Understanding the Shape of non-Gaussianity

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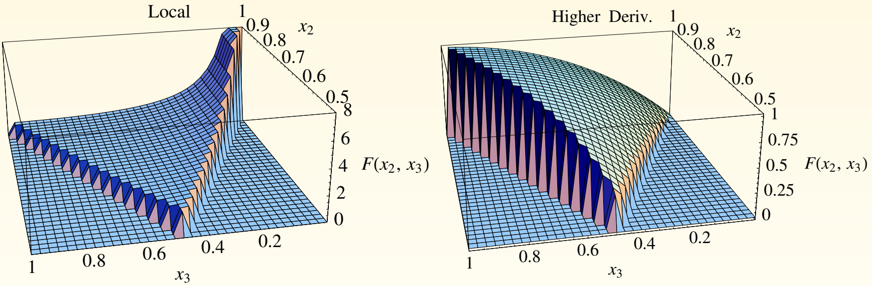
The Many Shapes of non-Gaussianity

Babich, Creminelli and Zaldarriaga [astro-ph/0405356]



But wait, how exactly k's got in there? Wave vector can enter through:

- gradient terms: $\vec{\partial}\phi \to \vec{k}\phi_k$
- time derivatives: $\dot{\phi} \to |k|\phi_k$
- spatial correlations: $P(\vec{k}) = 1/|k|^3$

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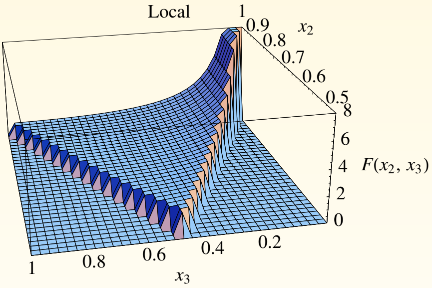

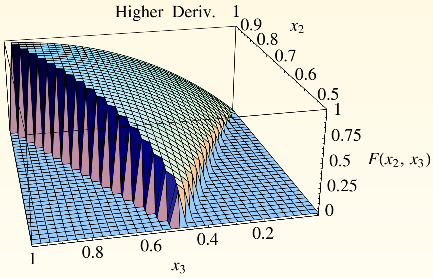

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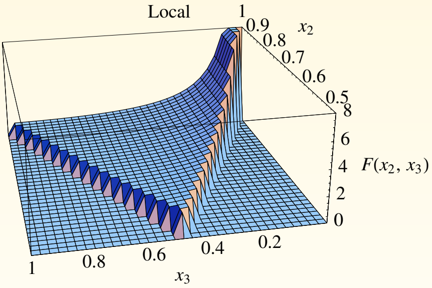

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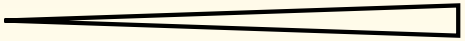
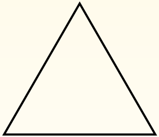
The Many Shapes of non-Gaussianity (Cont.)

$$\langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle = f_{\mathrm{NL}}\, B(\vec{k}_1,\vec{k}_2,\vec{k}_3)\,\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3) + \text{connected}$$

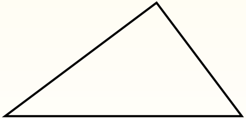
$$B_{\mathrm{loc}}(\vec{k}_i) = \frac{1}{k_1^3 k_2^3} + \frac{1}{k_1^3 k_3^3} + \frac{1}{k_2^3 k_3^3}$$



$$B_{\dot\sigma^3}(\vec{k}_i) = \frac{1}{k_1 k_2 k_3 k_t^3}, \quad k_t = k_1 + k_2 + k_3$$



$$B_{\dot\sigma(\nabla\sigma)^2}(\vec{k}_i) = \frac{\vec{k}_1\cdot\vec{k}_2}{k_1^3 k_2^3 k_3}\left(\frac{1}{k_t} + \frac{k_1+k_2}{k_t^2} + 2\frac{k_1 k_2}{k_t^3}\right) + \text{permutations}$$



The Many Shapes of non-Gaussianity (cont.)

$$\langle\Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle = f_{\rm NL}\, B(\vec{k}_1,\vec{k}_2,\vec{k}_3)\,\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3) + \text{connected}$$

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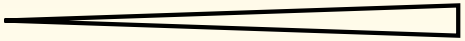

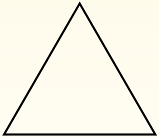

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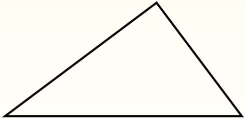
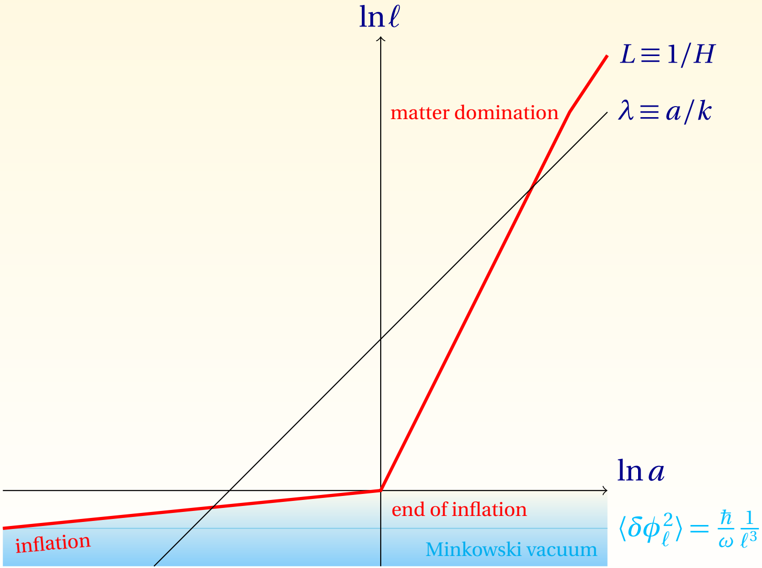


Quantum What & Where, or Map to Inflation



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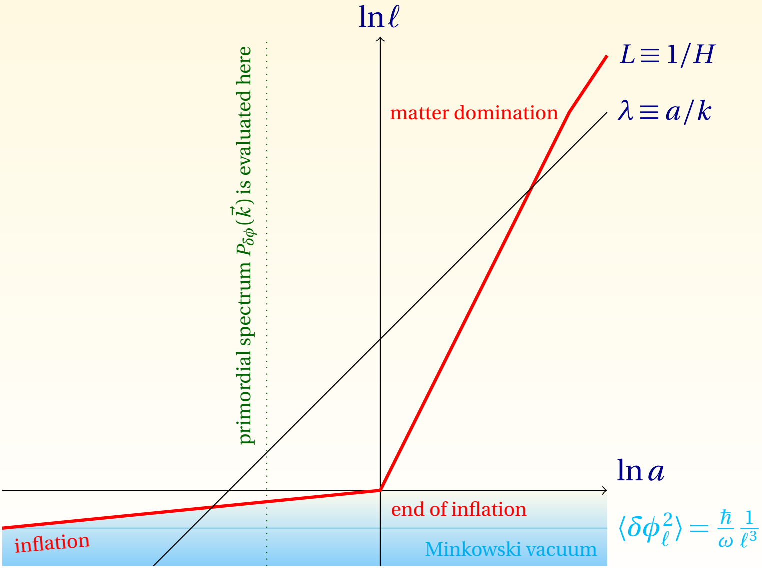
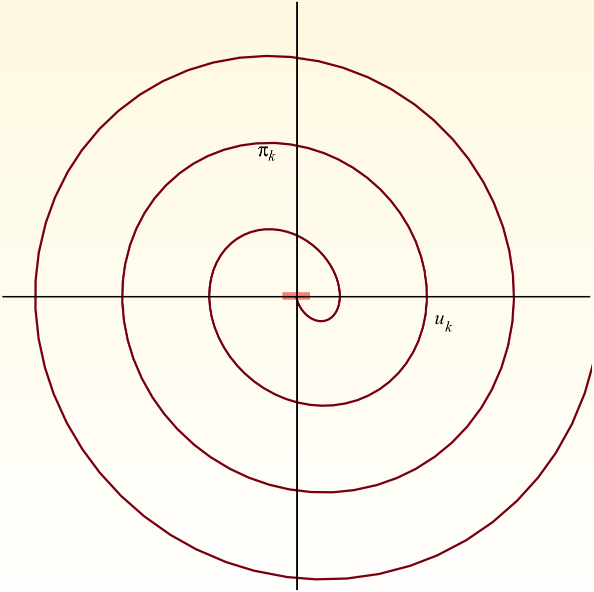
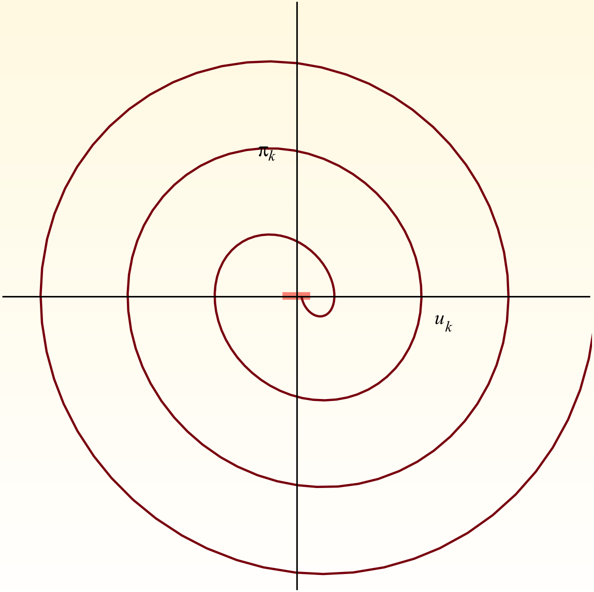



π_k
u_k

Following Comoving Mode (in a nutshell)



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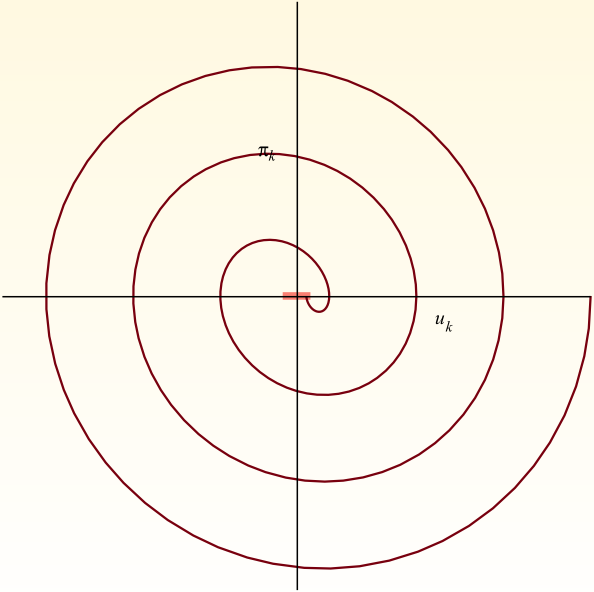

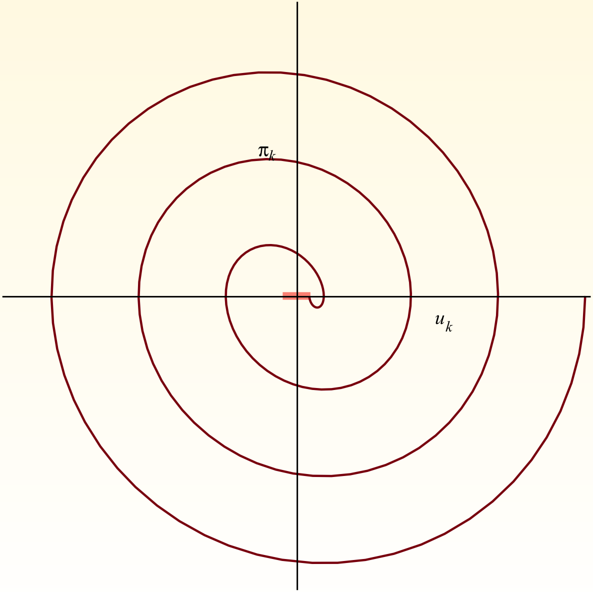
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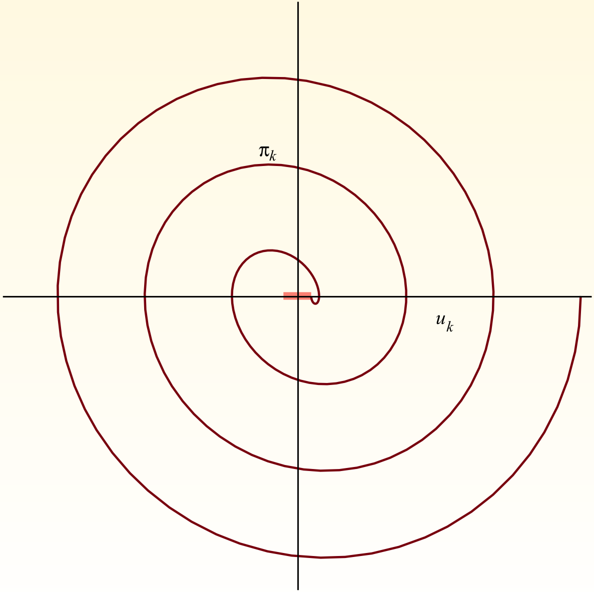
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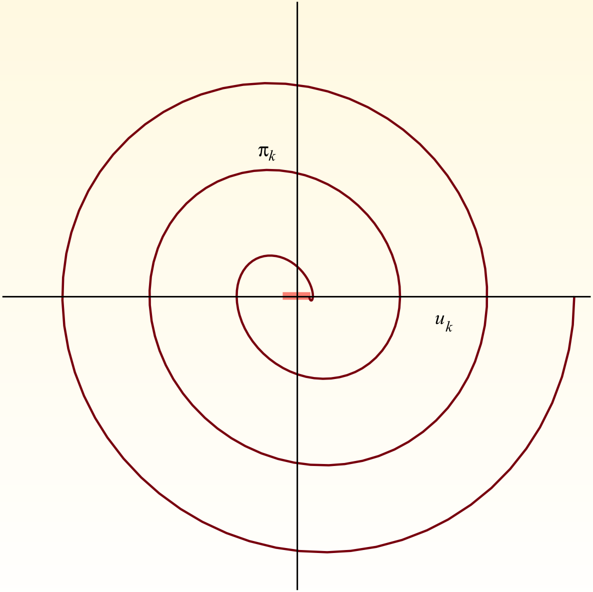

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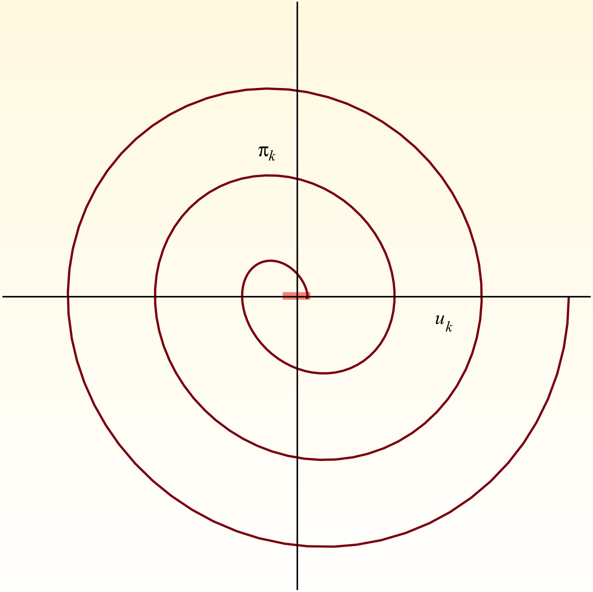

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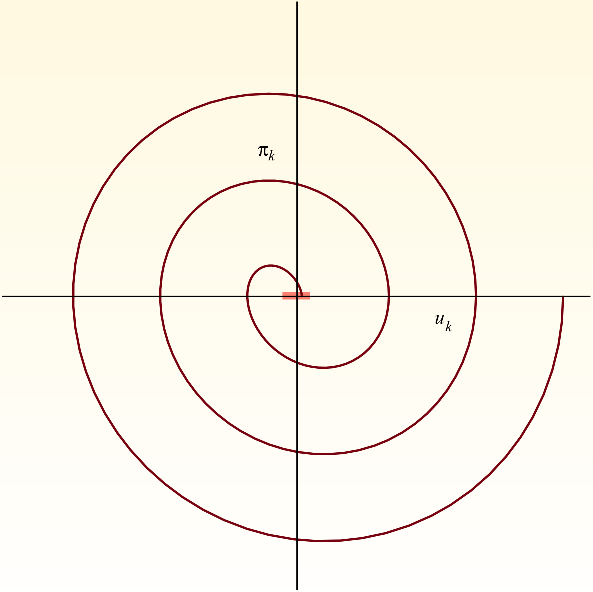

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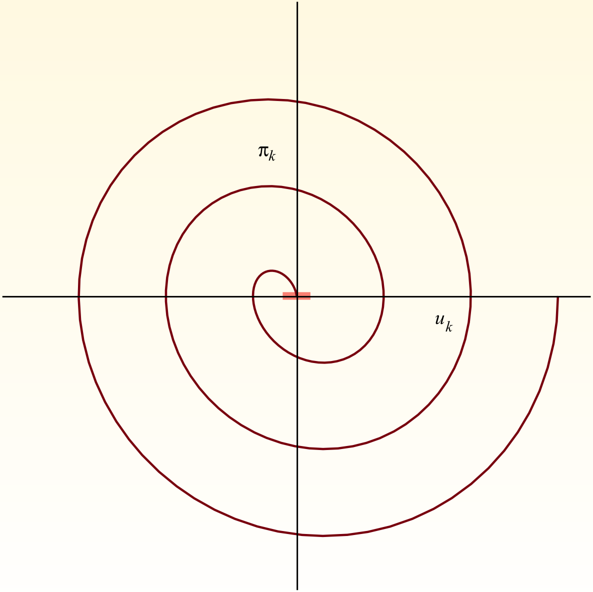


Massive Field in de Sitter (now with math)

Wave equation for mode φ_k with mass m, wavevector k in FRW metric:

$$\ddot{\varphi}_k + 3H\dot{\varphi}_k + \left(m^2 + \frac{k^2}{a^2}\right)\varphi_k = 0$$

Use conformal time $H\eta = -1/a$, switch variables to $x = -k\eta = p/H$:

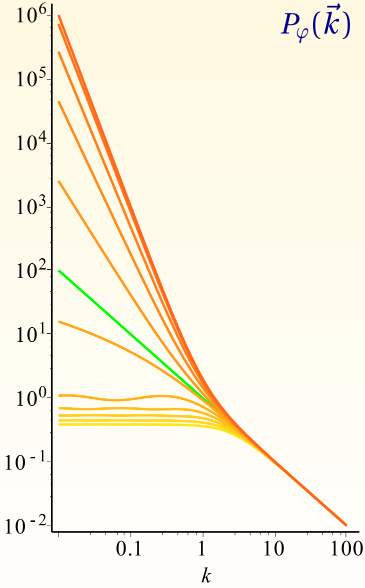
$$x^2\varphi_k'' - 2x\varphi_k' + \left(\frac{m^2}{H^2} + x^2\right)\varphi_k = 0$$

$$\varphi_k(x) = c_1 x^{\frac{3}{2}} H_\mu^{(1)}(x) + c_2 x^{\frac{3}{2}} H_\mu^{(2)}(x), \quad \mu^2 = \frac{9}{4} - \frac{m^2}{H^2}$$

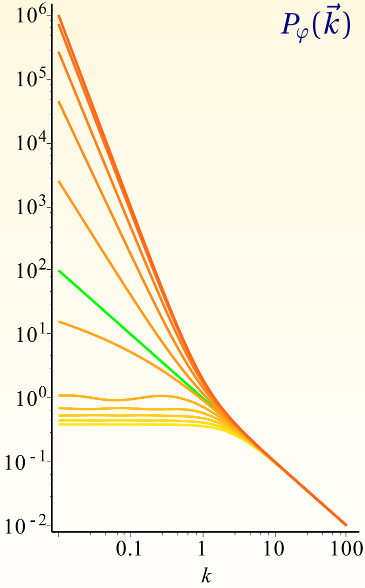
Match to Minkowski vacuum $\frac{e^{-ipt}}{\sqrt{2p}}$ as $p \to \infty$ to fix values of c_1 and c_2:

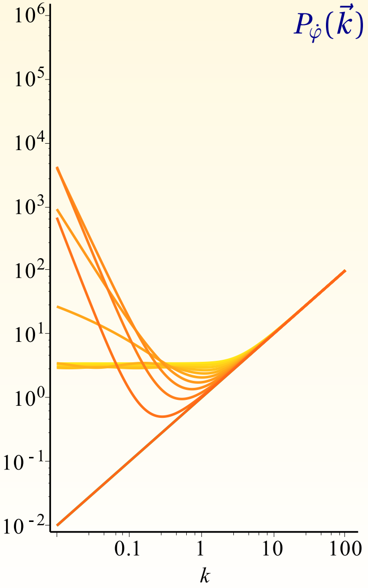
$$\varphi_k(\eta) = \frac{\pi^{\frac{1}{2}}}{2} H(-\eta)^{\frac{3}{2}} c_\mu H_\mu^{(1)}(-k\eta), \quad c_\mu = e^{i\frac{\pi}{2}(\mu+\frac{1}{2})} \text{ is pure phase}$$

de Sitter Vacuum Spectra



de Sitter Vacuum Spectra, with a Surprise!





Non-Linearity Sources non-Gaussianity

- Vacuum state deformation
- Mode freeze-out on (sound) horizon exit
- Transfer to curvature fluctuations

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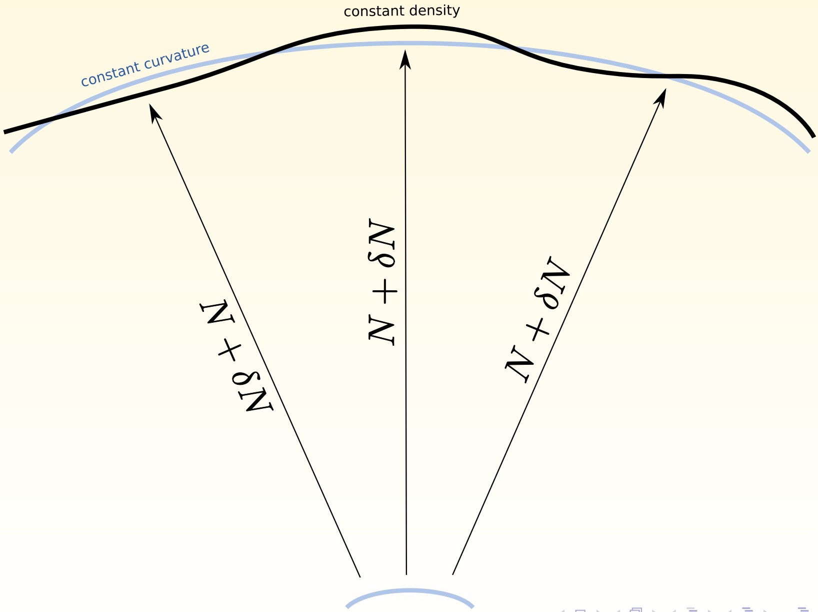
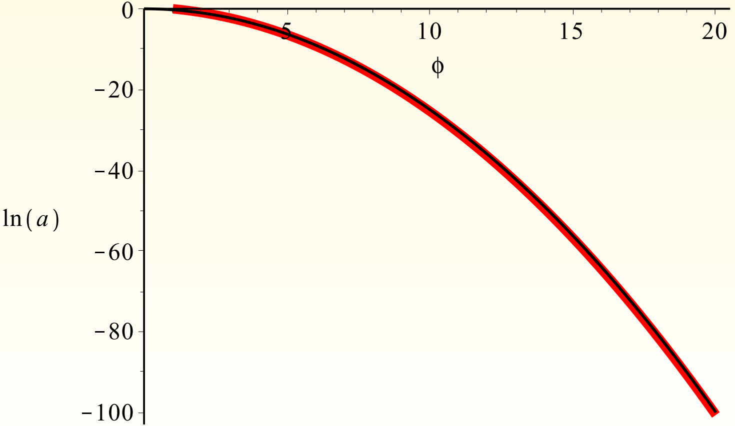
Transfer Field to Curvature Fluctuations



$$\Phi(\vec{x}) = F(\varphi(\vec{x}))$$



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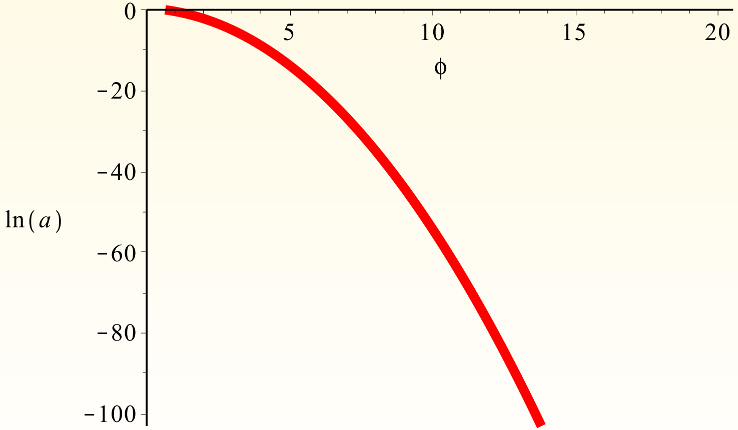

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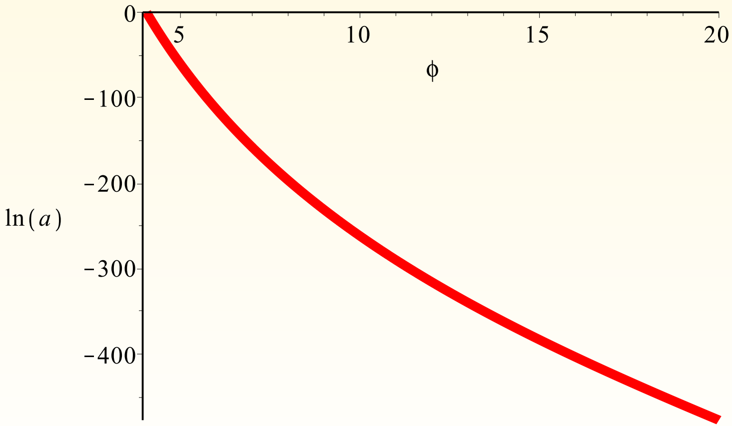
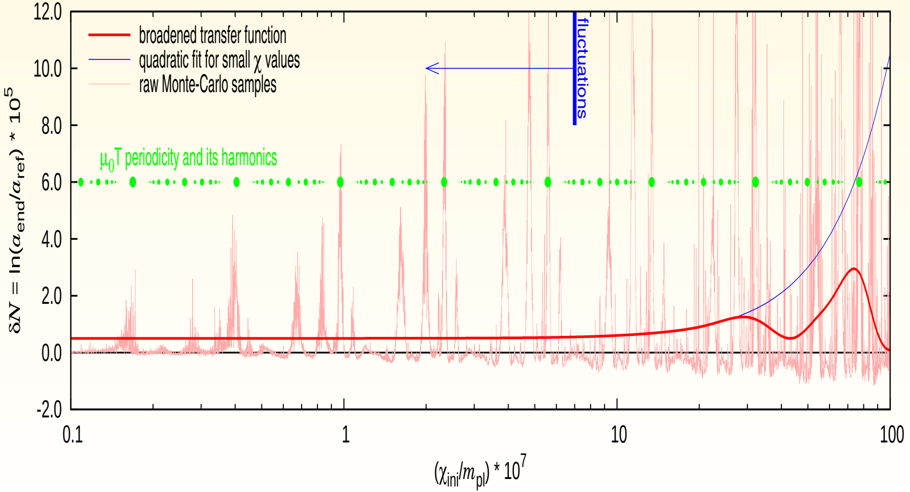


TRANSFER FUNCTION [RESONANT PREHEATING]

$$\Phi(\vec{x}) = F(\chi(\vec{x}))$$



Generating Correlations with non-Linearity

$$\Phi(\vec{x}) = F(\varphi(\vec{x}))$$

Non-linear transfer function can be expanded (if smooth enough):

$$F(\varphi) = \sum_{n=1}^{\infty} f_n \varphi^n, \quad f_n \equiv \frac{F^{(n)}}{n!}$$

Odd-point correlators of Gaussian random field vanish:

$$\langle \varphi(\vec{x})\varphi(\vec{y})\rangle = \xi(\vec{x},\vec{y})$$
$$\langle \varphi(\vec{x})\varphi(\vec{y})\varphi(\vec{z})\rangle = 0$$

But non-linearity generates them (from even-point correlators of GRF):

$$\langle \Phi(\vec{x})\Phi(\vec{y})\Phi(\vec{z})\rangle \neq 0$$

This is How It Works...

$$
\begin{aligned}
\langle \Phi(\vec{x})\Phi(\vec{y})\Phi(\vec{z})\rangle &= \left(\sum_{a=1}^{\infty} f_a \varphi^a(\vec{x})\right)\left(\sum_{b=1}^{\infty} f_b \varphi^b(\vec{y})\right)\left(\sum_{c=1}^{\infty} f_c \varphi^c(\vec{z})\right) \\
&= \sum_{n=1}^{\infty} \sum_{a+b+c=n} f_a f_b f_c \underbrace{\langle \varphi^a(\vec{x})\varphi^b(\vec{y})\varphi^c(\vec{z})\rangle}_{\text{only even } n \text{ contribute}} \\
&= f_1^2 f_2 \;\; \langle \varphi^2(\vec{x})\varphi(\vec{y})\varphi(\vec{z})\rangle + \text{permutations} \\
&\quad + f_2^3 \;\; \langle \varphi^2(\vec{x})\varphi^2(\vec{y})\varphi^2(\vec{z})\rangle \\
&\quad + f_1 f_2 f_3 \;\; \langle \varphi^3(\vec{x})\varphi^2(\vec{y})\varphi(\vec{z})\rangle + \text{permutations} \\
&\quad + f_1^2 f_4 \;\; \langle \varphi^4(\vec{x})\varphi(\vec{y})\varphi(\vec{z})\rangle + \text{permutations} \\
&\quad + \;\; \dots
\end{aligned}
$$

Back to Fourier Domain

$$\langle\varphi(\vec{k}_1)\varphi(\vec{k}_2)\rangle = P(\vec{k}_1)\delta^3(\vec{k}_1+\vec{k}_2)$$

$$\begin{aligned}
\langle\Phi(\vec{k}_x)\Phi(\vec{k}_y)\Phi(\vec{k}_z)\rangle &= \iiint_{\vec{x},\vec{y},\vec{z}} \langle\Phi(\vec{x})\Phi(\vec{y})\Phi(\vec{z})\rangle e^{i(\vec{k}_x\vec{x}+\vec{k}_y\vec{y}+\vec{k}_z\vec{z})} \\
\langle\varphi^2(\vec{x})\varphi(\vec{y})\varphi(\vec{z})\rangle &\to \iiiint_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} \langle\varphi(\vec{k}_1)\varphi(\vec{k}_2)\rangle\langle\varphi(\vec{k}_3)\varphi(\vec{k}_4)\rangle\times \\
&\qquad \delta^3(\vec{k}_1+\vec{k}_2-\vec{k}_x)\delta^3(\vec{k}_3-\vec{k}_y)\delta^3(\vec{k}_4-\vec{k}_z) \\
&= \iiiint_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} P(\vec{k}_1)P(\vec{k}_3)\times\delta^3(\vec{k}_1+\vec{k}_2)\delta^3(\vec{k}_3+\vec{k}_4) \\
&\qquad \delta^3(\vec{k}_1+\vec{k}_2-\vec{k}_x)\delta^3(\vec{k}_3-\vec{k}_y)\delta^3(\vec{k}_4-\vec{k}_z) \\
&= P(k_x)P(k_y)\delta^3(\vec{k}_x+\vec{k}_y+\vec{k}_z) \\
&\quad + \text{permutations} + \text{connected pieces}
\end{aligned}$$

More Shapes of Local non-Gaussianity!

$$\langle \varphi^4(\vec{x})\varphi(\vec{y})\varphi(\vec{z})\rangle \quad \to \quad \int d^3k\, P(\vec{k})\left(P(k_x)P(k_y) + \text{permutations}\right)$$

$$\langle \varphi^3(\vec{x})\varphi^2(\vec{y})\varphi(\vec{z})\rangle \quad \to \quad \int d^3k\, P(\vec{k})\left(P(\vec{k}_x - \vec{k})P(k_y) + \text{permutations}\right)$$

$$\langle \varphi^2(\vec{x})\varphi^2(\vec{y})\varphi^2(\vec{z})\rangle \quad \to \quad \int d^3k\, P(\vec{k})\left(P(\vec{k}_x - \vec{k})P(\vec{k}_y + \vec{k}) + \text{permutations}\right)$$

How do these shapes look like?

More Shapes of Local non-Gaussianity!

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How do these shapes look like?

Is non-"Local" Shape Really Local?

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