Developments in massive gravity

Kurt Hinterbichler (Perimeter Institute)

(ArXiv:1105.3735)

IPMU, October 9, 2012

The cosmological constant problem

One motivation: the cosmological constant problem:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu} \qquad \frac{\Lambda}{M_P^2} \sim 10^{-122}$$

Really small

Two aspects to the problem:

- existence of the small number (naturalness)
- stability under quantum corrections (technical naturalness)

Two roads to take:

- Take GR, the CC and known rules of QFT seriously ($\rightarrow$ anthropics, landscape)
- Modify things

Modifying gravity

- Lorentz-Invariance $\rightarrow$ degrees of freedom are classified by mass and spin/ helicity

- Should be an infrared modification, to say something about the cosmological constant without messing up solar system tests of gravity

- GR is the unique theory of an interacting massless helicity-2 at low energies $\rightarrow$ to modify gravity is to change the degrees of freedom

First thought: make the graviton massive

IR modification scale

$$V(r) \sim \frac{M}{M_P^2}\frac{1}{r}e^{-mr}, \quad m \sim H$$

Extra DOF: 5 massive spin states as opposed to 2 helicity states

Other motivations

1) It is an interesting field theoretic question: is it possible to have a consistent theory of an interacting massive spin-2 particle?

2) It gives general lessons about GR:
- Nicely illustrates the generic obstacles encountered when attempting to modifying gravity in the IR.
- Appreciation for why GR is special

3) It shows us new mechanisms: massive gravity is a deformation of GR $\rightarrow$ pathologies should go away as mass term goes to zero $\rightarrow$ new mechanisms for curing pathologies

Linear theory

Massive spin 2 particle: 5 degrees of freedom (as opposed to 2 for massless helicity 2)

<u>Fierz-Pauli action</u>: Fierz, Pauli (1939)

$$\mathcal{L} = \boxed{-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h}\boxed{-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)} + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

Einstein-Hilbert (massless) part.
Gauge symmetry: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Mass term breaks gauge symmetry.
Fierz-Pauli tuning ensures 5 D.O.F.

Equations of motion: $(\Box - m^2)h_{\mu\nu} = 0, \quad \partial^\mu h_{\mu\nu} = 0, \quad h = 0$

Hamiltonian Formulation:

$$S = \int d^D x\ \pi_{ij}\dot{h}_{ij} - \mathcal{H}(h_{ij}, \pi_{ij}) + 2h_{0i}\left(\partial_j \pi_{ij}\right) + m^2 h_{0i}^2 + h_{00}\left(\vec{\nabla}^2 h_{ii} - \partial_i\partial_j h_{ij} - m^2 h_{ii}\right)$$

Auxiliary variables

Lagrange Multiplier

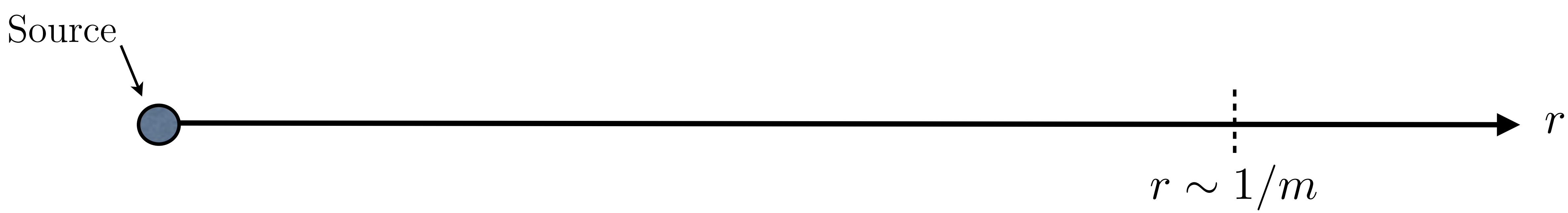
Linear solutions around sources

Amplitude for interaction of two conserved sources:

$$\mathcal{A} \sim \frac{1}{M_P} \int d^4p \frac{1}{p^2 + m^2} \left[T'^{\mu\nu}(p) T_{\mu\nu}(p) - \frac{1}{3} T'(p) T(p) \right]$$

For GR this would be 1/2

Newtonian Potential: $\phi_N = -\frac{4}{3}\frac{GM}{r} e^{-mr}$



Massless gravity vs. massless limit of massive gravity: the vDVZ discontinuity

van Dam, Veltman, and Zakharov (1970)

	$m \to 0$	$m = 0$
Newtonian potential	$\phi_N = -\frac{4}{3}\frac{GM}{r}$	$\phi_N = -\frac{GM}{r}$
Light bending angle (at impact parameter b)	$\alpha = \frac{4GM}{b}$	$\alpha = \frac{4GM}{b}$

Non-linearities

Take interactions to be those of GR: $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$

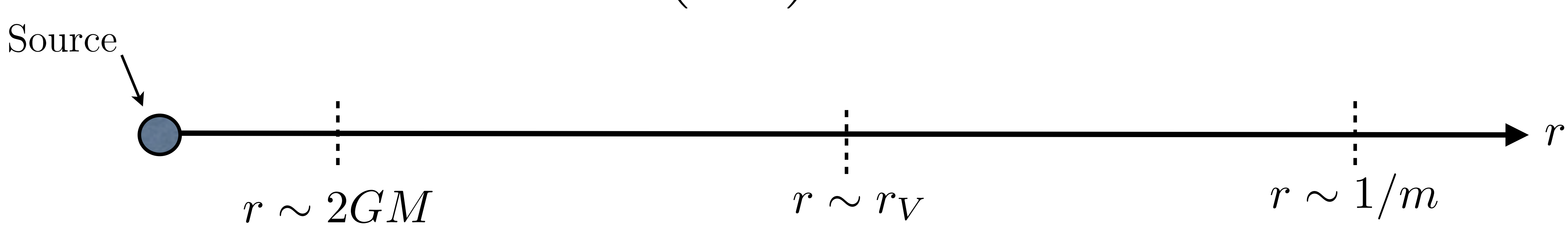
$$S = \frac{M_P^2}{2} \int d^4x \left[(\sqrt{-g}R) - \sqrt{-\eta}\frac{1}{4}m^2\eta^{\mu\alpha}\eta^{\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\nu\beta}\right)\right]$$

Non-linearity expansion of the potential: $\phi(r) \quad = \quad \phi_0(r) + \epsilon\phi_1(r) + \epsilon^2\phi_2(r) + \cdots$

$$\phi(r) = -\frac{4}{3}\frac{GM}{r}\left(1 - \frac{1}{6}\frac{GM}{m^4 r^5} + \cdots\right)$$

Non-linearity become important at the <u>Vainshtein radius</u>: Vainshtein (1972)

$$\epsilon \sim \left(\frac{r_V}{r}\right)^5, \quad r_V \equiv \left(\frac{GM}{m^4}\right)^{1/5}$$



vDVZ discontinuity could possibly be cured by non-linearities

The Boulware-Deser ghost

Boulware, Deser (1972)

ADM variables:
$$\begin{aligned} g_{00} &= -N^2 + g^{ij} N_i N_j, \\ g_{0i} &= N_i, \\ g_{ij} &= g_{ij}. \end{aligned}$$

Hamiltonian:

$$S = \frac{M_P^2}{2} \int d^4x p^{ab} \dot{g}_{ab} - N\mathcal{C} - N_i \mathcal{C}^i$$

In GR, lapse and shift are lagrange multipliers enforcing gauge constraints

In massive GR, they are auxiliary variables

Phase space DOF = 6 spatial metric + 6 canonical momentum - 0 constraints = 12 $\rightarrow$ 6 real space DOF

Extra non-linear D.O.F. is the <u>Boulware-Deser ghost</u>

Hamiltonian is unbounded.

Stükelberg analysis, linear theory

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

Two problems:

1) Massless limit is not smooth (DOF are lost)

2) Propagator looks bad at high energy

$$\mathcal{D}_{\alpha\beta,\sigma\lambda} = \frac{-i}{p^2+m^2}\left[\frac{1}{2}\left(P_{\alpha\sigma}P_{\beta\lambda} + P_{\alpha\lambda}P_{\beta\sigma}\right) - \frac{1}{D-1}P_{\alpha\beta}P_{\sigma\lambda}\right] \sim \frac{p^2}{m^4}$$

$$P_{\alpha\beta} \equiv \eta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m^2}$$

Familiar power-counting doesn't work

Stükelberg analysis, linear theory

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

Restore the gauge invariance broken by the mass term by introducing a <u>Stükelberg field</u>

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$$

$$\mathcal{L}_{m=0} - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) - \frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} - 2m^2\left(h_{\mu\nu}\partial^\mu A^\nu - h\partial_\mu A^\mu\right) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

There is now a gauge symmetry

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad \delta A_\mu = -\xi_\mu$$

Unitary gauge $A_\mu = 0$ recovers the original lagrangian

Canonically normalize, $A_\mu \sim \frac{1}{m}\hat{A}_\mu$ $\quad$ m=0 limit is still not smooth

Stükelberg analysis, linear theory

Introduce a further Stükelberg field $A_\mu \to A_\mu + \partial_\mu \phi$

$$\mathcal{L}_{m=0} - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) - \frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} - 2m^2\left(h_{\mu\nu}\partial^\mu A^\nu - h\partial_\mu A^\mu\right) - 2m^2\left(h_{\mu\nu}\partial^\mu\partial^\nu\phi - h\partial^2\phi\right)$$
$$+\frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

There is now a further gauge symmetry $\delta A_\mu = \partial_\mu \Lambda, \quad \delta\phi = -\Lambda$

Canonically normalize $A_\mu \sim \frac{1}{m}\hat{A}_\mu, \quad \phi \sim \frac{1}{m^2}\hat{\phi}$ massless limit

$$\mathcal{L}_{m=0} - \frac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - 2\left(h_{\mu\nu}\partial^\mu\partial^\nu\hat{\phi} - h\partial^2\hat{\phi}\right) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

Diagonalize kinetic terms $h_{\mu\nu} = h'_{\mu\nu} + \hat{\phi}\,\eta_{\mu\nu}$

This is the vDVZ discontinuity: scalar fifth force

$$\mathcal{L}_{m=0}(h') - \frac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - 3\,\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi} + \frac{1}{M_P}h'_{\mu\nu}T^{\mu\nu} + \frac{1}{M_P}\hat{\phi}T$$

In massless limit, Stükelberg fields are helicity 1 and 0 parts of the massive graviton

Propagators are now well behaved $\sim 1/p^2$

De-gravitation

Arkani-Hamed, Dimopoulos, Dvali, Gabadadze (2002)
Dvali, Hofmann, Khoury (2007)

$$\mathcal{L}_{m=0} - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) - \frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} - 2m^2\left(h_{\mu\nu}\partial^\mu A^\nu - h\partial_\mu A^\mu\right) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

Integrate out the vector field

$$\frac{1}{2}h_{\mu\nu}\left(1 - \frac{m^2}{\Box}\right)\mathcal{E}^{\mu\nu,\alpha\beta}h_{\alpha\beta} + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

Equations of the motion now look like gravity seen through a high-pass filter

$$\mathcal{E}^{\mu\nu,\alpha\beta}h_{\alpha\beta} = -\frac{1}{M_P}\boxed{\left(1 - \frac{m^2}{\Box}\right)^{-1}}T^{\mu\nu}$$

~ 1 for $\partial \gg m$
$\ll 1$ for $\partial \ll m$

A massive graviton is supposed to be able to screen a large CC

Stükelberg analysis: interacting theory

Arkani-Hamed, Georgi and Schwartz (2003)

$$S = \frac{M_P^2}{2} \int d^4x \left[\left(\sqrt{-g}R\right) - \sqrt{-\eta}\frac{1}{4}m^2\eta^{\mu\alpha}\eta^{\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\nu\beta}\right) \right]$$

Must restore full non-linear diffs

$$g_{\mu\nu}(x) \to G_{\mu\nu} = \frac{\partial Y^\alpha}{\partial x^\mu}\frac{\partial Y^\beta}{\partial x^\nu} g_{\alpha\beta}\left(Y(x)\right)$$

There is now a diffeomorphism symmetry

$$g_{\mu\nu}(x) \to \frac{\partial f^\alpha}{\partial x^\mu}\frac{\partial f^\beta}{\partial x^\nu} g_{\alpha\beta}\left(f(x)\right), \quad Y^\mu(x) \to f^{-1}\left(Y(x)\right)^\mu$$

Expand around unitary gauge

$$Y^\alpha(x) = x^\alpha + A^\alpha(x)$$

Introduce scalar Stükelberg

$$A_\mu \to A_\mu + \partial_\mu\phi$$

Replacement becomes

$$h_{\mu\nu} \to H_{\mu\nu} = h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\partial_\mu\partial_\nu\phi + \partial_\mu A^\alpha\partial_\nu A_\alpha + \partial_\mu A^\alpha\partial_\nu\partial_\alpha\phi + \partial_\mu\partial^\alpha\phi\partial_\nu A_\alpha + \partial_\mu\partial^\alpha\phi\partial_\nu\partial_\alpha\phi + \cdots$$

The effective field theory

$$\hat{h} \sim M_P h, \quad \hat{A} \sim m M_P A, \quad \hat{\phi} \sim m^2 M_P \phi$$

There are now interaction terms:

$$m^2 M_P^2 h^{n_h} (\partial A)^{n_A} (\partial^2 \phi)^{n_\phi} \sim \Lambda_\lambda^{4-n_h-2n_A-3n_\phi} \hat{h}^{n_h} (\partial \hat{A})^{n_A} (\partial^2 \hat{\phi})^{n_\phi}$$

Various strong coupling scales: $\Lambda_\lambda = \left(M_P m^{\lambda-1}\right)^{1/\lambda}, \quad \lambda = \dfrac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2}$

The larger λ, the smaller the scale

The smallest scale is carried by a cubic scalar interaction:

$$\sim \frac{(\partial^2 \hat{\phi})^3}{\Lambda_5^5}, \qquad \Lambda_5 = (M_P m^4)^{1/5}$$

This is the (UV) strong coupling scale of the theory

Cubic lagrangian and the decoupling limit

Creminelli, Nicolis, Papucci, Trincherini (2005)

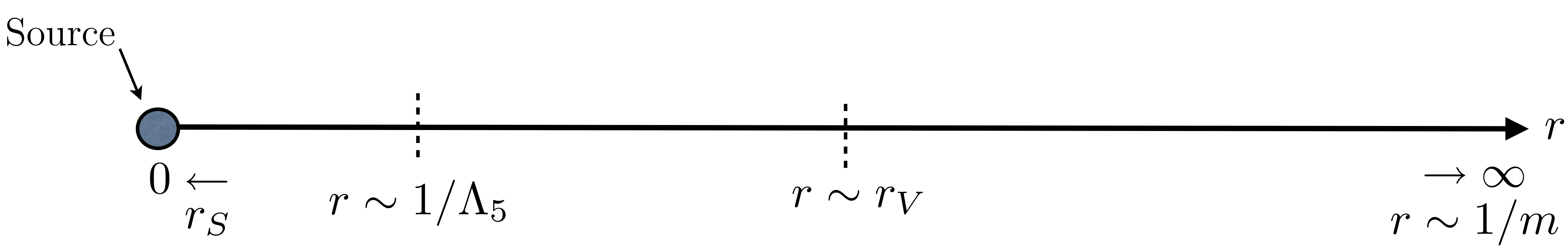
Decoupling limit: Massless limit where we focus in on the strong coupling scale

$$m \to 0, \quad M_P \to \infty, \qquad \Lambda_5 \text{ fixed}$$

All that survives is the leading cubic scalar interaction

$$-3(\partial\hat{\phi})^2 + \frac{2}{\Lambda_5^5}\left[(\Box\hat{\phi})^3 - (\Box\hat{\phi})(\partial_\mu\partial_\nu\hat{\phi})^2\right] + \frac{1}{M_P}\hat{\phi}T$$

The scalar non-linearities are responsible for the Vainshtein radius



Boulware Deser ghost (again)

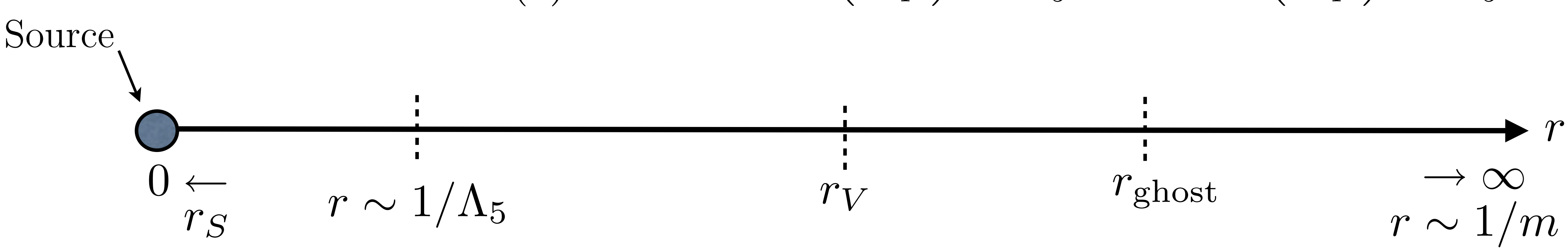
$$-3(\partial\hat{\phi})^2 + \frac{2}{\Lambda_5^5}\left[(\Box\hat{\phi})^3 - (\Box\hat{\phi})(\partial_\mu\partial_\nu\hat{\phi})^2\right] + \frac{1}{M_P}\hat{\phi}T$$

Higher derivative lagrangian, fourth order equations of motion $\rightarrow$ two degrees of freedom $\rightarrow$ manifestation of the Boulware-Deser ghost

Expand around the spherical background: $\phi = \Phi(r) + \varphi$

$$\sim -(\partial\varphi)^2 + \frac{(\partial^2\Phi)}{\Lambda_5^5}(\partial^2\varphi)^2 + \text{interactions}$$

$$m^2_{\text{ghost}}(r) \sim \frac{\Lambda_5^5}{\partial^2\Phi(r)} \qquad r_{\text{ghost}} \sim \left(\frac{M}{M_P}\right)^{1/3}\frac{1}{\Lambda_5} \gg r_V \sim \left(\frac{M}{M_P}\right)^{1/5}\frac{1}{\Lambda_5}$$



The Vainshtein mechanism

Scalar field profile for a spherical solution around a source of mass M

$$\begin{cases} \hat{\phi} \sim \frac{M}{M_P}\frac{1}{r}, & r \gg r_V, \\ \hat{\phi} \sim \left(\frac{M}{M_P}\right)^{1/2} \Lambda_5^{5/2} r^{3/2}, & r \ll r_V. \end{cases}$$

5-th force on a test particle is suppressed inside the Vainshtein radius:

$$\frac{F_\phi}{F_{\rm Newton}} = \frac{\hat{\phi}'(r)/M_P}{M/M_P^2 r^2} = \begin{cases} \sim \left(\frac{r}{r_V}\right)^{5/2} & r \ll r_V \\ \sim 1 & r \gg r_V \end{cases}$$

Can re-write the 4th order scalar lagrangian as two second order scalars $\hat{\phi} = \tilde{\phi} - \psi$

Deffayet, Rombouts (2005)

$$\mathcal{L} = -(\partial\tilde{\phi})^2 + (\partial\psi)^2 + \Lambda_5^{5/2}\psi^{3/2} + \frac{1}{M_P}\tilde{\phi}T + \frac{1}{M_P}\psi T$$

<u>The Vainshtein mechanism</u>: The ghost cancels the force of the longitudinal mode, restoring continuity with GR.

Quantum corrections

A small graviton mass is technically natural: gauge symmetry is restored when m=0. Quantum corrections to the mass are proportional to m.

In the decoupling limit, we should generate all operators with the symmetry $\hat{\phi} \to \hat{\phi} + c + c_\mu x^\mu$

$$c_{p,q}\partial^q h^p \sim \frac{\partial^q(\partial^2\hat{\phi})^p}{\Lambda_5^{3p+q-4}}. \qquad c_{p,q} \sim \Lambda_5^{-3p-q+4} M_P^p m^{2p} = \left(m^{16-4q-2p} M_P^{2p-q+4}\right)^{1/5}$$

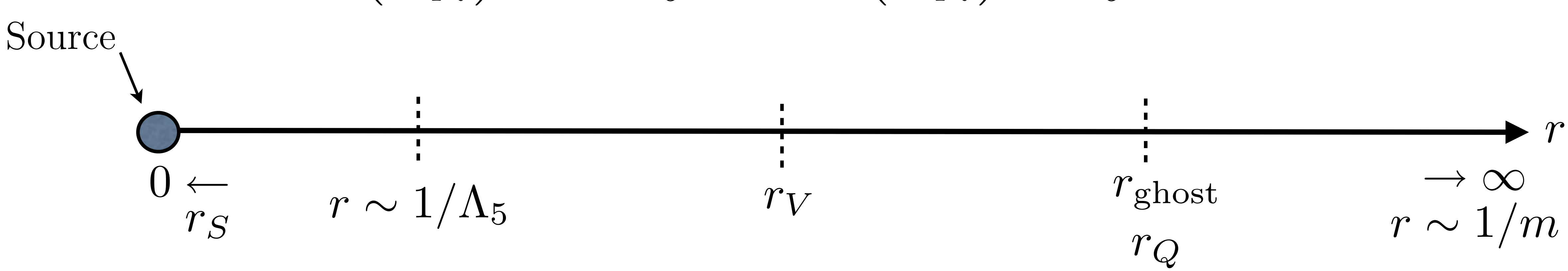
This includes a small mass correction

$$\delta m^2 = m^2 \left(\frac{m^2}{\Lambda_5^2}\right)$$

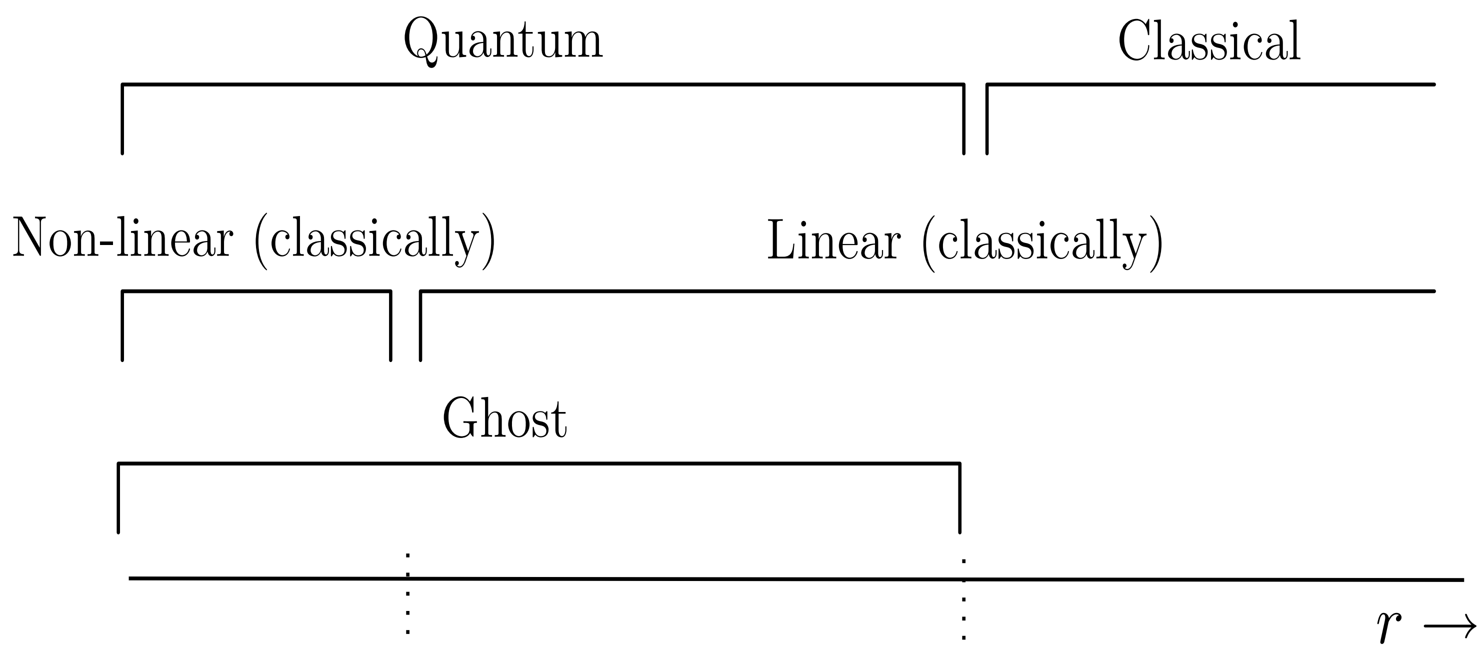
And a detuning of the Fierz-Pauli mass term, with ghost at $m_g \sim \Lambda_5$

Radius at which quantum operators become important:

$$r_{p,q} \sim \left(\frac{M}{M_{Pl}}\right)^{\frac{p-2}{3p+q-4}} \frac{1}{\Lambda_5} \to r_Q \sim \left(\frac{M}{M_{Pl}}\right)^{1/3} \frac{1}{\Lambda_5}$$



“Bad” massive gravity



$$r_V \sim \left(\frac{M}{M_P}\right)^{1/5} \frac{1}{\Lambda_5} \qquad r_Q \sim \left(\frac{M}{M_P}\right)^{1/3} \frac{1}{\Lambda_5}$$

Other non-linear interactions

$$\frac{M_P^2}{2}\int d^4x\ \left[(\sqrt{-g}R) - \sqrt{-g}\frac{1}{4}m^2 V(g,h)\right],$$

$$V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + V_5(g,h) + \cdots ,$$

$$\begin{aligned}
V_2(g,h) &= \langle h^2\rangle - \langle h\rangle^2, \\
V_3(g,h) &= +c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3, \\
V_4(g,h) &= +d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4, \\
V_5(g,h) &= +f_1\langle h^5\rangle + f_2\langle h^4\rangle\langle h\rangle + f_3\langle h^3\rangle\langle h\rangle^2 + f_4\langle h^3\rangle\langle h^2\rangle + f_5\langle h^2\rangle^2\langle h\rangle \\
&\quad +f_6\langle h^2\rangle\langle h\rangle^3 + f_7\langle h\rangle^5, \\
&\vdots
\end{aligned}$$

After Stükelberg-ing, $h_{\mu\nu} \to h_{\mu\nu} + 2\,\partial_\mu\partial_\nu\phi - \partial_\mu\partial_\alpha\phi\,\partial_\nu\partial^\alpha\phi$

the bad terms, those with cutoffs $< \Lambda_3 \equiv (m^2 M_P)^{1/3}$ are the scalar self-interactions

$$(\partial^2\phi)^n$$

Raising the cutoff

At each order in phi, there is a total derivative combination (the characteristic polynomial)

$$
\begin{aligned}
\mathcal{L}_1^{\rm TD}(\Pi) &= [\Pi], \\
\mathcal{L}_2^{\rm TD}(\Pi) &= [\Pi]^2 - [\Pi^2], \\
\mathcal{L}_3^{\rm TD}(\Pi) &= [\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3], \\
\mathcal{L}_4^{\rm TD}(\Pi) &= [\Pi]^4 - 6[\Pi^2][\Pi]^2 + 8[\Pi^3][\Pi] + 3[\Pi^2]^2 - 6[\Pi^4],
\end{aligned}
$$

$$\Pi_{\mu\nu} \equiv \partial_\mu \partial_\nu \phi$$

$$\det(1+\Pi) = 1 + \mathcal{L}_1^{\rm TD}(\Pi) + \frac{1}{2}\mathcal{L}_2^{\rm TD}(\Pi) + \frac{1}{3!}\mathcal{L}_3^{\rm TD}(\Pi) + \frac{1}{4!}\mathcal{L}_4^{\rm TD}(\Pi) + \cdots$$

Can choose the interactions, order by order in h, so that the scalar self-interactions appear in these combinations. Arkani-Hamed, Georgi and Schwartz (2003)

There is a three-parameter family of ways to do this (graviton mass m plus 2 other parameters)

Once this is done, the cutoff of the theory will be $\Lambda_3 = (m^2 M_P)^{1/3}$

The Λ_3 theory

The operators carrying the scale Λ_3 are $\sim \frac{\hat{h}(\partial^2\hat{\phi})^n}{M_P^{n+1}m^{2n+2}}$

The decoupling limit is now $m \to 0, \quad M_P \to \infty, \quad \Lambda_3$ fixed

de Rham, Gabadadze (2010)

$$\frac{1}{2}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta} - \frac{1}{2}\hat{h}^{\mu\nu}\left[-4X^{(1)}_{\mu\nu}(\hat{\phi}) + \frac{4(6c_3-1)}{\Lambda_3^3}X^{(2)}_{\mu\nu}(\hat{\phi}) + \frac{16(8d_5+c_3)}{\Lambda_3^6}X^{(3)}_{\mu\nu}(\hat{\phi})\right] + \frac{1}{M_P}\hat{h}_{\mu\nu}T^{\mu\nu}$$

X tensors:

$$X^{(n)}_{\mu\nu} = \frac{1}{n+1}\frac{\delta}{\delta\Pi_{\mu\nu}}\mathcal{L}^{\rm TD}_{n+1}(\Pi)$$

$$\Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi$$

$$\begin{aligned}
X^{(0)}_{\mu\nu} &= \eta_{\mu\nu} \\
X^{(1)}_{\mu\nu} &= [\Pi]\,\eta_{\mu\nu} - \Pi_{\mu\nu} \\
X^{(2)}_{\mu\nu} &= \left([\Pi]^2 - \left[\Pi^2\right]\right)\eta_{\mu\nu} - 2\,[\Pi]\,\Pi_{\mu\nu} + 2\Pi^2_{\mu\nu} \\
X^{(3)}_{\mu\nu} &= \left([\Pi]^3 - 3\,[\Pi]\,\left[\Pi^2\right] + 2\left[\Pi^3\right]\right)\eta_{\mu\nu} - 3\left([\Pi]^2 - \left[\Pi^2\right]\right)\Pi_{\mu\nu} + 6\,[\Pi]\,\Pi^2_{\mu\nu} - 6\Pi^3_{\mu\nu} \\
&\vdots
\end{aligned}$$

They have the following properties, which ensures that the decoupling limit is ghost free

$$\partial^\mu X^{(n)}_{\mu\nu} = 0$$

$X^{(n)}_{ij}$ has at most two time derivatives,

$X^{(n)}_{0i}$ has at most one time derivative,

$X^{(n)}_{00}$ has no time derivatives.

Galileons

Diagonalize: $\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \hat{\phi}\hat{h}_{\mu\nu} + \frac{2(6c_3-1)}{\Lambda_3^3}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}$

$$
\begin{aligned}
&\frac{1}{2}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta} \\
&-3(\partial\hat{\phi})^2 + \frac{6(6c_3-1)}{\Lambda_3^3}(\partial\hat{\phi})^2\Box\hat{\phi} - 4\frac{(6c_3-1)^2-4(8d_5+c_3)}{\Lambda_3^6}(\partial\hat{\phi})^2\left([\hat{\Pi}]^2-[\hat{\Pi}^2]\right) \\
&-\frac{40(6c_3-1)(8d_5+c_3)}{\Lambda_3^9}(\partial\hat{\phi})^2\left([\hat{\Pi}]^3-3[\hat{\Pi}^2][\hat{\Pi}]+2[\hat{\Pi}^3]\right)
\end{aligned}
$$

Longitudinal mode is described by Galileon interactions:

$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}(\partial\phi)^2 \ , \\
\mathcal{L}_3 &= -\frac{1}{2}(\partial\phi)^2[\Pi] \ , \\
\mathcal{L}_4 &= -\frac{1}{2}(\partial\phi)^2\left([\Pi]^2-[\Pi^2]\right) \ , \\
\mathcal{L}_5 &= -\frac{1}{2}(\partial\phi)^2\left([\Pi]^3-3[\Pi][\Pi^2]+2[\Pi^3]\right)
\end{aligned}
$$

- Equations of motion are second order (no ghost)
- Symmetry under shifts of the field and its derivative $\phi(x) \to \phi(x) + c + c_\mu x^\mu$
- Not renormalized at any loop (no quantum corrections in the decoupling limit)

Vainshtein Mechanism in Λ_3 theory

$$\mathcal{L} = -3(\partial\hat{\phi})^2 - \frac{1}{\Lambda_3^3}(\partial\hat{\phi})^2\Box\hat{\phi} + \frac{1}{M_4}\hat{\phi}T$$

Solution around point source of mass M:

$$\hat{\phi}(r) \sim \begin{cases} \Lambda_3^3 {r_V^{(3)}}^2 \left(\frac{r}{r_V^{(3)}}\right)^{1/2} & r \ll r_V^{(3)}, \\ \Lambda_3^3 {r_V^{(3)}}^2 \left(\frac{r_V^{(3)}}{r}\right) & r \gg r_V^{(3)}. \end{cases}$$

Vainshtein radius: $r_V^{(3)} \equiv \left(\frac{M}{M_{Pl}}\right)^{1/3} \frac{1}{\Lambda_3}$

5-th force on a test particle, relative to gravity:

$$\frac{F_\phi}{F_{\rm Newton}} = \frac{\hat{\phi}'(r)/M_P}{M/(M_P^2 r^2)} = \begin{cases} \sim \left(\frac{r}{r_V^{(3)}}\right)^{3/2} & r \ll r_V^{(3)}, \\ \sim 1 & r \gg r_V^{(3)}. \end{cases}$$

$\hat{\phi} = \Phi + \varphi, \quad T = T_0 + \delta T$

$$-3(\partial\varphi)^2 + \boxed{\frac{2}{\Lambda^3}\left(\partial_\mu\partial_\nu\Phi - \eta_{\mu\nu}\Box\Phi\right)}\,\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{\Lambda^3}(\partial\varphi)^2\Box\varphi + \frac{1}{M_4}\varphi\delta T$$

$$\sim \left(\frac{r_V^{(3)}}{r}\right)^{3/2}$$

Kinetic terms are enhanced, which means that, after canonical normalization, the coupling to δT is suppressed. The non-linear coupling scale is also raised.

This is known as a <u>Screening mechanism</u>

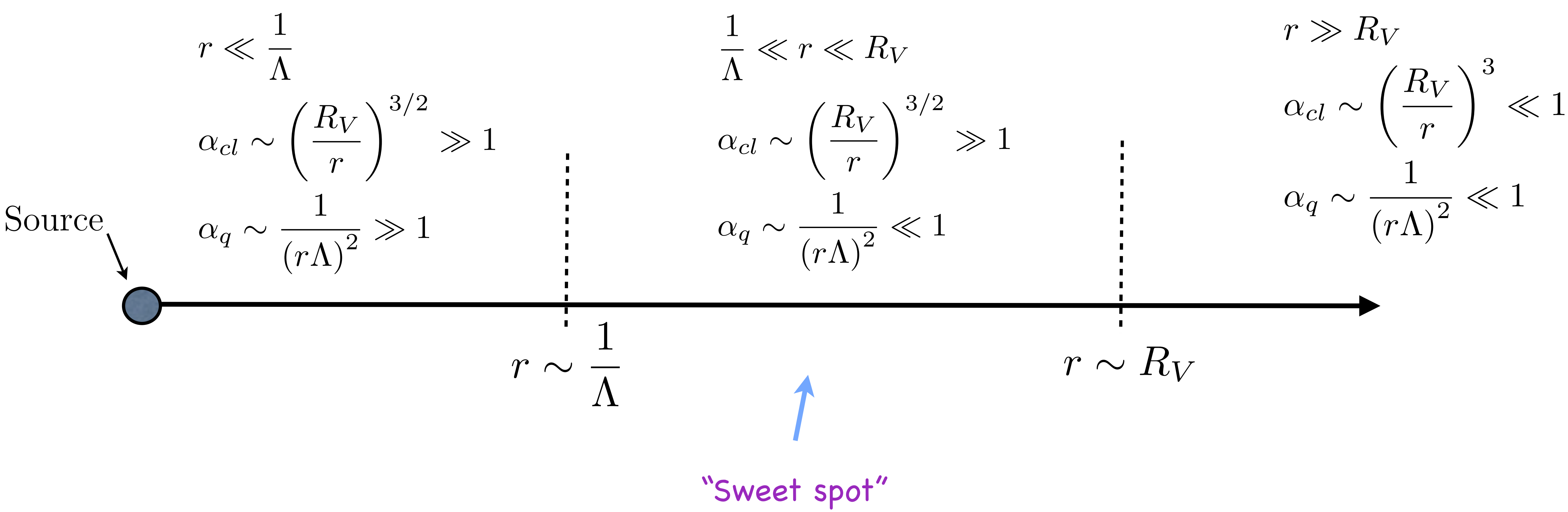
Quantum corrections and the effective field theory

Non-renormalizable effective theory with a cutoff Λ. Must include all terms compatible with galilean symmery, suppressed by powers of the cutoff

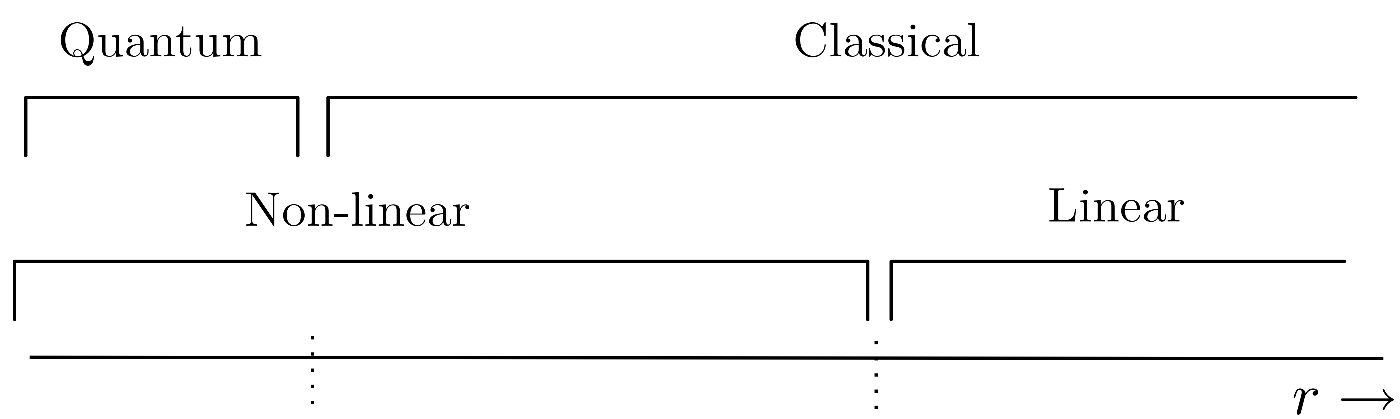
$$\mathcal{L} \sim (\partial\pi)^2 + \frac{1}{\Lambda^{3n}}(\partial\pi)^2(\partial\partial\pi)^n + \frac{1}{\Lambda^{m+3n-4}}\partial^m(\partial\partial\pi)^n$$

Galileon terms $\alpha_{cl} \equiv \frac{\partial\partial\pi}{\Lambda^3}$

Terms with at least two derivatives per field $\alpha_q \equiv \frac{\partial^2}{\Lambda^2}$



“Good” massive gravity



$$r_Q \sim \frac{1}{\Lambda_3} \qquad\qquad r_V \sim \left(\frac{M}{M_P}\right)^{1/3} \frac{1}{\Lambda_3}$$

- Higher cutoff
- Free of the Boulware-Deser ghost, to all orders beyond the decoupling limit Hassan, Rosen (2011)
- Possesses a screening mechanism in the non-linear regime, which is under control quantum mechanically, and restores continuity with GR as m approaches 0.

The Λ_3 theory (dRGT theory)

de Rham, Gabadadze, Tolley (2011)

The theory with this choice can be re-summed

$$\frac{M_P^{D-2}}{2}\int d^D x\,\sqrt{-g}\left[R-\frac{m^2}{4}\sum_{n=0}^{D}\beta_n S_n(\sqrt{g^{-1}\eta})\right]$$

Characteristic Polynomials

$$S_n(M)=\frac{1}{n!(D-n)!}\,\tilde{\epsilon}_{A_1A_2\cdots A_D}\,\tilde{\epsilon}^{B_1B_2\cdots B_D}\,M^{A_1}_{\ B_1}\cdots M^{A_n}_{\ B_n}\delta^{A_{n+1}}_{\ B_{n+1}}\cdots\delta^{A_D}_{\ B_D}$$

$$\begin{aligned}
S_0(M) &= 1\,,\\
S_1(M) &= [M]\,,\\
S_2(M) &= \frac{1}{2!}\left([M]^2-[M^2]\right)\,,\\
S_3(M) &= \frac{1}{3!}\left([M]^3-3[M][M^2]+2[M^3]\right)\,,\\
&\ \vdots
\end{aligned}$$

$$S_D(M)=\det M\,,$$

Vielbein formulation of ghost-free massive gravity

KH, Rachel Rosen (arXiv:1203.5783)

Or in terms of vierbeins $g_{\mu\nu} = e_\mu{}^A e_\nu{}^B \eta_{AB}$

$$\frac{M_P^{D-2}}{2} \int d^D x \, |e| R[e] - m^2 \sum_n a_n \int \epsilon_{A_1 \cdots A_D} e^{A_1} \wedge \cdots \wedge e^{A_n} \wedge 1^{A_{n+1}} \wedge \cdots \wedge 1^{A_n}$$

Ghost-free mass terms are simply all possible ways of wedging a vierbein and background vierbein:

$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge e^{A_4}$$
$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge 1^{A_4}$$
$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge 1^{A_3} \wedge 1^{A_4}$$
$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge 1^{A_2} \wedge 1^{A_3} \wedge 1^{A_4}$$
$$\epsilon_{A_1 A_2 A_3 A_4} 1^{A_1} \wedge 1^{A_2} \wedge 1^{A_3} \wedge 1^{A_4}$$

Vielbein formulation of massive gravity

KH, Rachel Rosen (arXiv:1203.5783)

Vielbein formulation makes it easy to see that the theory is ghost free:

Parametrize vierbeins as an upper triangular vierbein times a boost

$$\hat{E}_\mu{}^A = \begin{pmatrix} N & N^i e_i{}^a \\ 0 & e_i{}^a \end{pmatrix} \qquad \Lambda(p)^A{}_B = \begin{pmatrix} \gamma & p^a \\ p_b & \delta^a{}_b + \frac{1}{\gamma+1} p^a p_b \end{pmatrix}$$

$$E_\mu{}^A = \Lambda(p)^A{}_B \hat{E}_\mu{}^B = \begin{pmatrix} N\gamma + N^i e_i{}^a p_a & Np^a + N^i e_i{}^b(\delta_b{}^a + \frac{1}{\gamma+1} p_b p^a) \\ e_i{}^a p_a & e_i{}^b(\delta_b{}^a + \frac{1}{\gamma+1} p_b p^a) \end{pmatrix}$$

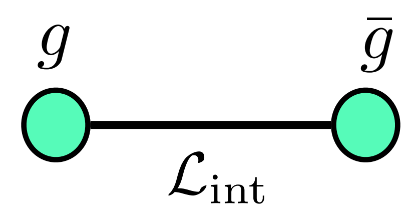
Due to structure of epsilons in the wedge product, mass terms are manifestly linear in lapse and shift:

$$N\mathcal{C}^{\rm m}(e,p) + N^i \mathcal{C}_i^{\rm m}(e,p) + \mathcal{H}(e,p)$$

Ghost free bi-gravity

Hassan, Rosen (2011)

Two-site model: bi-gravity



$$\frac{M_g^2}{2}\sqrt{-g}R[g] + \frac{M_{\bar{g}}^2}{2}\sqrt{-\bar{g}}R[\bar{g}] - \sqrt{-g}\frac{1}{4}m^2 M_{\rm eff}^2 \sum_n \mathcal{L}_n^{\rm TD}(\sqrt{g^{-1}\bar{g}})$$

$$M_{\rm eff}^2 \equiv \left(\frac{1}{M_g^2} + \frac{1}{M_{\bar{g}}^2}\right)^{-1}$$

- Linear theory: massless graviton + massive graviton of mass m ($=$ 7 DOF).
- One diff. invariance $\rightarrow$ generically 12 - 4 $=$ 8 DOF non-linearly
- Special constraint from absence of DB ghost $\rightarrow$ 7 DOF non-linearly

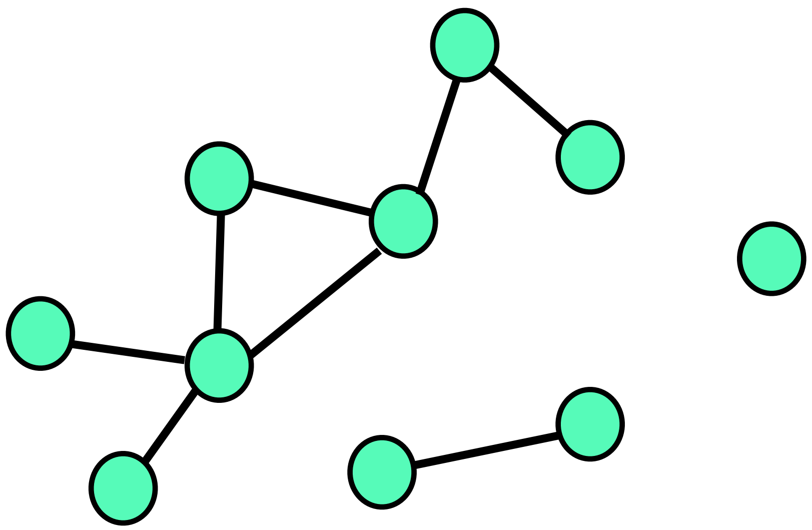
Vierbein formulation: KH, Rachel Rosen (arXiv:1203.5783)

$$\sim \sum_n a_n \epsilon_{A_1 \cdots A_D} e_{(1)}^{A_1} \wedge \cdots \wedge e_{(1)}^{A_n} \wedge e_{(2)}^{A_{n+1}} \wedge \cdots \wedge e_{(2)}^{A_D}$$

Ghost free multi-gravity

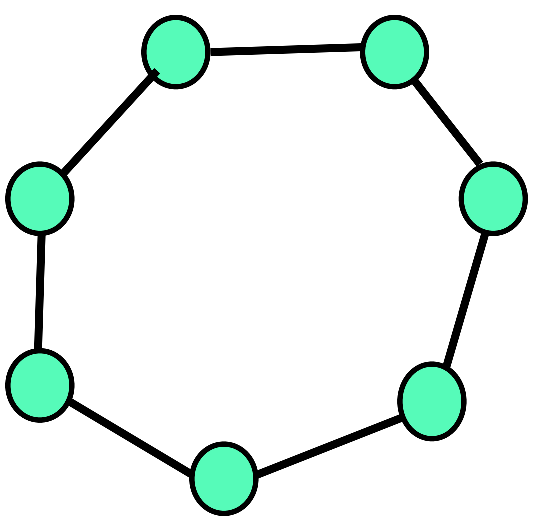
Multi-metric theory graph: one massless graviton per connected component + tower of massive gravitons

KH, Rachel Rosen (arXiv:1203.5783)



Ghost-free deconstructed gravitational dimensions

Arkani-Hamed, Georgi and Schwartz (2003)



Ghost free multi-gravity

KH, Rachel Rosen (arXiv:1203.5783)

Most general ghost-free potential interaction of multiple gravitons

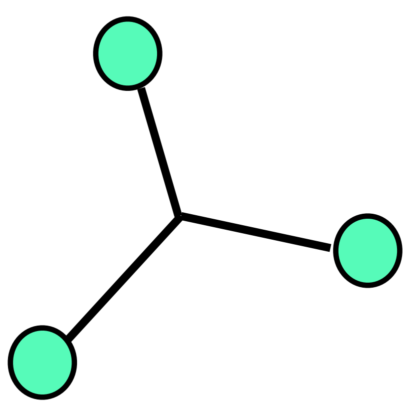
$$\sim T^{I_1 I_2 \cdots I_D} \epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(I_1)} \wedge^{A_2}_{(I_2)} \wedge \cdots \wedge e^{A_D}_{(I_D)}$$

New ghost-free multi-metric interactions in 4-dimensions:

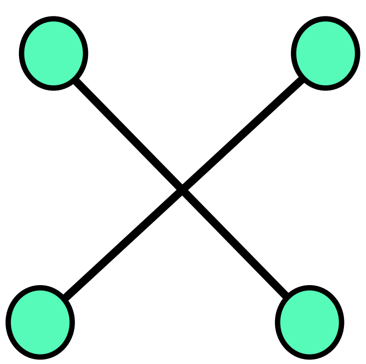
$$\epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(1)} \wedge e^{A_2}_{(1)} \wedge e^{A_3}_{(2)} \wedge e^{4}_{(3)}$$

$$\epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(1)} \wedge e^{A_2}_{(2)} \wedge e^{A_3}_{(2)} \wedge e^{4}_{(3)}$$

$$\epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(1)} \wedge e^{A_2}_{(2)} \wedge e^{A_3}_{(3)} \wedge e^{4}_{(3)}$$



$$\epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(1)} \wedge e^{A_2}_{(2)} \wedge e^{A_3}_{(3)} \wedge e^{4}_{(4)}$$



Interaction of longitudinal modes $\rightarrow$ multi-galileon interactions

Summary and open issues

- Λ_3 massive gravity is the best behaved IR modification of gravity proposed so far
- ~ 40 year old problem of the Boulware-Deser ghost has been solved
- Makes use of galileons, scalar theories with interesting and promising properties
- New signals for cosmology/potential for model building
- Still some underlying topological structure yet to be articulated
- Still the issue of UV completion