War on Lambda

fighting the cosmological constant problem

Tony Padilla,
The Royal Society & University of Nottingham

The Cosmological Constant Problem

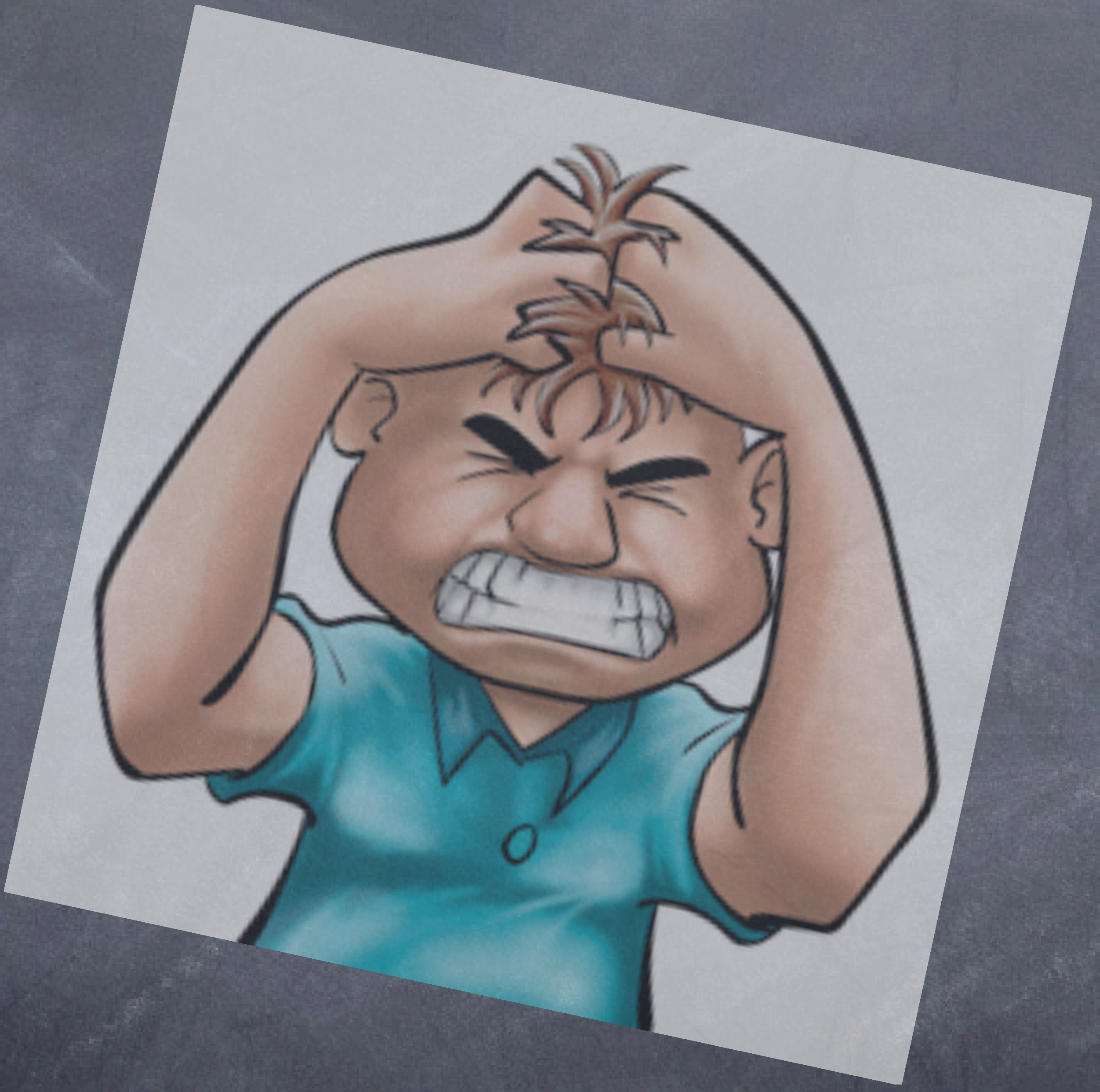
The Fab Four

w/ Charmousis, Copeland & Saffin
PRL 108 (2012) 051101
PRD 85 (2012) 104040

w/ Copeland & Saffin
arXiv:1208.3373

Cleaning up Λ

w/ Kimpton
arXiv 1203.1040
(to appear in JHEP)



The Cosmological Constant Problem

In GR, the vacuum energy gravitates

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{16\pi G} - \rho_{\rm vac}\right)$$

$$G_{\mu\nu} = -8\pi G \rho_{\rm vac} g_{\mu\nu}$$

$$\rho_{\mathrm{vac}}^{\mathrm{obs}} \ll \rho_{\mathrm{vac}}^{\mathrm{theory}}$$

Universe would not even extend to the moon!

$$\rho_{\rm vac}^{theory} \sim \rho_{\Lambda}^{bare}$$

+

zero point energies of each particle

+

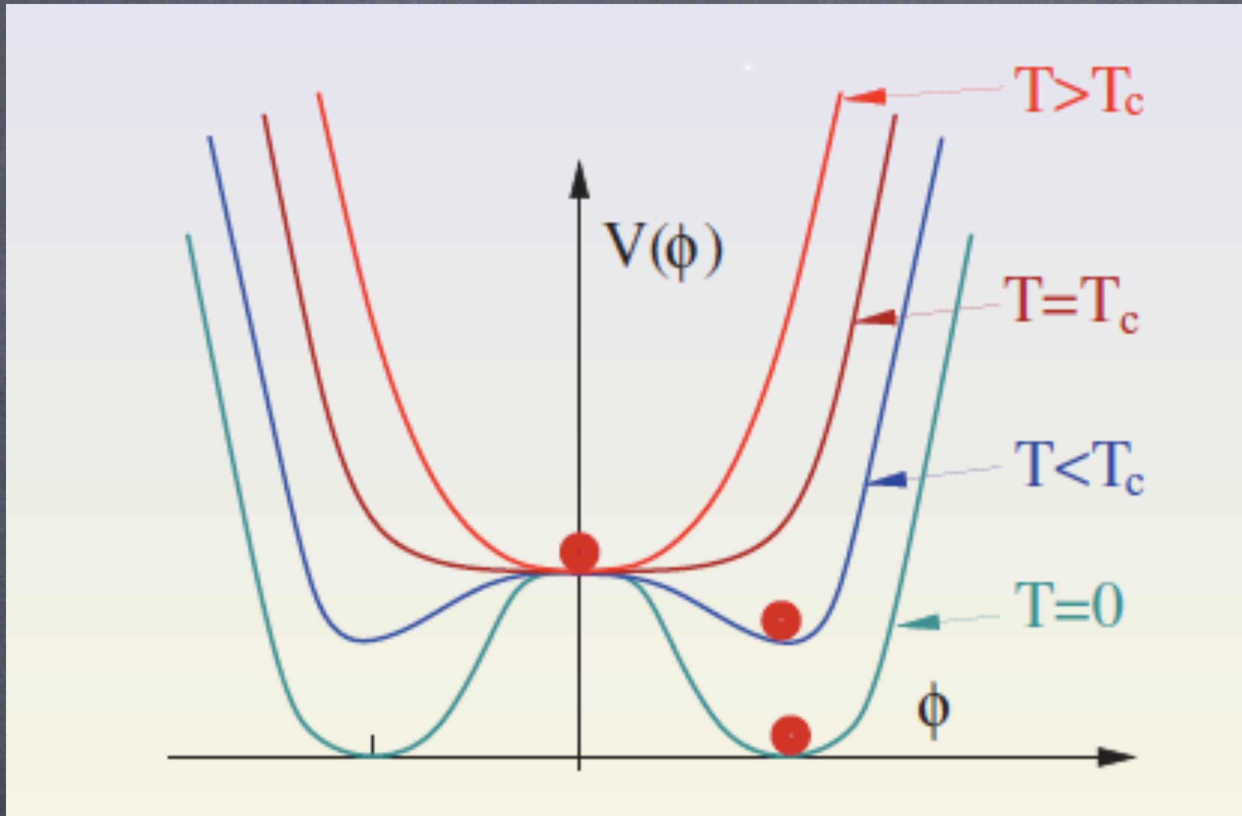
contributions from phase transitions in early universe

zero point energies of each particle

$$\rho_{\rm vac} \supset \quad \sum_m \int d^3k \frac{1}{2}\hbar\sqrt{k^2+m^2}$$

$$\sim c_\nu m_\nu^4 + c_e m_e^4 + c_\mu m_\mu^4 + \ldots + M_{\rm cut-off}^4$$

contributions from phase transitions in early universe



Effective potential $V_{eff}(\phi_0)$

$$\Delta V_{EW} \sim (200\ GeV)^4$$

$$\Delta V_{QCD} \sim (0.3\ GeV)^4$$

Quantum Gravity cut-off	$(10^{18}\ GeV)^4$	fine tuning to 120 decimal places
SUSY cut-off	$(TeV)^4$	fine tuning to 60 decimal places
EW phase transition	$(200\ GeV)^4$	fine tuning to 56 decimal places
QCD phase transition	$(0.3\ GeV)^4$	
muon	$(100 MeV)^4$	fine tuning to 44 decimal places
electron	$(MeV)^4$	fine tuning to 36 decimal places
	$(meV)^4$	observed value


Ask not why the vacuum energy is so small...


..but why it hardly gravitates.

Self tuning:

vacuum energy does not gravitate at all!


NO GO AREA!

Weinberg's no go theorem

Self tuning = Poincare invariant solution for any vacuum energy.

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$$S[\pi, g_{\mu\nu}] = \int d^4x\sqrt{-g}R + \Delta L(\pi, g_{\mu\nu}, \text{derivatives})$$

$$\Delta L|_{g,\pi=\text{const}} = -V_0\sqrt{-g}$$

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On shell field eqns: $\left.\dfrac{\partial \Delta L}{\partial g_{\mu\nu}}\right|_{g,\pi=\text{const}} = 0, \ \left.\dfrac{\partial \Delta L}{\partial \pi}\right|_{g,\pi=\text{const}} = 0$

Scalar eqn $\implies$ trace of gravity eqn

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If $g_{\mu\nu}$ and π are constant then ΔL is invariant under

$$\delta g_{\mu\nu} = \epsilon g_{\mu\nu}, \ \delta\phi = -\epsilon$$

where we define $\phi = \int \frac{d\pi}{f(\pi)}$

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where we define $\phi = \int \frac{d\pi}{f(\pi)}$

Then $\Delta L = \sqrt{-\hat{g}}\mathcal{L}(\hat{g}_{\mu\nu}, \text{derivatives})$ where $\hat{g}_{\mu\nu} = e^\phi g_{\mu\nu}$.

$$\Longrightarrow \left.\frac{\partial \Delta L}{\partial g_{\mu\nu}}\right|_{g,\pi=\mathrm{const}} = \left.\frac{1}{2} g^{\mu\nu} \Delta L\right|_{g,\pi=\mathrm{const}}$$

recall $\Delta L_{g,\pi=\mathrm{const}} = -V_0\sqrt{-g}$

$$\Longrightarrow \left.\frac{\partial \Delta L}{\partial g_{\mu\nu}}\right|_{g,\pi=\text{const}} = \frac{1}{2} g^{\mu\nu} \Delta L\Big|_{g,\pi=\text{const}}$$

recall $\Delta L_{g,\pi=\text{const}} = -V_0\sqrt{-g}$

EOMs imply LHS $= 0$, so RHS $= 0 \implies V_0 = 0$

Scalar eqn $\implies$ trace of gravity eqn

$$g_{\mu\nu}\frac{\partial \Delta L}{\partial g_{\mu\nu}} - f(\pi)\frac{\partial \Delta L}{\partial \pi} = \partial_\mu J^\mu$$

Sufficient but not necessary

If $g_{\mu\nu}$ and π are constant then ΔL is invariant under

$$\delta g_{\mu\nu} = g_{\mu\nu}, \;\; \delta\phi = -\epsilon$$

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Self tuning = Poincare invariant solution for any vacuum energy.

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“The Beatles [have made] negligible contributions to cosmological theory”

PRL, June 2011


THURSDAY NITE
IS....
FIELDY AND LITTLE COL
THE PLACE
JAGERBULL JUGS
CHEEKY
GUINNESS
STELLA ARTOIS
FOSTER'S

Horndeski in

Self tuning filter

Fab Four out

Horndeski in

a panoptic scalar tensor theory

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+ G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

G. W. Horndeski, Int. J. Theor. Phys. 10 (1974) 363-384.

Deffayet et al Phys.Rev. D80 (2009) 064015

Kobayashi et al 1105.5723 [hep-th])

a panoptic scalar tensor theory

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Aside: for multi-scalar generalisation see AP & V. Sivanesan arXiv 1210.4206

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+ matter minimally coupled to metric only

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Self-Portrait as Fertilizer with Irises
acrylic on linen, masonite and wood
42 x 36 x 3 $7,000

The self tuning filter

the theory should admit a Minkowski vacuum

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$\Downarrow$

this should remain true before and after any phase transition where the cosmological constant jumps instantaneously by a finite amount.

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the theory should permit a non-trivial cosmology

Scalar EOM should be trivial on Minkowski for any scalar

Scalar is completely determined by vacuum Friedmann eqn

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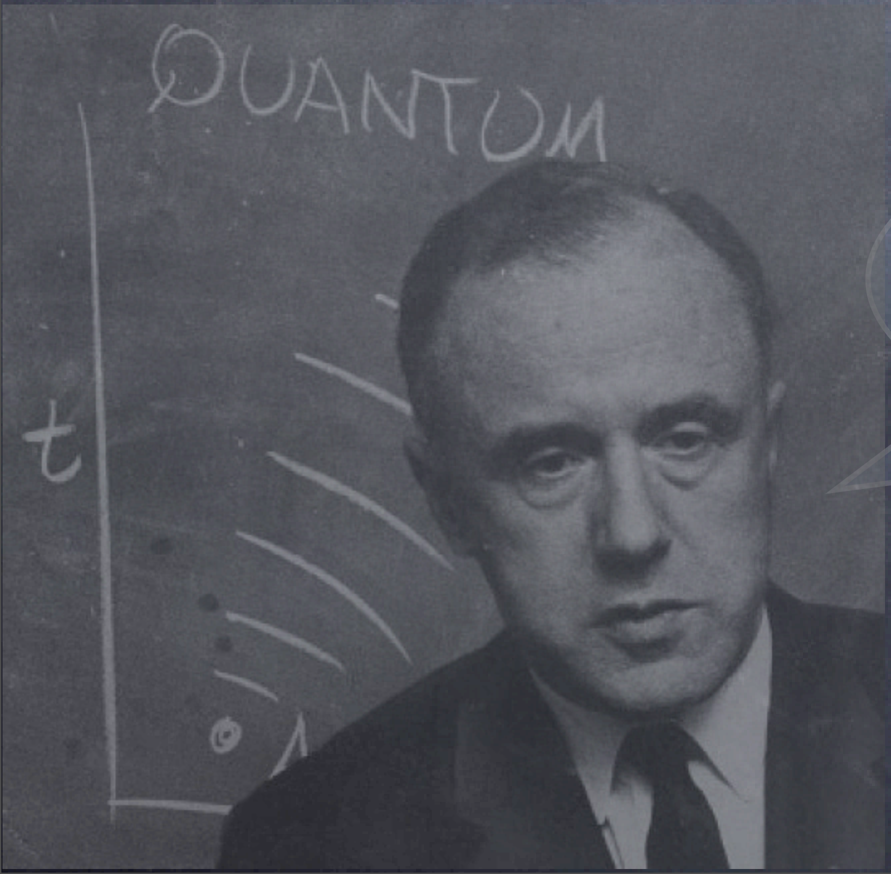

"matter tells scalar how to move, scalar tells spacetime not to curve"

Other constraints

"matter tells scalar how to move"

gravity equation should NOT be trivial on Minkowski

"scalar tells spacetime not to curve, BUT ONLY IN VACUUM"

Scalar equation should be a dynamical equation with a Minkowski fixed point

The Fab Four

$$
\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
\mathcal{L}_{paul} &= \sqrt{-g}V_{paul}(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi \\
\mathcal{L}_{george} &= \sqrt{-g}V_{george}(\phi)R \\
\mathcal{L}_{ringo} &= \sqrt{-g}V_{ringo}(\phi)\hat{G}
\end{aligned}
$$

where $\hat{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ and $P^{\mu\nu\alpha\beta} = -\frac{1}{4}\varepsilon^{\mu\nu\lambda\sigma}\, R_{\lambda\sigma\gamma\delta}\, \varepsilon^{\alpha\beta\gamma\delta}$

Something deeper?

George and Ringo both take the form

$$V(\phi)(\text{Euler density})$$

John and Paul both take the form

$$V(\phi)\nabla_\mu\phi\nabla_\nu\phi\frac{\delta W}{\delta g^{\mu\nu}}$$

with

$$\begin{aligned} W_{john} &= \int d^4x\sqrt{-g}R \\ W_{paul} &= -\frac{1}{4}\int d^4x\sqrt{-g}\phi\hat{G} \end{aligned}$$

Fab Four cosmology

$$\mathcal{H}_{john} + \mathcal{H}_{paul} + \mathcal{H}_{george} + \mathcal{H}_{ringo} = -\left[\rho_\Lambda + \rho_{\text{matter}}\right]$$

$$\mathcal{E}_{john} + \mathcal{E}_{paul} + \mathcal{E}_{george} + \mathcal{E}_{ringo} = 0$$

$$\mathcal{H}_{john} = 3V_{john}(\phi)\dot{\phi}^2\left(3H^2 + \frac{\kappa}{a^2}\right))$$

$$\mathcal{H}_{paul} = -3V_{paul}(\phi)\dot{\phi}^3 H\left(5H^2 + 3\frac{\kappa}{a^2}\right)$$

$$\mathcal{H}_{george} = -6V_{george}(\phi)\left[\left(H^2 + \frac{\kappa}{a^2}\right) + H\dot{\phi}\frac{V'_{george}}{V_{george}}\right]$$

$$\mathcal{H}_{ringo} = -24V'_{ringo}(\phi)\dot{\phi}H\left(H^2 + \frac{\kappa}{a^2}\right)$$

$$\mathcal{E}_{john} = 6\frac{d}{dt}\left[a^3 V_{john}(\phi)\dot{\phi}\Delta_2\right] - 3a^3 V'_{john}(\phi)\dot{\phi}^2\Delta_2$$

$$\mathcal{E}_{paul} = -9\frac{d}{dt}\left[a^3 V_{paul}(\phi)\dot{\phi}^2 H\Delta_2\right] + 3a^3 V'_{paul}(\phi)\dot{\phi}^3 H\Delta_2$$

$$\mathcal{E}_{george} = -6\frac{d}{dt}\left[a^3 V'_{george}(\phi)\Delta_1\right] + 6a^3 V''_{george}(\phi)\dot{\phi}\Delta_1 + 6a^3 V'_{george}(\phi)\Delta_1^2$$

$$\mathcal{E}_{ringo} = -24V'_{ringo}(\phi)\frac{d}{dt}\left[a^3\left(\frac{\kappa}{a^2}\Delta_1 + \frac{1}{3}\Delta_3\right)\right]$$

where $\Delta_n = H^n - \left(\frac{\sqrt{-\kappa}}{a}\right)^n$

Matter without Matter

$$V_{john}(\phi) = c_1 \phi^{\hat{n}+4}, \qquad V_{paul}(\phi) = 0, \qquad V_{george}(\phi) = c_2 \phi^{\hat{n}+3}, \qquad V_{ringo}(\phi) = -\frac{(\hat{n}+3)(2\hat{n}+5)}{8(2\hat{n}+7)(\hat{n}+6)} c_1 \phi^{\hat{n}+6}.$$

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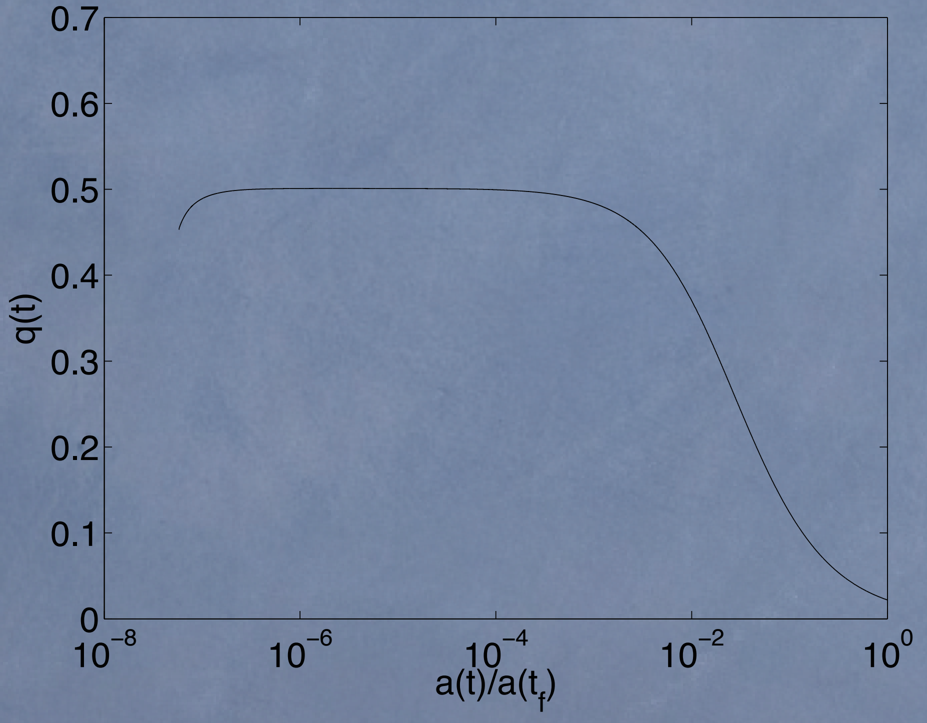

$k = -0.1,\ \phi_{initial} = 0.1,\ \rho_\Lambda = 1,\ \hat{n} = 2,\ c_1 = 1,\ c_2 = 8$

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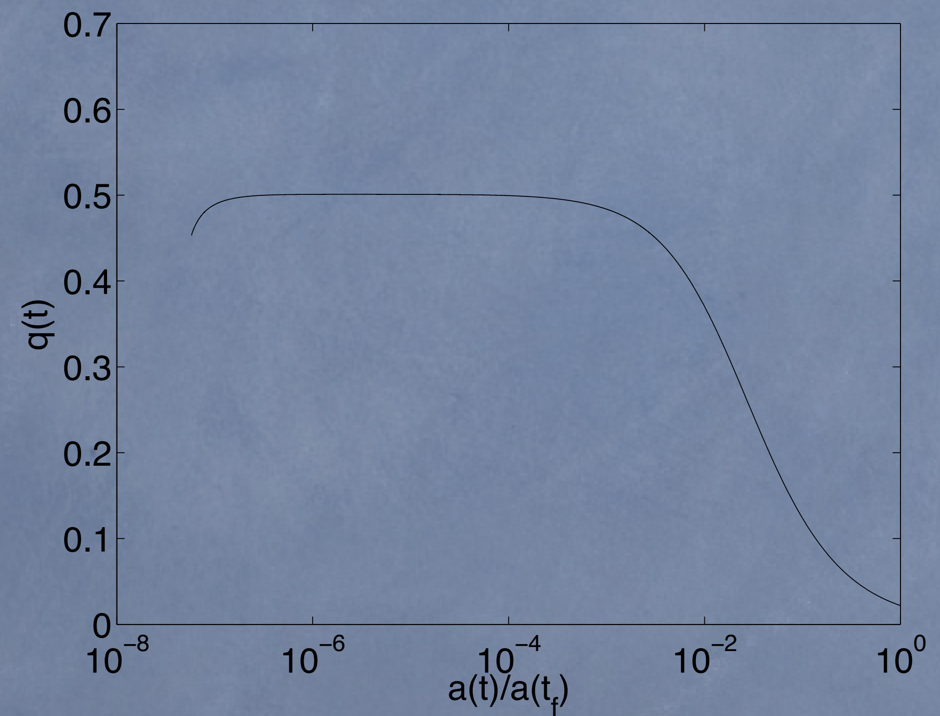

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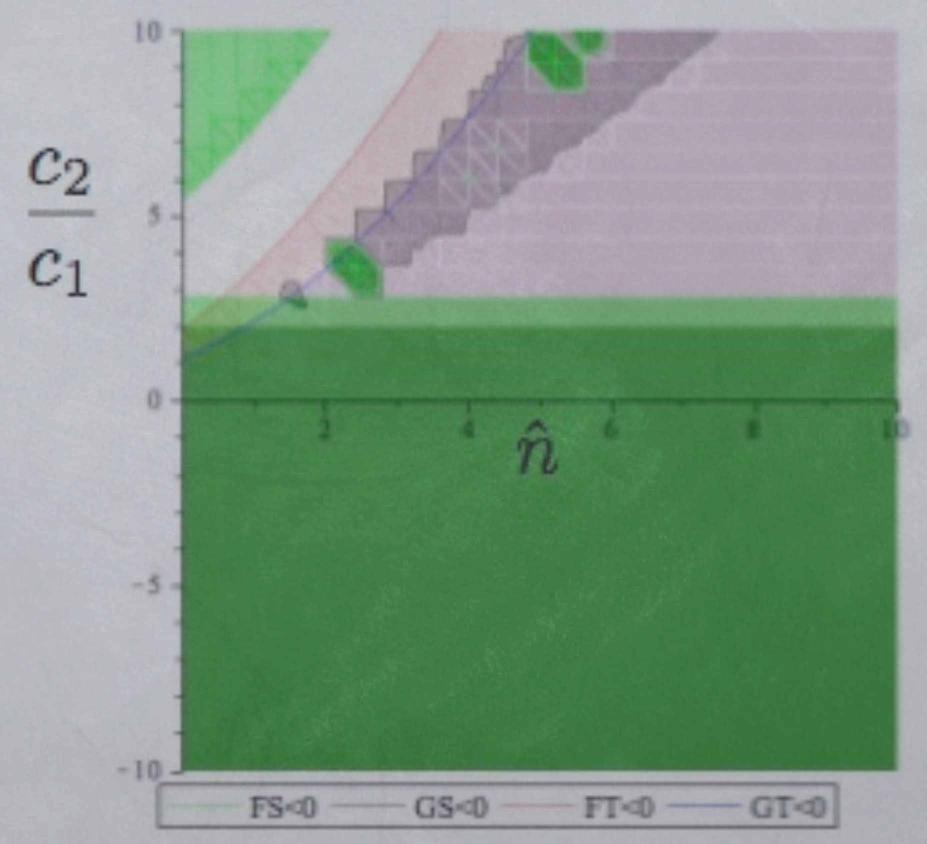

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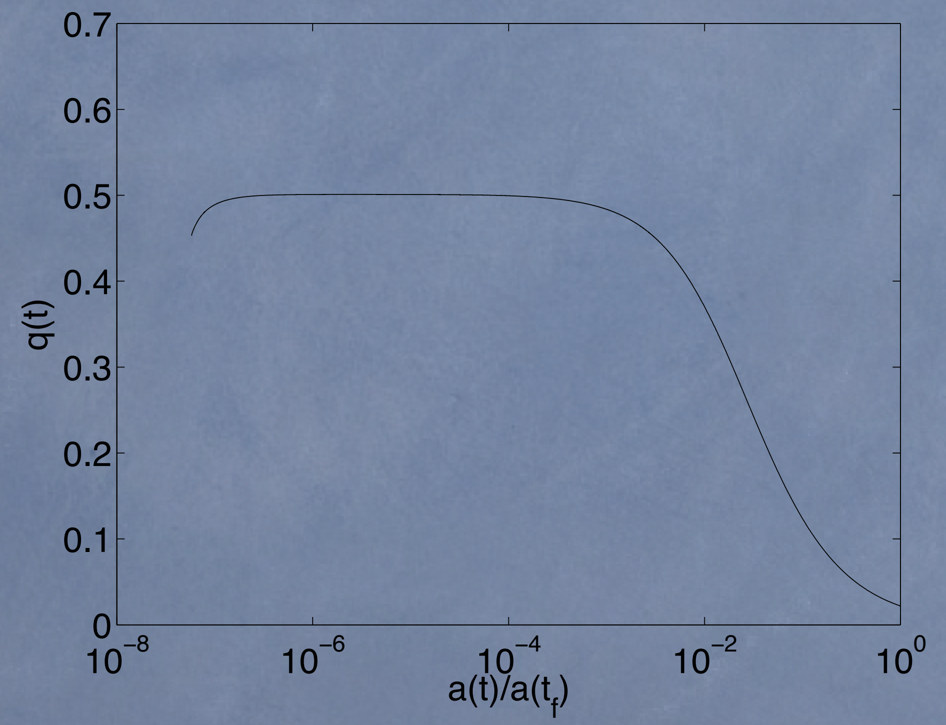

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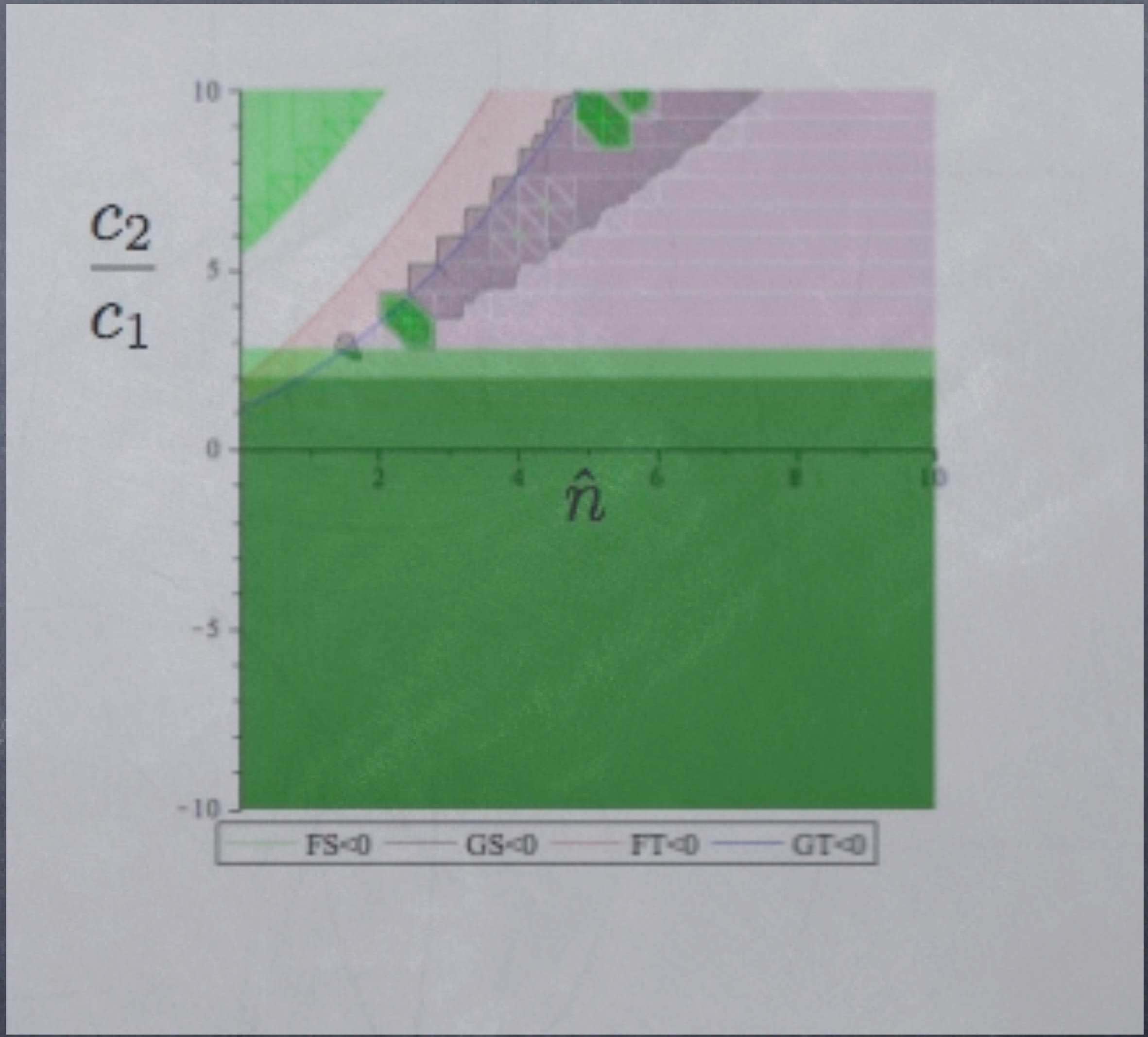

Also radiation, inflation, ...

Plenty to think about...

Solar system tests and good cosmology?

Vainshtein effects from John & Paul?

Radiative stability?

Significance of geometrical structure?

Stability stability?

ghosts etc?

Cleaning up Λ

w/ Kimpton
arXiv 1203.1040
(to appear in JHEP)

Mission Statement

"To prevent the vacuum energy from gravitating, allowing us to set the vacuum curvature as a boundary condition"

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Reminiscent of unimodular gravity

The plan

- Matter is minimally coupled to a *physical metric* $\tilde{g}_{ab} = \tilde{g}_{ab}(\phi_i, \partial\phi_i, \ldots)$.
- It is a composite of the fundamental fields ϕ_i, with the property that

$$\frac{\delta}{\delta\phi_i}\int d^4x\sqrt{-\tilde{g}} = 0$$

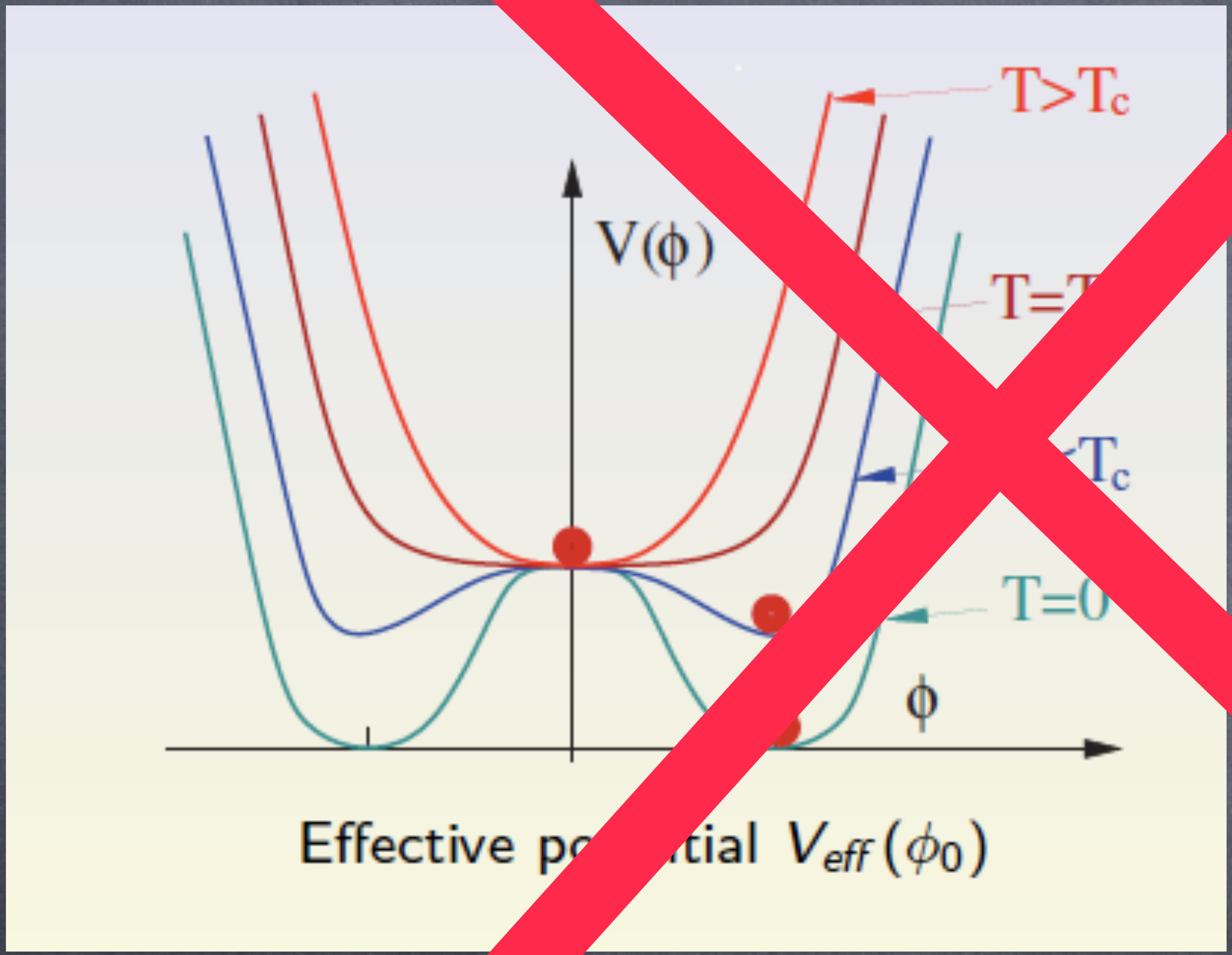
- As the vacuum energy contributes a term of the form $(constant)\int d^4x\sqrt{-\tilde{g}}$ it cannot enter the dynamics!

zero point energies of each particle

$$\rho_{\rm vac} \supset \sum_m \int d^3k \frac{1}{2}\sqrt{k^2 + m^2}$$

$$\sim c_\nu m_\nu^4 + c_e m_e^4 + c_\mu m_\mu^4 + \ldots + M^4_{\rm cut-off}$$

contributions from phase transitions in early universe



$$\Delta V_{EW} \sim (200\ GeV)^4$$

$$\Delta V_{QCD} \sim (0.3\ GeV)^4$$

Do NOT explain why Λ_{obs} goes like H_0^2

Example: conformally related metrics

Physical metric is conformally related to fundamental metric

$$\tilde{g}_{ab} = \Omega(\phi_i, \partial\phi_i, \ldots) g_{ab}$$

with

$$\frac{\delta}{\delta\phi_i} \int d^4x \; \Omega^2 \sqrt{-g} = 0$$

Possible examples: $\Omega^2 = \Box\Phi$ or $\Omega^2 =$ Gauss-Bonnet combination, etc.

Kinetic term?

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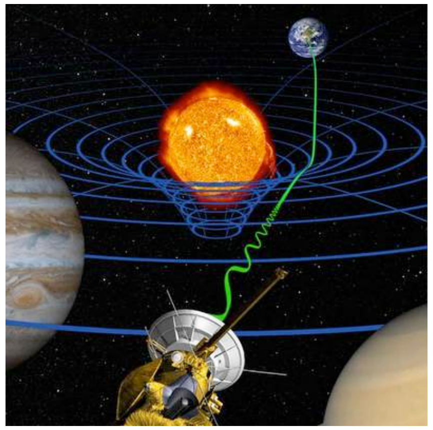
Obvious choice: standard kinetic terms for fundamental fields



issues with solar system tests of gravity

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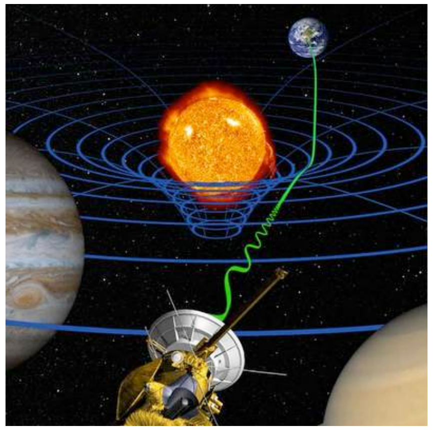

issues with solar system tests of gravity

Alternative choice: Einstein-Hilbert action for *physical metric*

automatically compatible with solar system tests
possible issue with Ostrogradski ghosts

$$S[\phi_i; \Psi_n] = \frac{1}{16\pi G} \int d^4x \sqrt{-\tilde{g}} R(\tilde{g}) + S_m[\phi_i; \Psi_n]$$

matter minimally coupled
to physical metric
$\tilde{g}_{ab} = \Omega(\phi_i, \partial\phi_i, \ldots) g_{ab}$

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$\tilde{g}_{ab} = \Omega(\phi_i, \partial\phi_i, \ldots)g_{ab}$

$$\frac{\delta S}{\delta\phi_i} = \sqrt{-g}\Omega\left(\tilde{E}^{ab} - \frac{1}{4}\tilde{E}\tilde{g}^{ab}\right)\frac{\partial g_{ab}}{\partial\phi_i} + \frac{1}{2}\mathcal{O}_i(\tilde{E}) = 0$$

where

$$\tilde{E}^{ab} = \frac{1}{\sqrt{-\tilde{g}}}\frac{\delta S}{\delta\tilde{g}_{ab}} = -\frac{1}{16\pi G}\left[\tilde{G}_{ab} - 8\pi G\tilde{T}_{ab}\right]$$

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$$\begin{aligned}\mathcal{O}_i(Q) &= \int d^4y Q(y)\frac{\delta}{\delta\phi_i(x)}\sqrt{-\tilde{g}(y)}\\ &= Q(x)\frac{\partial\sqrt{-\tilde{g}(x)}}{\partial\phi_i(x)} - \frac{\partial}{\partial x^a}\left(Q(x)\frac{\partial\sqrt{-\tilde{g}(x)}}{\partial\partial_a\phi_i(x)}\right)\\ &\quad + \frac{\partial^2}{\partial x^a\partial x^b}\left(Q(x)\frac{\partial\sqrt{-\tilde{g}(x)}}{\partial\partial_a\partial_b\phi_i(x)}\right) + \ldots\end{aligned}$$

For a constant, c, it is clear that

$$\mathcal{O}_i(c) = c\frac{\delta}{\delta\phi_i(x)}\int d^4y\sqrt{-\tilde{g}(y)} = 0$$

$$S[\phi_i;\Psi_n] = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}R(\tilde{g}) + S_m[\phi_i;\Psi_n]$$

matter minimally coupled
to physical metric
$\tilde{g}_{ab} = \Omega(\phi_i, \partial\phi_i, \ldots)g_{ab}$

$$\frac{\delta S}{\delta\phi_i} = \sqrt{-g}\Omega\left(\tilde{E}^{ab} - \frac{1}{4}\tilde{E}\tilde{g}^{ab}\right)\frac{\partial g_{ab}}{\partial\phi_i} + \frac{1}{2}\mathcal{O}_i(\tilde{E}) = 0$$

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- vacuum energy $\tilde{T}^{ab} = -\sigma\tilde{g}^{ab}$
- vacuum curvature $\tilde{R}_{ab} = \tilde{\Lambda}\tilde{g}_{ab}$

DO NOT ENTER DYNAMICS!

$$\mathcal{O}_i(c) = c\frac{\delta}{\delta\phi_i(x)}\int d^4y\sqrt{-\tilde{g}(y)} = 0$$

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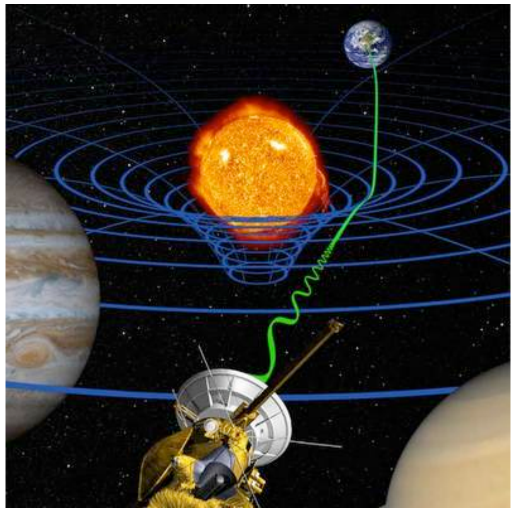


ANY SOLUTION TO GR (WITH ANY VACUUM CURVATURE) IS A SOLUTION TO OUR THEORY

$$\mathcal{O}_i(c) = c\frac{\delta}{\delta\phi_i(x)}\int d^4y\sqrt{-\tilde{g}(y)} = 0$$

GHOSTS?

GHOSTS?

generically "yes"...but not always

Ghost-free theories

$\Omega^2 = \frac{R_{GB}(\Xi, g)}{\mu^4} \implies$

Effective action about de Sitter vacuum is

$$\delta_2 S = \frac{\bar{\Omega}}{16\pi G} \delta_2 S_{GR}[e^{\psi/2} g] + \int d^4x \lambda (\delta R(g) + \nabla_a X^a - 2\Lambda\psi)$$

where $X^a = B^{ab}{}_b - B_b{}^{ba}$ and

$$B^c{}_{ab} = \delta \left[\Xi^c{}_{ab} - \frac{1}{2} g^{cd} (g_{da,b} + g_{db,a} - g_{ab,d}) \right]$$

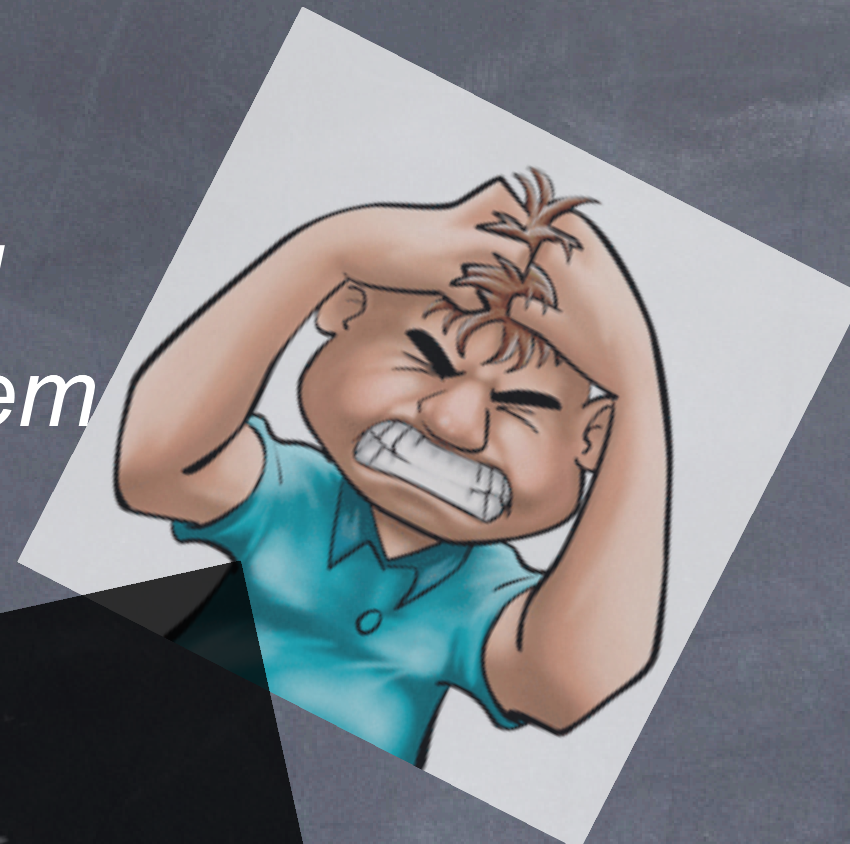
Summary:

- couple matter to a composite metric, such that covariant measure for composite is non-dynamical
- VACUUM ENERGY DOES NOT GRAVITATE!
- build the EH action out of composite aswell
- solar system tests easily satisfied
- Ostrogradski ghosts can be avoided

Future directions:

- extra solutions: cosmology, solar system?
- radiative stability of choice of Ω ?
- disformally related metrics?
- selection criteria?

The Cosmological Constant Problem



The Fab Four

ALL self tuning scalar tensor theories, stable matter-like cosmologies

Cleaning up Λ

generalises unimodular gravity

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"Those of you in the cheaper seats, clap your hands... And the rest of you... just rattle your jewellary."

John Lennon

Friedmann eqn:

$$\mathcal{H}(a, \dot{a}, \phi, \dot{\phi}) = \frac{1}{a^3}\left[\dot{a}\frac{\partial L_H^{\text{eff}}}{\partial \dot{a}} + \dot{\phi}\frac{\partial L_H^{\text{eff}}}{\partial \dot{\phi}} - L_H^{\text{eff}}\right] = -\rho_m$$

At most cubic in H $\quad \mu_3 H^3 + \mu_2 H^2 + \mu_1 H + \mu_0 = \rho_m$

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Scalar eqn:

$$\mathcal{E}(a,\dot{a},\ddot{a},\phi,\dot{\phi},\ddot{\phi}) = \frac{d}{dt}\left[\frac{\partial L_H^{\rm eff}}{\partial \dot{\phi}}\right] - \frac{\partial L_H^{\rm eff}}{\partial \phi} = 0$$

Linear in both $\ddot{\phi}$ and $\ddot{a}$

1. Vacuum solution is always Minkowski whatever the vacuum energy

2. Solution remains Minkowski even after a phase transition

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2. Solution remains Minkowski even after a phase transition

$$\langle \rho_m \rangle_{\rm vac} = \rho_\Lambda, \qquad H^2 = -\frac{\kappa}{a^2}, \qquad \phi = \phi_\Lambda(t)$$

piecewise constant but discontinuous at transition

always true

continuous everywhere but certainly not constant

“On-shell-in-a” EOMs

$$\bar{\mathcal{H}}(\phi_\Lambda, \dot{\phi}_\Lambda, a) = -\rho_\Lambda \qquad \bar{\mathcal{E}} = f(\phi_\Lambda, \dot{\phi}_\Lambda, a)\ddot{\phi}_\Lambda + g(\phi_\Lambda, \dot{\phi}_\Lambda, a) = 0$$

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continuous   continuous

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continuous continuous

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continuous | continuous

discontinuous | discontinuous

contains delta function

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contains
delta function

contains no
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continuous continuous

discontinuous discontinuous

contains delta function

contains no delta function

$$\Longrightarrow \quad \begin{aligned} f(\phi_\Lambda, \dot{\phi}_\Lambda, a) &= 0 \\ g(\phi_\Lambda, \dot{\phi}_\Lambda, a) &= 0 \end{aligned}$$

$$f(\phi_\Lambda, \dot{\phi}_\Lambda, a) = 0$$

If LHS depends on $\dot{\phi}_\Lambda$ it must be discontinouus.

But RHS is not discontinuous, so this means that LHS cannot be either.

Thus $f = f(\phi_\Lambda, a)$.

$$f(\phi_\Lambda, a) = 0$$

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Take derivatives wrt time and use $H^2 = -\kappa/a^2$,

$$\frac{\partial f}{\partial \phi_\Lambda}\dot{\phi}_\Lambda + \frac{\partial f}{\partial a}\sqrt{-\kappa} = 0$$

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Identical logic gives $\frac{\partial f}{\partial \phi_\Lambda} = 0$, or equivalently $f = f(a)$.

Also $g = g(a)$.

Stability?

Cosmological perturbations in Horndeski (Kobayashi et al 1105.5723 [hep-th])

tensors $$\int dt d^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right]$$

scalars $$\int dt d^3x a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla \zeta)^2\right]$$