2012/5/15
ACP seminar
@Kavli IPMU

Gravitational-wave Backgrounds from ground- and space-based interferometers

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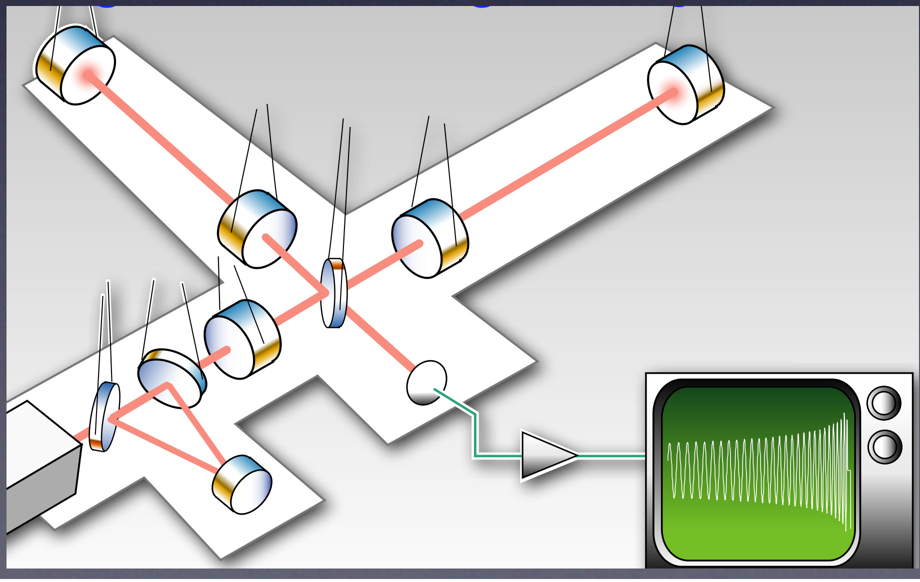
Introduction

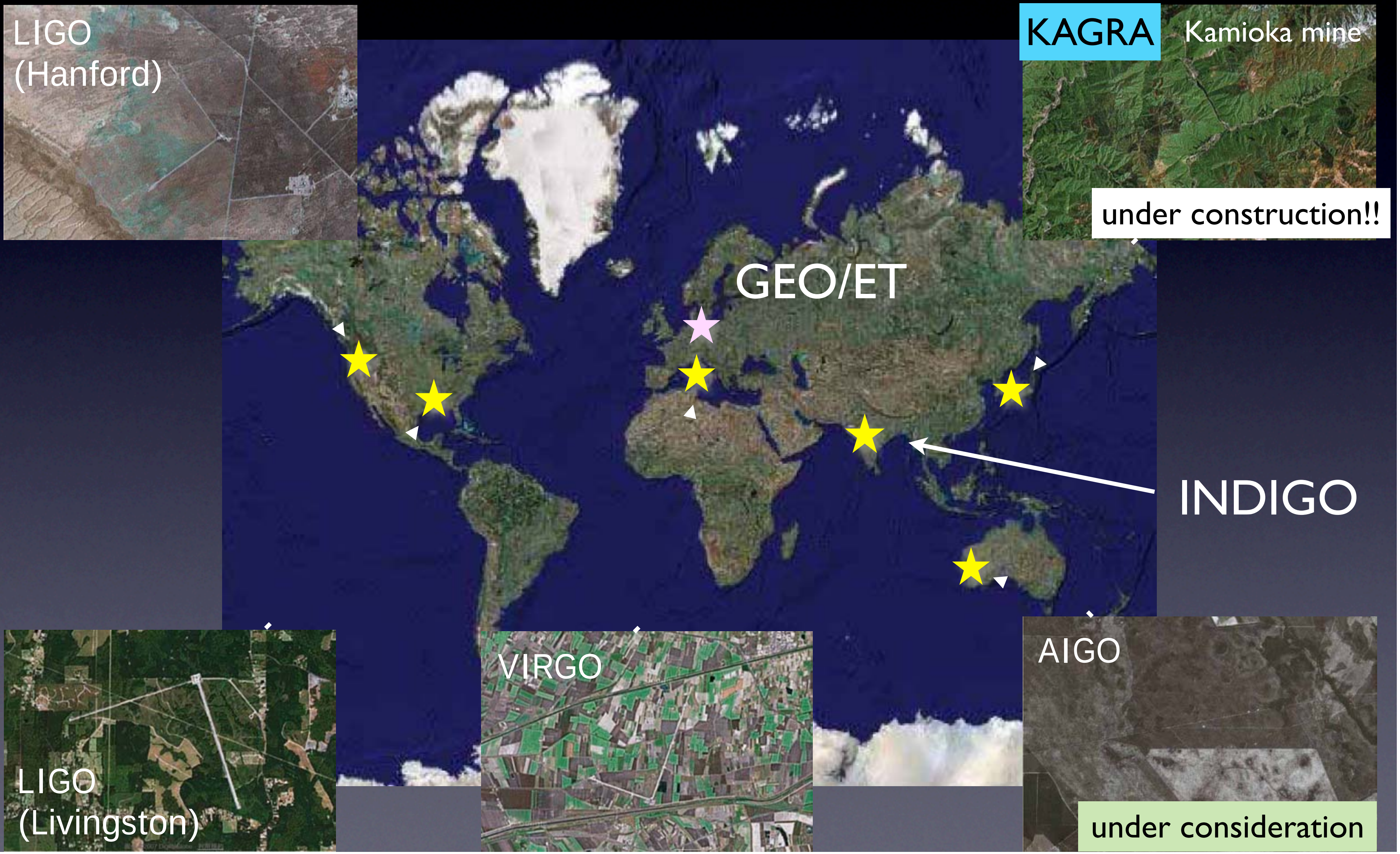
Gravitational waves (GWs)

- Spacetime fluctuations predicted by general relativity
- Indirect observation using binary pulsar (Hulse & Taylor)
- Many ongoing/upcoming missions to detect GWs

Laser interferometers
(Fabry-Perot)



Laser interferometers in the world



Laser interferometers in space

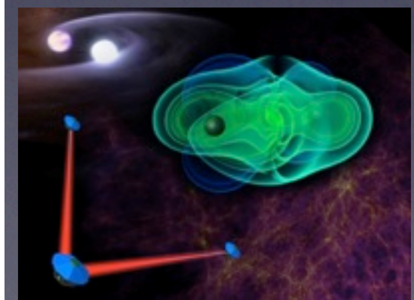
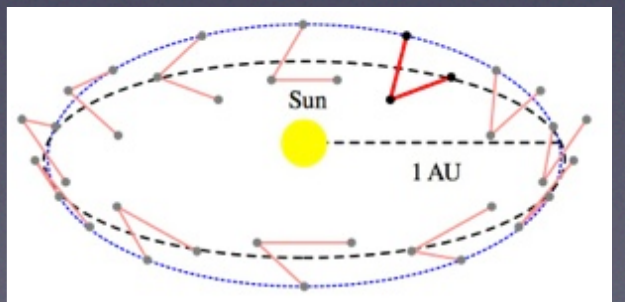
NGO (**N**ew **G**ravitationa-wave **O**bservatory) 2020+???

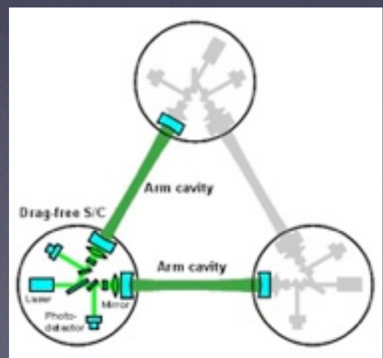
BBO (**B**ig-**B**ang **O**bserver) 2025+

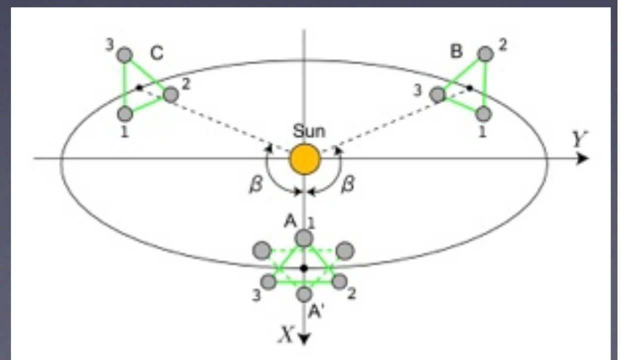
DECIGO 2027+
(**DEC**i-hertz **I**nterferometer **G**ravitational wave **O**bservatory)

Advantage of space

Free from seismic noises ⟶ low-frequency GWs
$(f = 10^{-4} - 1\,\mathrm{Hz})$





Targets

GW observations as astronomical & cosmological tool

- Coalescence of BHs
- Inspiral/merging of binary systems
  (WD-WD, NS-NS, BH-NS, ...)
- Stochastic backgrounds :

  Phase transition, cosmic string, ...

  Inflation $(\Omega_{\rm gw}h^2 \sim 10^{-16})$ **BBO/DECIGO**

  $\longrightarrow$ a large benefit to '*early universe physics*'

Plan of this talk

- ✦ Detection & characterization of GWBs from ground-based detectors
- ✦ Cosmology with space-based detectors : dark energy and primordial GWs

Refs. PRL 99, 121101 (‘07); PRD 77, 103001 (‘08)
PRD 85, 044047 (‘12)

Collaborators

S. Kawamura, A. Nishizawa, S. Saito, M. Sakagami,
N. Seto, T. Tanaka, K. Yagi

Statistical description of GWBs

GWBs: defined as stationary and random superposition of plane waves

$$ds^2 = -dt^2 + [\delta_{ij} + h_{ij}]dx^i dx^j$$

polarization tensor

$$h_{ij}(\vec{x},t) = \sum_{A=\times,+} \int_{-\infty}^{\infty} df \int d\hat{\Omega}\, \underline{\underline{h_A(f,\hat{\Omega})}}\, e^{i\,2\pi\, f(t-\hat{\Omega}\cdot\vec{x}/c)}\, e_{ij}^{A}(\hat{\Omega})$$

random variable

transverse & traceless

ensemble average (un-polarized)

$$\left\langle h_A(f,\hat{\Omega}) \right\rangle = 0$$

$$\left\langle h_A(f,\hat{\Omega})\, h_{A'}(f',\hat{\Omega}') \right\rangle = \frac{\delta(f-f')}{2}\, \frac{\delta^2(\hat{\Omega},\hat{\Omega}')}{4\pi}\, \delta_{AA'}\, S_h(|f|,\hat{\Omega})$$

Spectrum of GWBs

$S_h(|f|, \hat{\Omega})$: Power spectral density [1/Hz]

anisotropies

In most of cosmological origins, GWB is isotropic:

Strain amplitude $h(f) = S_h(f)^{1/2}$ [Hz$^{-1/2}$]

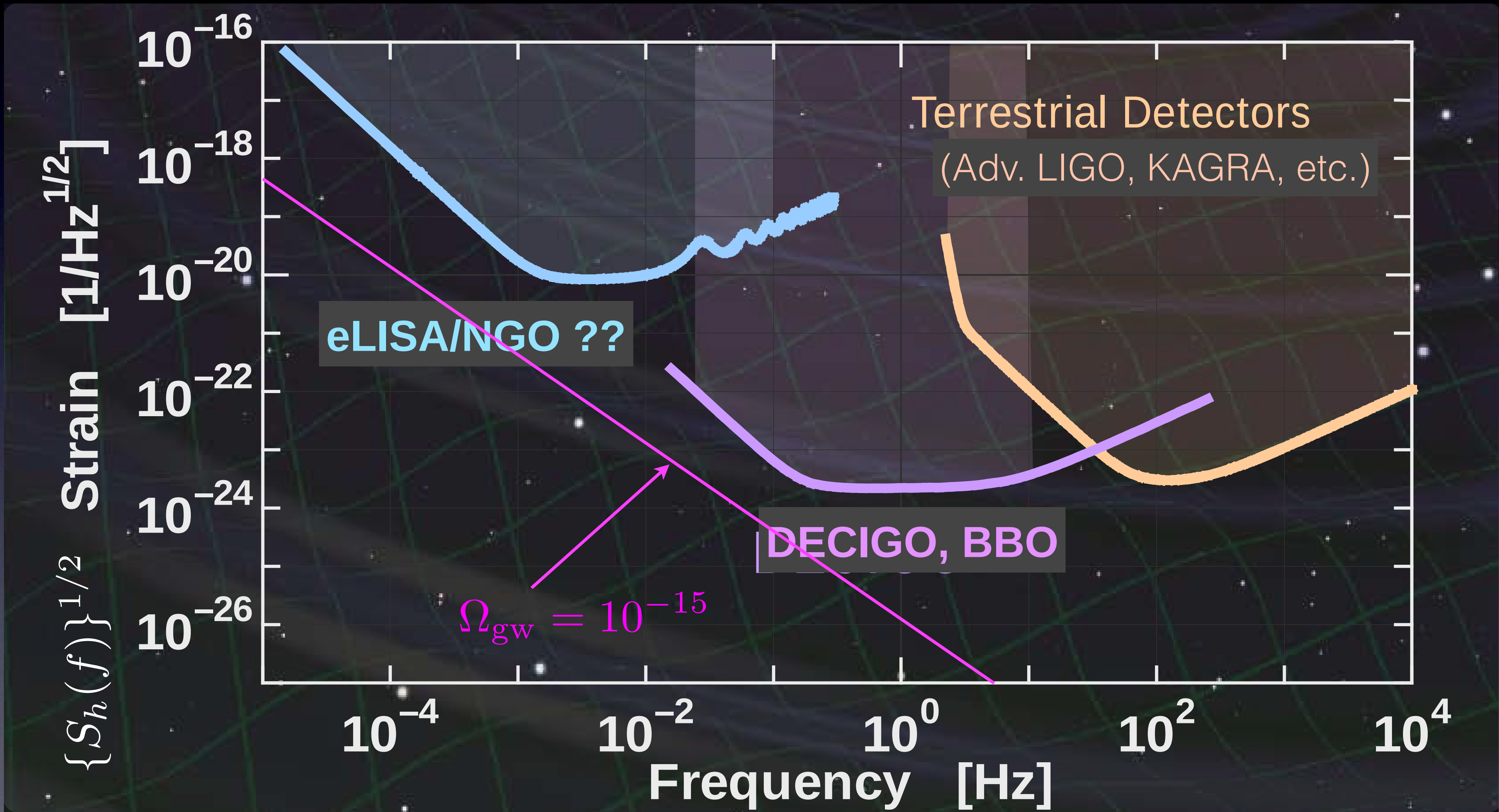
Density parameter $\Omega_{gw}(f) \equiv \dfrac{1}{\rho_{crit}} \dfrac{d\rho_{gw}}{d(\log f)} = \dfrac{4\pi^2}{3H_0^2} f^3 S_h(f)$

Amplitude is supposed to be small, indistinguishable from detector noises:

★ Cross-correlation with multiple detectors

★ Long-term monitoring (months~years)

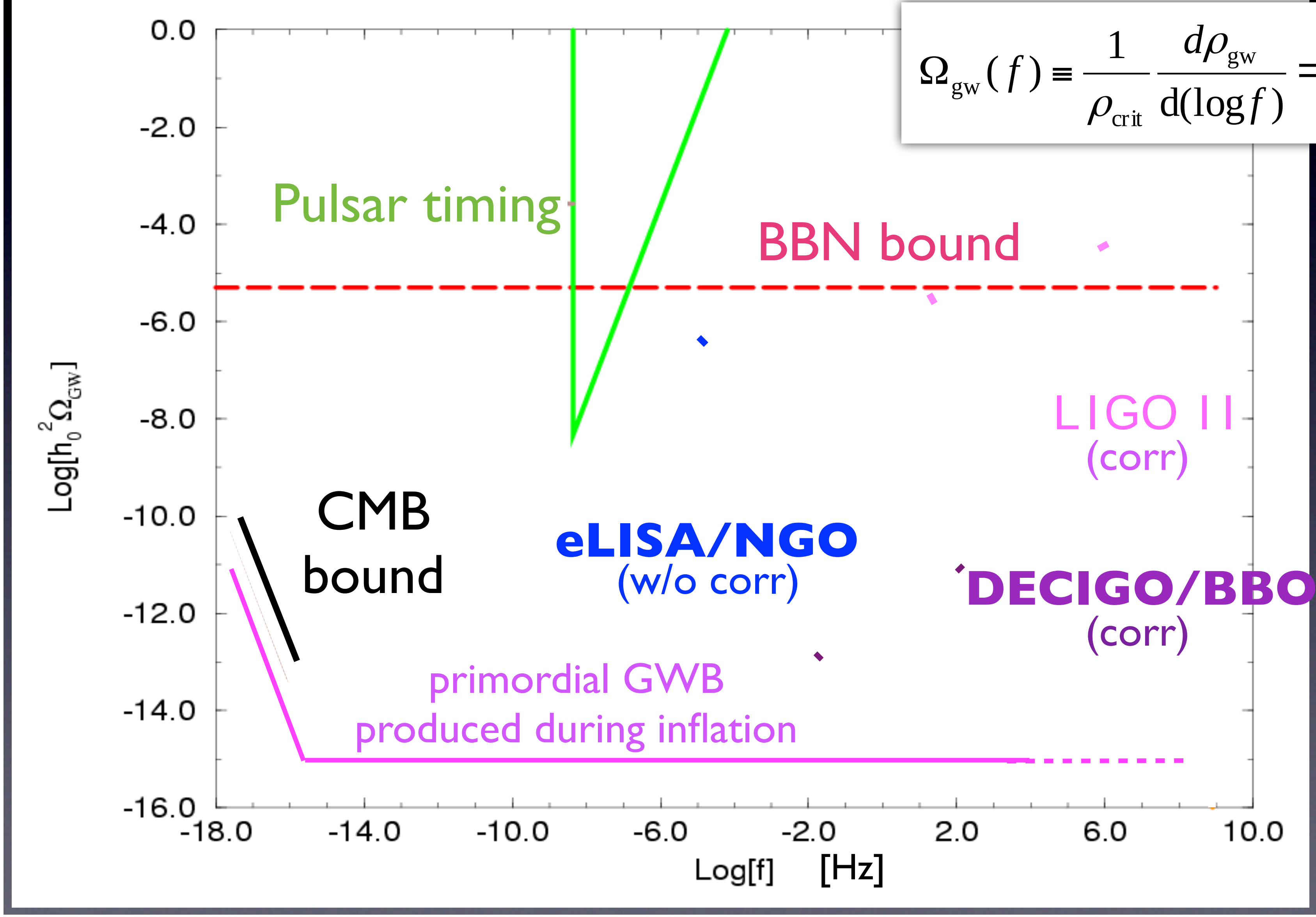
Observational window



viewgraph by M. Ando

Observational window

$$\Omega_{gw}(f) \equiv \frac{1}{\rho_{crit}} \frac{d\rho_{gw}}{d(\log f)} = \frac{4\pi^2}{3H_0^2} f^3 S_h(f)$$



Maggiore ('00)

Cross correlation analysis

Allen & Romano ('99)

Interferometric signals

(scalar)

$$\begin{cases} s_1(t) = h_1(t) + n_1(t) \\ s_2(t) = h_2(t) + n_2(t) \end{cases}$$

h(t): GW signal

n(t): detector's noise

$$h_i(t) = \sum_{A=\times,+} \int df \int d\hat{\Omega}\, \underline{D_i^{ab}(f,\hat{\Omega})}\, \underline{e_{ab}^{A}(\hat{\Omega})}\, \underline{h_A(f)}\, e^{i\,2\pi\, f(t-\hat{\Omega}\cdot\vec{x}_i/c)}$$

Detector's response tensor

Polarization tensor

GW amplitude

Quadratic estimator

$$S \equiv \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt'\, s_1(t) s_2(t')\, Q(t-t')$$

Optimal filter (given later)

Cross correlation analysis

ensemble average
- mean $\mu \equiv \langle S \rangle$
- dispersion $\sigma^2 \equiv \langle [S - \langle S \rangle]^2 \rangle$

Assumption

- Detector's noises >> GWB signals
- Two detectors' noises are uncorrelated, and are described as stationary Gaussian statistics

$$\mu \approx T \left\langle h_1(t) h_2(t) \right\rangle$$ ---------------------- GW signal is dominant

$$\sigma^2 \approx \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \left\langle n_1(t) n_1(t') \right\rangle \left\langle n_2(t) n_2(t') \right\rangle$$ -------- dominated by noise $(\propto T)$

$$\mathrm{SNR} \equiv \frac{\mu}{\sigma}$$: signal-to-noise ratio

Signal-to-noise ratio

$$\mathrm{SNR} = \frac{2}{5}\sqrt{T}\left[\int_{-\infty}^{\infty} df\, \frac{\{\gamma(f)S_h(f)\}^2}{S_{n,1}(f)S_{n,2}(f)}\right]^{1/2}$$

optimal filter

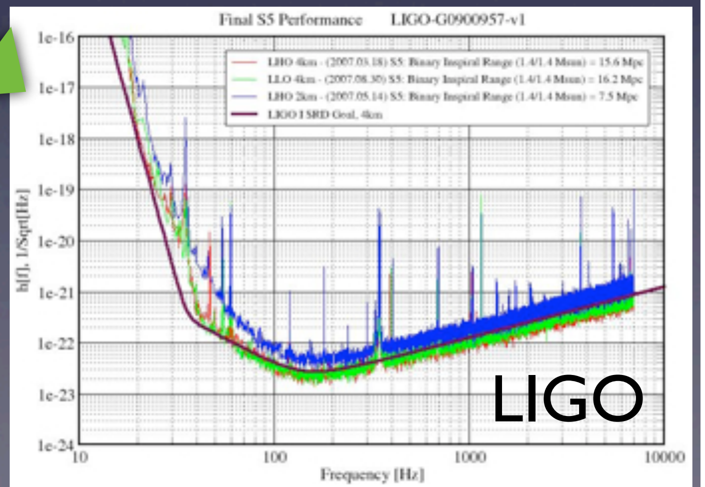
$$\tilde{Q}(f) = \frac{\frac{2}{5}\gamma(f)S_h(f)}{S_{n,1}(f)S_{n,2}(f)}$$

$S_h(f)$: power spectral density [1/Hz]

$$\Omega_{gw}(f) = \frac{4\pi^2}{3H_0^2} f^3 S_h(f)$$

$S_{n,i}(f)$: noise spectral density for detector "*i*"

$\gamma(f)$: overlap reduction function



To increase SNR,

- Long-term observation is necessary
- detector's noise is not the only important factor (next slide)

Overlap reduction function

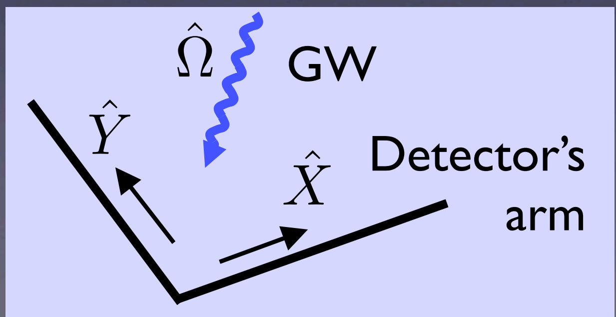
$$\gamma(f) = \frac{5}{8\pi}\int d\hat{\Omega}\, D_1^{ab} D_2^{cd}\left[e_{ab}^{+}(\hat{\Omega})e_{cd}^{+}(\hat{\Omega}) + e_{ab}^{\times}(\hat{\Omega})e_{cd}^{\times}(\hat{\Omega})\right] e^{i\,2\pi f\hat{\Omega}\cdot\Delta\vec{x}/c}$$

Response tensor

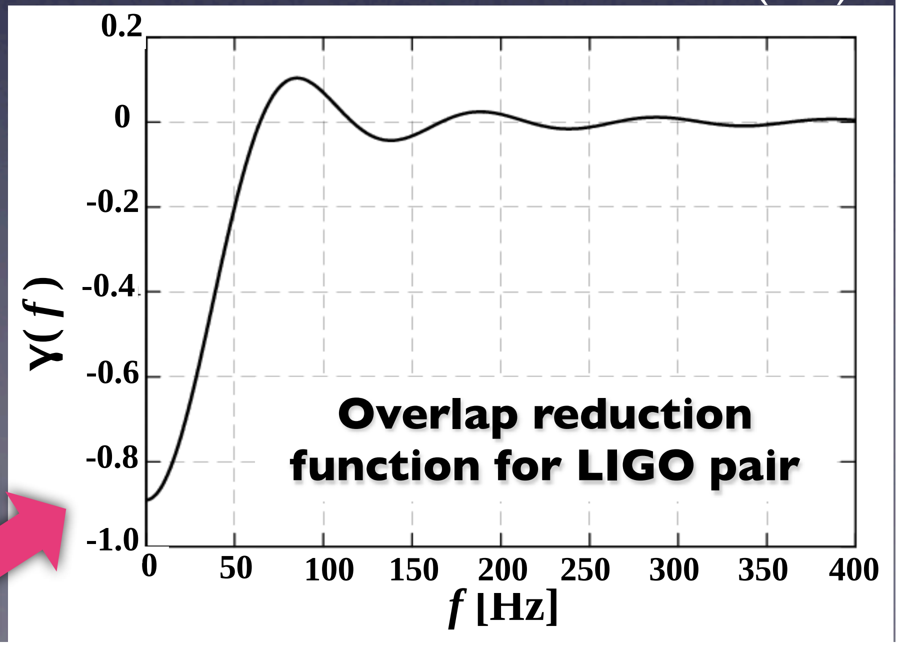
Displacement btw. two detectors

e.g., for LIGO pair $|\Delta\vec{x}| = 3001\mathrm{km}$

In most case, $D_i^{ab} = \frac{1}{2}(\hat{X}_i^a\hat{X}_i^b - \hat{Y}_i^a\hat{Y}_i^b)$

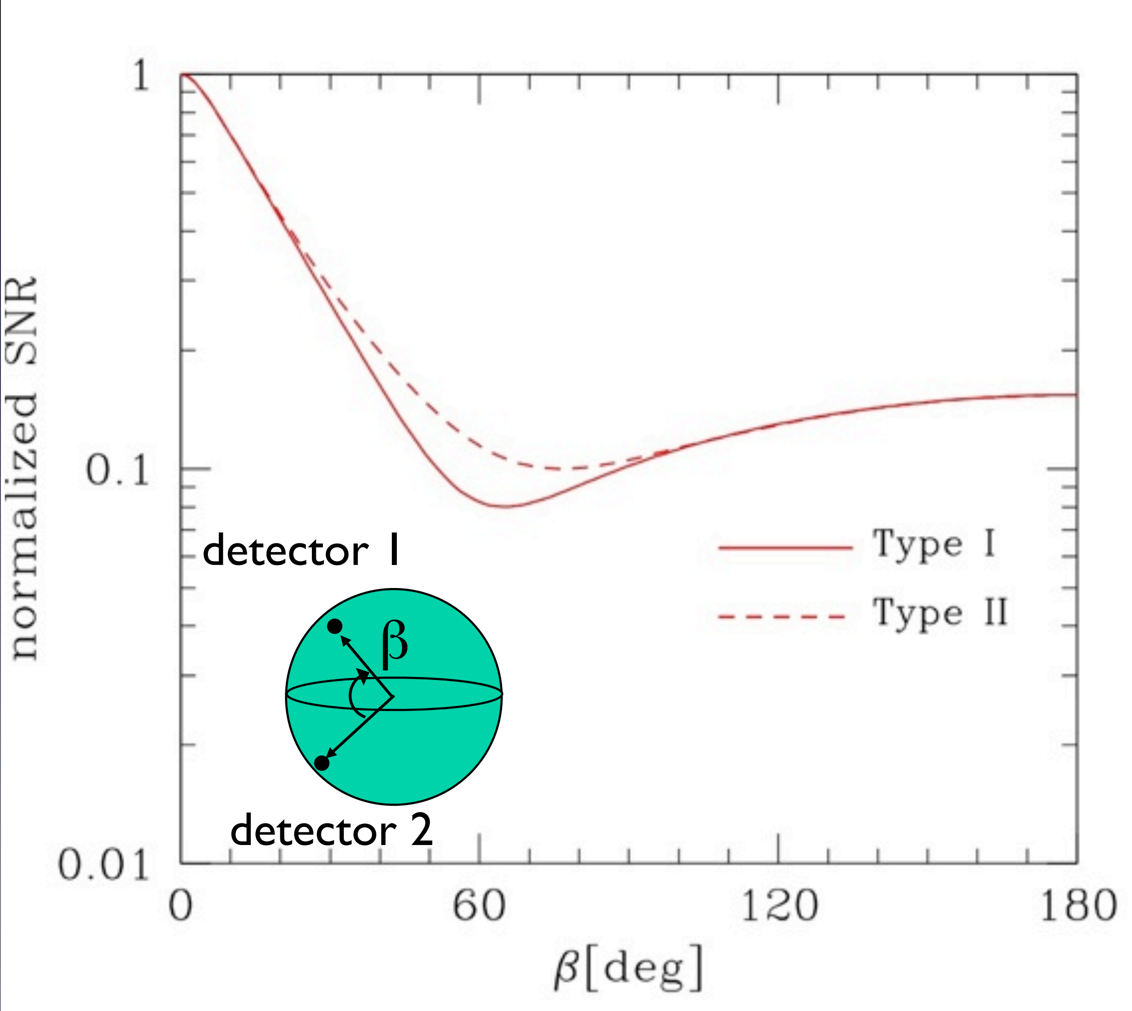


Abbott et al. ('04)

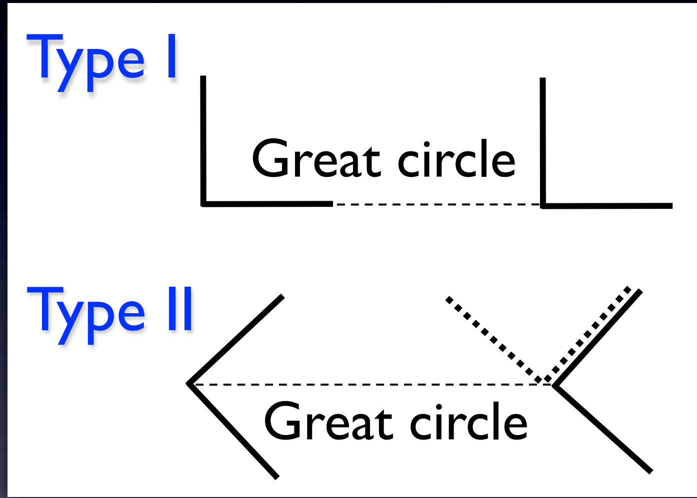


Reduction in sensitivity at $f \geq c/(2|\Delta\vec{x}|)$ (irrespective of detector's sensitivity)

SNR for ground-based detectors



Optimal configuration



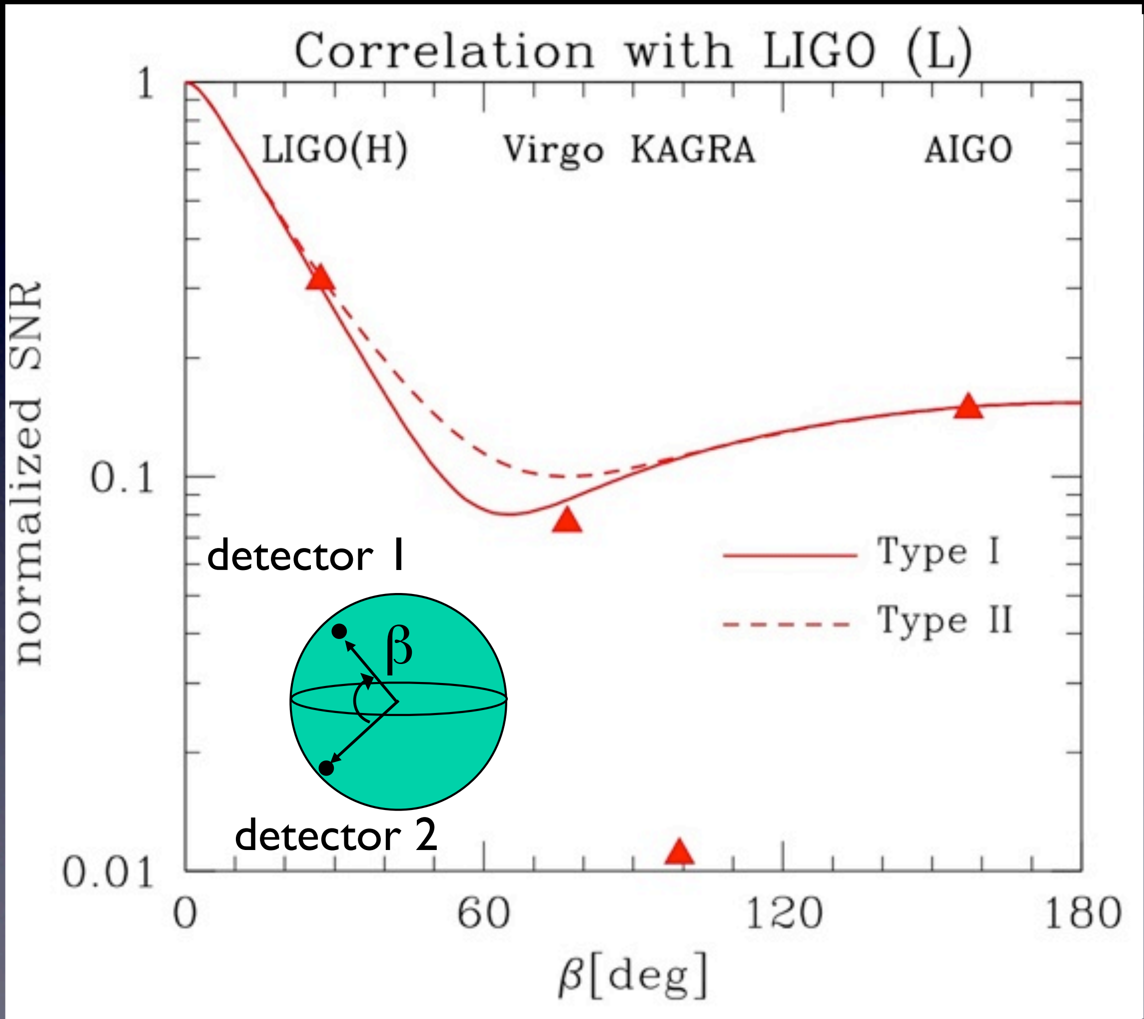
Flanagan ('93), Seto & AT ('08)

normalized w.r.t.

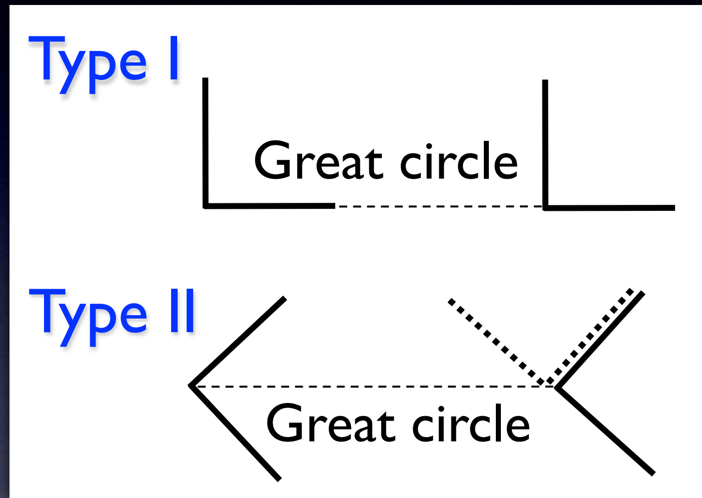
$$\mathrm{SNR}_0 = 5.5 \left(\frac{\Omega_{\rm gw}}{10^{-9}}\right)\left(\frac{T_{\rm obs}}{1\,\mathrm{yr}}\right)^{1/2}$$

co-located & co-aligned detector pair

SNR for ground-based detectors



Optimal configuration



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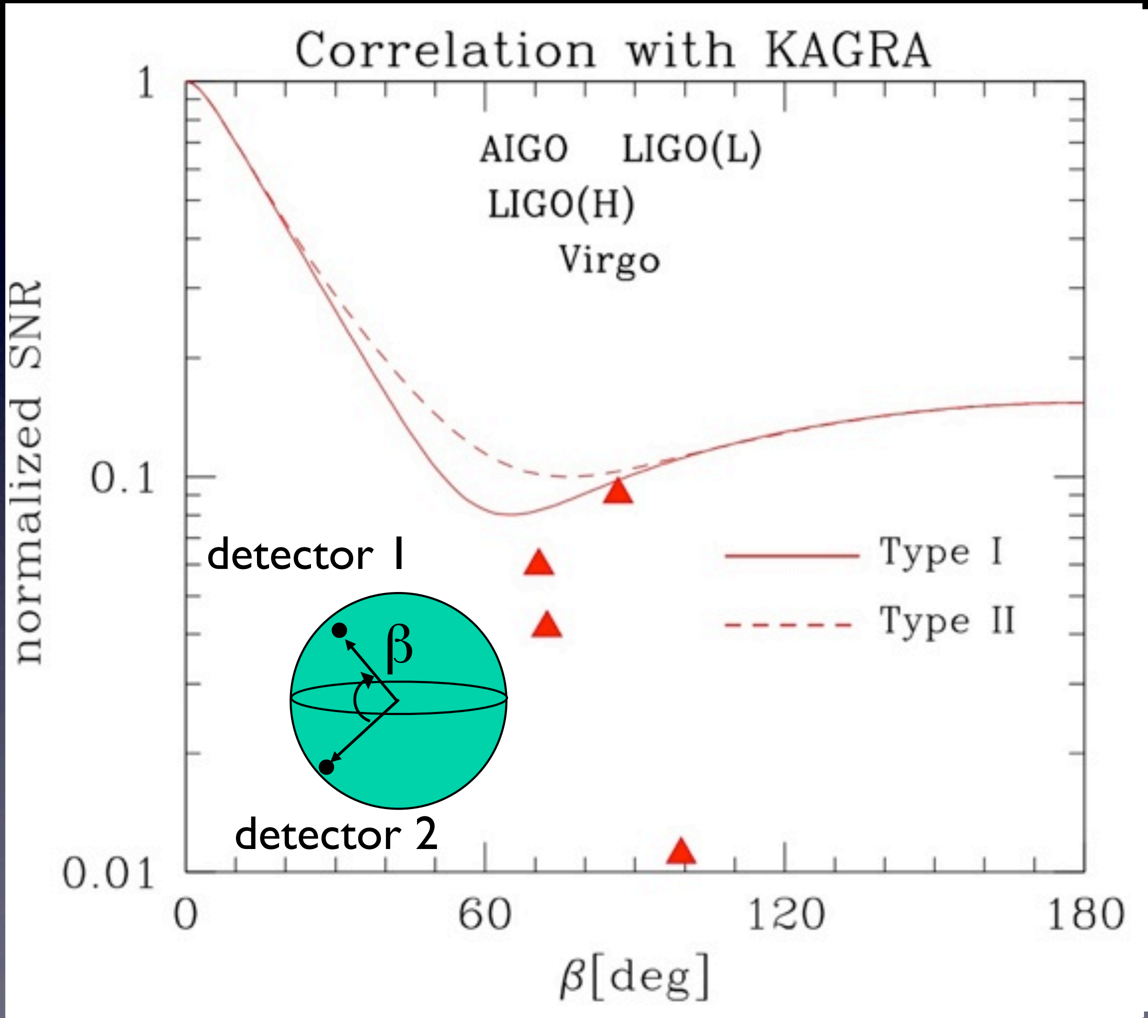
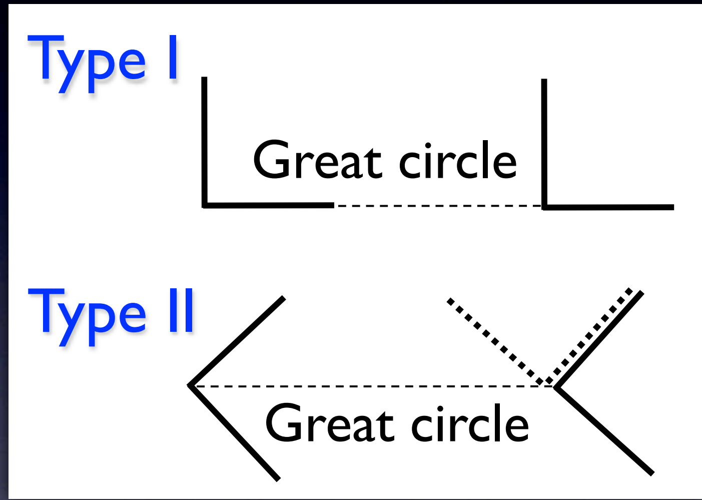
SNR for ground-based detectors



Optimal configuration



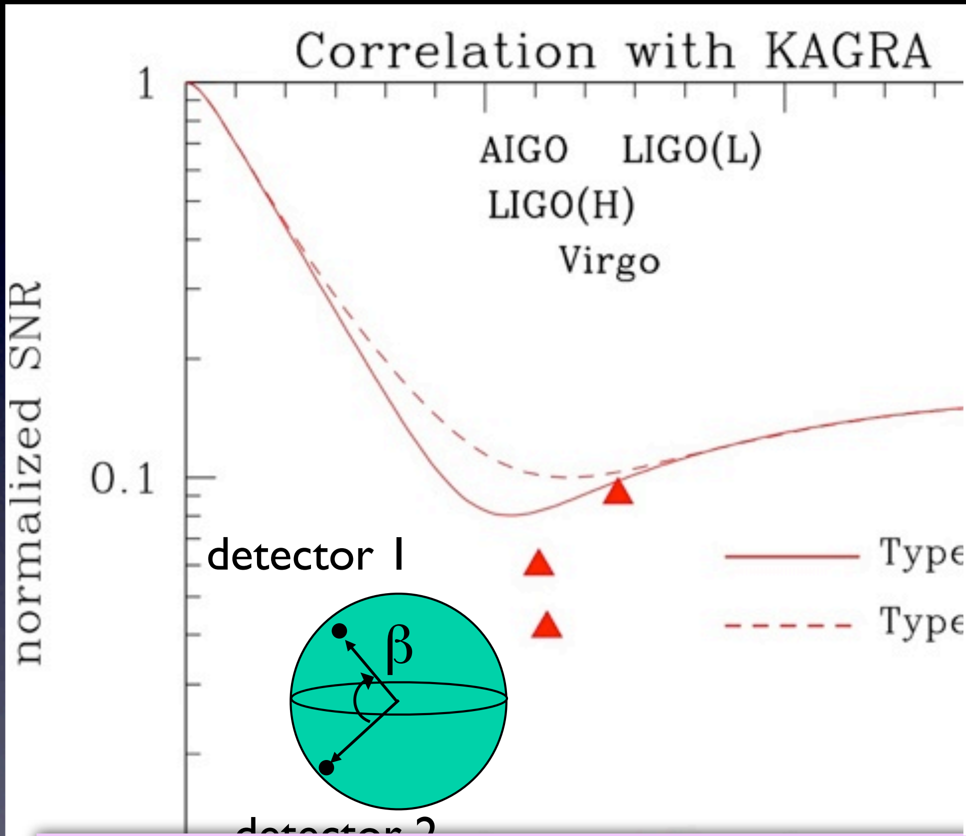
Flanagan ('93), Seto & AT ('08)

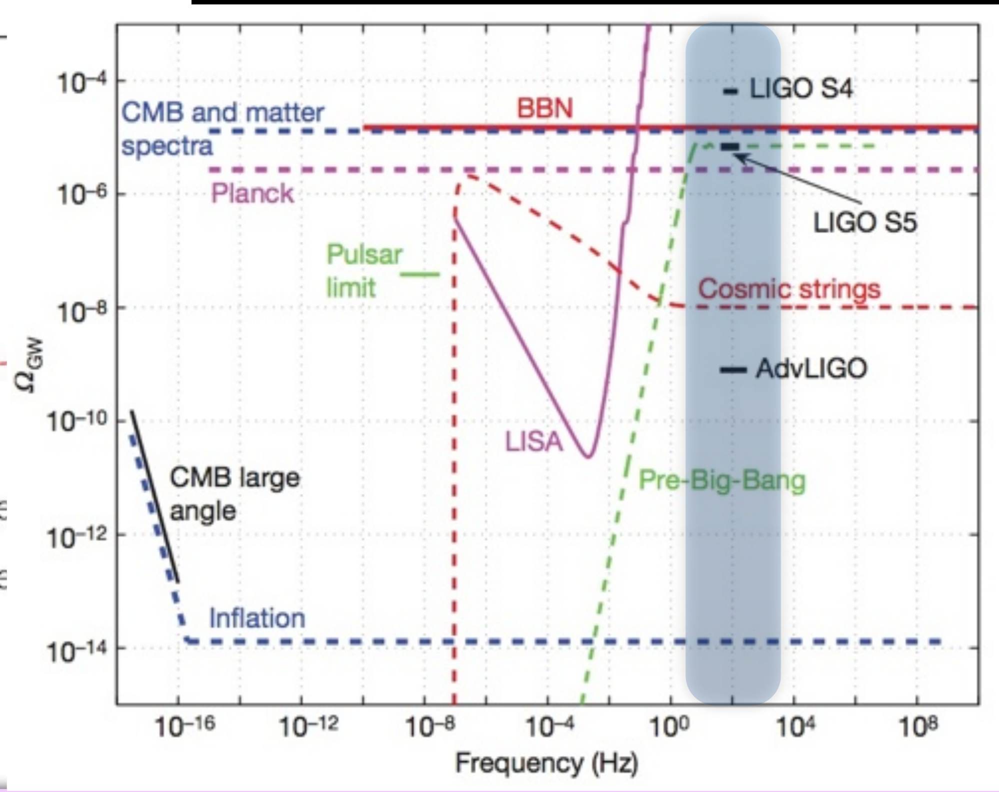
normalized w.r.t.

$$\mathrm{SNR}_0 = 5.5 \left(\frac{\Omega_{\rm gw}}{10^{-9}}\right) \left(\frac{T_{\rm obs}}{1\,\mathrm{yr}}\right)^{1/2}$$

← co-located & co-aligned detector pair

SNR for ground-based detectors





Combining all advanced detectors,

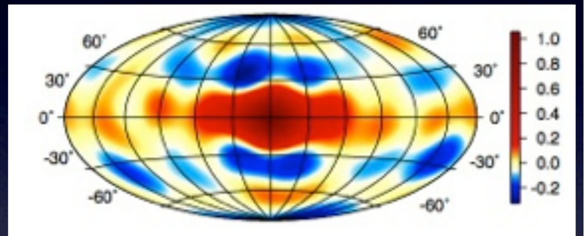
$$\Omega_{\rm gw} h^2 < 2.2\times 10^{-9}\left(\mathrm{SNR}_I/5\right)\left(T_{\rm obs}/1\,\mathrm{year}\right)^{-1/2}$$

c.f. LIGO+VIRGO: $\Omega_{\rm gw} \leq 6.9 \times 10^{-6}$ (Abbott et al. '09)

From detection to characterization

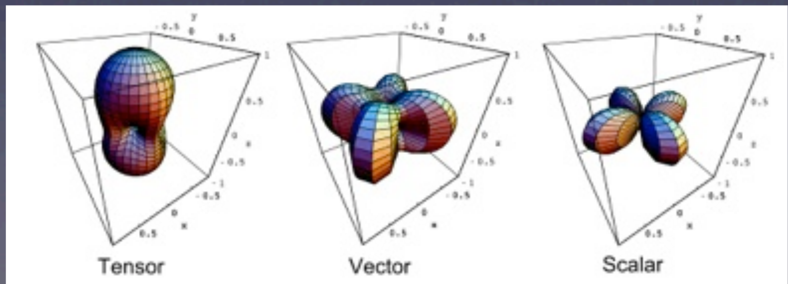
Relaxing several simplifications, characterization of stochastic GW is made possible

Isotropic GWB ⇨ Mapping anisotropies



Kudoh & AT ('05); AT & Kudoh ('05); AT ('06)

Tensor component ⇨ Detecting scalar/vector/tensor GWBs

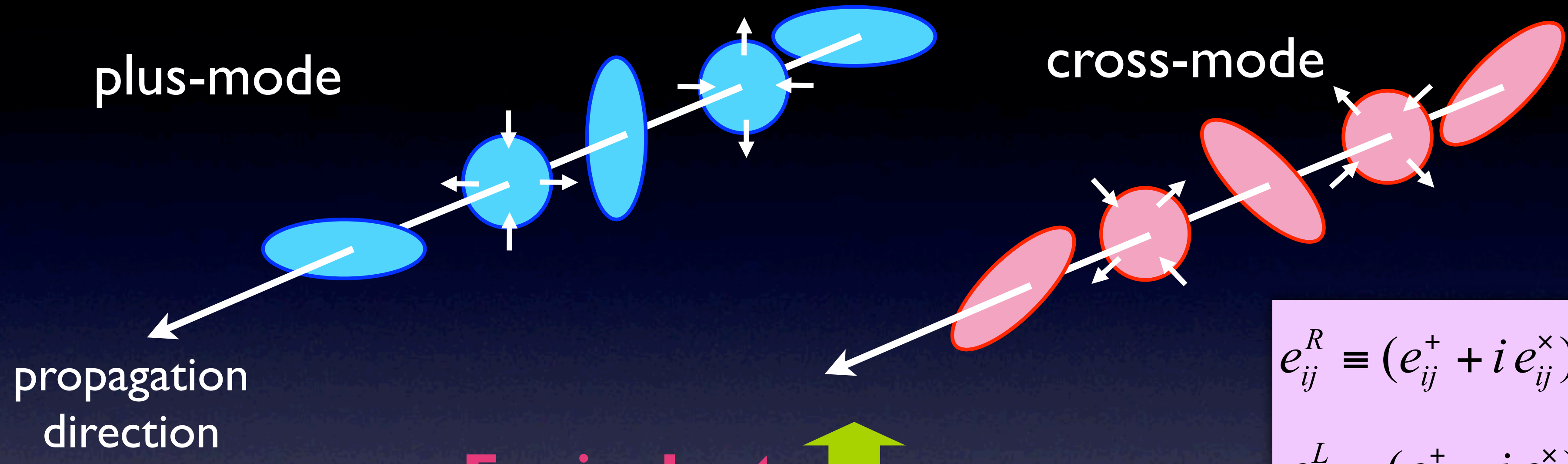


Nishizawa, AT et al. ('09, '10)

Un-polarized state ⇨ Probing polarized states of GWB

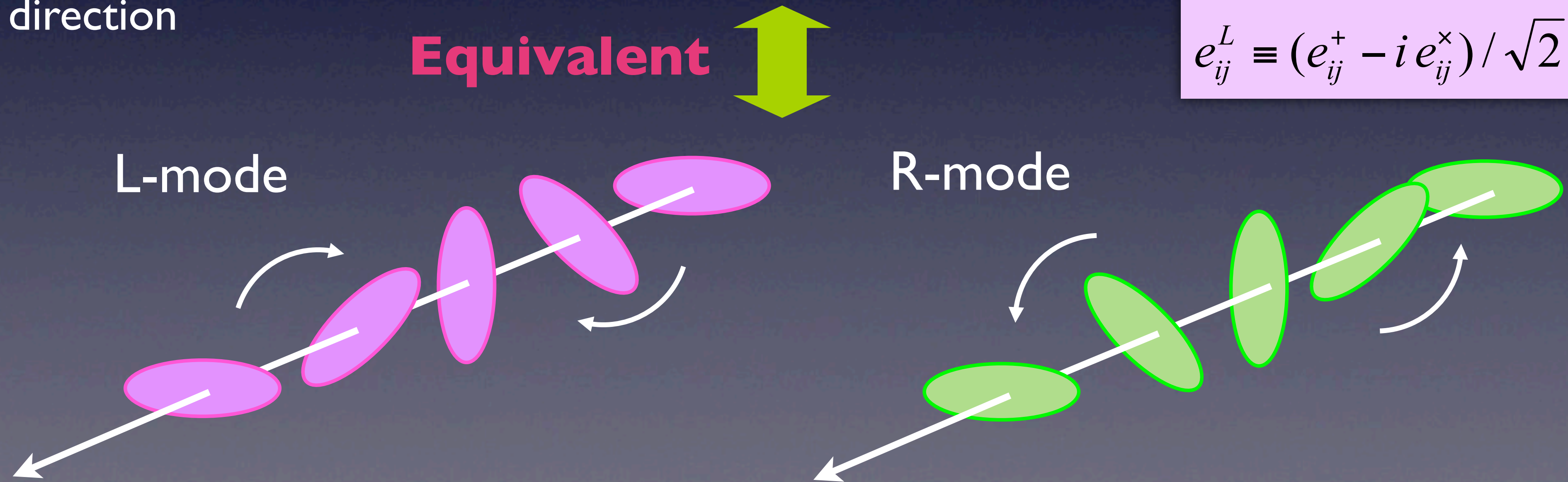
Seto ('07); Seto & AT ('07, '08)

Polarization basis



$$e_{ij}^{R} \equiv (e_{ij}^{+} + i\, e_{ij}^{\times})/\sqrt{2}$$

$$e_{ij}^{L} \equiv (e_{ij}^{+} - i\, e_{ij}^{\times})/\sqrt{2}$$



Unpolarized : $\langle h_R h_R^* \rangle = \langle h_L h_L^* \rangle$ Circularly polarized: $\langle h_R h_R^* \rangle \neq \langle h_L h_L^* \rangle$

Stokes parameters

GW in Fourier space

spin-2 polarization tensor (L,R)

$$h_{ij}(\vec{x},t) = \sum_{A=L,R} \int_{-\infty}^{\infty} df \int d\hat{\Omega}\, h_A(f,\hat{\Omega})\, e^{i\,2\pi\, f(t-\hat{\Omega}\cdot\vec{x}/c)}\, e_{ij}^{A}(\hat{\Omega})$$

Ensemble average

$$\begin{pmatrix} \langle h_R h_R^* \rangle & \langle h_L h_R^* \rangle \\ \langle h_R h_L^* \rangle & \langle h_L h_L^* \rangle \end{pmatrix} = \frac{\delta(f-f')}{2}\,\frac{\delta(\hat{\Omega},\hat{\Omega}')}{4\pi} \begin{pmatrix} I+V & Q+iU \\ Q-iU & I-V \end{pmatrix}$$

I : intensity (spin 0)

$Q \pm iU$: linear polarization (spin ±4)

V : *circular polarization* (spin 0)

Note--.

Linear polarization states are inevitably anisotropic $(\ell \geq 4)$

I- and *V*-modes possess isotropic component

How well we can detect and identify ?

SNR with non-zero polarization

$$\mathrm{SNR} = \sqrt{T}\left(\frac{3H_0^2}{10\pi^2}\right)\left[\int_{-\infty}^{\infty} df\, \frac{\{\gamma_I(f)\Omega_{\rm gw}^I(f) + \gamma_V(f)\Omega_{\rm gw}^V(f)\}^2}{f^6\, S_{n,1}(f)S_{n,2}(f)}\right]^{1/2}$$

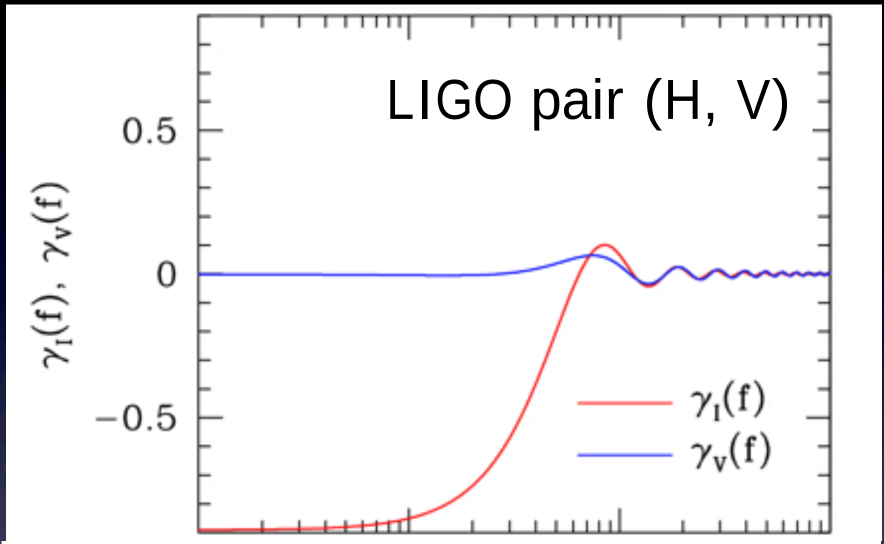
$\Omega_{\rm gw}^{I,V}(f)$: energy density spectrum of GWB for *I*-, *V*-modes

$\gamma_{I,V}(f)$: Overlap reduction functions for *I*-, *V*-modes

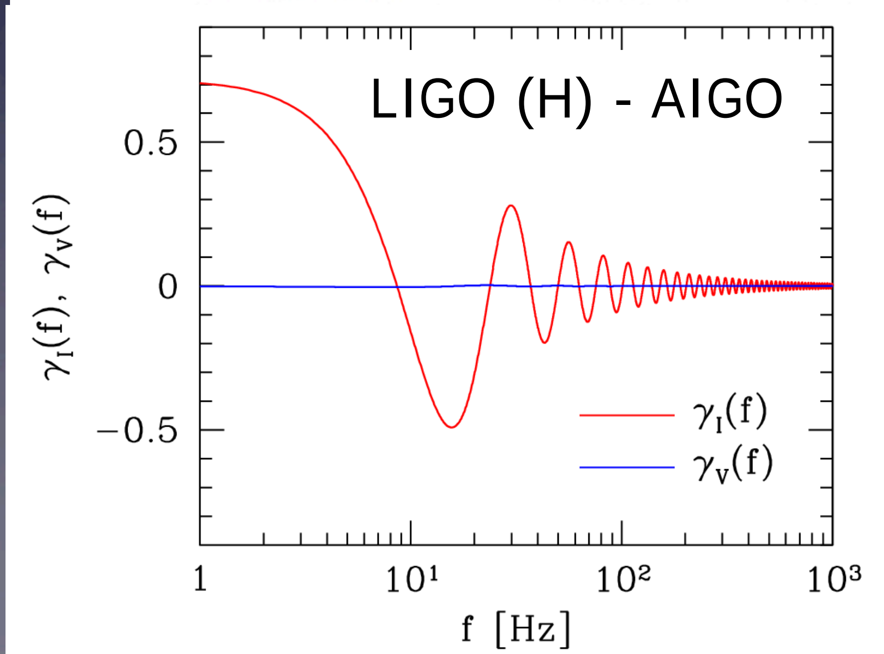
$$\gamma_I(f) = \frac{5}{8\pi}\int d\hat{\Omega}\, D_1^{ab} D_2^{cd}\left[e_{ab}^{+}(\hat{\Omega})e_{cd}^{+}(\hat{\Omega}) + e_{ab}^{\times}(\hat{\Omega})e_{cd}^{\times}(\hat{\Omega})\right] e^{i\,2\pi f\hat{\Omega}\cdot\Delta\vec{x}/c}$$

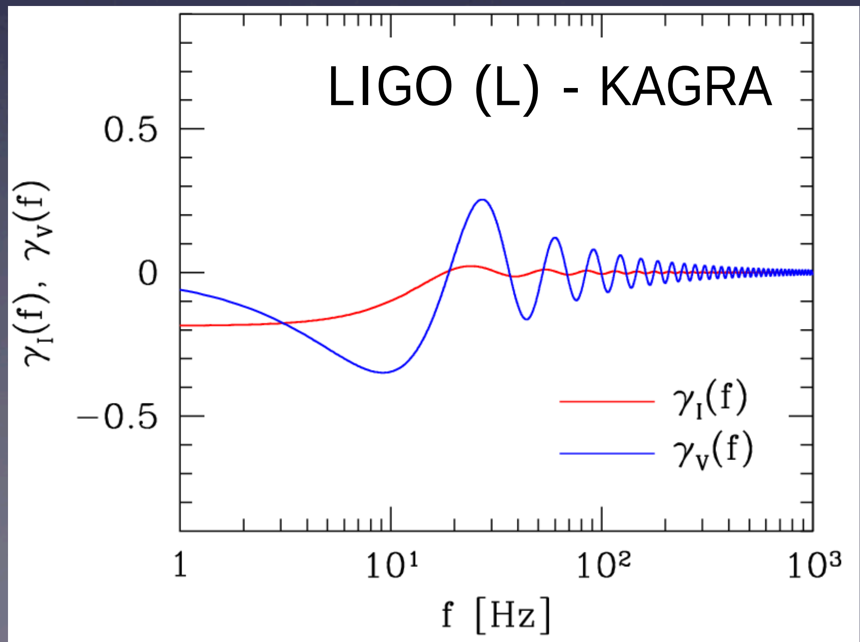
$$\gamma_V(f) = i\frac{5}{8\pi}\int d\hat{\Omega}\, D_1^{ab} D_2^{cd}\left[e_{ab}^{\times}(\hat{\Omega})e_{cd}^{+}(\hat{\Omega}) - e_{ab}^{+}(\hat{\Omega})e_{cd}^{\times}(\hat{\Omega})\right] e^{i\,2\pi f\hat{\Omega}\cdot\Delta\vec{x}/c}$$

Overlap reduction functions

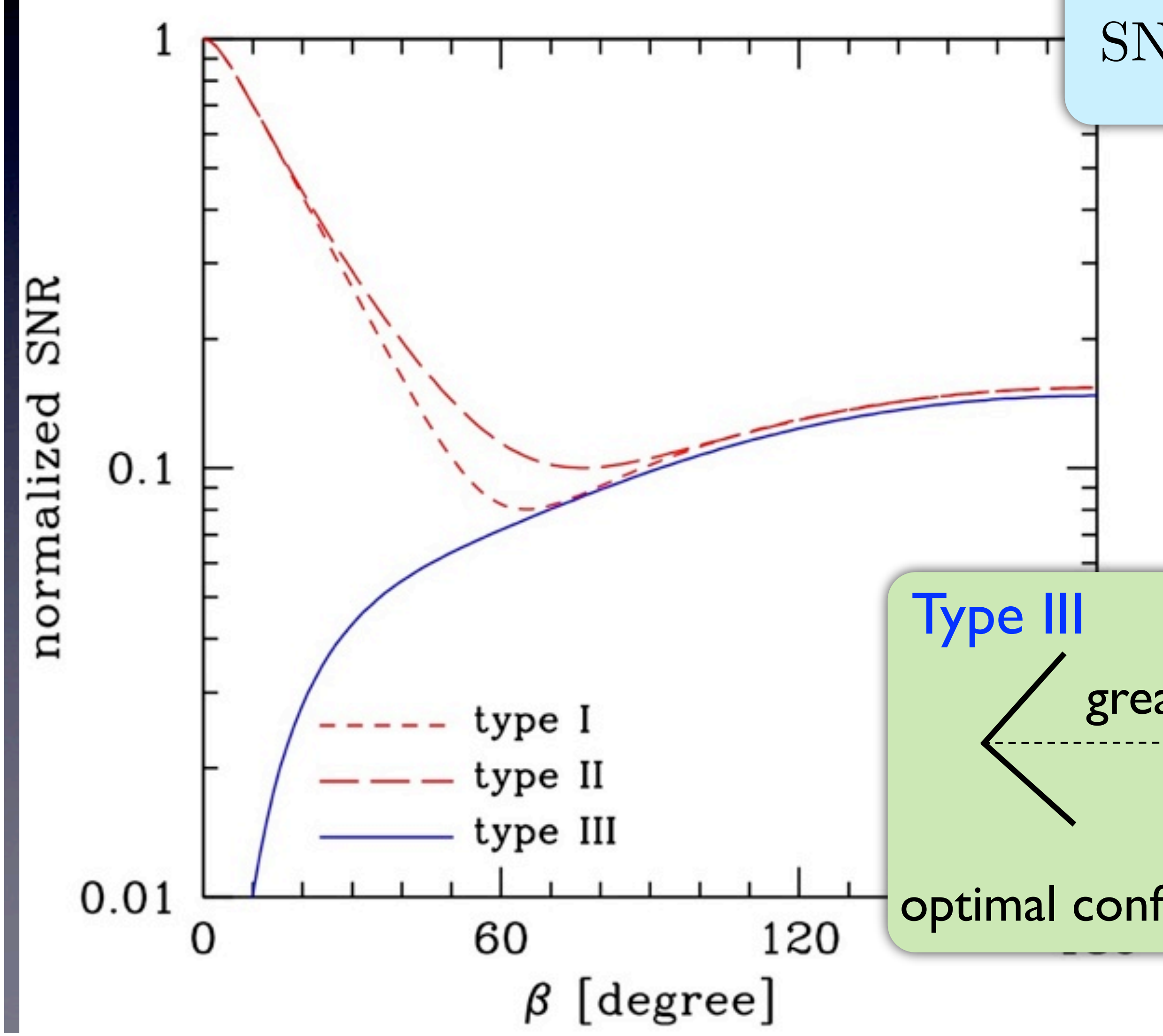


Even if the detector pair is insensitive to I-modes, it could be sensitive to V-mode





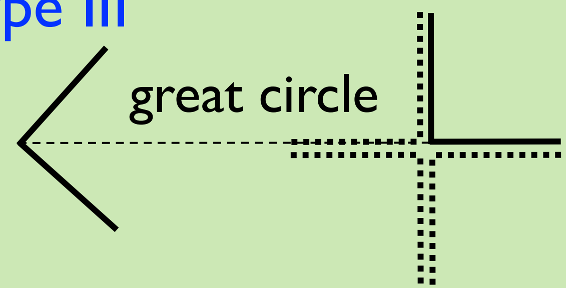
Sensitivity to polarized GWBs



normalized w.r.t.

$$\mathrm{SNR}_0 = 5.5 \left(\frac{\Omega_{\mathrm{gw}}}{10^{-9}} \right) \left(\frac{T_{\mathrm{obs}}}{1\,\mathrm{yr}} \right)^{1/2}$$

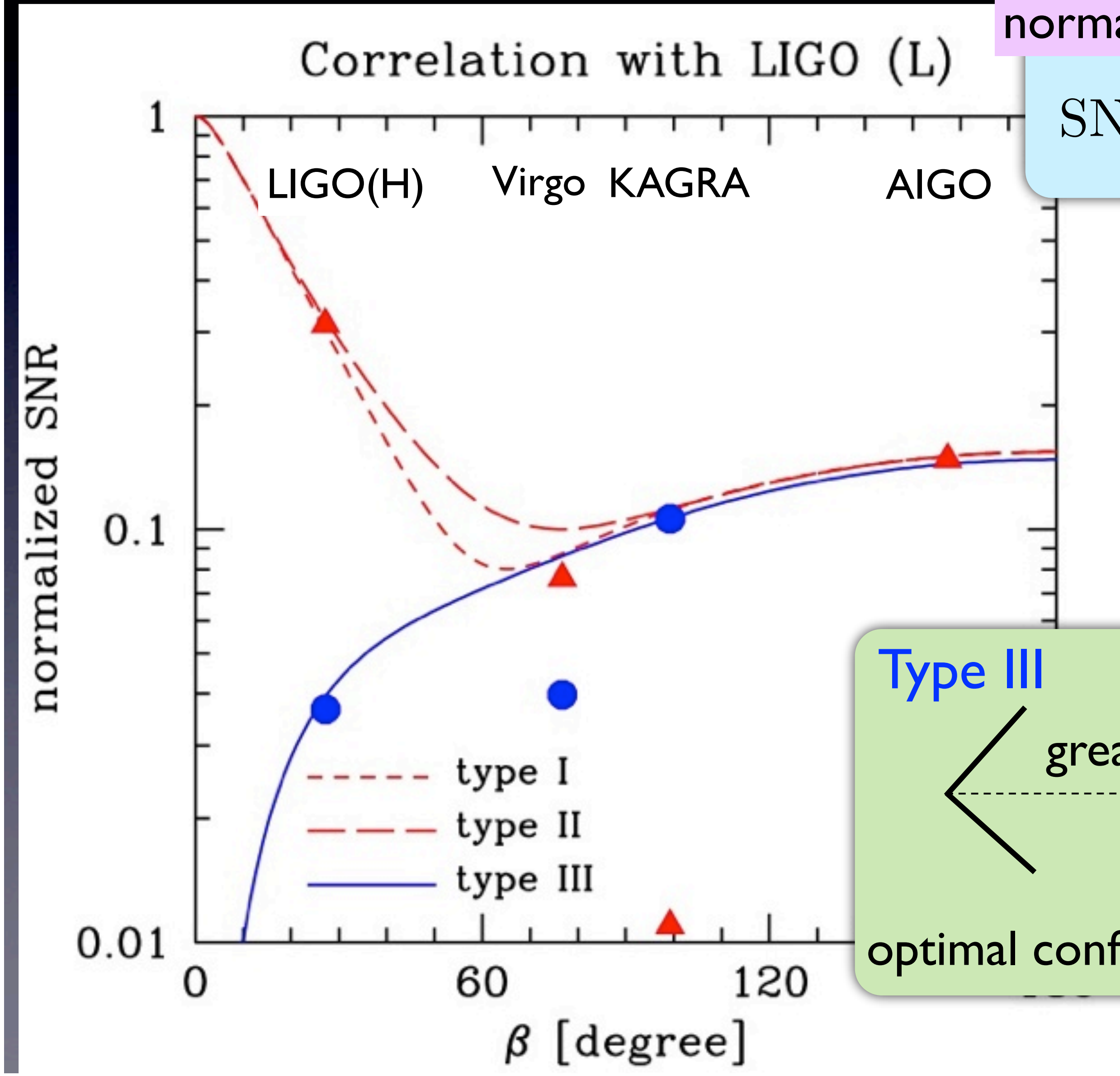
Type III



optimal configuration for *V*-mode

Seto & AT ('07, '08)

Sensitivity to polarized GWBs



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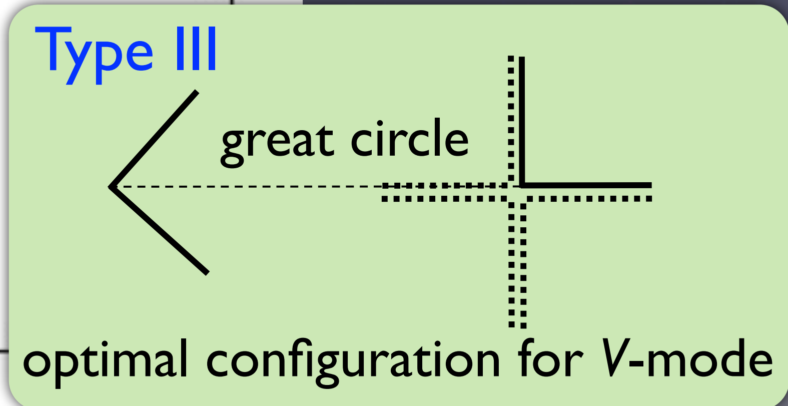
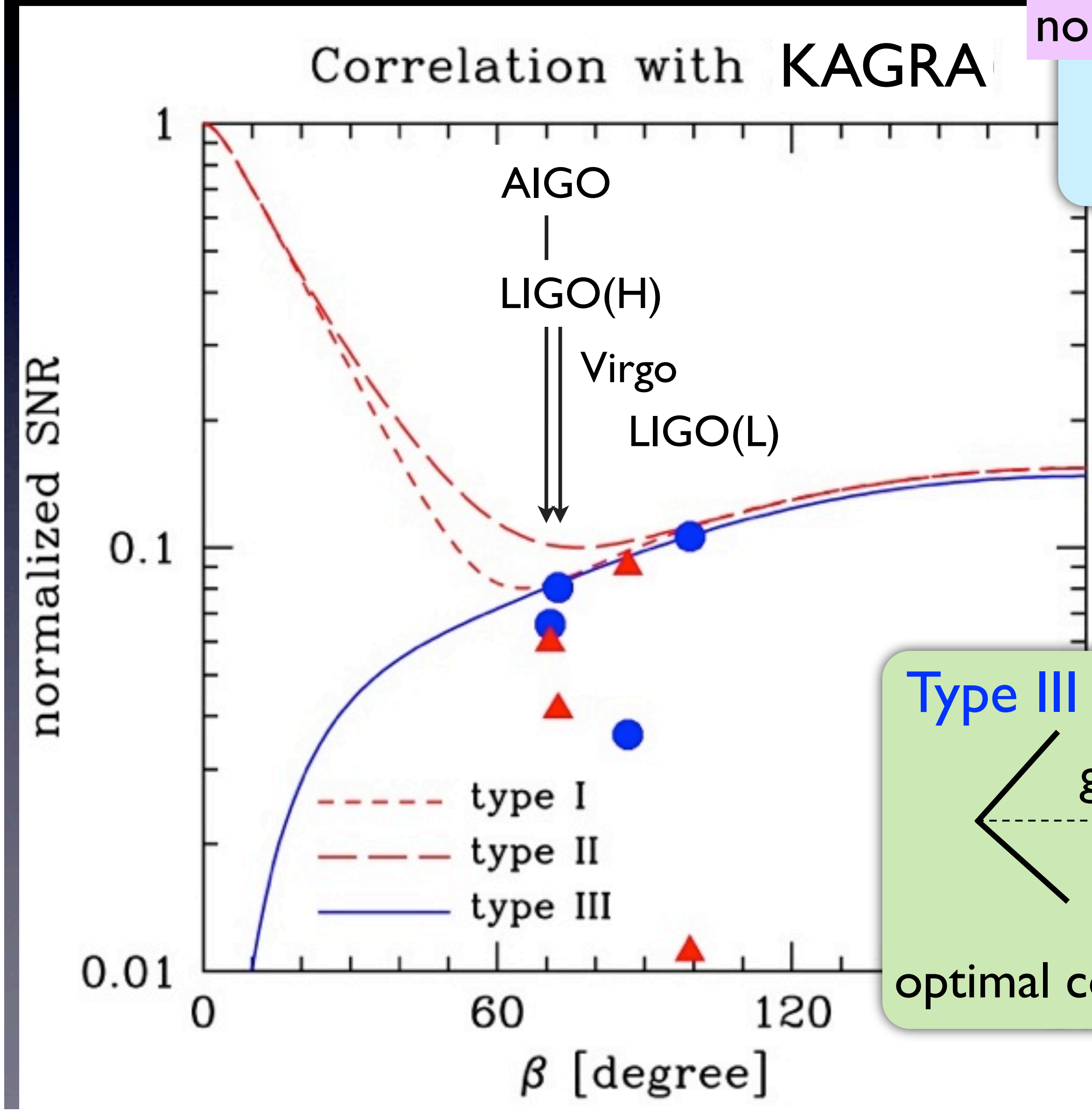


Seto & AT ('07, '08)

Sensitivity to polarized GWBs



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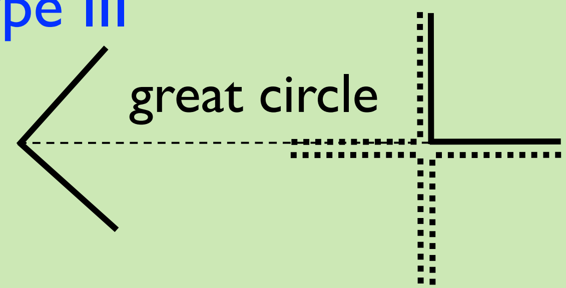


optimal configuration for *V*-mode

Seto & AT ('07, '08)

Separate detection of *I*-, *V*-modes

Basic idea

Taking a linear combination of 2 correlation signals out of 4 detectors

$$\mu_{ab} \equiv \langle s_a s_b \rangle \propto \gamma^I_{ab} I + \gamma^V_{ab} V, \qquad \mu_{cd} \equiv \langle s_c s_d \rangle \propto \gamma^I_{cd} I + \gamma^V_{cd} V$$

$$\begin{cases} \gamma^I_{ab}\mu_{cd} - \gamma^I_{cd}\mu_{ab} \propto (\gamma^I_{ab}\gamma^V_{cd} - \gamma^I_{cd}\gamma^V_{ab})\, I \cdots\cdots \text{I-mode} \\ \gamma^V_{ab}\mu_{cd} - \gamma^V_{cd}\mu_{ab} \propto (\gamma^V_{ab}\gamma^I_{cd} - \gamma^V_{cd}\gamma^I_{ab})\, V \cdots\cdots \text{V-mode} \end{cases}$$

With more than 4 detectors,

separate detection of *I*-, and *V*-modes can be optimized:

Minimum. detectable amplitude (5 detectors)

$$\Omega_{\mathrm{gw}} h^2 = \begin{cases} 2.2\times 10^{-9}\left(\mathrm{SNR}_I/5\right)\left(T_{\mathrm{obs}}/1\mathrm{year}\right)^{-1/2} \cdots\cdots \text{I-mode} \\ 4.5\times 10^{-9}\left(\mathrm{SNR}_V/5\right)\left(T_{\mathrm{obs}}/1\mathrm{year}\right)^{-1/2} \cdots\cdots \text{V-mode} \end{cases}$$

Seto & AT ('08)

From ground to space

Short summary With cross correlation & long-term obs,

- detection capability of GWBs sensitively depends on separation & geometric configuration of detectors
- a network of ground-based detecters would provide a nearly optimal detection & characterization with $\Omega_{\rm gw} h^2 \sim 10^{-9}$

(polarization states and/or scalar/vector/tensor modes)

For space-based detectors,

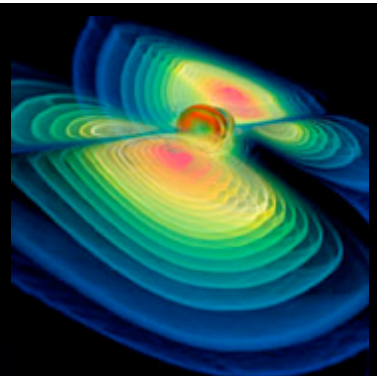
detection & characterization further depends on

orbital trajectory & constellation of space crafts

Seto ('06); Kudoh et al. ('06); Seto ('07); Nishizawa et al.('10)

foreground removal Cutler & Harms ('06); Yagi & Seto ('11)

Dark clouds over deci-Hz



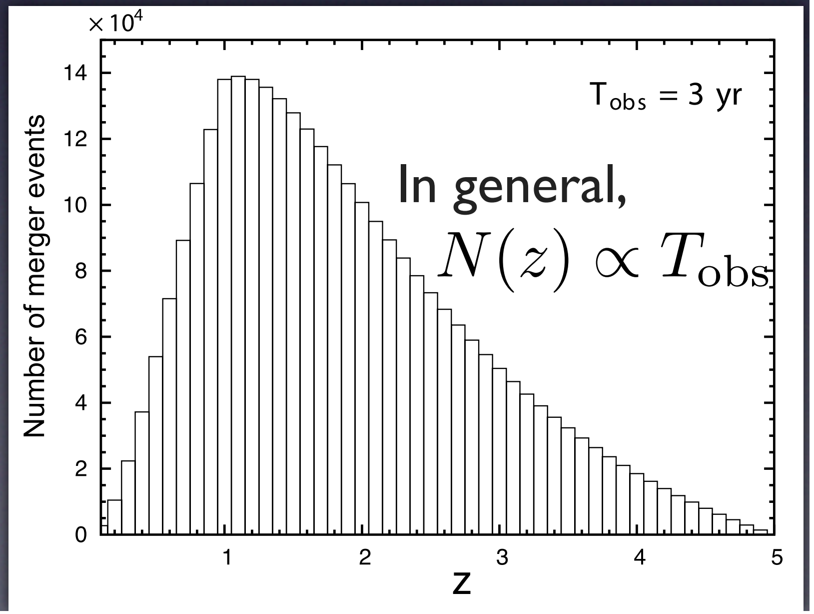
A large number of <u>NS-NS binaries</u> will be detected at ***0.1~1Hz***

………… sources of confusion noise to be subtracted

On the other hand,

The system can be treated as a point-particle system until last 3 min of coalescence → very "*clean*" system (free from systematics)

# of NS-NS binaries



Q

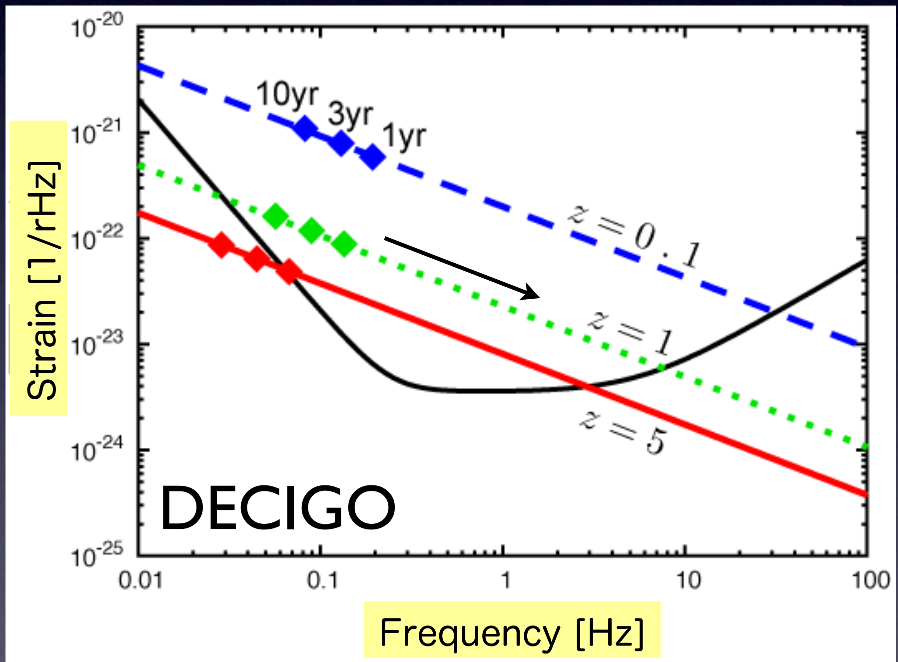
Safely subtracting these systems to detect GWBs, how well we can use them as cosmological tool ?

NS-NS binary as a standard siren

Schutz ('86)

Quadrupole formula

$$h_{+\times}(t) \propto \frac{[(1+z)M_c]^{5/3} f^{2/3}}{D_L}, \qquad \dot{f}(t) \propto [(1+z)M_c]^{5/3} f^{11/3},$$



chirp mass: $M_c = \dfrac{(m_1\, m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$

$h_{+,\times}$ and $f,\ \dot{f}$

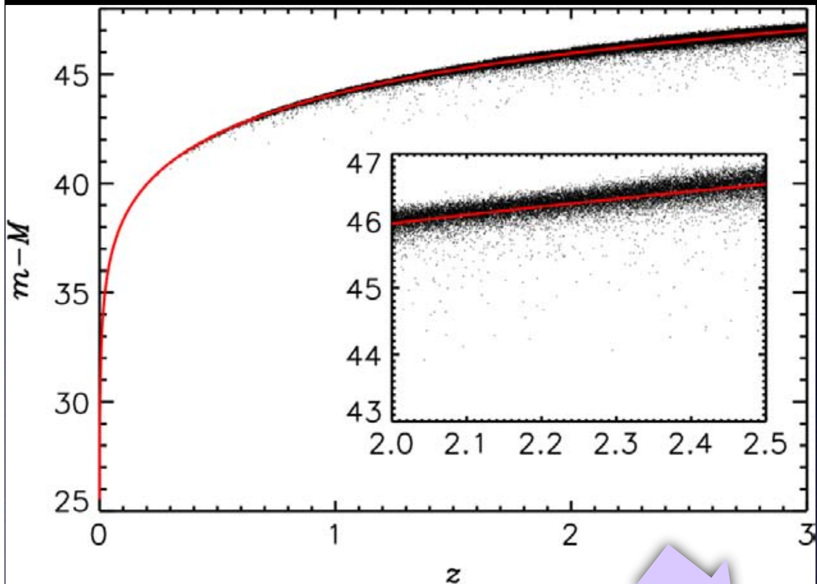
$$M_z = (1+z)M_c$$

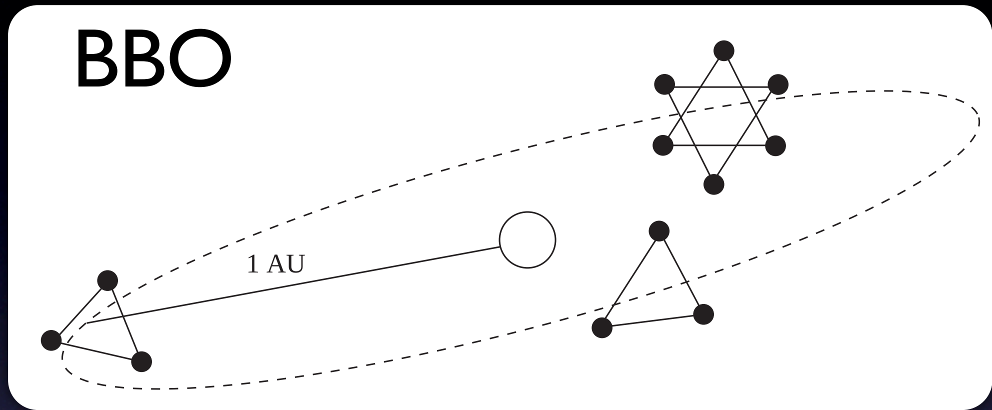
$$D_L(z)$$

Provided redshift information for host galaxies from optical follow-up, NS-NS binaries can be used as "*standard siren*" to trace cosmic expansion like SNe Ia

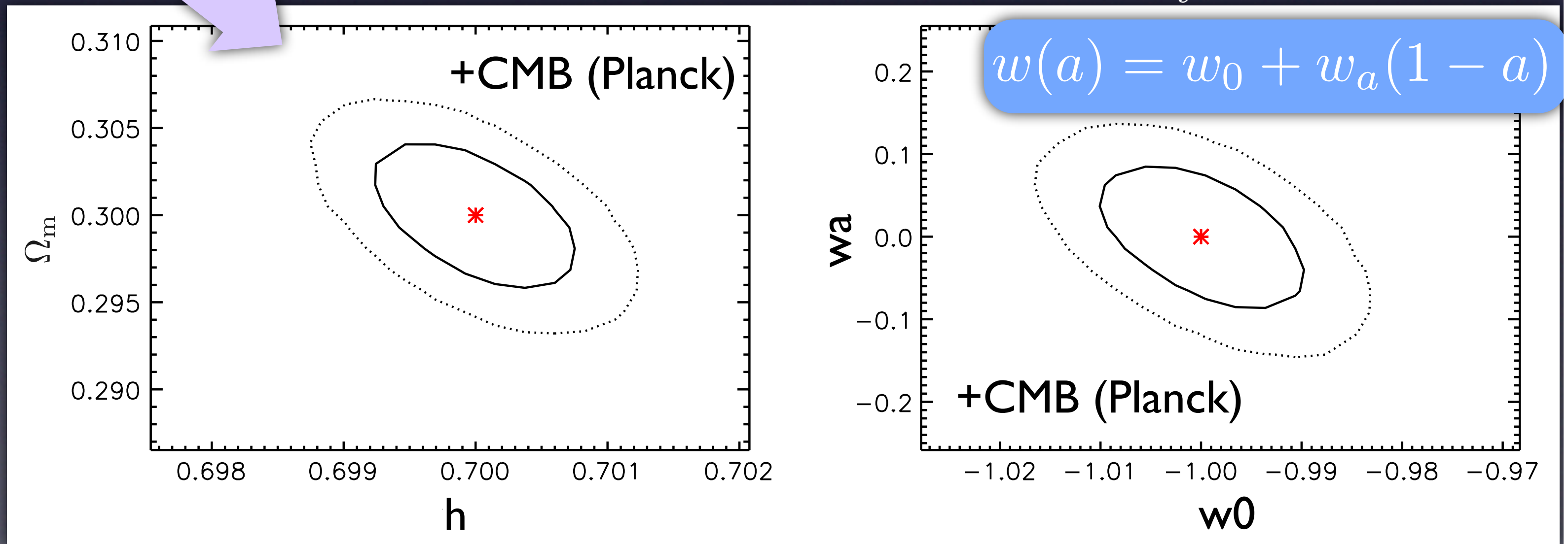
Ultra precision cosmology

Cutler & Holz ('09)





$N_{\rm binary} = 2.5 \times 10^5$



$w(a) = w_0 + w_a(1-a)$

But,

Big assumption--Redshifts of binaries are all measured by follow-up obs.

Standard sirens w/o follow-up obs.

Nishizawa, Yagi, AT & Tanaka ('12)

Key idea redshift drift : $\Delta t = \Delta T + X(z) \Delta T^2$ $\quad X(z) \equiv \frac{1}{2}\left[H_0 - \frac{H(z)}{1+z}\right]$

Δt : time to coalescence at observer

$\Delta T = (1+z)\Delta t_e$: redshifted time to coalescence at source

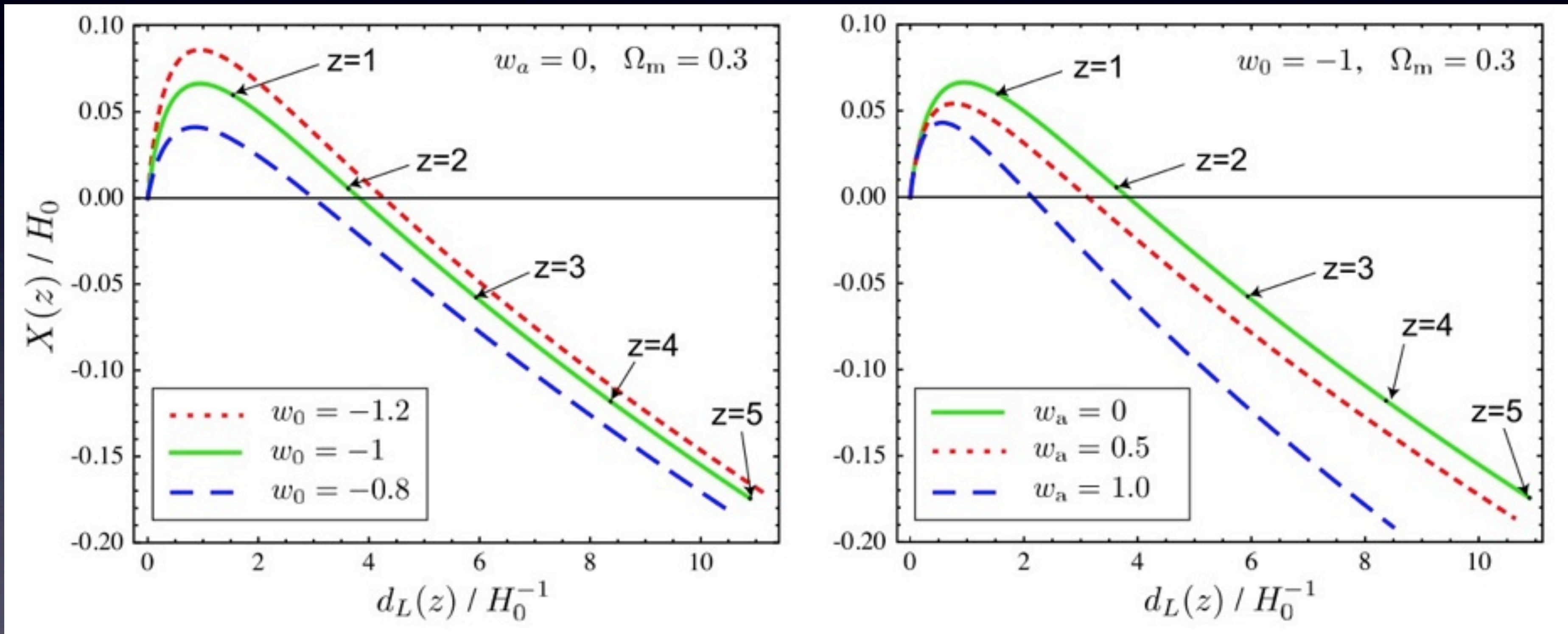
Sandage ('62), Loeb ('98)

$$\tilde{h}(f) = \frac{A}{d_L(z)} M_z^{5/6} f^{-7/6} e^{i\Psi(f)} \,; \quad \begin{aligned} \Psi(f) \;&=\; 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\pi M_z f)^{-5/3} \\ &\quad - \frac{25}{32768} X(z) (\pi f)^{-13/3} M_z^{-10/3} \end{aligned}$$

Seto, Kawamura & Nakamura ('01)

Cosmic acceleration from d_L & X

amplitude $\longrightarrow$ $d_L(z)$ phase $\longrightarrow$ $X(z)$



But, low S/N for phase drift measurement:

$$\left(\frac{\Delta d_L}{d_L}\right)_1 \sim 10^{-2} \qquad \left(\frac{\Delta X}{X}\right)_1 = 10^3 \sim 10^4$$

- long observation time (=observe many binaries)
- improved detector sensitivity

Self-consistent analysis

Nishizawa, Yagi, AT & Tanaka ('12)

Taking the binary confusion noise into account,

What is the required detector sensitivity for cosmological science on dark energy & primordial GWB ?

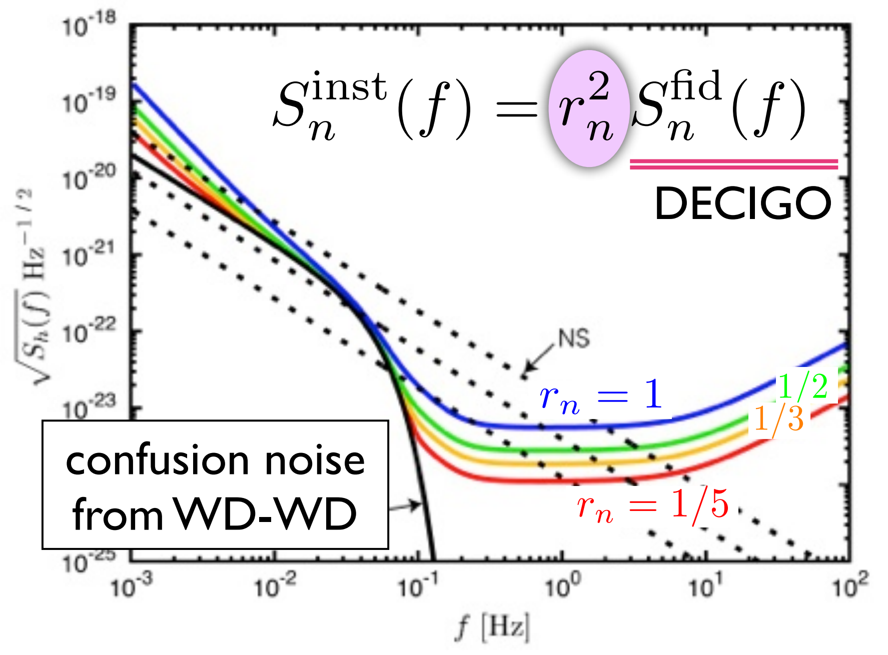
step1

Assume detector's sensitivity

$$S_n^{\rm inst}(f) = r_n^2\, S_n^{\rm fid}(f)$$

step2

Estimate fraction of un-resolved NS-NS binaries out to z=5 $\mathcal{R}_{\rm NS}(f)$

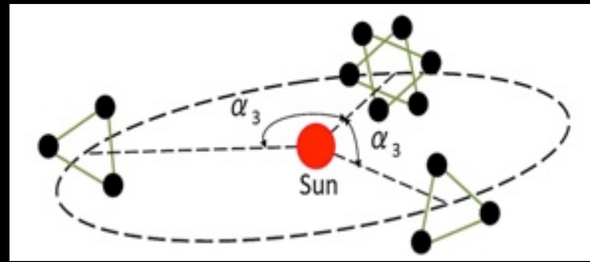


※ BBO corresponds to $r_n = 1/3$

step3

Estimate SNR for primordial GWB & expected constraint on dark energy in the *presence of residual noises:* $S_n(f) = S_n^{\rm inst}(f) + S_n^{\rm WD}(f) + S_n^{\rm NS}(f)\mathcal{R}_{\rm NS}(f)$

Result (I)



Fraction of un-resolved NS-NS binaries $\mathcal{R}_{\rm NS}$ below *SNR=20*

r_n	1 year	3 year	5 year
1	4.66×10^{-3}	4.64×10^{-3}	4.62×10^{-3}
1/2	0	0	0
1/3	0	0	0
1/5	0	0	0

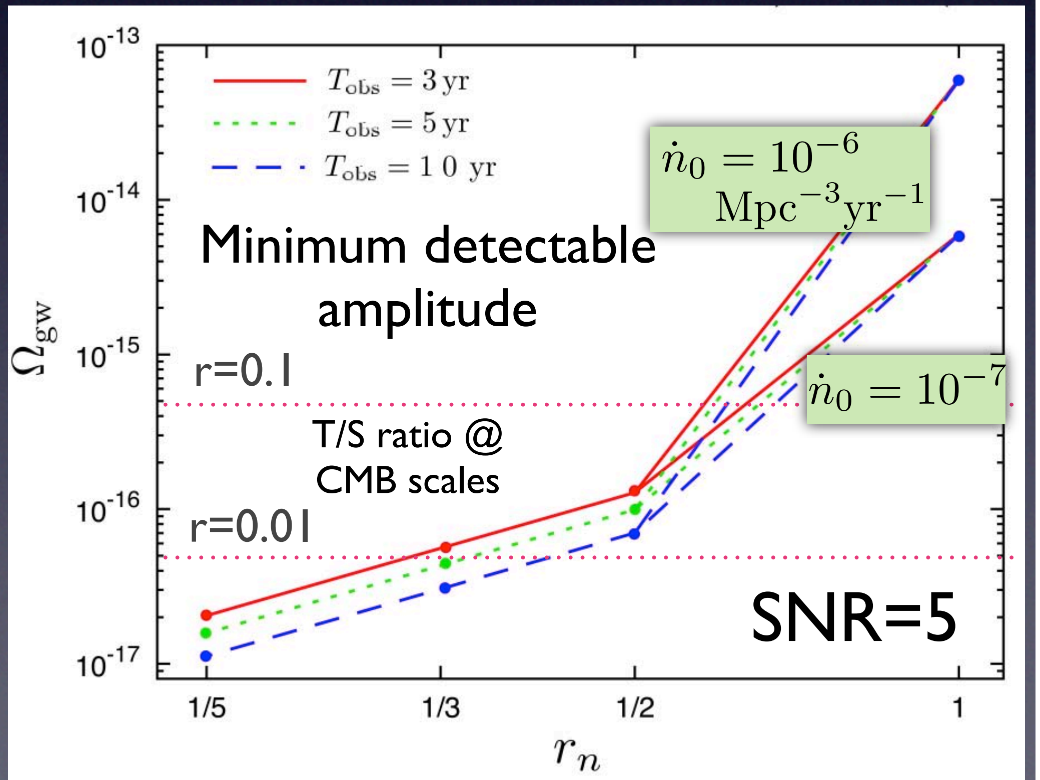
based on the Monte Carlo results by Yagi & Seto ('11)

SNR for Primordial GWB

$$\mathrm{SNR} = \frac{9H_0^2}{20\sqrt{2}\pi^2}\sqrt{N_{\rm corr}T_{\rm obs}} \times \left[\int_{f_{\rm min}}^{f_{\rm max}} df \frac{\Omega_{\rm gw}^2 \Theta[\Omega_{\rm gw} - \Omega_{\rm gw}^{\rm NS}(f)\mathcal{R}_{\rm NS}]}{f^6\{S_n(f)\}^2}\right]^{1/2}$$

$N_{\rm corr} = 2$

$$S_n(f) = S_n^{\rm inst}(f) + S_n^{\rm WD}(f) + S_n^{\rm NS}(f)\mathcal{R}_{\rm NS}(f)$$



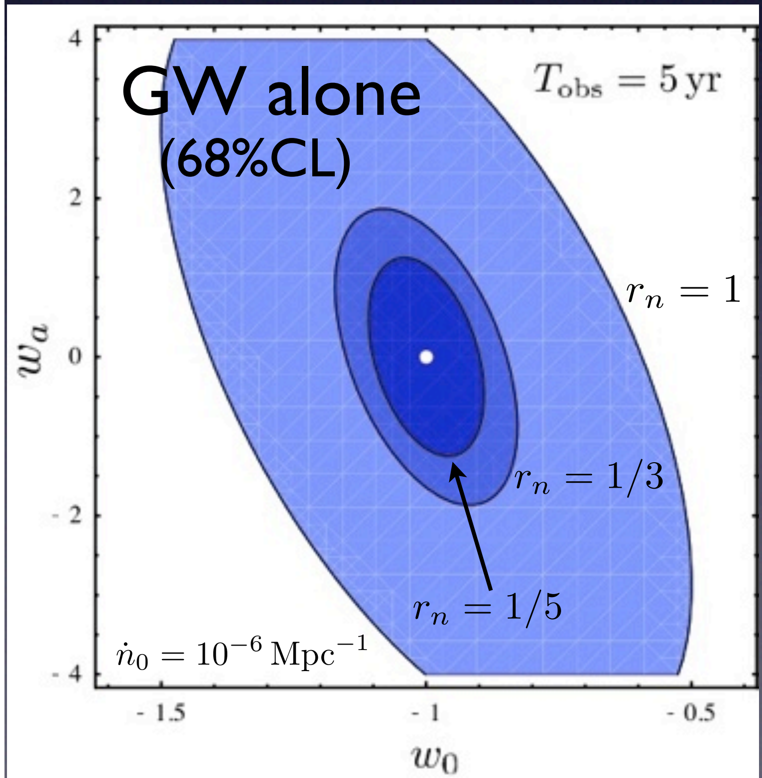
Result (2)

Fisher matrix

6 params including d_L, X

$$\Gamma_{ab} = 4\sum_{i=1}^{8} \mathrm{Re}\int_{f_{\min}}^{f_{\max}} \frac{\partial_a \tilde{h}^*_{(i)}(f)\,\partial_b \tilde{h}_{(i)}(f)}{S_n(f)} df \quad \left(\tilde{h}(f):\ \text{GW waveform (restricted 1.5PN)}\right)$$

The errors in $d_L(z_i)$ & $X(z_i)$ obtained from Fisher matrix are translated to the uncertainties in cosmological parameters $(\Omega_{\rm m},\ w_0,\ w_a)$



Dark energy E.O.S

$w(a) = w_0 + w_a(1-a)$

Result (2)

6 params including d_L, X

Fisher matrix

$$\Gamma_{ab} = 4\sum_{i=1}^{8} \mathrm{Re}\int_{f_{\min}}^{f_{\max}} \frac{\partial_a \tilde{h}^*_{(i)}(f)\,\partial_b \tilde{h}_{(i)}(f)}{S_n(f)}df \quad \left(\tilde{h}(f): \text{GW waveform (restricted 1.5PN)}\right)$$

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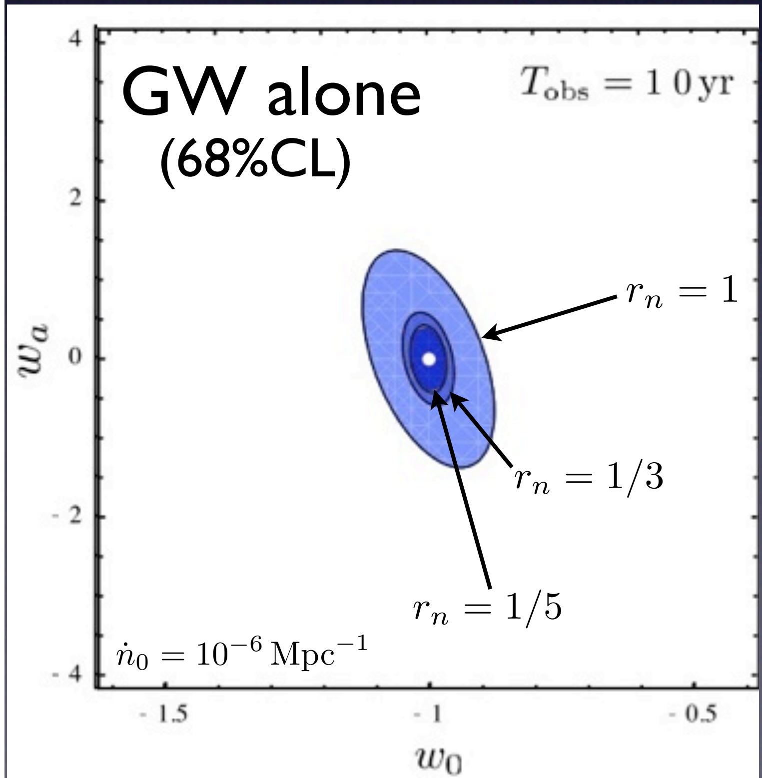

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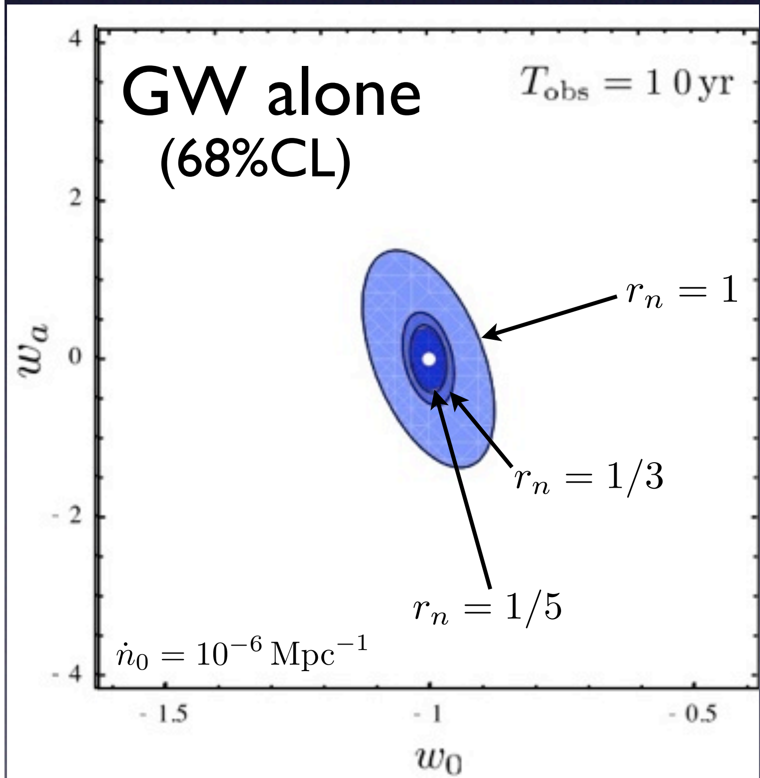


Dark energy E.O.S
$w(a) = w_0 + w_a(1-a)$

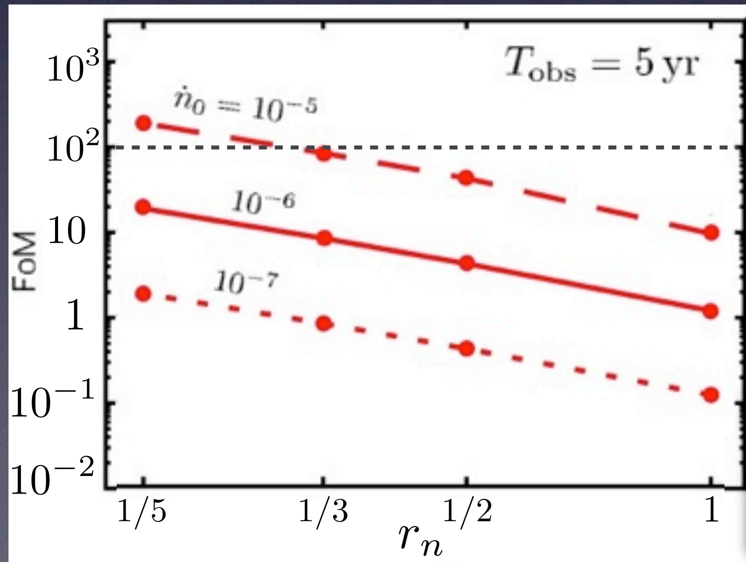


Figure-of-Merit for Dark energy

$$\mathrm{FoM} = [\sigma(w_0)\sigma(w_a)]^{-1}$$

$\dot{n}_0 = 10^{-5}\,\mathrm{Mpc}^{-3}\mathrm{yr}^{-1}$ (dashed)
10^{-6} (solid)
10^{-7} (dotted)

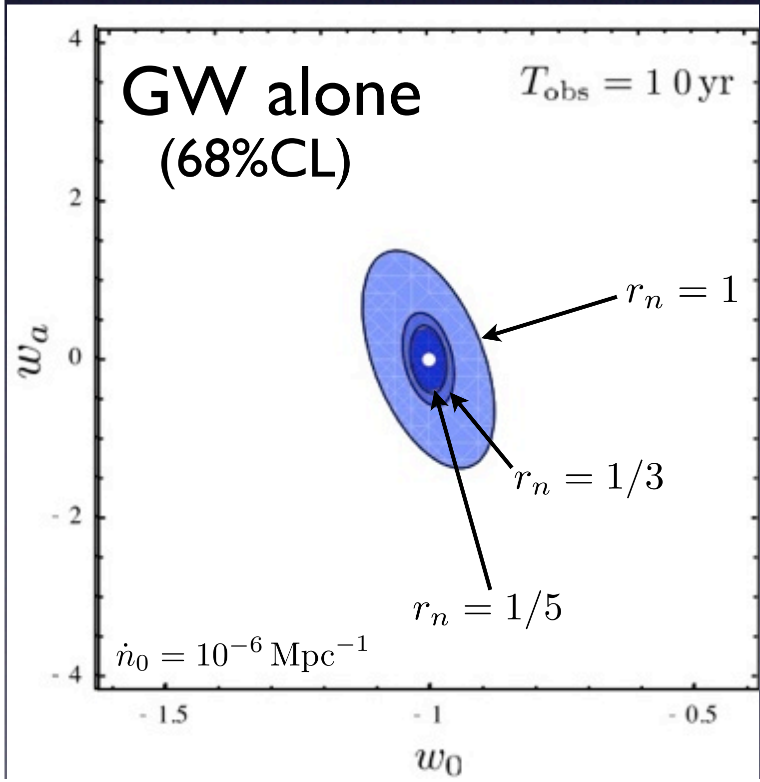
Result (2)

Fisher matrix

6 params including d_L, X

$$\Gamma_{ab} = 4\sum_{i=1}^{8} \mathrm{Re}\int_{f_{\min}}^{f_{\max}} \frac{\partial_a \tilde{h}^*_{(i)}(f)\,\partial_b \tilde{h}_{(i)}(f)}{S_n(f)}df \quad \left(\tilde{h}(f):\ \text{GW waveform (restricted 1.5PN)}\right)$$

The errors in $d_L(z_i)$ & $X(z_i)$ obtained from Fisher matrix are translated to the uncertainties in cosmological parameters $(\Omega_{\rm m},\ w_0,\ w_a)$



Dark energy E.O.S
$w(a) = w_0 + w_a(1-a)$

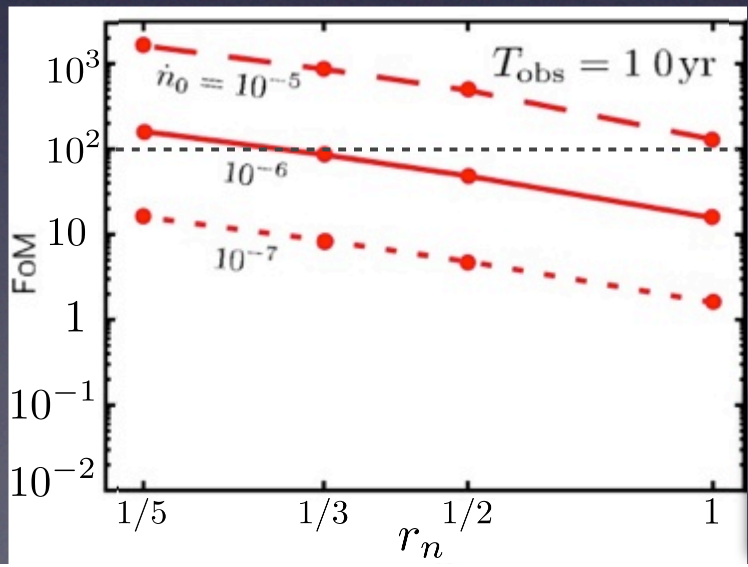


Figure-of-Merit for Dark energy

$$\mathrm{FoM} = [\sigma(w_0)\sigma(w_a)]^{-1}$$

$\dot{n}_0 = 10^{-5}\,{\rm Mpc}^{-3}{\rm yr}^{-1}$

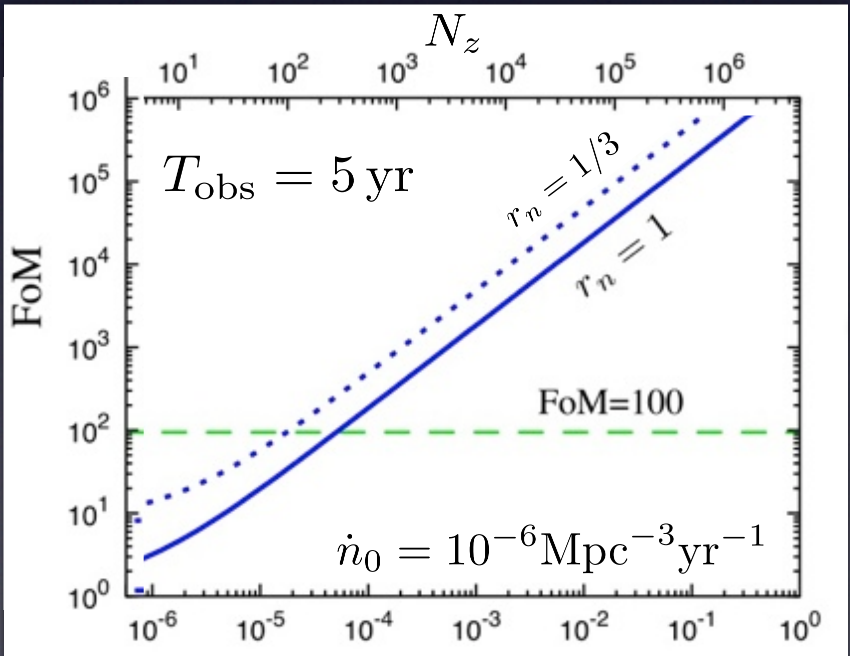
10^{-6}

10^{-7}

GW cosmology in space

With a detector sensitivity comparable to BBO $(r_n \lesssim 1/3)$

- Foreground removal will be made satisfactory
- Detectable primordial GWB will be down to $\Omega_{\rm gw} \lesssim 10^{-16}$ (r<0.01)
- Long-term obs. (10 yrs) will lead to a tight constraint on dark energy FoM~100



fraction of optical follow-up

Of course,

adding optical follow-up samples further improves FoM

※ Even a partial follow-up (<0.1% of total #) is enough to reach FoM=100

Summary

Detection & characterization of GWBs
and cosmology from space-based detectors

Ground-based

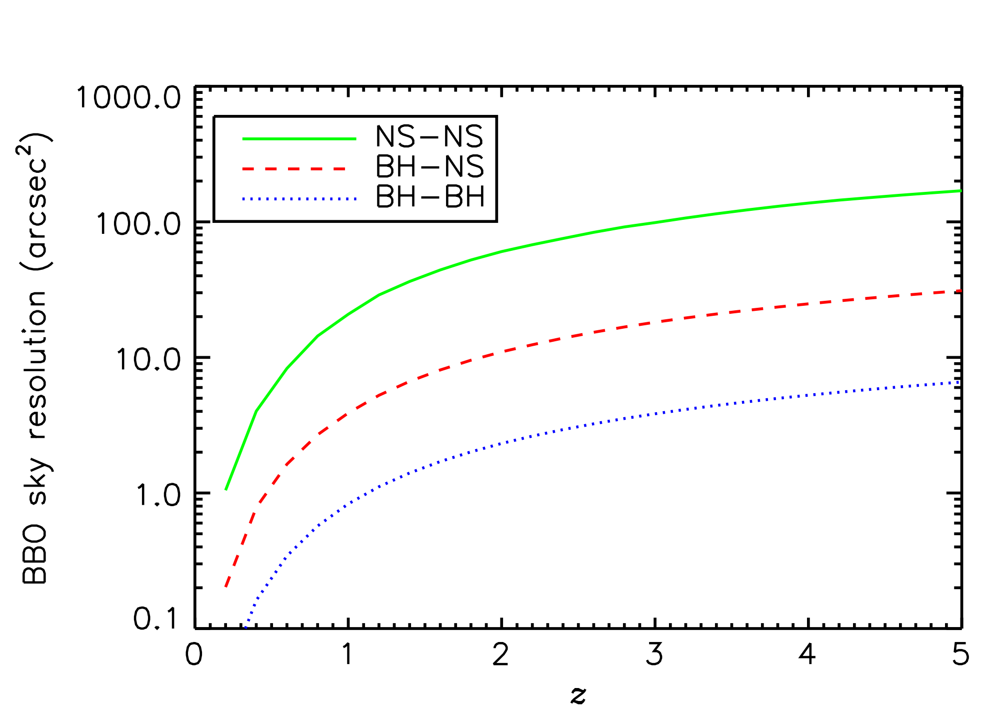
network of multiple detectors would give a nearly optimal characterization of GWBs (*polarization states*)

Space-based

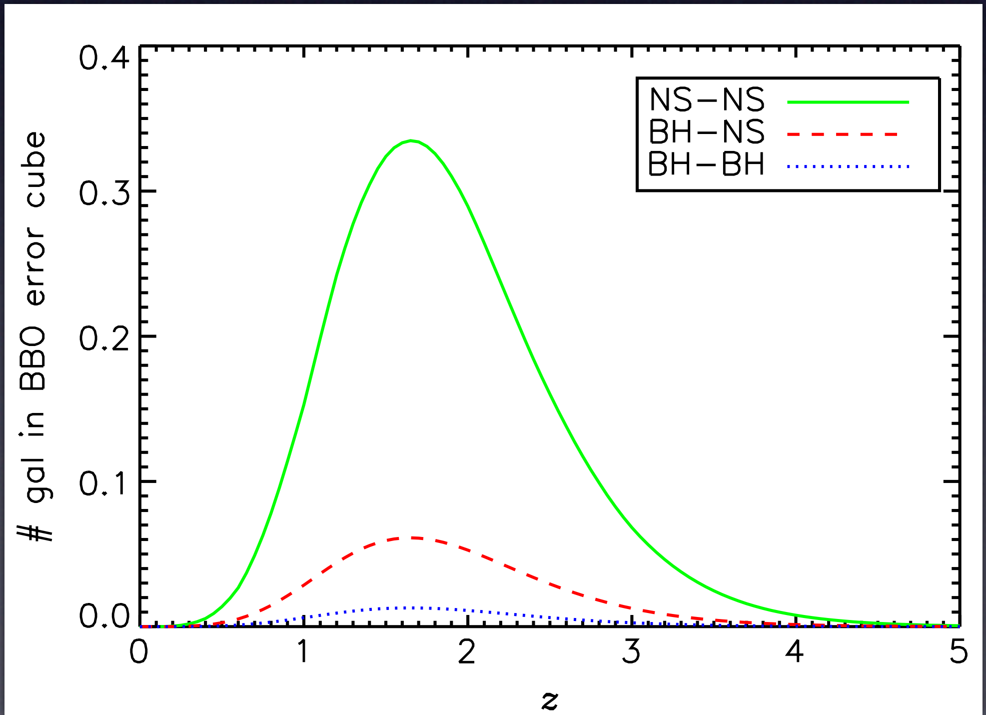
it will provide a way to directly detect primordial GWBs. Further, a large amount of NS binaries may be used as alternative cosmological probe *w/o optical follow-up,* though we still need a further investigation

Appendix

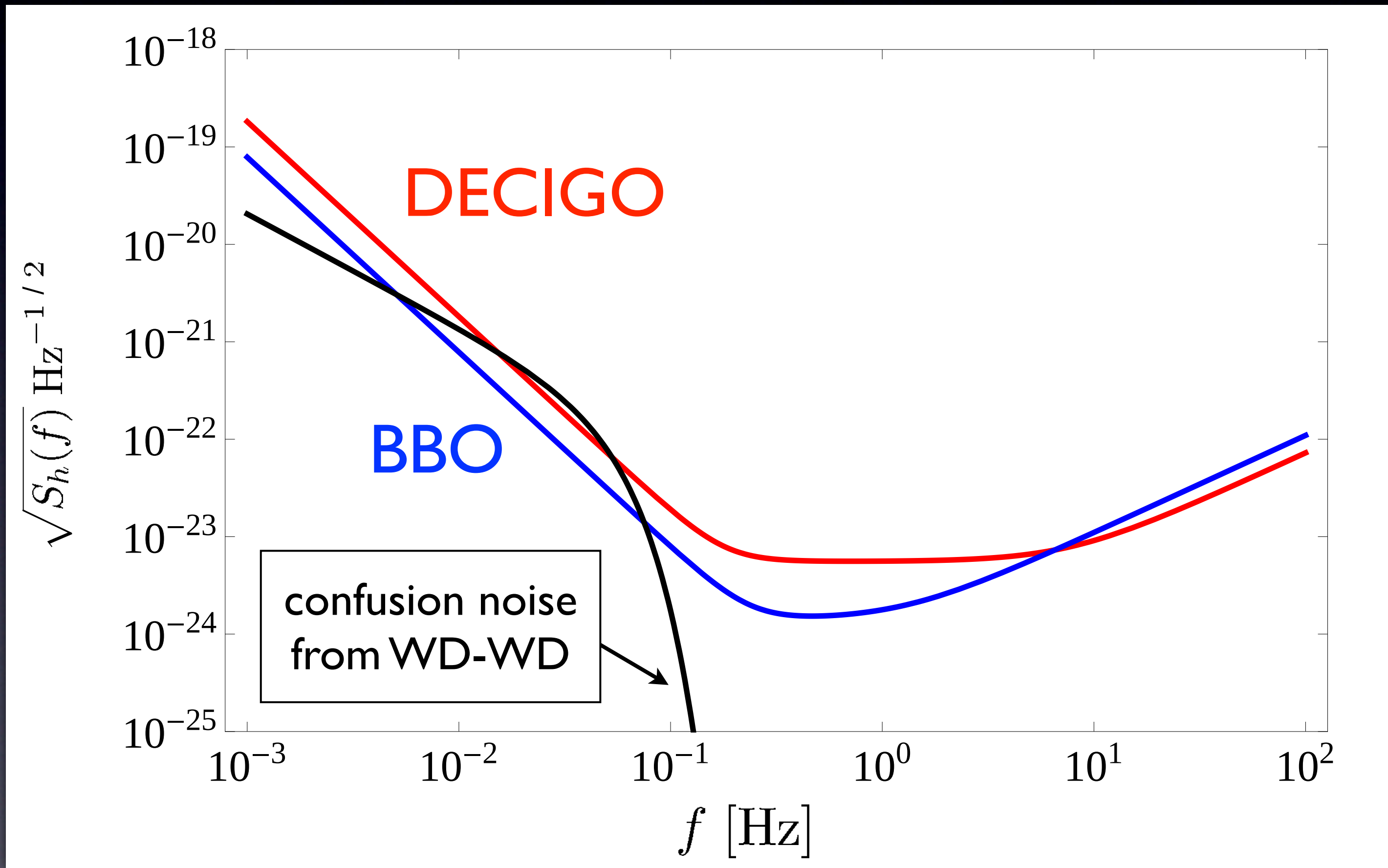
Angular resolution of BBO



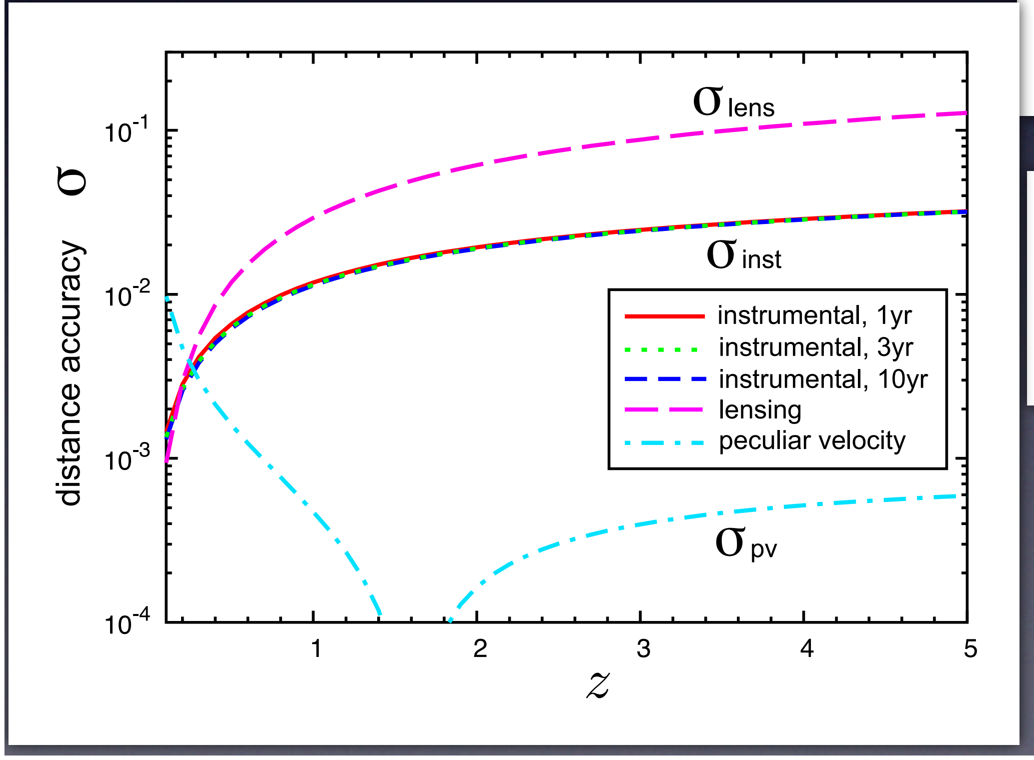
Cutler & Holz ('09)

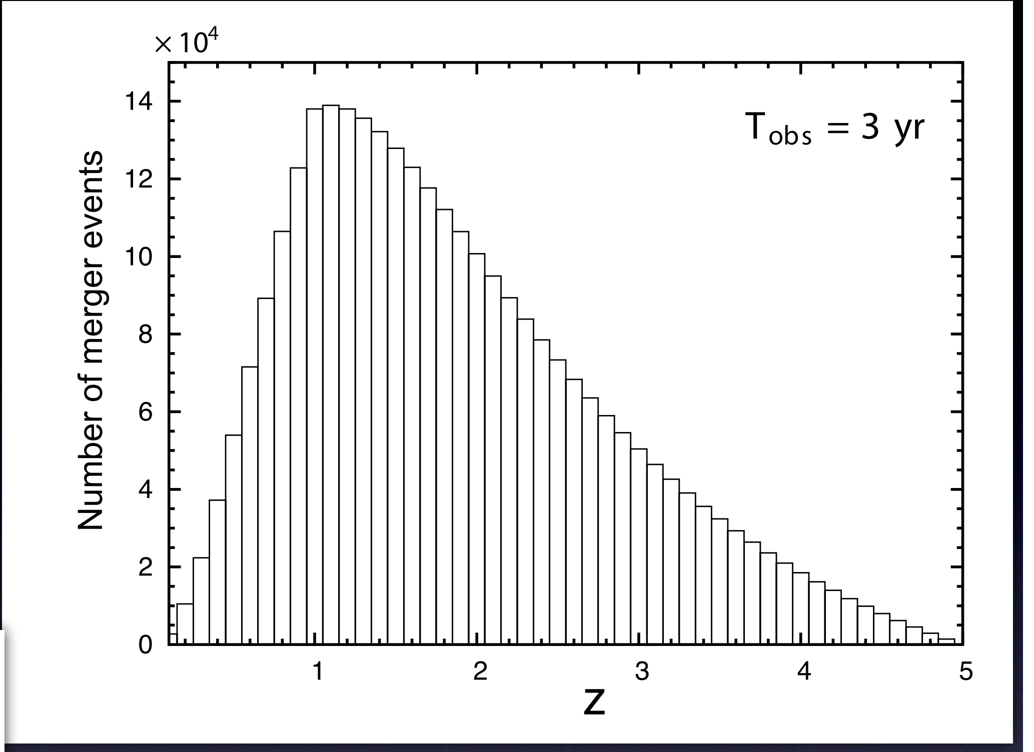


Noise curves: DECIGO & BBO



For single binary





$$\dot{n}(z) = \dot{n}_0\, s(z)\ ; \quad s(z) = \begin{cases} 1+2z & (z \le 1) \\ \frac{3}{4}(5-z) & (1 < z \le 5) \\ 0 & (5 < z) \end{cases}$$

Schneider et al. ('01)

Cutler & Harm ('06)

Waveform of GWs

$$\tilde{h}(f) = \frac{\overset{(\sqrt{6}\,\pi^{2/3})^{-1}}{\overset{\shortparallel}{A}}}{d_L(z)} M_z^{5/6} f^{-7/6} e^{i\Psi(f)}$$

A: sky-averaged amplitude over inclination of binary system
(detector response is also taken into account)

restricted 1.5PN

$$\begin{aligned}\Psi(f) &= 2\pi f\, t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\pi M_z f)^{-5/3} \\ &\quad \times \left[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right)\eta^{-2/5}(\pi M_z f)^{2/3} - 16\pi\eta^{-3/5}(\pi M_z f)\right]\end{aligned}$$

M_z : redshifted chirp mass $\quad (1+z)\mu^{3/5} m_{\rm tot}^{2/5}$

η : symmetric mass ratio $\quad \mu/m_{\rm tot}$

$t_c,\ \phi_c$: time & phase at coalescence