A Stringy Mechanism for a Small Cosmological Constant

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This talk is based on work with **Yoske Sumitomo** :

arXiv:1204.5177 (JCAP **1208** (2012) 032) and
arXiv:1209.5086

Applied to :
Large Volume Flux Compactification Scenario in Type IIB String Theory

in particular : M. Rummel and A. Westphal, arXiv:1107.2115

Also : A. Aazami and R. Easther, hep-th/0512102;
X. Chen, G. Shiu, Y. Sumitomo and S.-H.H. Tye, arXiv:1112.3338;
T. Bachlechner, D. Marsh, L. McAllister and T. Wrase, arXiv:1207.2763.

Background

- There is very strong evidence that we are living in a de-Sitter vacuum with a very small positive cosmological constant Λ,

$$\Lambda \sim +10^{-122} M_P^4$$

- Why dark energy contributes 70% of the content of our universe ?

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- Why dark energy contributes 70% of the content of our universe ? Why not 99.999999....999999% ?
- There is strong evidence that our universe has gone through an inflationary period, when the vacuum energy is below the Planck scale but much higher than the TeV scale.

What is considered to be a natural explanation for the observed dark energy ?

- Given the scale of the underlying theory, how the observed value emerges ?
  E.g., In gravity, we have M_P, so we have to explain why

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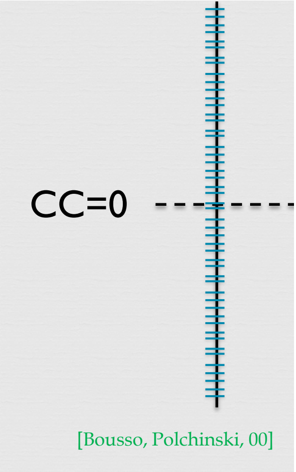
  E.g., String theory has string scale M_S, so it must generate both M_P and Λ from M_S.

The situation in string theory : J types of 4-form fluxes $F^i_{\mu\nu\rho\sigma}$



$$\lambda = \lambda_{\text{bare}} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2$$

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Why nature picks such a very small positive Λ ?

- We present a possible Stringy Mechanism why a very small Λ may be preferred.
- We use a simple but non-trivial model to illustrate the main idea. We believe this is the direction to go.

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- Find the probability distribution $P(\Lambda)$ for $\Lambda(a_j)$ as we sweep through allowed $\{a_j\}$.
- As we shall see in examples, $P(\Lambda)$ tends to peak at $\Lambda = 0$.
- It is simpler to find the value $w_0(a_k)$ of the superpotential $W(a_k, u_p)$ at the supersymmetric solution; then we see that $P(w_0)$ tends to peak at $w_0 = 0$ as well.

This peaking behavior of $P(\Lambda)$ at $\Lambda = 0$ is quiet generic.

The Basic Idea is very simple :

It is based on the properties of the probability distribution of functions of random variables.

Does Λ has the right functional form ? Do the random parameters have the right range and distribution ?

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An example :

Consider a set of random variables x_i $(i = 1, 2, ..., n)$. Let the probability distribution of each x_i be uniform in the range $[-1, +1]$. What is the probability distribution of their product z ?

Suppose we have n random variables x_i $(i = 1, 2, \cdots, n)$, each with probability distribution $P_i(x_i)$, where $\int dx_i P_i(x_i) = 1$. Let

$$z = f(x_1, x_2, \cdots, x_n)$$

Then the probability distribution $P(z)$ of z is given by

$$P(z) = \int dx_1 P_1(x_1)\, dx_2 P_2(x_2) \cdots dx_n P_n(x_n)\, \delta\left(f(x_i) - z\right)$$

$$\int P(z) dz = 1$$

so the probability distribution $P(z)$ of z can always be properly normalized, even when $P(z)$ diverges at $z = 0$ and/or elsewhere.

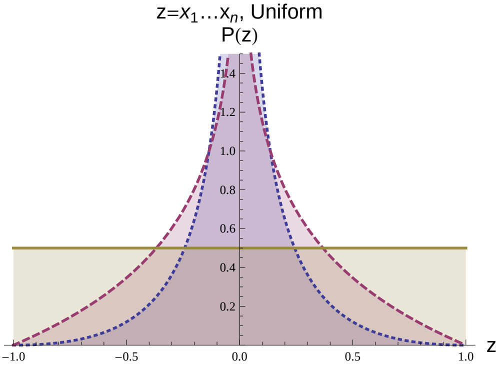
$z = x_1 x_2$

Let x_j to have a uniform distribution $P(x_j) = 1$ between 0 and 1. What is the probability distribution $P(z)$ of the product $z = x_1 x_2$?

$$P(z) = \int_0^1 dx_1 \int_0^1 dx_2\, \delta(x_1 x_2 - z) = \int_z^1 dx_1 \frac{1}{x_1} = \ln\left(\frac{1}{z}\right)$$

for $0 \leq z \leq 1$.

Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$



$$P(z) = \frac{1}{2(n-1)!} \left(\ln \frac{1}{|z|} \right)^{n-1}$$

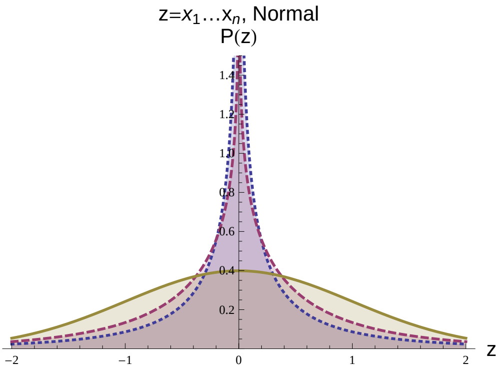


Figure: The product distribution $P(z)$ is for $z = x_1$ (solid brown curve for normal distribution), $z = x_1 x_2$ (red dashed curve), and $z = x_1 x_2 x_3$ (blue dotted curve), respectively. In general, the curves are given by the Meijer-G function.

Basic Properties

Let x_j to have a uniform distribution $P(x_j) = 1/L$ between 0 and L. What is the probability distribution $P(z)$ of the product $z = x_1 x_2$?

$$P(z) = \int_0^L \frac{dx_1}{L} \int_0^L \frac{dx_2}{L} \, \delta(x_1 x_2 - z) = \frac{1}{L^2} \ln\left(\frac{L^2}{z}\right)$$

for $0 \leq z \leq L^2$.

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For $z = x_1 x_2 ... x_n$, we have

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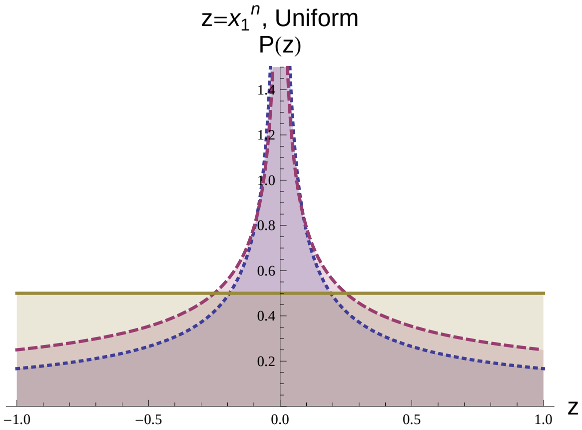
Since $< x_j >= \int_0^L dx_j \, (x_j/L) = L/2$, so $< z >= (L/2)^n$.

Probability distribution $P(z)$ for $z = x_1^n$

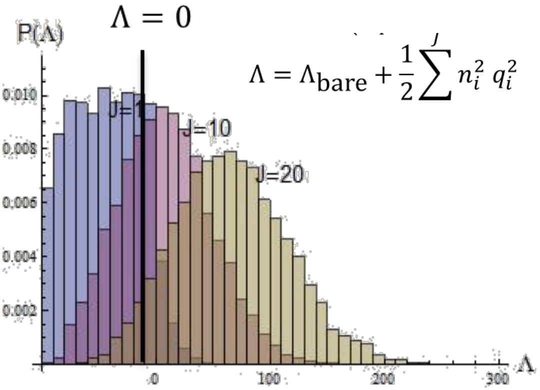


$$P(z) \sim z^{-1+1/n}$$

Probability distribution $P(z)$

z	Asymptote of $P(z)$ at $z = 0$
$x_1 \cdots x_n$	$(\ln(1/\|z\|))^{n-1}$
x_1^n	$z^{-1+1/n}$
$x_1^n \cdots x_m^n$	$z^{-1+1/n}(\ln(1/\|z\|))^{m-1}$
$x_1^m x_2^n$	$(z^{-1+1/m} - z^{-1+1/n})/(m-n)$
$x_1 \cdots x_m \; / \; y_1 \cdots y_n$	$(\ln(1/\|z\|))^{m-1}$
x_1^m / y_1^n	$z^{-1+1/m}$
$x_1^{n_1} + \cdots + x_m^{n_m}$	$z^{-1+1/n_1 + \cdots 1/n_m}$
$x_1 x_2$, $0 < c = x_1/x_2 < \infty$	smooth
$x_1 x_2$, $0 \leq c = x_1/x_2$ or $c \leq \infty$	$\ln(1/\|z\|)$

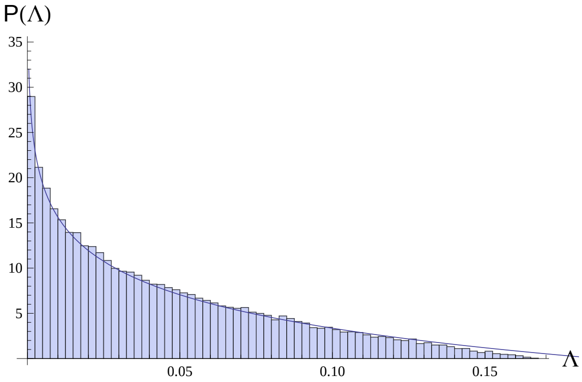
No peaking behavior for $P(\Lambda)$ if Λ is a sum of terms.



$$V(\phi) = a\phi - \frac{b}{2}\phi^2 + \frac{c}{3!}\phi^3$$

If ϕ is arbitrary $\rightarrow P(V = \Lambda)$ is smooth at $\Lambda = 0$.

Preference for Small Λ



$$
\begin{aligned}
V =& e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right), \\
K =& -2\ln(\mathcal{V} + \hat{\xi}/2) - \ln(S + \bar{S}) - \sum_j \ln(U_j + \bar{U}_j) \\
\mathcal{V} =& Vol/\alpha'^3 = \gamma_1 (T_1 + \bar{T}_1)^{3/2} - \sum_{i=2} \gamma_i (T_i + \bar{T}_i)^{3/2}, \\
\hat{\xi} =& -\frac{\zeta(3)\chi(M)}{4\sqrt{2}(2\pi)^3} \left(\frac{S + \bar{S}}{2} \right)^{3/2}, \\
W =& W_0(U_i, S) + \sum_{i=1}^{N_K} A_i e^{-a_i T_i}, \\
W_0(U_i, S) =& c_1 + \sum_j b_j U_j - s(c_2 + \sum_j d_j U_j)
\end{aligned}
$$

Typical Manifolds Studied

$$\chi(M) = 2(h^{1,1} - h^{2,1})$$

Manifold	$N_K = h^{1,1}$	$N_{cs} = h^{2,1}$	χ
$\mathcal{P}^4_{[1,1,1,6,9]}$	2	272	-540
$\mathcal{F}_{11}$	3	111	-216
$\mathcal{F}_{18}$	5	89	-168
$\mathcal{CP}^4_{[1,1,1,1,1]}$	1	$\mathcal{O}(100)$	$\mathcal{O}(-200)$

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- All parameters introduced are treated as random variables with some probability distributions.
- Find the supersymmetric solution $w_0 = W_0|_{\min}$ of W_0 for the complex structure moduli and insert this w_0 into V to stabilize the Kähler moduli.
- The functional form of $\Lambda = V_{\min}$ (and $w_0 = W_0|_{\min}$) in terms of the parameters are non-trivial.

Single Kähler Modulus Model

- $T_1 = t_1 + i\tau_1$, with $\tau_1 = 0$
- Consider the superpotential

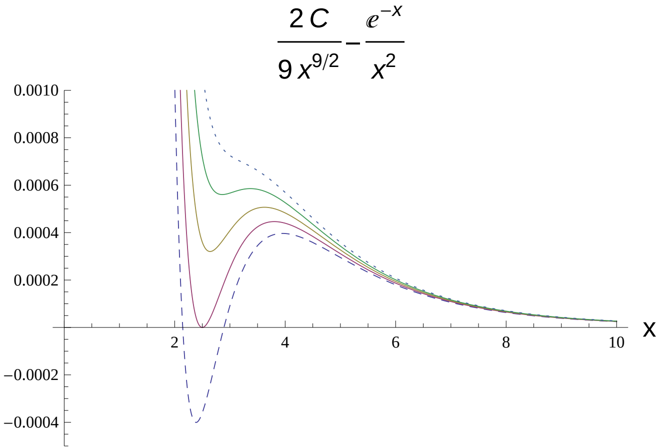
$$W = W_0 - A_1 e^{-x}$$

  where W_0 and A are (random) parameters and $x = a_1 t_1$ is the Kähler modulus.
- A stable vacuum can exist at $x = x_m$

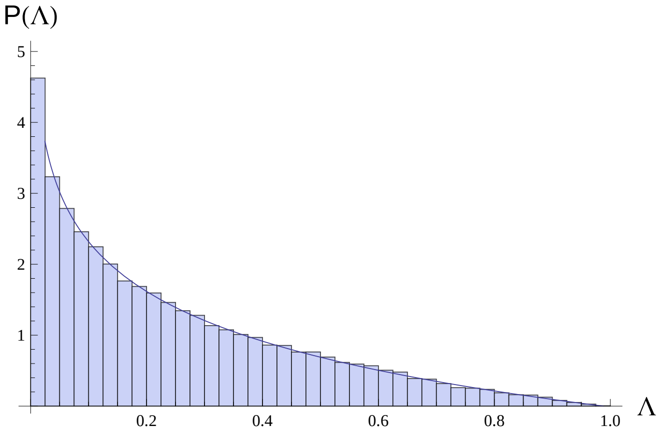
$$\Lambda = V_{min} = BW_0 A_1 \hat{\xi}(x_m - 2.5)$$

  where B is a constant.
- Let us treat W_0 and A_1 as random variables where $W_0/A_1 \sim C$ is constrained:
  $3.65 \lesssim C \lesssim 3.89, \quad 2.50 \leq x_m \lesssim 3.11$

The form of $V(x)$ with $W_0 A_1 \leq 0$



$P(\Lambda) \propto \ln(1/|\Lambda|)$ at $\Lambda \sim 0$



Multi-Complex Structure Moduli case

$$K = -2\ln(\mathcal{V} + \hat{\xi}/2) - \ln(S + \bar{S}) - \sum_j \ln(U_j + \bar{U}_j)$$

$$W = W_0(U_i, S) + \sum_{i=1}^{N_K} A_i e^{-a_i T_i}$$

$$W_0(U_i, S) = c_1 + \sum_j b_j U_j - s(c_2 + \sum_j d_j U_j)$$

Now consider the case with N_{cs} complex structure moduli U_i + the dilaton S + 1 Kähler modulus x. We solve for U_i and S at the supersymmetric point and then insert the resulting $w_0 = W_{0,min}$ into the Kähler uplift case to solve for x. There are $2n + 6$ parameters.

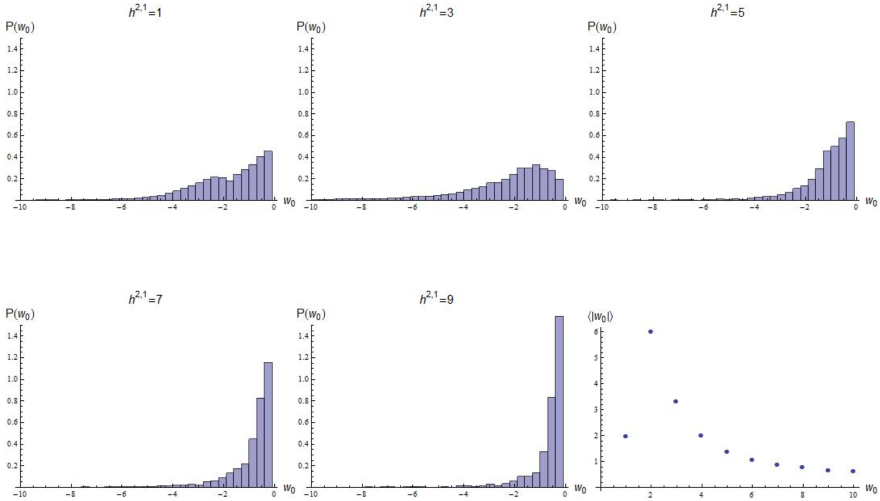
$$D_S W_0 = \partial_S W_0 + K_S W_0 = 0, \qquad D_i W_0 = 0$$
$$W_0(u_i, s) = c_1 + \sum_j b_j u_j - s(c_2 + \sum_j d_j u_j)$$

Solution :

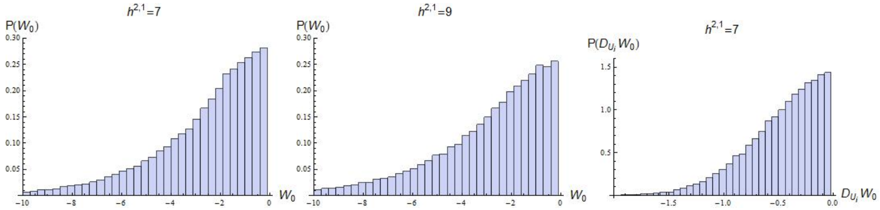
$$(N_{cs} - 2)\frac{c_1 + sc_2}{c_1 - sc_2} = \sum_{i=1}^{N_{cs}} \frac{b_i + sd_i}{b_i - sd_i}$$
$$w_0 = W_0|_{\min} = \frac{2(c_1 + sc_2)\Pi_1^n(b_i - sd_i)}{\sum_i (b_i + sd_i)\Pi_{j\neq i}(b_j - sd_j)}$$

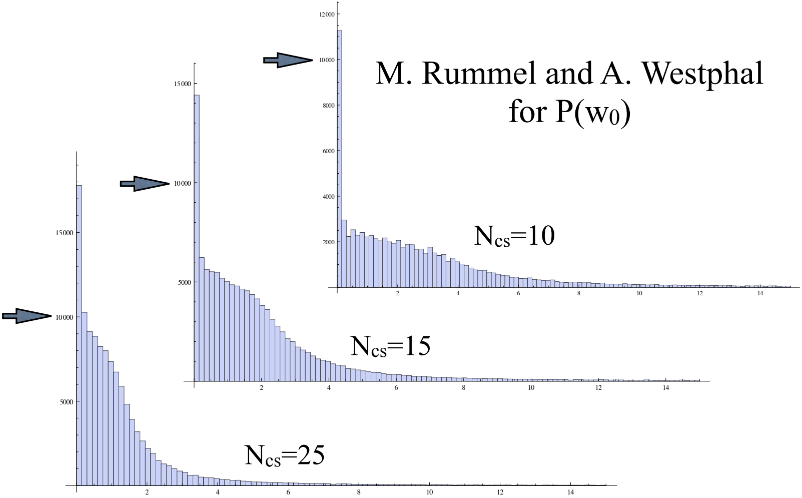
Then insert w_0 into the V for the Kähler moduli and solve :

$$\Lambda = \frac{e^{-5/2}}{9}\left(\frac{2}{5}\right)^2 \frac{-w_0 a_1^3 A_1}{\gamma_1^2}\left(x_m - \frac{5}{2}\right)$$

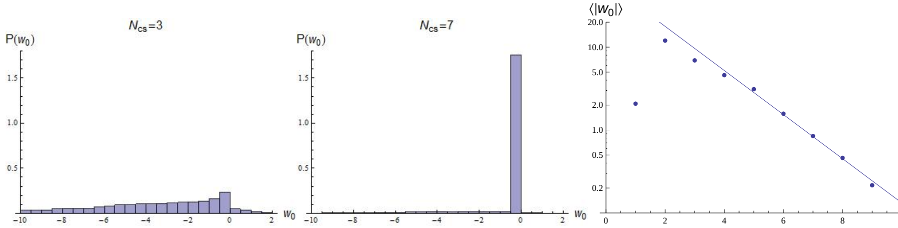

$h^{2,1}$=1
P(w_0)
w_0
$h^{2,1}$=3
P(w_0)
w_0
$h^{2,1}$=5
P(w_0)
w_0
$h^{2,1}$=7
P(w_0)
w_0
$h^{2,1}$=9
P(w_0)
w_0
⟨|w_0|⟩
w_0

If $P(W_0)$ and $P(D_i W_0)$ are truly independent :




M. Rummel and A. Westphal
for P(w0)
Ncs=10
Ncs=15
Ncs=25

$u_i = w_0/2b_i(1 - sr_i)$ leads to $b_i = -f(N_{cs})$



$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \overline{W} - 3 \left| W \right|^2 \right) \sim \frac{-w_0 a_1^3 A_1}{2\gamma_1^2 2^{N_{\rm cs}+1} s \prod u_i} \left(\frac{2C}{9x_1^{9/2}} - \frac{e^{-x_1}}{x_1^2} \right)$$

$$C = \frac{-27 w_0 \hat{\xi} a_1^{3/2}}{64\sqrt{2}\gamma_1 A_1}$$

$$x_1 = a_1 t_1$$

For $\Lambda = V_{min} \geq 0$ (with b_i fixed), $\langle \Lambda \rangle \sim e^{-2.56\, h^{2,1} + 7.40}$
For $h^{2,1} = N_{cs} = 113$, we'll have a small enough Λ.

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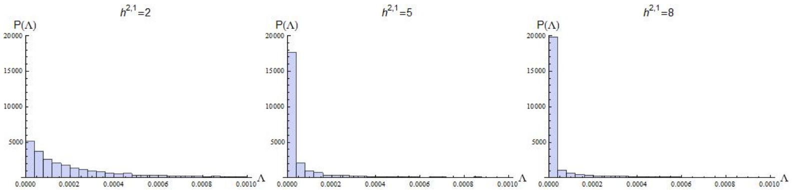
However, for larger $h^{2,1}$, the drop of $\langle \Lambda \rangle$ slows down appreciably.

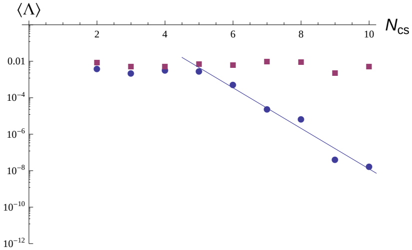
Pointed out to us by Rummel and Westphal

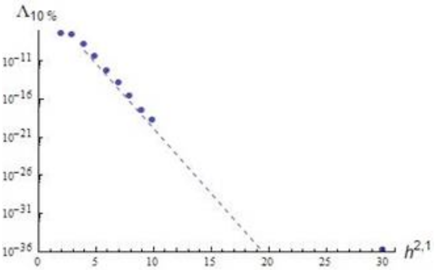
$P(\Lambda)$ as a function of $h^{2,1}$

Imposing the conditions $V_{barrier} \leq 1$, $s > 1$ and $u_i \geq 0$, for meta-stable vacua :



$P(\Lambda)$ is sharply peaked at $\Lambda = 0$ but with a long tail. So we ask : what is the cut-off $\Lambda_{10\%}$ if $\int_0^{\Lambda_{10\%}} P(\Lambda)\, d\Lambda = 10\%$?





$< \Lambda >$ versus $\Lambda_{10\%}$

$$\int_0^{\Lambda_{10\%}} P(\Lambda)\, d\Lambda = 10\%$$

That is, there is a 10% chance that $\Lambda_{10\%} \geq \Lambda \geq 0$.

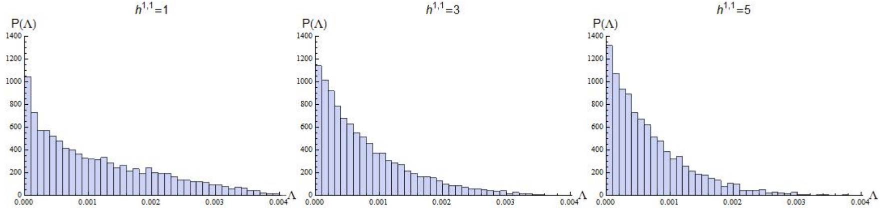
At $h^{2,1} = 10$, $< \Lambda > \sim 10^{-8}$ while $\Lambda_{10\%} \sim 10^{-19}$

At $h^{2,1} = 30$, $< \Lambda > \sim 10^{-11}$ while $\Lambda_{10\%} \sim 10^{-36}$

That is, for 30 complex structure moduli, there is a 10% chance that Λ is smaller than 10^{-36}.

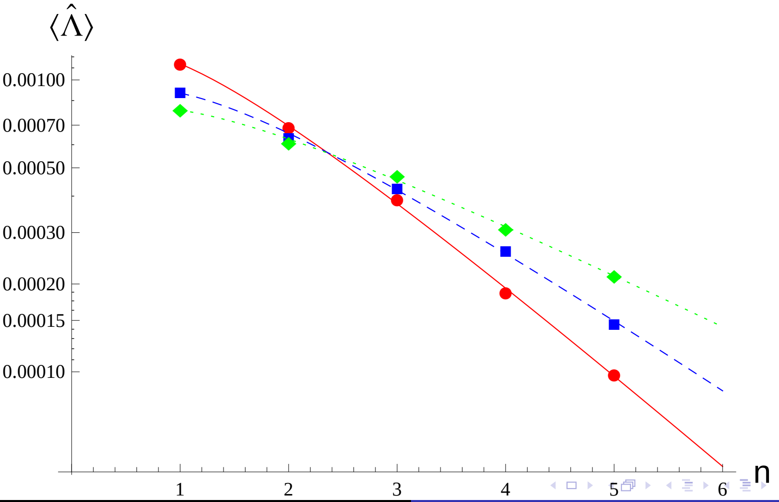
$P(\Lambda)$ as a function of $h^{1,1}$

When W_0 and A_i in the model are treated as random variables with uniform distributions in the range $[-1, 1]$:



$P(\Lambda = 0)$ is increasing (slowly) as $h^{1,1}$ increases.

Expectation value of Λ with peaking W_0



Expectation value of Λ with peaking W_0

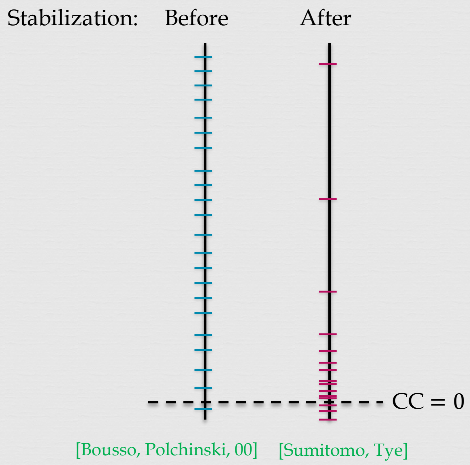
$$\begin{aligned}
\langle|\Lambda|\rangle_{N_K=1} =& 0.00251 n^{0.436} e^{-0.791 n}, \\
\langle|\Lambda|\rangle_{N_K=2} =& 0.00170 n^{0.457} e^{-0.633 n}, \\
\langle|\Lambda|\rangle_{N_K=3} =& 0.00125 n^{0.342} e^{-0.464 n}.
\end{aligned}$$

If the parameters A_i for the Kähler moduli are also peaked, as expected, then we need a lot less moduli.

Summary

- At high vacuum energies, no meta-stable vacua (because most extrema are unstable)
- At very low vacuum energies, meta-stable vacua begin to appear

New Picture


Stabilization:
Before
After
CC = 0
[Bousso, Polchinski, 00]
[Sumitomo, Tye]

Summary and Remarks

- At high vacuum energies, no stable vacua (because most extrema are unstable)
- At very low vacuum energies, meta-stable vacua begin to appear

Technical questions to be further studied :

- What is the back-reaction due to SUSY breaking ?
- What about higher (α' and loop) corrections ?
- How about the cosmological light moduli problem ?

The picture is very encouraging: many directions to be explored.