

# Residues of Chern classes

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The talk is concerned with localization of characteristic classes of vector bundles (or sheaves) and the associated residues.

If we have some geometric object, certain characteristic classes are localized at the singular set of the object, and we have “Residue Theorems”, the Poincaré-Hopf Index Theorem for vector fields being a model example.

The localization is done in the framework of the Chern-Weil theory adapted to the Čech-de Rham cohomology. Here I mainly talk about the localization of Chern classes of complex vector bundles on manifolds, or more generally singular varieties, by non-vanishing sections or frames. A direct comparison of the above viewpoint with the obstruction theoretic definition of Chern classes leads to many interesting results. The importance of the Thom class in this framework will also be emphasized.

In the first part, I start with the basic case of the order of a zero of a holomorphic function of one variable. This integer, which has various interesting interpretations, such as analytic, algebraic and topological ones, can be viewed as a residue of the first Chern class of a line bundle. Then I talk about the residues at an isolated singular point in the case of general vector bundles.

In the second part, I will describe the residues in a more general setting where the singular set may be non-isolated and the base space may also be singular. Then I explain how this can be applied to construct an analytic intersection theory on singular varieties.

I also mention the case of Atiyah classes if time permits.

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