Isocurvatons During Inflation

the Heavy, the Quasi-, and the Light

In collaboration with

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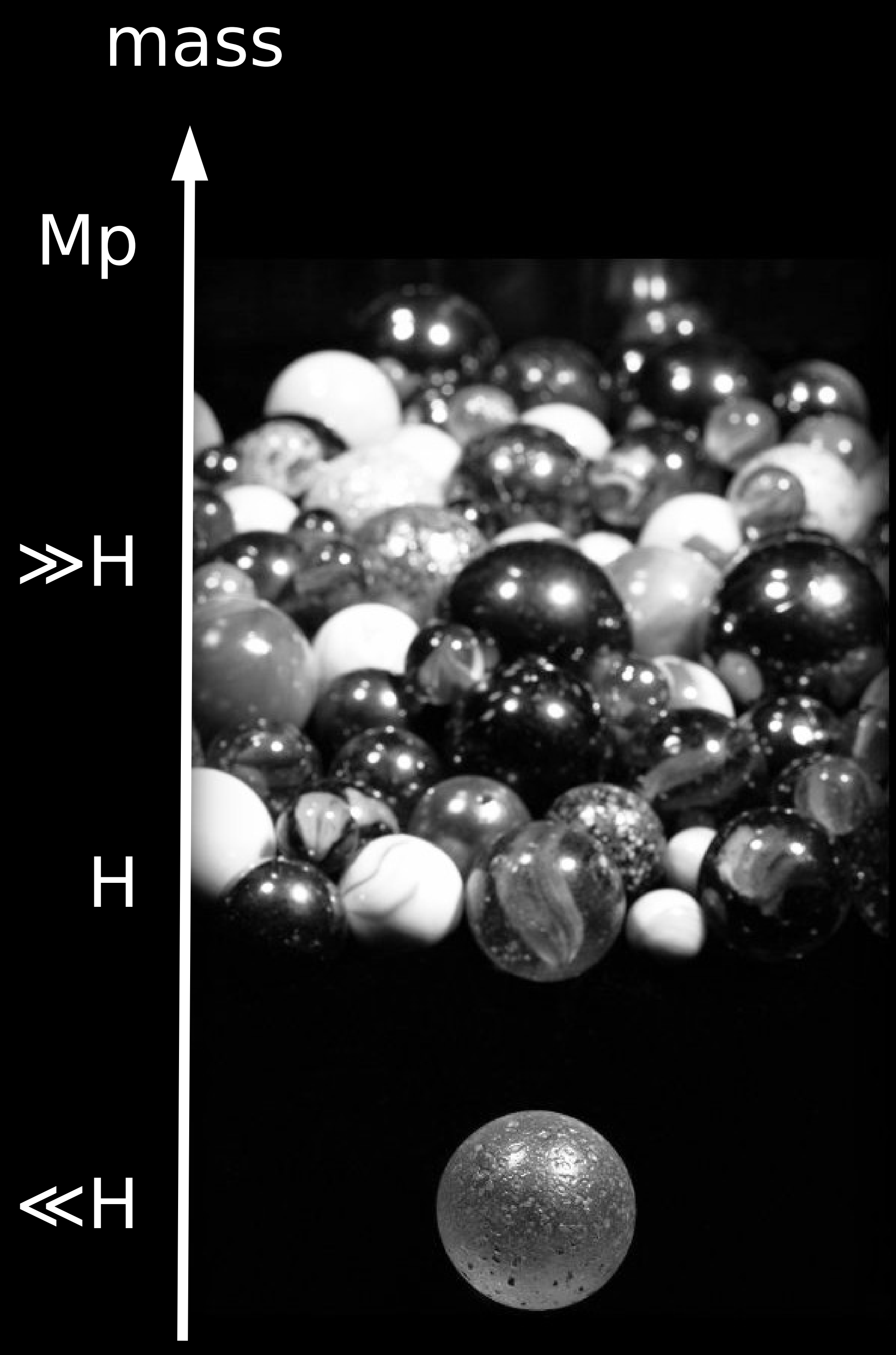
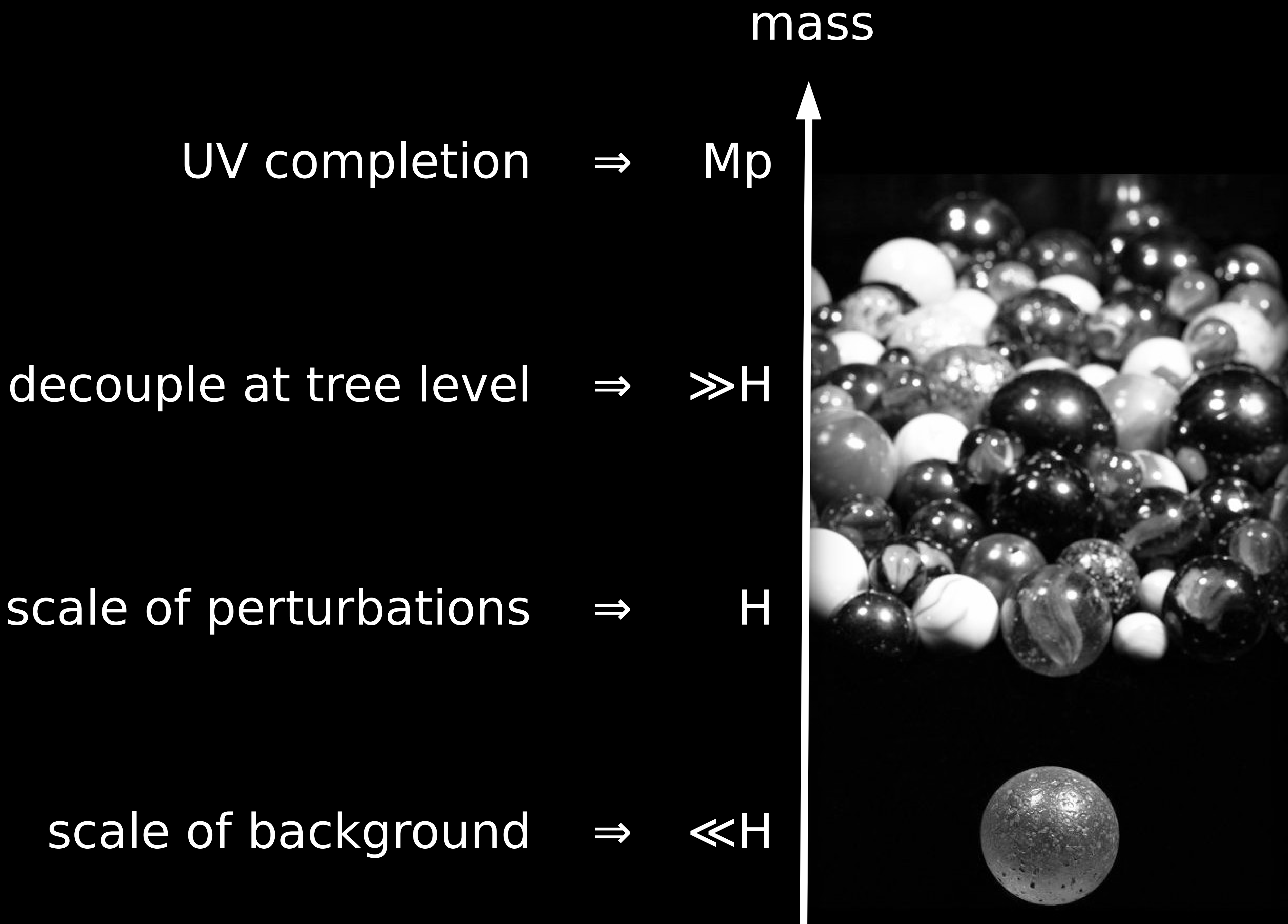
Yi Wang (王一), IPMU, Oct. 2, 2012

Isocurvatons during inflation




mass
UV completion ⇒ Mp
decouple at tree level ⇒ ≫H
scale of perturbations ⇒ H
scale of background ⇒ ≪H

Theories of

- the heavy
- the quasi-
- the light

mass

UV completion ⇒ M_p

decouple at tree level ⇒ $\gg H$

scale of perturbations ⇒ H

scale of background ⇒ $\ll H$

Part 1: Theory of the heavy:

Chen, YW (12)

single field inflation

+

$M \gg H$ directions

=

decouple or not?

mass

Mp

$\gg$H

H

$\ll$H

Why $M \gg H$ fields matter?

Why not decouple?

Not only propagators,

but also “coupling constant”,

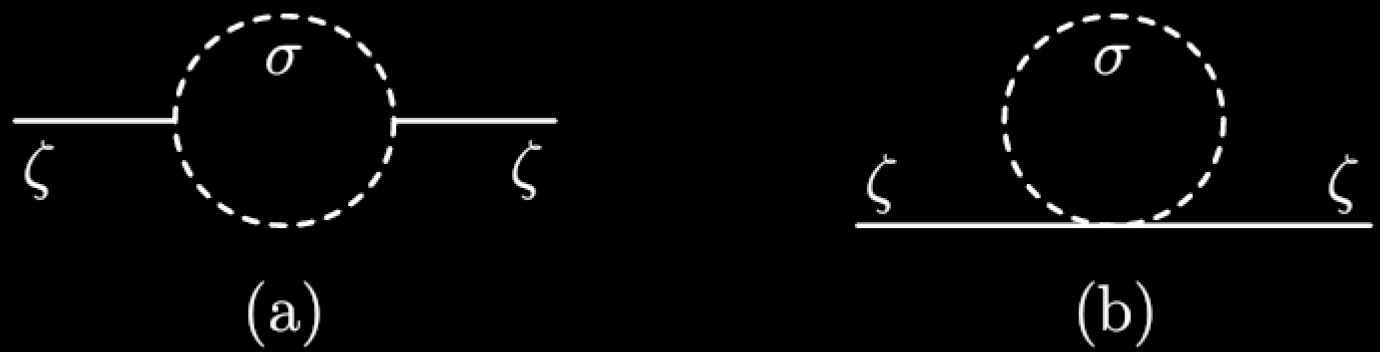
Gravity couples to energy.

depend on mass (and kinetic energy).

$$\mathcal{L}_m = \sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2 - V_{\rm sr}(\phi) - \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}M^2\sigma^2\right]$$

- the minimal model
- universal “lower bound” of general couplings

$$\mathcal{L}_m = \sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2 - V_{\rm sr}(\phi) - \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}M^2\sigma^2\right]$$



To show:

- for 1 massive field: $$\frac{\Delta P_\zeta}{P_\zeta} \sim P_\zeta \frac{\Lambda^4}{H^4} N_e$$
- for a KK-tower: $$\frac{\Delta P_\zeta}{P_\zeta} \sim \frac{\Lambda^4 N_e}{\epsilon H^2 M_{\rm Pl}^2} \times \frac{M_{\rm Pl}^2}{M_D^2} \sim \frac{M_D^2 N_e}{\epsilon H^2}$$

Steps:

- The pert. action up to 4th order
- The interaction Hamiltonian
- Calc. using in-in formalism
- Calc. integrals: $\int dt$ and $\int d^3p$

The pert. action up to 4th order

ζ-gauge

$$ds^2 = -\hat{N}^2 d\hat{t}^2 + \hat{h}_{ij}(d\hat{x}^i + \hat{N}^i d\hat{t})(d\hat{x}^j + \hat{N}^j d\hat{t})$$

$$\hat{h}_{ij} = a^2(\hat{t}) e^{2\zeta(\hat{t},\hat{\mathbf{x}})}\delta_{ij} \ , \ \hat{\phi}(\hat{t},\hat{\mathbf{x}}) = \phi_0(\hat{t}) \ , \ \hat{\sigma} = \hat{\sigma}(\hat{t},\hat{\mathbf{x}})$$

δφ-gauge

$$h_{ij} = a^2(t)\delta_{ij} \ , \ \phi = \phi_0(t) + \delta\phi(t,\mathbf{x}) \ , \ \sigma = \sigma(t,\mathbf{x})$$

gauge-trans $\delta t_{(1)} = -\zeta/H$

$$\sigma(t,x^i) = \hat{\sigma}(t,x^i) + \dot{\hat{\sigma}}(\delta t_{(1)} + \delta t_{(2)}) + \partial_i\hat{\sigma}\delta x^i_{(2)} + \frac{1}{2}\ddot{\hat{\sigma}}\delta t^2_{(1)}$$

The pert. action up to 4th order

ζ-gauge

- Pros:
  - Conservation for single field
  - Close relation to observables
- Cons:
  - Hard to calculate
  - Slow roll order not manifest

δφ-gauge

  - The opposite pros and cons

Simplified L_4 in the ζ-gauge

$$-\frac{9}{4} a^3 M^2 \zeta^2 \sigma^2 - \frac{3 a^3 M^2 \zeta \sigma^2 \partial_0\zeta}{2H} + \frac{9}{4} a^3 \zeta^2 (\partial_0\sigma)^2 - \frac{3a^3 \zeta \partial_0\zeta (\partial_0\sigma)^2}{2H} + \frac{a^3(\partial_0\zeta)^2(\partial_0\sigma)^2}{2H^2} - \frac{a\partial_0\zeta\partial_i\zeta\partial_0\sigma\partial_i\sigma}{H^2} - \frac14 a\zeta^2(\partial_i\sigma)^2 - \frac{a\zeta\partial_0\zeta(\partial_i\sigma)^2}{2H} + \frac{\partial_i\zeta\partial_j\zeta\partial_i\sigma\partial_j\sigma}{2aH^2} + \frac{aM^2\zeta\sigma^2\partial_i\partial_i\zeta}{4H^2} +$$

$$\frac{aM^2\sigma^2\partial_0\zeta\partial_i\partial_i\zeta}{4H^3} + \frac{a\zeta(\partial_0\sigma)^2\partial_i\partial_i\zeta}{4H^2} + \frac{a\partial_0\zeta(\partial_0\sigma)^2\partial_i\partial_i\zeta}{4H^3} + \frac{\zeta(\partial_j\sigma)^2\partial_i\partial_i\zeta}{4aH^2} + \frac{\partial_0\zeta(\partial_j\sigma)^2\partial_i\partial_i\zeta}{4aH^3} - \frac{aM^2\zeta\partial_j\partial_i\zeta\partial_j\partial_i(\partial^{-2}\sigma^2)}{4H^2} - \frac{aM^2\partial_0\zeta\partial_j\partial_i\zeta\partial_j\partial_i(\partial^{-2}\sigma^2)}{4H^3} -$$

$$\frac{a\zeta\partial_j\partial_i\zeta\partial_j\partial_i(\partial^{-2}(\partial_0\sigma)^2)}{4H^2} - \frac{a\partial_0\zeta\partial_j\partial_i\zeta\partial_j\partial_i(\partial^{-2}(\partial_0\sigma)^2)}{4H^3} - \frac{\zeta\partial_k\partial_j\zeta\partial_k\partial_j(\partial^{-2}(\partial_i\sigma)^2)}{4aH^2} - \frac{\partial_0\zeta\partial_k\partial_j\zeta\partial_k\partial_j(\partial^{-2}(\partial_i\sigma)^2)}{4aH^3} - \frac{3a\zeta\partial_k\partial_j\zeta\partial_k\partial_j(\partial^{-2}(\partial^{-2}(\partial_i\sigma\partial_i\partial_0\sigma)))}{2H} -$$

$$\frac{3a\partial_0\zeta\partial_k\partial_j\zeta\partial_k\partial_j(\partial^{-2}(\partial^{-2}(\partial_i\sigma\partial_i\partial_0\sigma)))}{2H^2} - \frac{3a\zeta\partial_k\partial_j\zeta\partial_k\partial_j(\partial^{-2}(\partial^{-2}(\partial_0\sigma\partial_i\partial_i\sigma)))}{2H} - \frac{3a\partial_0\zeta\partial_k\partial_j\zeta\partial_k\partial_j(\partial^{-2}(\partial^{-2}(\partial_0\sigma\partial_i\partial_i\sigma)))}{2H^2} + \frac{aM^2\partial_i\zeta\partial_j\partial_j\zeta\partial_i(\partial^{-2}\sigma^2)}{2H^2} -$$

$$\frac{a\partial_0\sigma\partial_j\sigma\partial_j(\partial^{-2}(\partial_i\zeta)^2)}{2H} + \frac{a\partial_i\zeta\partial_j\partial_j\zeta\partial_i(\partial^{-2}(\partial_0\sigma)^2)}{2H^2} + \frac{\partial_j\zeta\partial_k\partial_k\zeta\partial_j(\partial^{-2}(\partial_i\sigma)^2)}{2aH^2} + \frac{3a\partial_0\sigma\partial_j\sigma\partial_j(\partial^{-2}(\zeta\partial_i\partial_i\zeta))}{2H} + \frac{a\partial_0\sigma\partial_j\sigma\partial_j(\partial^{-2}(\partial_0\zeta\partial_i\partial_i\zeta))}{H^2} + \frac{\partial_0\sigma\partial_k\sigma\partial_k(\partial^{-2}(\partial_j\partial_i\zeta)^2)}{4aH^3} -$$

$$\frac{\partial_0\sigma\partial_k\sigma\partial_k(\partial^{-2}(\partial_i\partial_i\zeta\partial_j\partial_j\zeta))}{4aH^3} + \frac{3a\partial_0\sigma\partial_k\sigma\partial_k(\partial^{-2}(\partial^{-2}(\partial_j\partial_i\zeta)^2))}{2H} + \frac{3a\partial_j\zeta\partial_k\partial_k\zeta\partial_j(\partial^{-2}(\partial^{-2}(\partial_i\sigma\partial_i\partial_0\sigma)))}{H} + \frac{3a\partial_j\zeta\partial_k\partial_k\zeta\partial_j(\partial^{-2}(\partial^{-2}(\partial_0\sigma\partial_i\partial_i\sigma)))}{H} +$$

$$\frac{2a\partial_0\zeta\partial_k\partial_k\zeta\partial_j(\partial^{-2}(\partial^{-2}(\partial_j\partial_0\sigma\partial_i\partial_i\sigma)))}{H^2} - \frac{2a\partial_0\zeta\partial_k\partial_k\zeta\partial_j(\partial^{-2}(\partial^{-2}(\partial_i\partial_0\sigma\partial_j\partial_i\sigma)))}{H^2} - \frac{3a\partial_0\sigma\partial_k\sigma\partial_k(\partial^{-2}(\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_i\partial_0\zeta)))}{2H^2} + \frac{3a\partial_0\sigma\partial_k\sigma\partial_k(\partial^{-2}(\partial^{-2}(\partial_j\partial_i\zeta\partial_j\partial_i\partial_0\zeta)))}{2H^2} -$$

$$\frac{2a\partial_0\zeta\partial_k\partial_k\zeta\partial_j(\partial^{-2}(\partial^{-2}(\partial_j\sigma\partial_i\partial_i\partial_0\sigma)))}{H^2} + \frac{2a\partial_0\zeta\partial_k\partial_k\zeta\partial_j(\partial^{-2}(\partial^{-2}(\partial_i\sigma\partial_j\partial_i\partial_0\sigma)))}{H^2} - \frac{3a\partial_0\sigma\partial_k\sigma\partial_k(\partial^{-2}(\partial^{-2}(\zeta\partial_j\partial_j\partial_i\partial_i\zeta)))}{2H} + \frac{2a\partial_0\sigma\partial_j\sigma\partial^{-2}(\partial_j\zeta\partial_i\partial_i\zeta)}{H} -$$

$$\frac{2a\partial_0\sigma\partial_j\sigma\partial^{-2}(\partial_j\partial_0\zeta\partial_i\partial_i\zeta)}{H^2} - \frac{2a\partial_0\sigma\partial_j\sigma\partial^{-2}(\partial_i\zeta\partial_j\partial_i\zeta)}{H} + \frac{2a\partial_0\sigma\partial_j\sigma\partial^{-2}(\partial_i\partial_0\zeta\partial_j\partial_i\zeta)}{H^2} - \frac{a(\partial_j\zeta)^2\partial^{-2}(\partial_i\sigma\partial_i\partial_0\sigma)}{H} + \frac{2a\zeta\partial_j\partial_j\zeta\partial^{-2}(\partial_i\sigma\partial_i\partial_0\sigma)}{H} + \frac{3a\partial_0\zeta\partial_j\partial_j\zeta\partial^{-2}(\partial_i\sigma\partial_i\partial_0\sigma)}{2H^2} +$$

$$\frac{3a\partial^{-2}(\partial_k\partial_j\zeta)^2\partial^{-2}(\partial_i\sigma\partial_i\partial_0\sigma)}{2H} - \frac{a(\partial_j\zeta)^2\partial^{-2}(\partial_0\sigma\partial_i\partial_i\sigma)}{H} + \frac{2a\zeta\partial_j\partial_j\zeta\partial^{-2}(\partial_0\sigma\partial_i\partial_i\sigma)}{H} + \frac{3a\partial_0\zeta\partial_j\partial_j\zeta\partial^{-2}(\partial_0\sigma\partial_i\partial_i\sigma)}{2H^2} + \frac{3a\partial^{-2}(\partial_k\partial_j\zeta)^2\partial^{-2}(\partial_0\sigma\partial_i\partial_i\sigma)}{2H} -$$

$$\frac{3a\partial^{-2}(\partial_k\sigma\partial_k\partial_0\sigma)\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_i\partial_0\zeta)}{2H^2} - \frac{3a\partial^{-2}(\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_i\partial_0\zeta)}{2H^2} + \frac{3a\partial^{-2}(\partial_k\sigma\partial_k\partial_0\sigma)\partial^{-2}(\partial_j\partial_i\zeta\partial_j\partial_i\partial_0\zeta)}{2H^2} + \frac{3a\partial^{-2}(\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\partial_j\partial_i\zeta\partial_j\partial_i\partial_0\zeta)}{2H^2} -$$

$$\frac{3a\partial^{-2}(\partial_k\sigma\partial_k\partial_0\sigma)\partial^{-2}(\zeta\partial_j\partial_j\partial_i\partial_i\zeta)}{2H} - \frac{3a\partial^{-2}(\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\zeta\partial_j\partial_j\partial_i\partial_i\zeta)}{2H} + \frac{4a\partial^{-2}(\partial_k\partial_0\sigma\partial_k\partial_i\sigma)\partial^{-2}(\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_a\partial_a\zeta))}{H} - \frac{4a\partial^{-2}(\partial_i\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_a\partial_a\zeta))}{H} -$$

$$\frac{2a\partial^{-2}(\partial_k\partial_0\sigma\partial_k\partial_i\sigma)\partial^{-2}(\partial^{-2}(\partial_j\partial_j\partial_0\zeta\partial_i\partial_a\partial_a\zeta))}{H^2} + \frac{2a\partial^{-2}(\partial_i\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\partial^{-2}(\partial_j\partial_j\partial_0\zeta\partial_i\partial_a\partial_a\zeta))}{H^2} - \frac{4a\partial^{-2}(\partial_k\partial_0\sigma\partial_k\partial_j\sigma)\partial^{-2}(\partial^{-2}(\partial_i\partial_a\zeta\partial_j\partial_i\partial_a\zeta))}{H} +$$

$$\frac{4a\partial^{-2}(\partial_j\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\partial^{-2}(\partial_i\partial_a\zeta\partial_j\partial_i\partial_a\zeta))}{H} + \frac{2a\partial^{-2}(\partial_k\partial_0\sigma\partial_k\partial_j\sigma)\partial^{-2}(\partial^{-2}(\partial_i\partial_a\partial_0\zeta\partial_j\partial_i\partial_a\zeta))}{H^2} - \frac{2a\partial^{-2}(\partial_j\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\partial^{-2}(\partial_i\partial_a\partial_0\zeta\partial_j\partial_i\partial_a\zeta))}{H^2} -$$

$$\frac{4a\partial_0\sigma\partial_j\sigma\partial^{-2}(\partial^{-2}(\partial_k\partial_k\zeta\partial_j\partial_i\partial_i\zeta))}{H} + \frac{2a\partial_0\sigma\partial_j\sigma\partial^{-2}(\partial^{-2}(\partial_k\partial_k\partial_0\zeta\partial_j\partial_i\partial_i\zeta))}{H^2} + \frac{4a\partial_0\sigma\partial_k\sigma\partial^{-2}(\partial^{-2}(\partial_j\partial_i\zeta\partial_k\partial_j\partial_i\zeta))}{H} - \frac{2a\partial_0\sigma\partial_k\sigma\partial^{-2}(\partial^{-2}(\partial_j\partial_i\partial_0\zeta\partial_k\partial_j\partial_i\zeta))}{H^2} +$$

$$\frac{4a\partial^{-2}(\partial_k\partial_k\zeta\partial_j\partial_j\partial_i\zeta)\partial^{-2}(\partial^{-2}(\partial_i\sigma\partial_a\partial_a\partial_0\sigma))}{H} - \frac{2a\partial^{-2}(\partial_k\partial_k\partial_0\zeta\partial_j\partial_j\partial_i\zeta)\partial^{-2}(\partial^{-2}(\partial_i\sigma\partial_a\partial_a\partial_0\sigma))}{H^2} - \frac{4a\partial^{-2}(\partial_k\partial_j\zeta\partial_k\partial_j\partial_i\zeta)\partial^{-2}(\partial^{-2}(\partial_i\sigma\partial_a\partial_a\partial_0\sigma))}{H} +$$

$$\frac{2a\partial^{-2}(\partial_k\partial_j\partial_0\zeta\partial_k\partial_j\partial_i\zeta)\partial^{-2}(\partial^{-2}(\partial_i\sigma\partial_a\partial_a\partial_0\sigma))}{H^2} - \frac{4a\partial^{-2}(\partial_k\partial_k\zeta\partial_j\partial_j\partial_i\zeta)\partial^{-2}(\partial^{-2}(\partial_a\sigma\partial_i\partial_a\partial_0\sigma))}{H} + \frac{2a\partial^{-2}(\partial_k\partial_k\partial_0\zeta\partial_j\partial_j\partial_i\zeta)\partial^{-2}(\partial^{-2}(\partial_a\sigma\partial_i\partial_a\partial_0\sigma))}{H^2} +$$

$$\frac{4a\partial^{-2}(\partial_k\partial_j\zeta\partial_k\partial_j\partial_i\zeta)\partial^{-2}(\partial^{-2}(\partial_a\sigma\partial_i\partial_a\partial_0\sigma))}{H} - \frac{2a\partial^{-2}(\partial_k\partial_j\partial_0\zeta\partial_k\partial_j\partial_i\zeta)\partial^{-2}(\partial^{-2}(\partial_a\sigma\partial_i\partial_a\partial_0\sigma))}{H^2} - \frac{2a\partial^{-2}(\partial_k\partial_0\sigma\partial_k\partial_i\sigma)\partial^{-2}(\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_a\partial_a\partial_0\zeta))}{H^2} +$$

$$\frac{2a\partial^{-2}(\partial_i\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_a\partial_a\partial_0\zeta))}{H^2} + \frac{2a\partial^{-2}(\partial_k\sigma\partial_k\partial_i\partial_0\sigma)\partial^{-2}(\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_a\partial_a\partial_0\zeta))}{H^2} - \frac{2a\partial^{-2}(\partial_i\sigma\partial_k\partial_k\partial_0\sigma)\partial^{-2}(\partial^{-2}(\partial_j\partial_j\zeta\partial_i\partial_a\partial_a\partial_0\zeta))}{H^2} +$$

$$\frac{2a\partial^{-2}(\partial_k\partial_0\sigma\partial_k\partial_j\sigma)\partial^{-2}(\partial^{-2}(\partial_i\partial_a\zeta\partial_j\partial_i\partial_a\partial_0\zeta))}{H^2} - \frac{2a\partial^{-2}(\partial_j\partial_0\sigma\partial_k\partial_k\sigma)\partial^{-2}(\partial^{-2}(\partial_i\partial_a\zeta\partial_j\partial_i\partial_a\partial_0\zeta))}{H^2} - \frac{2a\partial^{-2}(\partial_k\sigma\partial_k\partial_j\partial_0\sigma)\partial^{-2}(\partial^{-2}(\partial_i\partial_a\zeta\partial_j\partial_i\partial_a\partial_0\zeta))}{H^2} +$$

$$\frac{2a\partial^{-2}(\partial_j\sigma\partial_k\partial_k\partial_0\sigma)\partial^{-2}(\partial^{-2}(\partial_i\partial_a\zeta\partial_j\partial_i\partial_a\partial_0\zeta))}{H^2} + \frac{2a\partial_0\sigma\partial_j\sigma\partial^{-2}(\partial^{-2}(\partial_k\partial_k\zeta\partial_j\partial_i\partial_i\partial_0\zeta))}{H^2} - \frac{2a\partial_0\sigma\partial_k\sigma\partial^{-2}(\partial^{-2}(\partial_j\partial_i\zeta\partial_k\partial_j\partial_i\partial_0\zeta))}{H^2}$$

The pert. action up to 4th order

δφ-gauge:

$$\mathcal{L}_2 = \frac{a^3}{2}\delta\dot{\phi}^2 - \frac{a}{2}(\partial_i\delta\phi)^2 + \frac{a^3}{2}\dot{\sigma}^2 - \frac{a}{2}(\partial_i\sigma)^2 - \frac{a^3}{2}M^2\sigma^2$$

higher orders: slow roll suppressed.

Note that for σ, gauge transformation is

$$\hat{\sigma} = \sigma - \frac{\zeta\dot{\sigma}}{H} + \mathcal{O}(\zeta^3, \zeta\sigma^2)$$

Not slow roll restricted!

c.f. $\delta\varphi^2 \sim \epsilon\zeta^2$

The pert. action up to 4th order

$$\mathcal{L}_2 = \frac{a^3}{2}\delta\dot{\phi}^2 - \frac{a}{2}(\partial_i\delta\phi)^2 + \frac{a^3}{2}\dot{\sigma}^2 - \frac{a}{2}(\partial_i\sigma)^2 - \frac{a^3}{2}M^2\sigma^2$$

$$\hat{\sigma} = \sigma - \frac{\zeta\dot{\sigma}}{H} + \mathcal{O}(\zeta^3, \zeta\sigma^2)$$

$$\mathcal{L}_3 = \mathcal{O}(\epsilon\zeta\sigma^2)\ ,$$

$$\mathcal{L}_4 = \frac{a^3}{2H^2}\left[(\partial_t(\dot{\sigma}\zeta))^2 - \frac{1}{a^2}\left(\partial_i(\dot{\sigma}\zeta)\right)^2 - M^2(\dot{\sigma}\zeta)^2\right]$$

(up to total derivatives)

The interaction Hamiltonian

$$F^{ab} f^{(0)}_{bc} = \delta^a_c$$

$$\begin{aligned}\mathcal{L} =& f^{(0)}_{ab}\dot\alpha^a\dot\alpha^b + j_2 \\ &+ g^{(0)}_{abc}\dot\alpha^a\dot\alpha^b\dot\alpha^c + g^{(1)}_{ab}\dot\alpha^a\dot\alpha^b + g^{(2)}_a\dot\alpha^a + j_3 \\ &+ h^{(0)}_{abcd}\dot\alpha^a\dot\alpha^b\dot\alpha^c\dot\alpha^d + h^{(1)}_{abc}\dot\alpha^a\dot\alpha^b\dot\alpha^c + h^{(2)}_{ab}\dot\alpha^a\dot\alpha^b + h^{(3)}_a\dot\alpha^a + j_4\end{aligned}$$

$$\begin{aligned}\mathcal{H} =& f^{(0)}_{ab}\dot\alpha^a_I\dot\alpha^b_I - j_2 \\ &- g^{(0)}_{abc}\dot\alpha^a_I\dot\alpha^b_I\dot\alpha^c_I - g^{(1)}_{ab}\dot\alpha^a_I\dot\alpha^b_I - g^{(2)}_a\dot\alpha^a_I - j_3 \\ &- h^{(0)}_{abcd}\dot\alpha^a_I\dot\alpha^b_I\dot\alpha^c_I\dot\alpha^d_I - h^{(1)}_{abc}\dot\alpha^a_I\dot\alpha^b_I\dot\alpha^c_I - h^{(2)}_{ab}\dot\alpha^a_I\dot\alpha^b_I - h^{(3)}_a\dot\alpha^a_I - j_4 \\ &+ \frac{9}{4}F^{ab}g^{(0)}_{acd}g^{(0)}_{bef}\dot\alpha^c_I\dot\alpha^d_I\dot\alpha^e_I\dot\alpha^f_I + 3F^{ab}g^{(0)}_{acd}g^{(1)}_{be}\dot\alpha^c_I\dot\alpha^d_I\dot\alpha^e_I + \frac{3}{2}F^{ab}g^{(0)}_{acd}g^{(2)}_b\dot\alpha^c_I\dot\alpha^d_I \\ &+ F^{ab}g^{(1)}_{ac}g^{(1)}_{bd}\dot\alpha^c_I\dot\alpha^d_I + F^{ab}g^{(1)}_{ac}g^{(2)}_b\dot\alpha^c_I + \frac{1}{4}F^{ab}g^{(2)}_a g^{(2)}_b\ .\end{aligned}$$

The interaction Hamiltonian

(only listed a few sample terms)

$$\mathcal{H}_0 = \epsilon a^3 \dot{\zeta}^2 + \epsilon a (\partial\zeta)^2 + \frac{a^3}{2}\dot{\sigma}^2 + \frac{a}{2}(\partial_i\sigma)^2 + \frac{a^3}{2}M^2\sigma^2 \ ,$$

$$\mathcal{H}_I = -\frac{3a^3}{H}\zeta\dot{\zeta}\dot{\sigma}^2 + \frac{a}{4}\zeta^2(\partial_i\sigma)^2 + \frac{9a^3}{4}M^2\zeta^2\sigma^2 + \cdots \ .$$

Calc. using in-in formalism

$$\begin{aligned}\langle Q\rangle &= \langle\Omega|F^{-1}(t,t_0)Q^I(t)F(t,t_0)|\Omega\rangle\ , \\ &= \langle\Omega|\left[\bar{T}\exp\left(i\int_{t_0}^{t}H_I(t)dt\right)\right]\ Q^I(t)\ \left[T\exp\left(-i\int_{t_0}^{t}H_I(t)dt\right)\right]|\Omega\rangle\end{aligned}$$

Calc. using in-in formalism

$$\zeta_{\mathbf{k}} = u_{\mathbf{k}} a_{\mathbf{k}} + u^*_{-\mathbf{k}} a^\dagger_{-\mathbf{k}} \ , \quad \sigma_{\mathbf{k}} = v_{\mathbf{k}} b_{\mathbf{k}} + v^*_{-\mathbf{k}} b^\dagger_{-\mathbf{k}}$$

$$u''_{\mathbf{k}} - \frac{2}{\tau} u'_{\mathbf{k}} + k^2 u_{\mathbf{k}} = 0$$

$$v''_{\mathbf{k}} - \frac{2}{\tau} v'_{\mathbf{k}} + k^2 v_{\mathbf{k}} + \frac{M^2}{H^2 \tau^2} v_{\mathbf{k}} = 0$$

$$u_{\mathbf{k}} = -\frac{H}{\sqrt{4\epsilon k^3}} (1 + ik\tau)\ e^{-ik\tau} \ ,$$

$$v_{\mathbf{k}} = -ie^{i\pi/2} e^{-\pi\mu/2} \frac{\sqrt{\pi}}{2} H(-\tau)^{3/2} H^{(1)}_{i\mu}(-k\tau)$$

$$\mu = \sqrt{M^2/H^2 - 9/4}$$

Calc. using in-in formalism

$$\Delta\langle\zeta^2\rangle \supset 2i\left(u_k^*(\tau_{\rm end})\right)^2 (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')$$

$$\times \int_{-\infty}^{\tau_{\rm end}} d\tau \left[-\frac{3a}{H}u_k u_k' \int \frac{d^3\mathbf{p}}{(2\pi)^3}|v_p'|^2\right.$$

$$\left. + \frac{a^2}{4}u_k^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3}p^2|v_p|^2 + \frac{9M^2a^4}{4}u_k^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3}|v_p|^2\right]$$

$$+ \text{c.c.} \; .$$

$$u_{\mathbf{k}} = -\frac{H}{\sqrt{4\epsilon k^3}}(1+ik\tau)\, e^{-ik\tau} \; ,$$

$$v_{\mathbf{k}} = -ie^{i\pi/2}e^{-\pi\mu/2}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H_{i\mu}^{(1)}(-k\tau)$$

Calc. integrals: ∫dt and ∫d³p

UV expansion:

$$H_{i\mu}^{(1)}(-p\tau) \to e^{-i\pi/4} e^{\pi\mu/2} \sqrt{-\frac{2}{\pi p \tau}} e^{-ip\tau}$$

$$\frac{\Delta P_\zeta}{P_\zeta} \supset \frac{\Lambda^4}{8\pi^2 H^2 M_{\rm Pl}^2 \epsilon} \left(\frac{19}{12} + \frac{3M^2}{2\Lambda^2} \right) \ln(-2k\tau_{\rm end})$$

IR expansion:

$$H_{i\mu}^{(1)}(-p\tau) \to e^{i\delta} e^{\pi\mu/2} \sqrt{\frac{2}{\pi \mu}} (-p\tau)^{i\mu}$$

$$\frac{\Delta P_\zeta}{P_\zeta} \supset \frac{\Lambda^4}{8\pi^2 H^2 M_{\rm Pl}^2 \epsilon} \left(\frac{3M}{\Lambda} + \frac{\Lambda}{15M} \right) \ln(-2k\tau_{\rm end})$$

Calc. integrals: ∫dt and ∫d³p

Full result:

$$i_1(\lambda,\mu) = \frac{e^{-\pi\mu}x^3}{\lambda^3}\int_0^{\lambda/x} dp\, p^2 |H^{(1)}_{i\mu}(px)|^2$$

$$i_2(\lambda,\mu) = \frac{e^{-\pi\mu}x^5}{\lambda^5}\int_0^{\lambda/x} dp\, p^4 |H^{(1)}_{i\mu}(px)|^2$$

$$i_3(\lambda,\mu) = \frac{e^{-\pi\mu}x^5}{\lambda^5}\int_0^{\lambda/x} dp\, p^2 |\partial_x H^{(1)}_{i\mu}(px)|^2$$

$$\begin{aligned} i_1(\lambda,\mu) =& e^{-\pi\mu}[1+\coth(\pi\mu)]\left[\frac{\cosh(\pi\mu) f(\frac{1}{2},\frac{3}{2};\frac{5}{2},1-i\mu,1+i\mu)}{3\pi\mu}\right. \\ &\left.+\frac{(2/\lambda)^{2i\mu} f(\frac{1}{2}-i\mu,\frac{3}{2}-i\mu;1-i\mu,\frac{5}{2}-i\mu,1-2i\mu)}{\sinh(\pi\mu)(-3+2i\mu)\Gamma^2(1-i\mu)}\right] + c.c. \end{aligned}$$

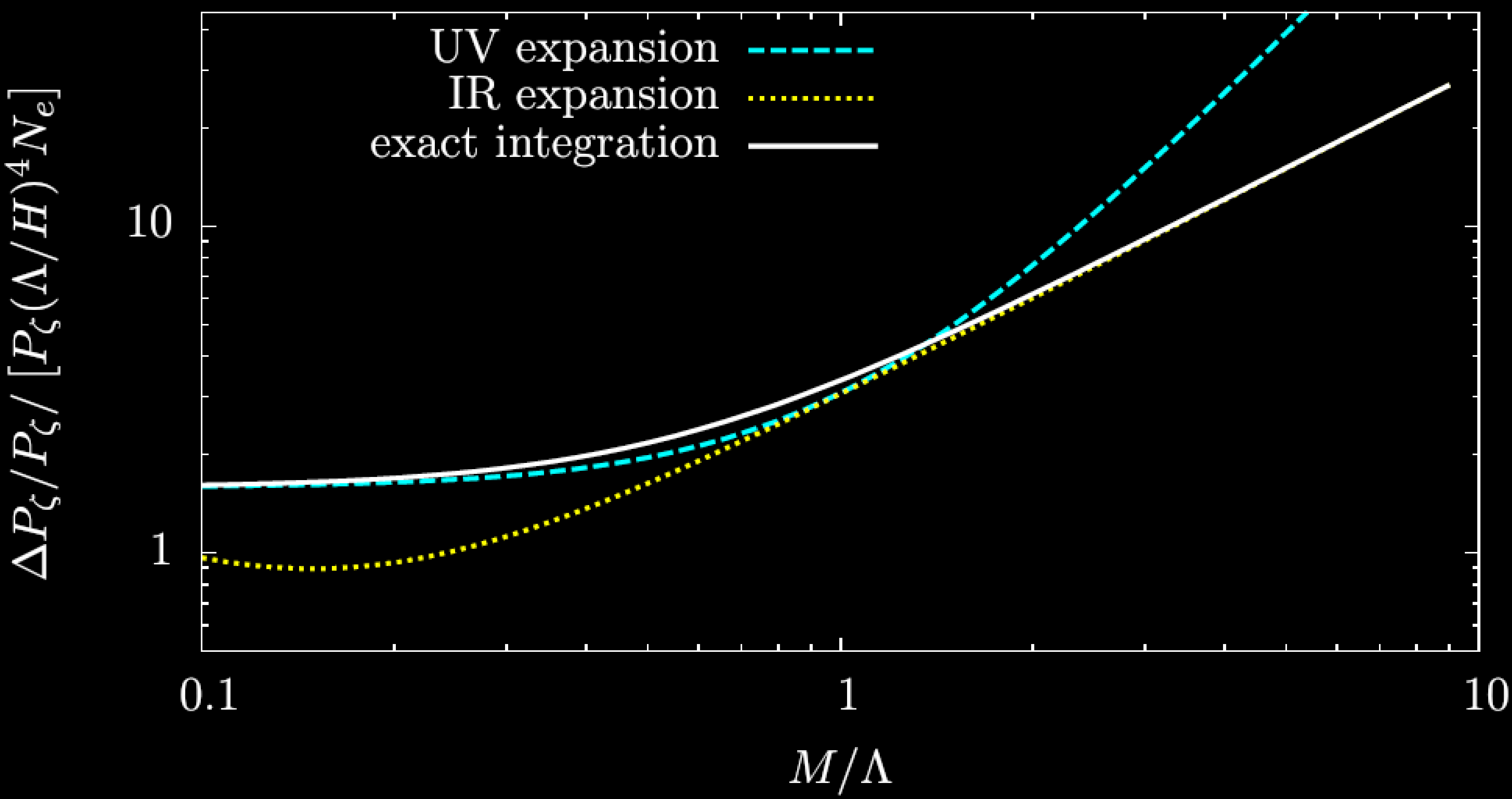
$$\begin{aligned} i_2(\lambda,\mu) =& e^{-\pi\mu}[1+\coth(\pi\mu)]\left[\frac{\cosh(\pi\mu) f(\frac{1}{2},\frac{5}{2};\frac{7}{2},1-i\mu,1+i\mu)}{5\pi\mu}\right. \\ &\left.+\frac{(2/\lambda)^{2i\mu} f(\frac{1}{2}-i\mu,\frac{5}{2}-i\mu;1-i\mu,\frac{7}{2}-i\mu,1-2i\mu)}{\sinh(\pi\mu)(-5+2i\mu)\Gamma^2(1-i\mu)}\right] + c.c. \end{aligned}$$

$$\begin{aligned} i_3(\lambda,\mu) =& e^{-\pi\mu}[1+\coth(\pi\mu)]\left\{-\frac{1}{6}J_{-i\mu}(\lambda)J_{i\mu}(\lambda)\coth(\pi\mu) - \frac{\coth(\pi\mu) f(\frac{1}{2},\frac{5}{2};\frac{7}{2},2-i\mu,i\mu)}{10\Gamma(2-i\mu)\Gamma(i\mu)}\right. \\ &+\frac{\mu\cosh(\pi\mu) f(-\frac{1}{2};-i\mu,i\mu)}{3\pi\lambda^2} - \frac{\lambda^2\cosh(\pi\mu) f(\frac{3}{2},\frac{7}{2};\frac{9}{2},2-i\mu,2+i\mu)}{42\pi\mu(1+\mu^2)} \\ &+\frac{2^{2i\mu} f(-\frac{1}{2}-i\mu,\frac{3}{2}-i\mu;\frac{5}{2}-i\mu,-1-2i\mu,-i\mu)}{\lambda^{2+2i\mu}(-3+2i\mu)\sinh(\pi\mu)\Gamma^2(-i\mu)} - \frac{J_{1-i\mu}(\lambda)[\lambda J_{1-i\mu}(\lambda)+2i\mu J_{-i\mu}(\lambda)]}{6\lambda\sinh(\pi\mu)} \\ &-\frac{2^{2i\mu}\mu f(\frac{1}{2}-i\mu,\frac{5}{2}-i\mu;\frac{7}{2}-i\mu,1-2i\mu,-i\mu)}{3(5i+2\mu)\lambda^{2i\mu}\sinh(\pi\mu)\Gamma^2(1-i\mu)} \\ &\left.+\frac{\lambda^{2-2i\mu} f(\frac{3}{2}-i\mu,\frac{7}{2}-i\mu;2-i\mu,\frac{9}{2}-i\mu,3-2i\mu)}{2^{4-2i\mu}(-7+2i\mu)\sinh(\pi\mu)\Gamma^2(2-i\mu)}\right\} + c.c. \end{aligned} \tag{E.6}$$

$$f(a_1,\ldots,a_p;b_1,\ldots,b_q) \equiv {}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;-\lambda^2)$$

$$\frac{\Delta P_\zeta}{P_\zeta} \supset \frac{\Lambda^4}{8\pi^2 H^2 M_{\rm Pl}^2 \epsilon}\left(\frac{19}{12}+\frac{3M^2}{2\Lambda^2}\right)\ln(-2k\tau_{\rm end})$$

$$\frac{\Delta P_\zeta}{P_\zeta} \supset \frac{\Lambda^4}{8\pi^2 H^2 M_{\rm Pl}^2 \epsilon}\left(\frac{3M}{\Lambda}+\frac{\Lambda}{15M}\right)\ln(-2k\tau_{\rm end})$$



How to understand the result?

$$\Delta\langle\zeta^2\rangle \supset 2i\left(u_k^*(\tau_{\rm end})\right)^2 (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')$$
$$\times \int_{-\infty}^{\tau_{\rm end}} d\tau \left[-\frac{3a}{H}u_k u_k' \int \frac{d^3\mathbf{p}}{(2\pi)^3}|v_p'|^2 \right.$$
$$\left. + \frac{a^2}{4}u_k^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3}p^2|v_p|^2 + \frac{9M^2a^4}{4}u_k^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3}|v_p|^2\right]$$
$$+\,\mathrm{c.c.}\ .$$

$\Delta P \supset \zeta\zeta(0)\ (\int a d\tau)\ (a^3\ \mathrm{Im}\zeta\zeta)\ (\int d^3p\ |\sigma^2|)$

↑ ↑ ↑ ↑

P N P $(\Lambda / H)^4$

Case study:

one massive field:

$$\frac{\Delta P_\zeta}{P_\zeta} \sim P_\zeta \frac{\Lambda^4}{H^4} N_e$$

non-perturbative when $\Lambda \sim O(100)\ H$

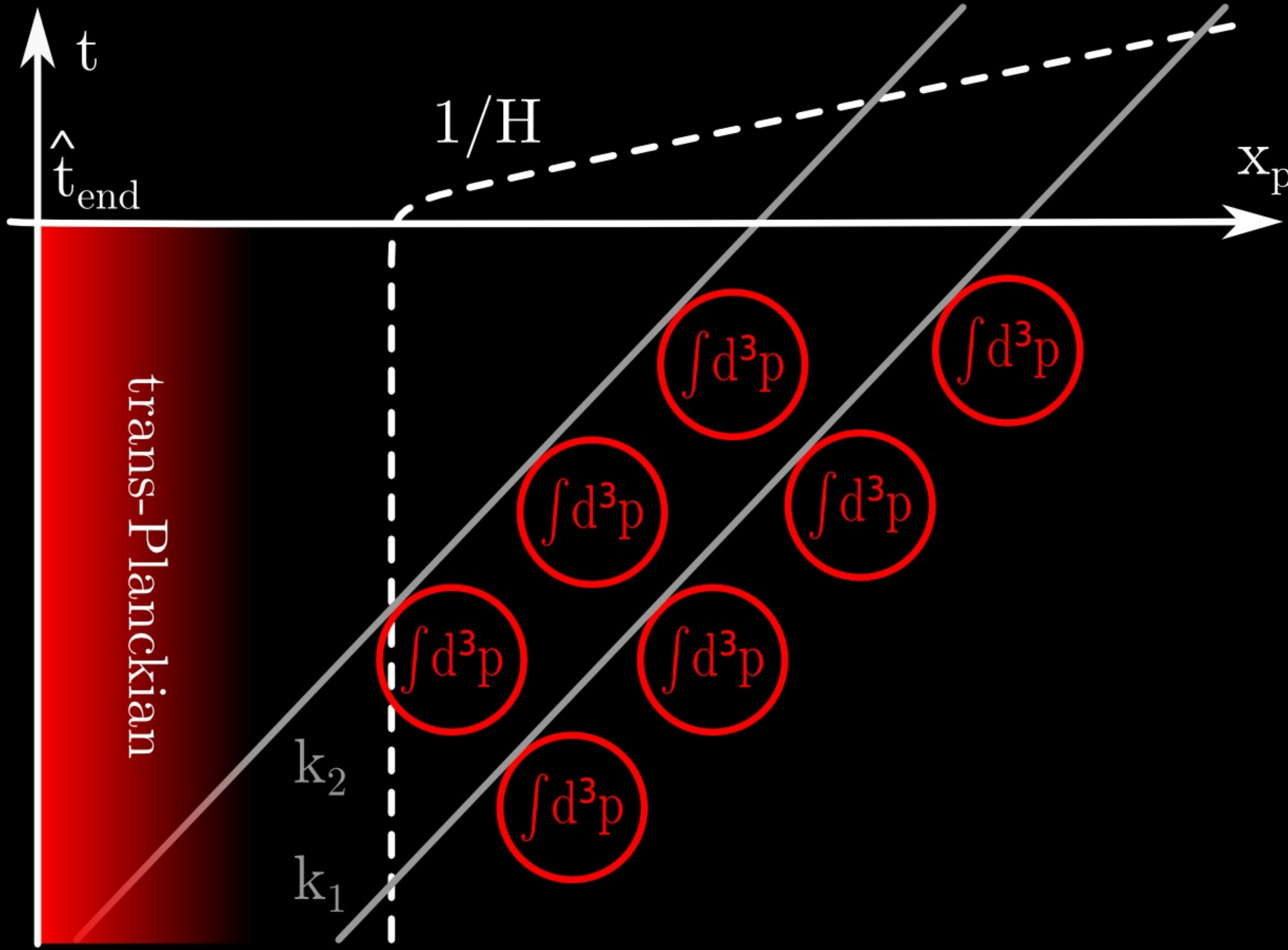
Case study:

KK tower is always non-perturbative!

$$M_{\rm Pl}^2 \sim M_D^{D-2} L^{D-4}$$

$$N_{\rm tot} \sim (M_D L)^{D-4} \sim \frac{M_{\rm Pl}^2}{M_D^2}$$

$$\frac{\Delta P_\zeta}{P_\zeta} \sim \frac{\Lambda^4 N_e}{\epsilon H^2 M_{\rm Pl}^2} \times \frac{M_{\rm Pl}^2}{M_D^2} \sim \frac{M_D^2 N_e}{\epsilon H^2}$$



Inflation is UV-sensitive

- η-problem → worse in pert
- trans-Planckian → non-linear level

Theory of the heavy –– conclusion:

- As a problem: vac fluct. in the pert level
- As a challenge: may be observed

Discussions:

- SUSY
- Resummation in non-pert regime
- Non-G? Tensor? …

Part 2: Theory of the quasi-:

- quasi-single field

  Chen, YW (09, 09)

  Baumann, Green (11)

mass

Mp

$\gg H$

scale of perturbations ⇒ H

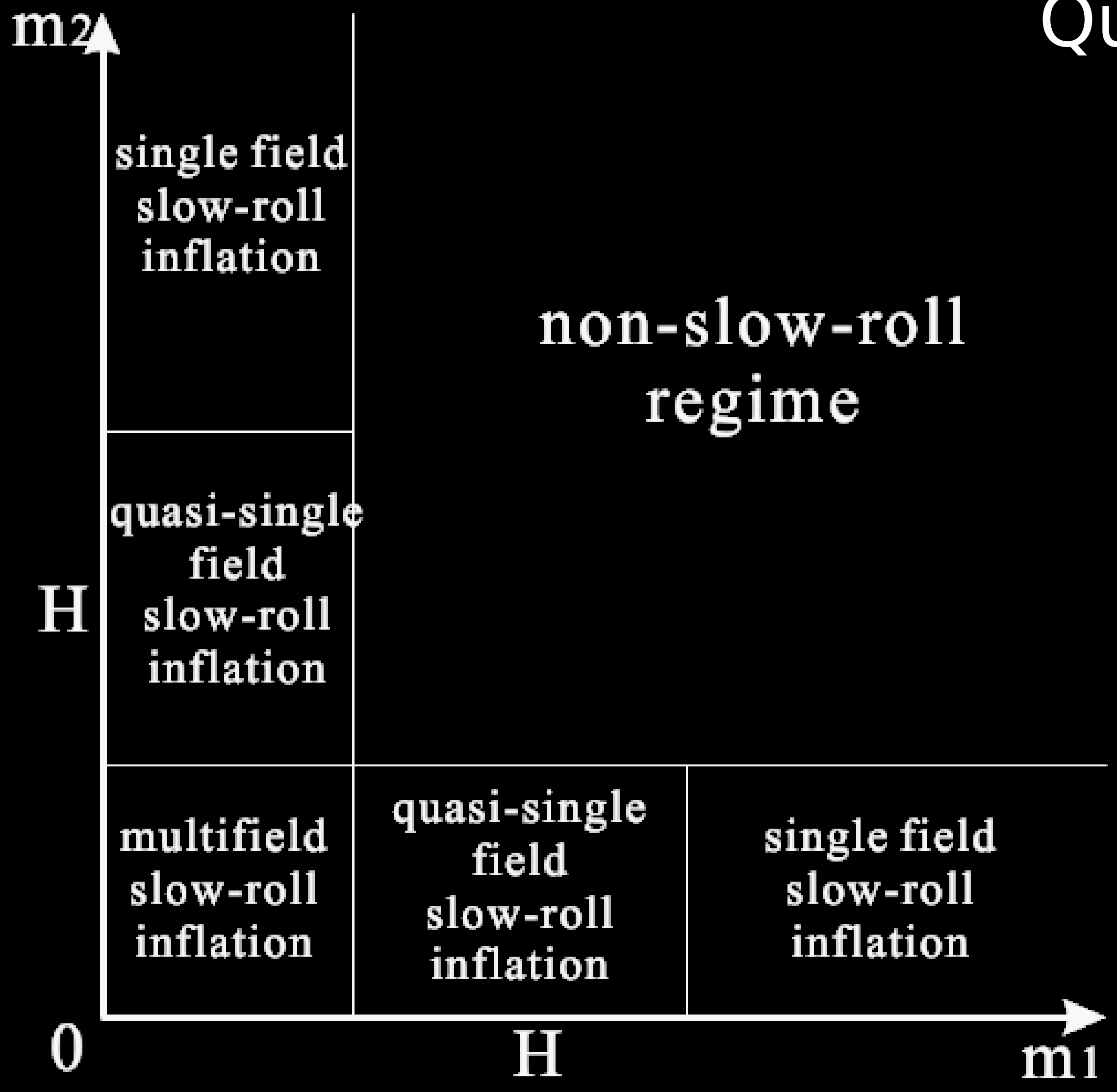
scale of background ⇒ $\ll H$

single field @ bgnd level

multiple fields @ pert level

$\Downarrow$

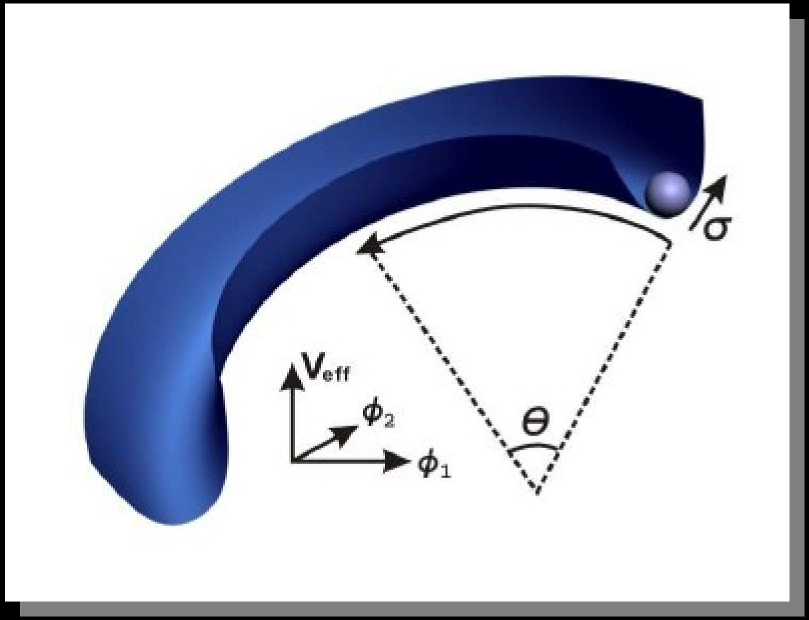
Quasi-single field inflation



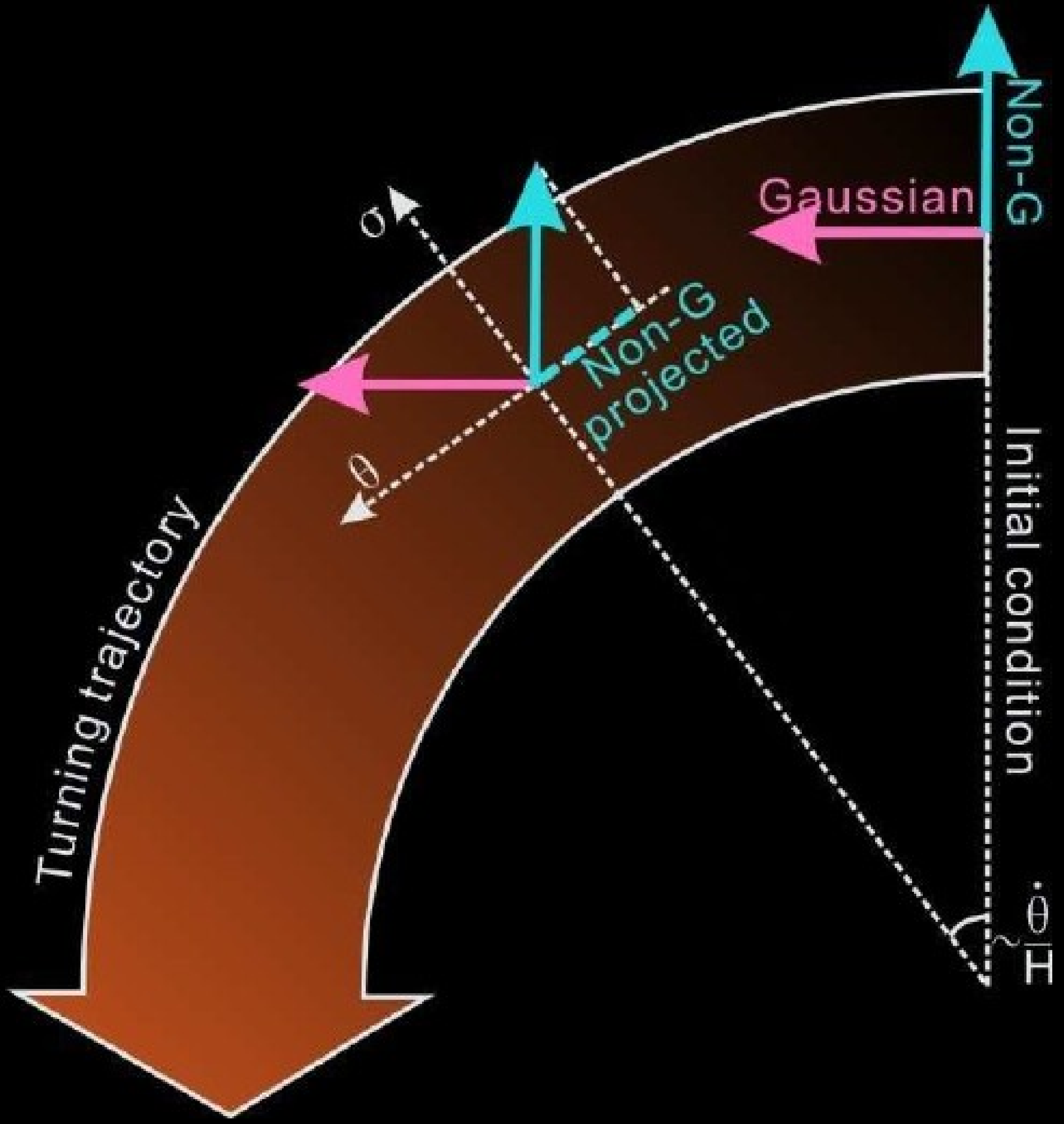
Why cares m~H?

- η-problem: $m \ll H \rightarrow m \sim H$
  - inflaton: may be fine tuned
  - isocurvaton: naturally $m \sim H$

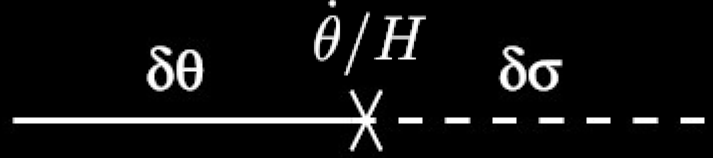
A simple model



$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\tilde{R}+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)\right]$$

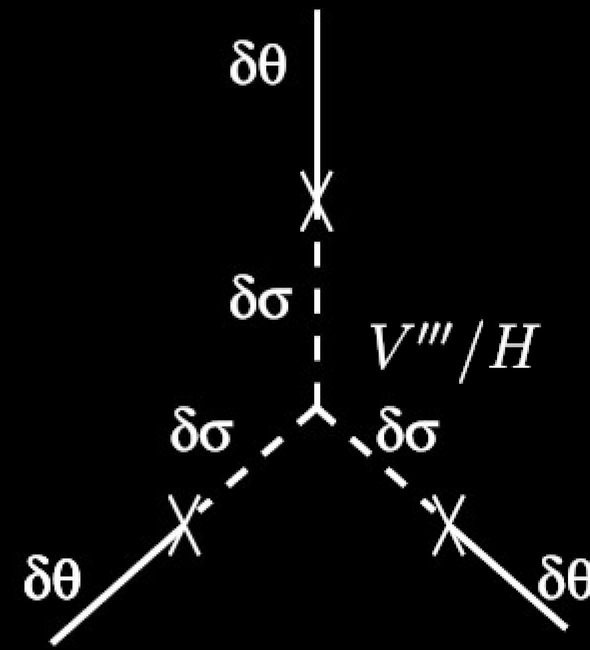


Non-G
Gaussian
σ
Non-G
projected
θ
Turning trajectory
Initial condition
$\dot{\theta}$
$\sim \frac{\dot{\theta}}{H}$



(a)

(b)



(c)

$$\delta P_\zeta \sim \left(\dot{\theta}/H\right)^2 P_\zeta$$

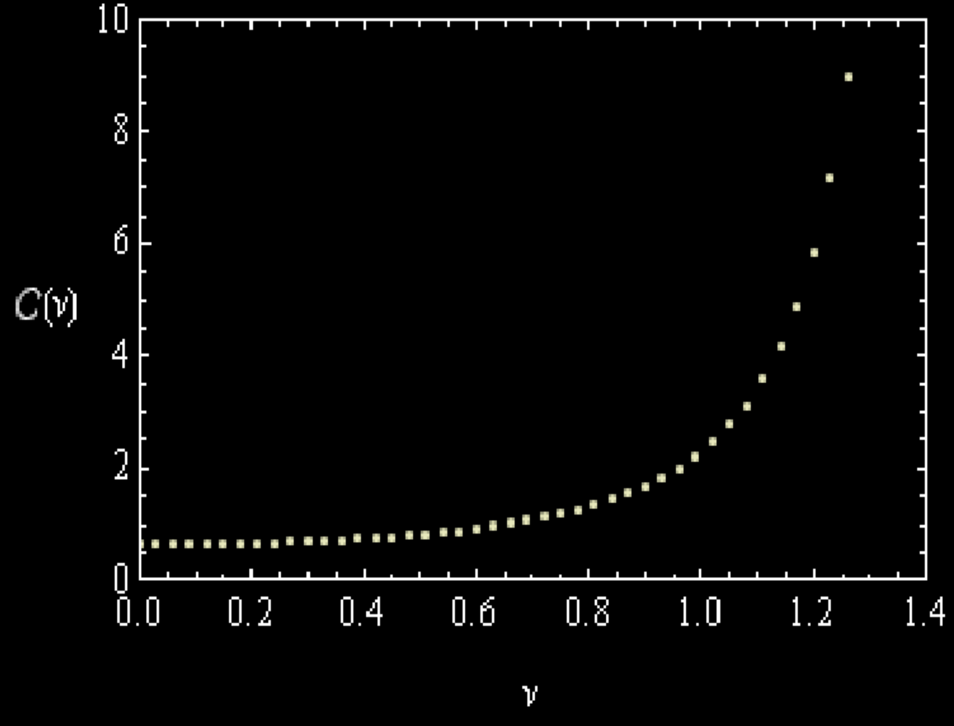
$$f_{NL} \sim P_\zeta^{-1/2} \left(\dot{\theta}/H\right)^3 (V'''/H)$$

$$P_\zeta = \frac{H^4}{4\pi^2 R^2 \dot{\theta}_0^2} \left[1 + 8\mathcal{C} \left(\frac{\dot{\theta}_0}{H} \right)^2 \right]$$

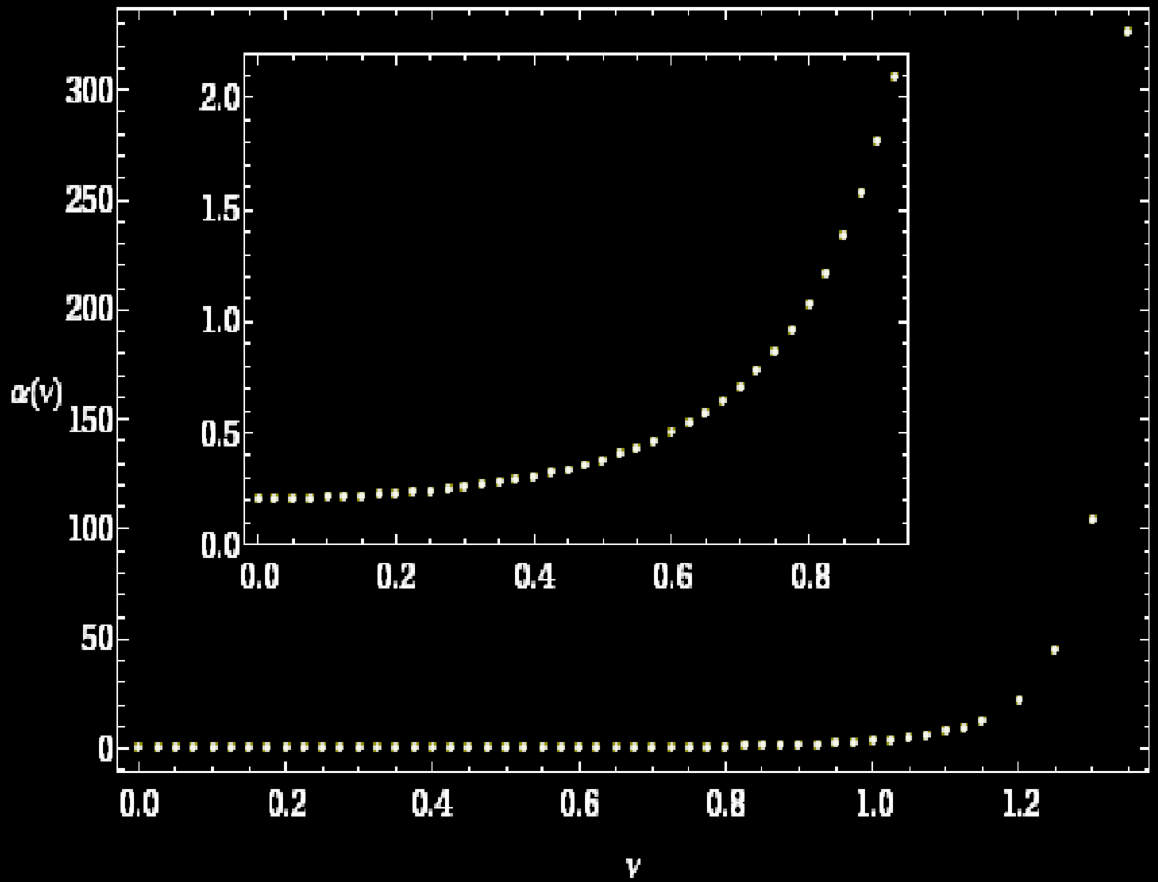
$$\mathcal{C}(\nu) \equiv \frac{\pi}{4} \mathrm{Re} \left[\int_0^\infty dx_1 \int_{x_1}^\infty dx_2 \left(x_1^{-1/2} H_\nu^{(1)}(x_1) e^{ix_1} x_2^{-1/2} H_\nu^{(2)}(x_2) e^{-ix_2} \right.\right.$$
$$\left.\left. - x_1^{-1/2} H_\nu^{(1)}(x_1) e^{-ix_1} x_2^{-1/2} H_\nu^{(2)}(x_2) e^{-ix_2} \right) \right] .$$

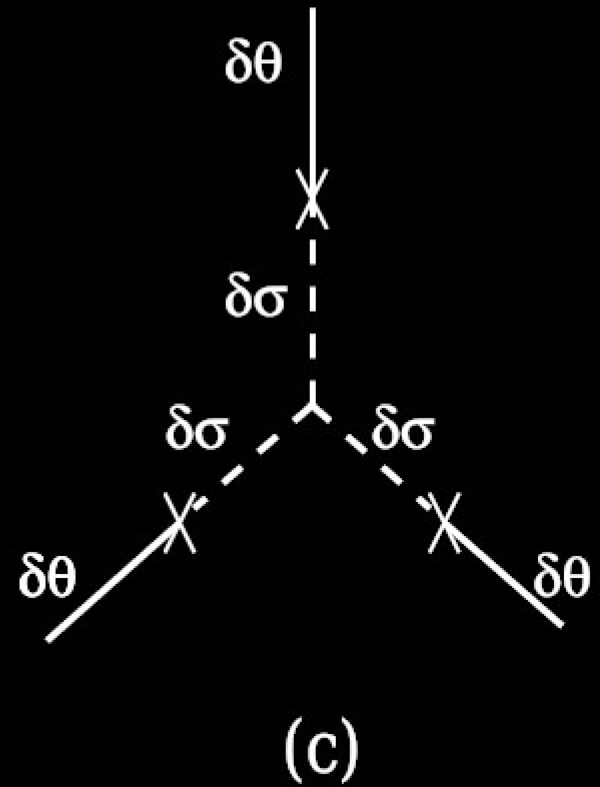
$$\nu = \sqrt{9/4 - m^2/H^2}$$

(b)



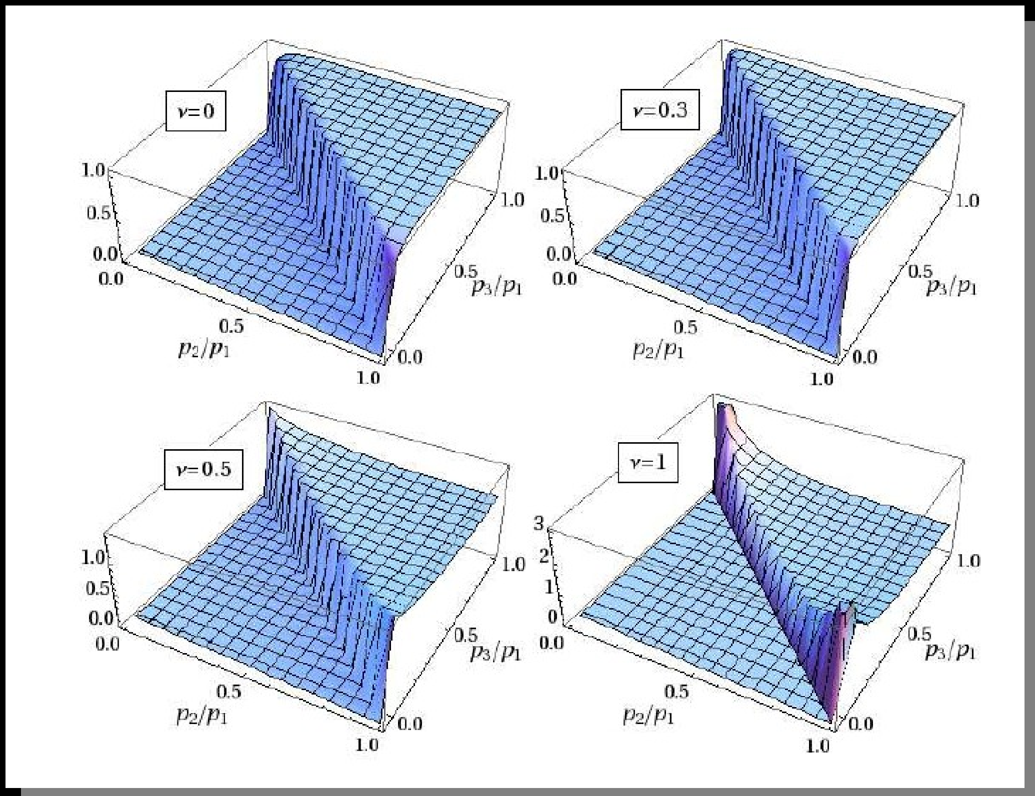
$$\langle \delta\theta^3 \rangle \equiv \langle 0 | \left[\bar{T} \exp\left(i \int_{t_0}^{t} dt' H_I(t') \right) \right] \delta\theta_I^3(t) \left[T \exp\left(-i \int_{t_0}^{t} dt' H_I(t') \right) \right] | 0 \rangle$$

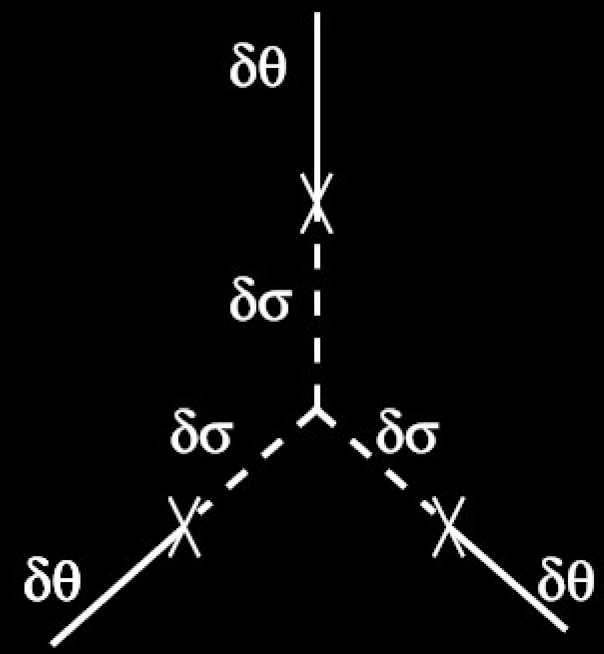




Perhaps they can be resummed, using the method by Burgess, Leblond, Holman and Shandera (2009)

$$\langle\delta\theta^3\rangle \equiv \langle 0| \left[\bar{T}\exp\left(i\int_{t_0}^{t} dt' H_I(t')\right)\right] \delta\theta_I^3(t) \left[T\exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right)\right] |0\rangle$$





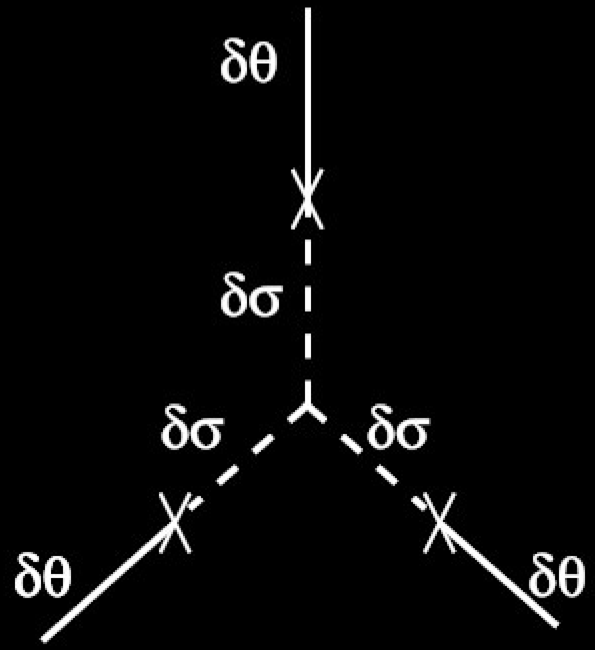
(c)

$$\langle\delta\theta^3\rangle \;\equiv\; \langle 0|\left[\bar{T}\exp\left(i\int_{t_0}^{t} dt' H_I(t')\right)\right]\delta\theta_I^3(t)\left[T\exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right)\right]|0\rangle$$

A continuous family of shapes

with continuous squeezed limit

not a superposition of knowns



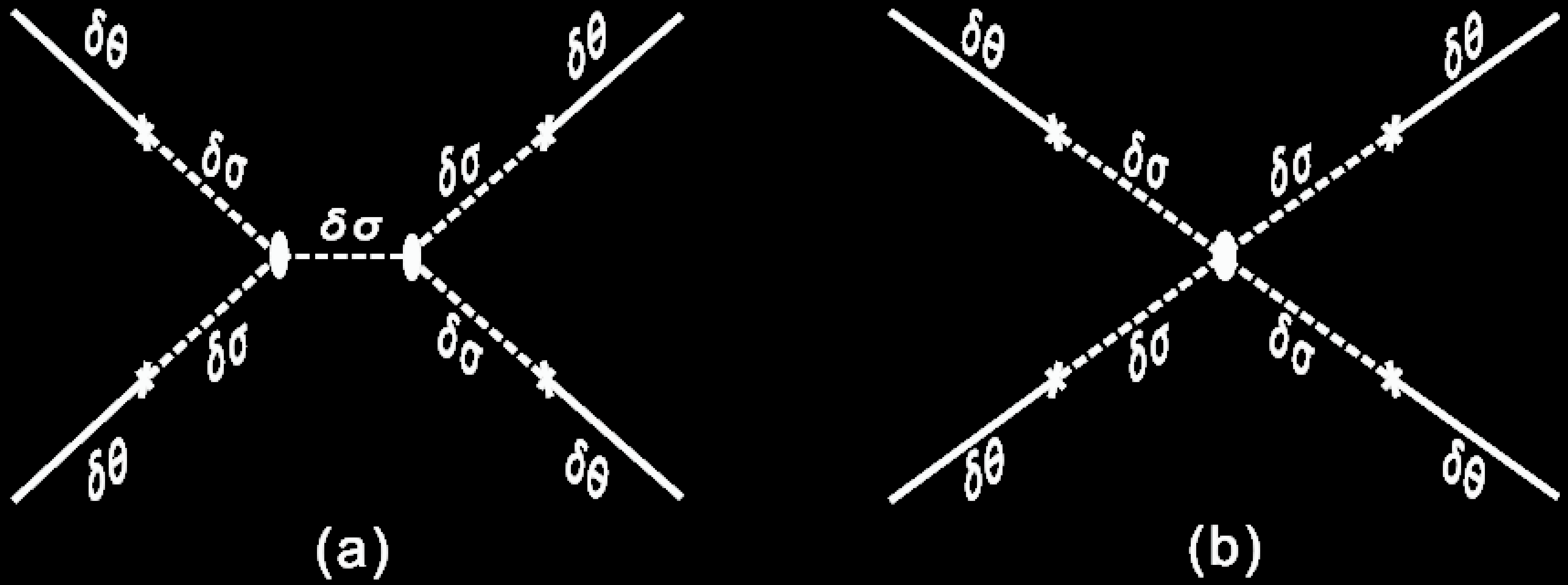
(c)

What if, say, local non-G detected?

Next logical step:

- if exactly local ($\sim 1/k^3$)
- or quasi-local ($\sim 1/k^{3+\delta}$, not from n_s)

QSFI: a (so far unique) model for quasi-local



$$t_{NL} \sim \max\left\{ P_\zeta^{-1}\left(\dot{\theta}/H\right)^4 (V'''/H)^2 \ ,\ P_\zeta^{-1}\left(\dot{\theta}/H\right)^4 V'''' \right\}$$

Connection to feeding mechanism (Barnaby & Shandera 2011)

Part 2.1: a bit heavier

- H/M order effects

  modifies c_s of inflaton

- energy injection

  brings in new scale

  may have oscillations

  Tolley, Wyman (09)

  Achucarro, et. al. (10)

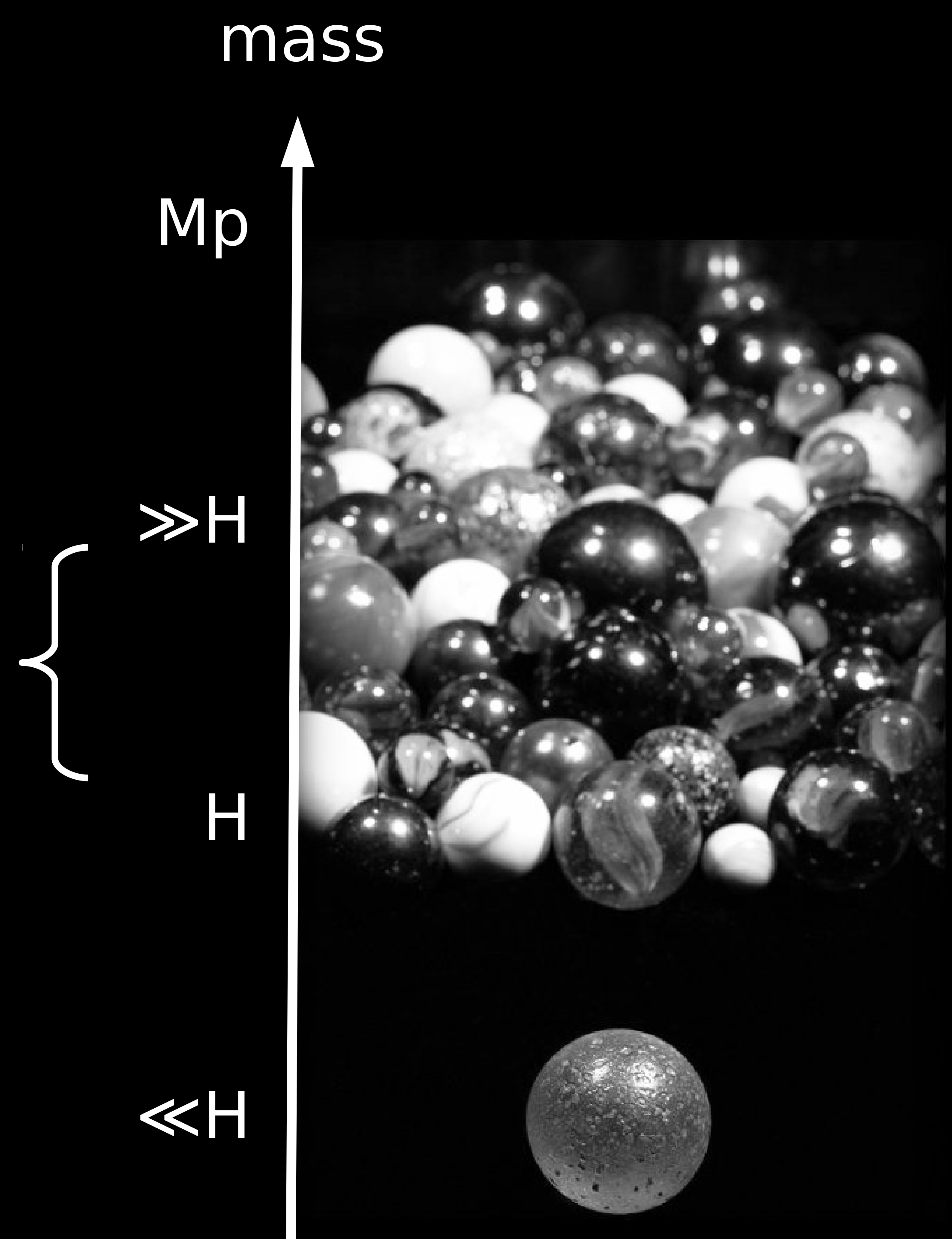
  Jackson, Schalm (10)

  Chen (11), Shiu, Xu (11)

  Gao, Langlois, Mizuno (12)

  Chen, YW (12)

  Sasaki, Pi (12) … …



Part 3: Theory of the light:

- single field
- multi-field
- random dynamics

  a landscape of fields:

  Easther, McAllister (05)

  Huang, Tye (08)

  ... ...

  Duplessis, YW, Brandenberger (12)

scale of background ⇒ ≪H

mass

Mp

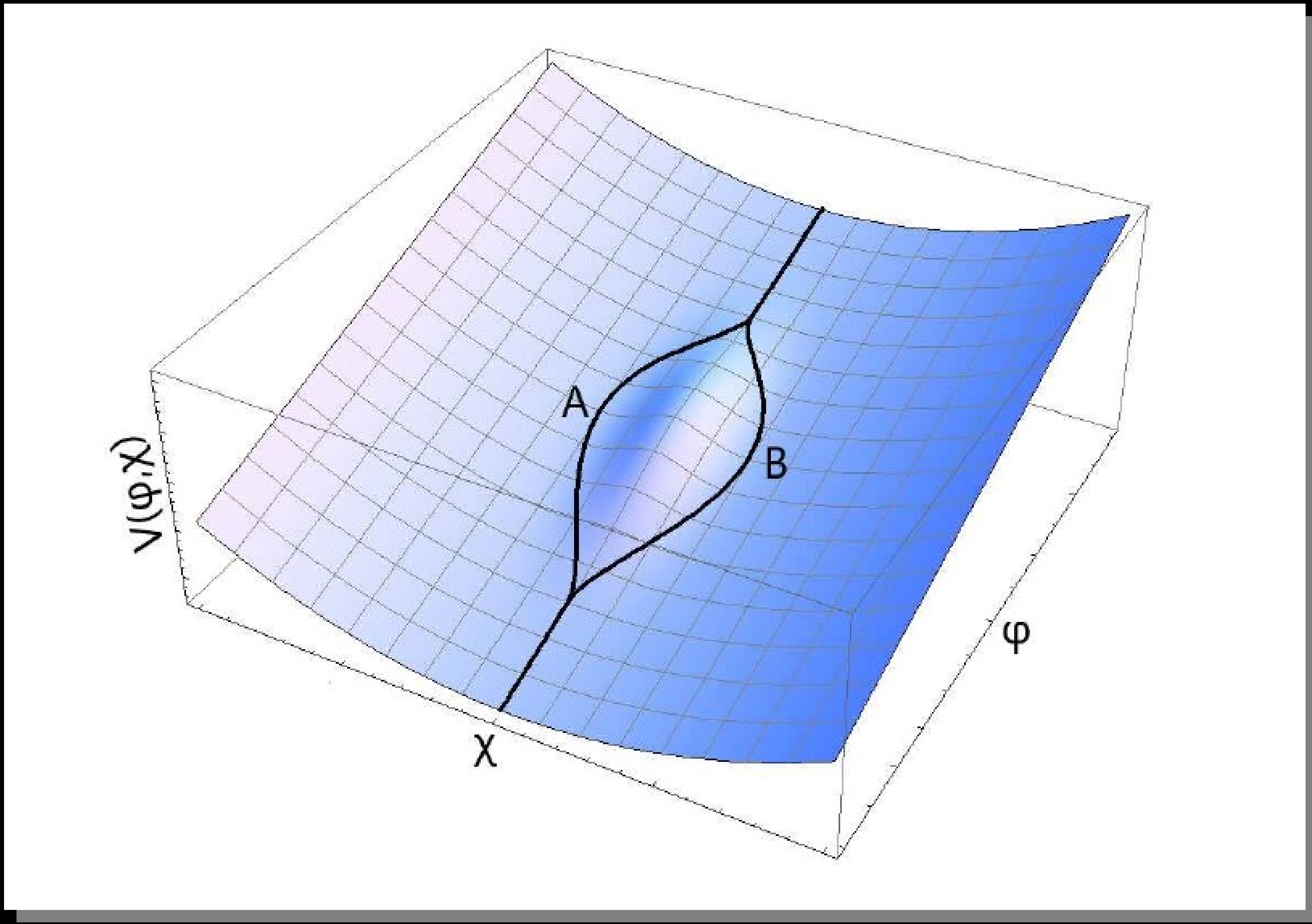
≫H

H

≪H

Li, YW (09), Afshordi, Slosar, YW (10)

Duplessis, YW, Brandenberger (12)



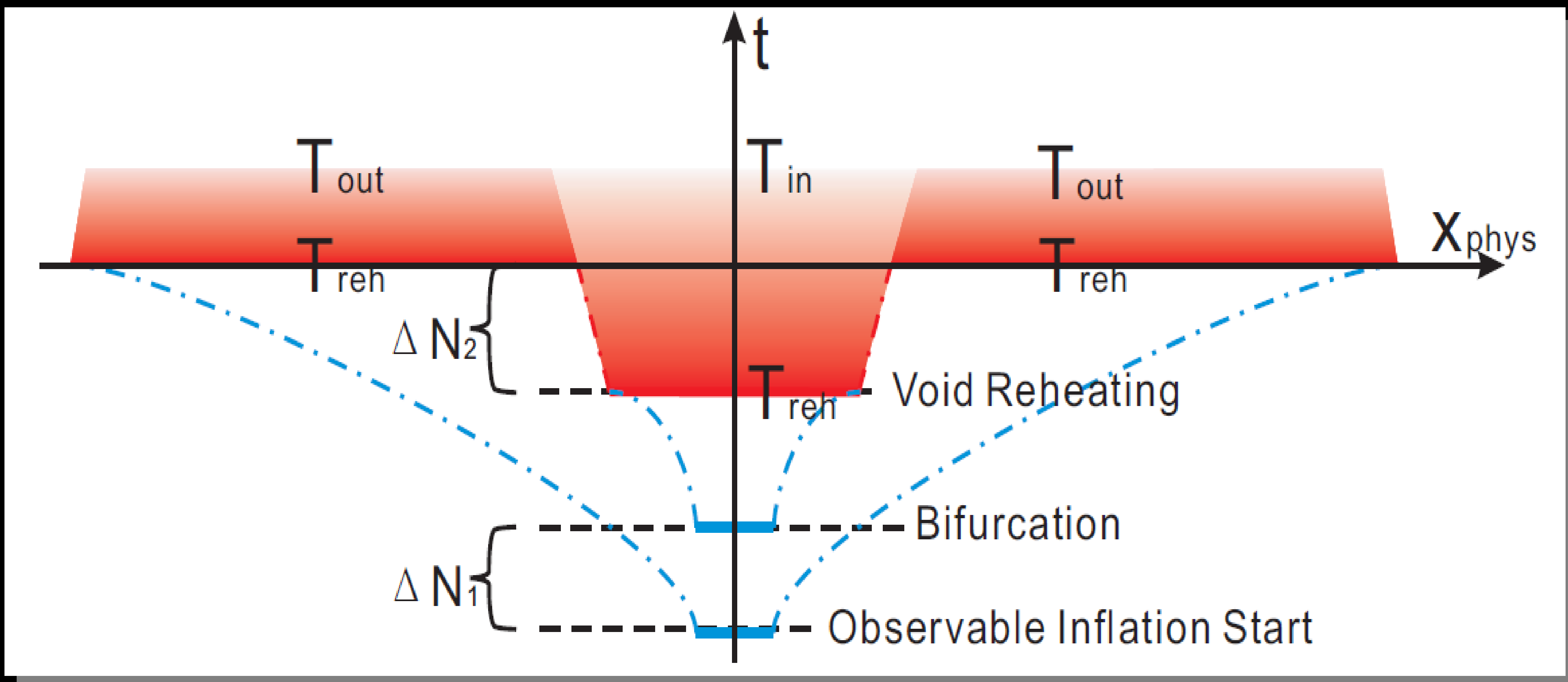
A
B
V(φ,χ)
φ
χ

Multi-stream inflation

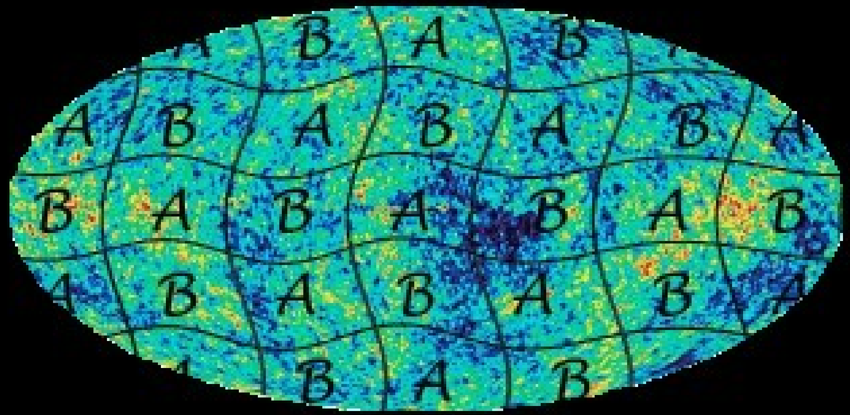
bifurcations {

under control ⇒ realistic models

not under control ⇒ constraints



t
T out
T in
T out
X phys
T reh
T reh
Δ N2
T reh
Void Reheating
Bifurcation
Δ N1
Observable Inflation Start



The nearly-symmetric case:

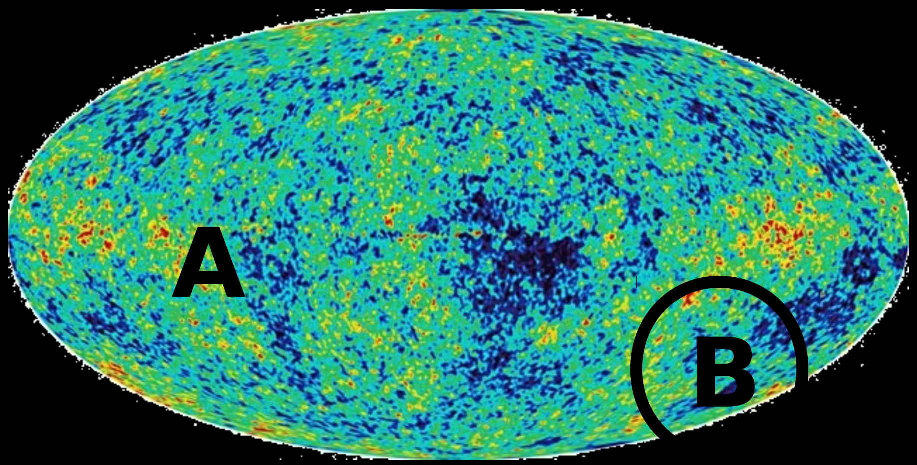
Bifurcation scale: $\zeta = \delta N$

Smaller scales: different path, different power

Their correlation: non-Gaussianity

The asymmetric case:

a spot on the CMB / LSS



The disaster case:

- bifurcations with large δN
- bifurcations with domain wall

They could happen in a random potential

$$\ddot{\varphi}_1 + 3H\dot{\varphi}_1 + \partial_1 V(\varphi_1) + \partial_1 U(\varphi_1, \varphi_2) = 0 \ ,$$

$$\ddot{\varphi}_2 + 3H\dot{\varphi}_2 + \partial_2 U(\varphi_1, \varphi_2) = 0 \ ,$$

$$\lambda \equiv \sqrt{\langle(\partial_1 U)^2\rangle}/|\partial_1 V|.$$

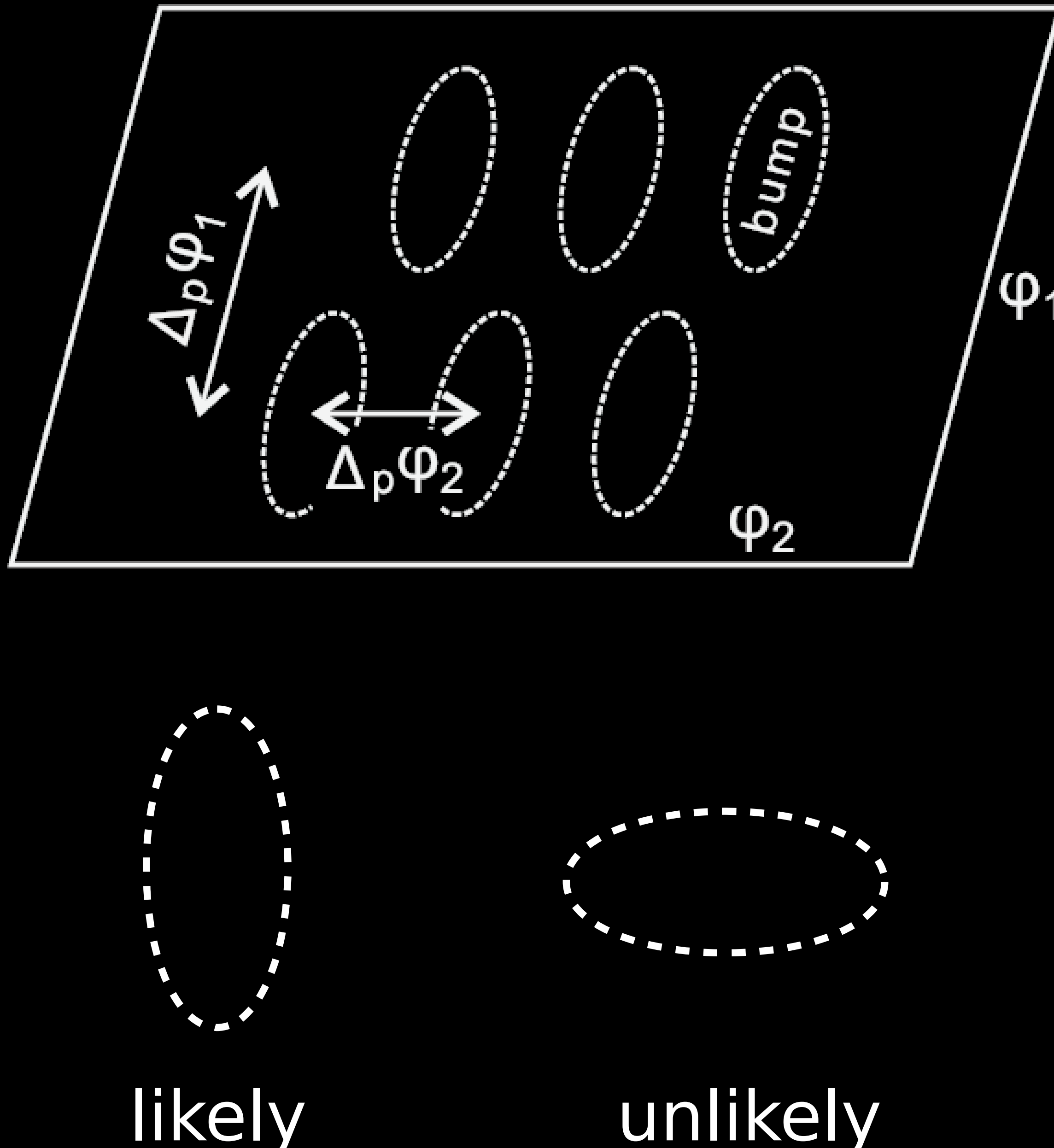
$$\delta \equiv \varphi_2^{(A)} - \varphi_2^{(B)}$$

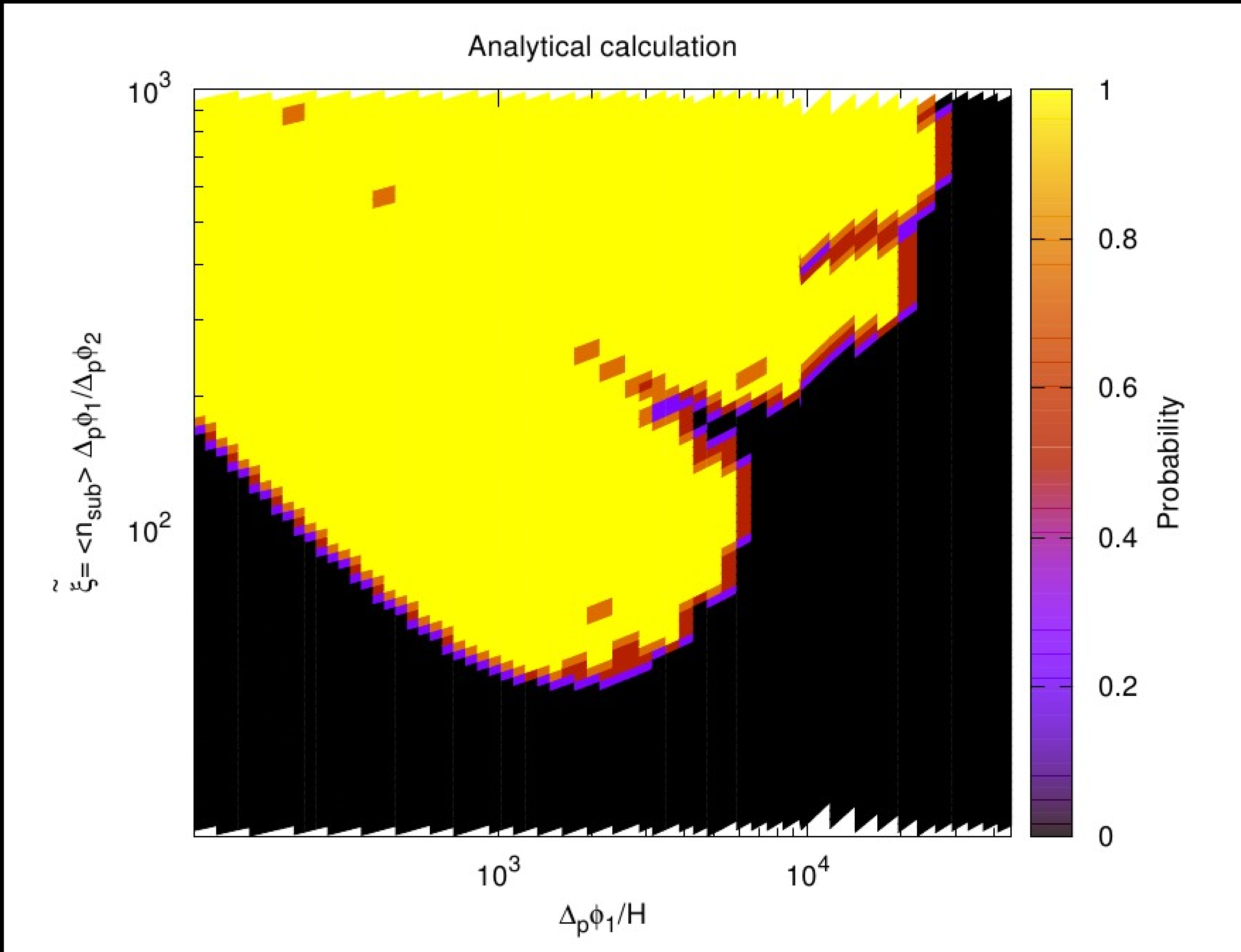
$$\ddot{\delta} + 3H\dot{\delta} + (\partial_2^2 U)\delta = 0$$

oscillate?

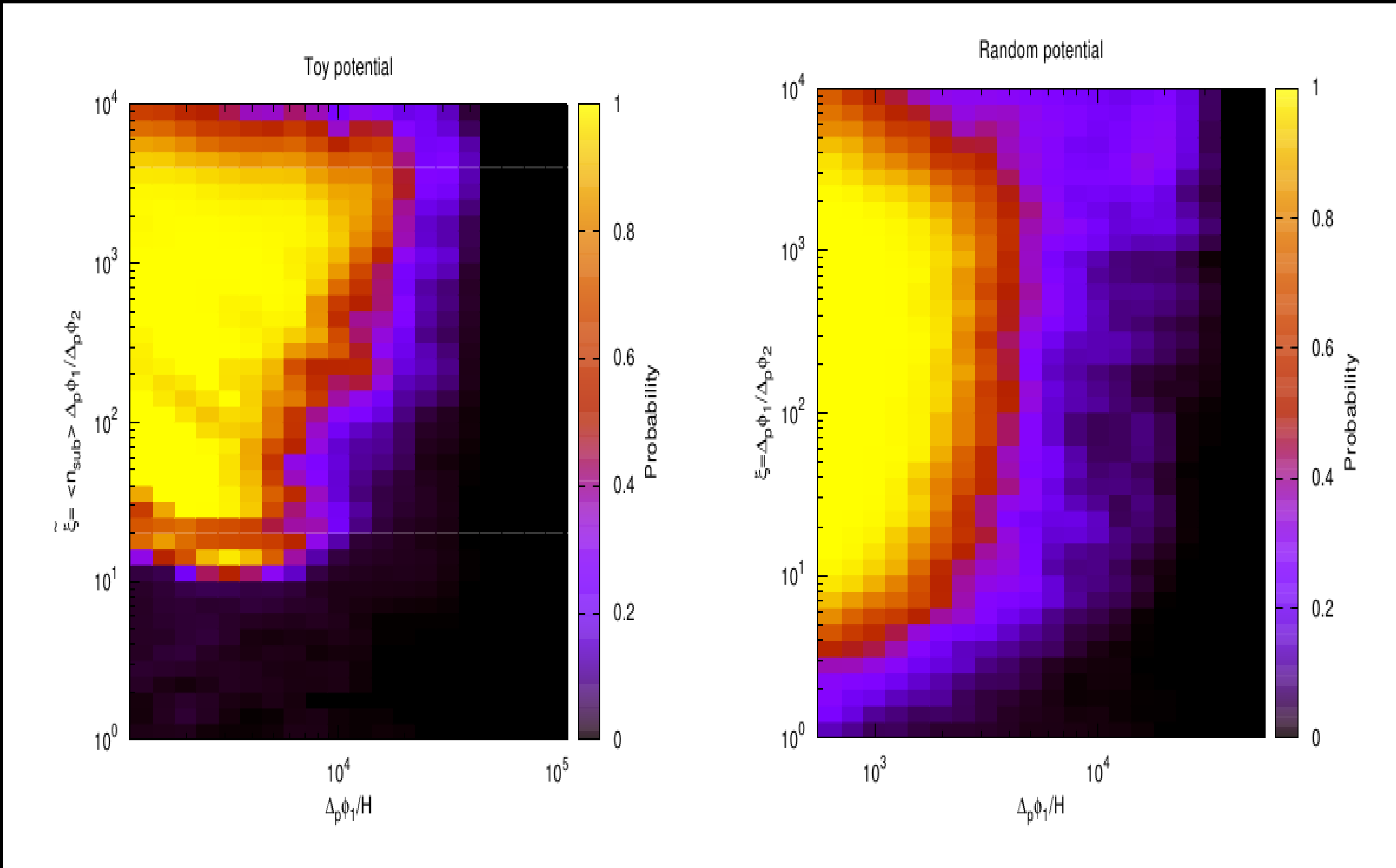
or grow?

$$\partial_2^2 U \simeq \frac{\lambda \xi \partial_1 V}{\Delta_p \varphi_2} \sin\left(\frac{2\pi\dot{\varphi}_1 t}{\Delta_p \varphi_1}\right)$$


bump
$\Delta_p\varphi_1$
$\Delta_p\varphi_2$
φ_1
φ_2
likely
unlikely

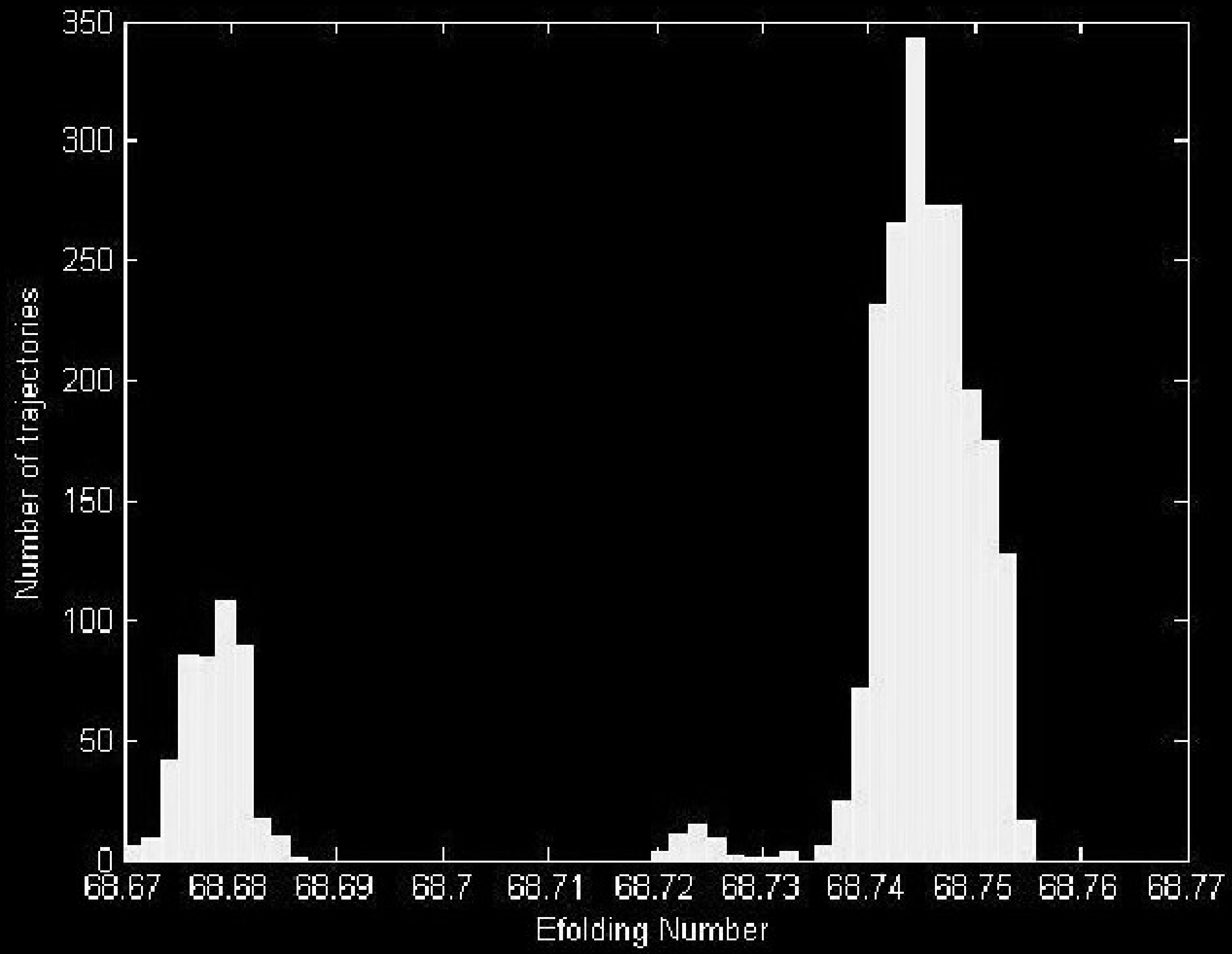


Analytical calculation
10^3
10^2
$\tilde{\xi} = \langle n_{sub} \rangle \, \Delta_p\phi_1/\Delta_p\phi_2$
10^3
10^4
$\Delta_p\phi_1/H$
1
0.8
0.6
0.4
0.2
0
Probability



Toy potential
Random potential
$\tilde{\xi} = \langle n_{sub} \rangle \Delta_p\phi_1/\Delta_p\phi_2$
$\xi = \Delta_p\phi_1/\Delta_p\phi_2$
$\Delta_p\phi_1/H$
$\Delta_p\phi_1/H$
Probability
Probability

The δN problem from the constraint



The above example:

2-dim random potential

Multiple-dim random potential:

bifurcation probability still unknown

– linear scaling?

– exponential scaling?

most probable bifurcate path?

To conclude:

- the heavy $\Rightarrow$ UV sensitivity from one loop
- the quasi- $\Rightarrow$ QSFI, shape of non-G
- the light $\Rightarrow$ bifurcations could happen

Thank you!

✉ FAQ: “Your effect is … ”

gauge artifact!

canceled by counter terms!

an artifact of Lorentz violating regulator!

calculated on an unstable background!

canceled by tadpole diagrams!

wrong in the decoupling limit!

wrong because it breaks scaling of dS!

Are you calculating a gauge artifact?

- General problem:
  - lack of local observables in QG
- What we calculate:
  - $\langle\zeta\zeta\rangle$ in the ζ-gauge
- What they measure?
  - comoving curvature perturbation
  - or things that are consistent
- The $\delta\varphi$ gauge + transformation way:
  - interactions seems $O(\varepsilon)$
  - however, vacuum shakes
  - ζ-obs + $\delta\varphi$-vac is not reasonable

Canceled by counter terms?

Our divergence: Λ^4, $M^2\Lambda^2$

Λ is UV cutoff in the EFT sense.
It parameterize our ignorance of UV theory.

As the case of c.c. / Higgs mass hierarchy,

It is not natural that the UV completion provides counter terms to cancel them.

And fine tuning is equivalent to lowering Λ.

Artifact of Lorentz violating regulator?

Counter arguments:

- c.c. analogue
- FRW breaks Lorentz anyway
- intuition: vac fluct gravitates

Explicit example:

$\int d^3p \,/\, p$

$\rightarrow \int d^{3-\delta}p \,/\, p \rightarrow a^{2-\delta} \int d^{3-\delta}q \,/\, q$ (q: physical)

$\rightarrow \sim a^{2-\delta}\, \delta^{-1} \sim a^2\, \delta^{-1}$, where $\delta^{-1} \sim \Lambda^2$

Calculated on an unstable background?

local expansion correction $M_p^2 \Delta(H^2) \sim \Lambda^4$

$\Delta(H^2) \sim H^2 \rightarrow (\Lambda / H)^4 (H^2 / M_p^2) \sim 1$

$\rightarrow \varepsilon (\Lambda / H)^4 P \sim 1$

compared with our result:

$\Delta P / P \sim (\Lambda / H)^4 P N_e \gg \varepsilon (\Lambda / H)^4 P$

our effect is perturbative → bkgd is stable

i.e. instability of bkgd < loop

Calculated on an unstable background?

inflaton mass correction $\Delta m^2 \sim \Lambda^4 / M_p^2$

Δm breaks slow roll → $\Delta m^2 \sim H^2$

→ $\varepsilon (\Lambda / H)^4 P \sim 1$

compared with our result:

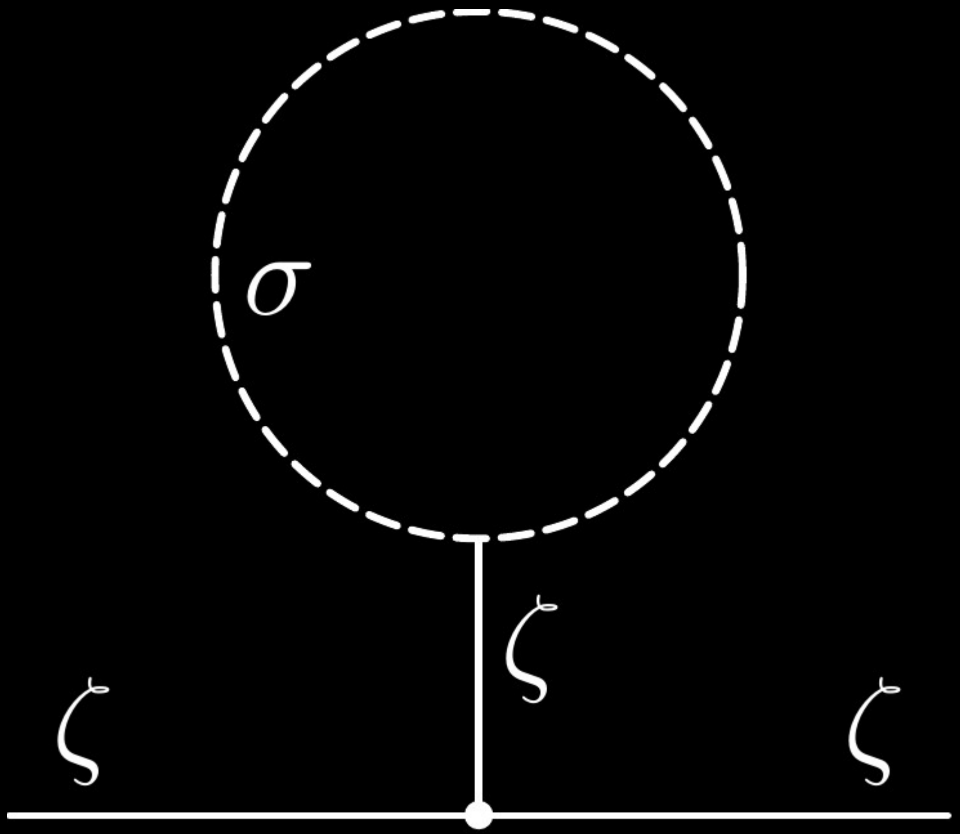
$\Delta P / P \sim (\Lambda / H)^4 P N_e \gg \varepsilon (\Lambda / H)^4 P$

our effect is perturbative → bkgd is stable

i.e. instability of bkgd < loop

Canceled by tadpole diagrams?

tadpole → small



Wrong in the decoupling limit?

Asked: $M_p \to \infty$, with P fixed, or Λ / M_p fixed

Not every $M_p \to \infty$ limit is a decoupling limit

Checked decoupling limit:

(✔) $M_p \to \infty$ with Λ, V fixed

(✔) $M_p \to \infty$ with Λ, H fixed

$\Delta P \sim P^2 N \log(-k\tau) \to 0$ (decouple)

Wrong because it breaks scaling of dS?

$$\frac{\Delta P_\zeta}{P_\zeta} \supset \frac{\Lambda^4}{8\pi^2 H^2 M_{\rm Pl}^2 \epsilon}\left(\frac{19}{12} + \frac{3M^2}{2\Lambda^2}\right)\ln(-2k\tau_{\rm end})$$

Assume reheat later: (a shift of $\tau_{\rm end}$)

Then momenta k need rescaling as well.

We observe the same CMB, with

- rescaled comoving k
- the same physical k

dS scaling respected