THE ACCELERATING UNIVERSE AND THE COSMIC MYSTERY OF DARK ENERGY

OIST

Grigoris Panotopoulos

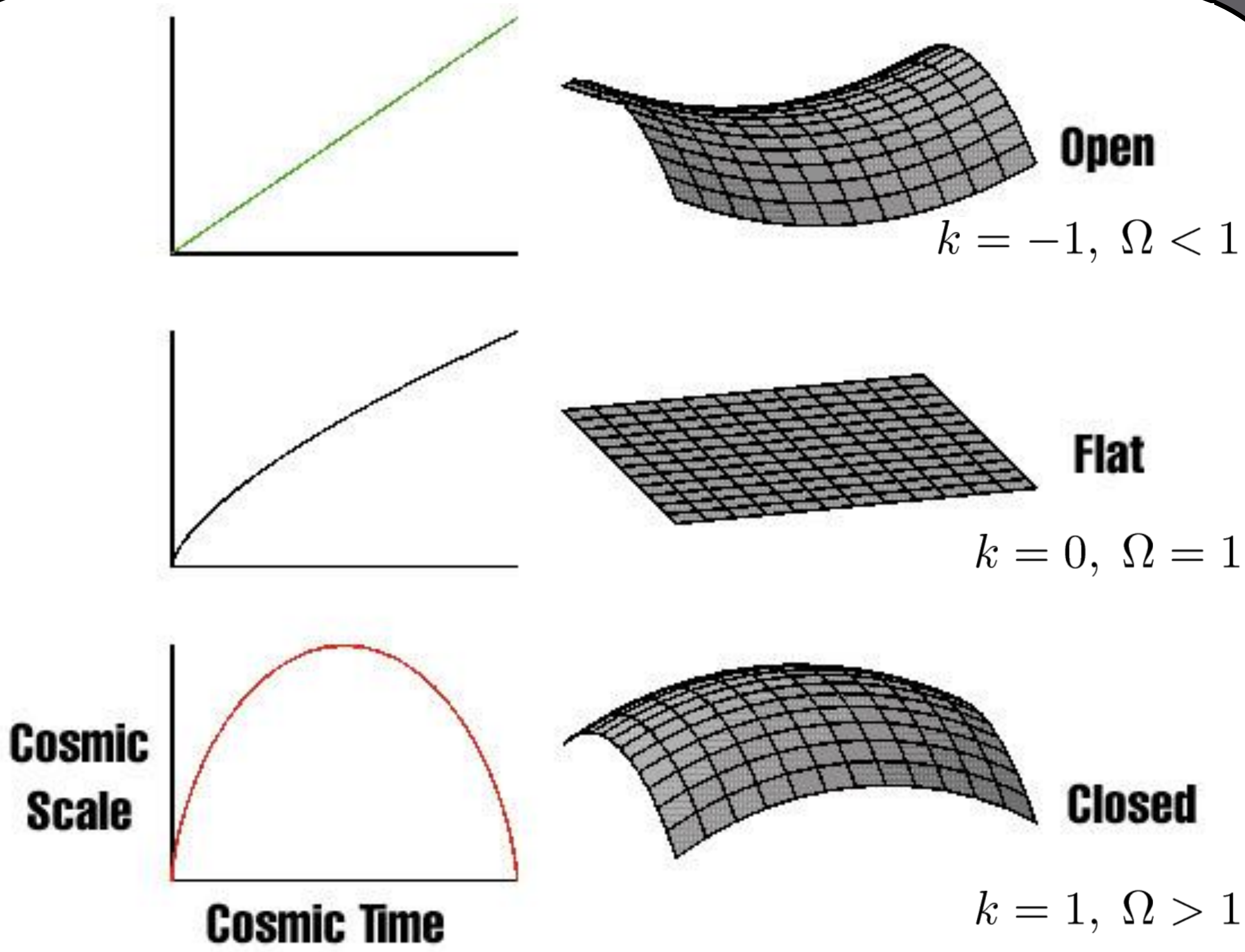
Okinawa Institute of Science and Technology Graduate University

IPMU, September 2012

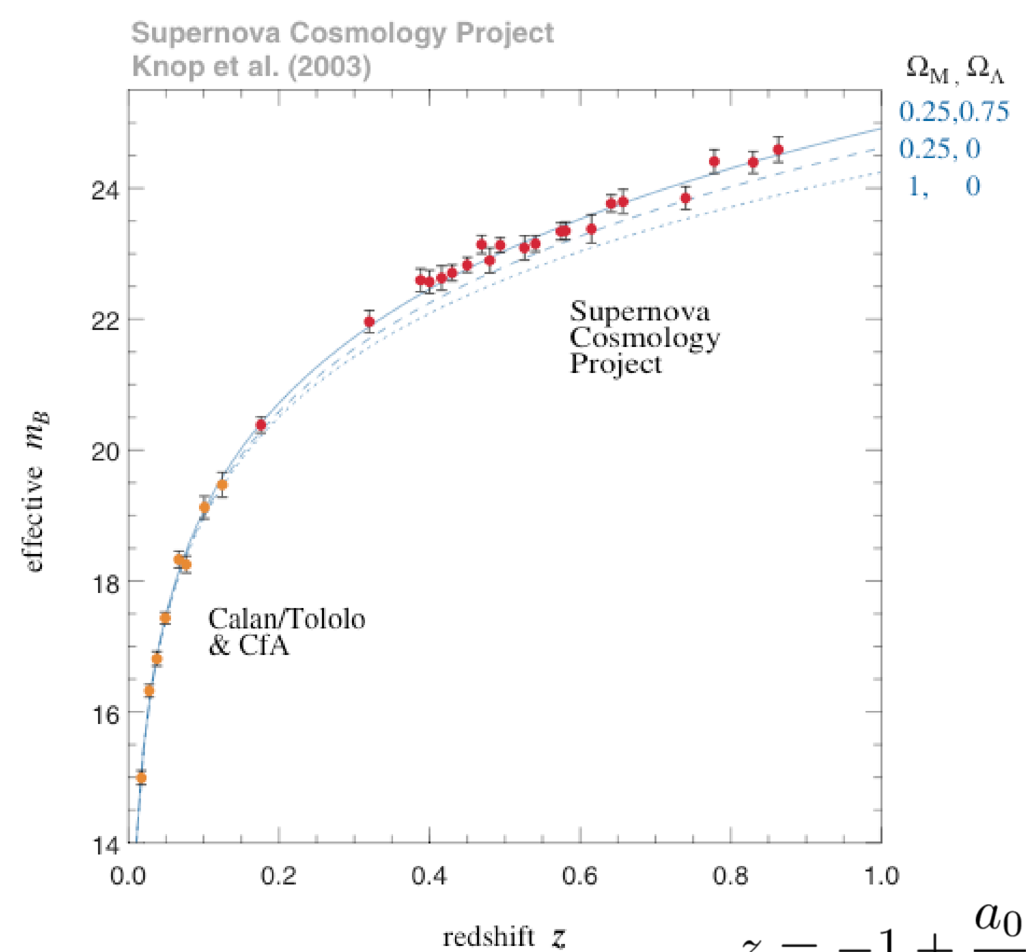
Outline

- Introduction - Motivation
- Dynamical dark energy
- Geometrical dark energy
- Statefinder diagnostics
- Conclusions

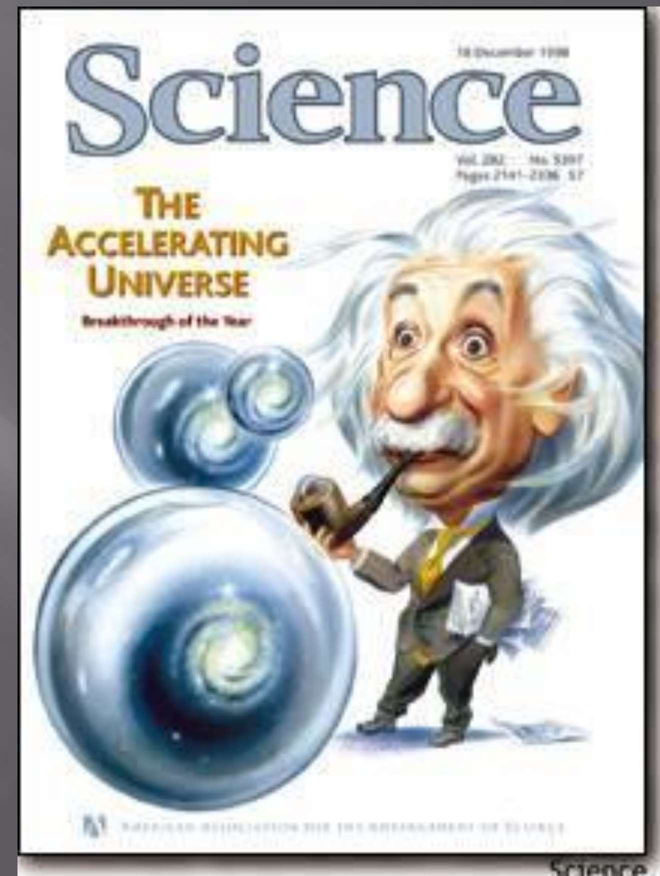
Evolution of the Universe



1998: The accelerating universe breakthrough of the year



$$z = -1 + \frac{a_0}{a}$$



Theory: Friedmann equations

- Derived from Einstein's theory of General Relativity
- First Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

Curvature

Cosmological Constant

- Second Friedmann equation

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

Energy density

Pressure

A. Friedmann
(1888-1925)

Acceleration (of Universe's expansion)

- 2nd Friedmann equation gives the acceleration / deceleration with which the universe expands

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\underbrace{(1+3w)}$$

$$p = w\rho$$

$w < -1/3$: acceleration
$w > -1/3$: deceleration

- Energy conservation (continuity)

$$\dot{\rho} = -3H\left(\rho + \frac{p}{c^2}\right)$$

Useful Quantities

- Critical energy density (the energy density for a flat universe)

$$\rho_c = \frac{3H^2}{8\pi G}$$

- Normalized densities

Matter

$$\Omega = \frac{\rho}{\rho_c}$$

$$\Omega_k = -\frac{k}{a^2 H^2}$$

Curvature

  - This way the 1st Friedmann equation can be written as

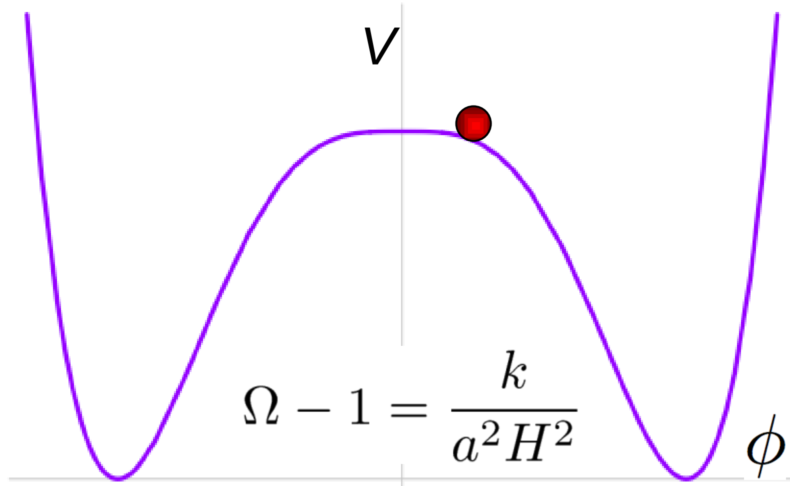
$$\Omega_k + \Omega = 1$$

Slow-roll inflation
A paradigm for the early Universe

- $\Omega = 1$ is a stable fixed point for small slow-roll parameters ($\varepsilon \ll 1, \eta \ll 1$)

$$\epsilon = \frac{m_{pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2$$

$$\eta = \frac{m_{pl}^2}{16\pi}\left(\frac{V''}{V}\right)$$



$$\Omega - 1 = \frac{k}{a^2 H^2}$$

$$H^2 \simeq \frac{8\pi V}{3m_{pl}^2}$$

$$\dot{\phi} \simeq -\frac{V'}{3H}$$

Basic prediction of inflation: The Universe is flat

Age of the Universe and Hubble constant

- Universe older than oldest objects: $t_0 > 12$ Gy
- Hubble Space Telescope: $H_0 = 72$ Km/(Mpc sec)

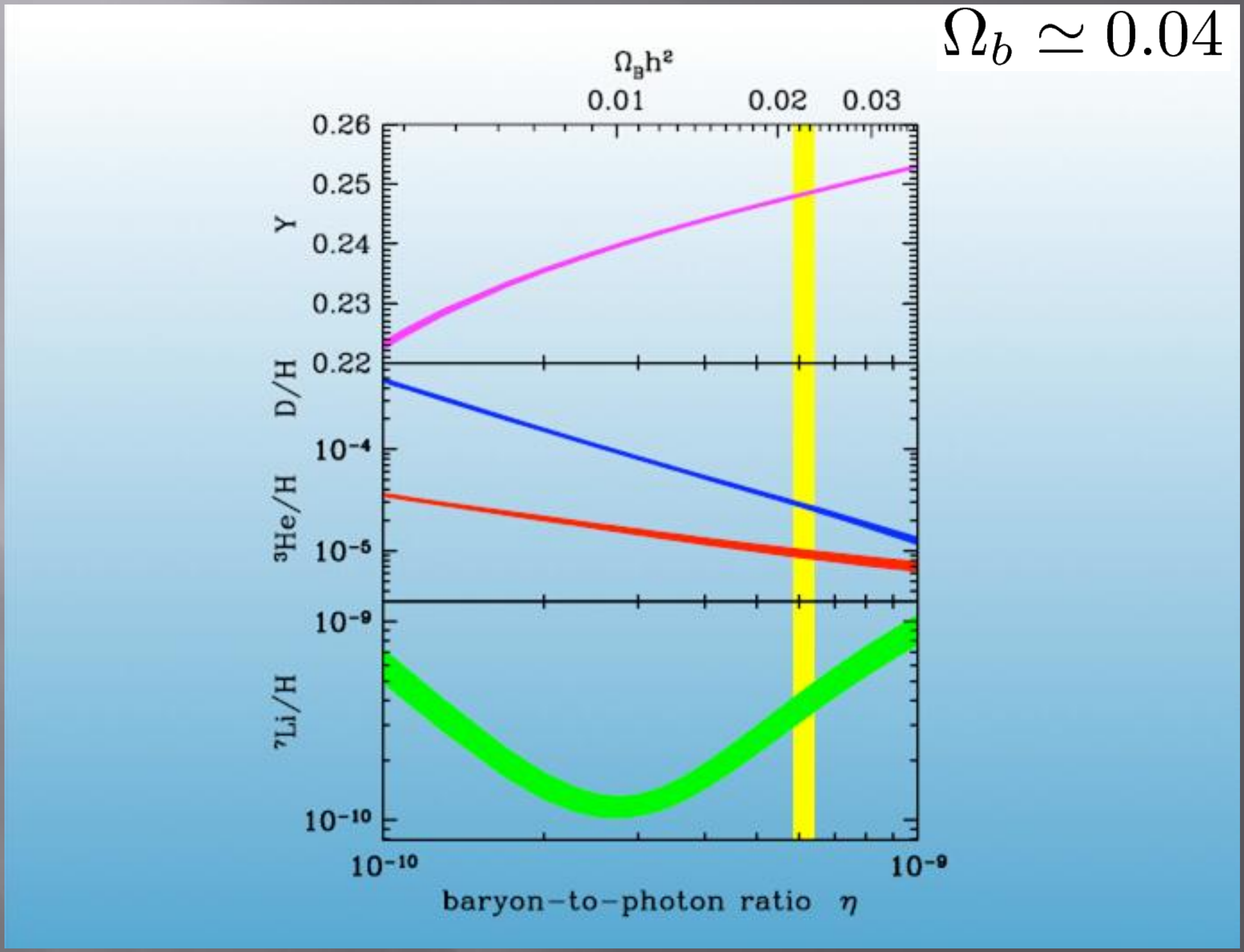
- With matter only, $H_0 t_0 = \frac{2}{3}$

  $\rightarrow$ Universe is too young

- With a cosmological constant $(\Omega_\Lambda = 0.7)$

  $H_0 t_0 \cong 0.96, \quad t_0 \cong 13.7$ Gy

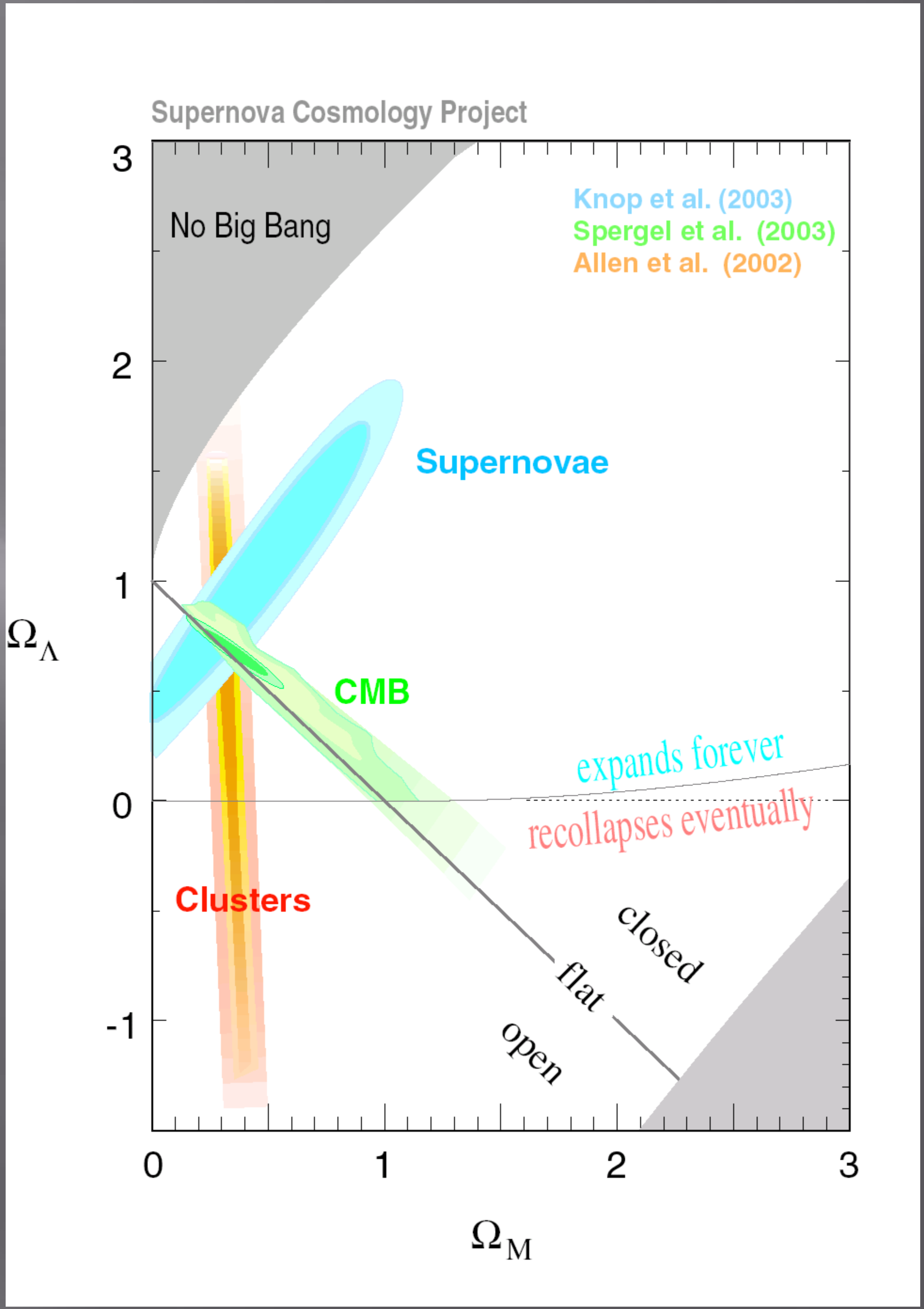
Primordial Nucleosynthesis



Today's picture of the Universe

- 3 independent data sets coincide
  - Supernovae
  - CMB
  - Galaxy surveys

Concordance Cosmological Model

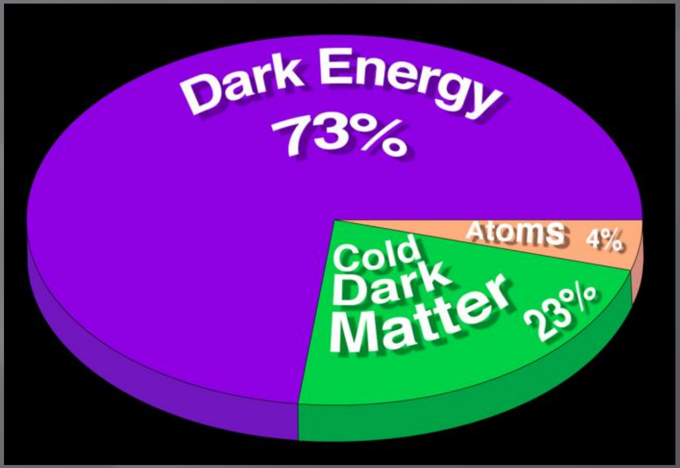


Today's picture of the Universe

- Dark energy dominates in the (flat) Universe

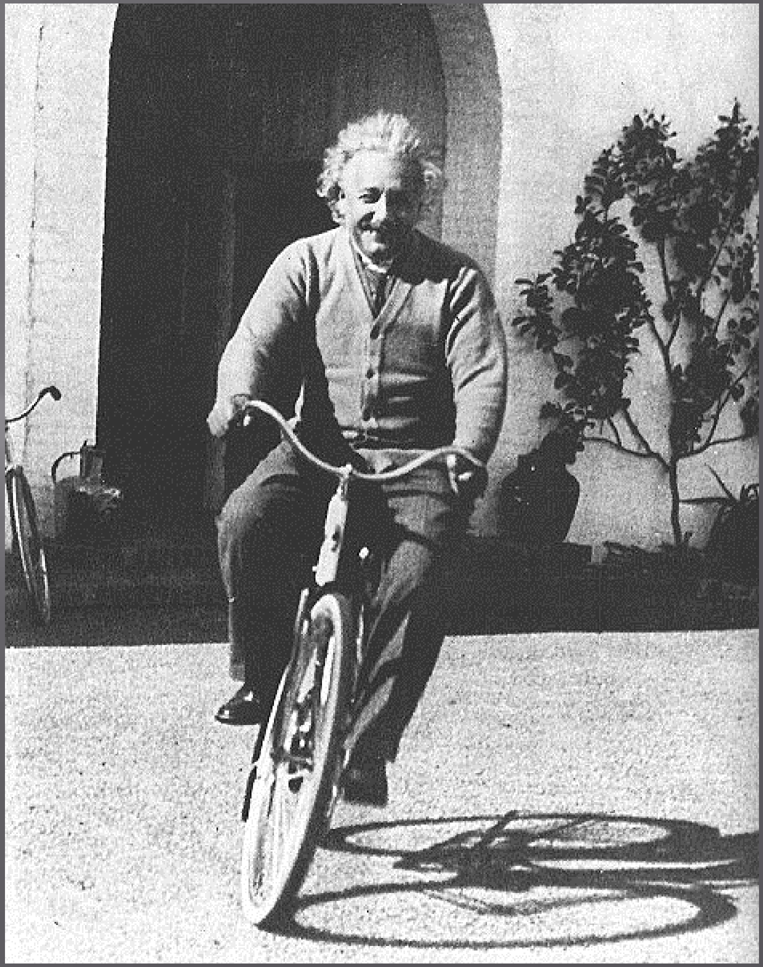
Energy in the Universe = Matter 27% (baryons + cold dark matter) + Dark Energy 73%

$$-1.1 < w < -0.9$$



What is Dark Energy?

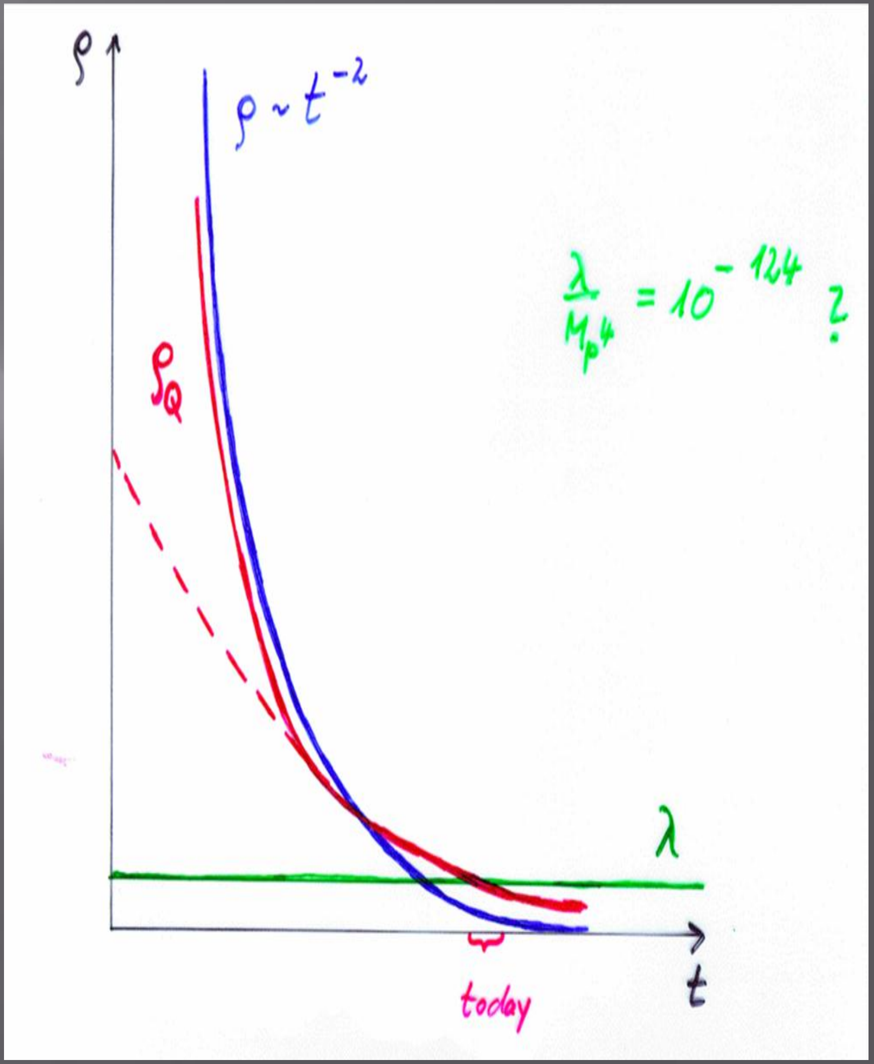
- In the simplest case: Cosmological constant
  - Introduced by Einstein for a static universe
  - Pros
    - Allowed by all symmetries
    - ΛCDM agrees with data
  - Cons
    - The cosmological and
    - the coincidence problems



Cosmological Constant

$$G_{\mu\nu} = -\Lambda g_{\mu\nu}$$

- Fluid with $w = -1$
- Very different evolution
- Value much lower than expected



Field equation for gravity

- Einstein's General Relativity with dust or radiation only

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- Theory
  - Decelerating expansion
- Observation
  - accelerating expansion

Disagreement between theory and observation!

Two choices

- Geometrical Dark Energy
  - Modify left hand side

$$G_{\mu\nu} + G_{\mu\nu}^{dark} = T_{\mu\nu}$$

→ New

gravitational theory

- Dynamical Dark Energy
  - Modify right hand side

$$G_{\mu\nu} = T_{\mu\nu} + T_{\mu\nu}^{dark}$$

→ New

dynamical component

A very active field

- S. Nojiri, S. D. Odintsov and M. Sami, arXiv:hep-th/0605039
- V. Sahni and Y. Shtanov, arXiv:astro-ph/0202346
- R. A. Brown, R. Maartens, E. Papantonopoulos and V. Zamarias, arXiv:gr-qc/0508116
- P. S. Apostolopoulos and N. Tetradis, arXiv:hep-th/0604014; arXiv:astro-ph/0605450;
- C. Wetterich, L. P. Chimento, R. Lazkoz, R. Maartens and I. Quiros, Nucl. Phys. B 302 (1988) 668;
- B.Ratra and P.J.E.Peebles, Phys. Rev. D 37 (1988) 3406;
- R. R. Caldwell, R. Dave and P. J. Steinhardt, arXiv:astro-ph/9708069

The Coincidence Problem

Q: Why are the Omegas of matter and dark energy so similar in magnitude?

A: There are two possibilities

1. Due to special initial conditions:

   if current universe is a finite point in phase-space

2. Due to the specific values of parameters:

   if current universe is close to a fixed point

Not so simple to realize!

- Cosmology of type $\boxed{H^2 = 2\gamma(\rho + \rho_{DE})}$
  - Without energy exchange

$$\boxed{\dot{\rho} + 3H(\rho + p) = 0}$$

Fixed point
→ deceleration

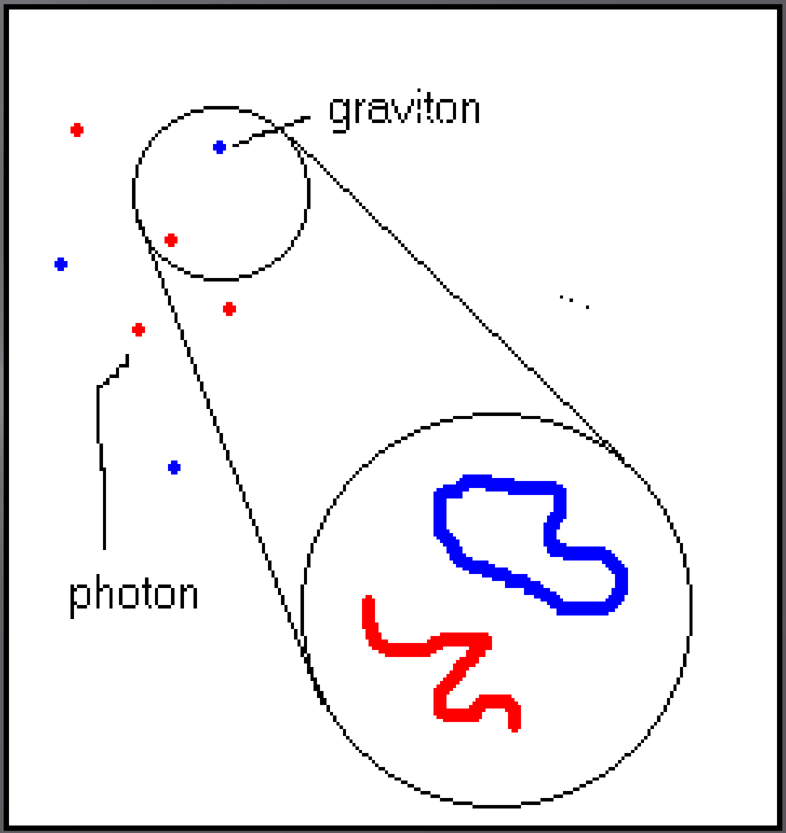
  - With energy exchange

$$\boxed{\begin{aligned} &\dot{\rho} + 3(1+w)H\rho = -T \\ &\dot{\rho}_{DE} + 3(1+w_{DE})H\rho_{DE} = T \end{aligned}}$$

Fixed point
→ acceleration

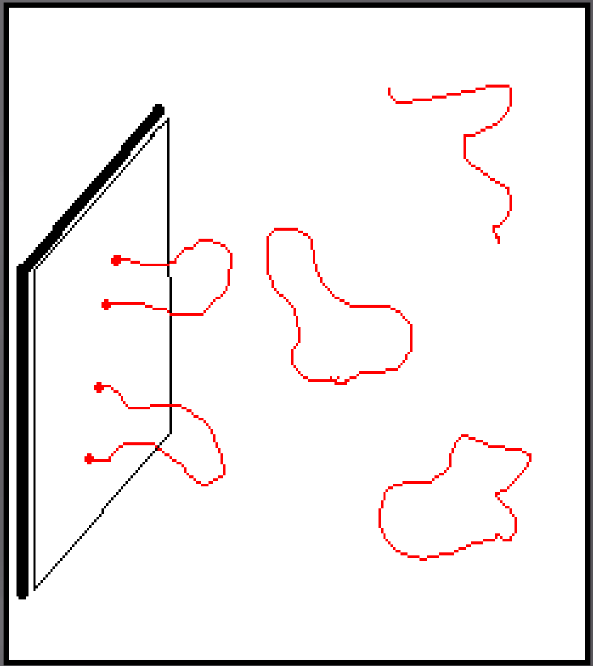
Superstring theory: basic idea

- Really fundamental objects are one-dimensional (strings)
- In low energies strings look like point-like particles
- All known particles are different oscillatory modes of the string



Extended Objects: Branes

- String theory does not contain strings only
- Normally, open strings satisfy Neumann boundary conditions
- Dirichlet boundary conditions also make sense



- End points are stuck on a hypersurface, interpreted as a heavy solitonic object, a D-brane
- Brane –world idea: we are confined on such an object.

A simple brane model (DGP)

- Action

$$S = \int d^5x\sqrt{-g}\, M^3 R + \int d^4x\sqrt{-h}\,(m^2\hat{R} - \mathcal{L}_{SM})$$

  - One extra dimension
  - Gravity in 5D, our world in 4D
  - Reduced to known gravity and cosmology in the early universe
  - New gravity and cosmology at late times

Dvali, Gabadadze, Porrati, hep-th/0005016; hep-th/0008054

Cosmology for DGP

- Friedmann equation

$$\boxed{\frac{H}{r_c} = H^2 - \frac{8\pi G}{3}\rho}$$

$$\boxed{r_c = \frac{m^2}{M^3}}$$

- For early times, we recover standard cosmology

$$H\, r_c \to \infty$$

- At recent times, $\rho \to 0, H \to 1/r_c$
- Same number of parameters as ΛCDM, $r_c \cong H_0^{-1}$

Deffayet, Dvali, Gabadadze, astro-ph/0105068

$$\Omega_{r_c} = \frac{1}{4r_c^2 H_0^2} = \left(\frac{1-\Omega_M}{2}\right)^2$$

A more realistic model

$$\boxed{S = \int d^5x\sqrt{-g}\,(M^3 R - \Lambda) + \int d^4x\sqrt{-h}\,(m^2 \hat{R} - V)}$$

$$\boxed{G_{AC} = \frac{1}{2M^3} T_{AC}|_{tot}}$$

+ Matter in 5 dimensions (unspecified)
+ Fluid on the brane

Kofinas, Panotopoulos, Tomaras, hep-th/0510207

Cosmological Solution

$$ds^2 = -n(t,y)^2 dt^2 + a(t,y)^2 \gamma_{ij} dx^i dx^j + b(t,y)^2 dy^2$$

$$G_{00} = 3\left\{\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right) - \frac{n^2}{b^2}\left[\frac{a''}{a}+\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)\right] + \frac{kn^2}{a^2}\right\}$$

$$G_{05} = 3\left(\frac{n'}{n}\frac{\dot{a}}{a} + \frac{a'}{a}\frac{\dot{b}}{b} - \frac{\dot{a}'}{a}\right)$$

$$G_{55} = 3\left\{\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right) - \frac{b^2}{n^2}\left[\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}-\frac{\dot{n}}{n}\right)\right] - \frac{kb^2}{a^2}\right\}$$

$$G_{ij} = \frac{a^2}{b^2}\gamma_{ij}\left\{\frac{a'}{a}\left(\frac{a'}{a}+\frac{2n'}{n}\right) - \frac{b'}{b}\left(\frac{n'}{n}+\frac{2a'}{a}\right) + \frac{2a''}{a}+\frac{n''}{n}\right\}$$
$$+\frac{a^2}{n^2}\gamma_{ij}\left\{\frac{\dot{a}}{a}\left(\frac{2\dot{n}}{n}-\frac{\dot{a}}{a}\right) - \frac{2\ddot{a}}{a} + \frac{\dot{b}}{b}\left(\frac{\dot{n}}{n}-\frac{2\dot{a}}{a}\right) - \frac{\ddot{b}}{b}\right\} - k\gamma_{ij}$$

Cosmological equations

$$\dot{\rho} + 3(1+w)H\rho = -T \qquad H^2 = \mu + 2\gamma\rho \pm \beta\psi - \frac{k}{a^2}$$

$$\dot{\psi} + 2H\left(\psi - \frac{\lambda + 6(1-3w)\gamma\rho}{\psi}\right) = \pm\frac{2\gamma T}{\beta} \qquad \boxed{T(\rho) = A\rho^{\nu}}$$

$$\frac{\ddot{a}}{a} = \mu - (1+3w)\gamma\rho \pm \beta\frac{\lambda + 6(1-3w)\gamma\rho}{\psi}$$

With new variables

$$\lambda = \frac{2V}{m^2} + \frac{12}{r_c^2} - \frac{\Lambda}{M^3} \quad \mu = \frac{V}{6m^2} + \frac{2}{r_c^2} \quad \gamma = \frac{1}{12m^2} \quad \beta = \frac{1}{\sqrt{3}r_c}$$

Phase-space analysis

$$\boxed{\omega_m + \omega_\psi = 1}$$

$$\boxed{{}' = \frac{1}{D}\frac{d}{dt}}$$

$$\omega_m' = \omega_m\Big[(1+3w)(\omega_m-1)Z - \frac{A}{\sqrt{|\mu|}}\Big(\frac{|\mu|\omega_m}{2\gamma}\Big)^{\nu-1}(1-Z^2)^{\frac{3}{2}-\nu} - 2Z(1-Z^2)\frac{1-Z^2-3(1-3w)\beta^2\mu^{-1}\omega_m}{1-\omega_m}\Big]$$

$$Z' = (1-Z^2)\Big[(1-Z^2)\frac{1-Z^2-3(1-3w)\beta^2\mu^{-1}\omega_m}{1-\omega_m} - 1 - \frac{1+3w}{2}\omega_m\Big]$$

New quantities for dynamical study

$$D = \sqrt{H^2-\mu} \qquad Z = \frac{H}{D} \qquad \omega_m = \frac{2\gamma\rho}{D^2} \qquad \omega_\psi = \frac{\beta\psi}{D^2}$$

Critical points and their stability

	$\nu < 3/2$	$\nu = 3/2$	$\nu > 3/2$
No. of F.P.	1	0 or 1	1
Nature	A	A	S

Table 1: The fixed points for w=0, influx

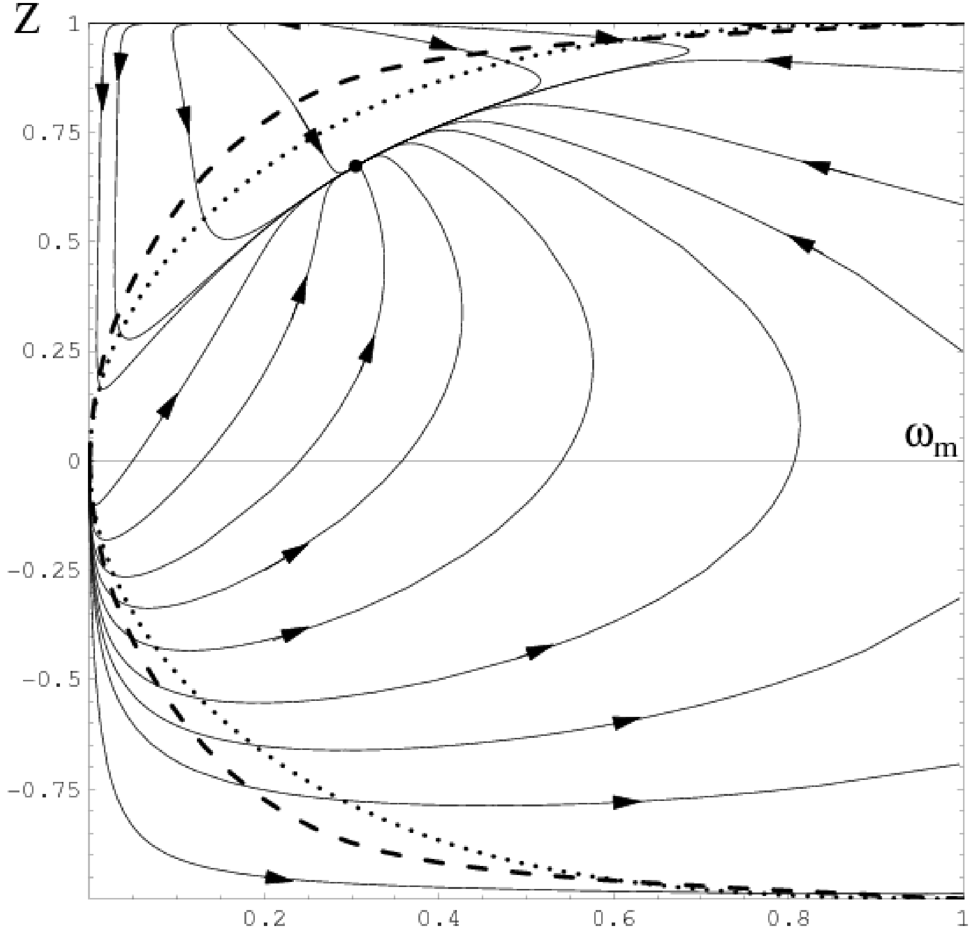
	$\nu<3/2$	$\nu=3/2$	$3/2<\nu<2$	$\nu=2$	$\nu>2$
No. of F.P.	1	0 or 1	0 or 2	0 or 1	1
Nature	A	A	A,S	S	S

Table 2: The fixed points for w=1/3, influx

Kofinas, Panotopoulos, Tomaras, hep-th/0510207

Numerical results for brane model

- Evolution in the $\omega_m - Z$ plane for $k = 0, w = 0, A < 0$



hep-th/
0510207

Coincidence problem with dynamical dark energy

- Cosmic acceleration → scalar field (idea from inflation)
- Exponential potential → cosmological scaling solutions ($\frac{\Omega_{DE}}{\Omega_M} = const.$)
- Interaction between DE and DM
  - Possibility that cannot be excluded
  - Can be realized in brane-models with brane-bulk energy exchange and in 4D models with -varying mass
  - We can have $w_{eff} < -1$ (allowed by data but problematic in theory) even though $w_{\phi} > -1$

Interacting dynamical dark energy: Quintessence

- Cosmological constant problem OK (not vacuum energy anymore)
- Coincidence problem → Interaction between DE & DM
- Usually assume source $Q \propto \rho_{dm}$ (linear)
- Model with $Q \propto \rho_{dm}\rho_{\phi}$ (0911.3089, quadratic)
- Our idea: Lagrangian description & comparison to data

Our model

- Dark energy

  → Canonical scalar field ϕ (Quintessence)
- Dark matter

  → Fermion Ψ
- Self-interaction potential $V(\phi)$
- Interaction

  → Lagrangian mass term for dark matter

$$m_{dm}(\phi)\bar{\Psi}\,\Psi$$

Mena, Honorez, Panotopoulos, 1009.5263

Equations of motion

- For dark matter
- For scalar field

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -Q/\dot{\phi}$$

- Source:

$$Q = \frac{\partial \ln m_{dm}(\phi)}{\partial \phi}\rho_{dm}\dot{\phi}$$

- Requirement $Q \propto \rho_{dm}\rho_{\phi} \approx \rho_{dm}V(\phi)$ satisfied by

$$V(\phi) = M^4 \exp[-\alpha\phi/M_{pl}]$$

$$m_{dm}(\phi) = \exp\left[\left(V(\phi)/\rho_{cr}^0\right)^n\right]$$

Phase-space analysis

- Define new dimensionless variables

$$y^2 = \frac{\kappa^2 V}{3H^2} \qquad x^2 = \frac{\kappa^2 \dot{\varphi}^2}{6H^2} \qquad z = \frac{H_0}{H + H_0}$$

$$\kappa^2 = 8\pi G$$

- 1st Friedmann eq. (constraint)

$$\Omega_{dm} = 1 - x^2 - y^2$$

- 2nd Friedmann eq. (dynamical)

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(1 + x^2 - y^2)$$

New dynamical equations

$$x' = -3x + \frac{3}{4}\frac{\alpha}{\sqrt{3\pi}}\,y^2 + \frac{3}{4}\frac{\alpha}{\sqrt{3\pi}}\,y^{2n}\frac{(1-z)^{2n}}{z^{2n}}(1-x^2-y^2) + \frac{3}{2}x(1+x^2-y^2)$$

$$y' = -\alpha\frac{\sqrt{3}}{4\sqrt{\pi}}\,xy + \frac{3}{2}y(1+x^2-y^2)$$

$$z' = \frac{3}{2}z(1-z)(1+x^2-y^2)$$

$$\boxed{N = \log(a)}$$

Stable fixed point → acceleration

$$x_* = \frac{\alpha}{4\sqrt{3\pi}} \qquad y_* = \frac{1}{4}\sqrt{16 - \frac{\alpha^2}{3\pi}} \qquad z_* = 1$$

- Existence: $\Omega_{\phi *} = x_*^2 + y_*^2 \leq 1$
- Stability: $x = x_* + u$

  $y = y_* + v$

  $$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \mathcal{M} \begin{pmatrix} u \\ v \end{pmatrix}$$

  $\mathrm{Re}\, \lambda_1, \lambda_2 < 0$

- Acceleration: $w_{T*} = x_*^2 - y_*^2 < -\frac{1}{3} \rightarrow \boxed{\alpha < 4\sqrt{\pi}}$

Comparison with data

- Supernovae

$$d_L(z) = c(1+z)\int_0^z H(z)^{-1} dz \qquad \mu = 5\log\left(\frac{d_L}{Mpc}\right) + 25$$

- CMB

$$R = 1.7 \pm 0.03 \qquad R = (\Omega_m H_0^2)^{1/2} \int_0^{1089} dz/H(z)$$

- BAO

$$A = 0.469 \pm 0.017$$

$$A = \sqrt{\Omega_m H_0^2}\left(\frac{d_L(z=0.35)^2}{H(z=0.35)(1+0.35)^2 0.35^2)}\right)^{1/3}$$

Global χ^2 analysis

- Supernovae

$$\chi^2_{SNIa}(c_i) = \sum_{z,z'} \left(\mu(c_i, z) - \mu_{obs}(z)\right) C^{-1}_{z:z'} \left(\mu(c_i, z') - \mu_{obs}(z')\right)$$

- CMB

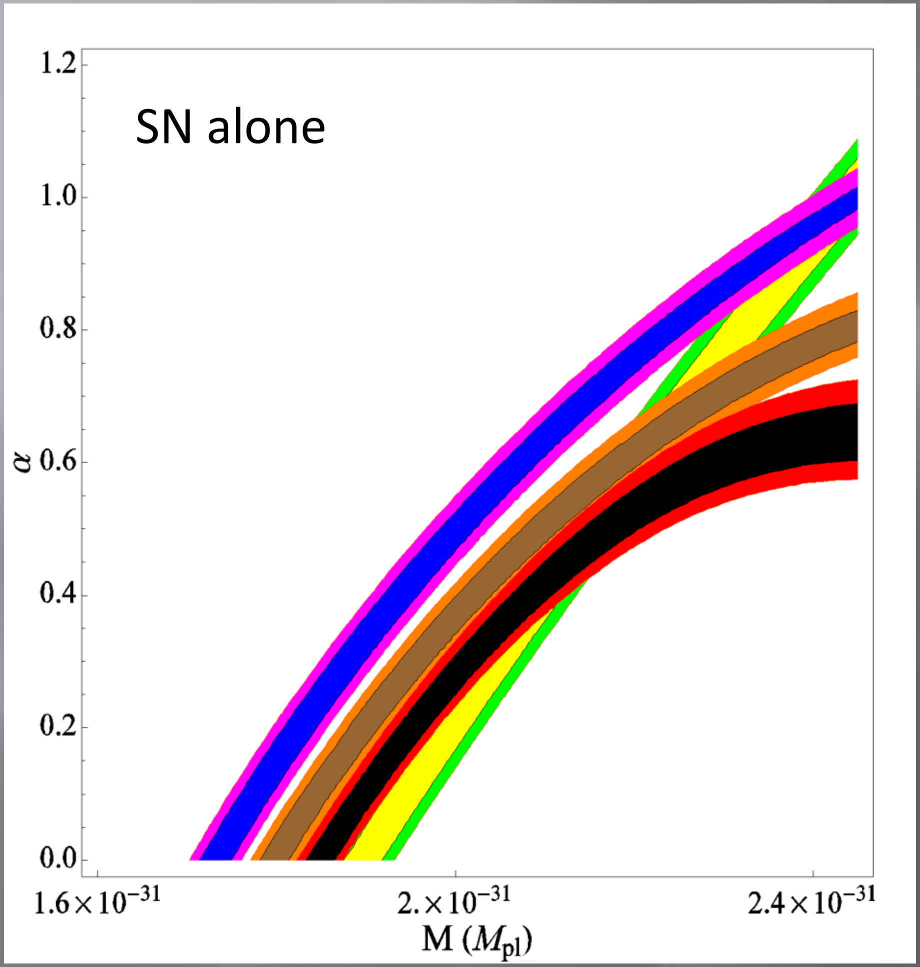
$$\chi^2_{CMB}(c_i) = \left[(R(c_i) - R)/\sigma_R\right]^2$$

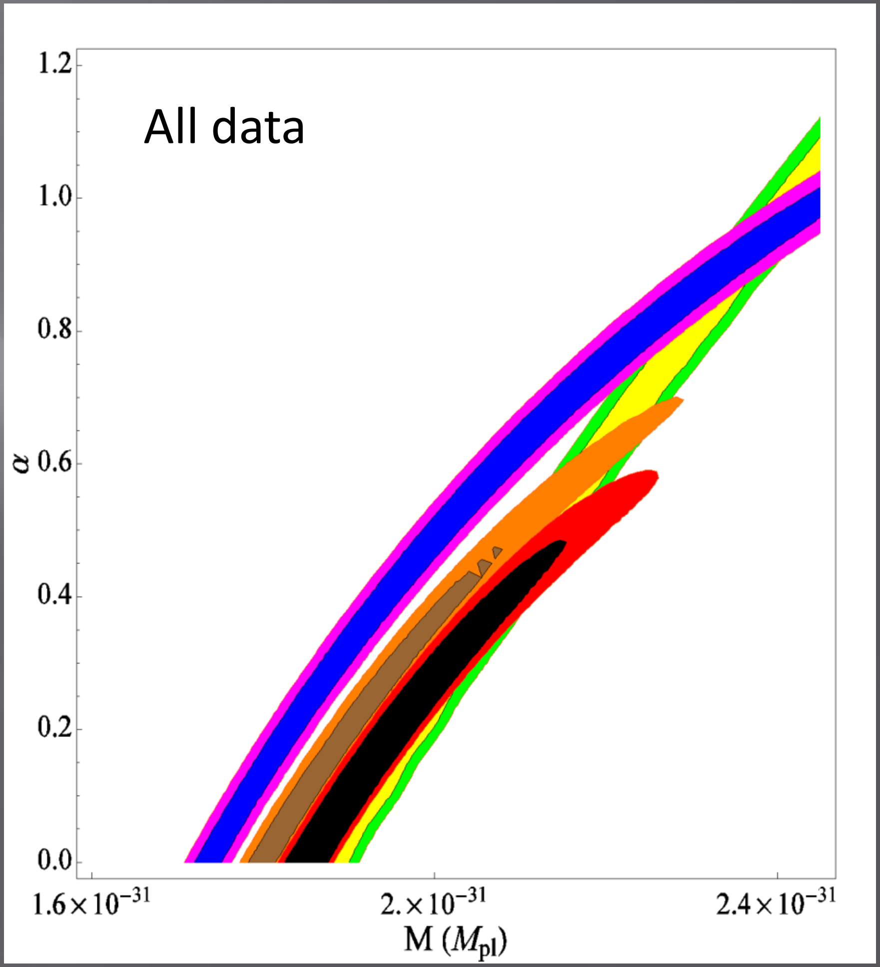
- BAO

$$\chi^2_{BAO}(c_i) = \left[(A(c_i, z = 0.35) - A)/\sigma_{A(z=0.35)}\right]^2$$

$$\chi^2_{tot}(c_i) = \chi^2_{SNIa}(c_i) + \chi^2_{BAO}(c_i) + \chi^2_{CMB}(c_i)$$

Numerical Results for 4D model





Diagnostic for different cosmological models

- Many models with similar expansion history that cannot be excluded by data
- Quantities that can be measured and computed within a model
- Appropriate quantities

$$r = \frac{\dddot{a}}{aH^3} \qquad s = \frac{r-1}{3(q-\frac{1}{2})}$$

$$H = \frac{\dot{a}}{a} \qquad q = -\frac{\ddot{a}}{aH^2}$$

Sahni, Saini, Starobinsky, Alam, astro-ph/0201498;
Alam, Sahni, Saini, Starobinsky, astro-ph/0303009

4D model with interacting dark energy

$$H^2 = \frac{\kappa^2}{3}\rho \qquad \dot{H} = -\frac{\kappa^2}{2}(\rho + p)$$

$$Q = \dot{\rho}_m + 3H\rho_m \qquad -Q = \dot{\rho}_X + 3H(\rho_X + p_X)$$

$$\rho = \rho_m + \rho_X \qquad p = p_m + p_X = p_X = w\rho_X$$

$$\kappa^2 = 8\pi G \qquad Q = \delta H \rho_m$$

- Use of dimensionless quantities

$$\Omega_m = \frac{\kappa^2 \rho_m}{3H^2} \qquad \Omega_X = \frac{\kappa^2 \rho_X}{3H^2} \qquad \Omega_m + \Omega_X = 1$$

- Upon comparison to observational data

$$\delta = -0.03 \qquad w = -1.02 \qquad \Omega_{X,0} = 0.73$$

astro-ph/0702015

Statefinders

$$s = 1 + w + \frac{\delta}{3}(\frac{1}{\Omega_X} - 1) \qquad r = 1 + \frac{9}{2}\, w\Omega_X s$$

$$\Omega_X' = -(1 - \Omega_X)(\delta + 3w\Omega_X) \qquad N = \ln a \qquad \Omega_m' = -\Omega_X'$$

- Critical points: two fixed points

Stable $\Omega_{*,1} = 1$ $\Omega_{*,2} = -\frac{\delta}{3w}$ Unstable

  - At the stable critical point

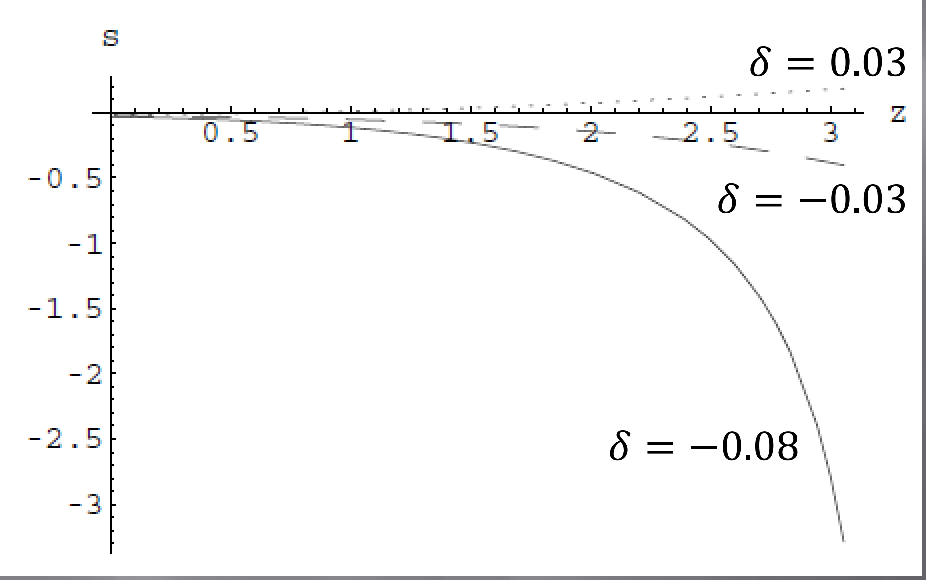
$$s = 1 + w \qquad r = 1 + \frac{9}{2}w(1 + w)$$

- Special case: cosmological constant

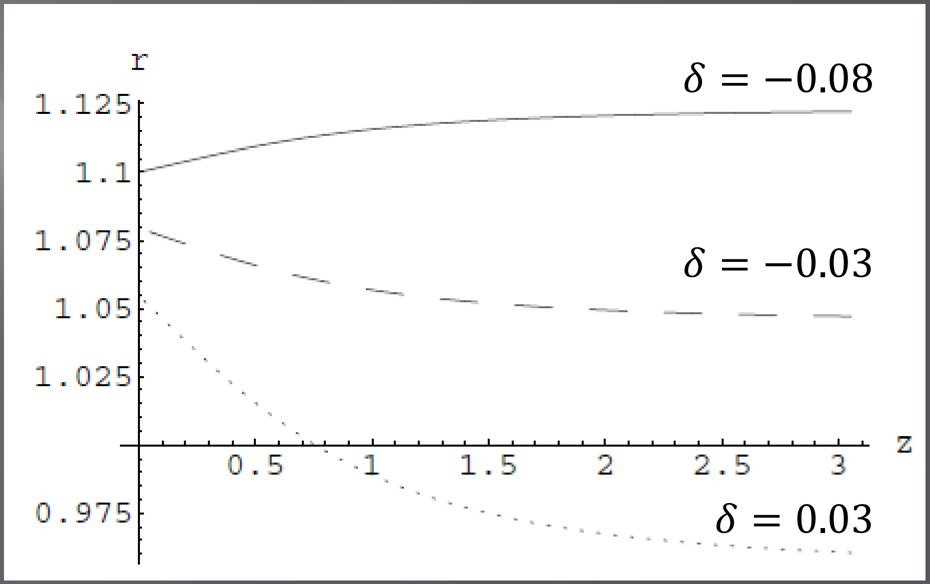
$$w = -1 \qquad \delta = 0 \qquad s = 0 \qquad r = 1$$

Numerical results for the 4D model

- Parameter s vs. red-shift



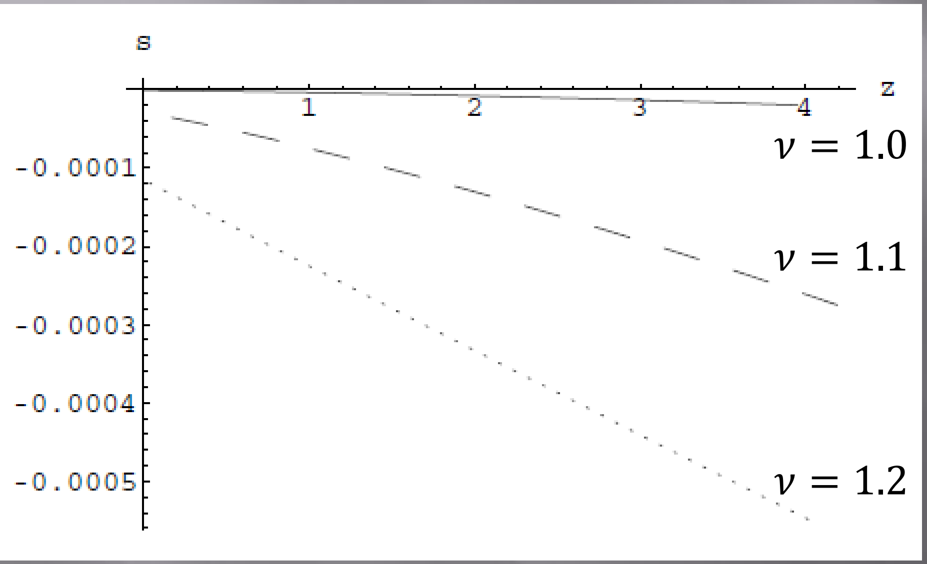
- Parameter r vs. red-shift

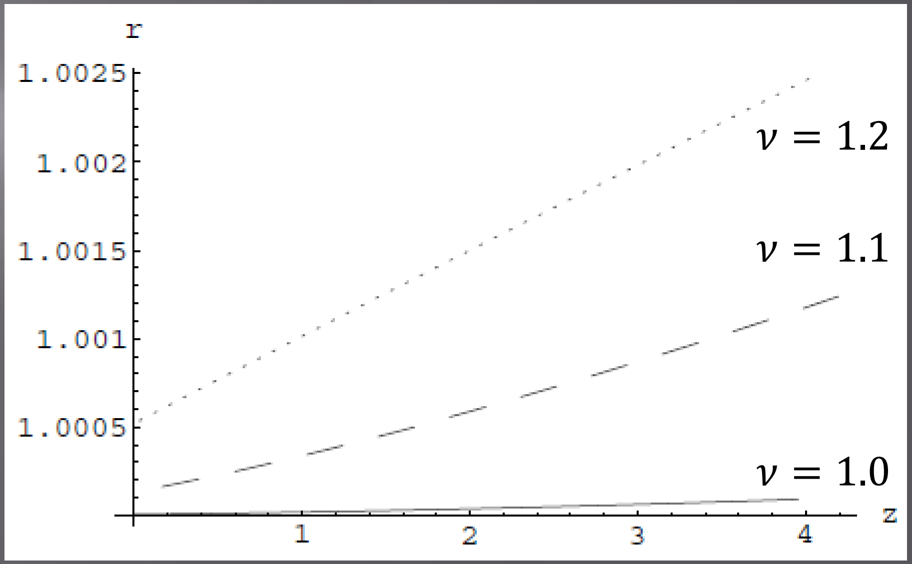


G. Panotopoulos, 0712.1177

Numerical results for the brane model

- Parameter s vs. red-shift
- Parameter r vs. red-shift

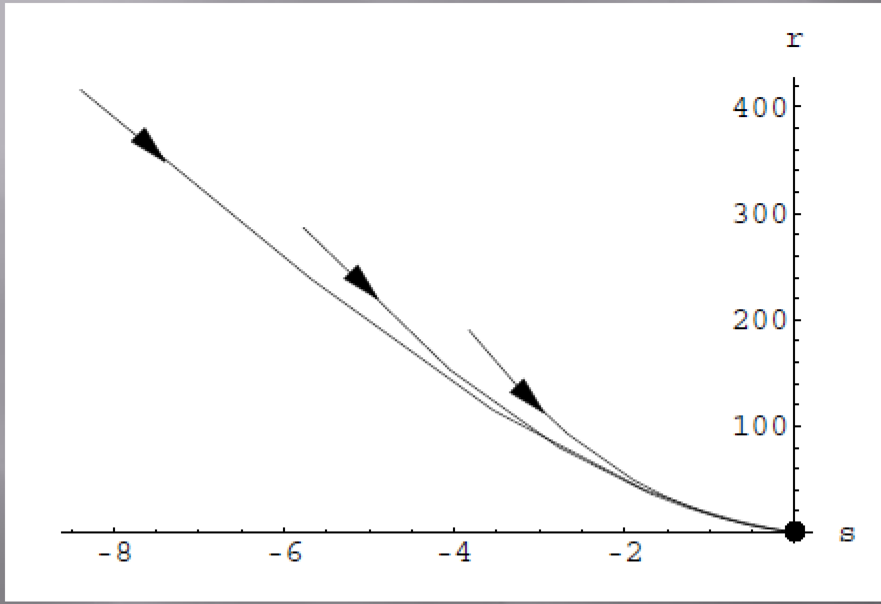




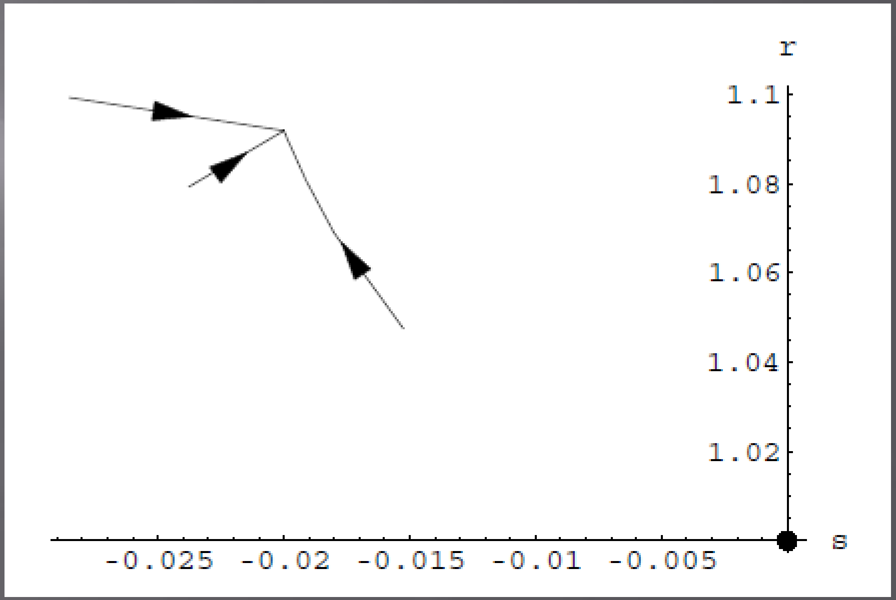
G. Panotopoulos , 0712.1177

(s-r) plane for both models

- 5D brane model



- 4D model in GR



G. Panotopoulos, 0712.1177

Conclusions

- Cosmic acceleration → Dark energy
- Cosmological constant & ΛCDM:
  most economical model
- Other possibilities
  - Dynamical dark energy (quintessence)
  - Geometrical dark energy (brane models)
- Statefinders: can discriminate between different dark energy models with same expansion history