Unified Description of Nambu-Goldstone Bosons without Lorentz Invariance

Haruki Watanabe, Hitoshi Murayama

Physical Review Letters **108**, 251602
Editor's Suggestion
chosen for Physics Synopsis

50-year old puzzle

- Nambu-Goldstone theorem (valid with Lorentz invariance)
  - for every broken symmetry, there is a massless excitation
  - $E = c\,p$ (linear dispersion relation)
- in Heisenberg ferromagnet
  - 1 gapless excitation for 2 broken symm
  - $E \propto p^2$ (quadratic dispersion relation)
- What is the generalized NG theorem?

Synopsis: Counting Broken Symmetries

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Unified Description of Nambu-Goldstone Bosons without Lorentz Invariance
Haruki Watanabe and Hitoshi Murayama
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From superconductivity in condensed-matter physics to electroweak symmetry breaking in particle physics, some of the richest phenomena in modern physics can be understood within the paradigm of spontaneous breaking of symmetries, i.e., the quantum mechanical scenario of the ground state of a quantum system being less symmetric than the corresponding Hamiltonian. A telltale sign of broken symmetries are massless bosonic excitations known as Nambu-Goldstone bosons (NGB). The number of such distinct bosonic excitations are generally taken to be a measure of how many of the original symmetries are dynamically broken. Despite decades of research on the subject, a general formula that predicts the number of different NGBs in a given dynamical system with broken symmetries has eluded theorists.

Now, in a paper in *Physical Review Letters*, Haruki Watanabe and Hitoshi Murayama (both jointly at the University of California, Berkeley, and the University of Tokyo, Japan) provide a general theorem that relates the number of NGBs to that of broken symmetries. The latter are characterized by the number of generators of the symmetry group of the Hamilonian (as well as their mutual commutation relations) that do not leave invariant the vacuum of the dynamical system under consideration. The power of this result lies in the fact that it applies to all dynamical systems subject to spontaneous symmetry breaking (under some minimal assumptions specified in the paper). While the relativistic analog of this result has been known for a long time, this is the first result of its kind that is applicable to *nonrelativistic* systems of interest in condensed-matter physics. This result should provide crucial insights, particularly in the study of quantum many-body systems and investigations into quantum criticality. – *Abhishek Agarwal*

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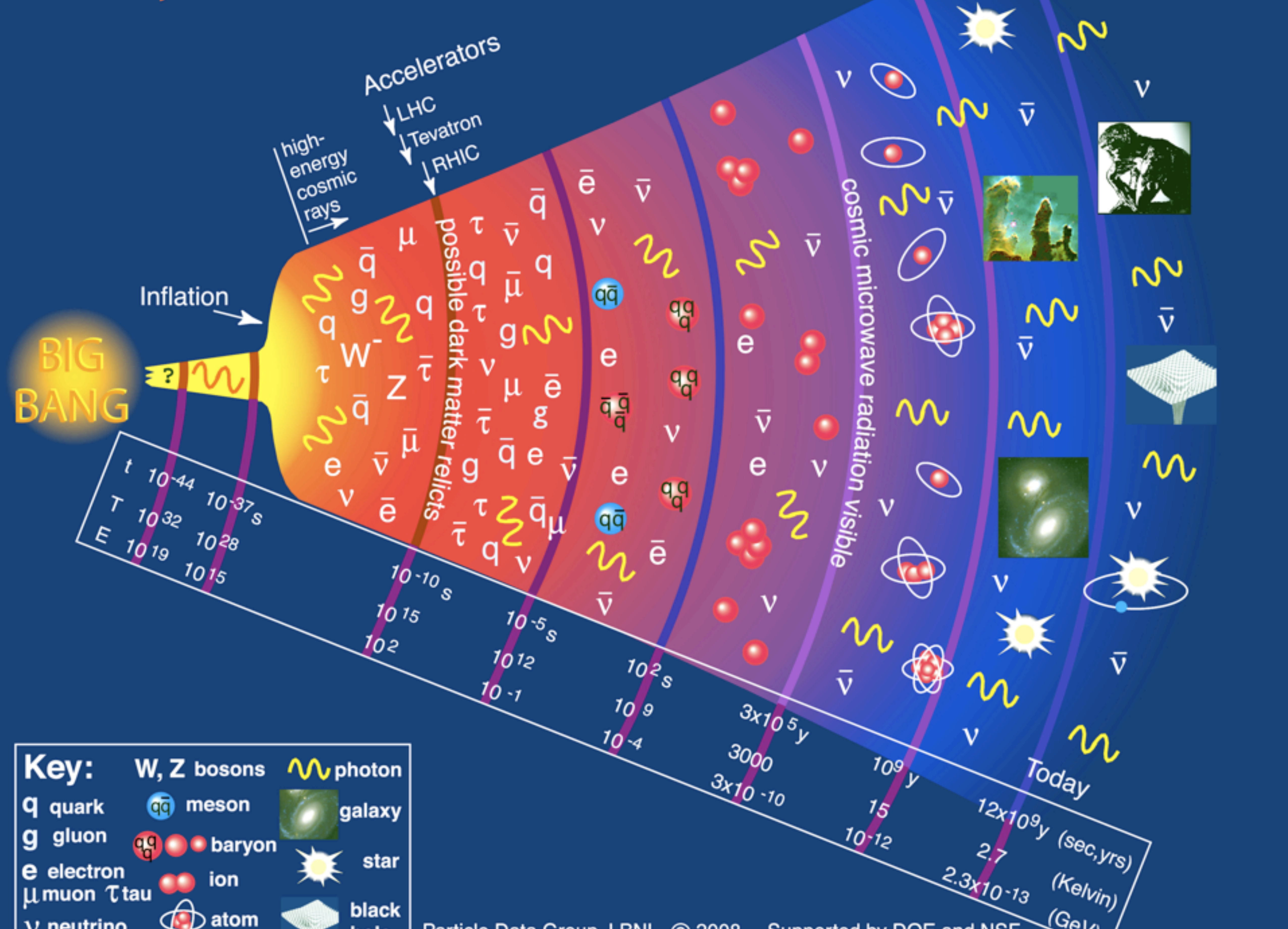
Introduction

History of the Universe



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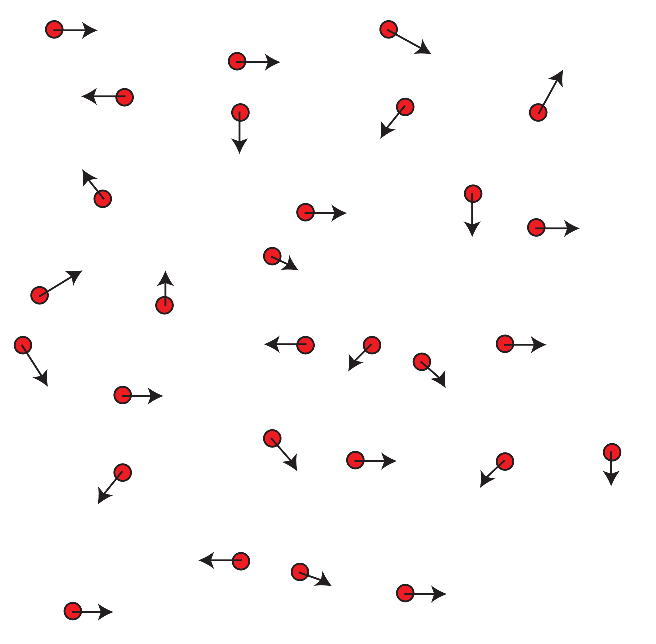
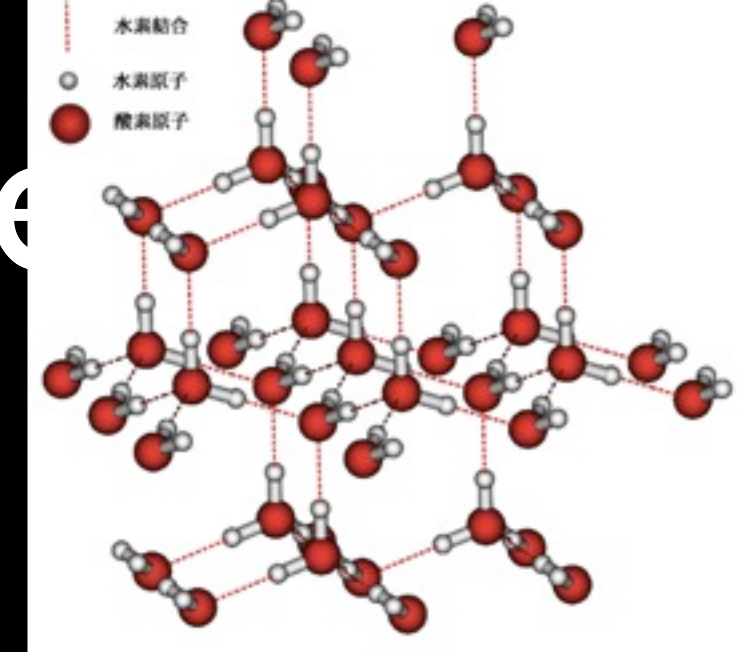


Universe has bee cooling



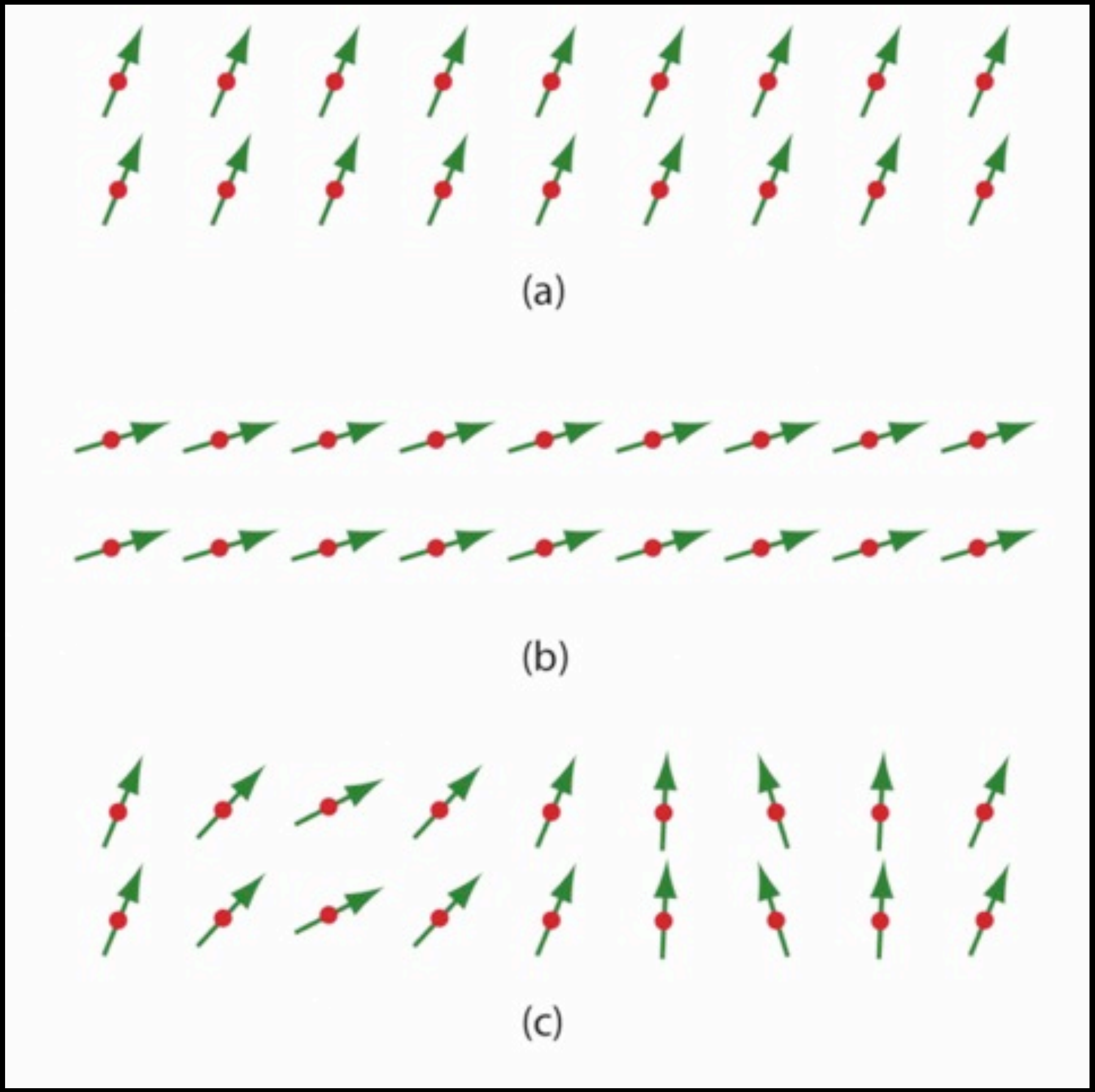
phase transitions ⇒ spontaneous symmetry breaking

Spontaneous Symmetry Breaking

- Yoichiro Nambu (1960)
- 2008 Nobel Prize in Physics
- "*For the discovery of the mechanism of spontaneous broken symmetry in subatomic physics*"
- long-range low-energy behavior determined by symmetries

Spontaneous Symmetry Breaking

$G = Z_2$
$H = 0$



$G = SO(3)$ $H = SO(2)$

Ubiquitous

- SSB is ubiquitous in nature
- chiral symm in strong interactions (QCD)
- crystals (spatial translations)
- superconductors, Higgs (gauge invariance)
- ^{4}He superfluid, scalar BEC (prtcle number)
- ^{3}He superfluid, spinor BEC, kaon condensation, color superconductivity, etc (a rich variety of symmetries)
- *what is the underlying unified description?*

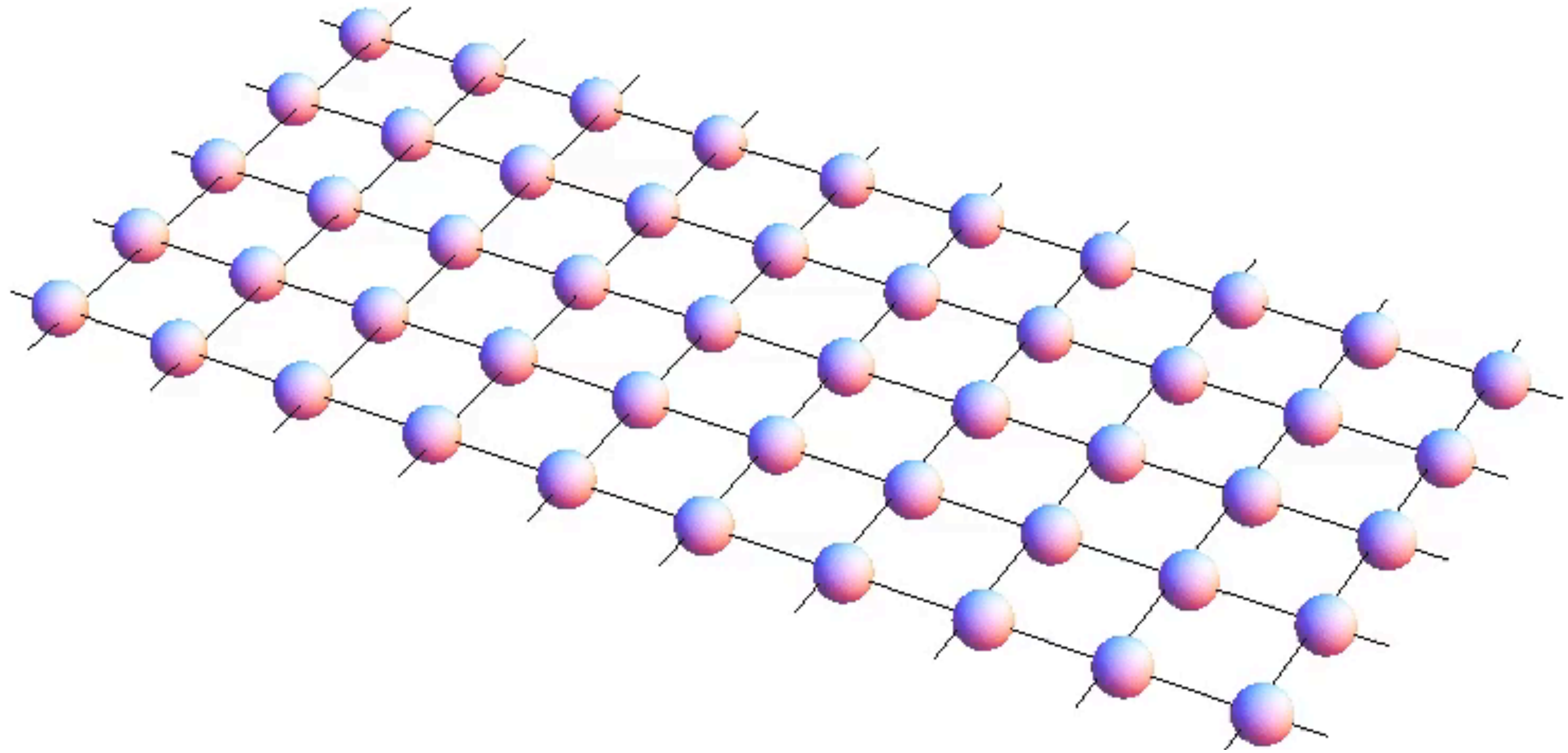
Nambu-Goldstone theorem

- When a continuous symmetry G is spontaneously broken to its subgroup H, there are massless bosons $E=c\,p$ for every broken generators.
- $n_{\mathrm{NGB}}=n_{\mathrm{BG}}$
- assumes Lorentz invariance

math counterpart

- consider $\pi: R^{3,1} \to G/H$
- Write G-invariant *action* $S = \int d^4x\, L(\pi, \partial\pi)$ up to a surface term
- expand in powers of derivative, keep low orders (often up to the second order)
- Lorentz invariance dictates the action to be $S = \int d^4x\, g_{ab}(\pi)\, \partial_\mu \pi^a\, \partial^\mu \pi^b$
- only data needed is G-inv metric on G/H

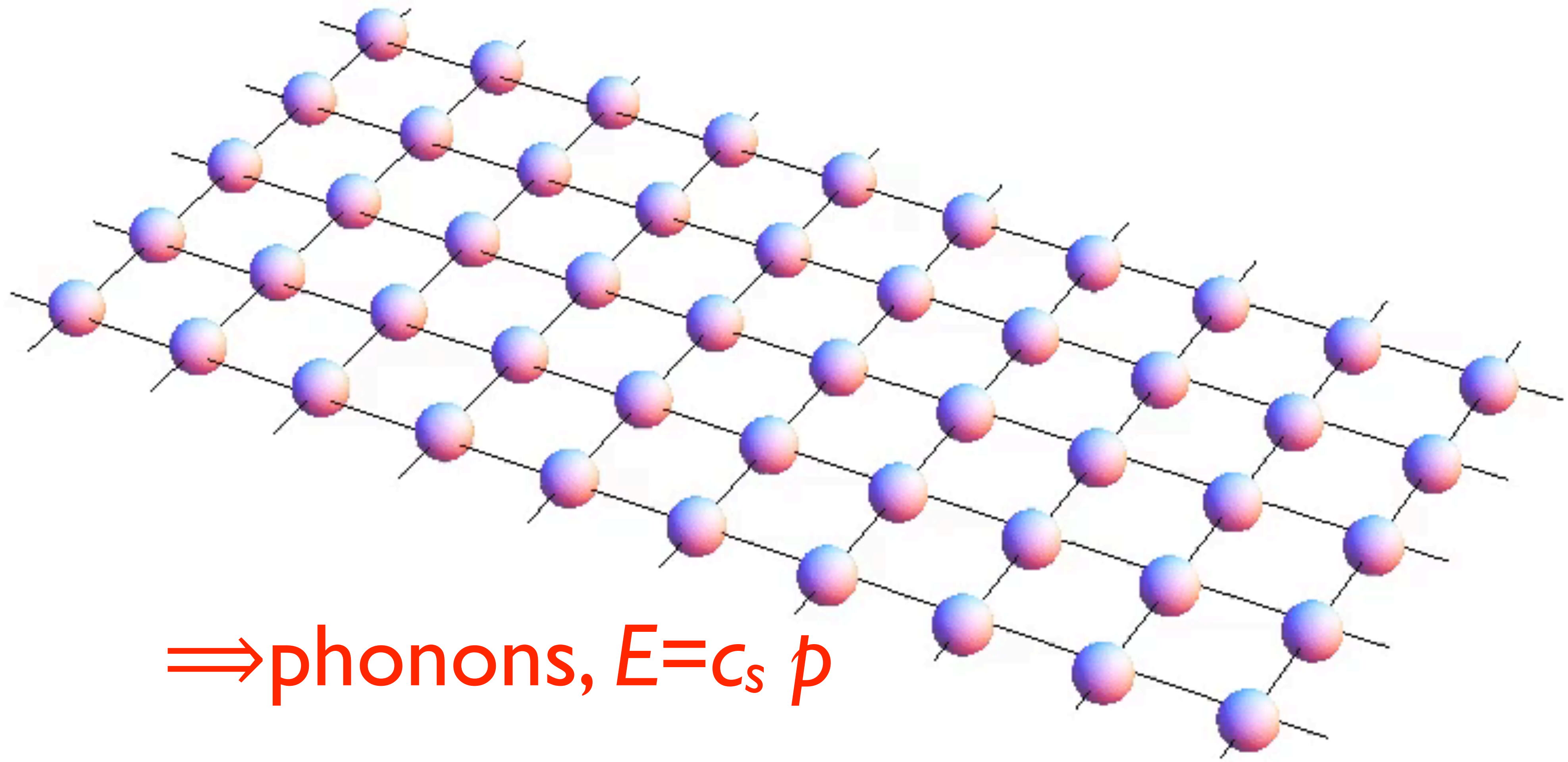
longitudinal



crystal

$G=R^2$
$H=0$

transverse



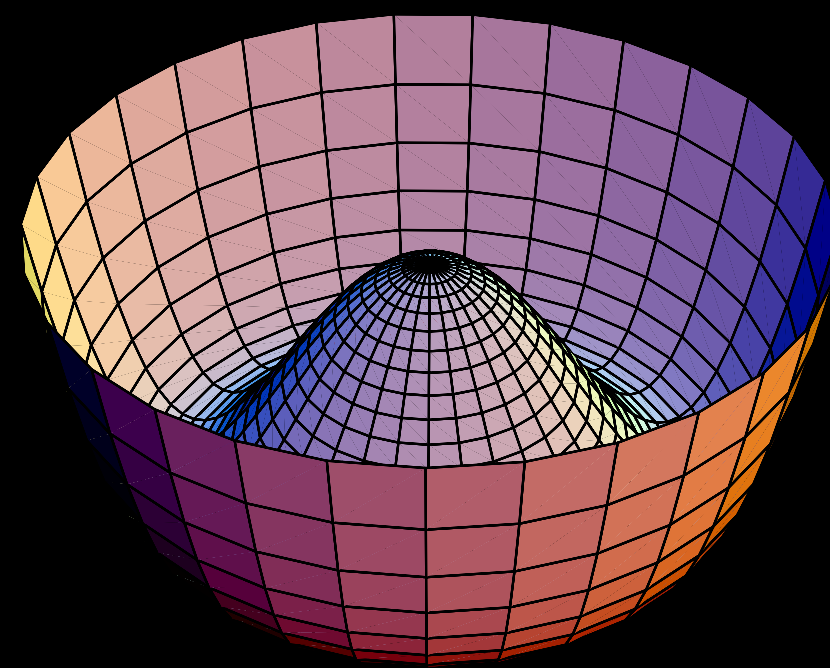
$\Longrightarrow$phonons, $E=c_s\, p$

Particle numbers

- U(1) symmetry

$$\psi(x) \to e^{i\theta}\psi(x)$$

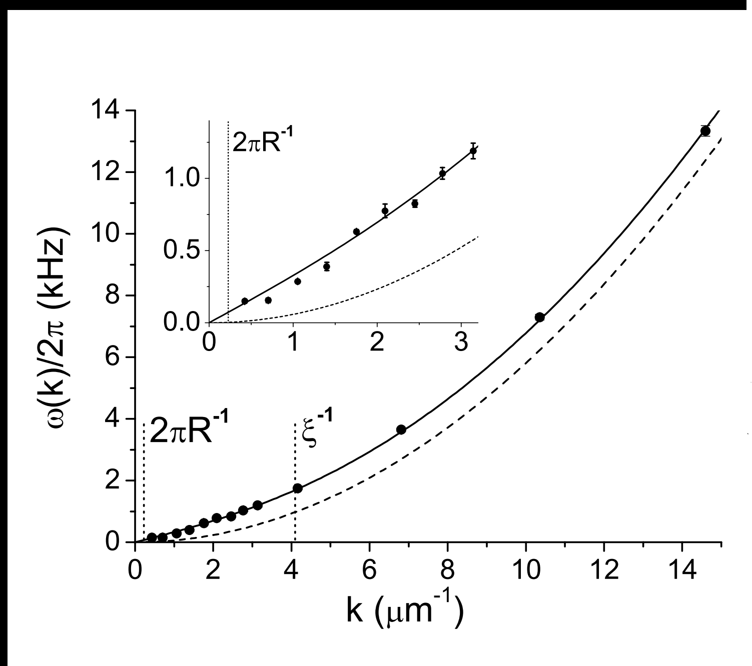
$$N = \int dx \psi^* \psi$$

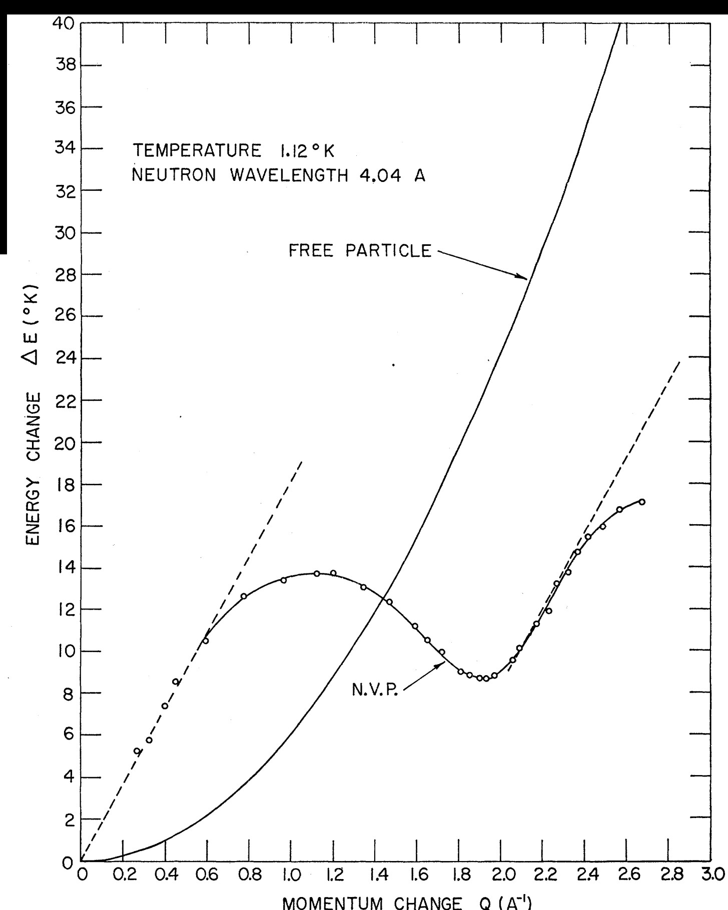


- Ginzburg-Landau theory

$$V = -\mu\psi^*\psi + \lambda(\psi^*\psi)^2 \qquad \langle 0|\psi|0\rangle \neq 0$$

- G=U(1), H=0
- ^{4}He superfluid
- scalar BEC





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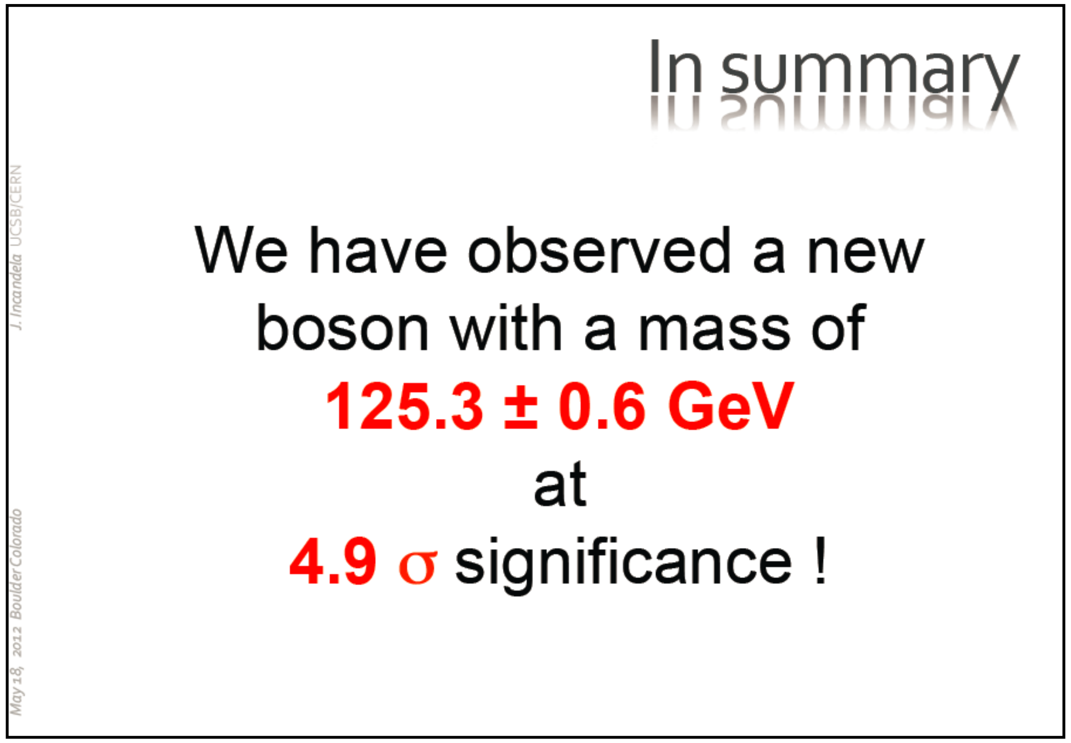
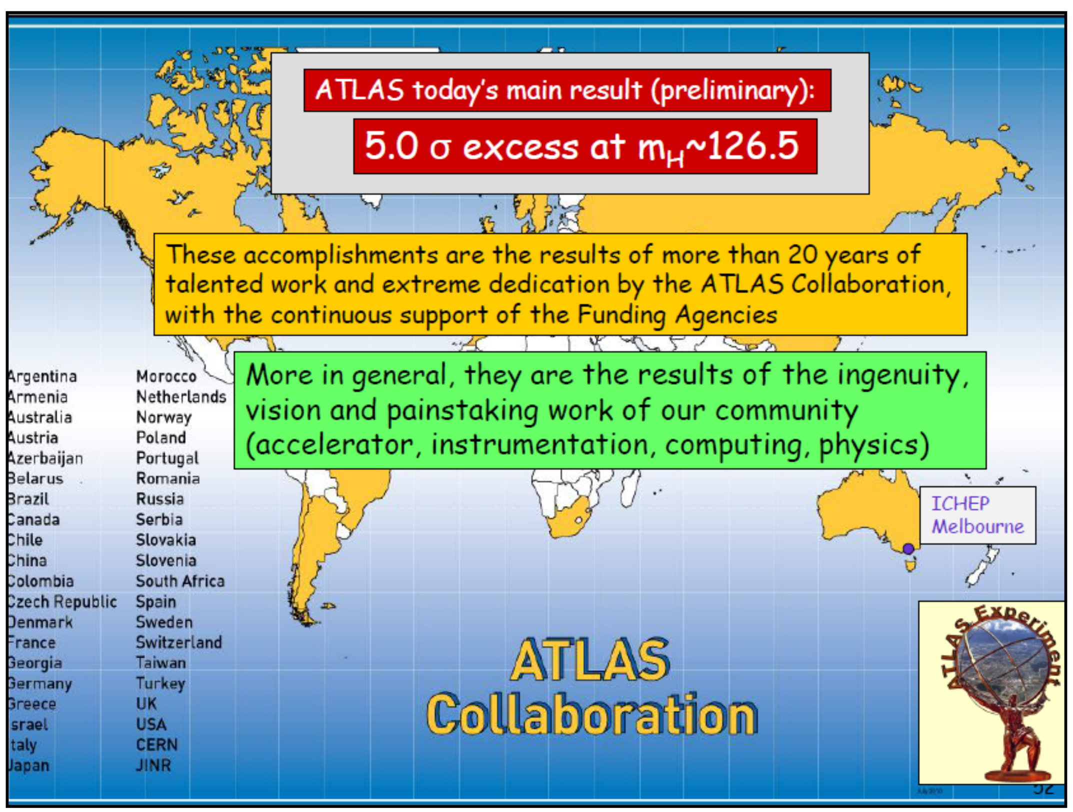




July 4, 2012

Heisenberg models

$$H = +J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$$

2 NGBs

$E \propto p$

$$H = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$$

1 NGB

$E \propto p^2$

Both G/H = SO(3)/SO(2) = S^2

experiment

- Dispersion relations can be tested experimentally
- specific heat
    - $E \propto p \Rightarrow C_V \propto T^3$
    - $E \propto p^2 \Rightarrow C_V \propto T^{3/2}$
- Plot $C_V/T^{3/2}$ vs $T^{3/2}$

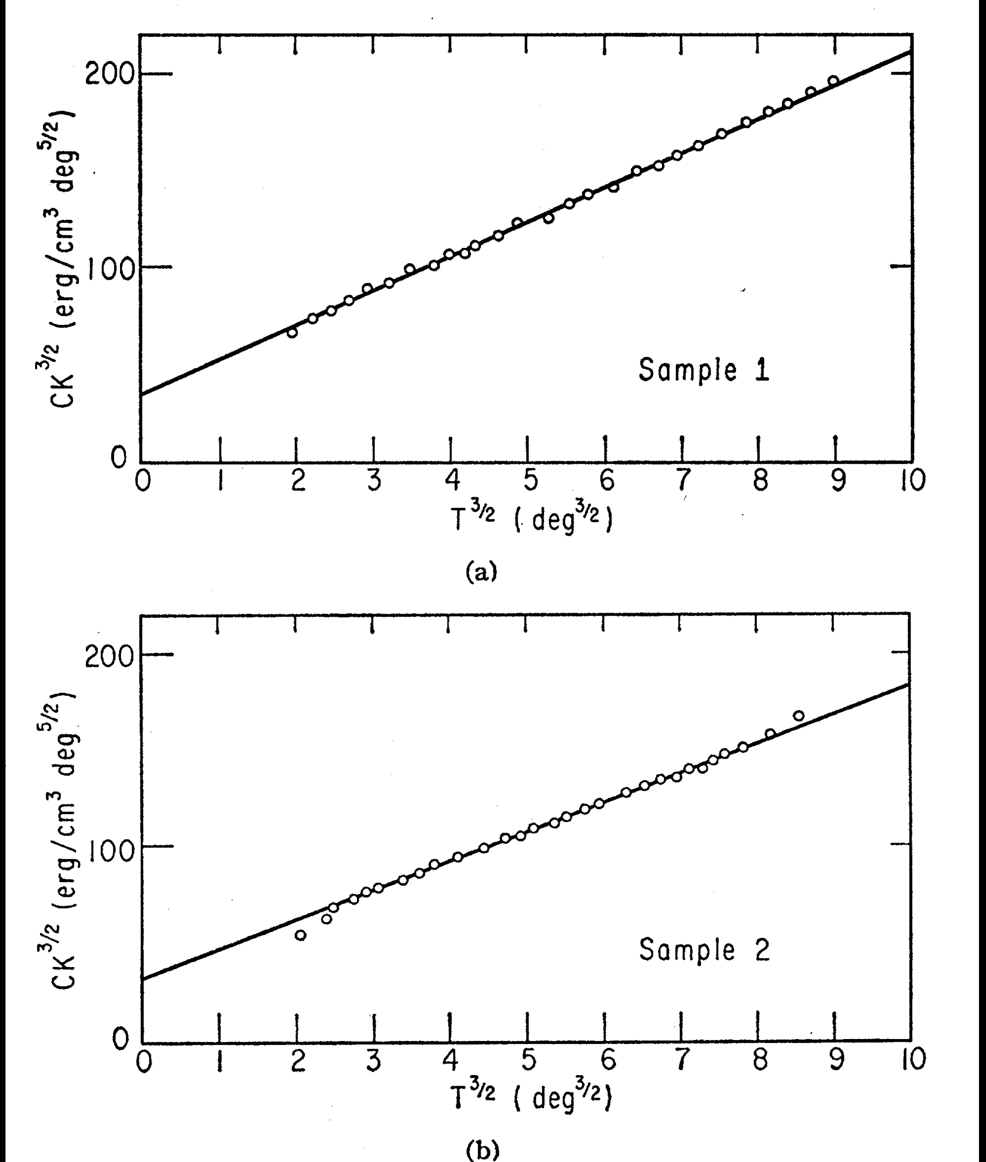
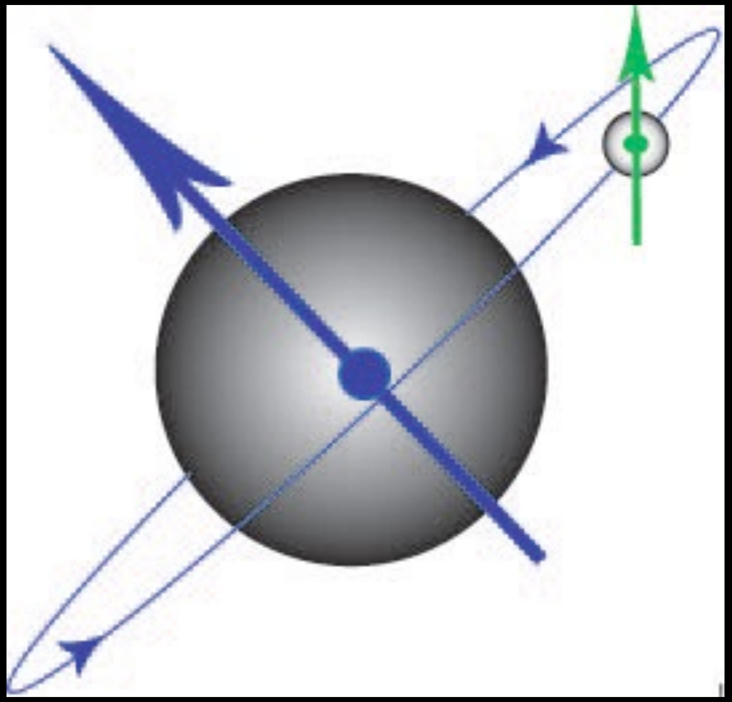


FIG. 1. The specific heat of the two YIG samples. The points for sample 1 give the results obtained in one experiment. The points for sample 2 give the results obtained in two separate experiments.

spinor BEC

- BEC of F=1 atoms (ferromagnetic)
- SO(3)xU(1) → SO(2)
- 3 broken generators
- 1 NGB with $E \propto p$
- 1 NGB with $E \propto p^2$



Spontaneous Breaking of Lie and Current Algebras

Yoichiro Nambu[1]

Received December 26, 2002; accepted January 29, 2003

The anomalous properties of Nambu–Goldstone bosons, found by Miransky and others in the symmetry breaking induced by a chemical potential, are attributed to the SSB of Lie and current algebras. Ferromagnetism, antiferromagnetism, and their relativistic analogs are discussed as examples.[2]

KEY WORDS: Symmetry breaking; Nambu–Goldstone boson; color superconductivity; chemical potential; ferromganetism; Lorentz symmetry; current algebra.

1. INTRODUCTION AND SUMMARY

In general the number of the Nambu–Goldstone (NG) bosons associated with a spontaneous symmetry breaking (SSB) $G \to H$ is equal to the number of symmetry generators Q_i in the coset G/H. In the absence of a gauge field, their energy ω goes as a power k^γ of wave number. In a relativistic theory, $\gamma = 1$ necessarily unless Lorentz invariance is broken.

There are, however, exceptions to the above "theorem."[(1–5)] Recently

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Spontaneous Symmetry Breaking with Abnormal Number of Nambu-Goldstone Bosons and Kaon Condensate

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PHYSICAL REVIEW D **70**, 014006 (2004)

Abnormal number of Nambu-Goldstone bosons in the color-asymmetric dense color superconducting phase of a Nambu−Jona-Lasinio−type model

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PHYSICAL REVIEW A **74**, 033604 (2006)

Superfluidity in a three-flavor Fermi gas with SU(3) symmetry

Lianyi He, Meng Jin, and Pengfei Zhuang
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(Received 26 April 2006; published 8 September 2006)

We investigate the superfluidity and the associated Nambu-Goldstone modes in a three-flavor atomic Fermi gas with SU(3) global symmetry. The s-wave pairing occurs in flavor antitriplet channel due to the Pauli principle, and the superfluid state contains both gapped and gapless fermionic excitations. Corresponding to the spontaneous breaking of the SU(3) symmetry to a SU(2) symmetry with five broken generators, there are only three Nambu-Goldstone modes, one is with linear dispersion law and two are with quadratic dispersion law. The other two expected Nambu-Goldstone modes become massive with a mass gap of the order of the fermion energy gap in a wide coupling range. The abnormal number of Nambu-Goldstone modes, the quadratic dispersion law, and the mass gap have significant effect on the low-temperature thermodynamics of the matter.

DOI: 10.1103/PhysRevA.74.033604 PACS number(s): 03.75.Ss, 05.30.Fk, 74.20.Fg, 34.90.+q

Heisenberg models

$$H = +J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$$

2 NGBs

$E \propto p$

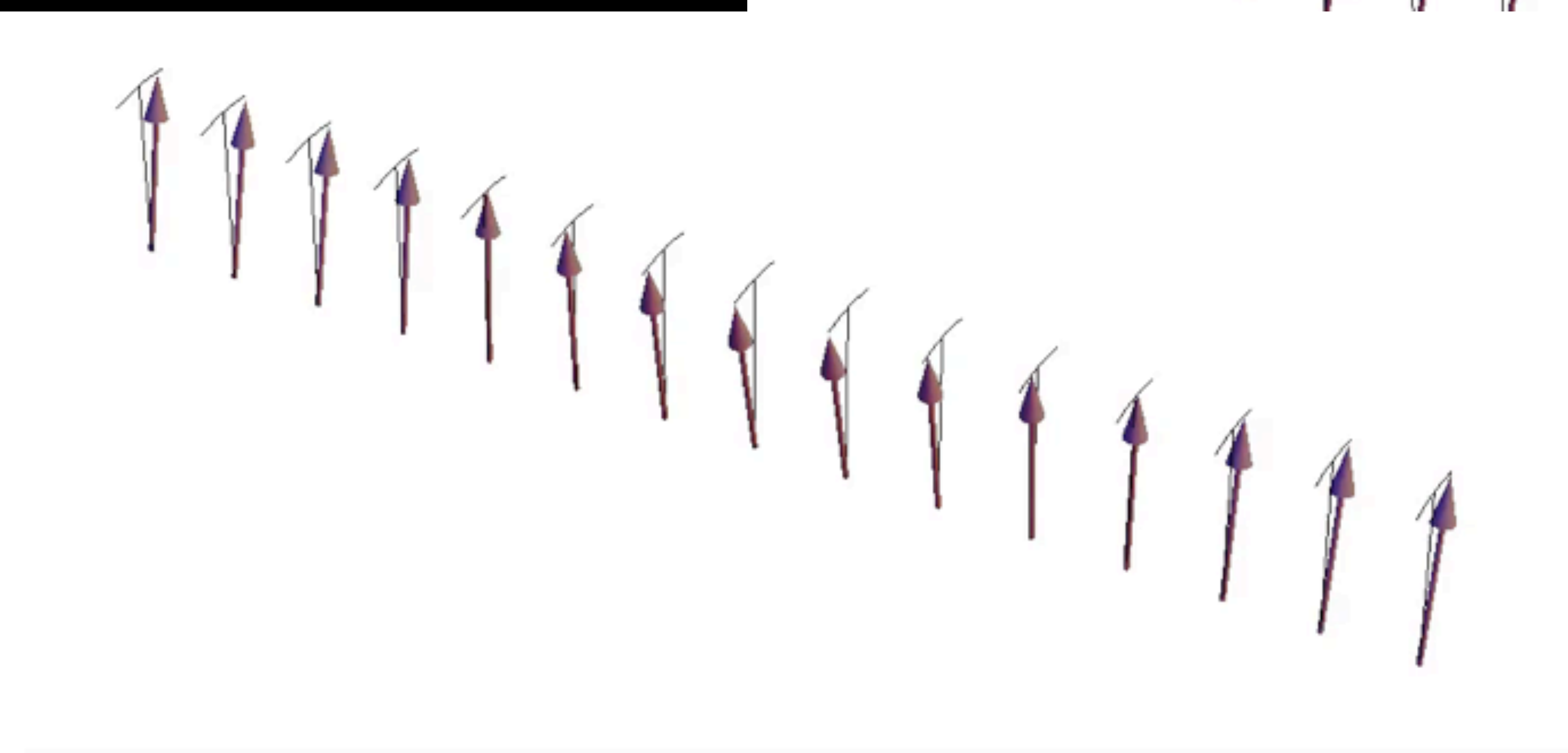
$$H = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$$

1 NGB

$E \propto p^2$

Both G/H = SO(3)/SO(2) = S^2

two NGBs?



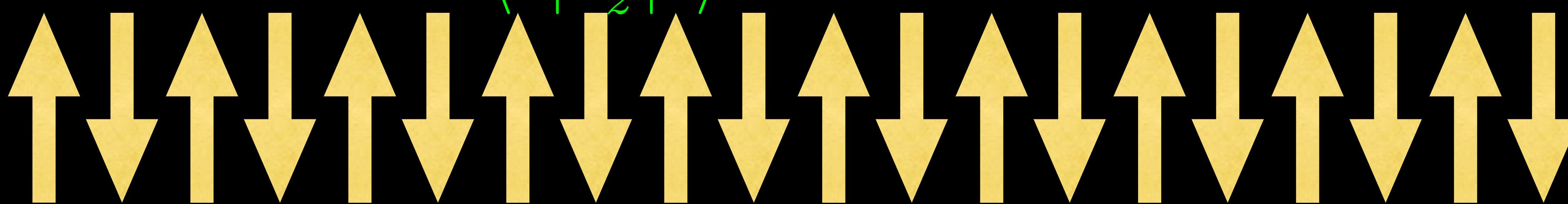
No!

Heisenberg models

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \frac{1}{2}\bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - \frac{1}{2}g_{ab}(\pi)\partial_r\pi^a\partial_r\pi^b$$

- anti-ferromagnet $H = +J\sum_{\langle i,j\rangle}\vec{s}_i\cdot\vec{s}_j$ 2 NGBs

  $\langle 0|J_z^0|0\rangle = 0$ $E \propto p$



- ferromagnet $H = -J\sum_{\langle i,j\rangle}\vec{s}_i\cdot\vec{s}_j$ 1 NGB

  $\langle 0|J_z^0|0\rangle = \langle 0|[J_x, J_y]|0\rangle \neq 0$ $E \propto p^2$

J_x and *J_y* canonically conjugate to each other describing the single degree of freedom *together*

No!

Goal

- consider $\pi: R^{3,1} \to G/H$ ("*pion field*")
- Write G-invariant *action* $S = \int d^4x\, L(\pi, \partial\pi)$ up to a surface term
- expand in powers of derivative, keep low orders (often up to the second order)
- don't assume Lorentz invariance SO(3,1), but only rotational invariance SO(3)
- *what geometrical data are needed?*

Bottomline

$$\mathcal{L}_{\text{eff}} = c_a(\pi)\dot{\pi}^a + \frac{1}{2}\bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - \frac{1}{2}g_{ab}(\pi)\partial_r\pi^a\partial_r\pi^b$$

- SSB leads to gapless excitations (NGBs)
- Lorentz invariance: $n_{\mathrm{NGB}}=n_{\mathrm{BG}}$, $E=cp$
- w/o Lorentz invariance: $\rho_{ab} = \langle [T^a, T^b] \rangle \neq 0$
  - Type A: $\rho_{ab}=0$, $E \propto p$
  - Type B: $\rho_{ab}\neq 0$, $E \propto p^2$
    - $n_{\mathrm{NGB}}=n_{\mathrm{A}}+n_{\mathrm{B}}$
    - $n_{\mathrm{BG}}=n_{\mathrm{A}}+2n_{\mathrm{B}}$
- explicit effective Lagrangian $\Longrightarrow$ interactions
- underlying partially symplectic geometry

Presymplectic Geometry

$U(N)/U(N-1)=S^{2N-1}$ $\qquad$ $U(1)$

closed G-inv ω_2 $\qquad G/H \longleftarrow F$ $\qquad$ Type A $E \propto p$

$\qquad\qquad \downarrow$

$\mathbb{C}P^N$

symplectic
homogeneous $\qquad B$ $\qquad$ Type B $E \propto p^2$

$$\omega(h_a, h_b) \propto \langle [T^a, T^b] \rangle$$

NGBs for generators *a* and *b* are symplectic pairs and describe a single degree of freedom

$$\mathrm{dim} G - \mathrm{dim} H = n_A + 2n_B$$

theorems

- Borel (1954): G compact semi-simple, $T \subset G$ a torus, H centralizer of T, then G/H Kähler
- Borel (1954): G simple non-compact, center reduced to $\{e\}$, K maximal compact subgroup, $U \subset K$ a centralizer of a torus in G, then G/H homogeneous symplectic
- Chu (1974): G simply-connected. A left-invariant closed two-form on G induces a symplectic homogenous space

Applications

$$n_{NGB} = n_{BG} - \frac{1}{2}\mathrm{rank}\rho$$

example	coset space	BG	NGB	rank ρ	theorem
anti-ferromagnet	O(3)/O(2)	2	2	0	2=2-0
ferromagnet	O(3)/O(2)	2	1	2	1=2-1
ferrimagnet	O(3)/O(2)	2	1	2	1=2-1
superfluid ^{4}He	U(1)	1	1	0	1=1-0
superfluid ^{3}He B phase	O(3)xO(3)xU(1)/O(2)	4	4	0	4=4-0
(in magnetic field)	O(2)xO(3)xU(1)/O(2)	4	3	2	3=4-1
BEC (F=0)	U(1)	1	1	0	1=1-0
BEC (F=1) planar	O(3)xU(1)/U(1)	3	3	0	3=3-0
BEC (F=1) ferro	O(3)xU(1)/SO(2)	3	2	2	2=3-1
3-comp. Fermi liquid	U(3)/U(2)	5	3	4	3=5-2
neutron star	U(1)	1	1	0	1=1-0
crystal	R^3	3	3	0	3=3-0
(in magnetic field)	R^3	3	2	2	2=3-1

§ Introduction

§ Setup

§ Presymplectic Structure

§ Trivial but Useful Examples

§ Realistic Examples

§ Space-Time Symmetries

remarks

§ Topological Solitons

$G/H \quad \{X_i\} \quad [X_i, X_j] = f_{ij}{}^k X_k \qquad X_i = X_i^a \frac{\partial}{\partial \pi^a}$

w/ Lorentz-inv.

$$\mathcal{L}_{eff} = \frac{1}{2} g_{ab}\, \partial_\mu \pi^a \partial^\mu \pi^b \qquad j_i^\mu = - g_{ab} X_i^a \partial^\mu \pi^b$$

$$\pounds_{X_i} g_{ab} = X_i^c \frac{\partial}{\partial \pi^c} g_{ab} + g_{ac} \frac{\partial}{\partial \pi^b} X_i^c + g_{cb} \frac{\partial}{\partial \pi^a} X_i^a = 0$$

w/o Lorentz-inv.

$$\mathcal{L}_{eff} = c_a \dot{\pi}^a + \frac{1}{2} \bar{g}_{ab} \dot{\pi}^a \dot{\pi}^b - \frac{1}{2} g_{ab} \vec{\nabla} \pi^a \vec{\nabla} \pi^b + O(\partial^3)$$

$$\int c_a \dot{\pi}^a \, dt = \int c_a \, d\pi^a \qquad \text{one-form on } G/H$$

G-inv up to a surface term

$$\pounds_{X_i} c = \underset{\substack{\| \\ de_i}}{X_i \lrcorner dc} + d(X_i \lrcorner c)$$

$e_X \in \mathfrak{g}^*$ "locally Hamiltonian"

if globally defined, "globally Hamiltonian"

$$j_i^0 = e_i - \bar{g}_{ab} h_i^a \dot{\pi}^b$$

$e_i \neq 0 \quad \Rightarrow \quad \langle 0 | j_i^0 | 0 \rangle = e_i \neq 0$

not possible if Lorentz inv.

$$\omega = dc \qquad \pounds_{X_i} \omega = d[\, de_i + d(X_i \lrcorner c)\,] = 0$$

Darboux' theorem

$$\omega = \sum_i dp^i \wedge dq^i$$

$$\mathcal{L}_{X_i} de_j = \mathcal{L}_{X_i}(X_j \lrcorner \omega) = [X_i, X_j] \lrcorner \omega + X_j \lrcorner \overset{0}{\overset{\shortparallel}{\mathcal{L}_{X_i}\omega}}$$

$$\overset{\shortparallel}{d(\mathcal{L}_{X_i} e_j)} \qquad = f_{ij}{}^k X_k \lrcorner \omega = f_{ij}{}^k de_k$$

$$\Rightarrow \mathcal{L}_{X_i} e_j = f_{ij}{}^k e_k + c_{ij}$$

$\shortparallel$ $\quad\uparrow$ central extension

$$\langle 0 | [X_i, j_j] | 0 \rangle = \frac{1}{\mathrm{Vol}} \langle 0 | [X_i, X_j] | 0 \rangle$$

~~semi-simple case $\Rightarrow c_{ij} = 0$~~

$$\begin{aligned} de_{[X,Y]} &= [X,Y] \lrcorner \omega \\ &= \mathcal{L}_X (Y \lrcorner \omega) - Y \lrcorner (\overset{0}{\mathcal{L}_X \omega}) \\ &= X \lrcorner d(Y \lrcorner \omega) + d(X \lrcorner (Y \lrcorner \omega)) \\ &= X \lrcorner (\underset{\overset{\shortparallel}{0}}{\mathcal{L}_Y \omega} - Y \lrcorner \underset{\overset{\shortparallel}{0}}{d\omega}) + d\omega(Y, X) \\ &= -d\omega(X,Y) \end{aligned}$$

$$c_{XY} + e_{[X,Y]} = -\omega(X,Y)$$

$+ \quad g_{ab} \vec{\nabla} \pi^a \vec{\nabla} \pi^b$

$\Rightarrow$ Type-A $\quad$ not part of ω $\quad$ $E \propto p$

Type-B $\quad$ part of ω $\quad$ $E \propto p^2$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi = \frac{1}{2}\left[\frac{1}{c^2}\dot{\phi}^2 - (\vec{\nabla}\phi)^2\right]$$

$G = \mathbb{R}$ $\quad \phi \to \phi + a$

$\langle 0|\phi|0\rangle = 0 \quad \Rightarrow \quad$ SSB

$G/H = \mathbb{R} \quad \phi$

w/ Lorentz-invariance

$$\mathcal{L} = \frac{1}{2}\left[\frac{1}{c_s^2}\dot{\phi}^2 - (\vec{\nabla}\phi)^2\right] \qquad \text{sound wave}$$

only one $\Rightarrow$ no symplectic structure possible

$G = ISO(2) = SO(2) \ltimes \mathbb{R}^2$

$U(1) \ltimes \mathbb{C}$

$\psi \to e^{i\theta}\psi, \quad \psi \to \psi + c$

$\langle 0|\psi|0\rangle = 0 \quad \Rightarrow \quad H = U(1)$

$G/H = \mathbb{C} \quad \to \psi$

$$[Q, R] = I, \quad [Q, I] = -R, \quad \underset{\Downarrow\, Z}{[R, I] = 0}$$

$$\mathcal{L} = \underset{\substack{\Uparrow \\ \text{central extension}}}{\psi^* i\hbar\dot{\psi}} + \frac{\hbar^2}{2m}\vec{\nabla}\psi^*\vec{\nabla}\psi$$