Surface operators and AdS/CFT correspondence

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based on
E. Koh, SY, arXiv:0812.1420 JHEP 0902:012,2009
arXiv:0904.1460

Introduction

Plan

- What is “surface operator”?
- Relation to string theory
- Very short summary of our work

Quantum field theory

Example: “free massless complex scalar field”

x^0, x^1, x^2, x^3 : Coordinates of 4-dim Euclidean space

$\Phi(x)$: Complex valued function “field”

$S[\Phi]$: Real valued functional. “Action”

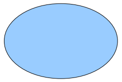
we choose $S[\Phi] = \int d^4x \, \partial_\mu \Phi \partial^\mu \bar{\Phi}$

Expectation value (correlation function)

$$\langle \bigcirc \rangle := \frac{1}{Z} \int_{x \to \infty, \Phi \to 0} D\Phi \bigcirc e^{-S[\Phi]}$$

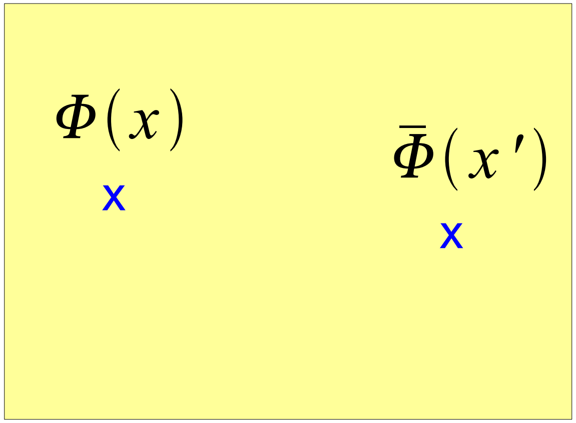
$$Z = \int D\Phi \, e^{-S[\Phi]}$$

Local operator

 : "operators"

Example of "local operator": $\Phi(x), \quad \bar{\Phi}(x), \quad \Phi^2(x), \quad \cdots$

$$\langle \Phi(x) \rangle = 0$$



$$\langle \Phi(x) \bar{\Phi}(x') \rangle = \frac{1}{2\pi^2 |x - x'|^2}$$

Line operator

Example of “line operator”

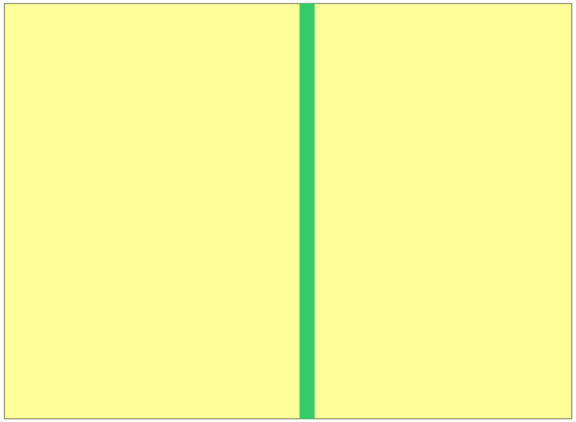
$$W_\alpha = \exp \int_{x^i=0} dx^0 (\bar{\alpha}\Phi(x) + \alpha\bar{\Phi}(x))$$

α : complex parameter

More generally

$$W_\alpha(C) = \exp \int_C |dx| (\bar{\alpha}\Phi(x) + \alpha\bar{\Phi}(x))$$

Introducing test particle source



$$\langle \bigcirc\, W_\alpha \rangle = Z^{-1} \int D\Phi \, \bigcirc\, \underbrace{W_\alpha e^{-S[\Phi]}}_{e^{-\tilde{S}[\Phi]}}$$

$$\tilde{S}[\Phi] = \int d^4 x \, \partial_\mu \Phi \, \partial^\mu \bar{\Phi} - \int_{x^i=0} dx^0 \left(\bar{\alpha} \Phi(x) + \alpha \bar{\Phi}(x) \right)$$

Saddle point $\dfrac{\delta \tilde{S}}{\delta \bar{\Phi}} = 0$

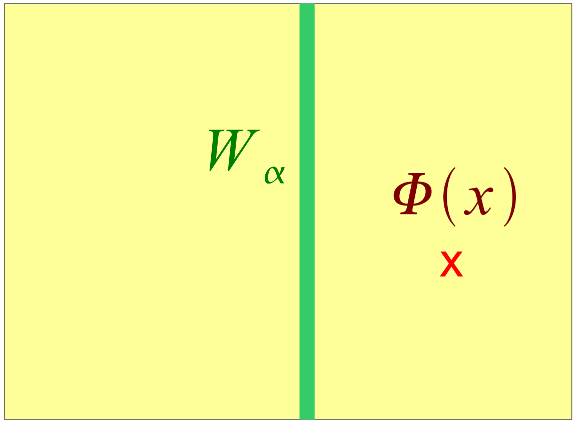
$$\partial_\mu \partial^\mu \Phi = - \alpha \, \delta^3 (x^i)$$

particle source for Φ

Solution $\Phi(x) = \Phi_0(x) = -\dfrac{\alpha}{4 \pi l}$ $\quad l := \sqrt{x_1^2 + x_2^2 + x_3^2}$

Correlation function with a local operator

$$\frac{\langle \Phi(x) W_\alpha \rangle}{\langle W_\alpha \rangle} = \Phi_0(x)$$



Change integration variable $\quad \Phi = \Phi_0 + \varphi$

$$\int D\Phi = \int D\varphi \qquad \tilde{S}[\Phi] = \tilde{S}[\Phi_0] + S[\varphi]$$

$$\langle \Phi(x) W_\alpha \rangle = \underbrace{\langle \Phi_0(x) W_\alpha \rangle}_{\Phi_0(x)\langle W_\alpha \rangle} + \underbrace{\langle \varphi(x) W_\alpha \rangle}_{0}$$

Surface operator

Insert string source $O_\beta(\Sigma) = \exp \pi \int_{z=0} dx^0 dx^1 (\beta \partial_z \bar{\Phi} + \bar{\beta} \partial_{\bar{z}} \Phi)$

↓

Classical solution (saddle point)

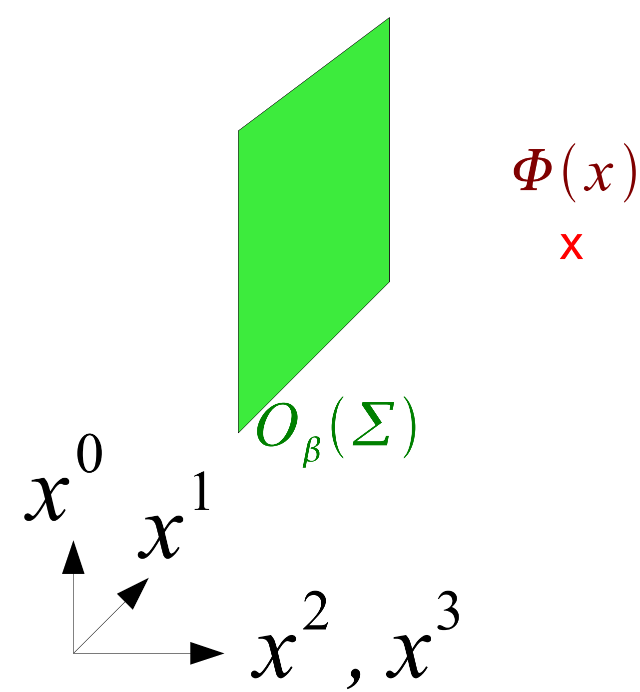
$$\Phi = \Phi_0 := \frac{\beta}{z}$$

$$z := x^2 + i x^3$$

β : constant

Correlation function with a local operator

$$\frac{\langle \Phi(x) O_\beta(\Sigma) \rangle}{\langle O_\beta(\Sigma) \rangle} = \Phi_0(x)$$



Motivation

- Phase structure of quantum field theories

- To understand “branes” in string theory via AdS/CFT

AdS/CFT correspondence

AdS / CFT

AdS = type IIB superstring AdS5 x S5

CFT = 4dim N=4 Super YM theory SU(N)

- conformal invariance
- massless scalars

some objects ⟺ (local or non-local) operaotors

ex. surface oprator

GKPW prescription ⟺ correlation function

Problem

What is gravity dual of a surface operator?

Correlation functions?

1/2 BPS surface operator

[Gukov, Witten '06]

1/2 BPS surface operator in N=4 SYM

Gravity dual= a configuration of D3-brane

[Constable, Erdmenger,Guralnik,Kirsch '02]

[Gomis, Matsuura '07]

Gravity dual= Bubbling AdS geometry (a classical solution)

[Lin, Lunin, Maldacena '04], [Lin, Maldacena '05]

[Drukker, Gomis, Matsuura '08]

Correlation functions are calculated in the gauge theory side and the gravity side

⇨ Agree!

Summary of our work

[Koh, SY '08, '09]

The case with branch cuts $\Phi = \frac{\beta}{z^{n/m}}$

- 1/4 BPS surface operators in N=4 SYM
- 1/2 BPS surface operators in Klebanov-Witten theory

Propose a gravity dual of these surface operators
Check the supersymmetry
Calculate the correlation function with local operators

→ Support AdS/CFT correspondence

Plan

- Review of 1/2 BPS surface operators in N=4 SYM
  - definition and symmetry
  - Gravity dual
- 1/4 BPS surface operators in N=4 SYM
  - definition and symmetry
  - Gravity dual
  - correlation function
- 1/2 BPS surface operators in Klebanov-Witten theory
  - definition and symmetry
  - Gravity dual
  - correlation function

4-dim N=4 SYM
1/2 BPS surface operator

Definition of a operator

Not all the operators can be written as functions of the fields in the Lagrangian

Example: 2-dim massless compact free boson

$$S=\frac{1}{2\pi}\int d^2 z\,\partial_z\phi\,\partial_{\bar{z}}\phi \qquad \phi\simeq\phi+2\pi R$$

Vertex operator of momentum p $\qquad O(z)=\exp(ip\phi)$

Winding modes?

Definition of a operator by a boundary condition

Winding mode operator can be written in terms of boundary condition (or OPE) as

$$\phi(z)\tilde{O}(0) \sim \frac{wR}{2i}(\log z - \log \bar{z})\tilde{O}(0)$$

- Correlation function can be defined by the path-integral under this boundary condition

Classical solution with singularity

$$\phi(z) = \frac{wR}{2i}(\log z - \log \bar{z})$$

→ an operator localized at the singularity

4-dim N=4 super Yang-Mills theory

- fields

$$A_\mu, \mu = 0,1,2,3 \qquad \psi \qquad \phi_i, i = 4,\cdots,9$$

- action

$$S_{YM} = \frac{2N}{\lambda} \int d^4x \, \mathrm{tr}\left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots \right]$$

$$\lambda = g_{YM}^2 N$$: 't Hooft coupling

- global symmetry

SO(2,4) x SO(6)

Supersymmetry

→ PSU(2,2|4)

1/2 BPS surface operator

[Gukov, Witten '06]

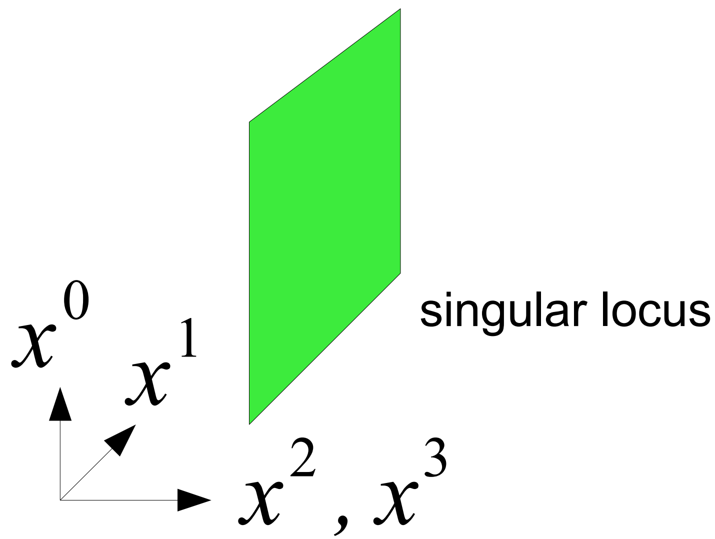
4-dim N=4 SYM

$$\Phi := \phi_4 + i\,\phi_5$$

$$\Phi = diag\left(\frac{\beta}{z}, 0, 0, \ldots, 0\right)$$

$$z = x^2 + i x^3$$

β : constant



- This configuration is a classical solution
- singular locus $z=0$

parallel to x^0, x^1 direction

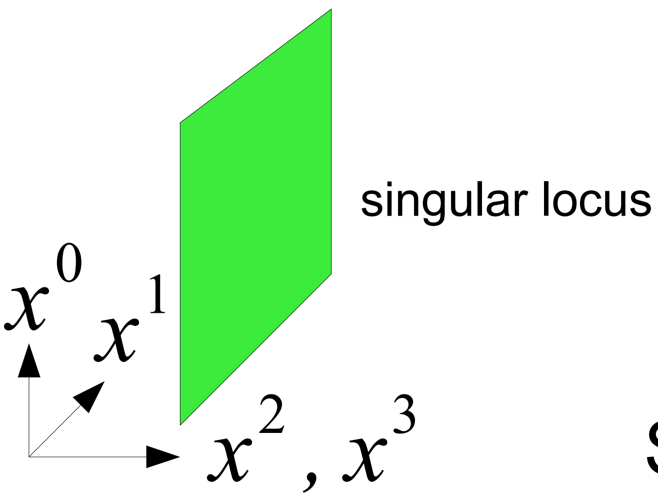
→ Operator localized at $z=0$

Symmetry of the classical solution

$$\Phi = diag(\frac{\beta}{z}, 0,0,\ldots,0)$$

$$\Phi := \phi_4 + i\,\phi_5$$

$$z = x^2 + ix^3$$



$\phi^6, \ldots, \phi^9$ rotation

SO(2,2) x SO(2) x SO(4)

SO(2,2): 2 dim global conformal symmetry

SO(2): diagnal subgrop of

- rotation of x^2, x^3
- rotation of ϕ^4, ϕ^5

Supersymmetry of the classical solution

The classsical solution preserves half of the supersymmetry

$$\delta \psi = D_{\mu} \phi_{I} \Gamma^{\mu I} \epsilon = 0 \Rightarrow (1 + \Gamma^{2345}) \epsilon = 0$$

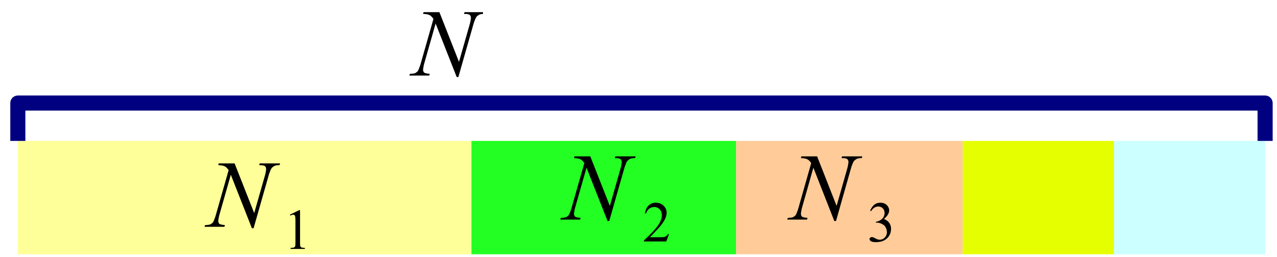
$\Rightarrow$ 1/2 BPS

Generalization

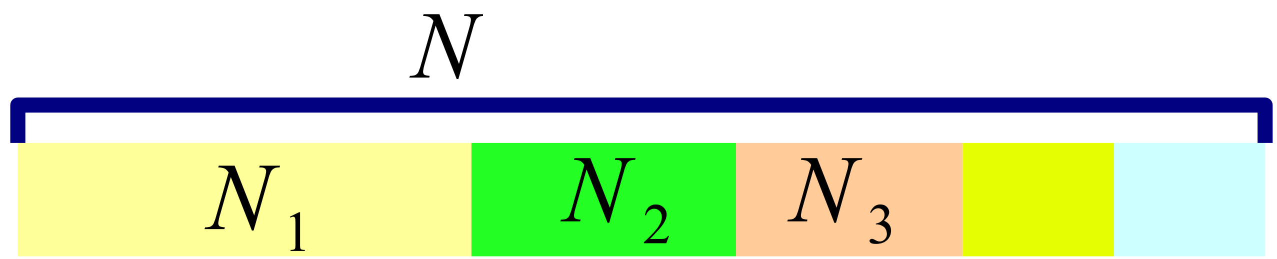
M : integer

$N_i, \quad i = 1, \ldots, M$: partition of N $\quad \sum_{i=1}^{M} N_i = N$

$$\Phi = \frac{1}{z} diag$$



$$A = \frac{dz}{2\pi i z} diag$$



Generalization

$$\Phi = \frac{1}{z} diag(\overbrace{\beta_1, \ldots, \beta_1}^{N_1}, \beta_2, \ldots, \beta_{M-1}, \overbrace{\beta_M, \ldots, \beta_M}^{N_M})$$

$$A = \frac{dz}{2\pi i z} diag(\overbrace{\alpha_1, \ldots, \alpha_1}^{N_1}, \alpha_2, \ldots, \alpha_{M-1}, \overbrace{\alpha_M, \ldots, \alpha_M}^{N_M})$$

$\exp[i \sum_i \eta_i \int_\Sigma tr_{N_i} F]$ insertion

α_i, η_i :real

β_i :complex

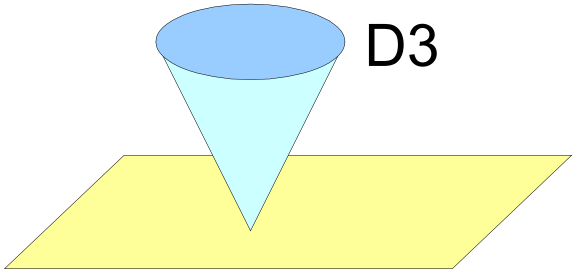
parameter $(\beta_i, \alpha_i, \eta_i), \quad i = 1, \ldots, M$

Gravity dual of 1/2 BPS surface operator

[Constable, Erdmenger,Guralnik,Kirsch '02], [Gukov, Witten '06], [Gomis, Matsuura '07], [Drukker, Gomis, Matsuura '08], [Lin, Lunin, Maldacena '04], [Lin, Maldacena '05]

- D3-brane probe

AdS3 x S1 shaped



SO(2,2) x SO(2) x SO(4)

supersymmetry

↕

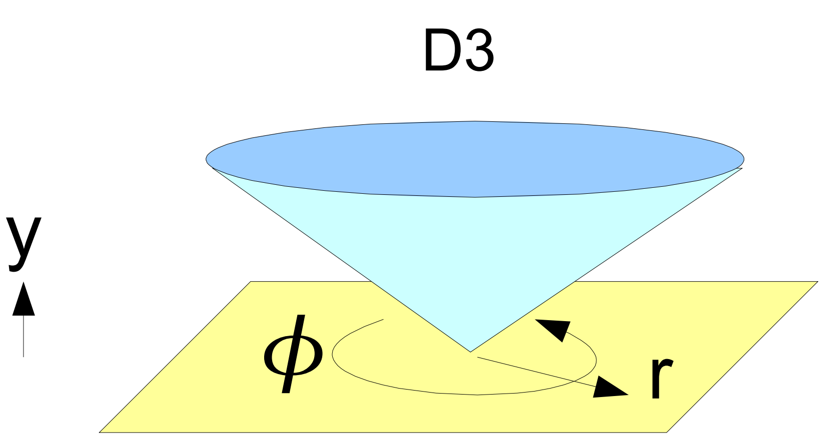
$$\Phi = diag\left(\frac{\beta}{z}, 0, 0, \ldots, 0\right)$$

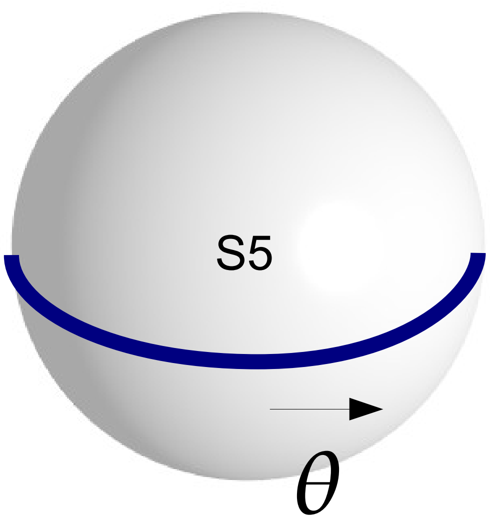
AdS5 coordinates $(y, r, \phi, x_{1,} x_2)$

Coordinate of a great circle in S5 θ

$$ds^2 = \frac{1}{y^2}(dy^2 + dr^2 + r^2 d\phi^2 + dx_0^2 + dx_1^2) + d\theta^2$$

D3-brane $\kappa y = r, \quad \theta = \phi$





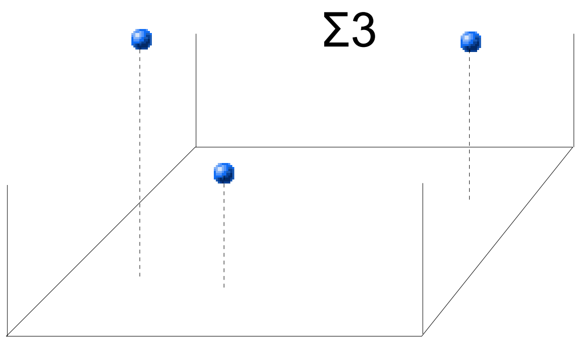
- Bubbling geometry

$$N_i \simeq N$$

Large number of D3-brane get together and back-reaction cannot be ignored

AdS3 x S3 x S1 x Σ3

SO(2,2) x SO(4) x SO(2)



Plan

- Review of 1/2 BPS surface operators in N=4 SYM
  - definition and symmetry
  - Gravity dual
- 1/4 BPS surface operators in N=4 SYM
  - definition and symmetry
  - Gravity dual
  - correlation function
- 1/2 BPS surface operators
  in Klebanov-Witten theory
  - definition and symmetry
  - Gravity dual
  - correlation function

1/4 BPS surface operator

Summary of the result

[Koh, SY]

- 1/4 or less BPS surface operators

- Identify gravity dual

- Check the supersymmetry
  in both the gauge theory side and gravity side

- Calculate the correlation functions with local operators
  in both sides and see they agree

How they agree between weak and strong coupling ?

1/2 BPS surface operator

4 dim N=4 SYM

$$\Phi := \phi_4 + i\phi_5$$

$$\Phi = diag\left(\frac{\beta}{z^1}, 0, 0, \ldots, 0\right)$$

$$z^1 = x^2 + ix^3$$

β : constant

- Supersymmetry

$$\delta\psi = D_\mu \phi_I \Gamma^{\mu I} \epsilon = 0 \Rightarrow (1 + \Gamma^{2345})\epsilon = 0$$

1/2 BPS

Holomorphy is important to preserve the supersymmetry!

- Dilatation symmetry

Φ has conformal dimension 1

Degree (-1) is important to preserve the dilatation symmetry

1/4 BPS surface operator

$$\Phi \sim \frac{1}{\sqrt{z^1 z^2}}$$

Multi-valued

$$z^1 = x^2 + ix^3$$
$$z^2 = x^0 + ix^1$$

Well-defined ??

Yes, in the following way.

$$\Phi = \mathrm{diag}\left(\frac{\beta}{\sqrt{z^1 z^2}}, -\frac{\beta}{\sqrt{z^1 z^2}}, 0, \cdots, 0\right), \qquad A_\mu = 0,$$

For expample for fixed z^2, there is monodroy around $z^1 = 0$

$$z^1 \to z^1 e^{2\pi i}$$

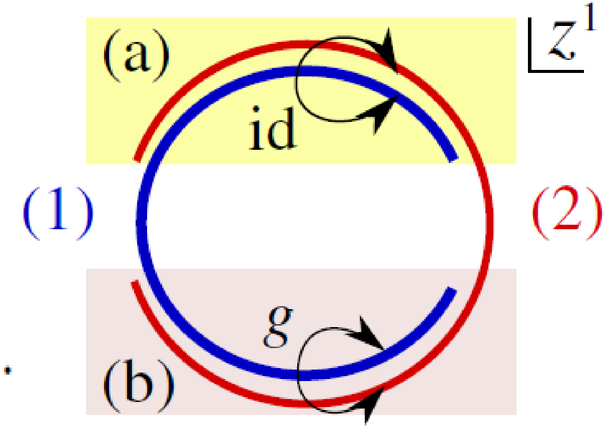
Cancel the monodromy by the gauge holonomy

Introduce two patches

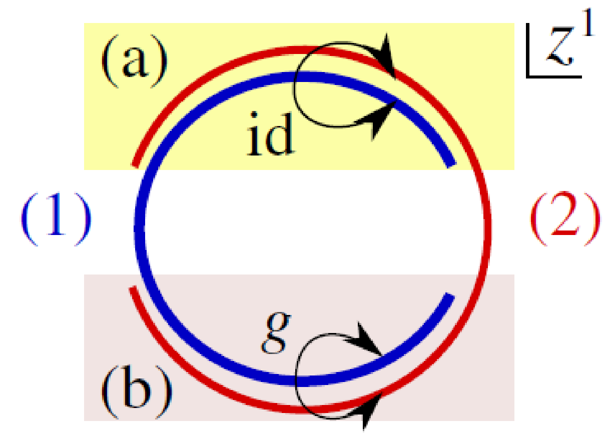
(1) $0 < \phi_1 < 2\pi$ (branch cut at $\phi_1 = \pi$).

(2) $-\pi < \phi_1 < \pi$ (branch cut at $\phi_1 = 0$).

$$z^1 = r_1 e^{i\phi_1}$$



In region (a) two patches are related by identity gauge transformation.



In region (b) two patches are related by the gauge transformation by the constant matrix g

$$g = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & I_{N-2} \end{pmatrix}$$

⇒ Cancel the monodromy and become a consistent configuration

Gravity dual

= a configuration of D3-brane

AdS5 x S5

complex coordinates $(z^1, z^2, \omega^1, \omega^2, \omega^3)$

$$ds^2 = \frac{1}{y^2}(|dz^1|^2 + |dz^2|^2) + y^2 \sum_{a=1}^{3} |d\omega^a|^2$$

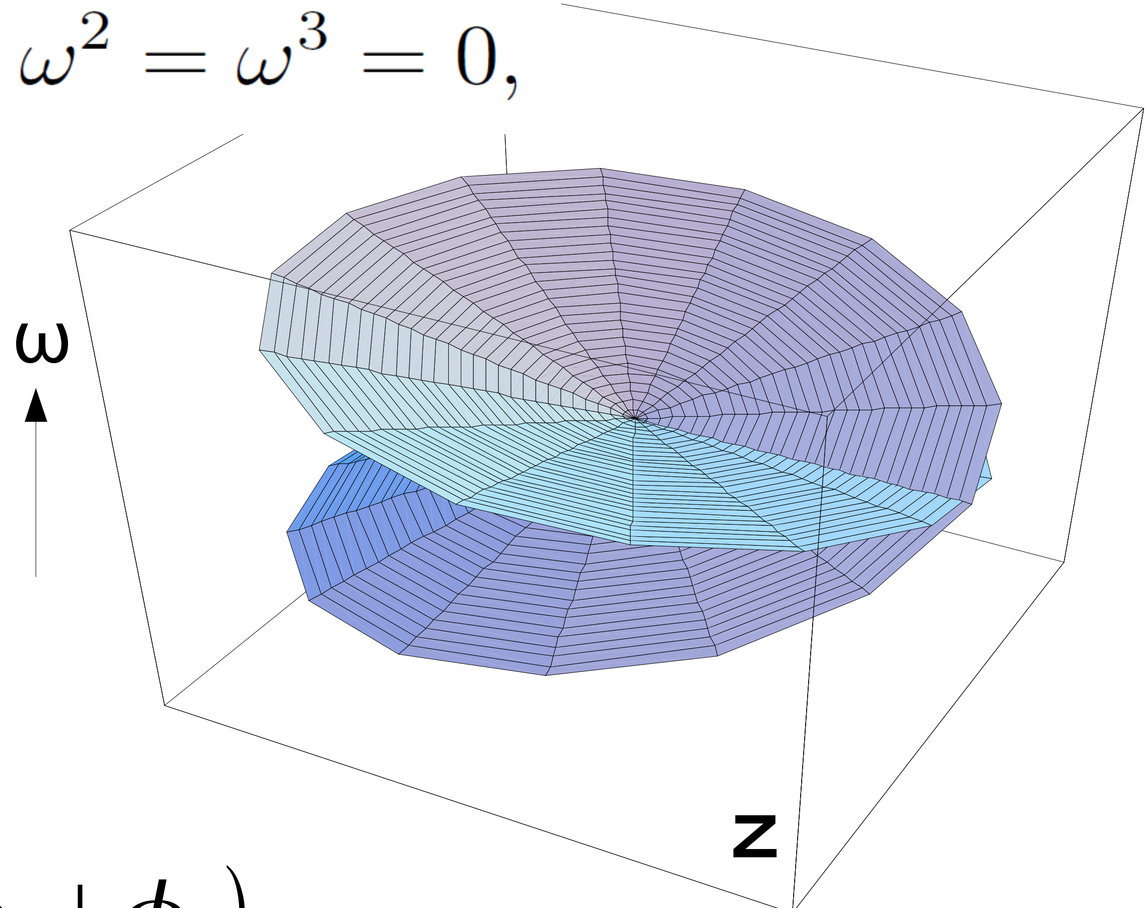
$$y^{-2} := \sum_{a=1}^{3} |\omega^a|^2$$

D3-brane wrapping the surface

$$z^1 z^2 (\omega^1)^2 - \kappa^2 = 0, \quad \omega^2 = \omega^3 = 0,$$

κ : constant related to β by $\kappa = \frac{2\pi\beta}{\sqrt{\lambda}}$

$$z^1 z^2 (\omega^1)^2 - \kappa^2 = 0, \quad \omega^2 = \omega^3 = 0,$$



$$\kappa\, y = \sqrt{r_1 r_2}, \quad \theta = \frac{1}{2}(\phi_1 + \phi_2)$$

$$\omega = y^{-1} e^{i\theta}, \quad z_j = r_j e^{i\phi_j}.$$

Supersymmetry of the gravity dual

- Kappa symmetry projection
- 12 dimensional formulation

[Mikhailov '00], [Kim, Lee '06]

Correlation function with a local operator— gauge theory side

surface operator

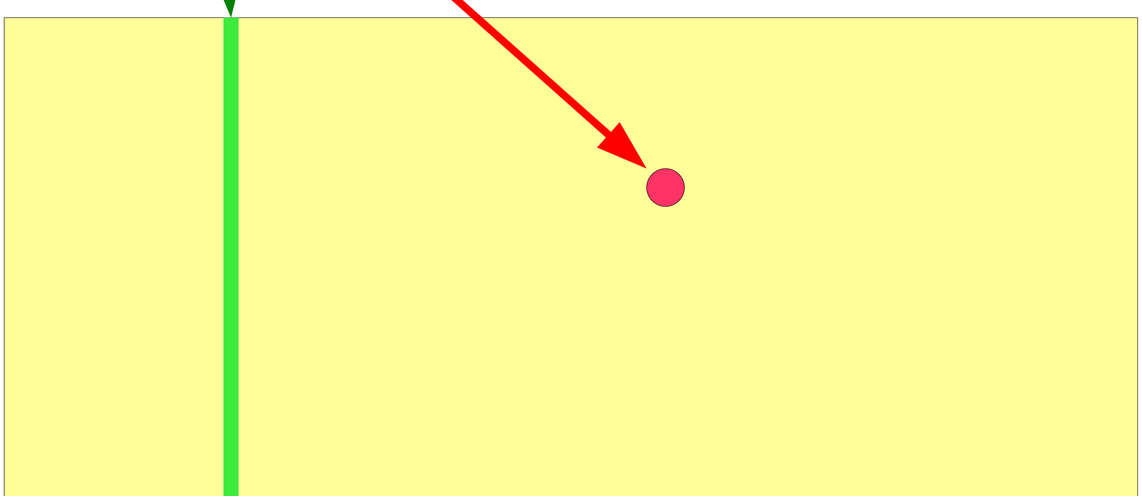
(For 1/2 BPS case [Drukker, Gomis, Matsuura])

$$\langle O_\beta \cdot O(z) \rangle$$

local operator
"chiral primary"

$$O(z) = C^{I_1 \cdots I_\Delta} tr[\phi_{I_1} \cdots \phi_{I_\Delta}]$$

Traceless, symmetric tensor



$$\frac{\langle \mathcal{O}_\Sigma \cdot \mathcal{O}(\zeta) \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \frac{1}{\langle \mathcal{O}_\Sigma \rangle} \int_{\text{boundary condition}} [DAD\psi D\phi] \, \mathcal{O}(\zeta) \, e^{-S}$$

$$\cong \mathcal{O}|_\Sigma(\zeta)$$

classical approximation
simply insert the classical solution

The result in the gauge theory side (classical)

$$\frac{\langle O_\beta(\Sigma) O_{\Delta,k}(\zeta) \rangle}{\langle O_\beta(\Sigma) \rangle} = \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_{\Delta,k} \frac{\beta^\Delta}{(\bar{\zeta}^1 \bar{\zeta}^2)^{(\Delta-k)/2} (\zeta^1 \zeta^2)^{(\Delta+k)/2}} (1+(-1)^\Delta)$$

Correlation function with a local operator— gravity side

Some field fluctuation of metric and RR4-form

$\Longleftrightarrow$ Chiral primary operators

GKPW: calculate the classical action of the solution with source inserted at boundary.

D3-brane is treated as probe

Action of the gravity side $S_{gravity} = S_{IIB\ sugra} + S_{D3}$

$$S_{D3} = S_{DBI} - S_{WZ}, \quad S_{DBI} = T_{D3} \int d^4\xi \sqrt{|\det G_{mn}|}, \quad S_{WZ} = T_{D3} \int_{\Sigma_4} C_4.$$

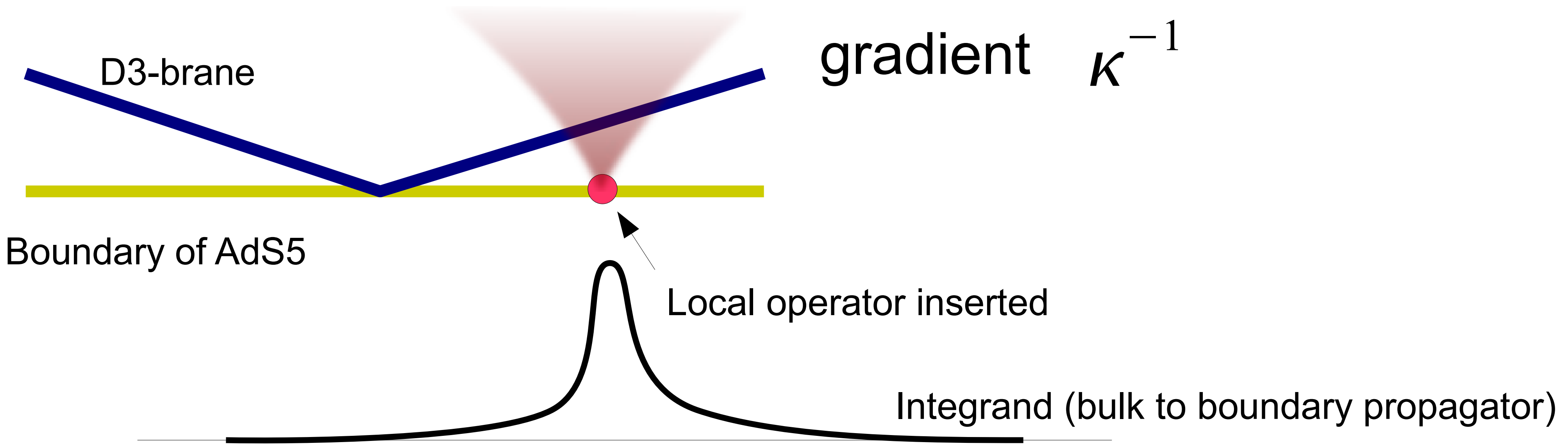
$$\frac{\langle O_\beta(\Sigma) O_{\Delta,k}(\zeta)\rangle}{\langle O_\beta(\Sigma)\rangle} = \frac{\delta S_{gravity}}{\delta s_0(\zeta)}\Big|_{s_0=0} = \frac{\delta S_{D3}}{\delta s_0(\zeta)}\Big|_{s_0=0}$$

$$= -2\Delta T_{D3}\, c(\Delta) C_{\Delta,k} \int d^4 z \frac{\omega^{-\frac{\Delta-k}{2}}(z)\bar{\omega}^{-\frac{\Delta+k}{2}}(\bar{z})}{L^{\Delta+2}} \frac{|\zeta^m \partial_m \omega(z)|^2}{|\omega(z)|^2}$$

$$L \equiv \sum_{m=1,2} |z^m - \zeta^m|^2 + |\omega|^{-2} \qquad \omega(z) = \frac{\kappa}{\sqrt{z^1 z^2}}$$

It is not easy to evaluate exactly this integral

Approximation $\kappa \to \infty$



The integrand has a SHARP PEAK in this limit!

The result in the gravity side $\kappa \to \infty$

$$\frac{\langle O_\beta(\Sigma) O_{\Delta,k}(\zeta)\rangle}{\langle O_\beta(\Sigma)\rangle} = \frac{2^{\Delta/2}}{\sqrt{\Delta}} C_{\Delta,k} \frac{\kappa^\Delta}{(\bar{\zeta}^1 \bar{\zeta}^2)^{(\Delta-k)/2} (\zeta^1 \zeta^2)^{(\Delta+k)/2}} (1+(-1)^\Delta)$$

The result in the gauge theory side

$$\frac{\langle O_\beta(\Sigma) O_{\Delta,k}(\zeta)\rangle}{\langle O_\beta(\Sigma)\rangle} = \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_{\Delta,k} \frac{\beta^\Delta}{(\bar{\zeta}^1 \bar{\zeta}^2)^{(\Delta-k)/2} (\zeta^1 \zeta^2)^{(\Delta+k)/2}} (1+(-1)^\Delta)$$

Agree with the classical calculation in the gauge theory side with the identification

$$\kappa = \frac{2\pi\beta}{\sqrt{\lambda}}$$

Correction

$$\frac{\langle O_\beta(\Sigma) O_{\Delta,k}(\zeta)\rangle}{\langle O_\beta(\Sigma)\rangle} = (leading)\left[1 + \frac{\lambda}{4\pi^2\beta^2}\frac{\Delta^2 - k^2}{16(\Delta-1)}\left(\frac{|\zeta^1|^2 + |\zeta^2|^2}{|\zeta^1\zeta^2|}\right) + \cdots\right]$$

This expression is positive power in λ !

The situation is similar to plane wave limit of BMN

Large β mimics the perturbative expansion in λ

To compare this term with the perturbative Yang-Mills calculation is an interesting problem.

Plan

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- 1/2 BPS surface operators in Klebanov-Witten theory
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  - Gravity dual
  - correlation function

1/2 BPS surface operators in the Klebanov-Witten Theory

An example of N=1 AdS/CFT

[Klebanov,Witten '98]

AdS / CFT

=

type IIB superstring
AdS5 x T1,1

=

Klebanov-Witten Theory
N=1 superconformal

Klebanov-Witten theory

N=1 gauge theory

- gauge group $SU(N) \times SU(N)$
- matters $A_1, A_2 \quad (N, \bar{N})$

  $B_1, B_2 \quad (\bar{N}, N)$
- superpotential $W = Tr[A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1]$

This theory has non-trivial fixed point

1/2 BPS surface operator

$A\,,B$: dimension 3/4

To preserve the scale invariance $A\,,B \sim \frac{1}{z^{3/4}}$

Example:

$$A_1 = B_1 = \frac{\beta}{z^{3/4}} diag(1, i\,, 0, \cdots, 0)\,, \quad A_2 = B_2 = 0.$$

Holonomy around z=0 ⇨ Well-defined configuration

$(g\,,\tilde{g}) \in SU(N) \times SU(N)$

$$g = \begin{pmatrix} \sigma_1 & 0 \\ 0 & e^{\frac{\pi i}{N-2}} I_{N-2} \end{pmatrix}, \qquad \tilde{g} = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & I_{N-2} \end{pmatrix},$$

Supersymmetry

$$A_1 = B_1 = \frac{\beta}{z^{3/4}} diag(1, i, 0, \cdots, 0), \quad A_2 = B_2 = 0.$$

$$F = 0, D = 0$$

(variation of fermions)=0

↓

Non-trivial condition $\sigma^\mu \partial_\mu A_1 \epsilon = 0$

↓

$$(\sigma^1 + i\sigma^2)\epsilon = 0$$

1/2 BPS

Gravity dual

$$AdS_5 \times T^{1,1}$$

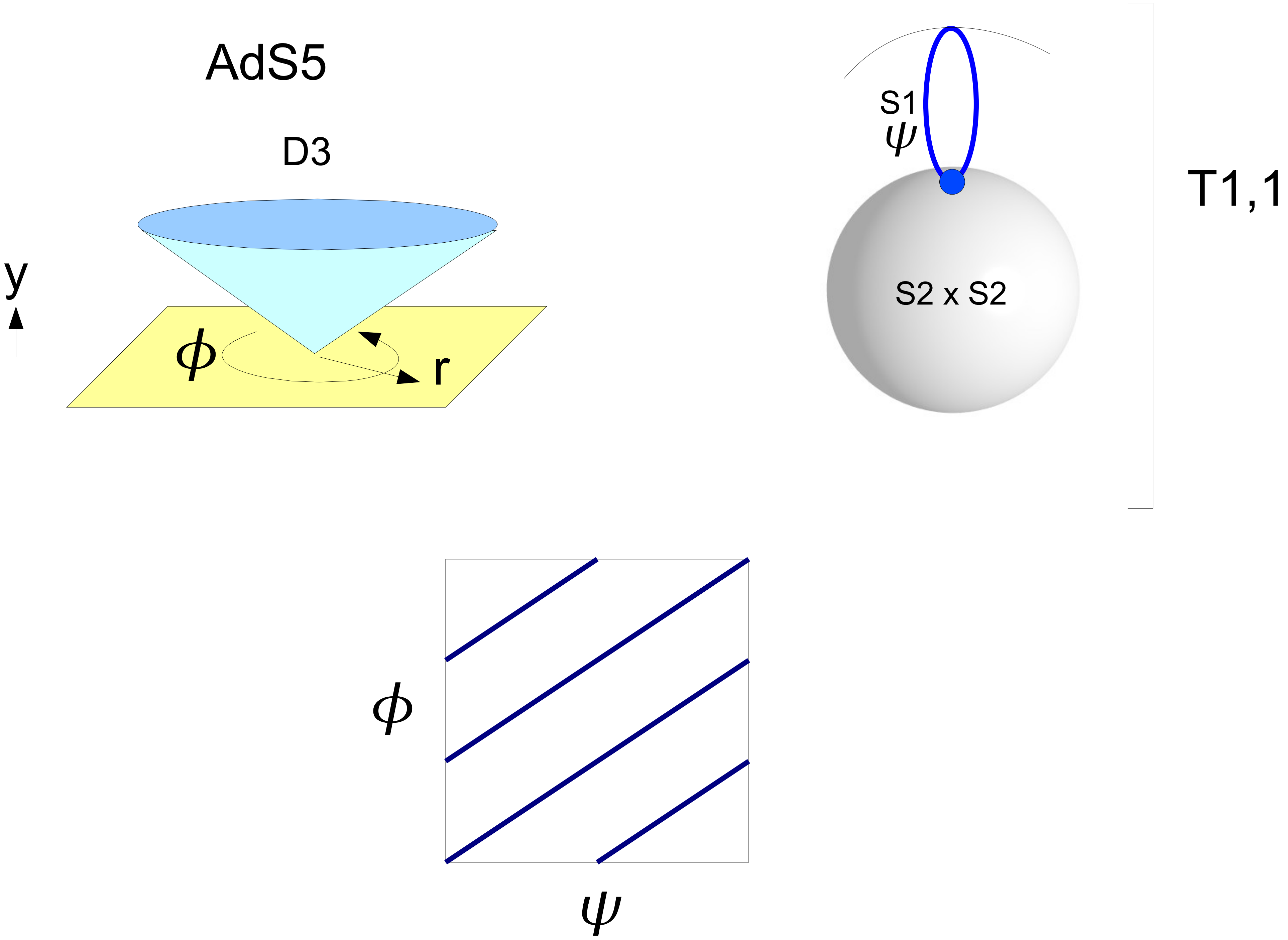
$$ds^2 = \frac{1}{y^2}(dy^2 + dr^2 + r^2 d\phi^2 + dx_0^2 + dx_1^2) + ds_{T^{1,1}}^2$$

$$ds_{T^{1,1}}^2 = \frac{1}{9}(d\psi + \cos\theta_1 d\nu_1 + \cos\theta_2 d\nu_2)^2 + \frac{1}{6}\sum_{i=1,2}(d\theta_i^2 + \sin^2\theta d\nu_i^2)$$

D3-brane wrapping on the surface

$$\kappa y = r, \quad \psi = -3\phi, \quad \theta_i = \pi, \nu_i = 0$$

Induced metric = AdS3 x S1


AdS5
D3
y
ϕ
r
S1
ψ
S2 x S2
T1,1
ϕ
ψ

Correlation function with chiral primary operator

Chiral primary operators

$$O_n^I = p_n^I C^{I\ (i_1,\cdots i_n)\ (j_1,\cdots,j_n)} Tr[A_{i_1} B_{j_1} A_{i_2} B_{j_2} \cdots A_{i_n} B_{j_n}]$$

Conformal dimension

$$\Delta = \frac{3}{2} n$$

Normalized as

$$\langle \bar{O}_n^I(x) O_n^I(0) \rangle = \frac{1}{|x|^2}$$

Operators

$$O_n = p_n Tr[(A_1 B_1)^n]$$

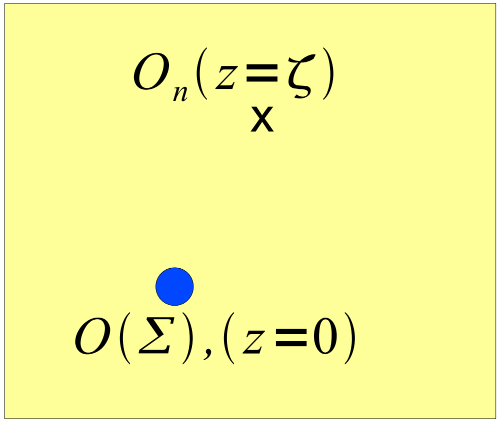
have non-trivial correlation function
with the surface operator

Gravity side GKPW calculation

$$\frac{\langle O(\Sigma) O_n(\zeta)\rangle}{\langle O(\Sigma)\rangle} = \frac{\sqrt{3}}{4}\frac{2\Delta+3}{\sqrt{\Delta(\Delta+1)(\Delta+2)}}\frac{\kappa^{\Delta}}{\zeta^{\Delta}}(1+(-1)^n)$$

Classical approximation (?) in the gauge theory

$$\frac{\langle O(\Sigma) O_n(\zeta)\rangle}{\langle O(\Sigma)\rangle} = p_n \frac{\beta^{2n}}{\zeta^{\Delta}}(1+(-1)^n)$$



- The position dependence is the same
- The relations (up to overall scaling of A,B)

$$\kappa = \beta^{4/3}, \qquad p_n = \frac{\sqrt{3}}{4}\frac{2\Delta+3}{\sqrt{\Delta(\Delta+1)(\Delta+2)}}$$

Summary

- 1/4 BPS surface operators in N=4 SYM

  - Definition.

  - The gravity dual as a certain configuration of a probe D3-brane.

  - Correlation functions with chiral primary operators

agree!

Gravity ⟷ Gauge

- 1/2 BPS surface operators in Klebanov-Witten theory

  - We defined a 1/2 BPS surface operators in KW.

  - Gravity dual, correlation function.

Thank you