New ways of searching for the primordial gravitational wave from Large Scale Structure

Donghui Jeong (Johns Hopkins University)

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References

Large-Scale clustering of galaxies in general relativity
DJ, Schmidt & Hirata [arXiv:1107.5427]

Clustering Fossils from the Early Universe
DJ & Kamionkowski [arXiv:1203.0302]

Cosmic Rulers
Schmidt & DJ [arXiv:1204.3625]

Large-Scale Structure with Gravitational Waves I: Galaxy Clustering
DJ & Schmidt [arXiv:1205.1512]

Large-Scale Structure with Gravitational Waves II: Shear
Schmidt & DJ [arXiv:1205.1514]

Introduction

Gravitational Wave 101

Gravitational wave (GW)

- is a traceless transverse (tensor) component of the metric perturbations:
  (Einstein convention + Greek=0-4, Latin=1-3)

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \{\delta_{ij} + h_{ij}(\eta, \boldsymbol{x})\}\, dx^i dx^j\right]$$

Traceless : $\mathrm{Tr}[h_{ij}] = h^i_i = g^{ij} h_{ij} = 0$

Transverse : $\nabla_i h_{ij} = 0$

- There are
  6 (symmetric 3x3 spatial matrix) - 3 (transverse) - 1(traceless)
  = 2 degrees of freedom = $h_\times$, h_+

Primordial Gravitational Wave

- de-Sitter space generates stochastic gravitational waves with amplitude of ($m_{pl} = \sqrt{G_N}$)

$$\Delta_h^2(k) = \frac{k^3 P_T(k)}{2\pi^2} = \frac{64\pi}{m_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

+ Friedmann equation: $3H^2 \sim 8\pi G\rho$

where power spectrum is defined as ($P_T = 4P_h$)

$$\langle h_{ij}(\boldsymbol{k})h^{ij}(\boldsymbol{k}')\rangle = (2\pi)^3 P_T(k)\delta^D(\boldsymbol{k}-\boldsymbol{k}')$$

- Gravitational wave amplitude = energy scale of inflation!

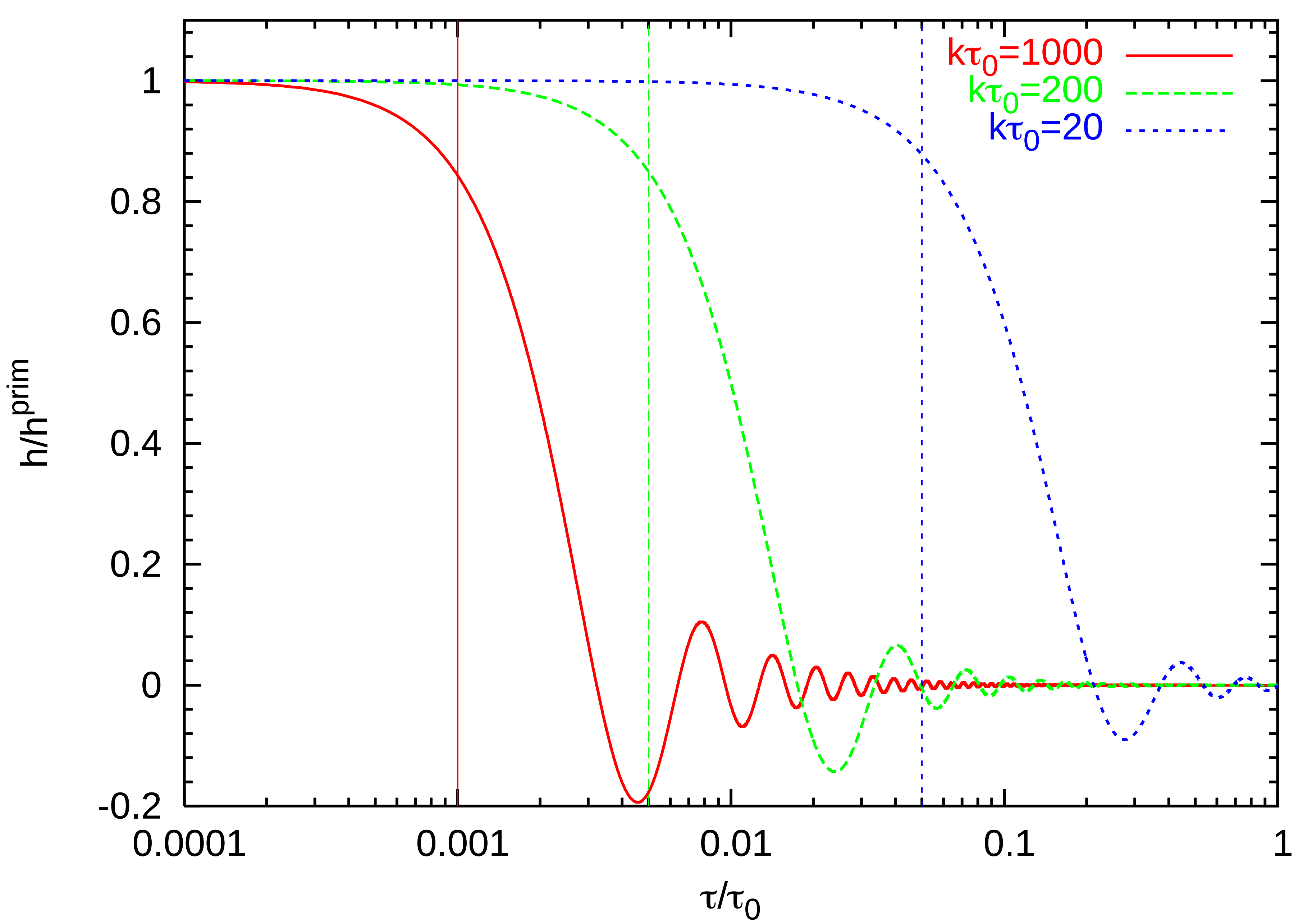
Evolution of GW

- Evolution of GW(p=+,x) are described by K-G equation sourced by anisotropic stress ($\mathscr{H}$=a'/a and ' = d/dη):

$$-h_{ij;\nu}{}^{;\nu} = h_p''(\boldsymbol{k}) + 2\mathcal{H}h_p'(\boldsymbol{k}) + k^2 h_p(\boldsymbol{k}) = 16\pi G a^2 \Pi_p(\boldsymbol{k})$$

Hubble damping term

Watanabe, Komatsu (2006)



- GW decays once the mode enters the horizon. As effect from Π_p is small,

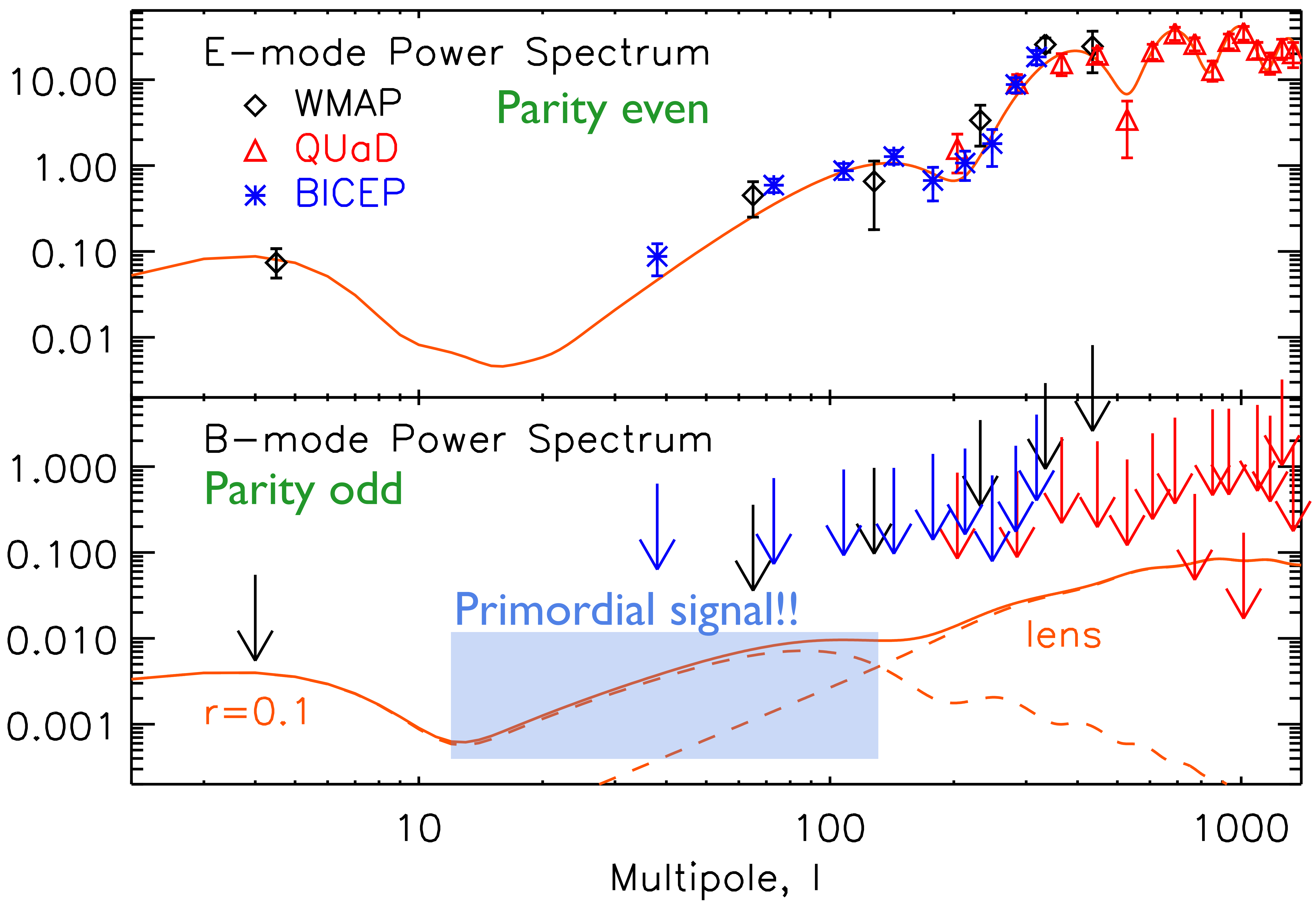
RD : $h_p(\boldsymbol{k},\eta) = j_0(k\eta)h_p^{\rm prim}$

MD : $h_p(\boldsymbol{k},\eta) = \dfrac{3j_1(k\eta)}{k\eta}h_p^{\rm prim}$

r=(tensor amplitude/scalar amplitude)^2 at k=0.002 [1/Mpc]

GW from CMB polarization

Steal from Eiichiro Komatsu's talk



- Parity-odd (B-mode) polarization is a window to the GW (or vector) in the primordial universe!
- No B-mode yet...
- B-mode experiments: Keck array, PIPER, CLASS, LiteBIRD, PIXIE, ...

(e.g. 5σ for r<10^{-3})

GW from Large Scale Structure

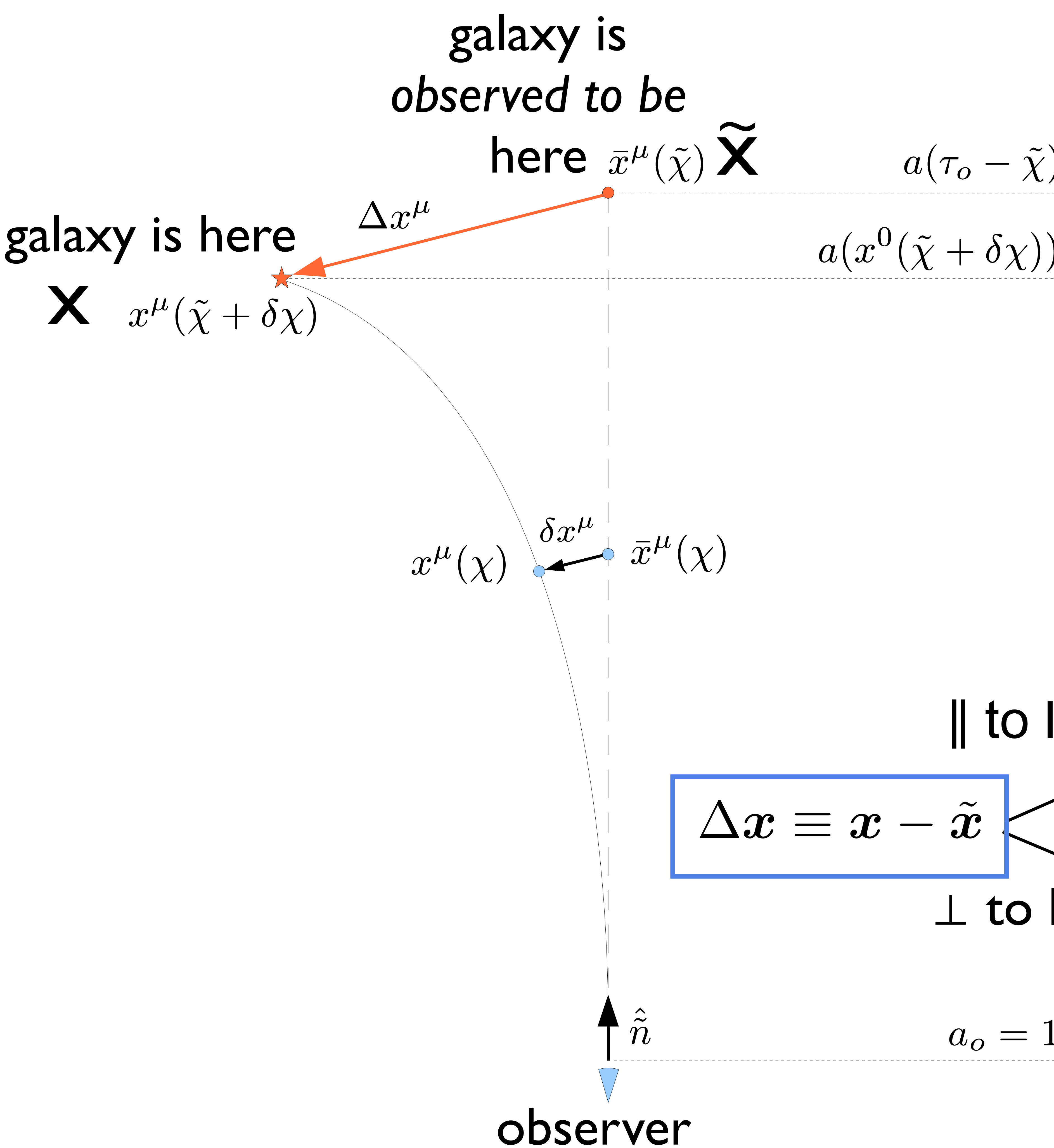
- Two effects:
  - At the location of galaxies (Source)
  - Deflection of light from galaxies (Line of sight)
- Three possible ways of detecting GW from Large Scale Structure :
  - Clustering of galaxies in large scale structure (S,L)
  - Distortion on shape of galaxies, or cosmic shear (S,L)
  - Fossil memory at the off-diagonal correlation (S)

Large-Scale Structure with GW I
: Galaxy clustering

Donghui Jeong, Fabian Schmidt & Christopher Hirata [arXiv:1107.5427]
Donghui Jeong & Fabian Schmidt [arXiv:1205.1512]

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Light deflection due to GW



- Deflection of photon changes the observed location of galaxies.
- From the geodesic equation, we calculate Δx: (Here, $h_\parallel = h_{ij}\tilde{n}^i\tilde{n}^j$)

$$\Delta \boldsymbol{x} \equiv \boldsymbol{x} - \tilde{\boldsymbol{x}}$$

time delay + l.o.s. displacement + redshift pert.

∥ to l.o.s.

$$\Delta x_\parallel = -\frac{1}{2}\int_0^{\tilde{\chi}} d\chi h_\parallel - \frac{1+\tilde{z}}{2H(\tilde{z})}\int_0^{\tilde{\chi}} d\chi h'_\parallel$$

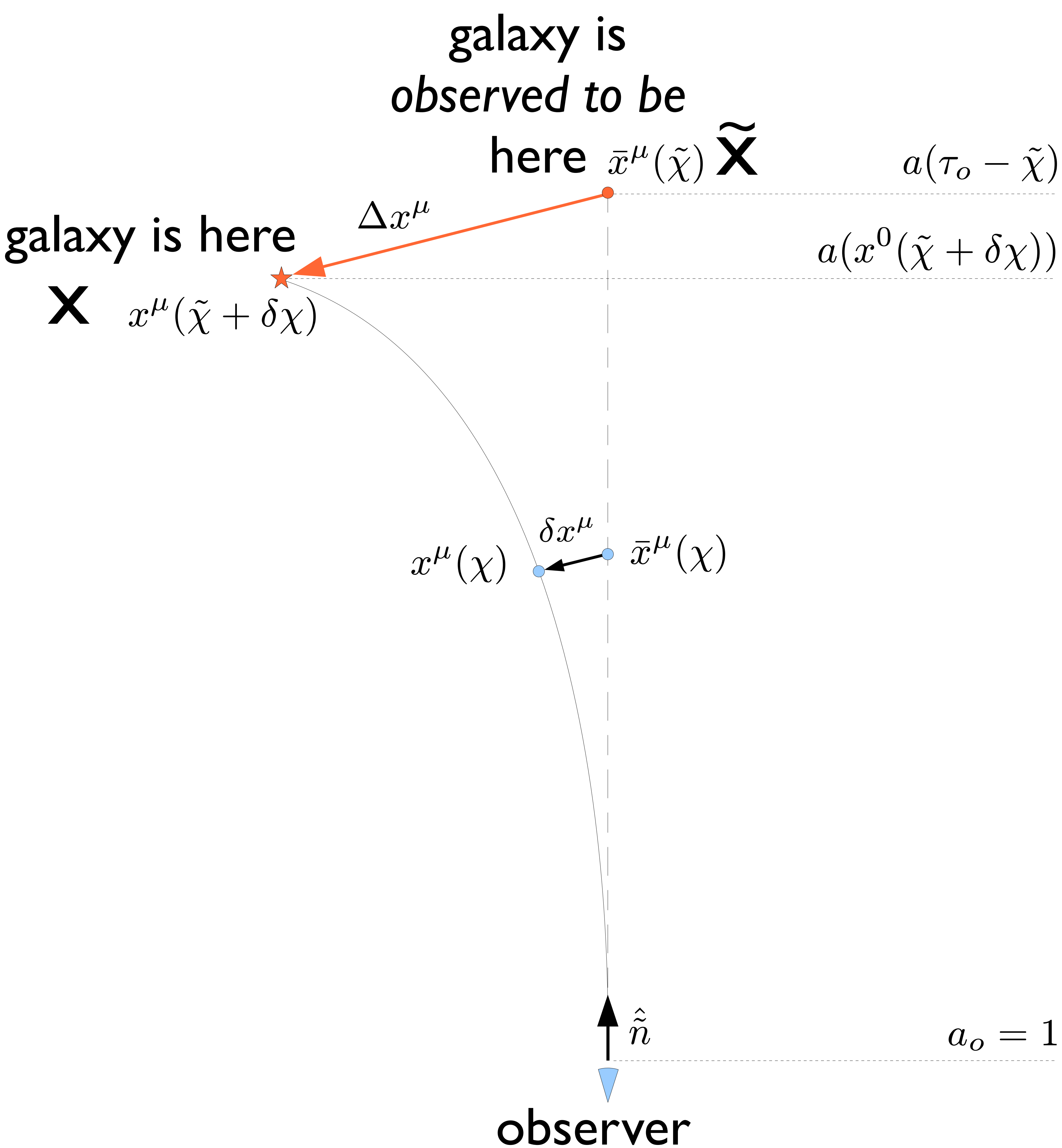
⊥ to l.o.s.

$$\Delta x^i_\perp = \frac{1}{2}\tilde{\chi}\left[(h_{ij})_0\,\hat{n}^j - (h_\parallel)_0\,\hat{n}^i\right] + \int_0^{\tilde{\chi}} d\chi \left\{\frac{\tilde{\chi}-\chi}{2}\partial^i_\perp h_\parallel + \frac{\tilde{chi}}{\chi}\left(h_\parallel \hat{n}^i - h_{ij}\hat{n}^j\right)\right\}$$

GW at the observer's position

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij}+h_{ij})dx^i dx^j\right]$$

GW effect 1. Volume distortion



- Then, the volume (number density) we inferred from the observed coordinate *is different from* the true volume (number density):

$$N = \int_{\tilde{V}} \sqrt{-g}\, n_g(x^\alpha) \frac{1}{a(x^0)} \left|\frac{\partial x^i}{\partial \tilde{x}^j}\right| d^3\tilde{x}$$

$$\boldsymbol{\Delta x} \equiv \boldsymbol{x} - \tilde{\boldsymbol{x}}$$

$$\left|\frac{\partial x^i}{\partial \tilde{x}^j}\right| = 1 + \frac{\partial \Delta x^i}{\partial \tilde{x}^i} = \partial_\parallel \Delta x_\parallel + \frac{2\Delta x_\parallel}{\tilde{\chi}} + \partial_{\perp i}\Delta x^i_\perp$$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

GW effect II. redshift perturbation

- Clustering measure: density contrast $\delta_g^{\rm obs}(\tilde{z},\hat{n}) = \dfrac{n(\tilde{z},\hat{n}) - \bar{n}(\tilde{z})}{\bar{n}(\tilde{z})}$
- But, the measured redshift is different from the ***true*** redshift!

$$1 + \tilde{z} = (1+\bar{z})(1+\delta z) \qquad \delta z = \frac{1}{2}\int_0^{\tilde{\chi}} d\chi h'_{\parallel}$$

- That is, we under-(over-) estimate the mean number density for positive (negative) δz [when there are more galaxies at lower redshifts].

$$\delta_g^{\rm obs}(\tilde{z},\hat{n}) = \delta_g^{\rm intrinsic} + b_e \delta z$$

$$b_e \equiv \left.\frac{d\ln(a^3\bar{n}_g)}{d\ln a}\right|_{\tilde{z}} = -(1+\tilde{z})\left.\frac{d\ln(a^3\bar{n}_g)}{dz}\right|_{\tilde{z}}$$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

GW effect III. Magnification

- If galaxies are selected by apparent magnitude, the magnification

$$\mathcal{M} \equiv \frac{D_L^{-2}}{\tilde{D}_L^{-2}(\tilde{z})} = \frac{D_A^{-2}}{\tilde{D}_A^{-2}(\tilde{z})}$$

also changes the density contrast (Q= -d ln $\tilde{n}_g$/d ln F_{cut}):

$$\tilde{\delta}_g = \tilde{\delta}_g(\text{no mag}) + \frac{\partial \ln \tilde{n}}{\partial \ln \mathcal{M}}(\mathcal{M} - 1) \equiv \tilde{\delta}_g(\text{no mag}) + \mathcal{Q}\delta\mathcal{M}$$

We shall talk more about the magnification later.

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Galaxy density contrast with GW

- If gravitational waves are the ONLY source of the distortion, the "observed" galaxy density contrast becomes

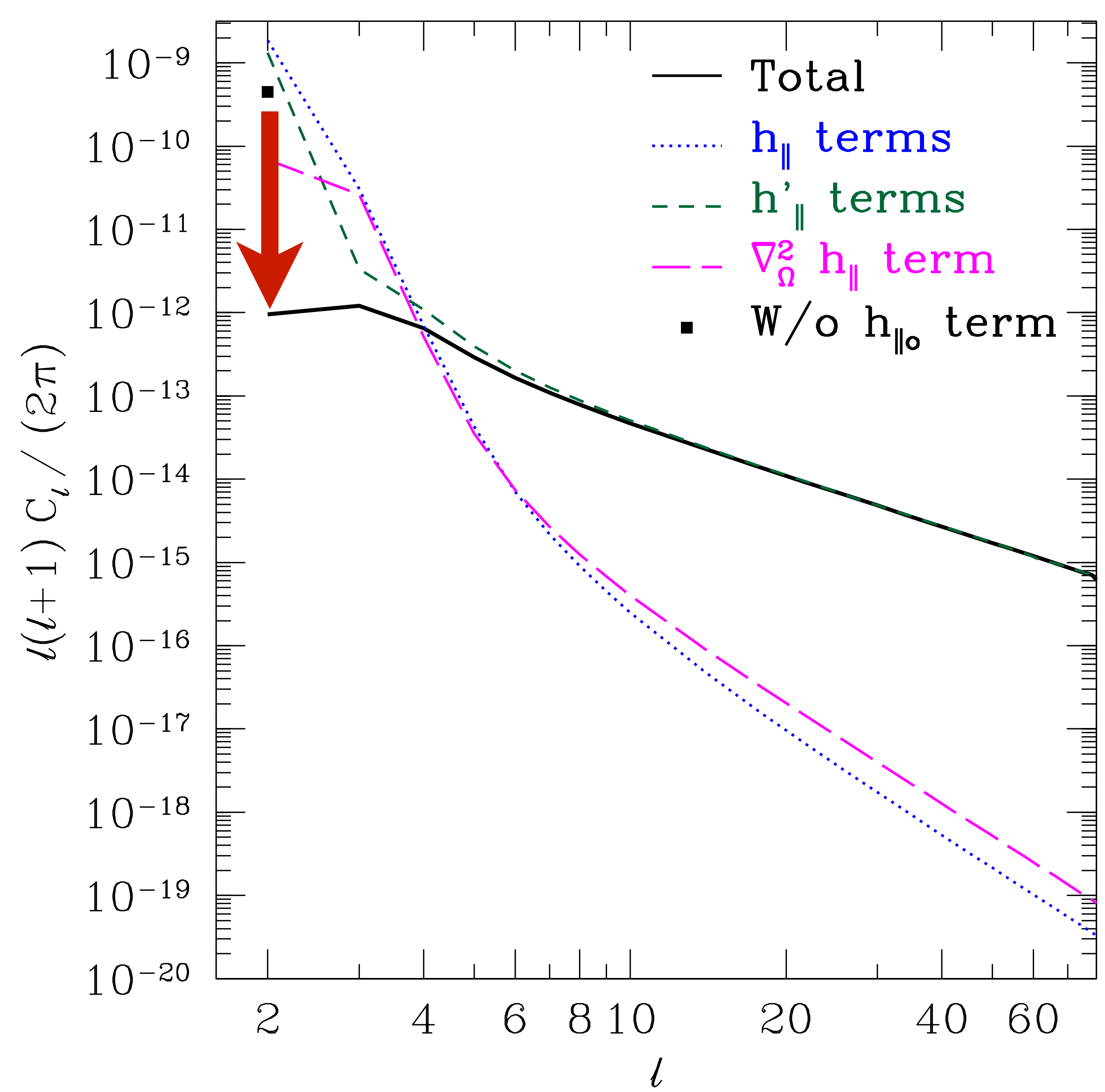
$$\begin{aligned}\tilde{\delta}_{gT} = &(b_e - 2\mathcal{Q})\delta z - 2(1-\mathcal{Q})\hat{\kappa} - \frac{1-\mathcal{Q}}{2}h_{\parallel} - \frac{1+\tilde{z}}{2H(\tilde{z})}h'_{\parallel} \\ &- \frac{1-\mathcal{Q}}{\tilde{\chi}}\left[\int_0^{\tilde{\chi}} d\chi h_{\parallel} + \frac{1+\tilde{z}}{H(\tilde{z})}\int_0^{\tilde{\chi}} d\chi h'_{\parallel}\right] \\ &- \frac{H(\tilde{z})}{2}\frac{\partial}{\partial\tilde{z}}\left[\frac{1+\tilde{z}}{H(\tilde{z})}\right]\int_0^{\tilde{\chi}} d\chi h'_{\parallel}.\end{aligned}$$

$$\hat{\kappa} = \frac{5}{4}h_{\parallel o} - \frac{1}{2}h_{\parallel} - \frac{1}{2}\int_0^{\tilde{\chi}} d\chi\left[h'_{\parallel} + \frac{3}{\chi}h_{\parallel}\right] - \frac{1}{4}\nabla^2_{\Omega}\int_0^{\tilde{\chi}} d\chi \frac{\tilde{\chi}-\chi}{\chi\tilde{\chi}}h_{\parallel}$$

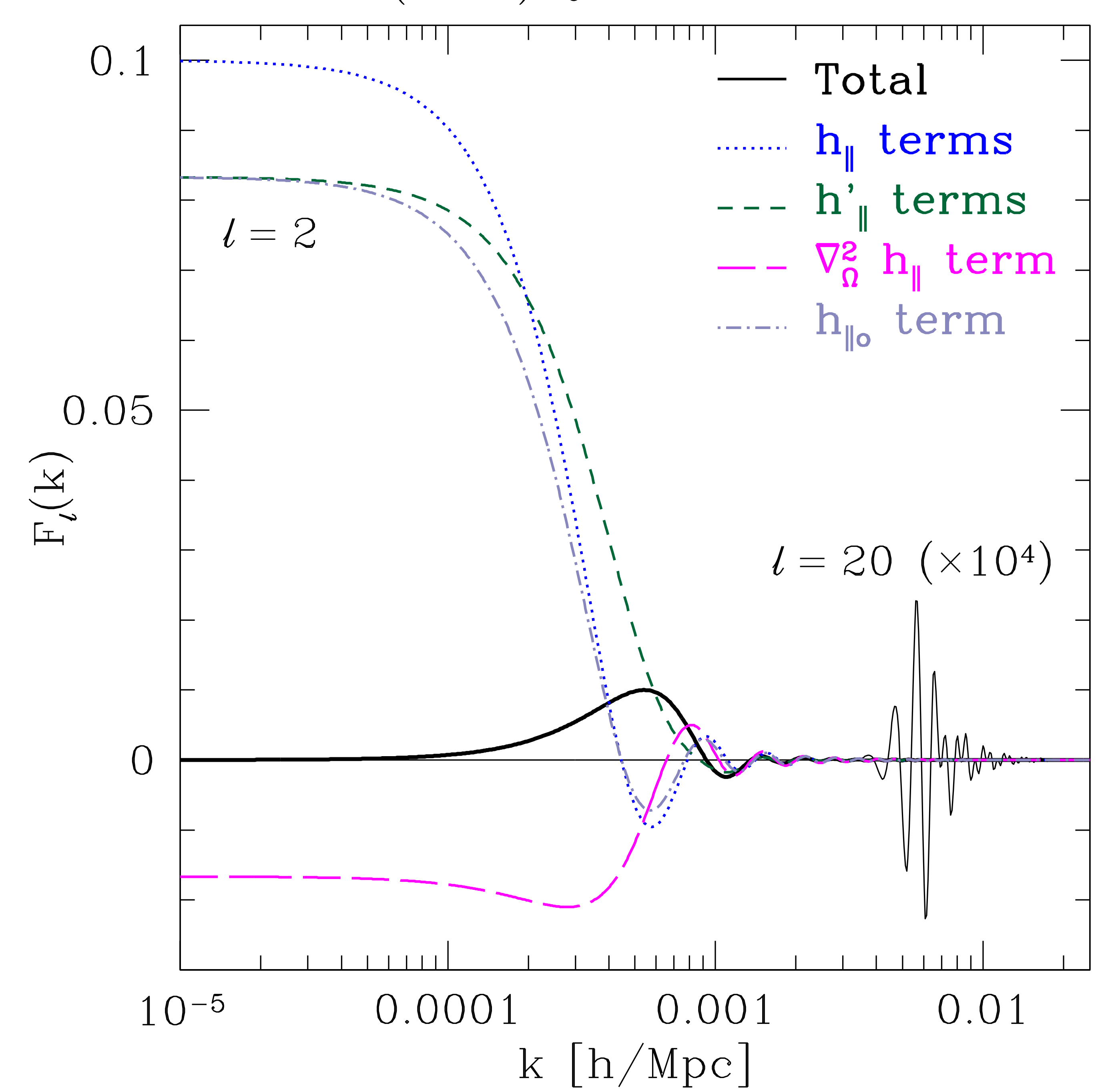
Observer term

Angular power spectrum with GW

- For the sharp redshift slice at z=2 with b_e=2.5,$\mathcal{Q}$=1.5



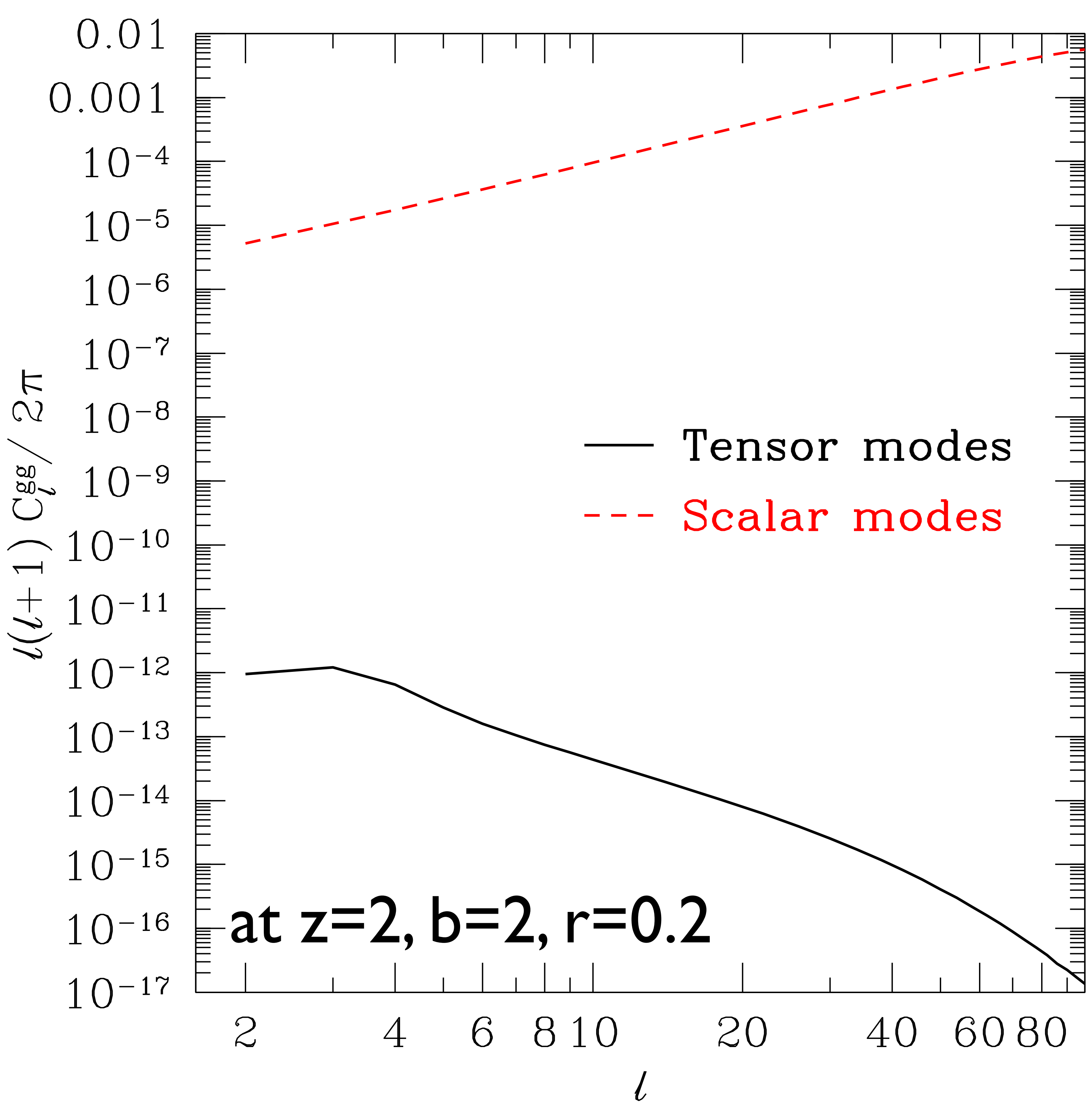
$$C_\ell = \frac{1}{2\pi}\frac{(\ell+2)!}{(\ell-2)!}\int k^2 dk P_T^{\rm prim}(k)\,[F_\ell(k)]^2$$

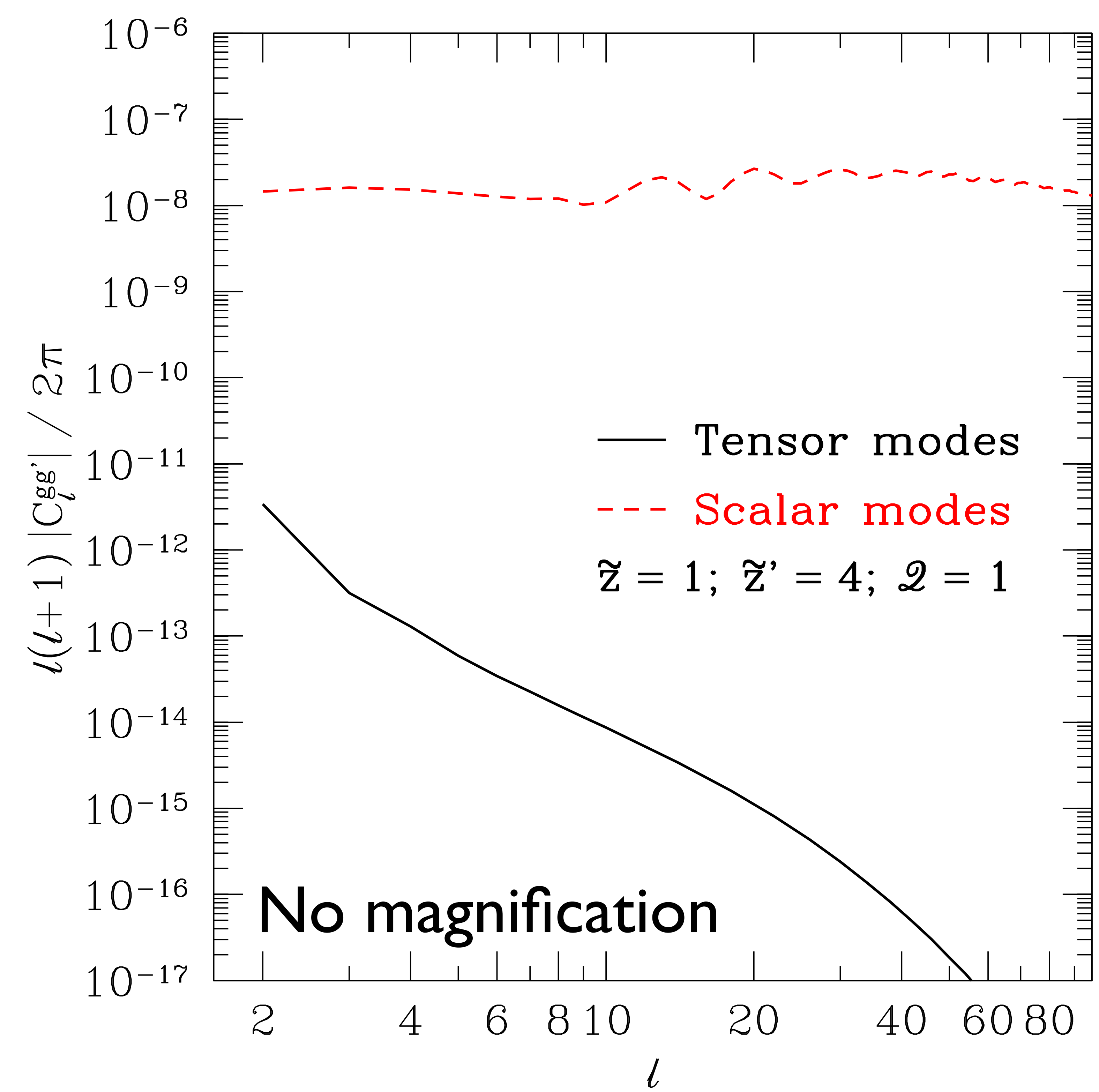


When including all effects, **NO** super horizon k-modes affect the sub-horizon clustering!! cf. Masui & Pen (2010)

We *enjoyed* physics, but...

- GW signal is way too small compared to the (1) intrinsic correlation and (2) the effect from scalar metric perturbations.





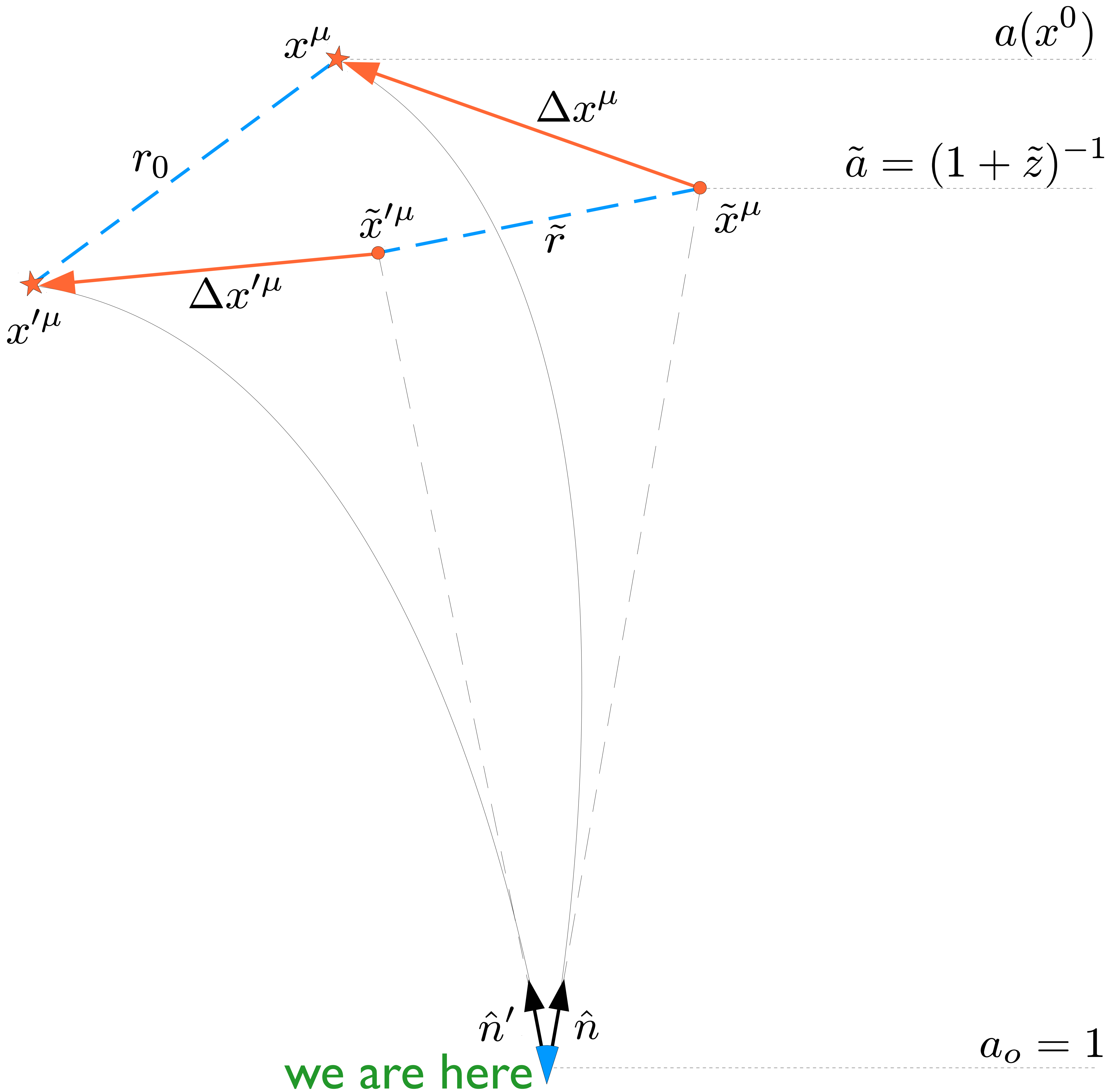
Cosmic Rulers

or, covariant formalism for the shape distortions

Fabian Schmidt & Donghui Jeong [arXiv:1204.3625]

$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

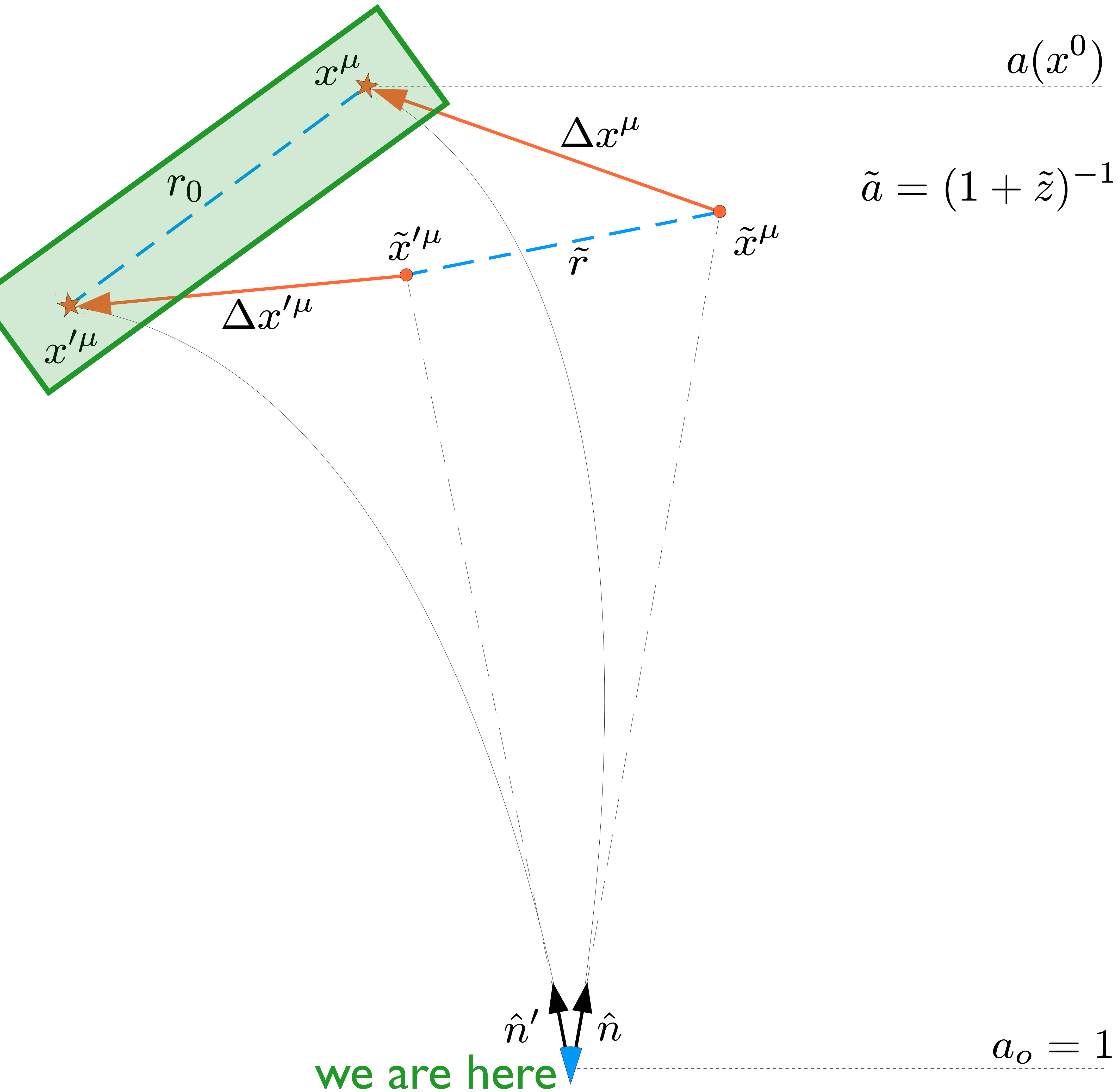
Cosmology with a high-z yardstick



- Consider a shining yardstick at high redshift, whose proper length is somehow known : r_0
- We observe (RA,DEC,z) for both ends of the stick, infer the length of the stick from them : $\tilde{r}$
- Due to perturbations, $\boxed{\tilde{r} \neq r_0}$ such a distortion to the size is an important tool to study perturbations!

$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

Who measures r_0?



- An (imaginary) observer moving with the stick measures the length of the ruler:

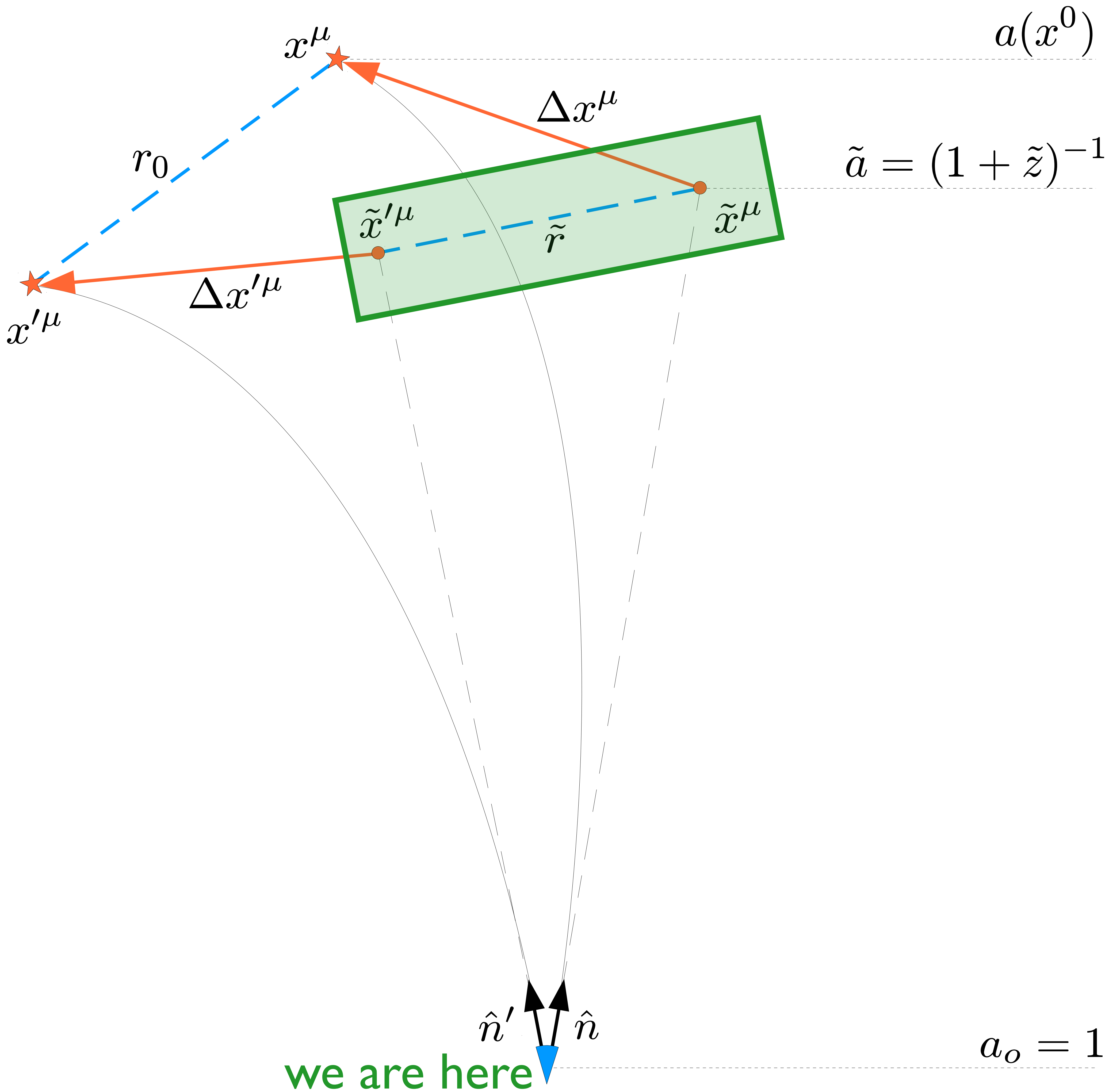
$$r_0^2 = \underbrace{\left[g_{\mu\nu} + u_\mu u_\nu\right]}(x^\mu - x'^\mu)(x^\nu - x'^\nu)$$

metric projected onto the constant-proper time hyper-surface of the comoving observer

$$g_{\mu\nu} + u_\mu u_\nu = a^2 \begin{pmatrix} 0 & -v_i \\ -v_i & \delta_{ij} + h_{ij} \end{pmatrix}$$

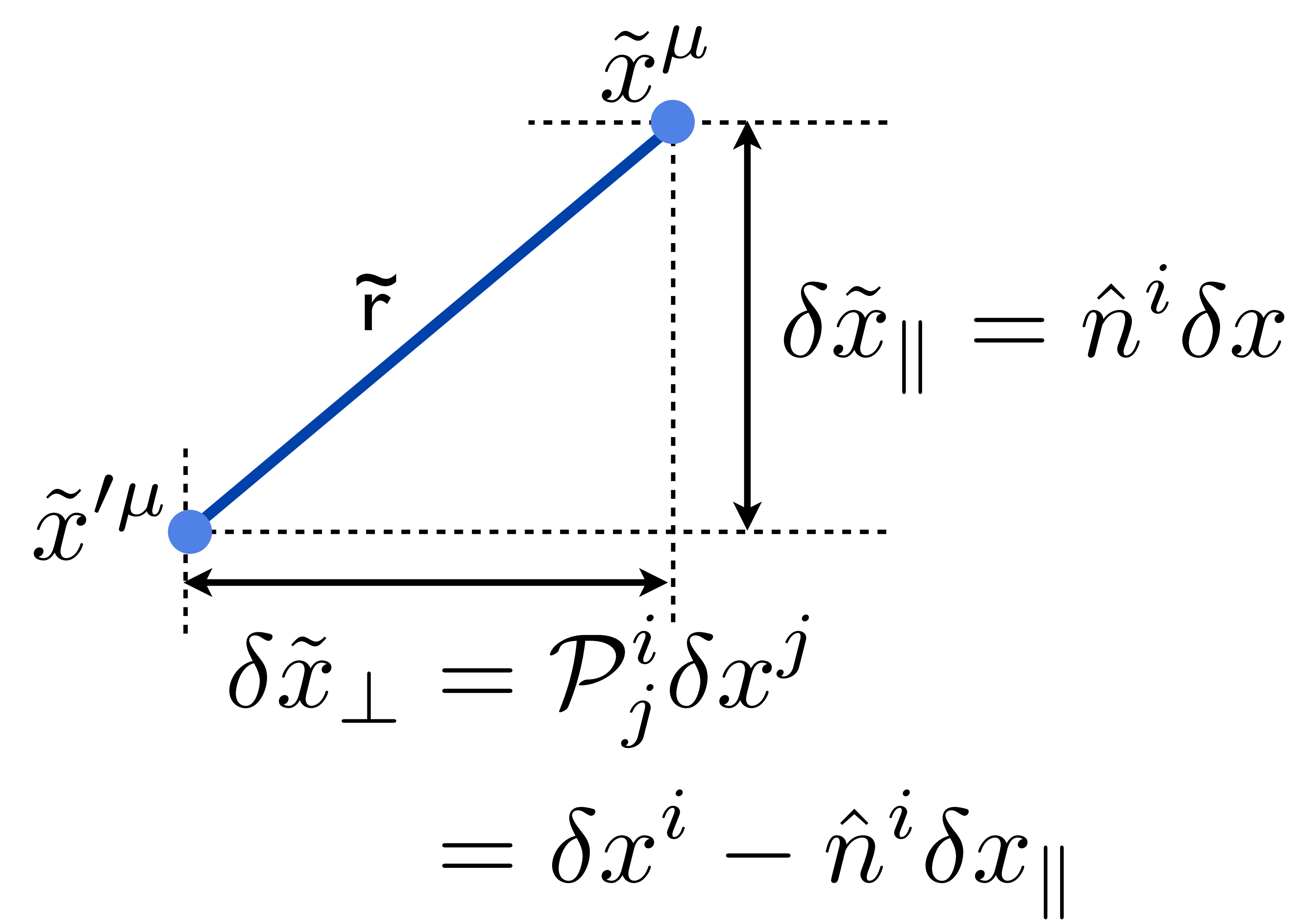
We assume a small ruler.

We measure r̃ !



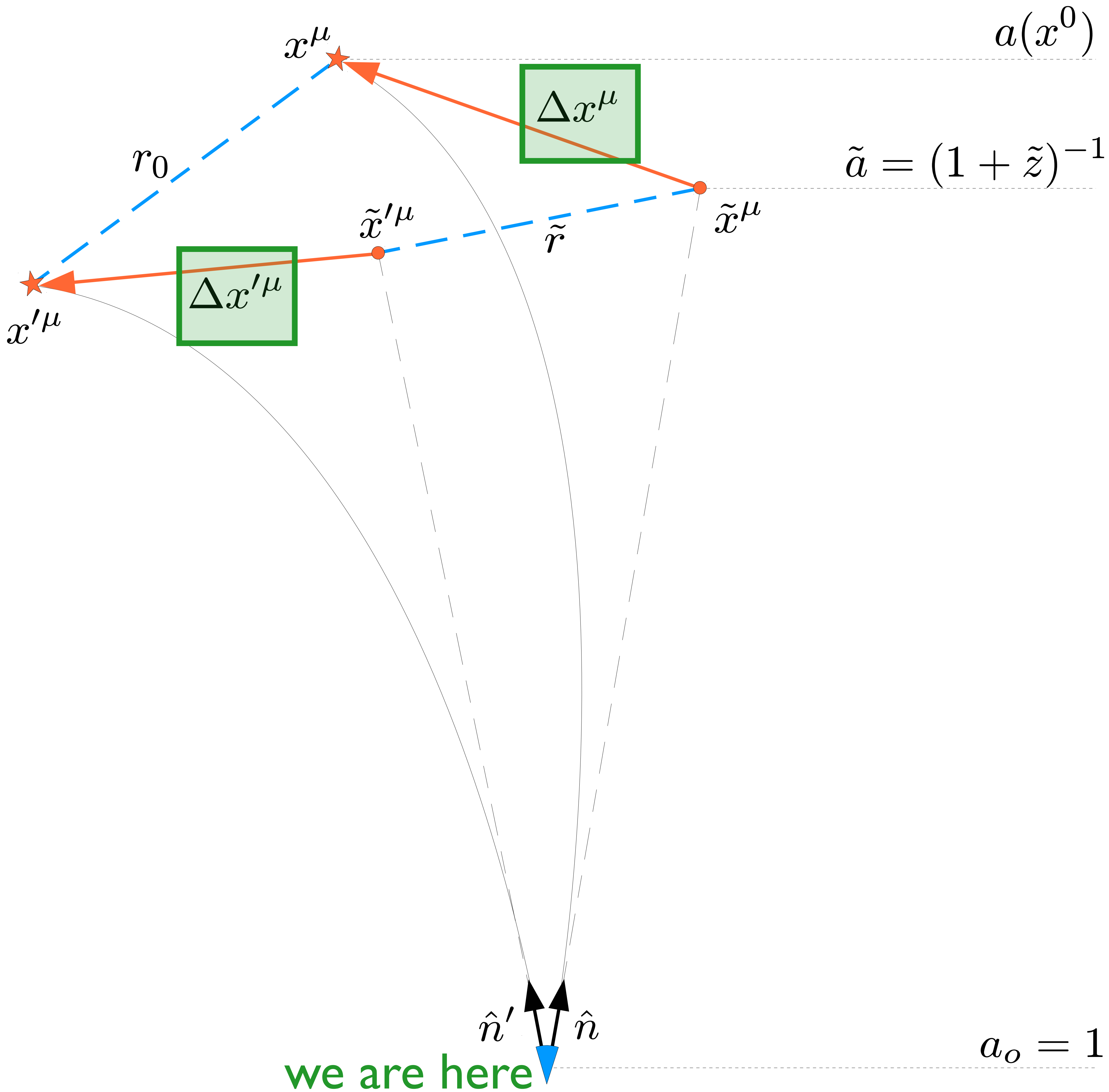
- We measure the angular and radial separations by using the unperturbed metric:

$$\tilde{r}^2 = \tilde{a}^2 \delta_{ij} \left(\tilde{x}^i - \tilde{x}'^i\right) \left(\tilde{x}^- \tilde{x}'^j\right)$$



$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

Δx is from geodesic equations



Shift along the line of sight direction

$$\Delta x_\parallel = \int_0^{\tilde{\chi}} d\chi \left[A - B_\parallel - \frac{1}{2}h_\parallel\right] - \frac{1+\tilde{z}}{H(\tilde{z})}\Delta \ln a$$

Shift along the perpendicular direction

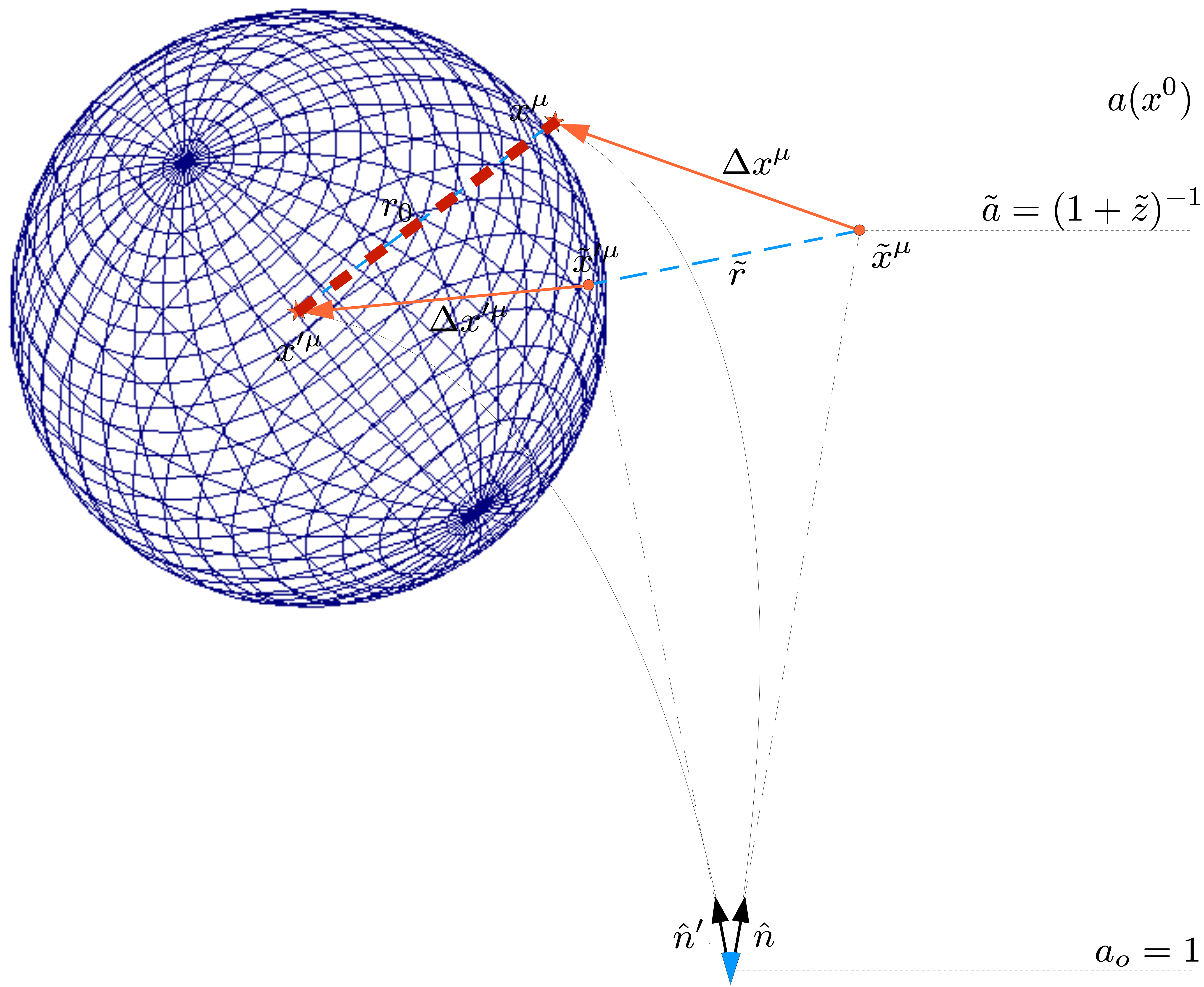
$$\Delta x_\perp^i = \left[\frac{1}{2}\mathcal{P}^{ij}(h_{jk})_o\,\hat{n}^k + B^i_{\perp o} - v^i_{\perp o}\right]\tilde{\chi} - \int_0^{\tilde{\chi}} d\chi \left[\frac{\tilde{\chi}}{\chi}\left(B^i_\perp + \mathcal{P}^{ij}h_{jk}\hat{n}^k\right) + (\tilde{\chi}-\chi)\partial^i_\perp\left(A - B_\parallel - \frac{1}{2}h_\parallel\right)\right]$$

perturbation to the scale factor at emission

$$\Delta \ln a = A_o - A + v_\parallel - v_{\parallel o} - \int_0^{\tilde{\chi}} d\chi \left[A - B_\parallel - \frac{1}{2}h_\parallel\right]'$$

Also, see Yoo et al. (2010)

Now, consider a spherical ruler

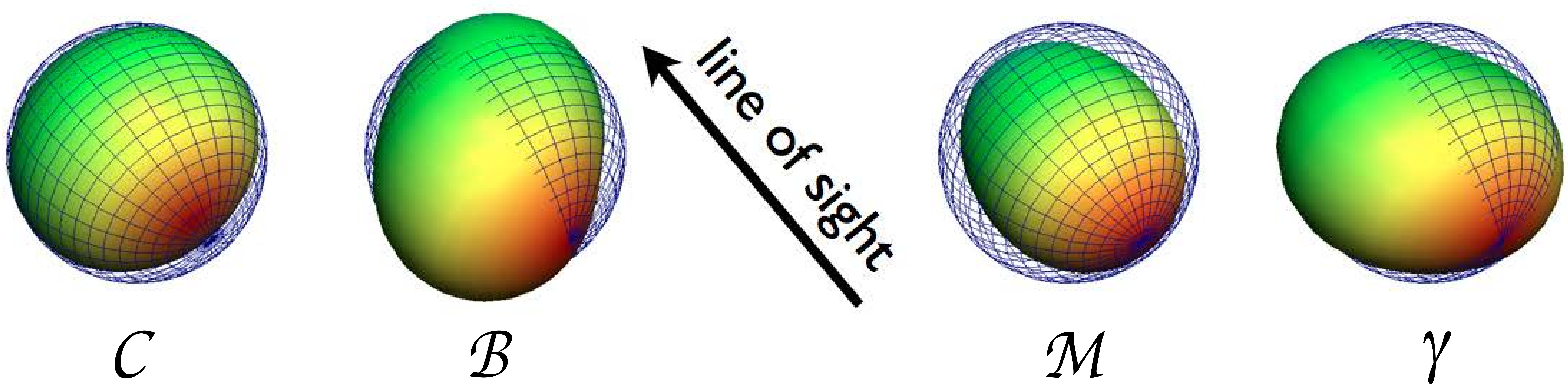


Classification of distortion

- We decompose the distortion as Scalar, Vector and Tensor according to their rotational property on sphere:

$$\frac{\tilde{r}-r_0}{\tilde{r}} = \mathcal{C}\frac{(\delta\tilde{x}_{\parallel})^2}{\tilde{r}_c^2} + \mathcal{B}_i\frac{\delta\tilde{x}_{\parallel}\delta\tilde{x}^i_{\perp}}{\tilde{r}_c^2} + \mathcal{A}_{ij}\frac{\delta\tilde{x}^i_{\perp}\delta\tilde{x}^j_{\perp}}{\tilde{r}_c^2}$$

longitudinal scalar — Vector — Magnification (trace) + shear (spin-2)



New!!

$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

Covariant formula for C, B, M

longitudinal scalar

$$\begin{aligned}\mathcal{C} =& -\Delta \ln a \left[1 - H(\tilde{z})\frac{\partial}{\partial \tilde{z}}\left(\frac{1+\tilde{z}}{H(\tilde{z})}\right) - \frac{\partial \ln r_0}{\partial \ln a}\right] \\ & - A - v_\parallel + B_\parallel \\ & + \frac{1+\tilde{z}}{H(\tilde{z})}\left(-\partial_\parallel A + \partial_\parallel v_\parallel + B'_\parallel - v'_\parallel + \frac{1}{2}h'_\parallel\right)\end{aligned}$$

Vector

$$\begin{aligned}\mathcal{B}_i =& -\mathcal{P}_i{}^j h_{jk}\hat{n}^k - v_{\perp i} - \partial_{\perp i}\Delta x_\parallel - \partial_{\tilde{\chi}}\Delta x_{\perp i} + \frac{\Delta x_{\perp i}}{\tilde{\chi}} \\ =& -v_{\perp i} + B_{\perp i} + \frac{1+\tilde{z}}{H(\tilde{z})}\partial_{\perp i}\Delta \ln a,\end{aligned}$$

Magnification (e.g. Yoo et al. 2009, Challinor&Lewis2011, Bonvin& Durrer2011, Jeong et al. 2011)

$$\mathcal{M} \equiv \mathcal{P}^{ij}\mathcal{A}_{ij} = -2\Delta \ln a \left[1 - \frac{\partial \ln r_0}{\partial \ln a}\right] - \frac{1}{2}\left(h^i{}_i - h_\parallel\right) + 2\hat{\kappa} - \frac{2}{\tilde{\chi}}\Delta x_\parallel.$$

convergence

$$\begin{aligned}\hat{\kappa} =& -\frac{1}{2}\left[\frac{1}{2}\left((h^i{}_i)_o - 3(h_\parallel)_o\right) - 2(B_\parallel - v_\parallel)_o\right] \\ & + \frac{1}{2}\int_0^{\tilde{\chi}} d\chi \left[\partial^k_\perp B_k - \frac{2}{\chi}B_\parallel + (\partial^l_\perp h_{lk})\hat{n}^k + \frac{1}{\chi}\left(h^i{}_i - 3h_\parallel\right) + (\tilde{\chi}-\chi)\frac{\chi}{\tilde{\chi}}\nabla^2_\perp\left\{A - B_\parallel - \frac{1}{2}h_\parallel\right\}\right]\end{aligned}$$

New!!

$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

... and γ!!

- First fully relativistic, covariant expression for the cosmic shear!!

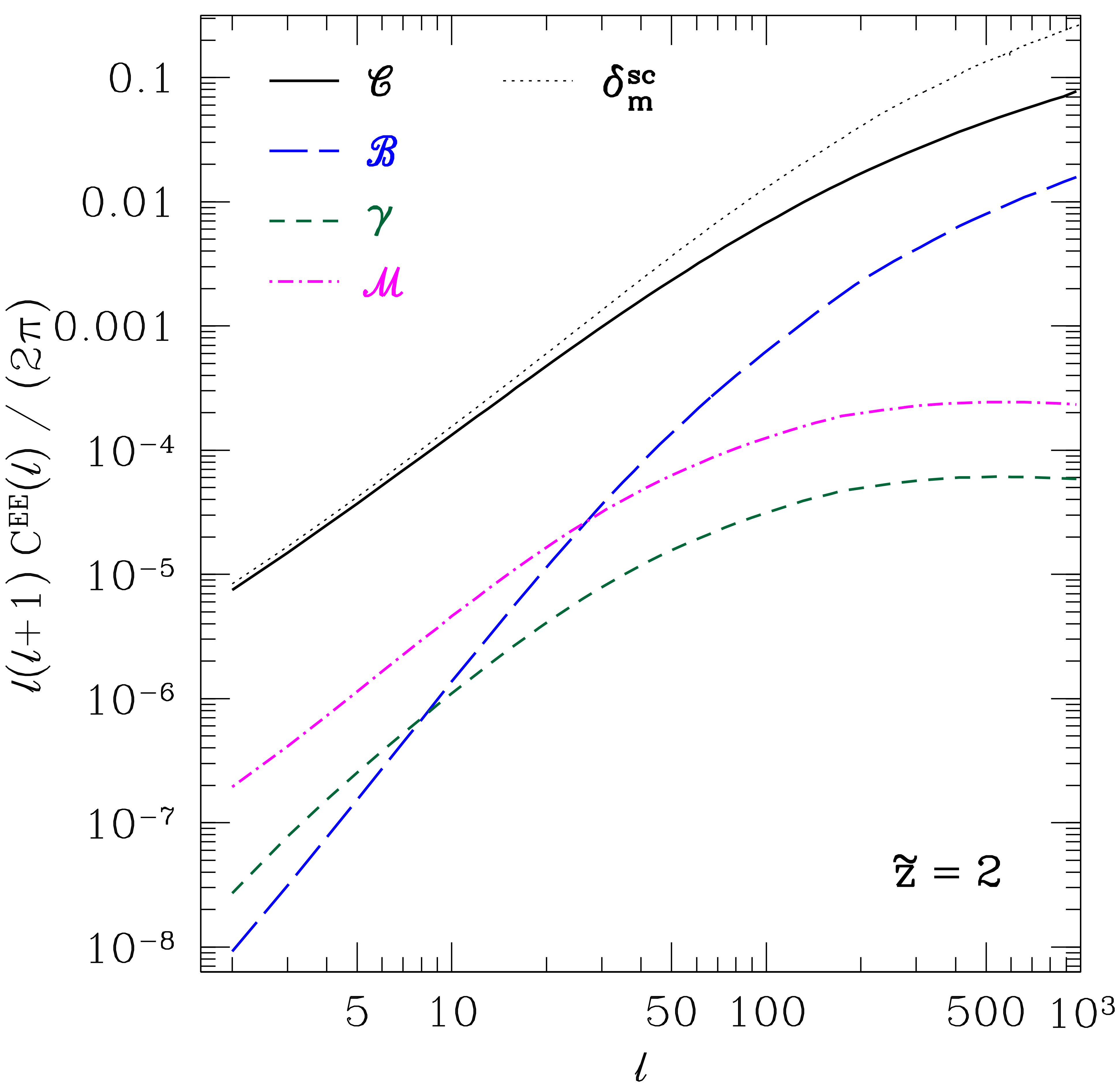
$$
\begin{aligned}
{}_{\pm2}\gamma \;=& -\frac{1}{2}h_\pm - \frac{1}{2}(h_\pm)_o - \int_0^{\tilde\chi} d\chi\Bigg[\left(1-2\frac{\chi}{\tilde\chi}\right)\left[m_\mp^k\partial_\pm B_k + (\partial_\pm h_{lk})m_\mp^l \hat n^k\right] - \frac{1}{\tilde\chi}h_\pm \\
&+ (\tilde\chi-\chi)\frac{\chi}{\tilde\chi}\left\{-m_\mp^i m_\mp^j\partial_i\partial_j A + \hat n^k m_\mp^i m_\mp^j \partial_i\partial_j B_k + \frac{1}{2}m_\mp^i m_\mp^j(\partial_i\partial_j h_{kl})\hat n^k \hat n^l\right\}\Bigg]
\end{aligned}
$$

Here, ${}_{\pm2}\gamma(\hat{\boldsymbol{n}}) \equiv m_\mp^i m_\mp^j \mathcal{A}_{ij}$ is a spin ±2 component of the shear, where

$\boldsymbol{m}_\pm = \frac{1}{\sqrt{2}}(\boldsymbol{e}_1 \mp i\boldsymbol{e}_2)$ are spin ±1 vector field on sphere in the sense that it transforms $\boldsymbol{m}_\pm \to \boldsymbol{m}'_\pm = e^{\pm i\psi}\boldsymbol{m}_\pm$ under the rotation $\boldsymbol{e}_i \to \boldsymbol{e}'_i$ with angle ψ.

- Conformal Newtonian gauge: ${}_{\pm2}\gamma(\hat{\boldsymbol{n}}) = \int_0^{\tilde\chi} d\chi(\tilde\chi-\chi)\frac{\chi}{\tilde\chi}m_\mp^i m_\mp^j\partial_i\partial_j(\Psi-\Phi)$

C, B, M, γ from scalar perturbations



- On small scales, C is dominated by line-of-sight velocity. When projecting onto sphere, velocity (solid line) and density (dotted line) have different slope
- Small scale B is dominated by perpendicular derivative of l.o.s. velocity.
- On small scales, |M| = 2|κ| = 2|γ|: $C_l^M = 4\ C_l^\gamma$

Large-Scale Structure with GW II : Shear

Donghui Jeong & Fabian Schmidt [arXiv:1205.1512]

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Cosmic shear with GW

- With only tensor perturbation, shear expression becomes

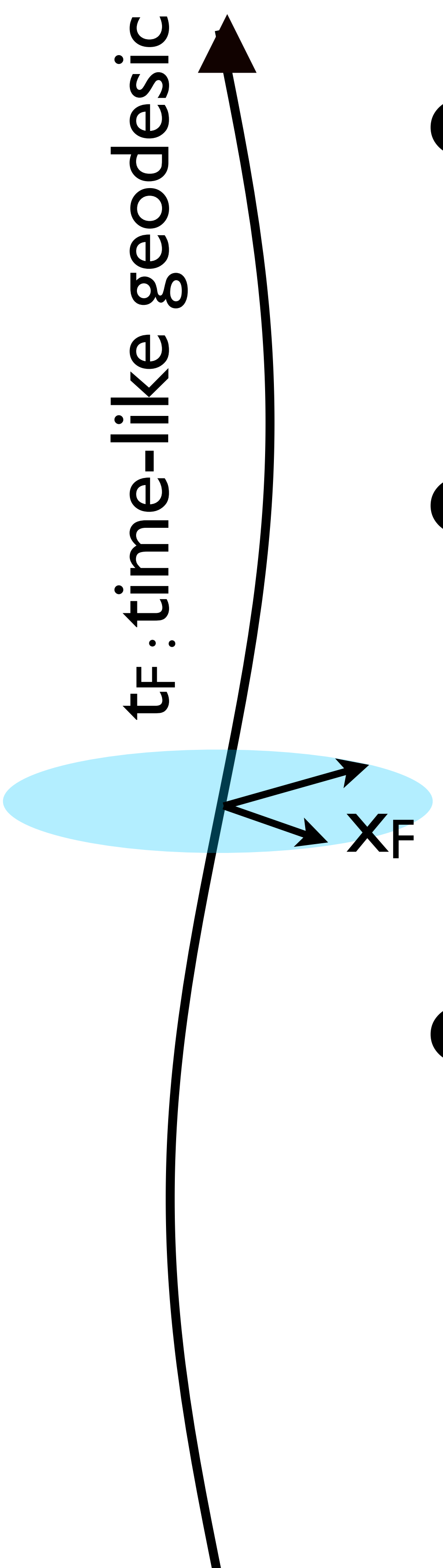
$${}_{\pm 2}\gamma(\hat{\boldsymbol{n}}) = \underbrace{-\frac{1}{2}h_{\pm o} - \frac{1}{2}h_{\pm}}_{\text{Metric Shear}} - \int_0^{\tilde{\chi}} d\chi \left\{ \frac{\tilde{\chi}-\chi}{2}\frac{\chi}{\tilde{\chi}}(m^i_{\mp} m^j_{\mp}\partial_i\partial_j h_{kl})\hat{n}^k\hat{n}^l + \left(1 - 2\frac{\chi}{\tilde{\chi}}\right)\hat{n}^l m^k_{\mp} m^i_{\mp}\partial_i h_{kl} - \frac{1}{\tilde{\chi}}h_{\pm} \right\}$$

Metric Shear

- Dodelson, Rozo & Stebbins (2003)
  "Assuming physical isotropy, we must add a 'metric shear' caused by the shearing of the coordinates with respect to physical space, i.e. $\Delta\gamma_{ij}$, which is just the traceless transverse projection of $-h_{ij}/2$"

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

What "metric shear" really is



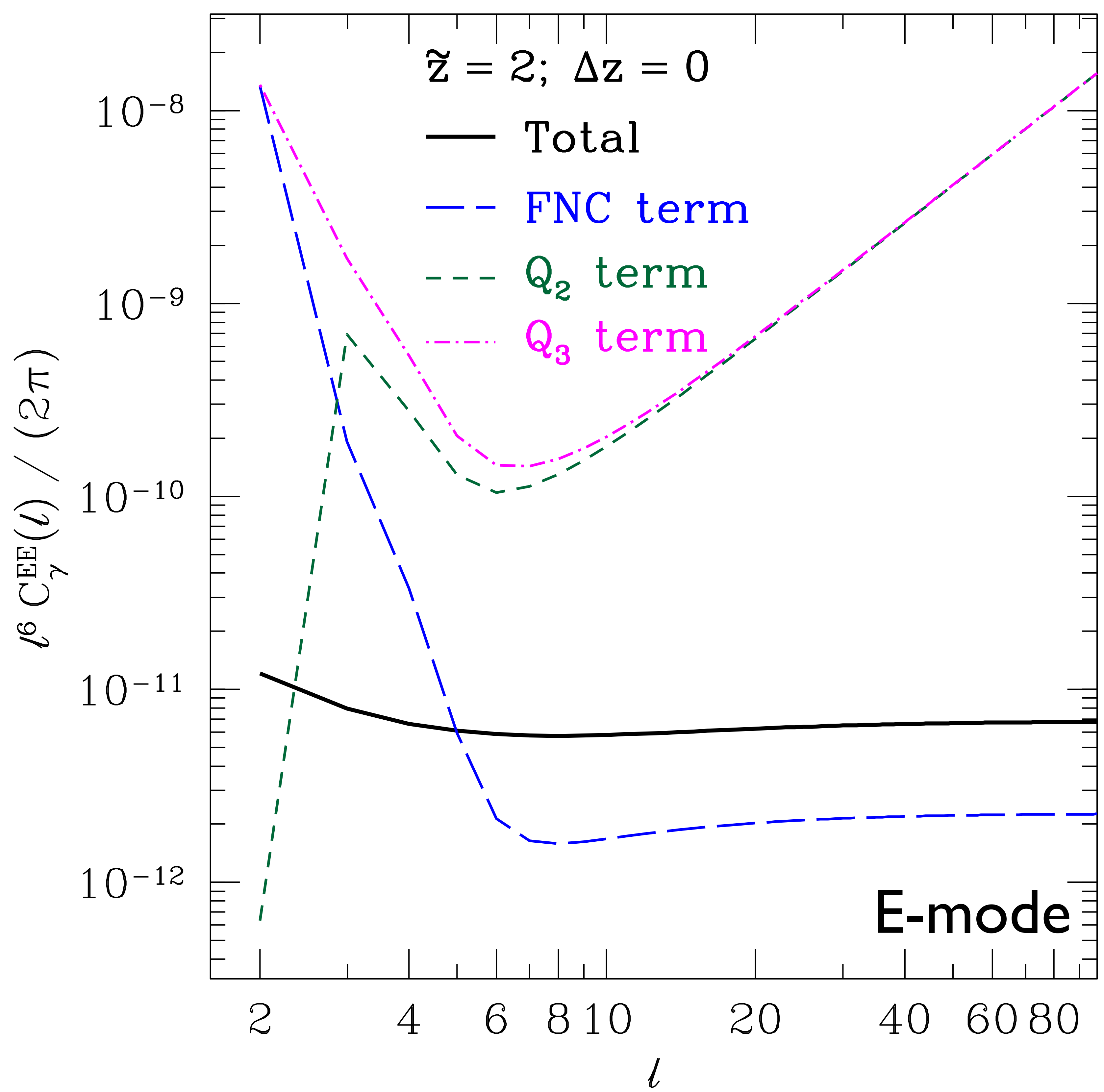
- The cosmic shear measurement are referenced to the frame within which galaxies are statistically round.
- The most natural choice of such coordinate is the local inertial frame defined along the time-like geodesic of the galactic center, or so called Fermi Normal Coordinate (FNC)!
- Coordinate transformation from FRW to FNC coordinate:

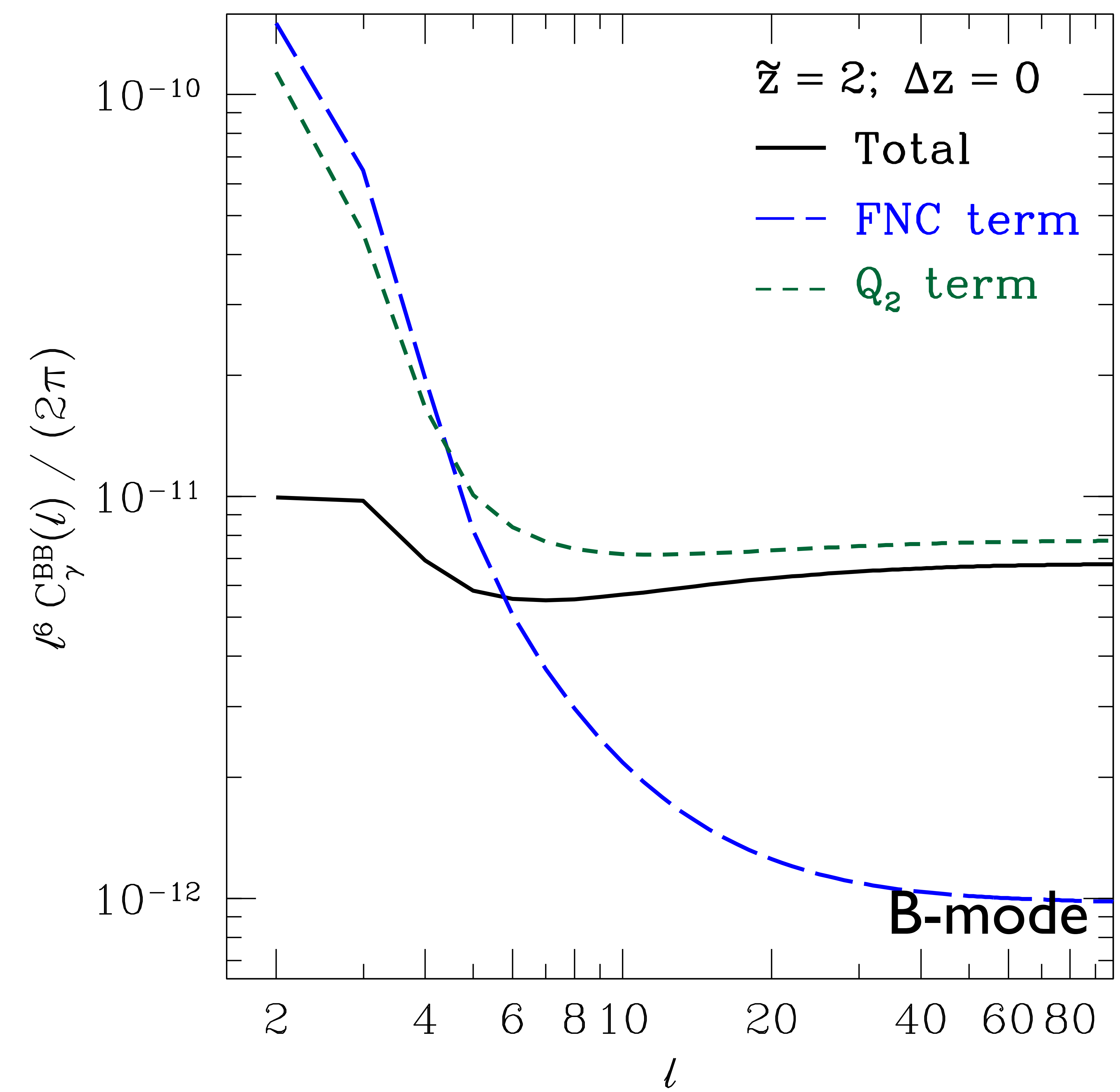
$$x_F^i = x^i - \frac{1}{2}h_{ij}x^j - \frac{1}{2}\Gamma^i_{jk}x^j x^k + \mathcal{O}(x^3)$$

FNC term

leads to an additional shear of $\underbrace{\partial_{\perp(i}\Delta x_{\perp j)}}_{\gamma_{ij}} \rightarrow \underbrace{\partial_{\perp(i}\Delta x_{\perp j)}}_{\gamma_{ij}} + \frac{1}{2}\mathcal{P}_i^k\mathcal{P}_j^l h_{kl} + \cdots$

Metric shear vs. l.o.s. integral





- They are about the same order of magnitude, but with opposite sign...

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

FNC metric and tide

- The metric in the Fermi Normal Coordinate is given by

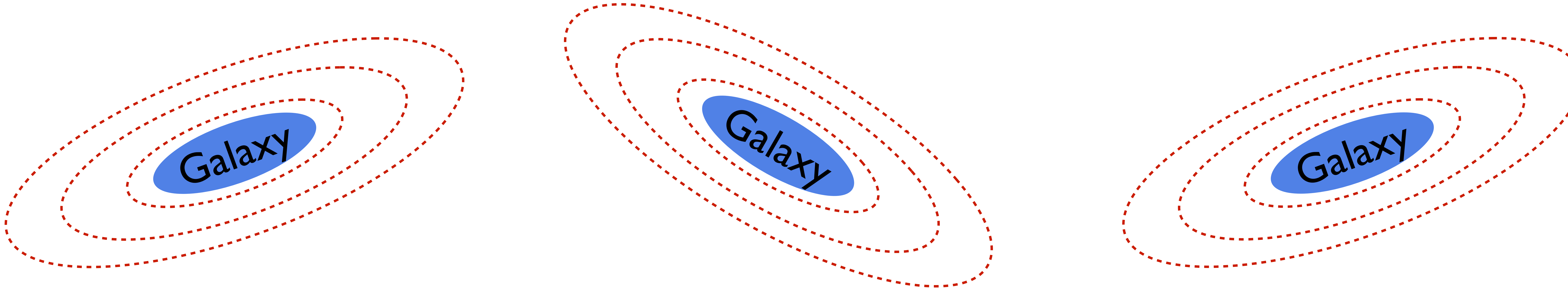
$$g^F_{00} = -1 + \left(\dot{H} + H^2\right) r_F^2 + \left[\frac{1}{2}\ddot{h}_{lm} + H\dot{h}_{lm}\right] x^l_F x^m_F.$$
$$g^F_{0i} = \frac{1}{3}\left(\nabla_i \dot{h}_{lm} - \nabla_m \dot{h}_{li}\right) x^l_F x^m_F$$
$$g^F_{ij} = \delta_{ij} + \frac{H^2}{3}\left[x^i_F x^j_F - r_F^2 \delta_{ij}\right] + \frac{1}{6}\left(\nabla_i\nabla_j h_{ml} + \nabla_l\nabla_m h_{ij} - \nabla_l\nabla_j h_{im} - \nabla_i\nabla_m h_{jl}\right) x^l_F x^m_F$$
$$+ \frac{H}{6}\left(\dot{h}_{lj} x^l_F x^i_F + \dot{h}_{im} x^m_F x^j_F - \dot{h}_{ij} r_F^2 - \dot{h}_{lm} x^l_F x^m_F \delta_{ij}\right).$$

- Equation of motion for non-relativistic body in FNC is determined by the effective gravitational potential $\Psi_{\rm eff} = -\delta g_{00}/2$.

- Ψ generates tidal force: $t_{ij} = \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Psi^F = -\left(\frac{1}{2}\ddot{h}_{lm} + H\dot{h}_{lm}\right)$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Intrinsic alignment (IA) model

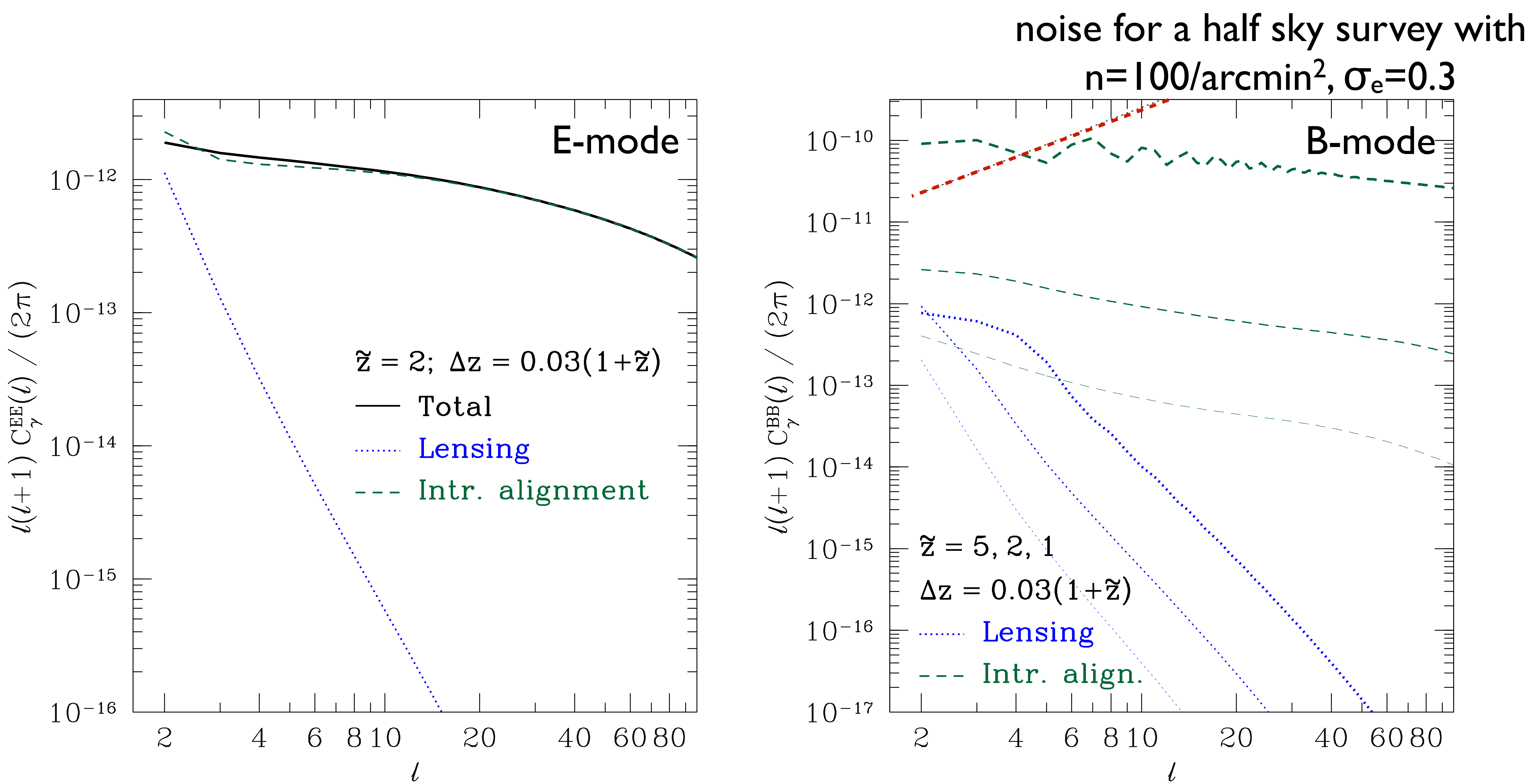


- Intrinsic alignment: tidal fields (anisotropic gravitational potential) tends to align galaxies
- Linear alignment model $\gamma_{ij}^{IA}(\boldsymbol{n}) = -\frac{C_1}{4\pi G}\mathcal{P}_{ik}\mathcal{P}_{jl}t^{kl} = -\frac{2}{3}\frac{C_1\rho_{\mathrm{cr}0}}{H_0^2}\mathcal{P}_{ik}\mathcal{P}_{jl}t^{kl}$
  - consistent with observations on large (>10 [Mpc/h]) scales

Blazek+(2011), Joachimi+(2011)

$$_{\pm 2}\gamma^{\mathrm{IA}}(\hat{\boldsymbol{n}}) = \frac{1}{3}\frac{C_1\rho_{\mathrm{cr}0}}{a^2H_0^2}\left(h''_{\pm} + aHh'_{\pm}\right)$$

Shear vs. intrinsic alignment

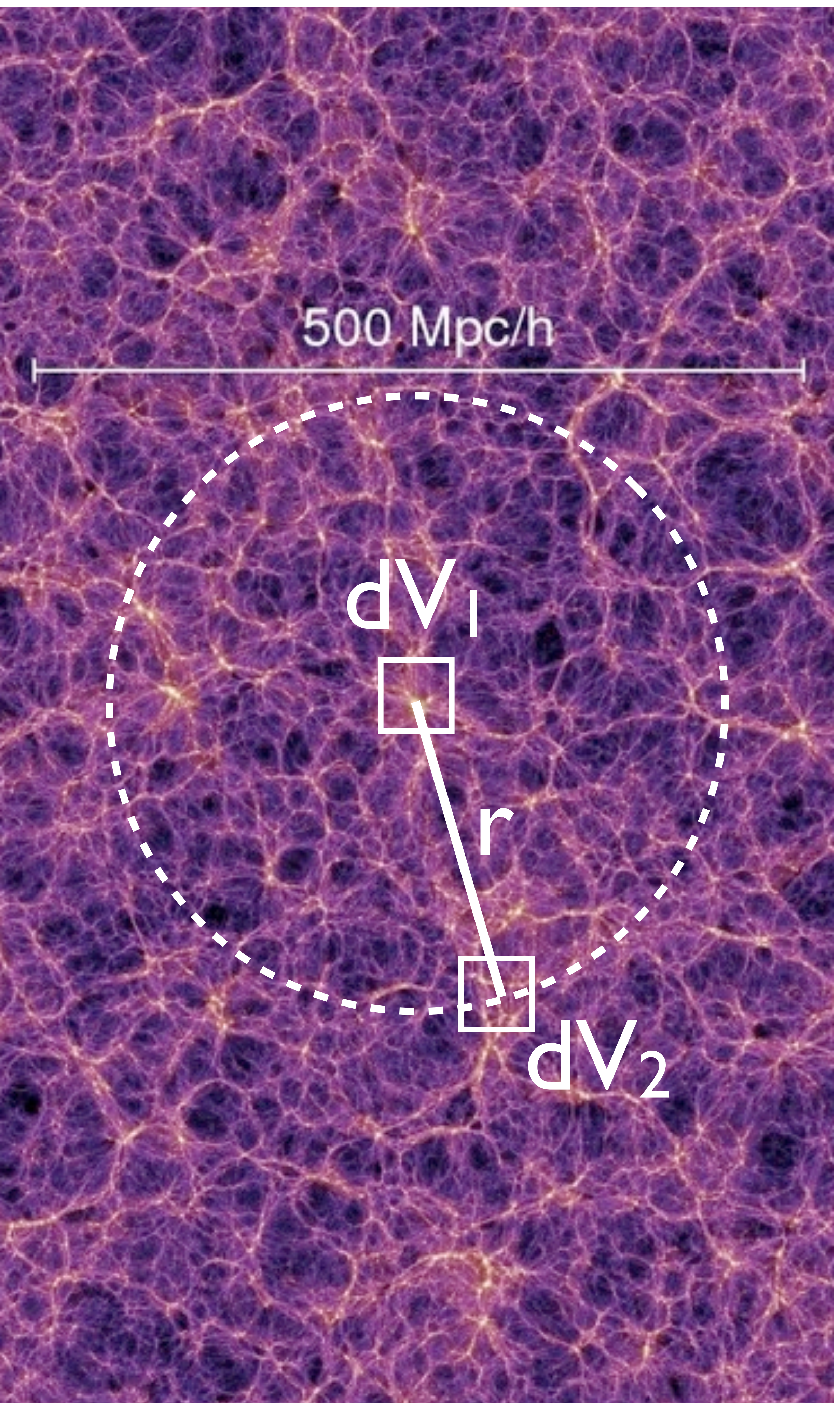


- Intrinsic alignment dominates over the lensing signal, and IA signal increases at higher redshifts!

Clustering Fossils from the Early Universe

Donghui Jeong & Marc Kamionkowski [arXiv:1203.0302]

Two-point correlation functions



- Probability of finding two galaxies at separation r is given by the two-point correlation function:

$$P_2(\boldsymbol{r}) = \bar{n}^2[1+\xi(\boldsymbol{r})]dV_1 dV_2$$

$$\xi(\boldsymbol{r}) \equiv \langle \delta(\boldsymbol{x})\delta(\boldsymbol{x}+\boldsymbol{r})\rangle$$

statistical homogeneity (translational invariance)

- Power spectrum is the Fourier transform of it:

$$P(\boldsymbol{k}) = \int d^3 r \xi(\boldsymbol{r}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

or in terms of density contrast,

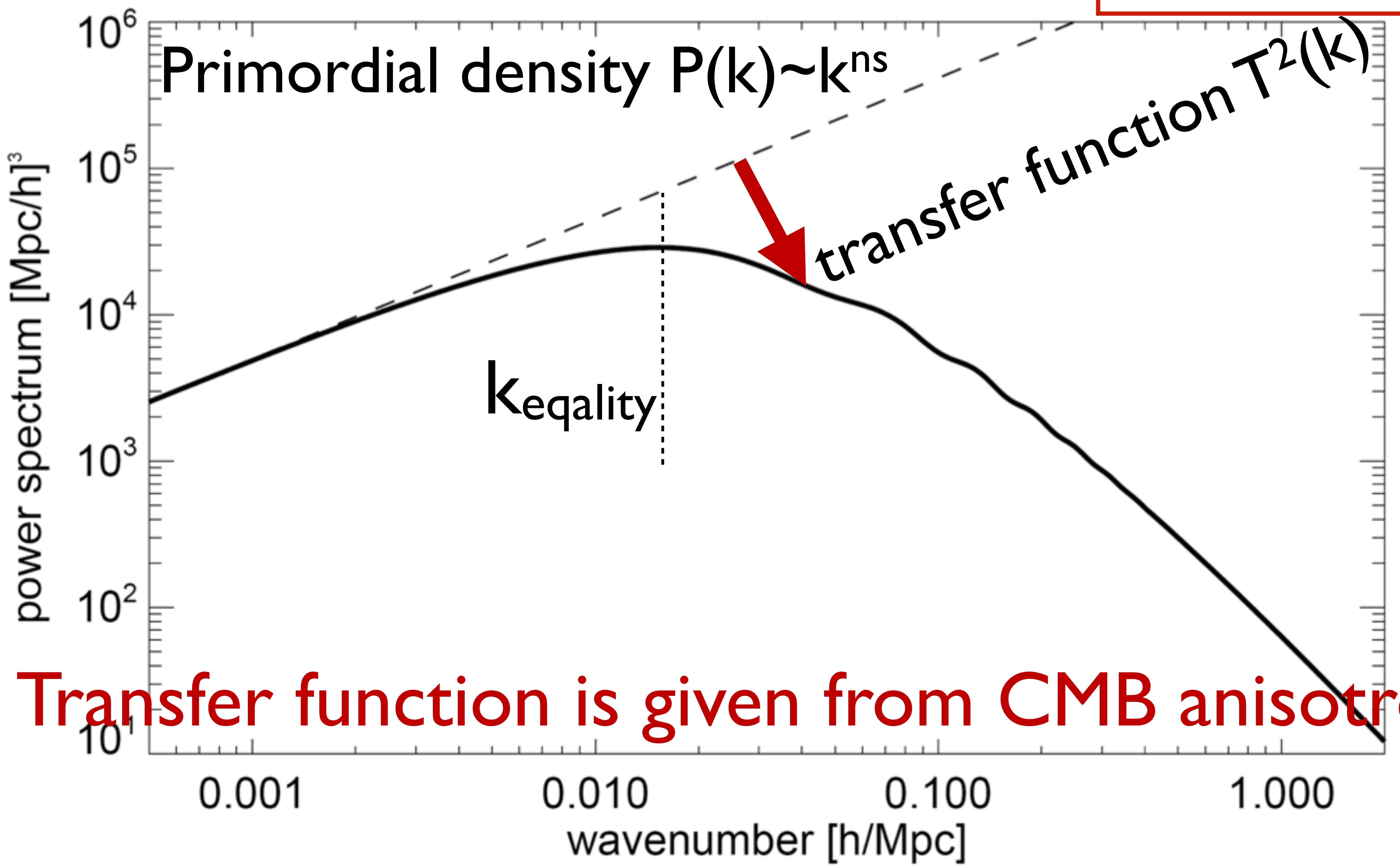
$$\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\rangle = (2\pi)^3 P(\boldsymbol{k})\delta^D(\boldsymbol{k}+\boldsymbol{k}')$$

Linear evolution of power spectrum

$$\langle \delta_i(\boldsymbol{k}) \delta_i(\boldsymbol{k}') \rangle = (2\pi)^3 P_{\text{initial}}(\boldsymbol{k}) \boxed{\delta^D(\boldsymbol{k}+\boldsymbol{k}')}$$

$$\langle \delta(\boldsymbol{k}) \delta(\boldsymbol{k}') \rangle = (2\pi)^3 \left[T(\boldsymbol{k})\right]^2 P_{\text{initial}}(\boldsymbol{k}) \boxed{\delta^D(\boldsymbol{k}+\boldsymbol{k}')}$$

Initial statistical homogeneity is sustained!



Transfer function is given from CMB anisotropy.

Jeong & Kamionkowski (2012)

Non-Gaussianity and (local) homogeneity

- IF we have a following non-linear coupling between primordial density fluctuations and new field h_p (JK coupling):

power spectrum of new field

$$\langle \delta_i(\boldsymbol{k}_1)\delta_i(\boldsymbol{k}_2)h_p(\boldsymbol{K})\rangle = (2\pi)^3 P_p(K) f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij}k^i_1k^j_2\delta^D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K})$$

coupling amplitude   polarization basis (scalar, vector, tensor)

- THEN, density power spectrum we observe now has ***non-zero off-diagonal*** components: **Fossil equation**

$$\langle \delta_i(\boldsymbol{k}_1)\delta_i(\boldsymbol{k}_2)\rangle|_{h_p(\boldsymbol{K})} = h_p(\boldsymbol{k}_1+\boldsymbol{k}_2) f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij}k^i_1k^j_2\delta^D_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K}}$$

Why worrying about new fields?

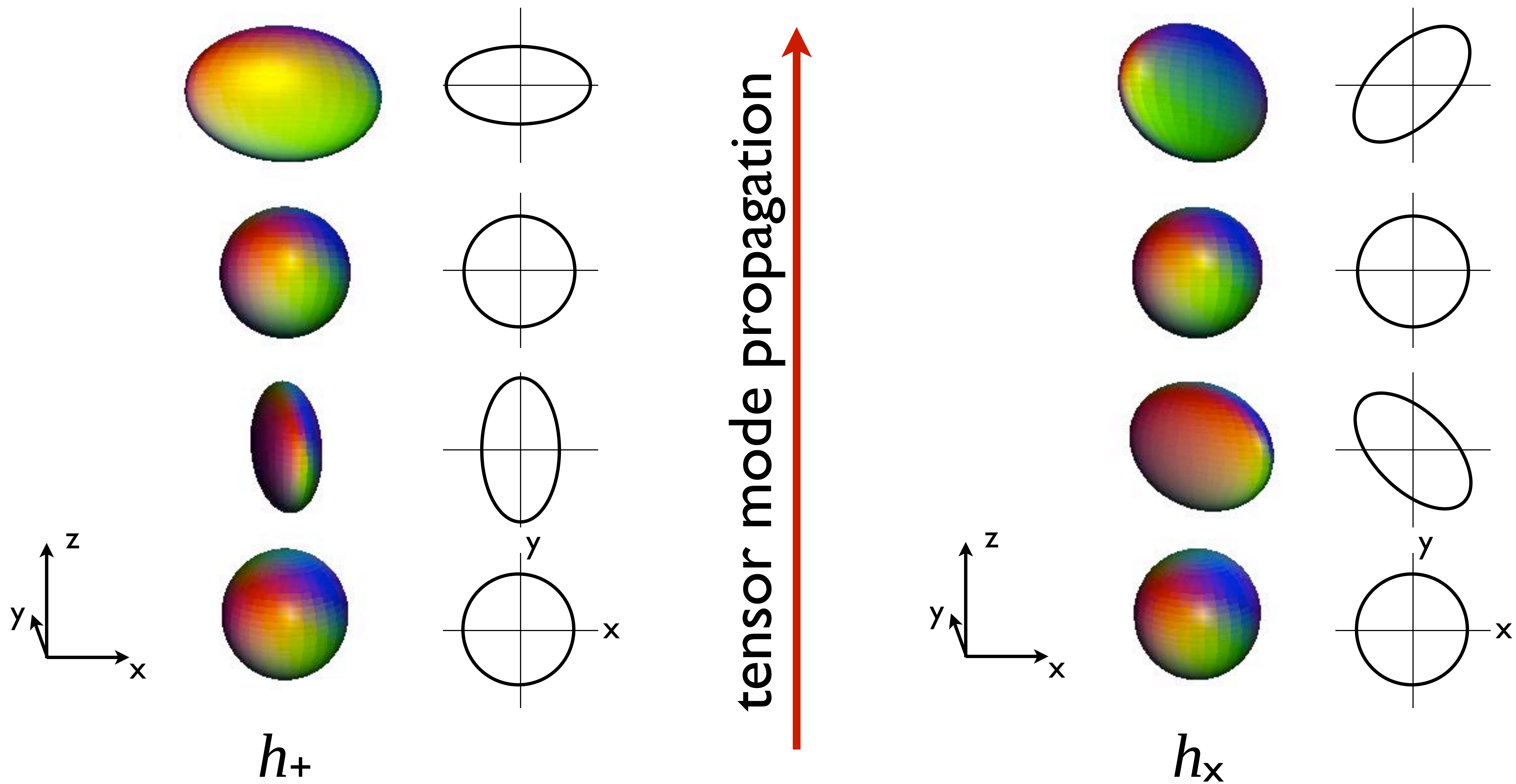
- Inflaton(s) : a scalar field(s) responsible for inflation
- But, inflaton might not be alone. Many inflationary models need/ introduce additional fields. But, direct detection of such fields turns out to be very hard:
  - Additional Scalar: not contributing to seed fluctuations
  - Vector: decays as 1/[scale factor]
  - Tensor: decays after coming inside of comoving horizon
- Off-diagonal correlation (Fossil equation) opens new way of detecting them!

$\varepsilon^p{}_{ij}$: six independent modes

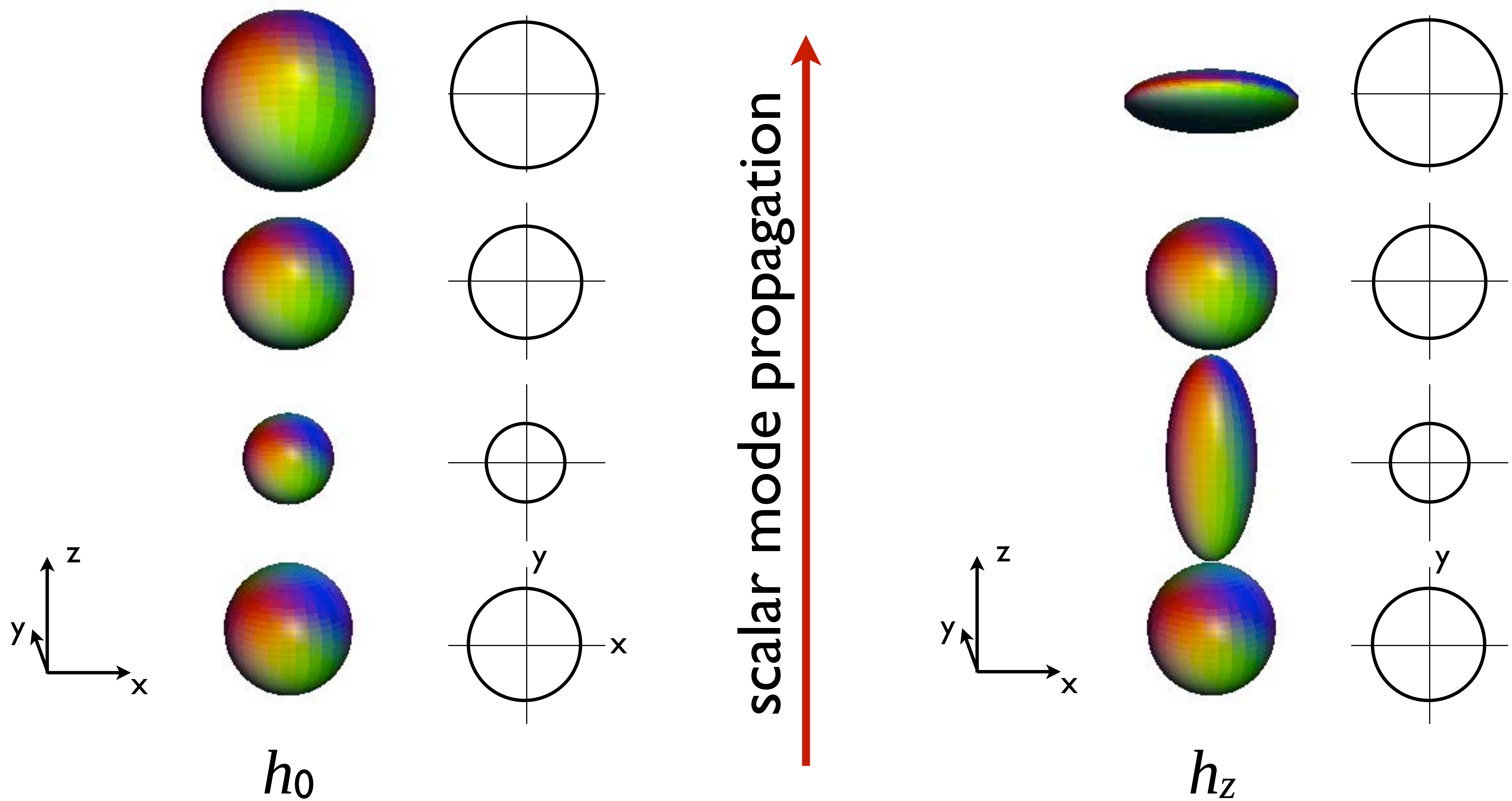
- In a symmetric 3x3 tensor, we have 6 degrees of freedom, which are further decomposed by Scalar, Vector and Tensor polarization modes.
- They are orthogonal: $\epsilon^p_{ij}\epsilon^{p',ij} = 2\delta_{pp'}$
  - Scalar (p=0,z): $\epsilon^0_{ij} \propto \delta_{ij} \qquad \epsilon^z_{ij}(\boldsymbol{K}) \propto K_iK_j - K^2/3$
  - Vector (p=x,y): $\epsilon^{x,y}_{ij}(\boldsymbol{K}) \propto \frac{1}{2}(K_ie_j + K_je_i)$ where $K_ie_i = 0$
  - Tensor (p=x,+): transverse and traceless

$$K_i\epsilon^{+,\times}_{ij}(\boldsymbol{K}) = 0 \qquad \delta_{ij}\epsilon^{+,\times}_{ij}(\boldsymbol{K}) = 0$$

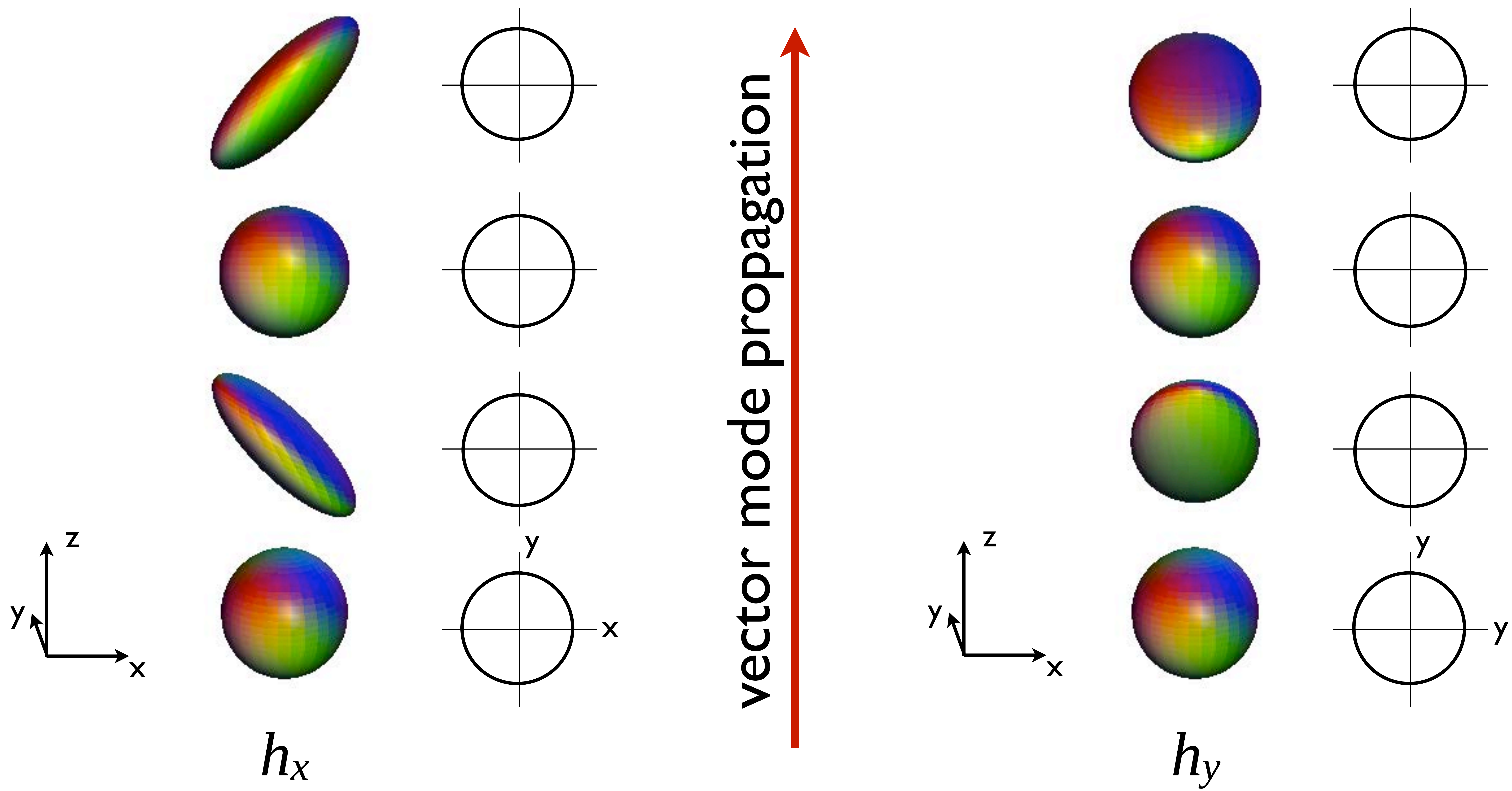
$\xi(\mathbf{r})$ with single tensor mode (p=+,x)



$\xi(\mathbf{r})$ with single scalar mode (p=0,z)



$\xi(\mathbf{r})$ with single vector mode (p=x,y)

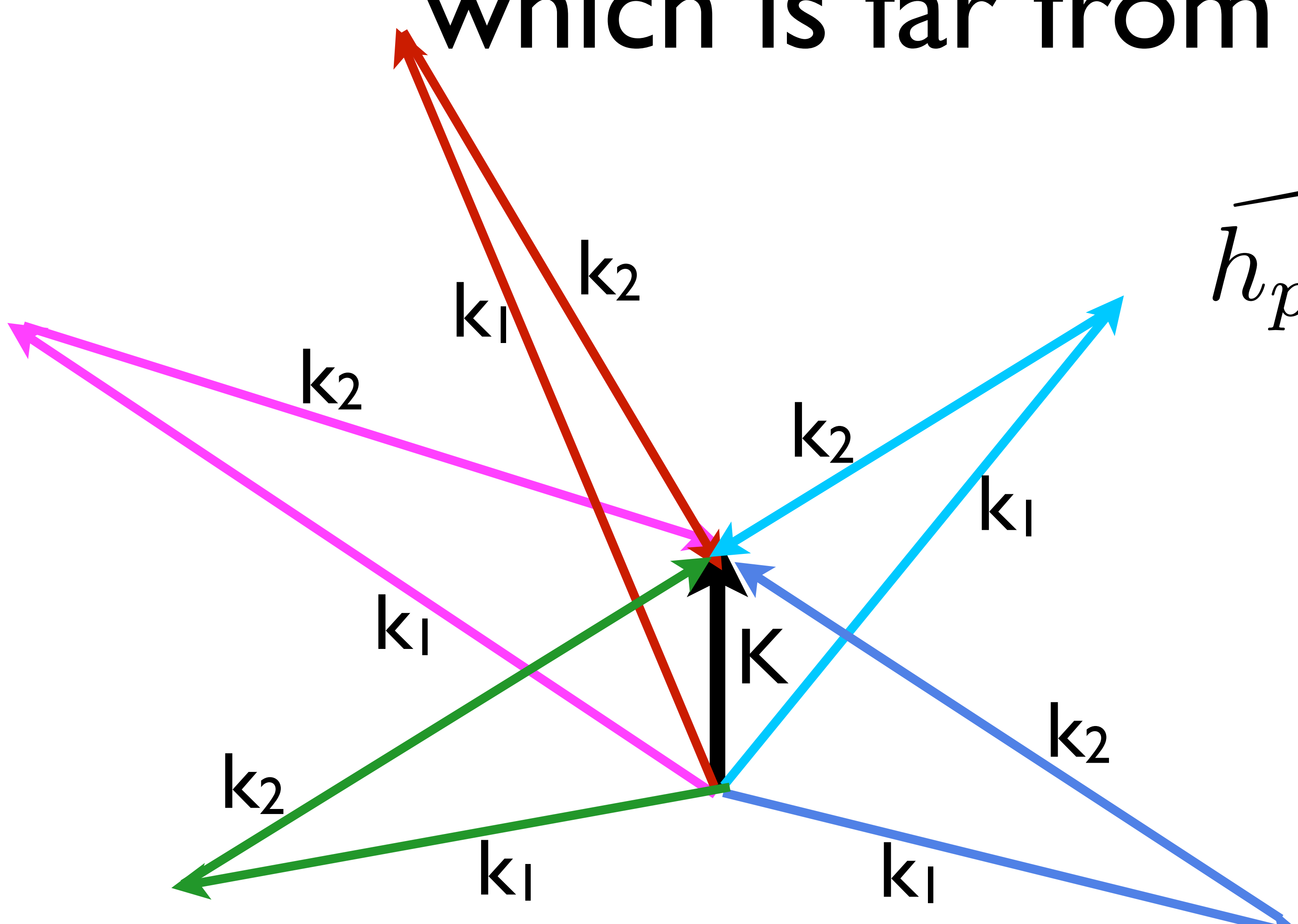


Naive estimator

- Let's start from Fossil equation

$$\langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle|_{h_p(\boldsymbol{K})} = h_p(\boldsymbol{k}_1+\boldsymbol{k}_2) f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij} k^i_1 k^j_2 \delta^D_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K}}$$

- Rearranging it a bit, we get a naive estimator for the new field, which is far from optimal:



$$\widehat{h_p(\boldsymbol{K})} = \sum_{\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{K}} \frac{\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)}{f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij} k^i_1 k^j_2}$$

Optimal estimator for a single mode

- Inverse-variance weighting gives an optimal estimator for a single mode

$$\widehat{h_p(\mathbf{K})} = P_p^n(\mathbf{K}) \sum_{\mathbf{k}} \frac{f_p^*(\mathbf{k}, \mathbf{K}-\mathbf{k}) \epsilon_{ij}^p k^i (K-k)^j}{2V P^{\rm tot}(k) P^{\rm tot}(|\mathbf{K}-\mathbf{k}|)} \delta(\mathbf{k}) \delta(\mathbf{K}-\mathbf{k})$$

- With a noise power spectrum ($P_{tot} = P_{galaxy} + P_{noise}$)

$$P_p^n(K) = \left[\sum_{\mathbf{k}} \frac{\left| f_p(\mathbf{k}, \mathbf{K}-\mathbf{k}) \epsilon_{ij}^p k^i (K-k)^j \right|^2}{2V P^{\rm tot}(k) P^{\rm tot}(|\mathbf{K}-\mathbf{k}|)} \right]^{-1}$$

Optimal estimator for the power amplitude A_h

- For a stochastic background of new fields with power spectrum $P_p(K)=A_h P_h^f(K)$, we optimally summed over different K-modes to estimate the amplitude by (w/ NULL hypothesis):

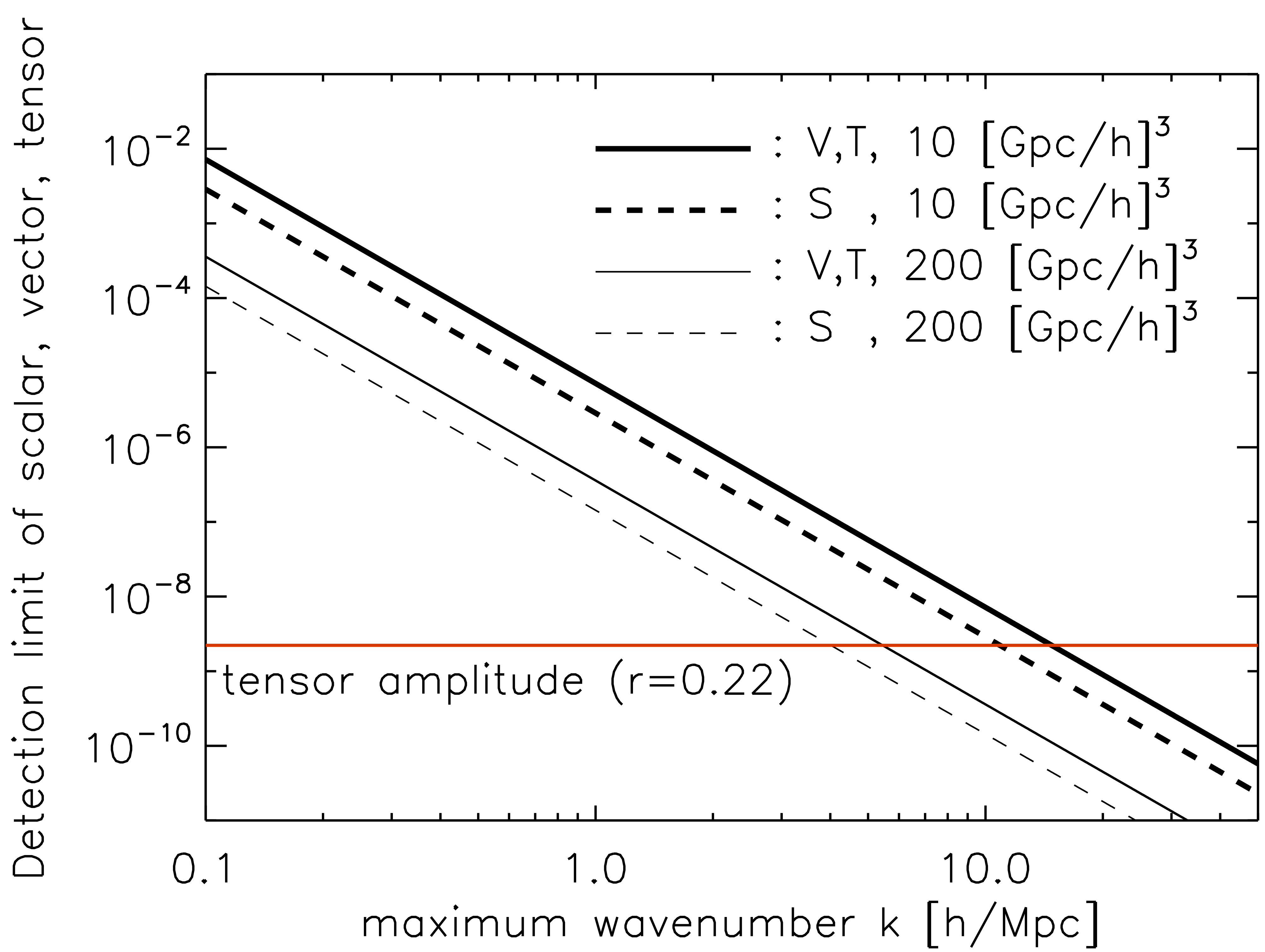
$$\widehat{A_h} = \sigma_h^2 \sum_{\boldsymbol{K},p} \frac{\left[P_h^f(K)\right]^2}{2\left[P_p^n(K)\right]^2} \left(\frac{\left|\widehat{h_p(\boldsymbol{K})}\right|^2}{V} - P_p^n(K) \right)$$

- Here, the minimum uncertainty of measuring amplitude is

$$\sigma_h^{-2} = \sum_{\boldsymbol{K},p} \left[P_h^f(K)\right]^2 / 2\left[P_p^n(K)\right]^2$$

When new "fields" are usual metric fluctuations

- Then, new field only rescales the wave-vector $k^2 \rightarrow k^2 - h_{ij}k_ik_j$, which reads f_p=-3/2P(k)/k^2 (Maldacena, 2003)



- projected 3-sigma (99% C.L.) detection limit with galaxy survey parameters
- To detect the gravitational wave, we need a large dynamical range
- Current survey (e.g. SDSS) should set a limit on primordial V and T!

Conclusion

- We present three different ways of detecting primordial GW. For all three methods, effect at the source location is important as GW itself decays in time.
  - Galaxy clustering: impossible to probe as the signal is too weak compared to that of scalar perturbations
  - Cosmic shear: a bit challenge, but possible to detect GW on large scales thanks to the intrinsic alignment effect!
  - Fossil equation: requires large dynamical range to beat the small signal (21cm map?). Interesting potential to detect primordial vector fields as well.