Non-Gaussianity and the Adiabatic Limit

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Inflation

- Inflation is the accelerated expansion of space which took place in the early universe.
- Space expanded by 26 orders of magnitude in about $10^{-32}$ seconds during inflation.
Inflation provides the flat, homogeneous, isotropic initial conditions for the subsequent evolution of the universe.
• Quantum fluctuations produced during inflation led to density perturbations which seeded all of the structure we see today
The cosmic microwave background carries an imprint of the quantum fluctuations produced during inflation and has travelled to us largely unimpeded for 13.7 billion years.
Large Scale Structure

- The same density fluctuations grew by gravitational instability and collapsed to form the large scale structure of the universe
• Measurements of the cosmic microwave background and large scale structure have allowed non-trivial tests of inflation
Power Spectrum and Inflation

- Data has begun to rule out some models of inflation, but many models remain viable
Non-Gaussianity

- Non-Gaussianity may provide a key tool for discriminating amongst models of inflation
Non-Gaussianity

\[ f_{\text{local}}^{\text{NL}} = 32 \pm 21 \]
\[ f_{\text{equil}}^{\text{NL}} = 26 \pm 140 \]

- Experiments have already begun to constrain certain types of non-gaussianity
- The Planck satellite and future large scale structure surveys will better constrain these parameters soon

\[ \Delta f_{\text{local}}^{\text{NL}} \sim 5 \]

Komatsu, Spergel (2001)
Inflation and Non-Gaussianity

Inflation Models

$f_{\text{NL}}^{\text{local}} \sim \mathcal{O}(10)$
Single Field Consistency Relation

• $f_{\text{local}}$ is always small in single field inflation

\[ f_{\text{NL}}^{\text{local}} = \frac{5}{12} (1 - n_s) \]

• A convincing detection of $f_{\text{NL}}^{\text{local}}$ would rule out \textit{all} models of single field inflation

Maldacena (2002)
Creminelli, Zaldarriaga (2004)
Ganc, Komatsu (2010)
Inflation and Non-Gaussianity

Multiple Field Inflation Models

Single Field Inflation Models

\[ f_{\text{NL}}^{\text{local}} \sim O(10) \]
Inflation and Non-Gaussianity

A convincing detection of $f_{\text{NL}}^{\text{local}}$ would rule out all models of single field inflation, and many models of multiple field inflation.
Lessons From Data

- It is unlikely that we will ever be able to pin down exactly which model of inflation was responsible for the universe we see.
- Observations can, however, teach us something about the physical principles which governed inflation.
• The scalar spectral index $n_s$ allows us to learn about the expansion history during inflation

$P_\zeta(k)$

• This allows a measurement of the steepness of the inflationary potential
Tensor-to-Scalar Ratio

- Primordial gravitational waves produce B mode polarization in the cosmic microwave background.

\[ P_h \sim \frac{H^2}{M_{\text{pl}}^2} \]

- A measurement of the tensor power allows a determination of the inflationary energy scale.

Seljak, Zaldarriaga (1997); Kamionkowski, et al. (1997)
Equilateral Non-Gaussianity

- The equilateral bispectrum measures the speed of sound during inflation

\[ f_{\text{NL}}^{\text{equil}} \sim \frac{1}{c_s^2} \]

- A small sound speed indicates new physics near the inflationary scale

Baumann, Green (2011)
Folded Non-Gaussianity

- The folded bispectrum contains information about the initial state of inflation

- Measurement of this limit would indicate a deviation from Bunch-Davies initial conditions

Chen, et al. (2006)
Local Non-Gaussianity

• The local bispectrum can only be produced from multiple field inflation

• What else can we learn from a detection of local non-Gaussianity?
Adiabaticity

- The curvature perturbation is conserved outside the horizon when the fluctuations are adiabatic.
- Single field inflation always produces purely adiabatic fluctuations.

• The curvature perturbation can evolve on superhorizon scales in the presence of non-adiabatic fluctuations
• Multiple field inflation naturally produces non-adiabatic fluctuations
Adiabaticity

- Non-adiabatic fluctuations which persist through the radiation-dominated era produce observable effects
- Observations show no evidence for non-adiabatic fluctuations
  - Uncorrelated: $\alpha_0 < 0.077$
  - Anti-correlated: $\alpha_{-1} < 0.0047$

Bucher, Moodley, Turok (2001); Komatsu, et al. (2010)
Approach to Adiabaticity

- Non-adiabatic fluctuations may become adiabatic in at least two ways:

  1. Effectively single field inflation
  2. Local thermal equilibrium

Any model with multiple dynamical fields is *incomplete* without an understanding of the evolution of the cosmological perturbations until they become adiabatic, or until they are observed.
Adiabaticity and Non-Gaussianity

• Are there general features of multiple field inflation models which predict $f_{NL}^{\text{local}} \sim \mathcal{O}(10)$ and a purely adiabatic power spectrum?

• We will focus on two field inflation models which pass through a short phase of effectively single field inflation before reheating
δN Formalism

- We use the δN formalism to calculate the evolution of observables outside the horizon

\[ N = \int_{*}^{C} H \, dt \]

\[ N_{,I} \equiv \frac{\partial N}{\partial \phi_{*}^{I}} \]

\[ \zeta = \delta N \simeq \sum_{I} N_{,I} \delta \phi_{*}^{I} + \sum_{IJ} N_{,IJ} \delta \phi_{*}^{I} \delta \phi_{*}^{J} \]

\[ \frac{6}{5} f_{\text{local}}^{\text{NL}} = \frac{\sum_{IJ} N_{,IN_{,J}N_{,IJ}}}{\left( \sum_{K} N_{,K}^{2} \right)^{2}} \]

Sasaki, Stewart (1996); Lyth, Rodriguez (2005)
Two Field Models

- Potentials admitting analytic treatment are of the form: \[ W(\phi, \chi) = F(U(\phi) + V(\chi)) \]

- For simplicity, focus on \[ W(\phi, \chi) = U(\phi) + V(\chi) \]

JM, Sivanandam (2010,2011)
Results

• We find for a sum-separable potential

\[
\frac{6}{5} f_{\text{NL}}^{\text{local}} = \frac{x^2}{\epsilon_*^2} \left( 2 - \frac{x \eta_\phi^*}{\epsilon_*} \right) + \frac{y^2}{\epsilon_*} \left( 2 - \frac{y \eta_\chi^*}{\epsilon_*} \right) + \frac{2 (U_c + V_c)^2}{(U_* + V_*^2)} \left( \frac{x}{\epsilon_*^2} - \frac{y}{\epsilon_*} \right)^2 \frac{\epsilon_c^2}{\epsilon_c} \left( \frac{\eta_{ss}^c}{\epsilon_c} - 1 \right)
\]

• Where we have used the definitions

\[
x \equiv \frac{1}{U_* + V_*} \left( U_* + \frac{V_c \epsilon_c^\phi - U_c \epsilon_c^\chi}{\epsilon_c} \right)
\]

\[
y \equiv \frac{1}{U_* + V_*} \left( V_* - \frac{V_c \epsilon_c^\phi - U_c \epsilon_c^\chi}{\epsilon_c} \right)
\]

\[
\eta_{ss} \equiv \frac{\epsilon_\chi \eta_\phi + \epsilon_\phi \eta_\chi}{\epsilon}
\]

Vernizzi, Wands (2006)
Effectively Single Field Inflation

- As the inflaton rolls through a steep valley, characterized by \((\eta^{ss} > 1)\) we find:
  
  - Entropy perturbations:  \(|\delta s| \sim \exp\left[-\frac{3}{2} \int H \, dt\right]\)
  
  - Non-Gaussianity:
    
    \[ f_{\text{NL}}^{\text{local}} \sim \mathcal{O}(\varepsilon_*) + \mathcal{O}(1) \times \eta^{ss} \exp\left[-2 \int C_\eta H \eta^{ss} \, dt\right] \]

JM, Sivanandam (2010); Watanabe (2012)
Higher Point Statistics

• Similar results also apply to the trispectrum after passing through a steep valley:

\[ \tau_{NL} \sim O(\varepsilon_*) \quad g_{NL} \sim O(\varepsilon_*) \]

• And in fact to all local form n-point statistics:

\[ F_{NL,i}^{(n)} \sim O(\varepsilon_*) \]

JM, Sivanandam (2011)
Observables in Adiabatic Limit

• After adiabaticity is achieved, the observables take the following form

\[
\frac{6}{5} f_{\text{NL}}^{\text{local}} \simeq \left( \frac{r x}{16 \epsilon_*^\phi} \right)^2 \left( 2 \epsilon_*^\phi - x \eta_*^\phi \right) + \left( \frac{r y}{16 \epsilon_*^\chi} \right)^2 \left( 2 \epsilon_*^\chi - y \eta_*^\chi \right)
\]

\[
n_s - 1 = -2 \epsilon_* - 2 \left( \frac{r x}{16 \epsilon_*^\phi} \right) \left( 2 \epsilon_*^\phi - x \eta_*^\phi \right) - 2 \left( \frac{r y}{16 \epsilon_*^\chi} \right) \left( 2 \epsilon_*^\chi - y \eta_*^\chi \right)
\]

• Recall the definitions

\[
x \equiv \frac{1}{U_* + V_*} \left( U_* + \frac{V_c \epsilon_*^\phi - U_c \epsilon_*^\chi}{\epsilon_c} \right)
\]

\[
y \equiv \frac{1}{U_* + V_*} \left( V_* - \frac{V_c \epsilon_*^\phi - U_c \epsilon_*^\chi}{\epsilon_c} \right)
\]
Conditions for Observable $f_{NL}$

- Generating local non-gaussianity which is preserved in the adiabatic limit seems to require (at least for simple potentials):
  - One *very* slowly rolling field at horizon exit
  - A finely-tuned trajectory through field space
  - One field with negligible contribution to the energy density at horizon exit

Byrnes, Choi, Hall (2008); Kim, Liddle, Seery (2010); Elliston, et al. (2011)
Open Questions

• How general are these results?

• How does reheating affect non-gaussianity?

  Leung et al. (2102)

• Can we understand the curvaton mechanism and modulated reheating in similar terms?
Curvaton Mechanism

- The energy density of the curvaton is entirely negligible at horizon exit
- The curvaton rolls extremely slowly during inflation, then decays after the inflaton reheats the universe

Linde, Mukhanov (1997); Lyth, Wands (2001); Moroi, Takahashi (2001); Enquist, Sloth (2001)
Modulated Reheating

\[ \Gamma = \Gamma(\sigma) \]

\[ \Gamma(\sigma) = \Gamma_0 + \delta \Gamma \]

- A field with negligible energy density modulates the decay rate of the inflaton
- The light field does not roll during inflation

Dvali, Gruzinov, Zaldarriaga (2003); Kofman (2003)
Conclusions

• Detection of local non-gaussianity rules out more than just single field inflation

• Sharp predictions require an understanding of the evolution until fluctuations become adiabatic, or until they are observed

• Two-field inflation models which pass through an effectively single field phase require tuning of parameters at horizon exit to produce local form non-gaussianity