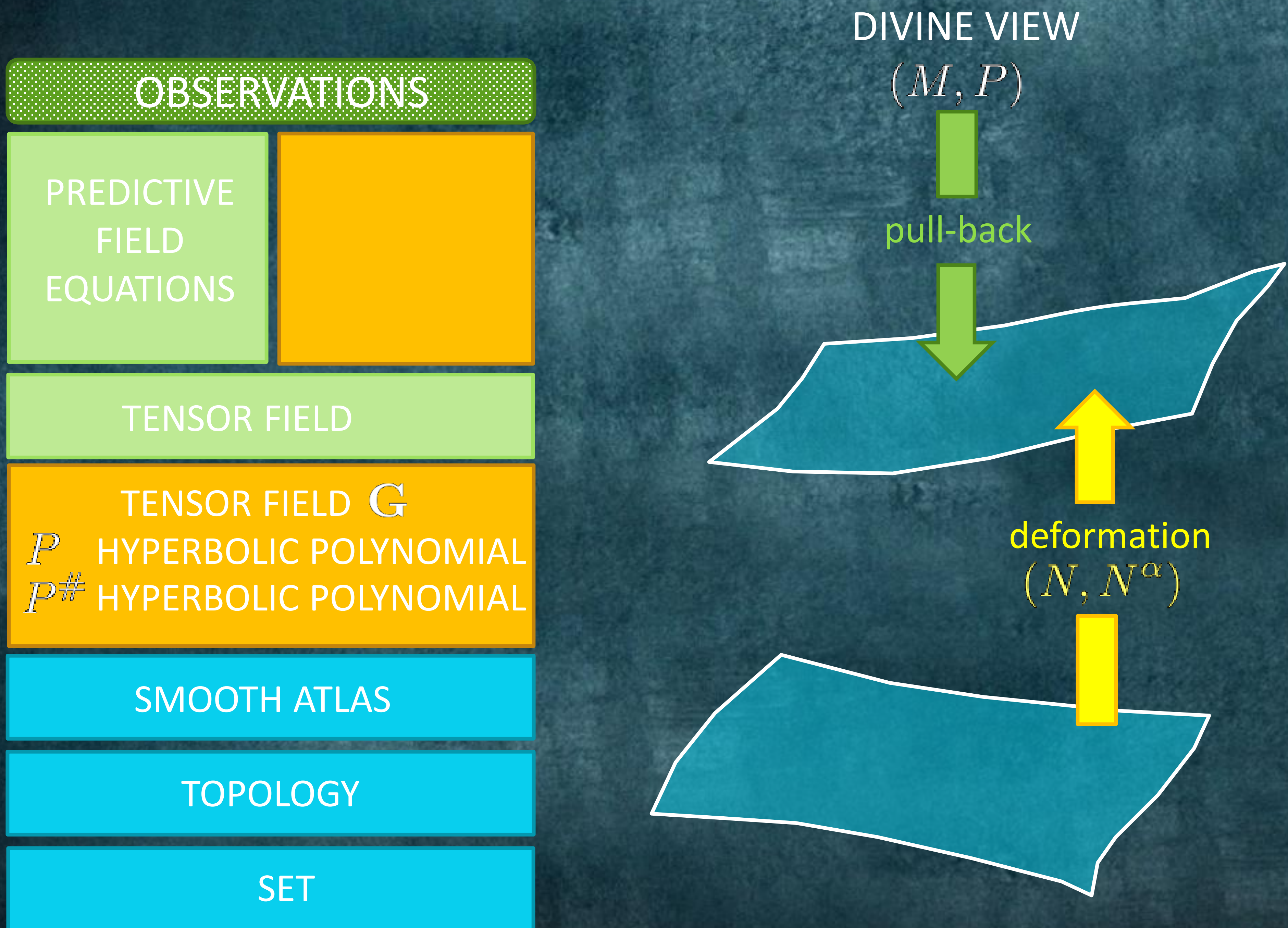
TOPOLOGY

SET

DIVINE VIEW

(M, P)




OBSERVATIONS
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TENSOR FIELD
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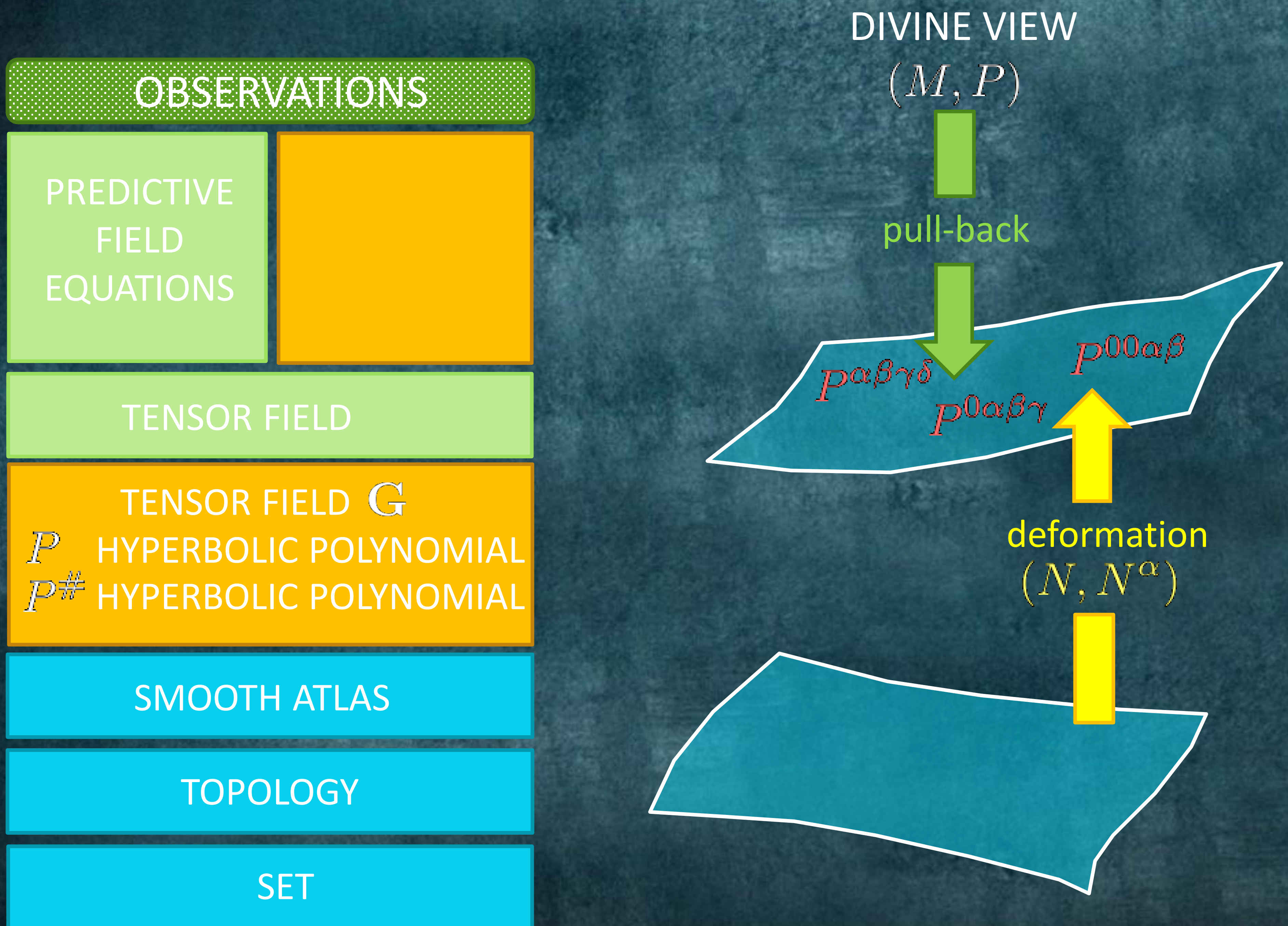
SMOOTH ATLAS
TOPOLOGY
SET
DIVINE VIEW
(M, P)
pull-back
deformation
(N, N^{α})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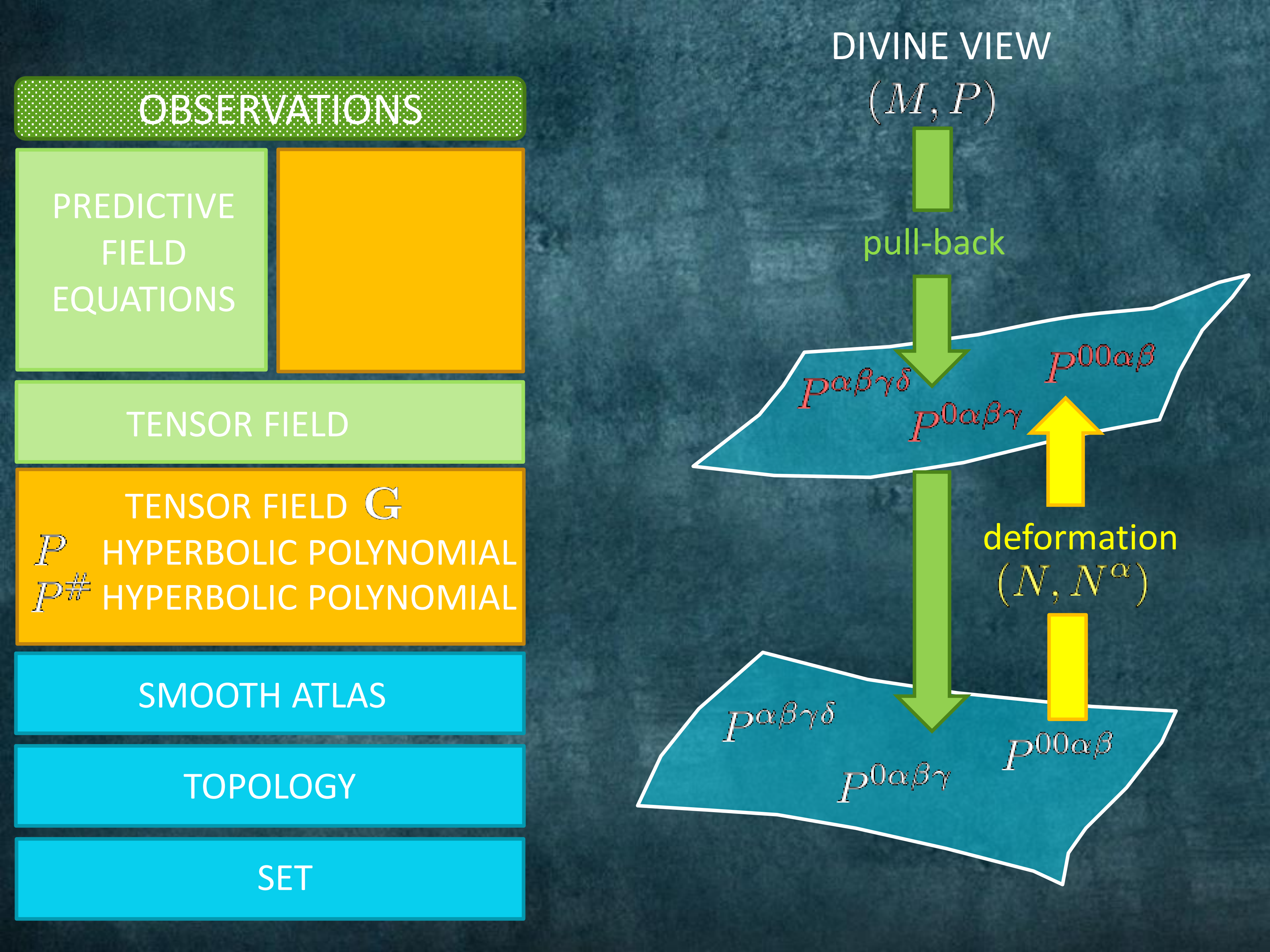
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL
SMOOTH ATLAS
TOPOLOGY
SET
DIVINE VIEW
(M, P)
pull-back
$P^{\alpha\beta\gamma\delta}$
$P^{00\alpha\beta}$
$P^{0\alpha\beta\gamma}$
deformation
(N, N^{α})



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TENSOR FIELD G
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL
SMOOTH ATLAS
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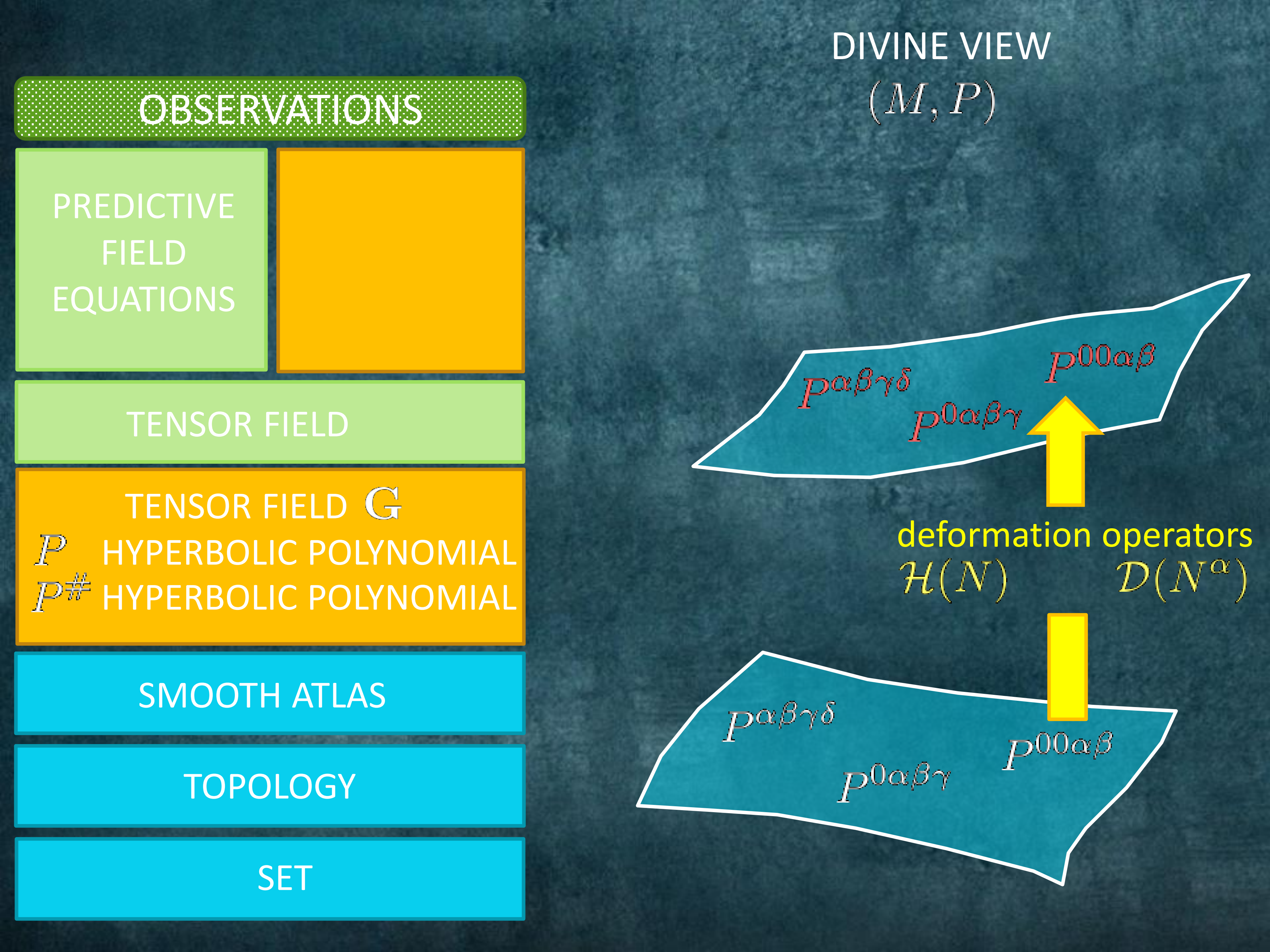
SET
DIVINE VIEW
(M, P)
pull-back
$P^{\alpha\beta\gamma\delta}$
$P^{0\alpha\beta\gamma}$
$P^{00\alpha\beta}$
deformation
(N, N^{α})
$P^{\alpha\beta\gamma\delta}$
$P^{0\alpha\beta\gamma}$
$P^{00\alpha\beta}$


DIVINE VIEW
(M, P)
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TENSOR FIELD G
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$P^{\#}$ HYPERBOLIC POLYNOMIAL
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SET
$P^{\alpha\beta\gamma\delta}$
$P^{00\alpha\beta}$
$P^{0\alpha\beta\gamma}$
deformation operators
$\mathcal{H}(N)$
$\mathcal{D}(N^{\alpha})$
$P^{\alpha\beta\gamma\delta}$
$P^{00\alpha\beta}$
$P^{0\alpha\beta\gamma}$

OBSERVATIONS

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TENSOR FIELD $\mathbf{G}$

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

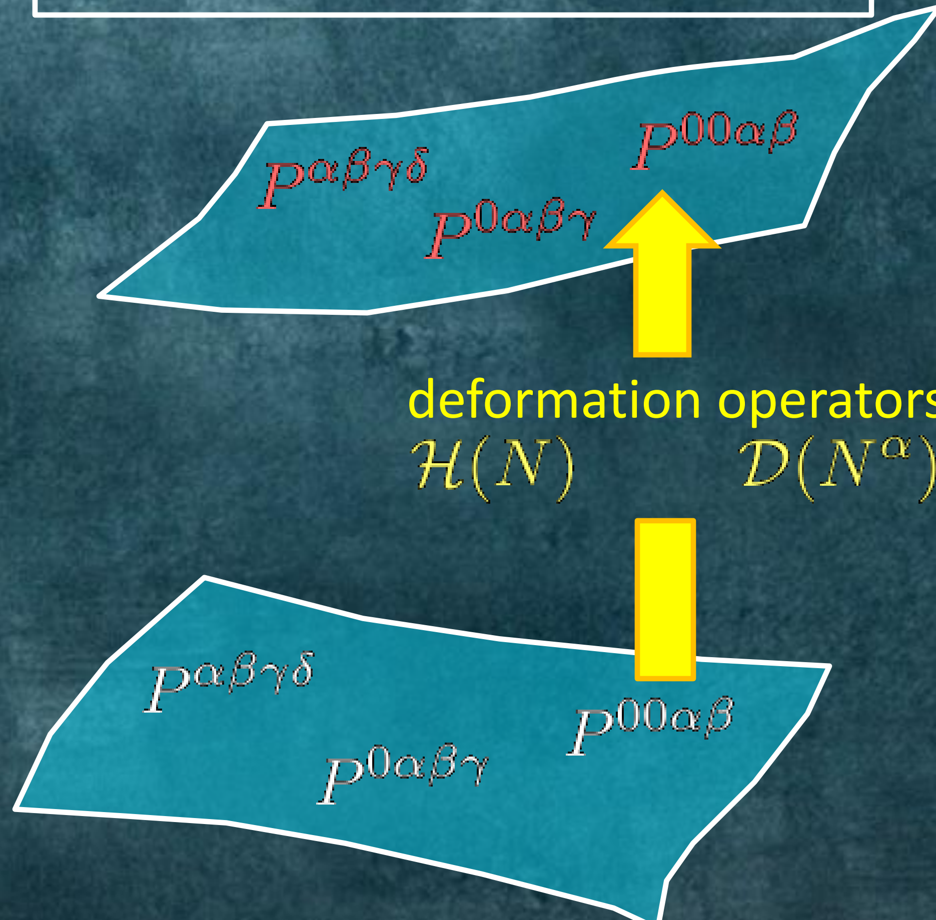
SET

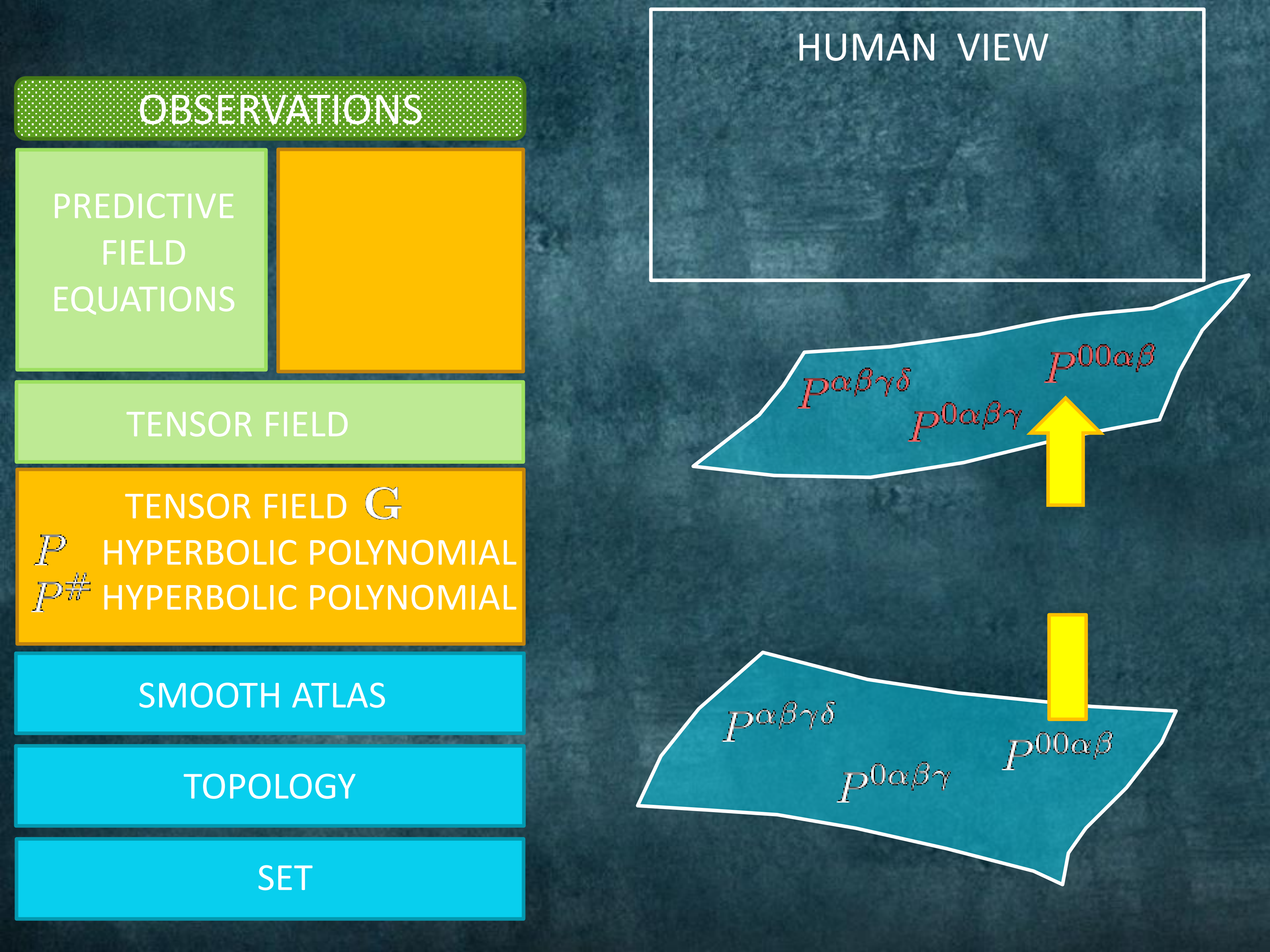
DIVINE VIEW

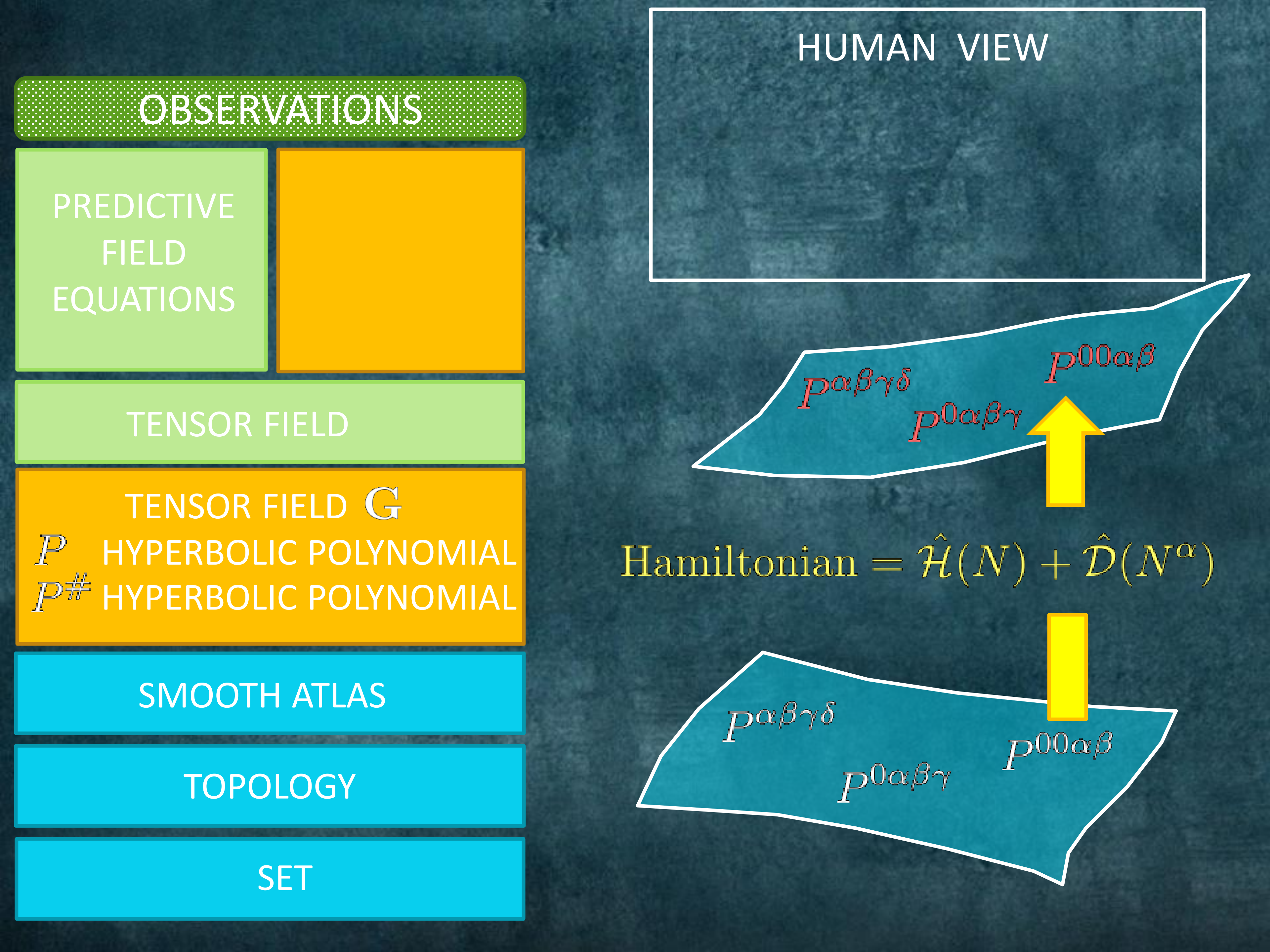
$$[\mathcal{H},\mathcal{H}] = P\,\mathcal{D}$$

$$[\mathcal{D},\mathcal{H}] = \mathcal{H}$$

$$[\mathcal{D},\mathcal{D}] = \mathcal{D}$$




HUMAN VIEW
OBSERVATIONS
PREDICTIVE FIELD EQUATIONS
TENSOR FIELD
TENSOR FIELD $\mathbf{G}$
P HYPERBOLIC POLYNOMIAL
$P^{\#}$ HYPERBOLIC POLYNOMIAL
SMOOTH ATLAS
TOPOLOGY
SET
$P^{\alpha\beta\gamma\delta}$
$P^{0\alpha\beta\gamma}$
$P^{00\alpha\beta}$
$P^{\alpha\beta\gamma\delta}$
$P^{0\alpha\beta\gamma}$
$P^{00\alpha\beta}$


HUMAN VIEW
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TENSOR FIELD G
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$P^{\#}$ HYPERBOLIC POLYNOMIAL
SMOOTH ATLAS
TOPOLOGY
SET
$P^{\alpha\beta\gamma\delta}$
$P^{00\alpha\beta}$
$P^{0\alpha\beta\gamma}$
$\text{Hamiltonian} = \hat{\mathcal{H}}(N) + \hat{\mathcal{D}}(N^{\alpha})$
$P^{\alpha\beta\gamma\delta}$
$P^{00\alpha\beta}$
$P^{0\alpha\beta\gamma}$

OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

TENSOR FIELD $\mathbf{G}$

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

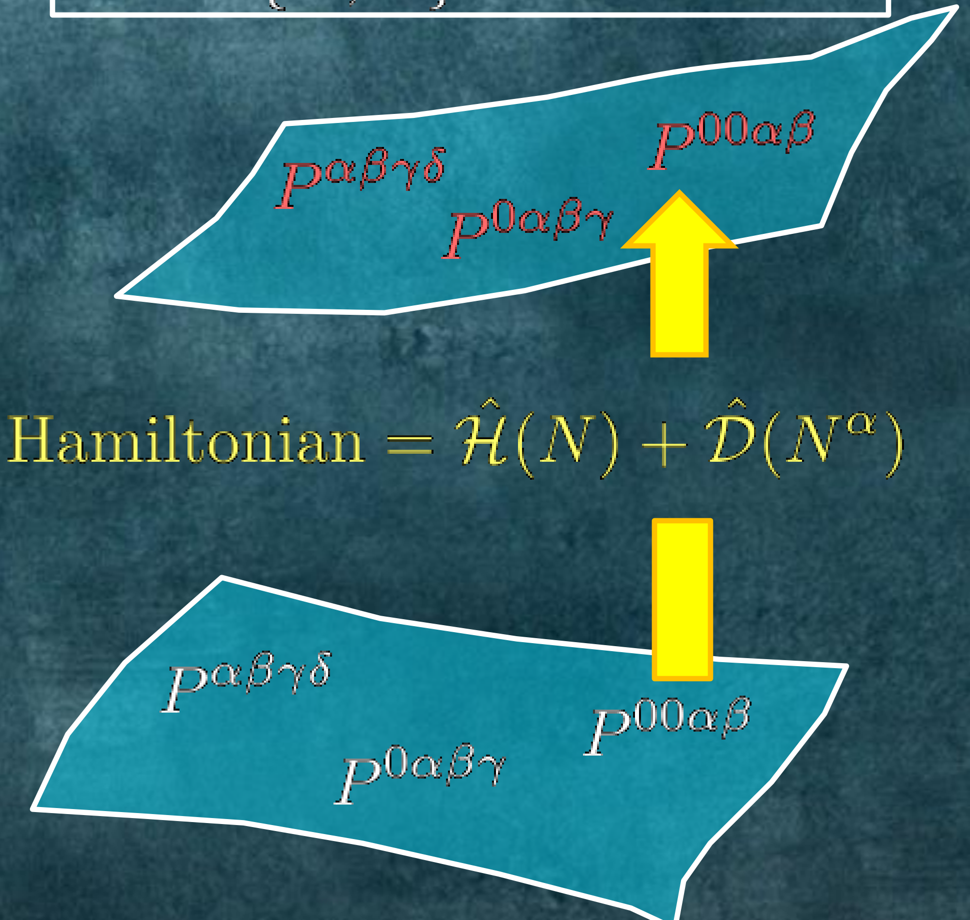
SET

HUMAN VIEW

$$\{\hat{\mathcal{H}}, \hat{\mathcal{H}}\} = P\,\hat{\mathcal{D}}$$

$$\{\hat{\mathcal{D}}, \hat{\mathcal{H}}\} = \hat{\mathcal{H}}$$

$$\{\hat{\mathcal{D}}, \hat{\mathcal{D}}\} = \hat{\mathcal{D}}$$



OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

TENSOR FIELD $\mathbf{G}$

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

SET

DIVINE VIEW

$$[\mathcal{H},\mathcal{H}] = P\,\mathcal{D}$$

$$[\mathcal{D},\mathcal{H}] = \mathcal{H}$$

$$[\mathcal{D},\mathcal{D}] = \mathcal{D}$$

HUMAN VIEW

$$\{\hat{\mathcal{H}},\hat{\mathcal{H}}\} = P\,\hat{\mathcal{D}}$$

$$\{\hat{\mathcal{D}},\hat{\mathcal{H}}\} = \hat{\mathcal{H}}$$

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OBSERVATIONS

PREDICTIVE FIELD EQUATIONS

TENSOR FIELD

TENSOR FIELD G

P HYPERBOLIC POLYNOMIAL

$P^{\#}$ HYPERBOLIC POLYNOMIAL

SMOOTH ATLAS

TOPOLOGY

SET

DIVINE VIEW

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hard (NON-LIE ALGEBRA) REPRESENTATION THEORY PROBLEM

[Rätzel, Rivera, FPS]

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only a HOMOGENEOUS LINEAR PDE PROBLEM

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Search for modified geometrodynamics

$\Updownarrow$

existence/uniqueness/solution of homogeneous linear PDE

hard (NON-LIE ALGEBRA) REPRESENTATION THEORY PROBLEM

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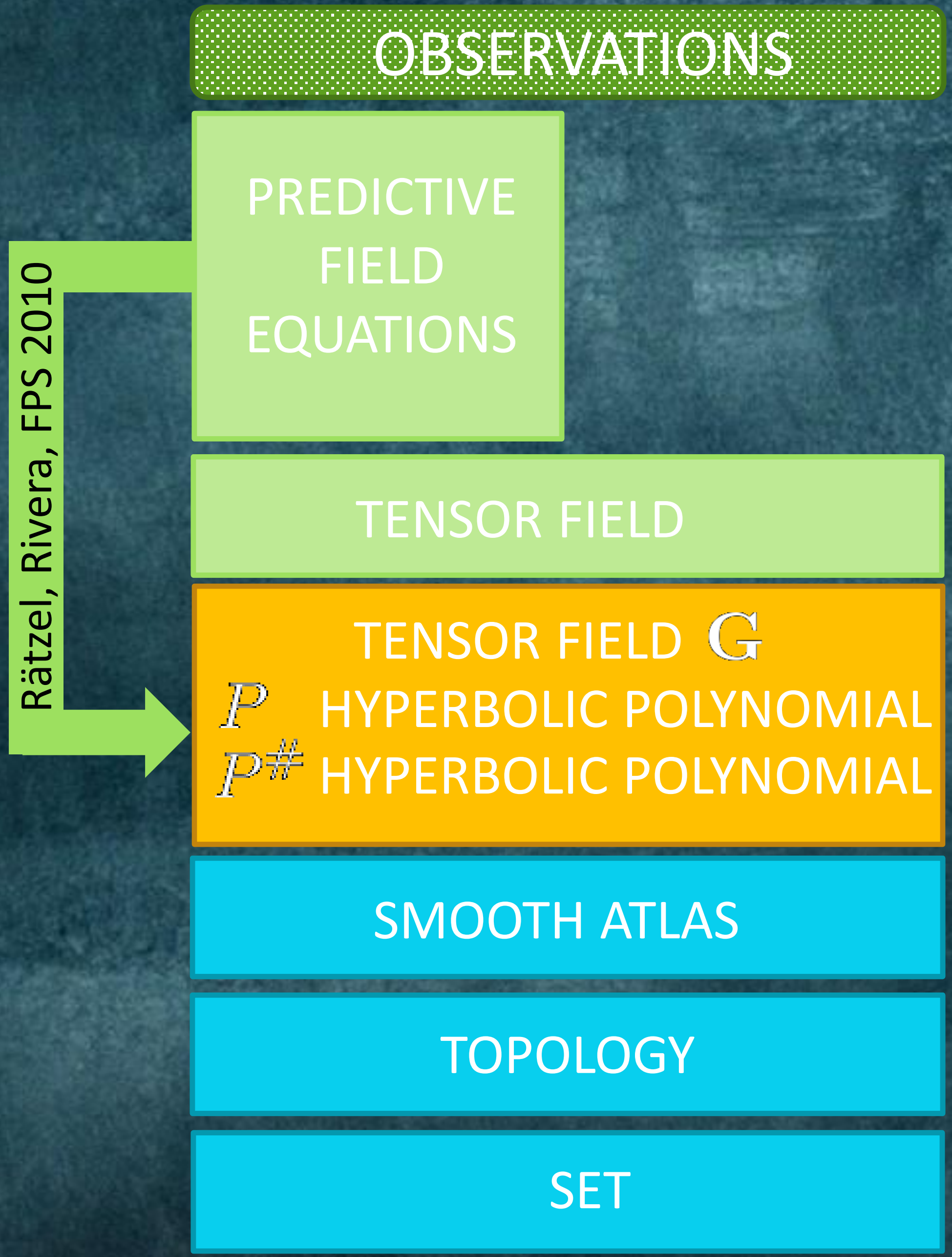

OBSERVATIONS
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Rätzel, Rivera, FPS 2010
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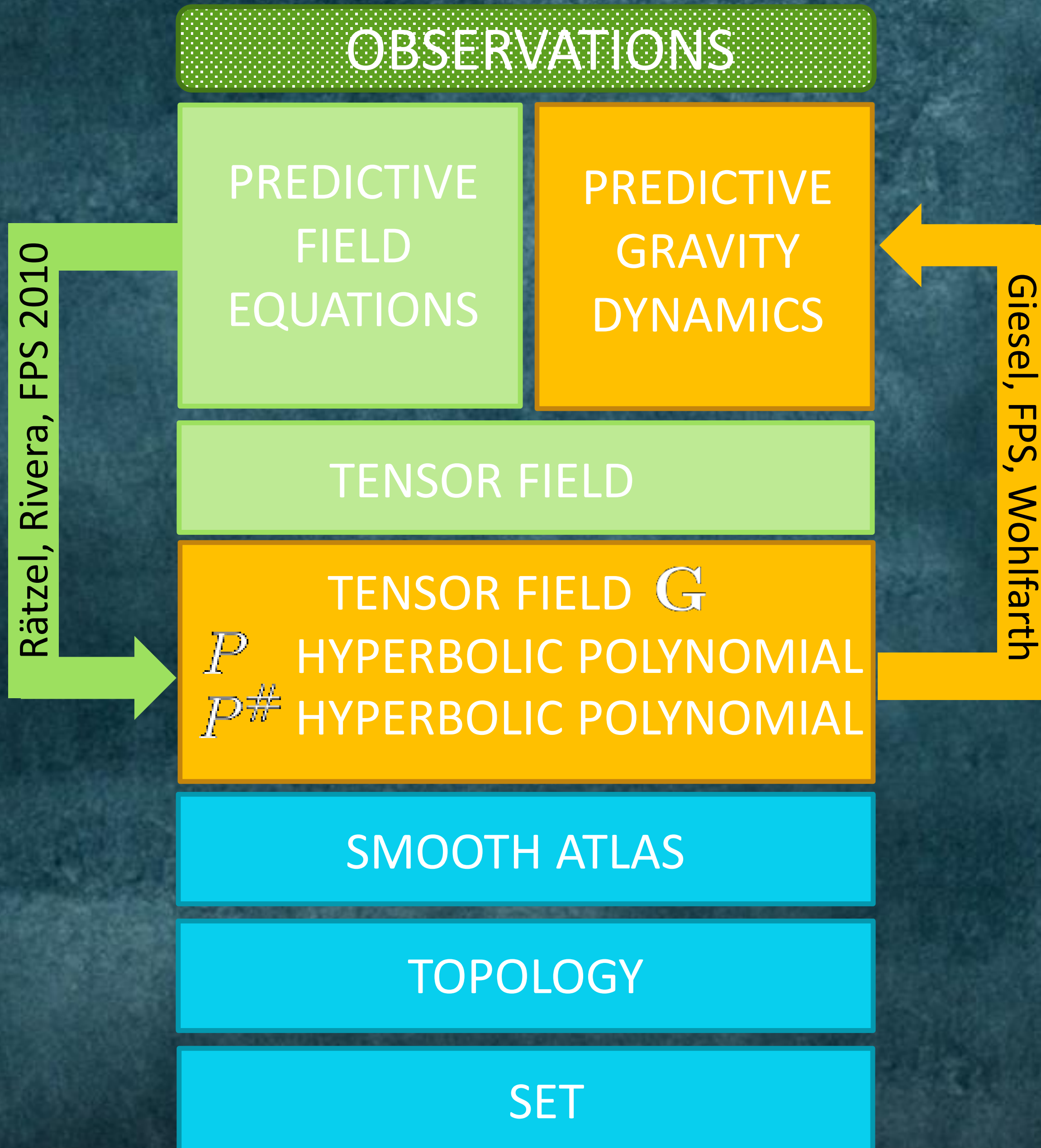
OBSERVATIONS
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TAKE-HOME MESSAGE

結果

- Lorentzian spacetimes are tailored to Maxwell theory.

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- Other geometries tailored to other matter severely restricted

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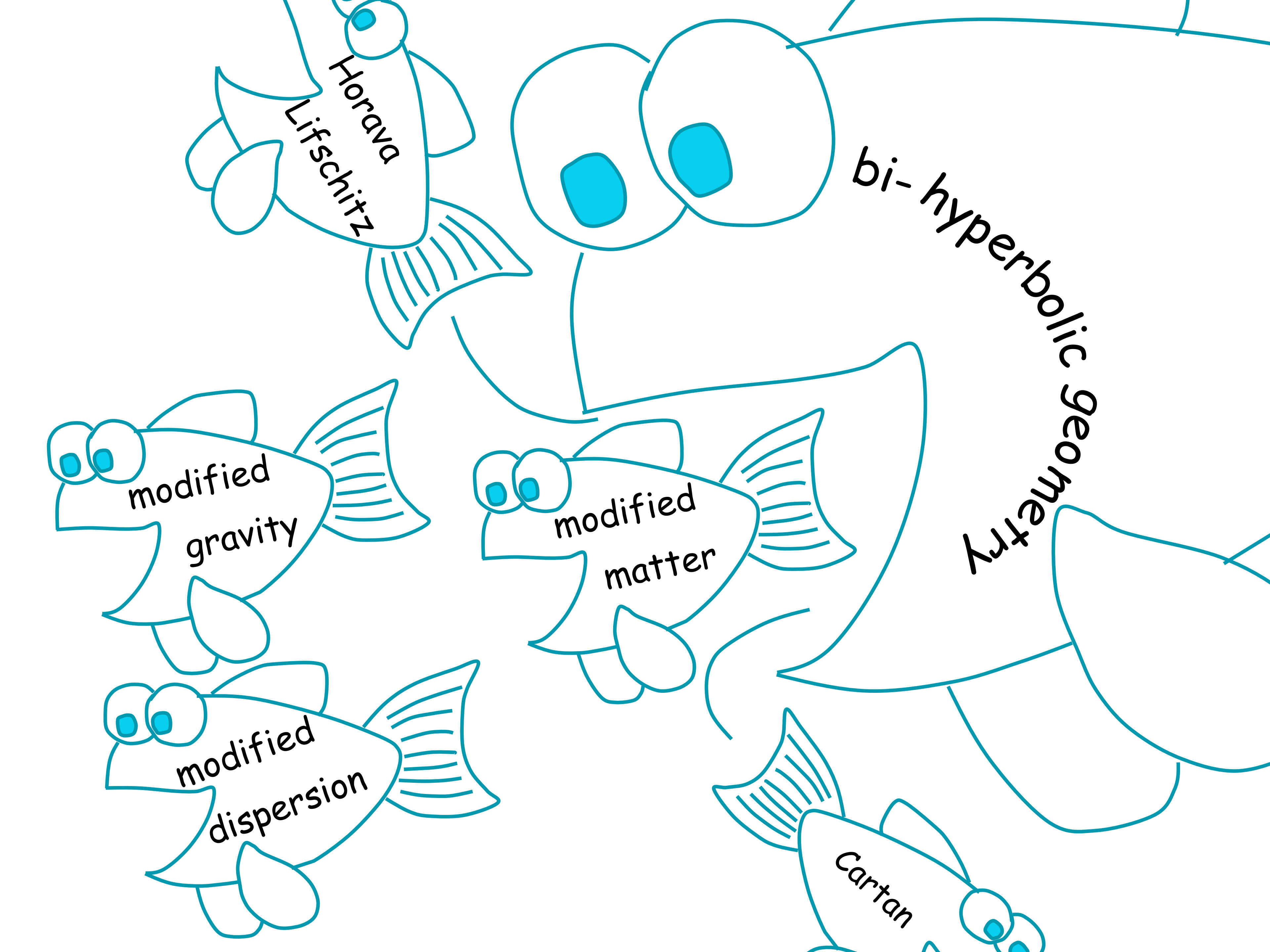
  - @ identification of all classical spacetime geometries
  - @ construction of all classical gravity dynamics
  - @ constraints for classical limits of quantum gravity
- It is all very simple.
- Mathematical certainty.


Horava Lifschitz
bi- hyperbolic geometry
modified gravity
modified matter
modified dispersion
Cartan

Example 1: **Metric geometry carrying Maxwell theory**

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$$G^{ab} = G^{(ab)}$$

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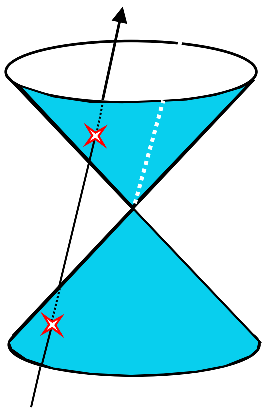

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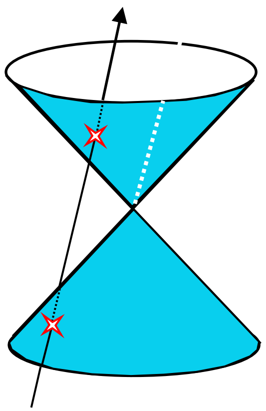

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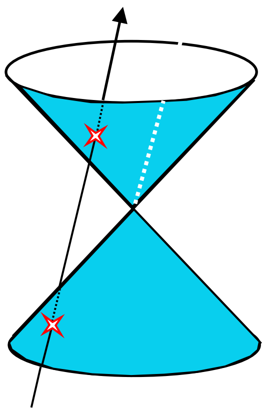

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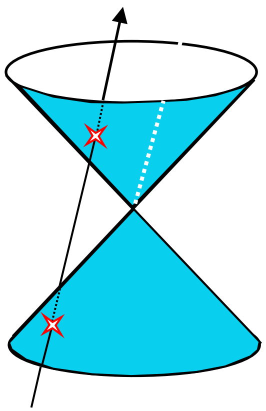

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Representation on 3-geometry phase space

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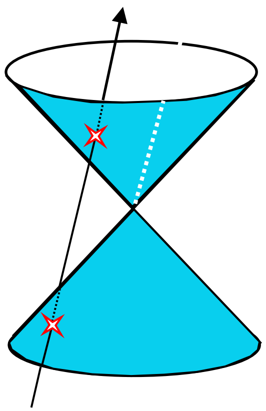

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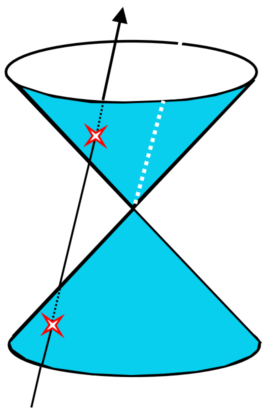
Representation on 3-geometry phase space

$$S[G] = \frac{1}{2\kappa} \int_M \omega_G\; (R[G] - 2\Lambda)$$

Example 2: Metric geometry carrying quartically interacting Proca



$$G^{ab} = G^{(ab)}$$

$$S[A] = \int vol_G \left[F_{ab}F^{ab} - \frac{1}{8}m^2 A_a A^a - \lambda(A_a A^a)^2 \right]$$

$$P^{(ijkl)} = G^{(ij|} \left[(1 + \lambda m^{-2} A_a A^a) G^{|kl)} + 2\lambda m^{-2} A^{|k} A^{l)} \right]$$

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Representation on 3-geometry phase space

Homogeneous linear PDE → modified gravity

solve

Example 3: **Area-metric geometry**

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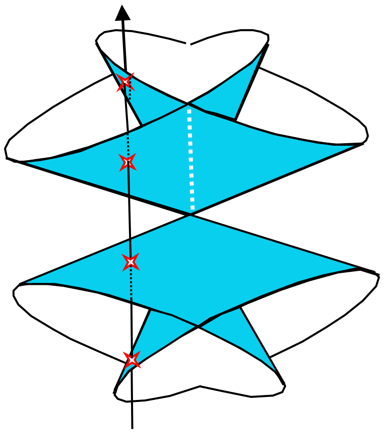

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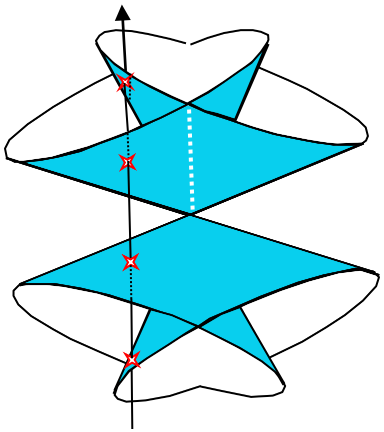

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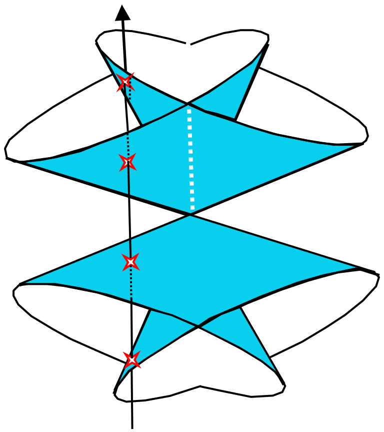

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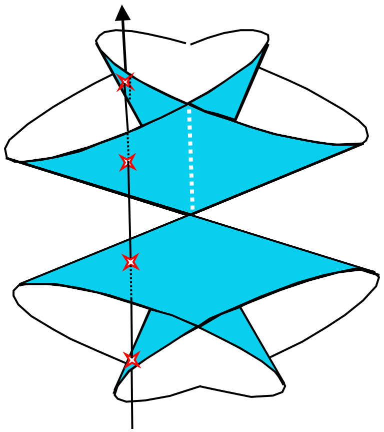

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Representation on 3-geometry phase space

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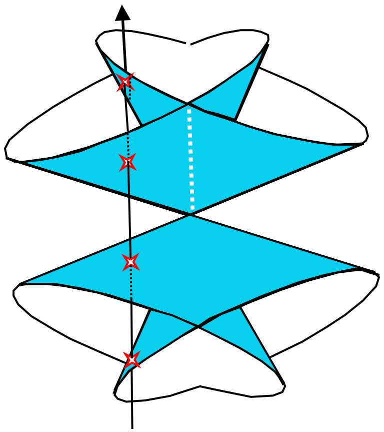

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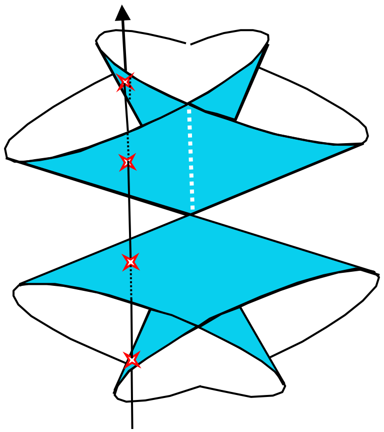
Representation on 3-geometry phase space

Homogeneous linear PDE

Example 3: **Area-metric geometry**



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Representation on 3-geometry phase space

Homogeneous linear PDE → **solve** → area metric gravity

good cosmology!

General case: **your favourite candidate geometry**

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$$(G_1, \ldots, G_m)$$

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$$(G_1, \ldots, G_m)$$

↓ **your favourite matter** $S[A_1, \ldots, A_n;\, G_1, \ldots, G_m]$

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$$P$$

$$P^\#$$

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P $\qquad \deg P$

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Representation on 3-geometry phase space

Homogeneous linear PDE

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Hypersurface deformation algebra

$$[\mathcal{H}(N), \mathcal{H}(M)] = -\mathcal{D}((\deg P - 1) P^{\alpha\beta}(M\partial_\beta N - N\partial_\beta M)\partial_\alpha)\,,$$

$$[\mathcal{D}(N^\alpha\partial_\alpha), \mathcal{H}(M)] = -\mathcal{H}(N^\alpha\partial_\alpha M)\,,$$

$$[\mathcal{D}(N^\alpha\partial_\alpha), \mathcal{D}(M^\beta\partial_\beta)] = -\mathcal{D}((N^\beta\partial_\beta M^\alpha - M^\beta\partial_\beta N^\alpha)\partial_\alpha)\,.$$

Representation on 3-geometry phase space

Homogeneous linear PDE → **solve** → $(G_1, \ldots, G_m)$ gravity