New Probes of Initial State of Quantum Fluctuations during Inflation

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ACP Seminar at IPMU, July 13, 2012

This talk is based on...

- Squeezed-limit bispectrum
  - *Ganc & Komatsu, JCAP, 12, 009 (2010)*
- Non-Bunch-Davies vacuum and CMB
  - *Ganc, PRD 84, 063514 (2011)*
- Scale-dependent bias and μ-distortion
  - *Ganc & Komatsu, PRD 86, 023518 (2012)*

Question

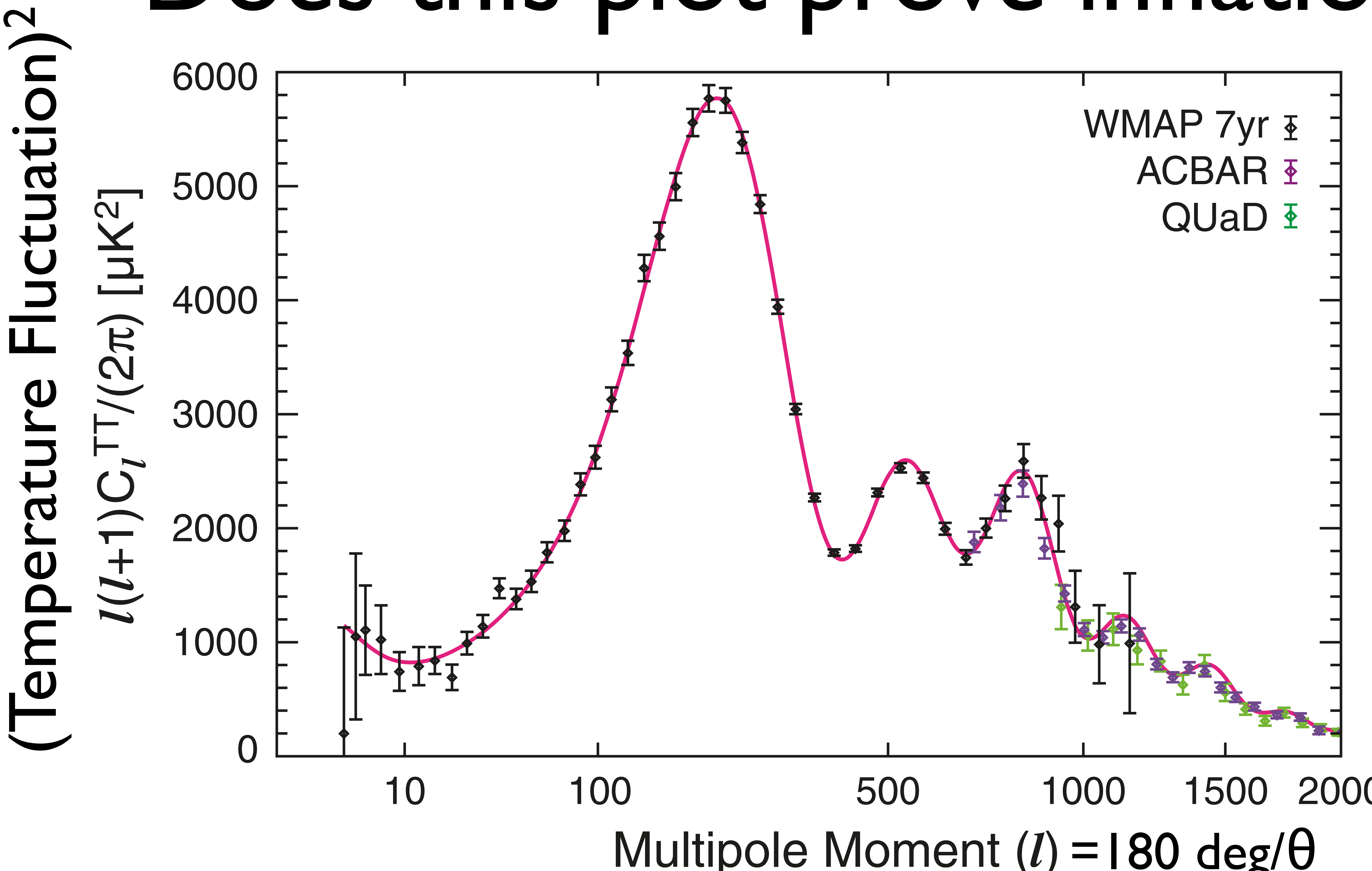
- Did inflation really occur?

Question

- Did inflation* really occur?

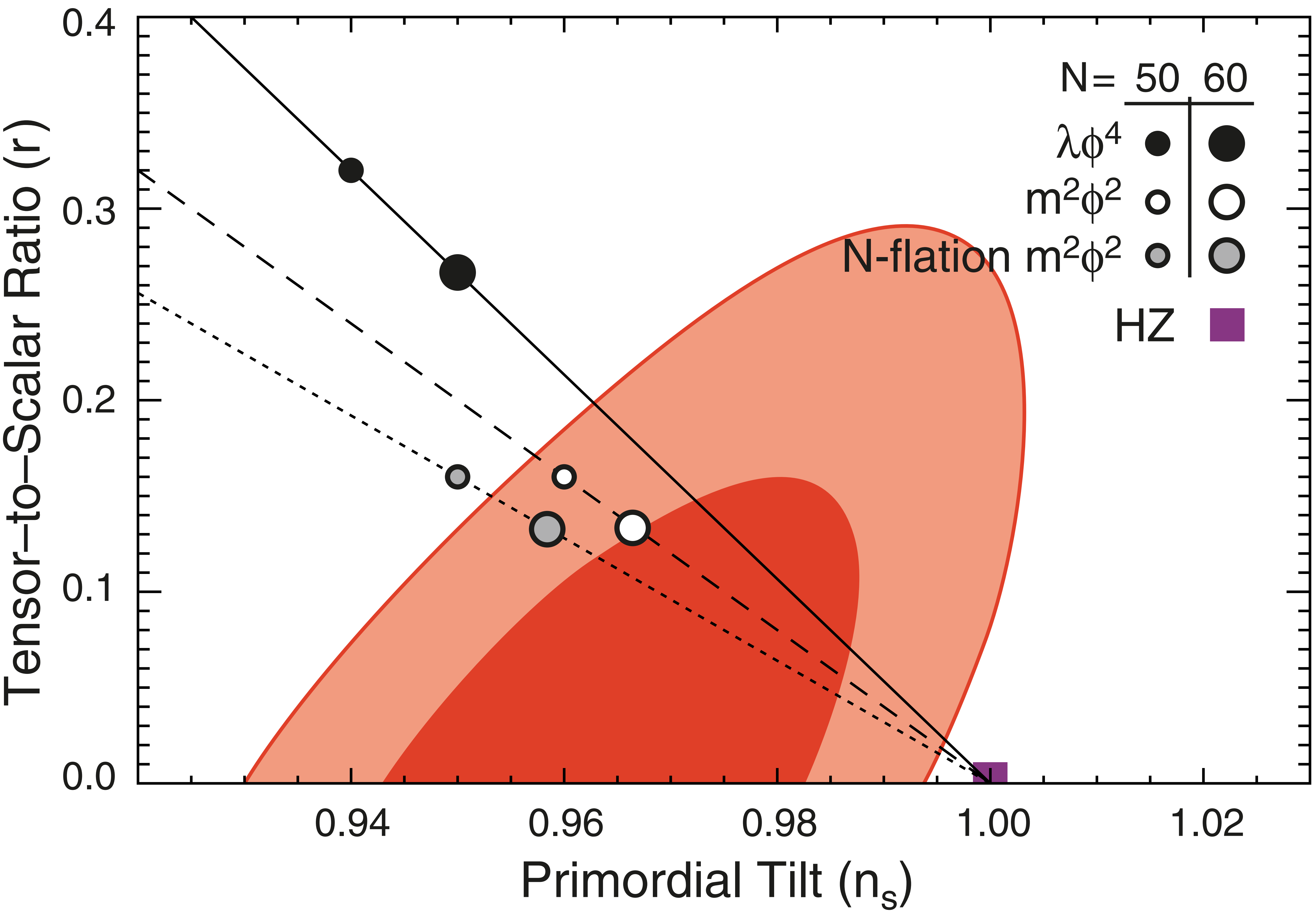
* By "inflation," I mean a period of the early universe during which the expansion of the universe accelerates. (Quasi-exponential expansion.)

Does this plot prove inflation?



Komatsu et al. (2011)

Inflation looks good (in 2-point function)



- $P_{scalar}(k) \sim k^{\mathbf{ns}-4}$
  - $\mathbf{n_s=0.968\pm0.012}$ (68%CL; WMAP7+BAO+H_0)
- $\mathbf{r}=4P_{tensor}(k)/P_{scalar}(k)$
  - $\mathbf{r < 0.24}$ (95%CL; WMAP7+BAO+H_0)

Motivation

- Can we falsify inflation?

Falsifying “inflation”

- We still need inflation to explain the flatness problem!
  - (Homogeneity problem can be explained by a bubble nucleation.)
- However, the observed fluctuations may come from different sources.
- So, what I ask is, “can we rule out inflation as a mechanism for generating the observed fluctuations?”

First Question:

- Can we falsify **single-field** inflation?

**I will not be talking about multi-field inflation today: for potentially ruling out multi-field inflation, see Sugiyama, Komatsu & Futamase, PRL, 106, 251301 (2011)*

An Easy One: Adiabaticity

- Single-field inflation = One degree of freedom.
  - Matter and radiation fluctuations originate from a single source.

$$\mathcal{S}_{c,\gamma} \equiv \underbrace{\frac{\delta\rho_c}{\rho_c}}_{\text{Cold Dark Matter}} - \underbrace{\frac{3\delta\rho_\gamma}{4\rho_\gamma}}_{\text{Photon}} = 0$$

*A factor of 3/4 comes from the fact that, in thermal equilibrium, $\rho_c \sim \rho_\gamma^{3/4}$

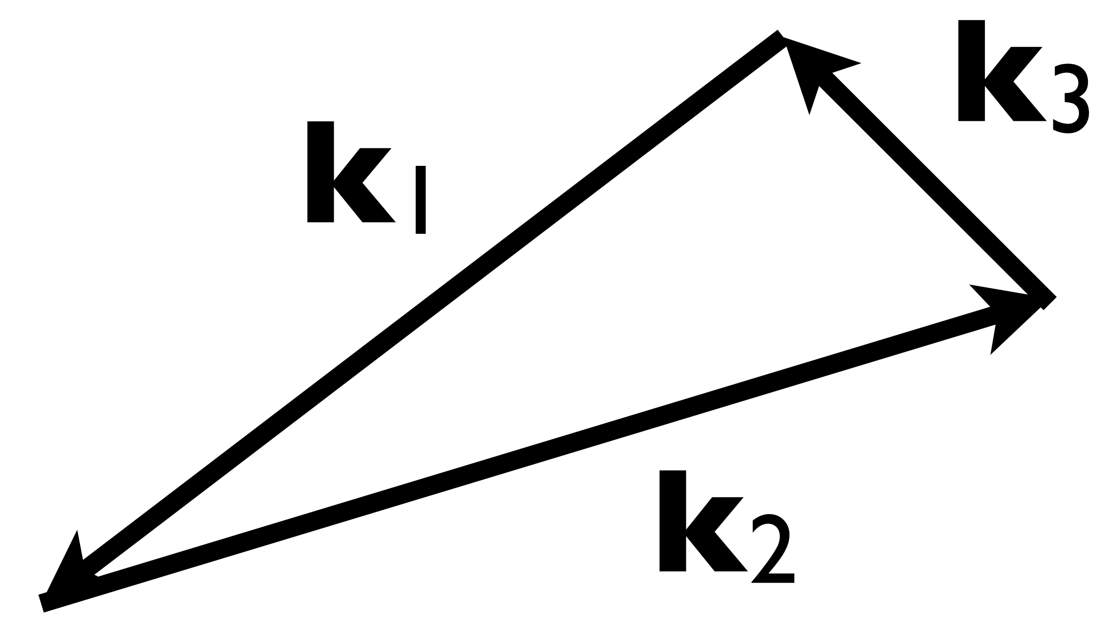
Non-adiabatic Fluctuations

- Detection of non-adiabatic fluctuations immediately rule out single-field inflation models.

The current CMB data are consistent with adiabatic fluctuations:

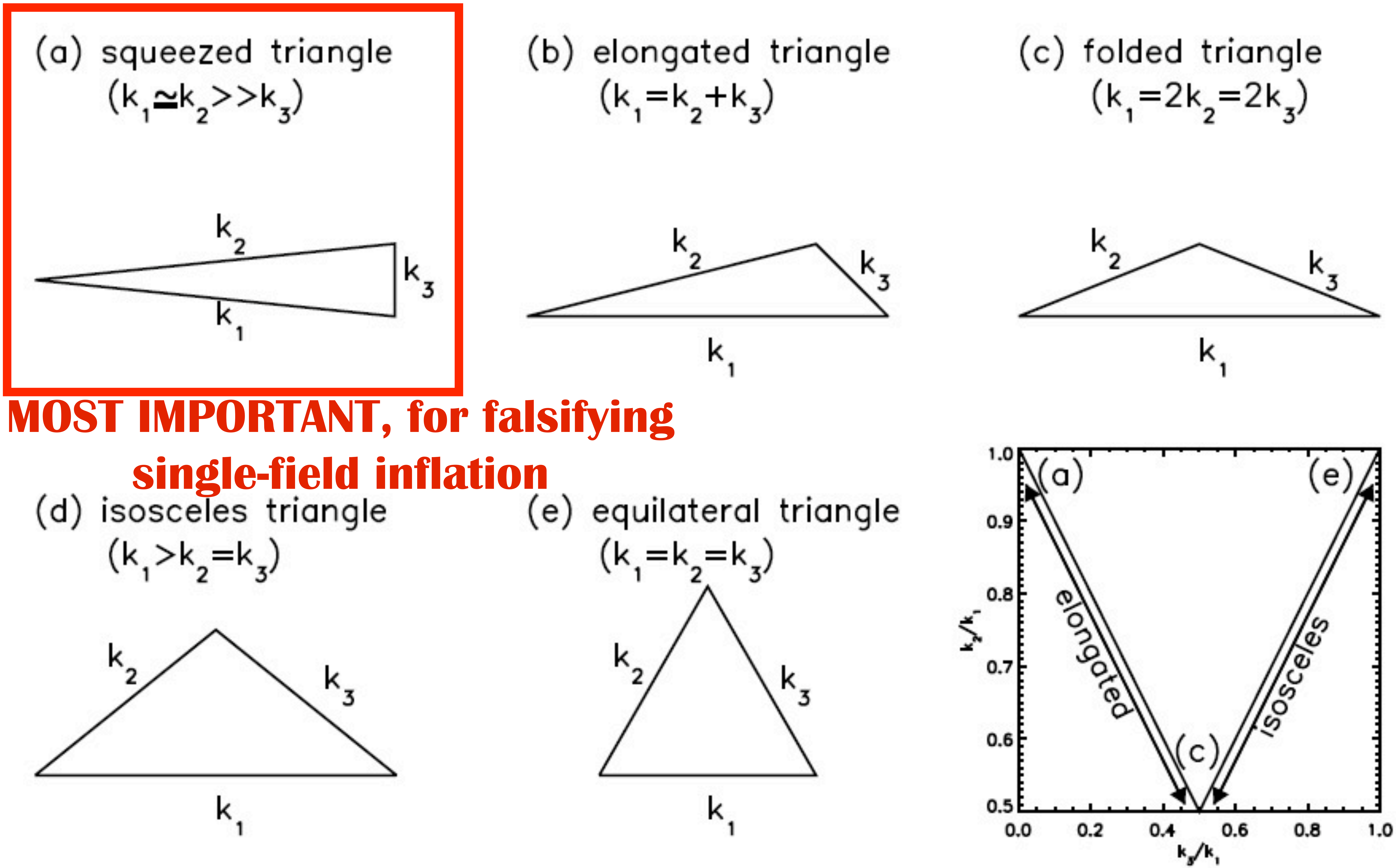
$$\frac{|\delta\rho_c/\rho_c - 3\delta\rho_\gamma/(4\rho_\gamma)|}{\frac{1}{2}[\delta\rho_c/\rho_c + 3\delta\rho_\gamma/(4\rho_\gamma)]} < 0.09 \quad \text{(95\% CL)}$$

Let's use 3-point function



- Three-point function (bispectrum)
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3} \rangle = (\text{amplitude}) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,b(k_1,k_2,k_3)$

  model-dependent function


(a) squeezed triangle
(k1≃k2>>k3)
MOST IMPORTANT, for falsifying
single-field inflation
(b) elongated triangle
(k1=k2+k3)
(c) folded triangle
(k1=2k2=2k3)
(d) isosceles triangle
(k1>k2=k3)
(e) equilateral triangle
(k1=k2=k3)
elongated
isosceles
k2/k1
k3/k1
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0

Curvature Perturbation

- In the gauge where the energy density is uniform, $\delta\rho=0$, the metric on super-horizon scales ($k \ll aH$) is written as

$$ds^2 = -N^2(x,t)dt^2 + a^2(t)e^{2\zeta(x,t)}dx^2$$

  - We shall call ζ the "curvature perturbation."
  - This quantity is independent of time, $\zeta(x)$, on super-horizon scales for single-field models.
  - The lapse function, $N(x,t)$, can be found from the Hamiltonian constraint.

Action

- Einstein's gravity + a canonical scalar field:
- $S=(1/2)\int d^4x\sqrt{-g}\,[R-(\partial\Phi)^2-2V(\Phi)]$

Maldacena (2003)

Quantum-mechanical Computation of the Bispectrum

$$\langle \zeta^3(\bar{t}) \rangle = -i \int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \langle 0 | [\zeta^3(\bar{t}), H_I^{(3)}(t')] | 0 \rangle$$

$$\partial^2 \chi = \frac{\dot{\phi}^2}{2\dot{\rho}^2}\dot{\zeta}$$
$$H \equiv \dot{\rho}$$

$$S_{\rm int}^{(3)} = \int \frac{1}{4}\frac{\dot{\phi}^4}{\dot{\rho}^4}[e^{3\rho}\dot{\zeta}^2\zeta + e^{\rho}(\partial\zeta)^2\zeta] - \frac{\dot{\phi}^2}{\dot{\rho}^2}e^{3\rho}\dot{\zeta}\partial_i\chi\partial_i\zeta + \\ - \frac{1}{16}\frac{\dot{\phi}^6}{\dot{\rho}^6}e^{3\rho}\dot{\zeta}^2\zeta + \frac{\dot{\phi}^2}{\dot{\rho}^2}e^{3\rho}\dot{\zeta}\zeta^2\frac{d}{dt}\left[\frac{1}{2}\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{4}\frac{\dot{\phi}^2}{\dot{\rho}^2}\right] + \frac{1}{4}\frac{\dot{\phi}^2}{\dot{\rho}^2}e^{3\rho}\partial_i\partial_j\chi\partial_i\partial_j\chi\zeta \\ + f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1$$

Initial Vacuum State

$$\zeta_{\mathbf{k}}(t) = u_k(t)a_{\mathbf{k}} + u_k^*(t)a^\dagger_{-\mathbf{k}}$$

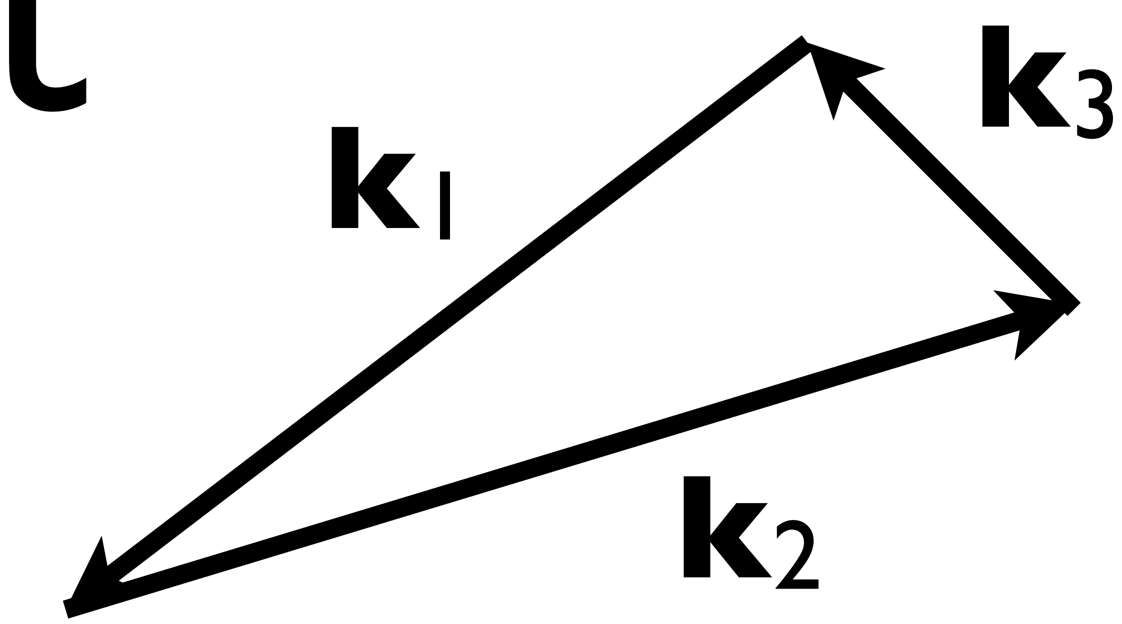
- Bunch-Davies vacuum, $a_{\mathbf{k}}|0\rangle=0$ with

$$u_k(\eta) = \frac{H^2}{\dot{\phi}}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}$$

[η: conformal time]

Maldacena (2003)

Result



- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle =$ (amplitude) x $(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$

- $b(k_1,k_2,k_3)= \dfrac{\dot\rho_*^4}{\dot\phi_*^4}\dfrac{H_*^4}{M_{pl}^4}\dfrac{1}{\prod_i(2k_i^3)}$

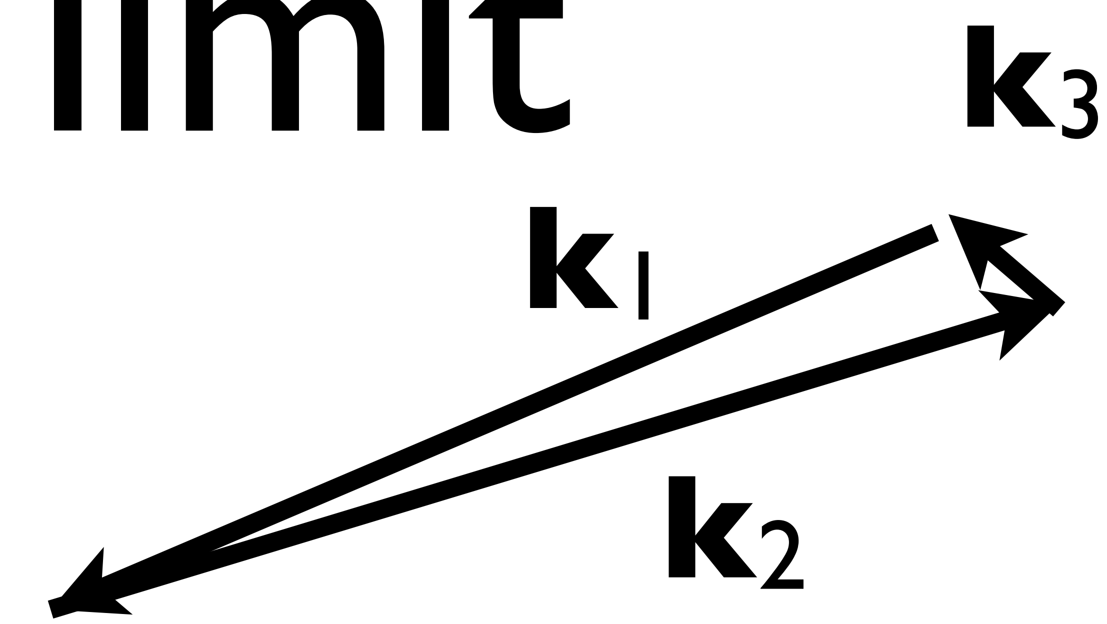
$$\boxed{H \equiv \dot\rho}$$

$$\times\left\{2\frac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}\sum_i k_i^3 + \frac{\dot\phi_*^2}{\dot\rho_*^2}\left[\frac{1}{2}\sum_i k_i^3 + \frac{1}{2}\sum_{i\neq j} k_i k_j^2 + 4\frac{\sum_{i>j} k_i^2 k_j^2}{k_t}\right]\right\}$$

Complicated? But...

Maldacena (2003)

Taking the squeezed limit
($k_3 << k_1 \approx k_2$)



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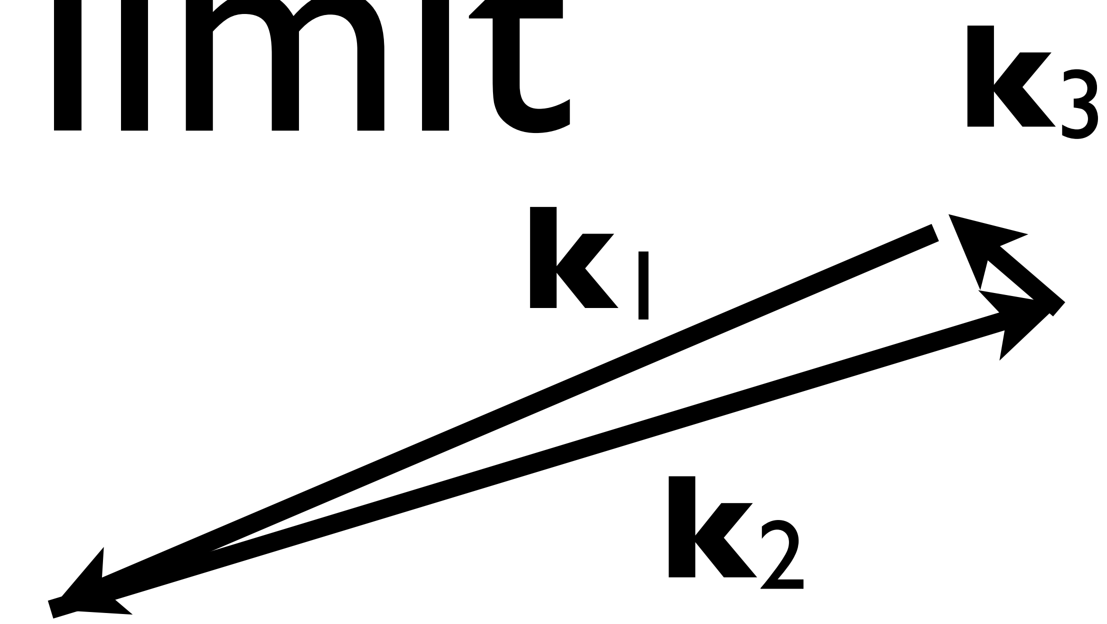
- $b(k_1,k_1,k_3 \to 0) = \dfrac{\dot\rho_*^4}{\dot\phi_*^4}\dfrac{H_*^4}{M_{pl}^4}\dfrac{1}{\prod_i (2k_i^3)}$

$$\times\left\{2\frac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}\sum_i k_i^3 + \frac{\dot\phi_*^2}{\dot\rho_*^2}\left[\frac{1}{2}\sum_i k_i^3 + \frac{1}{2}\sum_{i\neq j} k_i k_j^2 + 4\frac{\sum_{i>j} k_i^2 k_j^2}{k_t}\right]\right\}$$

$2k_1^3$ k_1^3 k_1^3 $2k_1^3$

Maldacena (2003)

Taking the squeezed limit ($k_3 \ll k_1 \approx k_2$)



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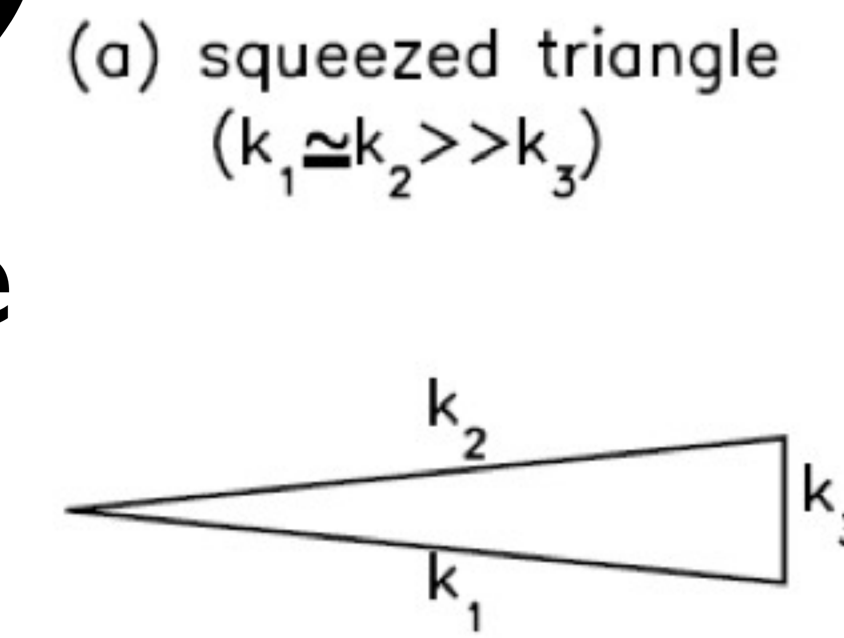
- $b(k_1,k_1,k_3\text{->}0) = \frac{\dot\rho_*^4}{\dot\phi_*^4}\frac{H_*^4}{M_{pl}^4}\, 2 \underbrace{\left[\frac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}+\frac{\dot\phi_*^2}{\dot\rho_*^2}\right]}_{=1-n_s} \frac{1}{k_1{}^3k_3{}^3}$

$$= (1-n_s)P_\zeta(k_1)P_\zeta(k_3)$$

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)



- For **ANY** single-field models*, the bispectrum in the squeezed limit ($k_3 << k_1 \approx k_2$) is given by
  - $B_\zeta(\mathbf{k}_1, \mathbf{k}_1, \mathbf{k}_3 \to 0) = (1 - n_s) \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times P_\zeta(k_1) P_\zeta(k_3)$

* for which the single field is solely responsible for driving inflation ***and*** generating observed fluctuations.

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

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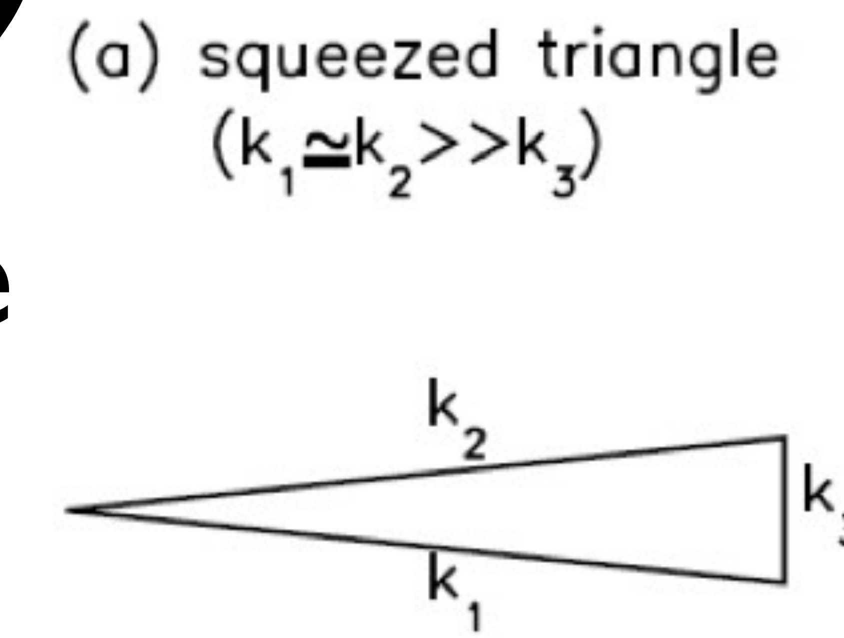

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$$\boxed{\frac{6}{5} f_{NL} \equiv \frac{B_\zeta(k_1,k_2,k_3)}{P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)P_\zeta(k_1)}}$$

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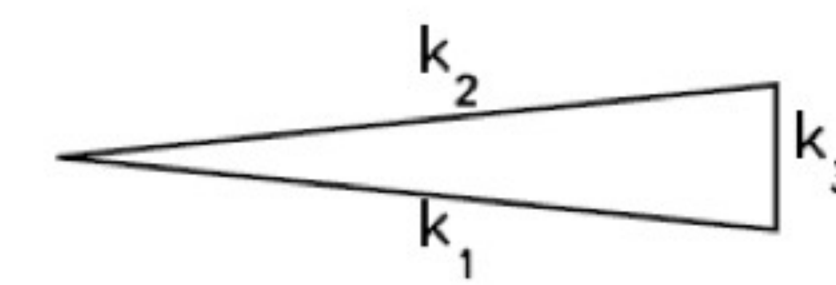
Single-field Theorem (Consistency Relation)

(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)



- For **ANY** single-field models*, the bispectrum in the squeezed limit ($k_3 << k_1 \approx k_2$) is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_1,\mathbf{k}_3\text{->}0) = (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$
  - Therefore, all single-field models predict $f_{NL} \approx (5/12)(1-n_s)$.
  - With the current limit $n_s=0.96$, $\underline{f_{NL}\text{ is predicted to be }0.017}$.

* for which the single field is solely responsible for driving inflation ***and*** generating observed fluctuations.

Limits on f_{NL}

$$\boxed{\frac{6}{5} f_{NL} \equiv \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1)}}$$

When f_{NL} is independent of wavenumbers, it is called the "**local type**."

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Komatsu&Spergel (2001)

Acoustic signatures in the primary microwave background bispectrum

Eiichiro Komatsu* and David N. Spergel†
Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey 08544

(Received 25 October 2000; published 13 February 2001)

If the primordial fluctuations are non-Gaussian, then this non-Gaussianity will be apparent in the cosmic microwave background (CMB) sky. With their sensitive all-sky observation, MAP and Planck satellites should be able to detect weak non-Gaussianity in the CMB sky. On a large angular scale, there is a simple relationship between the CMB temperature and the primordial curvature perturbation: $\Delta T/T = -\Phi/3$. On smaller scales, however, the radiation transfer function becomes more complex. In this paper, we present the angular bispectrum of the primary CMB anisotropy that uses the full transfer function. We find that the bispectrum has a series of acoustic peaks that change a sign and a period of acoustic oscillations is twice as long as that of the angular power spectrum. Using a single non-linear coupling parameter to characterize the amplitude of the bispectrum, we estimate the expected signal-to-noise ratio for COBE, MAP, and Planck experiments. In order to detect the primary CMB bispectrum by each experiment, we find that the coupling parameter should be larger than 600, 20, and 5 for COBE, MAP, and Planck experiments, respectively. Even for the ideal noise-free and infinitesimal thin-beam experiment, the parameter should be larger than 3. We have included effects from the cosmic variance, detector noise, and foreground sources in the signal-to-noise estimation. Since the simple inflationary scenarios predict that the parameter is an order of 0.01, the detection of the primary bispectrum by any kind of experiments should be problematic for those scenarios. We compare the sensitivity of the primary bispectrum to the primary skewness and conclude that, when we can compute the predicted form of the bispectrum, it becomes a ''matched filter'' for detecting the non-Gaussianity in the data and a much more powerful tool than the skewness. For example, we need the coupling parameter of larger than 800, 80, 70, and 60 for each relevant experiment in order to detect the primary skewness. We also show that MAP and Planck can separate the primary bispectrum from various secondary bispectra on the basis of the shape difference. The primary CMB bispectrum is a test of the inflationary scenario and also a probe of the non-linear physics in the very early universe.

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Komatsu et al. (2011)

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$$\boxed{\frac{6}{5} f_{NL} \equiv \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1)}}$$

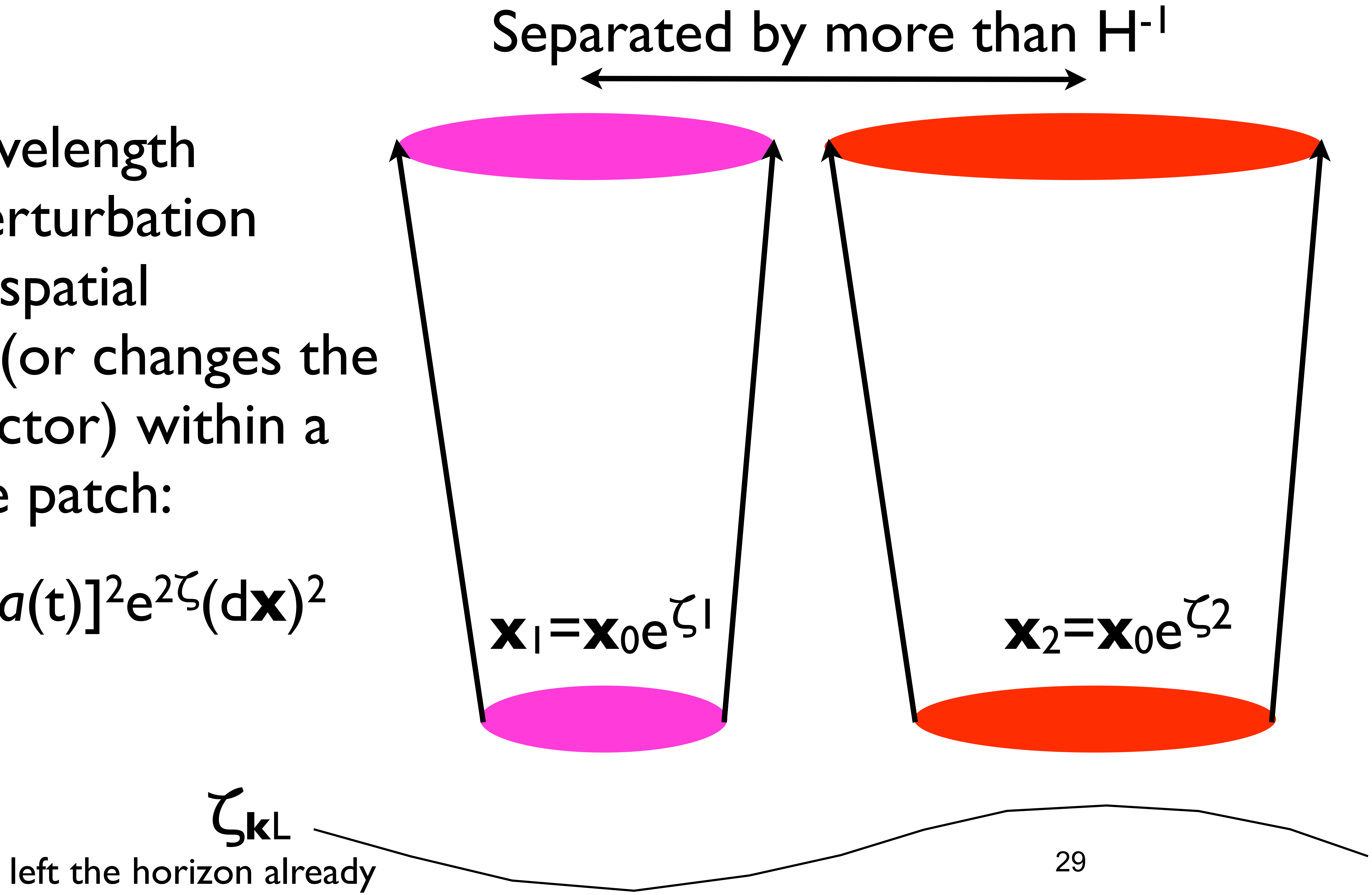
- **f_{NL} = 32 ± 21** (68%C.L.) from WMAP 7-year data
  - Planck's CMB data is expected to yield Δf_{NL}=5.
- **f_{NL} = 27 ± 16** (68%C.L.) from WMAP 7-year data combined with the limit from the large-scale structure (by Slosar et al. 2008)
  - Future large-scale structure data are expected to yield Δf_{NL}=1.

Understanding the Theorem

- First, the squeezed triangle correlates one very long-wavelength mode, k_L ($=k_3$), to two shorter wavelength modes, k_S ($=k_1 \approx k_2$):
  - $\langle \zeta_{\mathbf{k}1} \zeta_{\mathbf{k}2} \zeta_{\mathbf{k}3} \rangle \approx \langle (\zeta_{\mathbf{k}S})^2 \zeta_{\mathbf{k}L} \rangle$
- Then, the question is: "why should $(\zeta_{\mathbf{k}S})^2$ ever care about $\zeta_{\mathbf{k}L}$?"
  - The theorem says, "it doesn't care, if $\zeta_{\mathbf{k}}$ is exactly scale invariant."

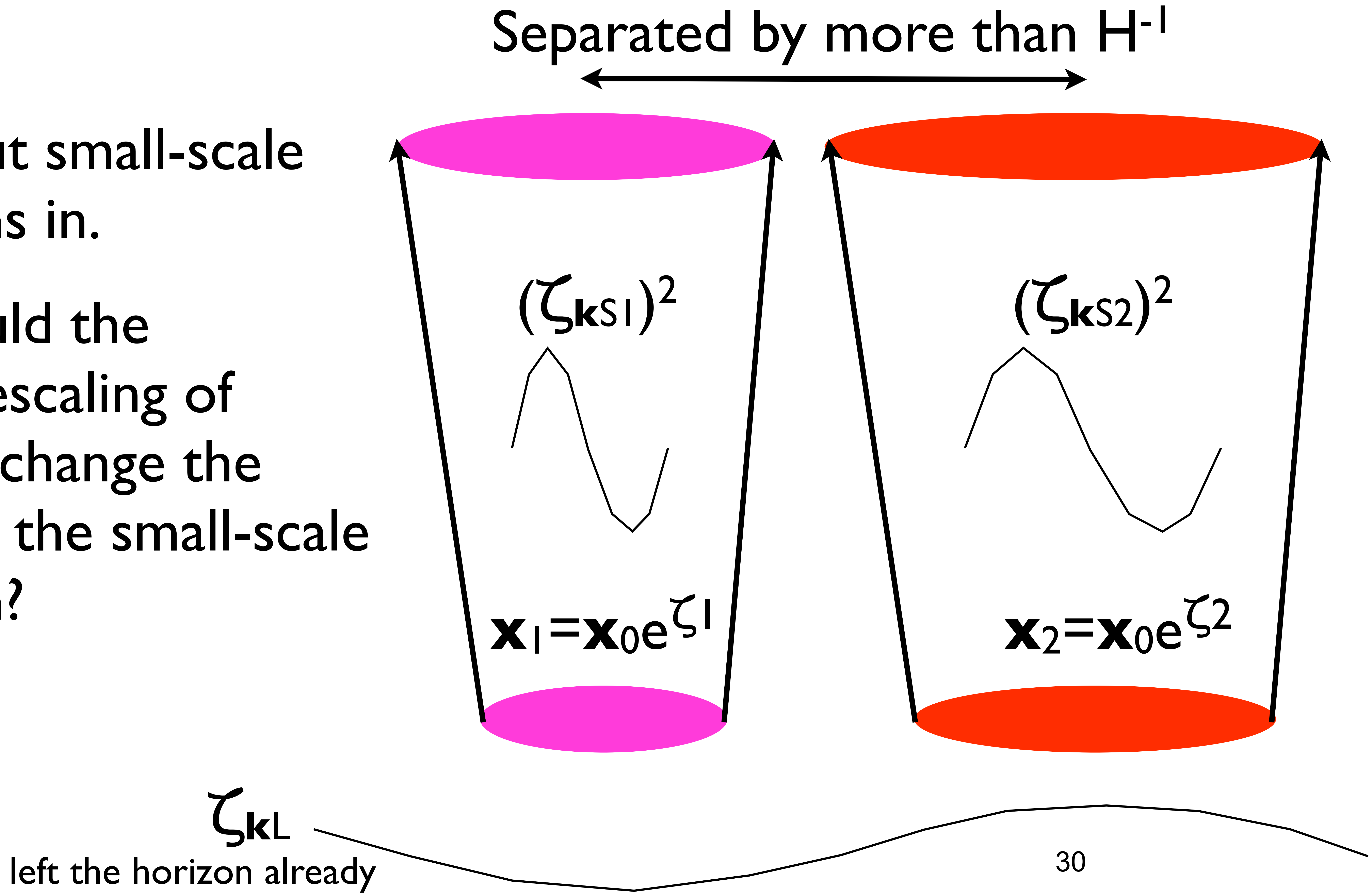
$\zeta_{\mathbf{k}L}$ rescales coordinates

- The long-wavelength curvature perturbation rescales the spatial coordinates (or changes the expansion factor) within a given Hubble patch:
  - $ds^2 = -dt^2 + [a(t)]^2 e^{2\zeta} (d\mathbf{x})^2$



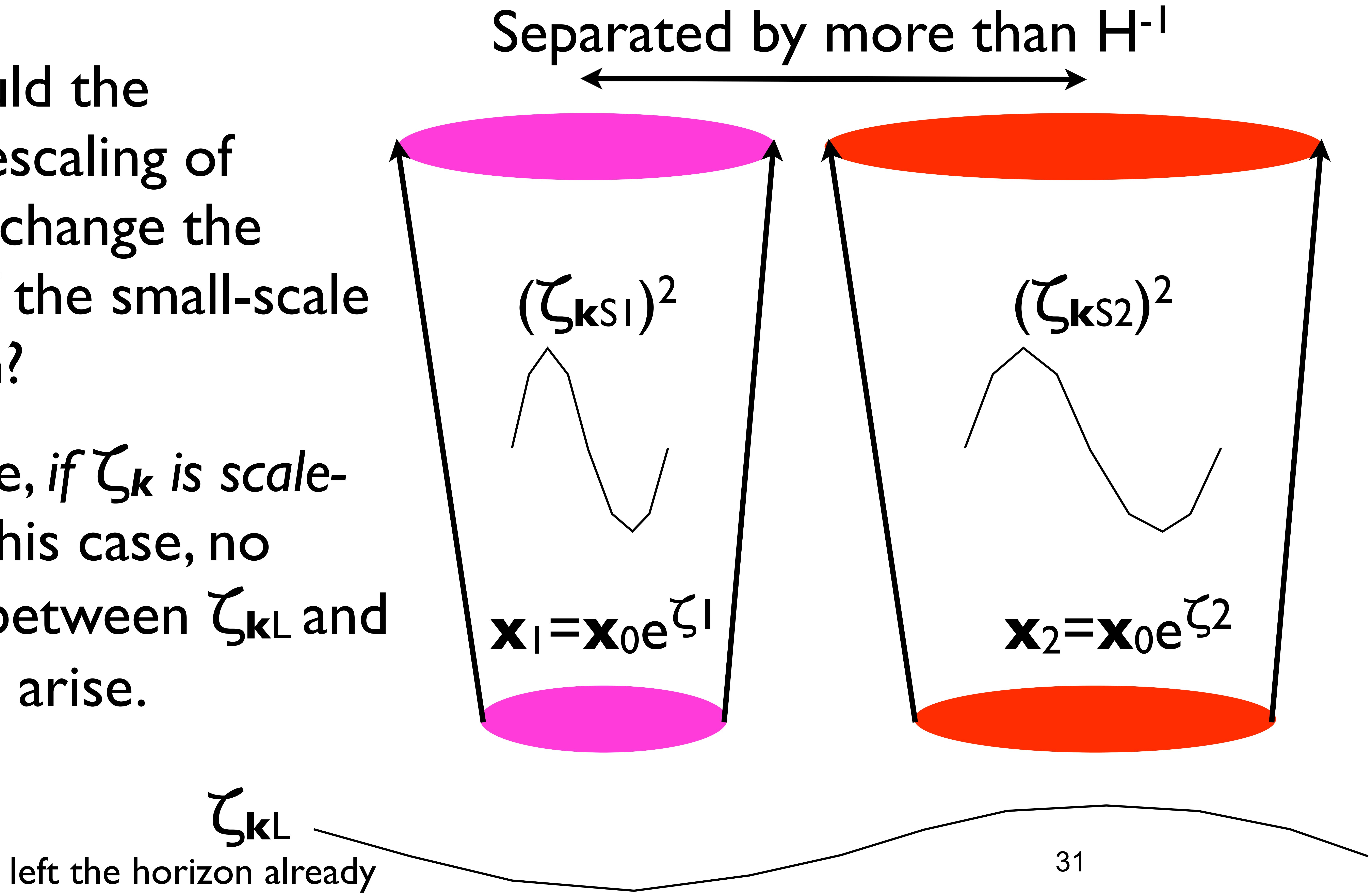
$\zeta_{\mathbf{k}L}$ rescales coordinates

- Now, let's put small-scale perturbations in.
- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?



$\zeta_{\mathbf{k}L}$ rescales coordinates

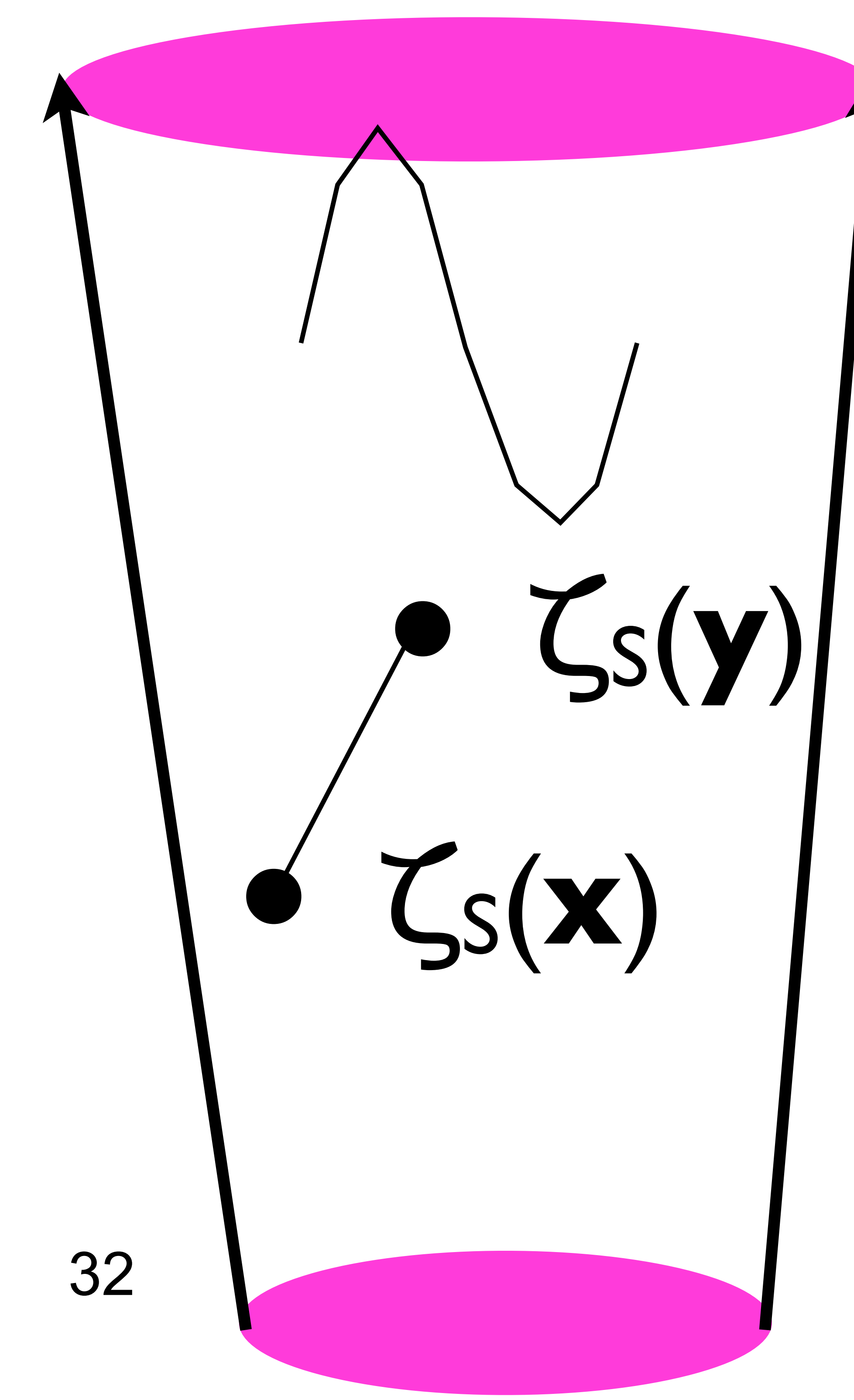
- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?
- A. No change, *if $\zeta_{\mathbf{k}}$ is scale-invariant*. In this case, no correlation between $\zeta_{\mathbf{k}L}$ and $(\zeta_{\mathbf{k}S})^2$ would arise.



Creminelli & Zaldarriaga (2004); Cheung et al. (2008)

Real-space Proof

- The 2-point correlation function of short-wavelength modes, $\xi = \langle \zeta_S(\mathbf{x}) \zeta_S(\mathbf{y}) \rangle$, within a given Hubble patch can be written in terms of its vacuum expectation value (in the absence of ζ_L), ξ_0, as:
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\, [d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\zeta_L]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\, [d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\ln|\mathbf{x}-\mathbf{y}|]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\, (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)$



$$
\begin{aligned}
\text{3-pt func.} &= \langle (\zeta_S)^2 \zeta_L \rangle = \langle \xi_{\zeta L} \zeta_L \rangle \\
&= (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)\langle \zeta_L^2 \rangle
\end{aligned}
$$

This is great, but...

- The proof relies on the following Taylor expansion:
  - $<\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_{\zeta_L} = <\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_0 + \zeta_L\,[d<\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_0/d\zeta_L]$
- Perhaps it is interesting to show this explicitly using the in-in formalism.
  - Such a calculation would shed light on the limitation of the above Taylor expansion.
  - Indeed it did - we found a non-trivial "counter-example" (more later)

An Idea

- How can we use the in-in formalism to compute the two-point function of short modes, given that there is a long mode, $\langle\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})\rangle_{\zeta_L}$?
- Here it is!

$$\left\langle\zeta_S^2(\bar{t})\right\rangle_{\zeta_L} = -i\int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \left\langle 0\left|\left[\zeta_S^2(\bar{t}), H_I^{(3)}(t')\right]\right|0\right\rangle$$

Ganc & Komatsu, JCAP, 12, 009 (2010)

Long-short Split of H_I

$$\langle \zeta_S^2(\bar{t}) \rangle_{\zeta_L} = -i \int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \langle 0 | [\zeta_S^2(\bar{t}), H_I^{(3)}(t')] | 0 \rangle$$

- Inserting $\zeta=\zeta_L+\zeta_S$ into the cubic action of a scalar field, and retain terms that have one ζ_L and two ζ_S's.

$$S_{\rm int}^{(3)} = \int d^4x \left[\left(\frac{1}{4}\frac{\dot{\phi}_0^4}{H^4} - \frac{1}{16}\frac{\dot{\phi}_0^6}{H^6} \right) a^3 \zeta_L \dot{\zeta}_S^2 + \frac{1}{4}\frac{\dot{\phi}_0^4}{H^4} a \zeta_L (\partial\zeta_S)^2 - \frac{\dot{\phi}_0^4}{2H^4} a^3 \dot{\zeta}_S \partial_i \zeta_S \partial_i \partial^{-2} \dot{\zeta}_L + \right.$$
$$+ \frac{1}{16}\frac{\dot{\phi}_0^6}{H^6} a^3 \partial_i \partial_j \partial^{-2} \dot{\zeta}_S \, \partial_i \partial_j \partial^{-2} \dot{\zeta}_S \, \zeta_L + 2 \frac{\dot{\phi}_0^2}{H^2} a^3 \zeta_L \frac{d}{dt}\left[\frac{1}{2}\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} + \frac{1}{4}\frac{\dot{\phi}_0^2}{H^2} \right] \dot{\zeta}_S \zeta_S$$
$$\left. - f(\zeta) \frac{\delta L_0}{\delta \zeta_S} \right],$$

Result

$$\langle \zeta_{S,\mathbf{k}_1}\zeta_{S,\mathbf{k}_2}\rangle_{\zeta_{\mathbf{k}_3}} = \zeta_{L,\mathbf{k}_1+\mathbf{k}_2}\left[K + \left(\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} + \frac{1}{2}\frac{\dot{\phi}_0^2}{H^2}\right)P(k_1)\right]$$

- where

$$K \equiv i u_{k_1}^2(\bar{\eta})\int_{-\infty(1-i\epsilon)}^{\bar{\eta}} d\eta\left[\frac{1}{2}\frac{\dot{\phi}_0^4}{H^4}a^2 u_{k_1}^{\prime *2}(\eta) + \frac{1}{2}\frac{\dot{\phi}_0^4}{H^4}a^2 k_1^2 u_{k_1}^{*2}(\eta) + \right.$$
$$\left. + 2\frac{\dot{\phi}_0^2}{H^2}a^3\frac{d}{dt}\left(\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} + \frac{1}{2}\frac{\dot{\phi}_0^2}{H^2}\right)u_{k_1}^{\prime *}(\eta)u_{k_1}^{*}(\eta)\right] + \text{c.c.}$$

Result

- Although this expression looks nothing like $(1-n_S)P(k_1)\zeta_{kL}$, we have verified that it leads to the known consistency relation for (i) slow-roll inflation, and (ii) power-law inflation.
- But, there was a curious case – Alexei Starobinsky's exact $n_S=1$ model.
  - If the theorem holds, we should get a vanishing bispectrum in the squeezed limit.

Starobinsky (2005)

Starobinsky's Model

- The famous Mukhanov-Sasaki equation for the mode function is

$$\frac{d^2u_k}{d\eta^2} + \left(k^2 - \frac{1}{z}\frac{d^2z}{d\eta^2}\right)u_k = 0$$

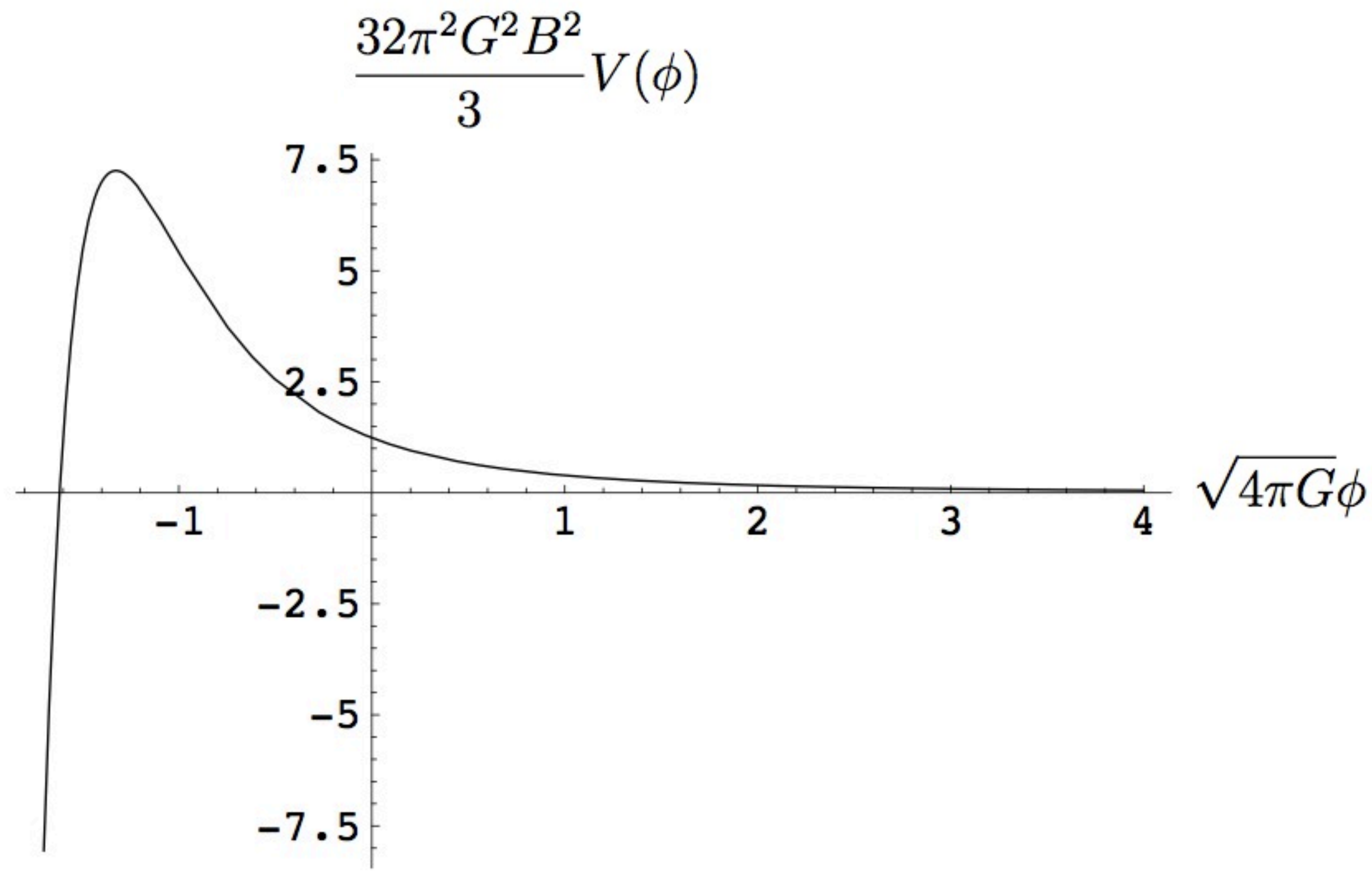
where

$$z = \frac{a\dot{\phi}}{H}$$

- The scale-invariance results when $\frac{1}{z}\frac{d^2z}{d\eta^2} = \frac{2}{\eta^2}$

So, let's write **$z=B/\eta$**

Starobinsky's Potential



- This potential is a one-parameter family; this particular example shows the case where inflation lasts very long: $\varphi_{end} \to \infty$

Result

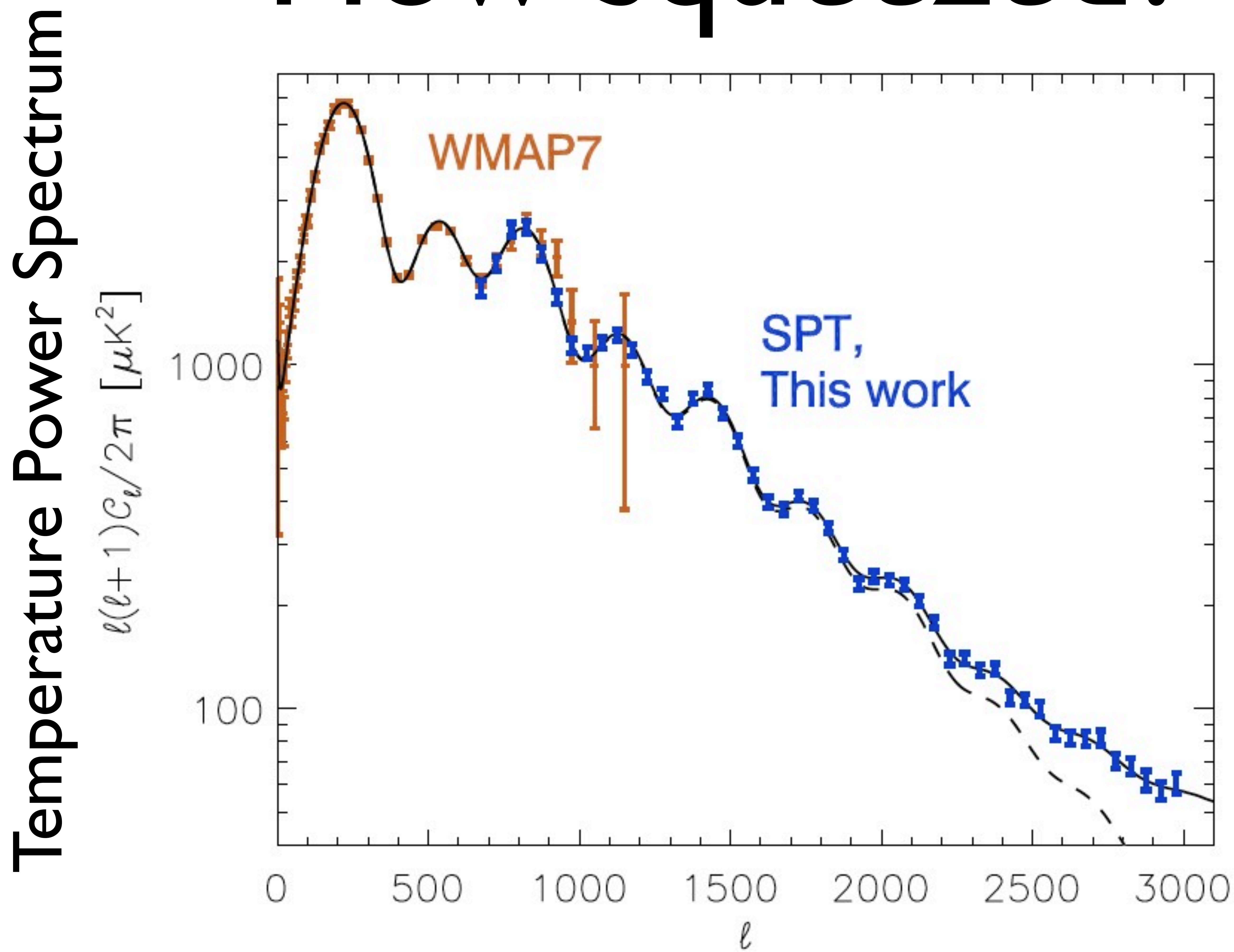
$$\langle \zeta_{S,\mathbf{k}_1}\zeta_{S,\mathbf{k}_2}\rangle_{\zeta_{\mathbf{k}_3}} = \zeta_{L,\mathbf{k}_1+\mathbf{k}_2} 4P(k_1)(k_1\eta_{\rm start})^2 e^{-\frac{1}{2}\phi_{\rm end}^2}$$

- **It does not vanish!**
- But, it approaches zero when $\Phi_{\rm end}$ is large, meaning the duration of inflation is very long.
  - In other words, this is a condition that **the longest wavelength that we observe, k_3, is far outside the horizon.**
  - In this limit, the bispectrum approaches zero.

Initial Vacuum State?

- What we learned so far:
  - The squeezed-limit bispectrum is proportional to $(1-n_S)P(k_1)P(k_3)$, provided that ζ_{k3} is far outside the horizon when k_1 crosses the horizon.
- What if the state that $\zeta_{\mathbf{k}3}$ sees is not a Bunch-Davies vacuum, but something else?
  - The exact squeezed limit (k_3->0) should still obey the consistency relation, but perhaps something happens when **k_3/k_1 is small but finite**.

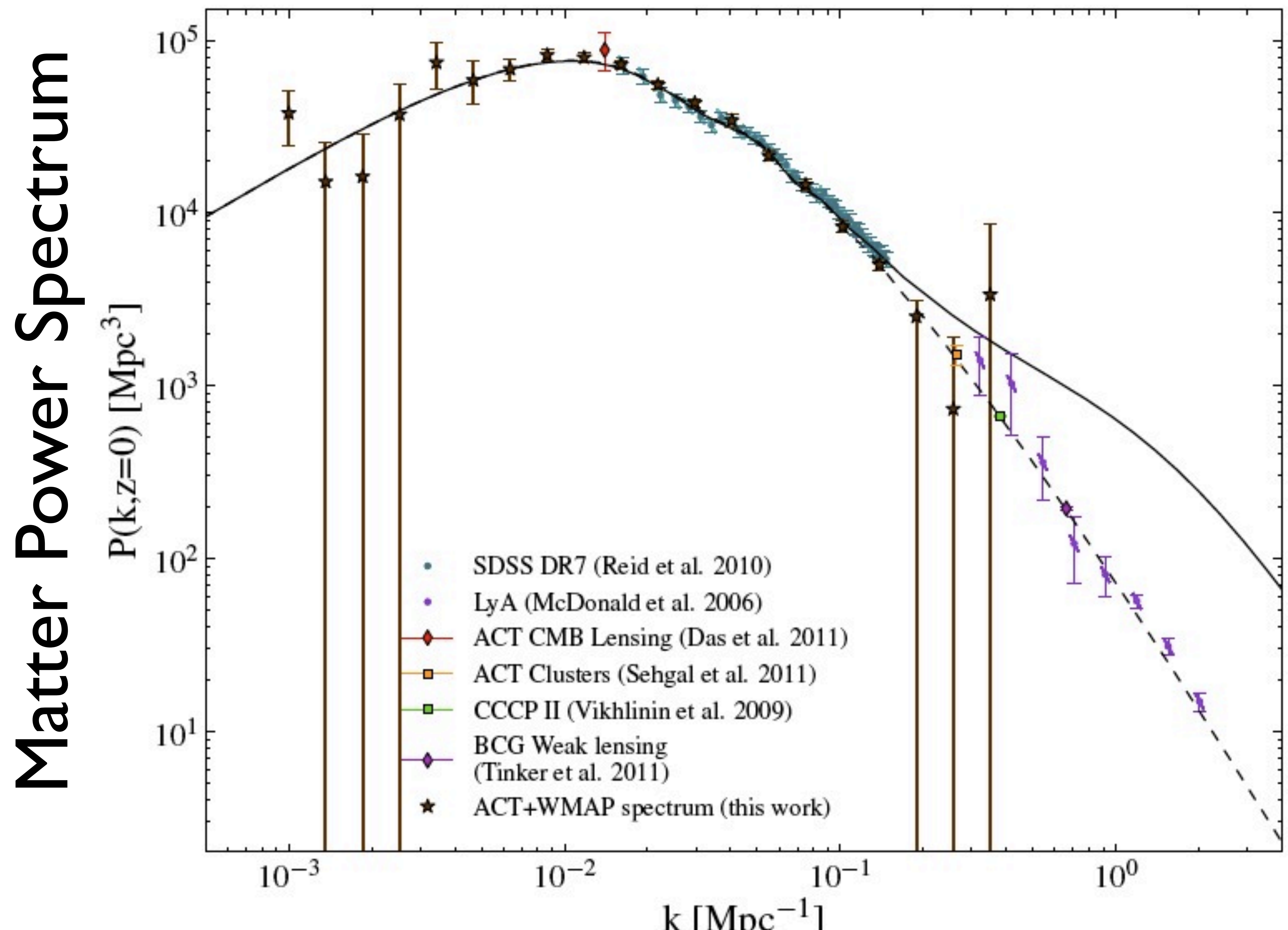
How squeezed?



Keisler et al. (2011)

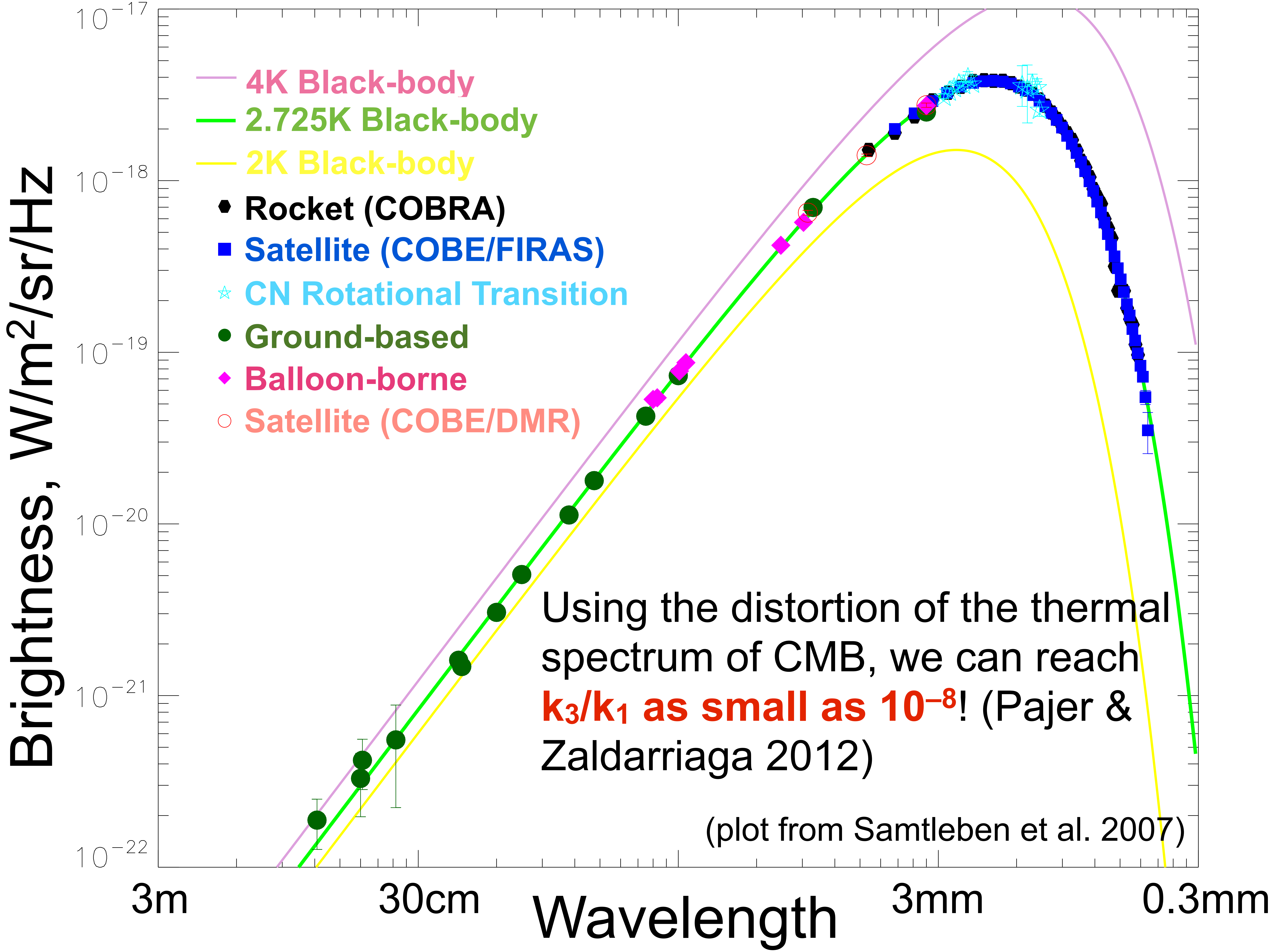
- With CMB, we can measure primordial modes in l=2–3000. Therefore, **k_3/k_1 can be as small as 1/1500**.

How squeezed?



Hlozek et al. (2011)

- With large-scale structure, we can measure primordial modes in $k=10^{-3}$–1 Mpc^{-1}. Therefore, **k_3/k_1 can be as small as 1/1000**.



Using the distortion of the thermal spectrum of CMB, we can reach **k_3/k_1 as small as 10^{-8}**! (Pajer & Zaldarriaga 2012)

(plot from Samtleben et al. 2007)

Back to in-in

$$\langle \zeta^3(t^*) \rangle = -i \int_{t_0}^{t^*} dt' \langle 0 \big| [\zeta^3(t^*), H_I(t')] \big| 0 \rangle$$

$$\Downarrow$$

$$B_\zeta(k_1, k_2, k_3) = 2i \frac{\dot{\phi}^4}{H^6} \sum_i \left(\frac{1}{k_i^2} \right) \tilde{u}_{k_1}(\bar{\eta}) \tilde{u}_{k_2}(\bar{\eta}) \tilde{u}_{k_3}(\bar{\eta}) \int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^3} {u'}^*_{k_1} {u'}^*_{k_2} {u'}^*_{k_3} + \text{c.c.}$$

- The Bunch-Davies vacuum: $u_k' \sim \eta e^{-ik\eta}$ (positive frequency mode)
  - The integral yields $1/(k_1+k_2+k_3)$ -> $1/(2k_1)$ in the squeezed limit

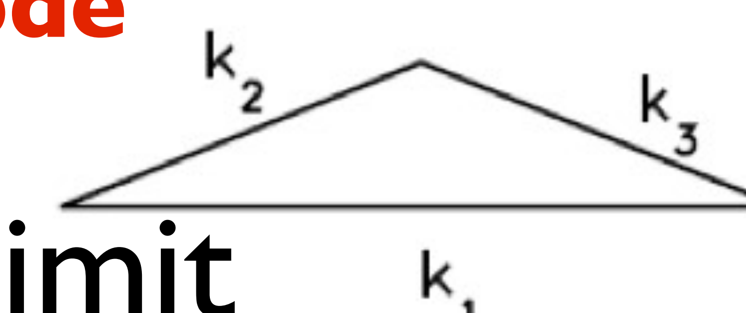
Back to in-in

$$\langle \zeta^3(t^*) \rangle = -i \int_{t_0}^{t^*} dt' \langle 0 | [\zeta^3(t^*), H_I(t')] | 0 \rangle$$

$$B_\zeta(k_1, k_2, k_3) = 2i \frac{\dot{\phi}^4}{H^6} \sum_i \left(\frac{1}{k_i^2} \right) \tilde{u}_{k_1}(\bar{\eta}) \tilde{u}_{k_2}(\bar{\eta}) \tilde{u}_{k_3}(\bar{\eta}) \int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^3} u'^*_{k_1} u'^*_{k_2} u'^*_{k_3} + \text{c.c.}$$

negative frequency mode

- Non-Bunch-Davies vacuum: $u_k' \sim \eta(A_k e^{-ik\eta} + B_k e^{+ik\eta})$
- The integral yields $1/(k_1-k_2+k_3)$, peaking in the folded limit



Chen et al. (2007); Holman & Tolley (2008)

- The integral yields $1/(k_1-k_2+k_3)$ -> $1/(2k_3)$ in the squeezed limit

Enhanced by k_1/k_3: this can be a big factor!

Agullo & Parker (2011)

Agullo & Parker (2011)

Enhanced Squeezed-limit Bispectrum

$$B_\zeta(k_1, k_2, k_3) \underset{k_3/k_1 \ll 1}{\longrightarrow} P_\zeta(k_1) P_\zeta(k_3) \left\{ (1 - n_s) + 4 \frac{\dot{\phi}^2}{H^2} \frac{k_1}{k_3} [1 - \cos(k_3 \eta_0)] \right\}$$

- The second term blows up as $k_1/k_3 \to 0$.
- Important consequences for observables!

An interesting possibility:

- What if $k_3\eta_0 = O(1)$?
- The squeezed bispectrum receives an enhancement of order $\varepsilon k_1/k_3$, which can be sizable.
- Most importantly, **the bispectrum grows faster than the local-form toward k_3/k_1 -> 0!**
  - $B_\zeta(k_1,k_2,k_3) \sim 1/k_3^3$ [Local Form]
  - $B_\zeta(k_1,k_2,k_3) \sim 1/k_3^{\mathbf{4}}$ [non-Bunch-Davies]
- This has an observational consequence – particularly a scale-dependent bias and distortion of CMB spectrum.

Power Spectrum of Galaxies

- Galaxies do not trace the underlying matter density fluctuations perfectly. They are **biased tracers**.
- "Bias" is operationally defined as
  - $\mathbf{b_{galaxy}{}^2(k) = \langle|\delta_{galaxy,k}|^2\rangle / \langle|\delta_{matter,k}|^2\rangle}$

Density-ζ Relation

- It is given by the Poisson equation:

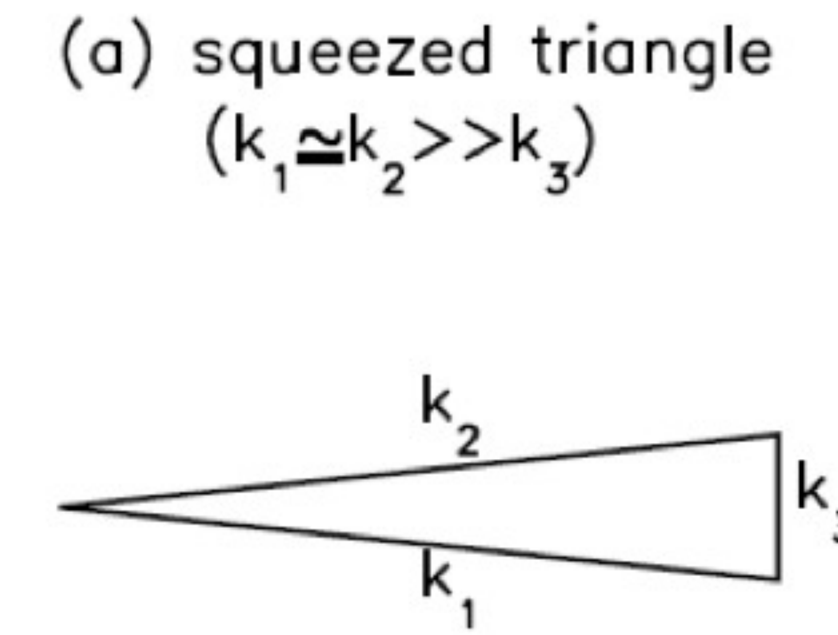
$$\delta_{m,\mathbf{k}}(z) = \frac{2k^2}{5H_0^2\Omega_m}\zeta_{\mathbf{k}}T(k)D(k,z)$$

$T(k) \to 1$ for $k \ll 10^{-2}$ Mpc^{-1}
$T(k) \to (\ln k)^2/k^4$ for $k \gg 10^{-2}$ Mpc^{-1}

$D(k,z)=1/(1+z)$ during the matter-dominated era

Positive ζ_k -> positive $\delta_{m,k}$!

Galaxy clustering modified by the squeezed limit



- The existence of long-wavelength ζ changes the small-scale power of δ_m.
- **A positive long-wavelength ζ -> more power on small scales**, for a positive squeezed-limit bispectrum.
- More power on small scales -> more galaxies formed.

Dalal et al. (2008); Matarrese & Verde (2008); Desjacques et al. (2011)

Scale-dependent Bias

$$\Delta b(k,R) = 2\frac{\mathcal{F}_R(k)}{\mathcal{M}_R(k)}\left[(b_1-1)\delta_c\right],$$

$M_R(k) \sim k^2$ for $k \ll 1/R$ and small for $k \gg 1/R$

$$\mathcal{F}_R(k) \approx \frac{1}{4\sigma_R^2 P_\zeta(k)}\int \frac{d^3k_1}{(2\pi)^3}\mathcal{M}_R^2(k_1)B_\zeta(k_1,k_1,k)$$

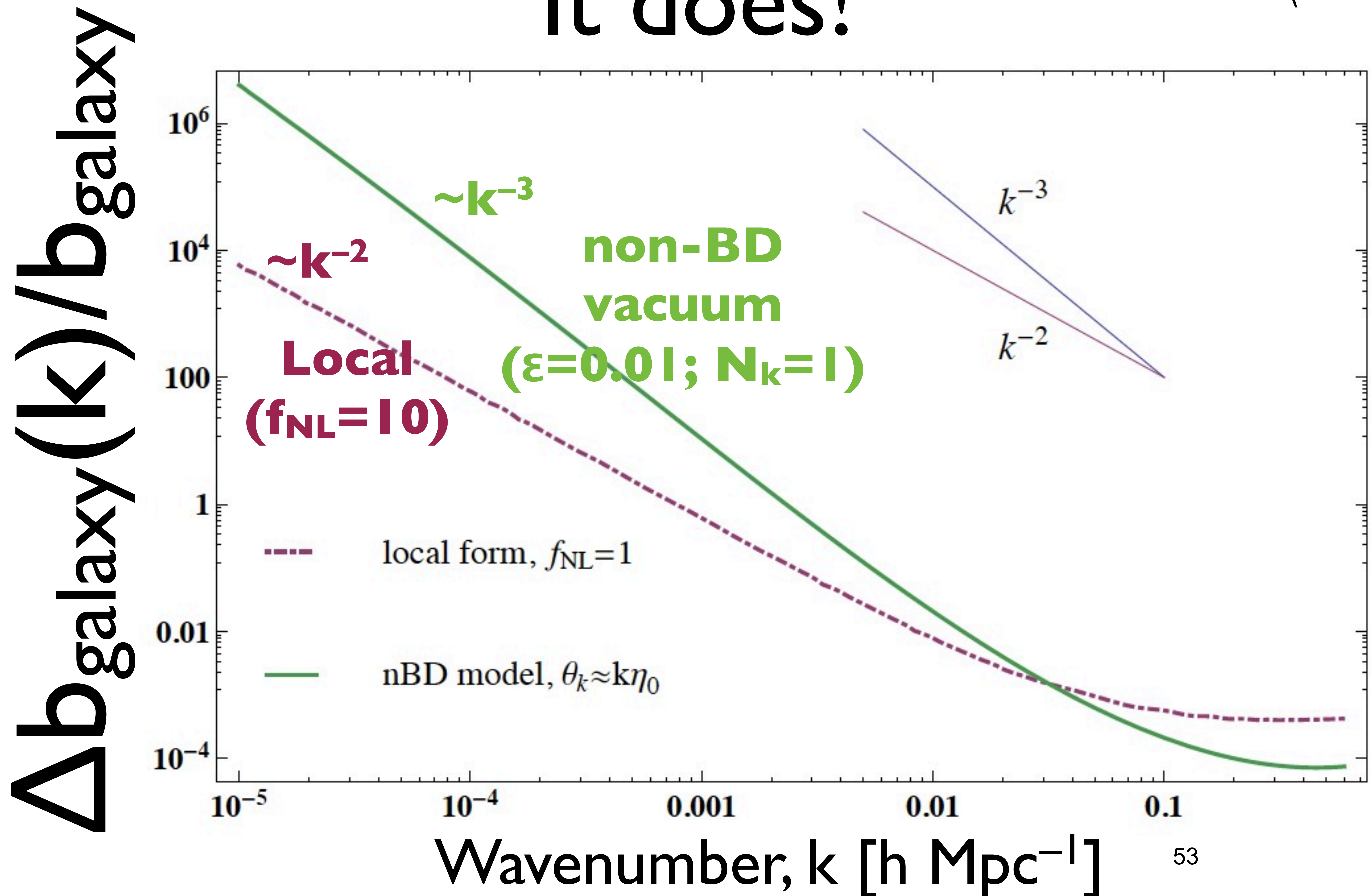
R is the linear size of dark matter halos

$$\sigma_R^2 \equiv \int \frac{d^3k}{(2\pi)^3}P_\zeta(k)\mathcal{M}_R^2(k)$$

- A rule-of-thumb:
  - For $B(k_1,k_2,k_3) \sim 1/k_3^{\mathbf{p}}$, the scale-dependence of the halo bias is given by $b(k) \sim 1/k^{\mathbf{p-1}}$
  - For a local-form (p=3), it goes like $b(k) \sim 1/k^2$
  - For a non-Bunch-Davies vacuum (p=4), would it go like $b(k) \sim 1/k^3$?

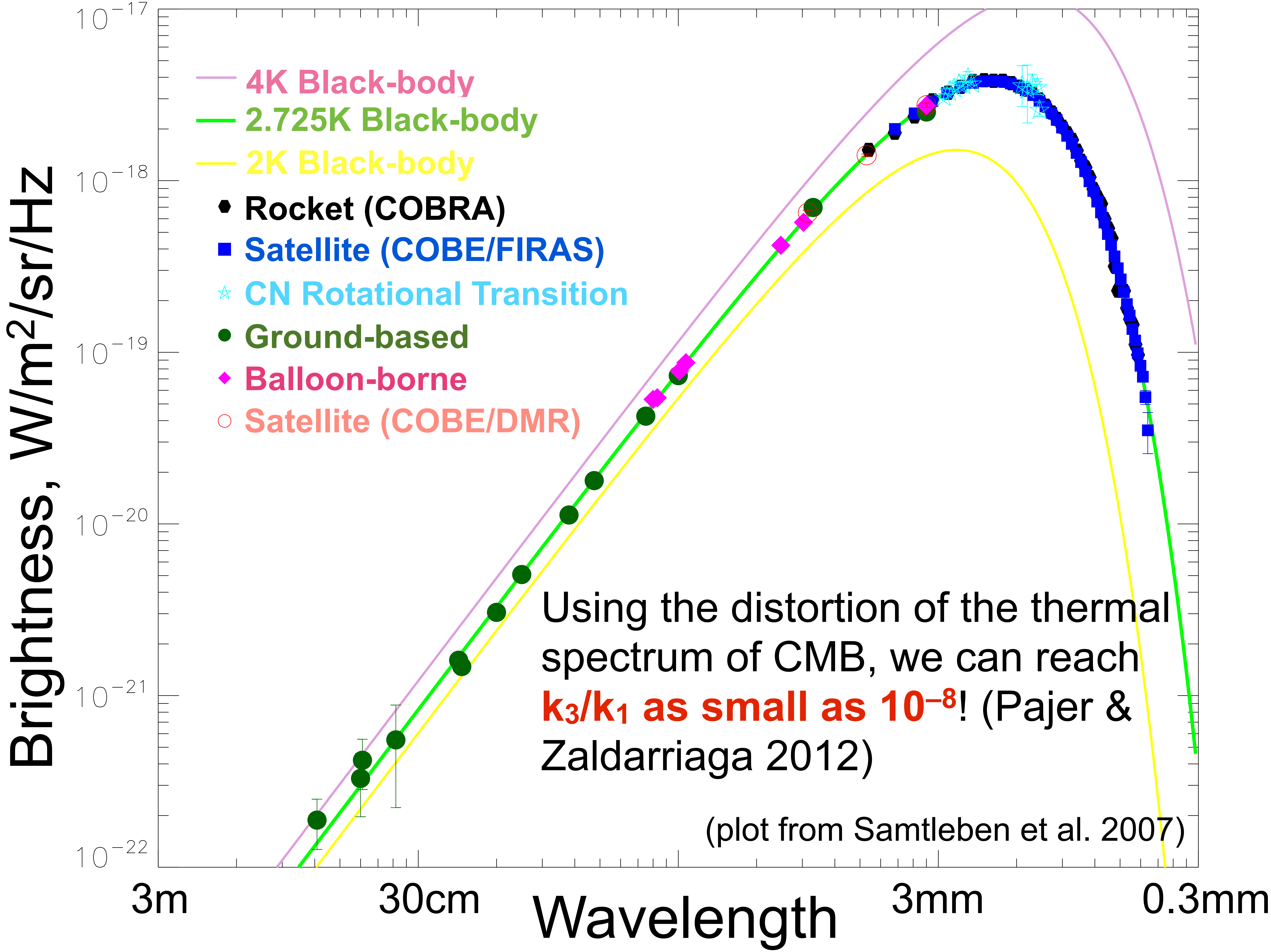
It does!

Ganc & Komatsu (2012)



CMB Bispectrum

- The expected contribution to f_{NL} as measured by the CMB bispectrum is typically $f_{NL} \approx 8(\varepsilon/0.01)$.
  - A lot bigger than $(5/12)(1-n_S)$, and could be detectable with Planck.
- Note that this does not mean a violation of the single-field consistency condition, which is valid in the exact squeezed limit, $k_3 \to 0$.
- We have an enhanced bispectrum in the squeezed configuration where **k_3/k_1 is small but finite**.

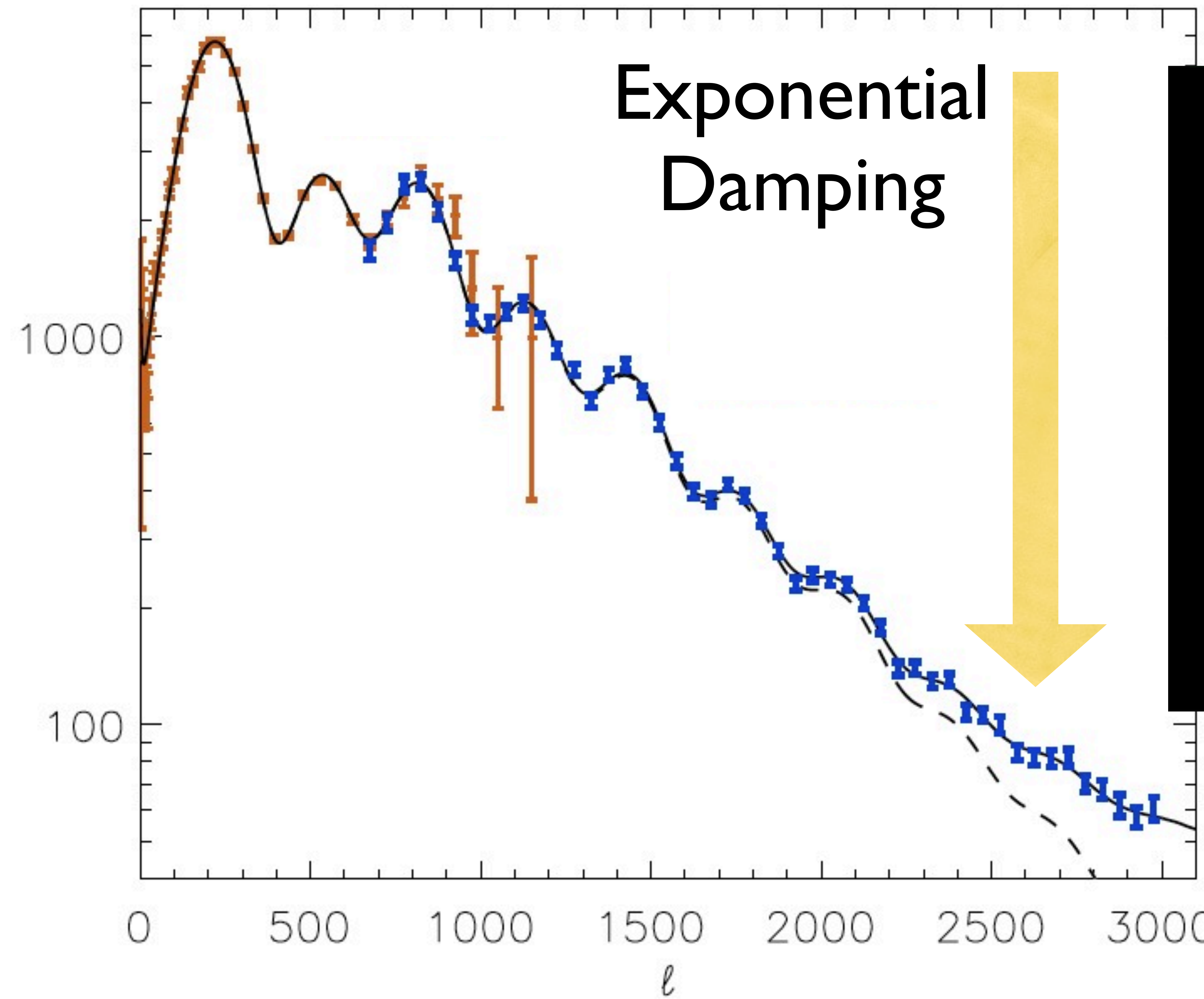


Using the distortion of the thermal spectrum of CMB, we can reach **k_3/k_1 as small as 10^{-8}**! (Pajer & Zaldarriaga 2012)

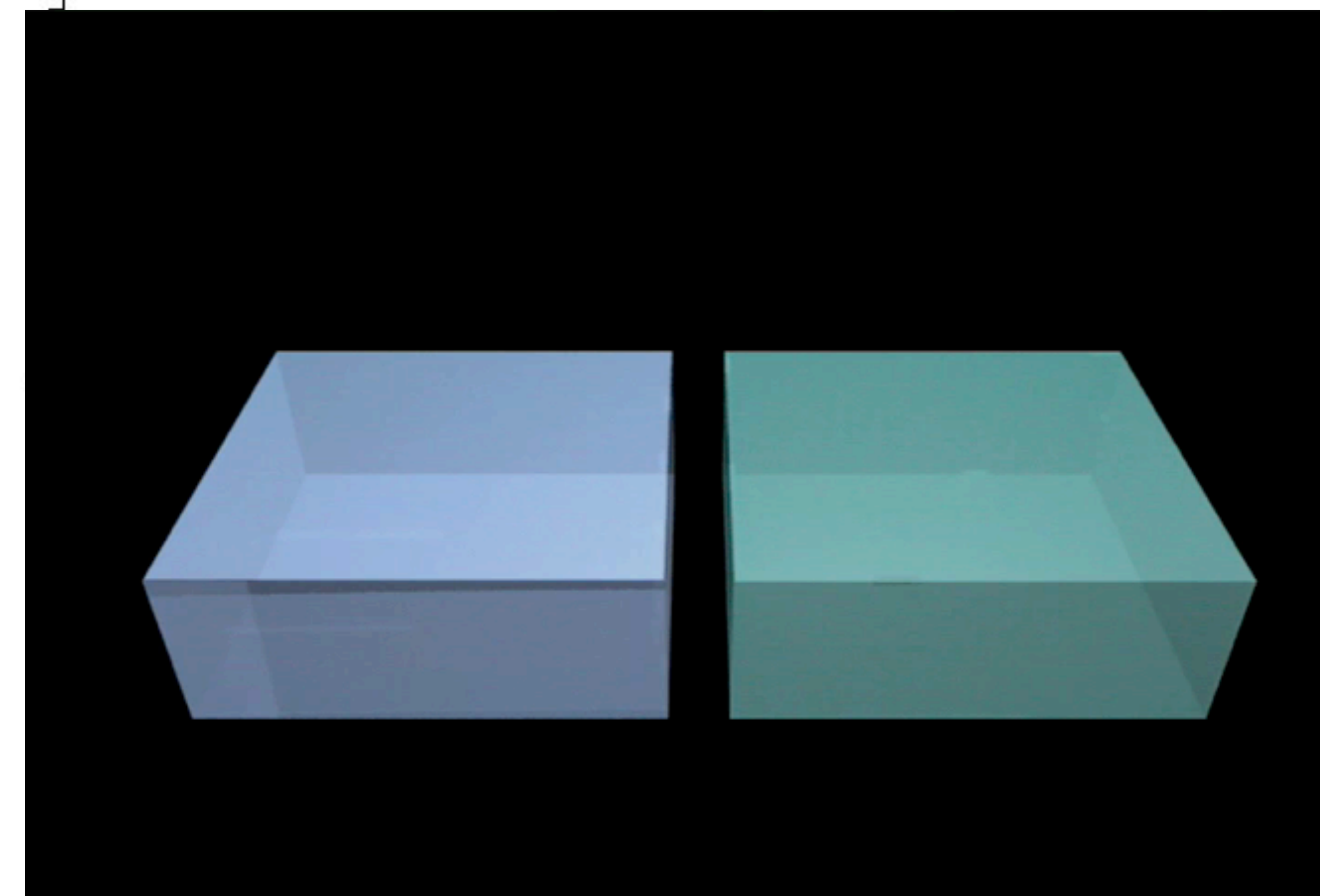
(plot from Samtleben et al. 2007)

Damping of Acoustic Waves

Temperature Power Spectrum



Exponential Damping



- Energy stored in the acoustic waves must go somewhere -> heating of CMB photons -> distortion of the thermal spectrum

Chemical potential from energy injection

- Suppose that some energy, ΔE, is injected into the cosmic plasma during the radiation dominated era.

- What happens? The thermal spectrum of CMB should be distorted!

Chemical potential from energy injection

- For $z > \mathbf{z_i = 2x10^6}$, double Compton scattering, $e^- + \gamma \to e^- + 2\gamma$, is effective, erasing the distortion of the thermal spectrum of CMB.

- Black-body spectrum is restored.

Chemical potential from energy injection

- For $z<z_i=2\times10^6$, double Compton scattering, $e^-+\gamma \to e^-+2\gamma$, freezes out.
- However, the elastic scattering, $e^-+\gamma \to e^-+\gamma$, remains effective [until **$z_f=5\times10^4$**]
- Black-body spectrum is not restored, but the spectrum relaxes to a Bose-Einstein spectrum with a non-zero chemical potential, μ, **for $z_f<z<z_i$**:

$$n(\nu)=\frac{1}{e^{h\nu/(k_BT)}-1}\to\frac{1}{e^{h\nu/(k_BT)+\mu}-1}$$

Chemical potential from energy injection

$$n(\nu) = \frac{1}{e^{h\nu/(k_B T)} - 1} \rightarrow \frac{1}{e^{h\nu/(k_B T)+\mu} - 1}$$

- Energy density is added to the plasma ($\mu \ll 1$):
  - $aT^4 + \Delta E/V = a(T')^4(1-1.11\mu)$
- Number density is conserved ($\mu \ll 1$):
  - $bT^3 = b(T')^3(1-1.37\mu)$
- Solving for μ gives
  - $\mu = 1.4[\Delta E/(aT^4V)]$ **$= 1.4(\Delta E/E)$**

How much energy?

- Only **1/3** of the total energy stored in the acoustic wave during radiation era is used to *heat* CMB (thus distorting the CMB spectrum) (papers by Jens Chluba):
  - $Q = (1/3)(9/4)c_s^2\rho_\gamma(\delta_\gamma)^2 = (1/4)\rho_\gamma(\delta_\gamma)^2$
- $\mu \approx 1.4 \int dz[(dQ/dz)/\rho_\gamma]$

  $=(1.4/4)[(\delta_\gamma)^2(z_i)-(\delta_\gamma)^2(z_f)]$
  - where $z_i=2\mathrm{x}10^6$ and $z_f=5\mathrm{x}10^4$

Bottom Line

- Therefore, the chemical potential is generated by the photon density perturbation **squared**.
- At what scale? The diffusion damping occurs at the mean free path of photons. In terms of the wavenumber, it is given by:

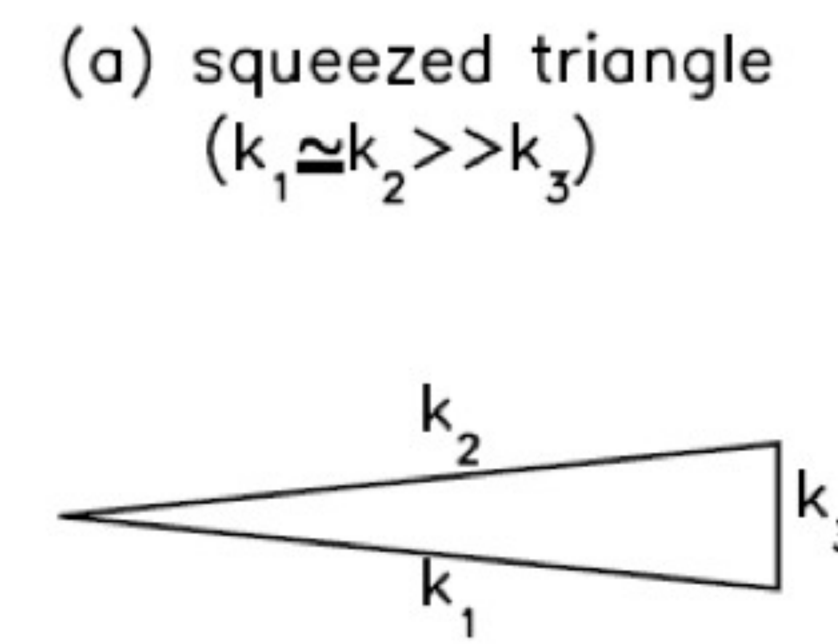
$$k_D \approx 130\ [(1+z)/10^5]^{3/2}\ \mathrm{Mpc}^{-1}$$

$$k_D(z_i) \approx 12000\ \mathrm{Mpc}^{-1}\ \ ;\ \ k_D(z_f) \approx 46\ \mathrm{Mpc}^{-1}$$

It's a very small scale!

(compared to the large-scale structure, k~1 Mpc^{-1})

μ-distortion modified by the squeezed limit



- The existence of long-wavelength ζ changes the small-scale power of δ_γ.
- **A positive long-wavelength ζ -> more power on small scales** for a positive squeezed-limit bispectrum.
- More power on small scales -> more μ-distortion.
- **μ-distortion becomes anisotropic on the sky!** (Pajer & Zaldarriaga 2012)

μ-T cross-correlation

- In real space:
  - $\mu = (1.4/4)[(\delta_\gamma)^2(z_i) - (\delta_\gamma)^2(z_f)]$ at $k_1 \sim O(10^2) - O(10^4)$
  - $\Delta T/T = -(1/5)\zeta$ at $k_3 \sim O(10^{-4})$ [in the Sachs-Wolfe limit]
- Correlating these will probe the bispectrum in the squeezed configuration with **$k_3/k_1 = O(10^{-6}) - O(10^{-8})$!!**

More exact treatment

- Going to harmonic space:
  - $\Delta T/T(\mathbf{n})=\sum a_{lm}{}^{T} Y_{lm}(\mathbf{n})$; $\mu(\mathbf{n})=\sum a_{lm}{}^{\mu} Y_{lm}(\mathbf{n})$

- $$a_{lm}^{T} = \frac{12\pi}{5}(-i)^l \int \frac{d^3k}{(2\pi)^3} \underline{\zeta(\mathbf{k})} g_{Tl}(k) Y_{lm}^*(\hat{k})$$

[$g_{Tl}(k)$ contains info about the acoustic oscillation]

- $$a_{lm}^{\mu} = 18\pi(-i)^l \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} Y_{lm}^*(\hat{k}) \underline{\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)} W\left(\frac{k}{k_s}\right) \times$$
$$\times\, j_l(kr_L) \langle \cos(k_1 r) \cos(k_2 r) \rangle_p \left[e^{-(k_1^2+k_2^2)/k_D^2(z)} \right]_{z_f}^{z_i}$$

μ-T cross-power spectrum

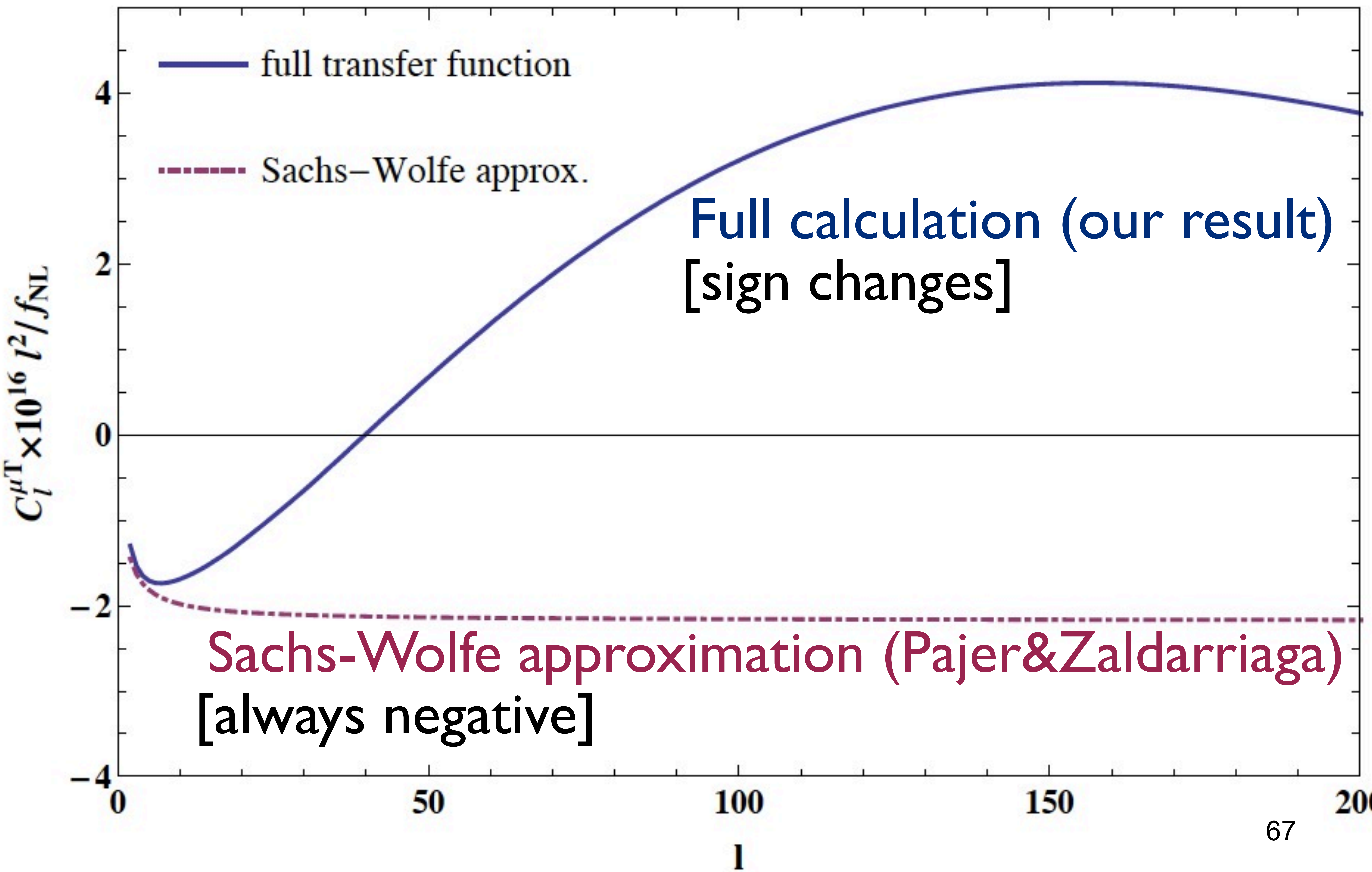
$$C_l^{\mu T} = \frac{27}{20\pi^3}\int_0^\infty k_1^2 dk_1 \left[e^{-2k_1^2/k_D^2(z)}\right]_{z_f}^{z_i} \times \int_0^\infty k^2 dk\, W\left(\frac{k}{k_s}\right) B_\zeta(k_1, k_2, k) j_l(kr_L) g_{Tl}(k)$$

- Here, the integral is dominated by $k_1 \approx k_2 \approx k_D$ (which is big) and $k \approx l/r_L$ (which is small because r_L=14000 Mpc)
- Very squeezed limit bispectrum

Ganc & Komatsu (2012)

Local-form Result

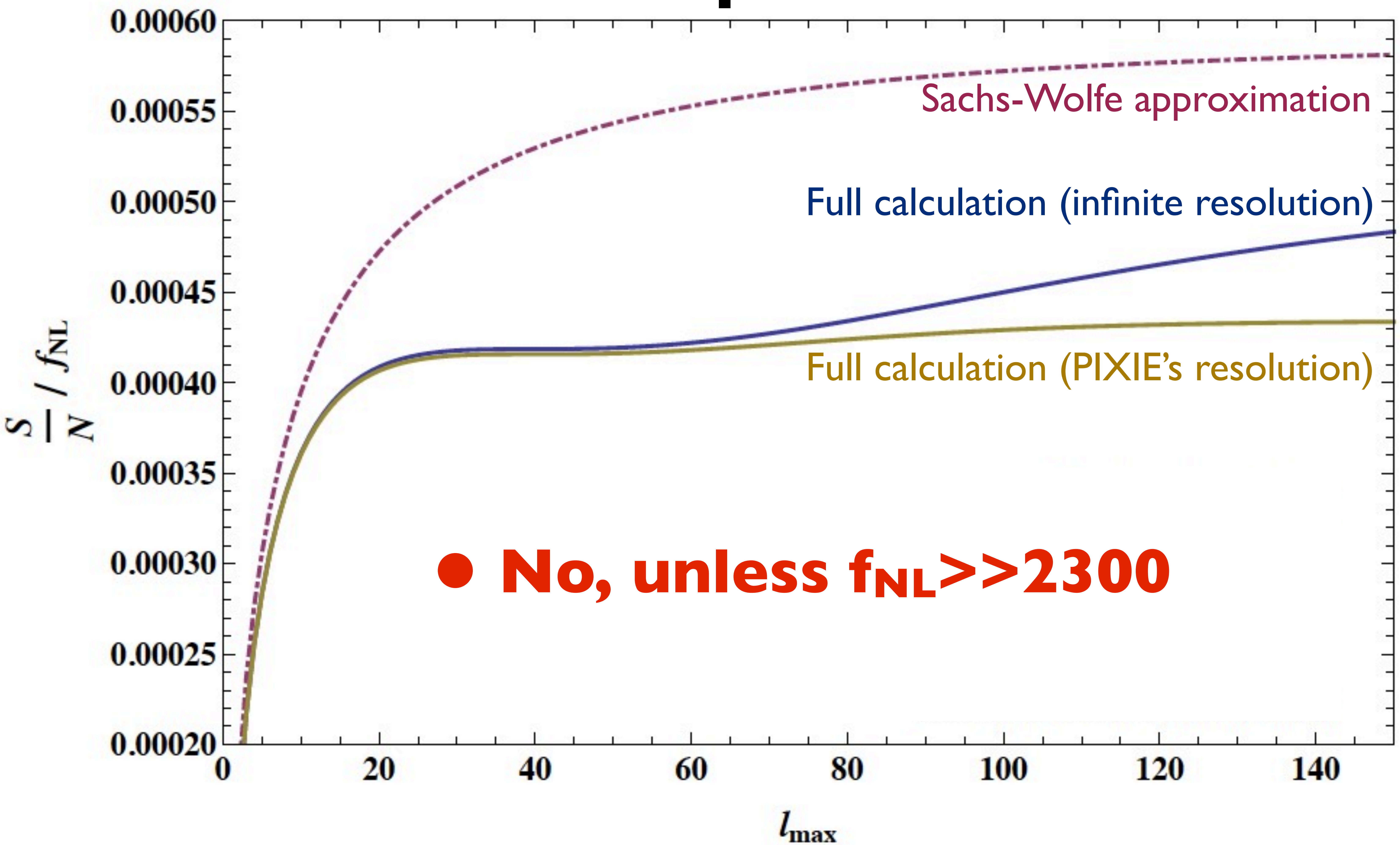
μ-T cross-correlation



Ganc & Komatsu (2012)

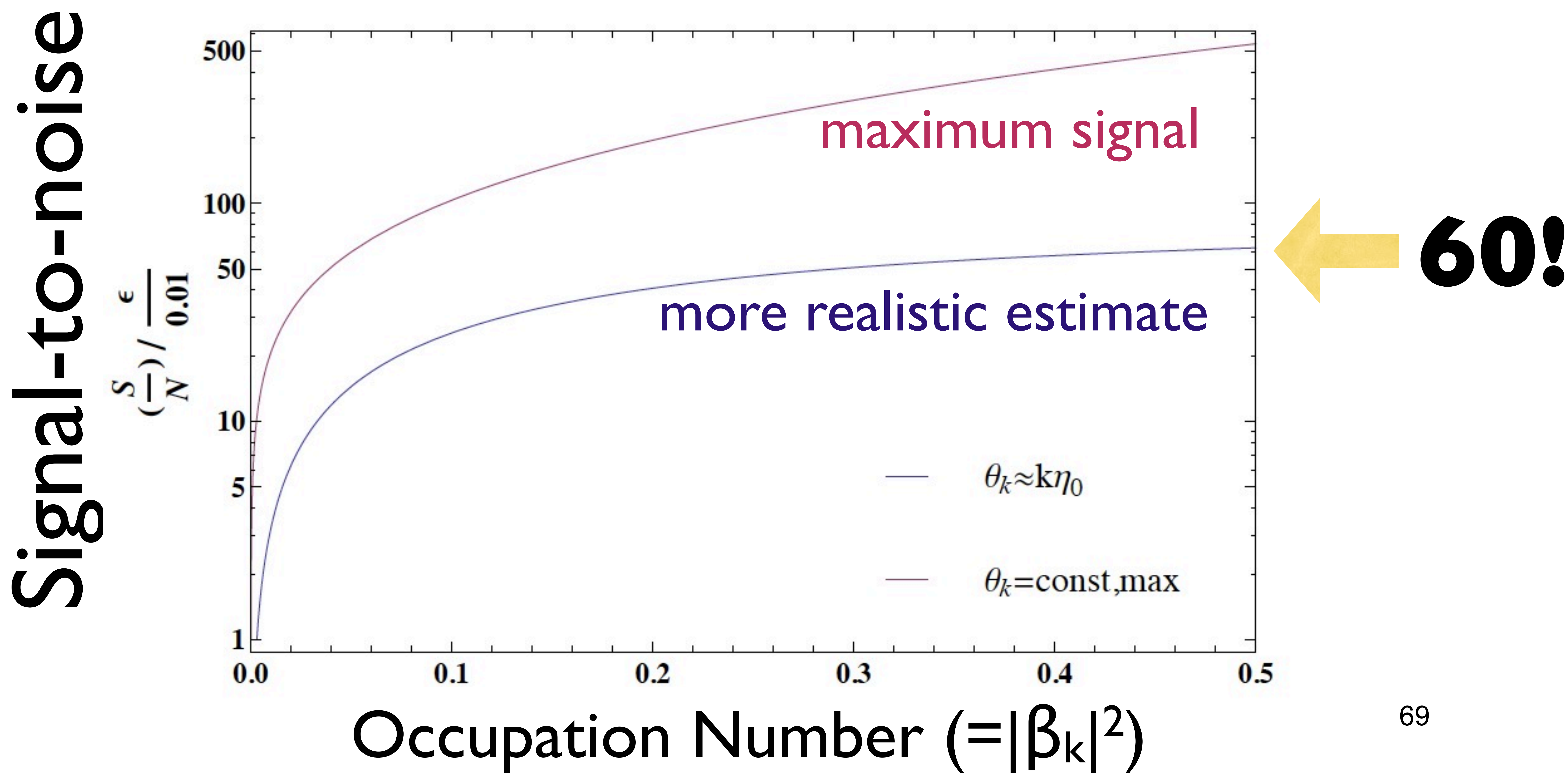
Can we detect the local-form bispectrum?

Signal-to-noise / f_{NL}



- **No, unless f_{NL}>>2300**

But, a modified initial state enhances the signal



Future Work

- All we did was to impose the following mode function at a finite past:
- $u_k = \frac{H^2}{\dot{\phi}} \frac{1}{\sqrt{2k^3}} [\alpha_k(1+ik\eta)e^{-ik\eta} + \beta_k(1-ik\eta)e^{ik\eta}]$
  - with the condition: $\beta_k \to 0$ for $k \to \infty$
- However, it is desirable to construct an explicit model which will give explicit forms of α_k and β_k, so that we do not need to put an arbitrary model function at an arbitrary time by hand.

Summary

New probes of initial state of quantum fluctuations!

- A more insight into the single-field consistency relation for the squeezed-limit bispectrum using in-in formalism.
- Non-Bunch-Davies vacuum can give an enhanced bispectrum in the $k_3/k_1 \ll 1$ limit, yielding a distinct form of the scale-dependent bias.
- The μ-type distortion of the CMB spectrum becomes anisotropic, and it can be detected by correlating μ on the sky with the temperature anisotropy.

Squeezed-limit bispectrum
= Test of single-field inflation
& initial state of quantum fluctuations