The Pseudo-Conformal Universe

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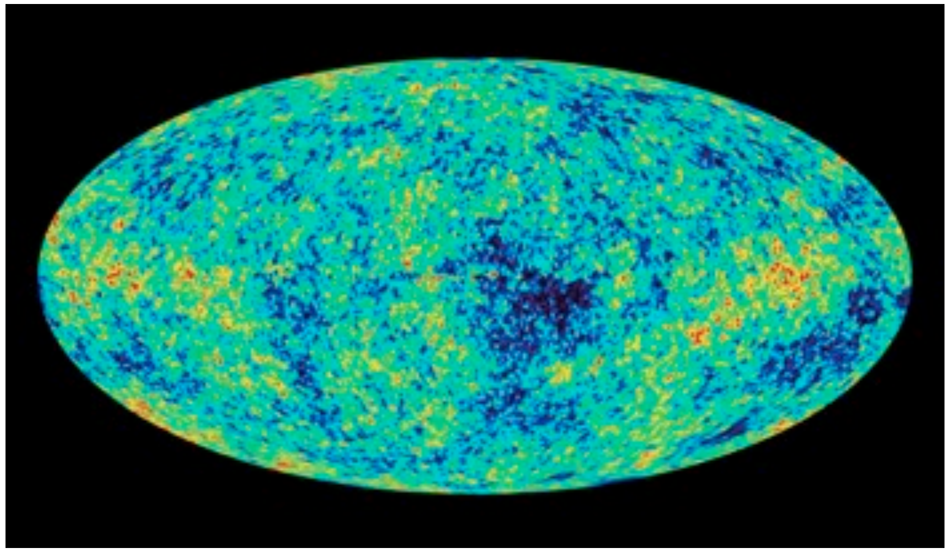
KH, Justin Khoury arXiv:1106.1428
KH, Justin Khoury, Austin Joyce arXiv:1202.6056
KH, Justin Khoury, Austin Joyce, Godfrey Miller arXiv:1209.5742

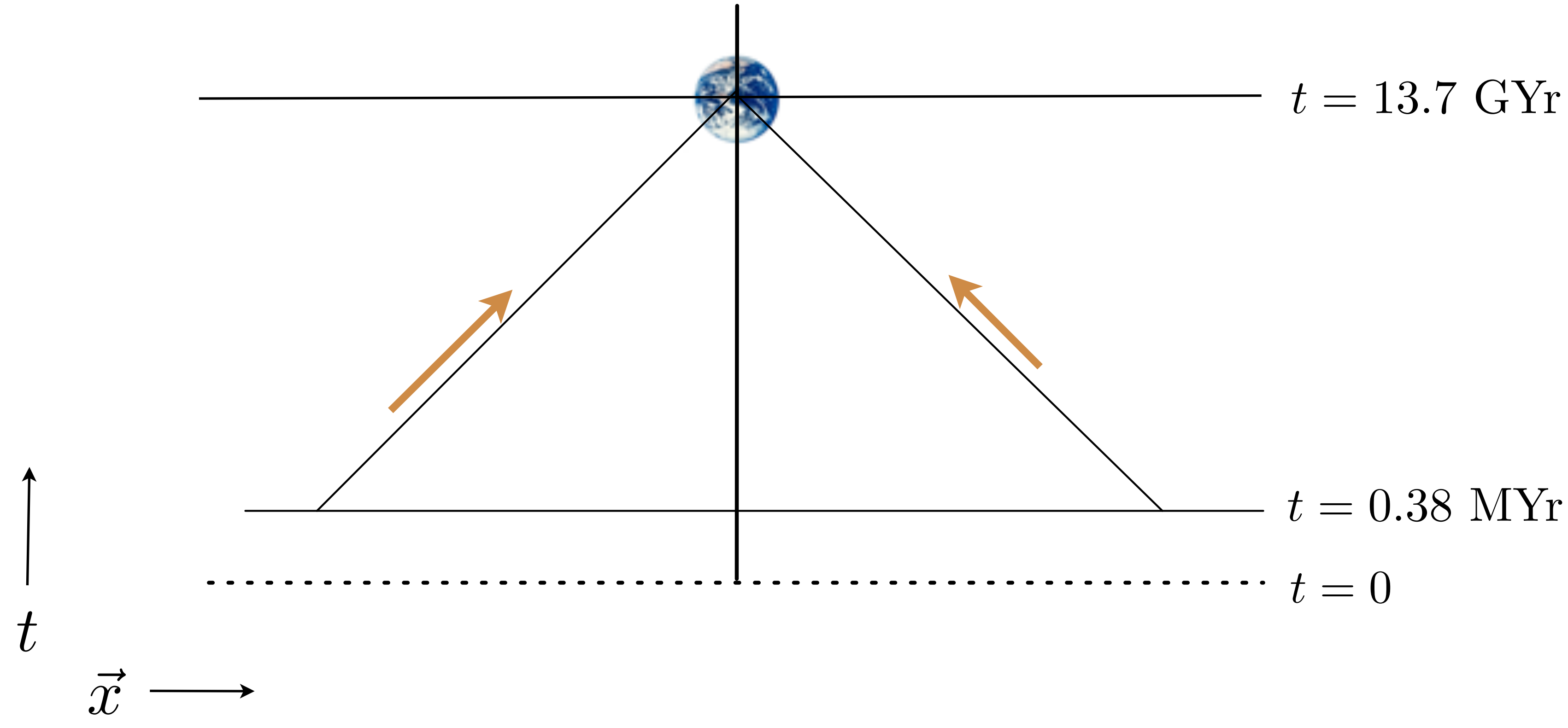
IPMU, October 16, 2012

The early universe

- Data tells us the early universe was very smooth and homogeneous, and nearly spatially flat.

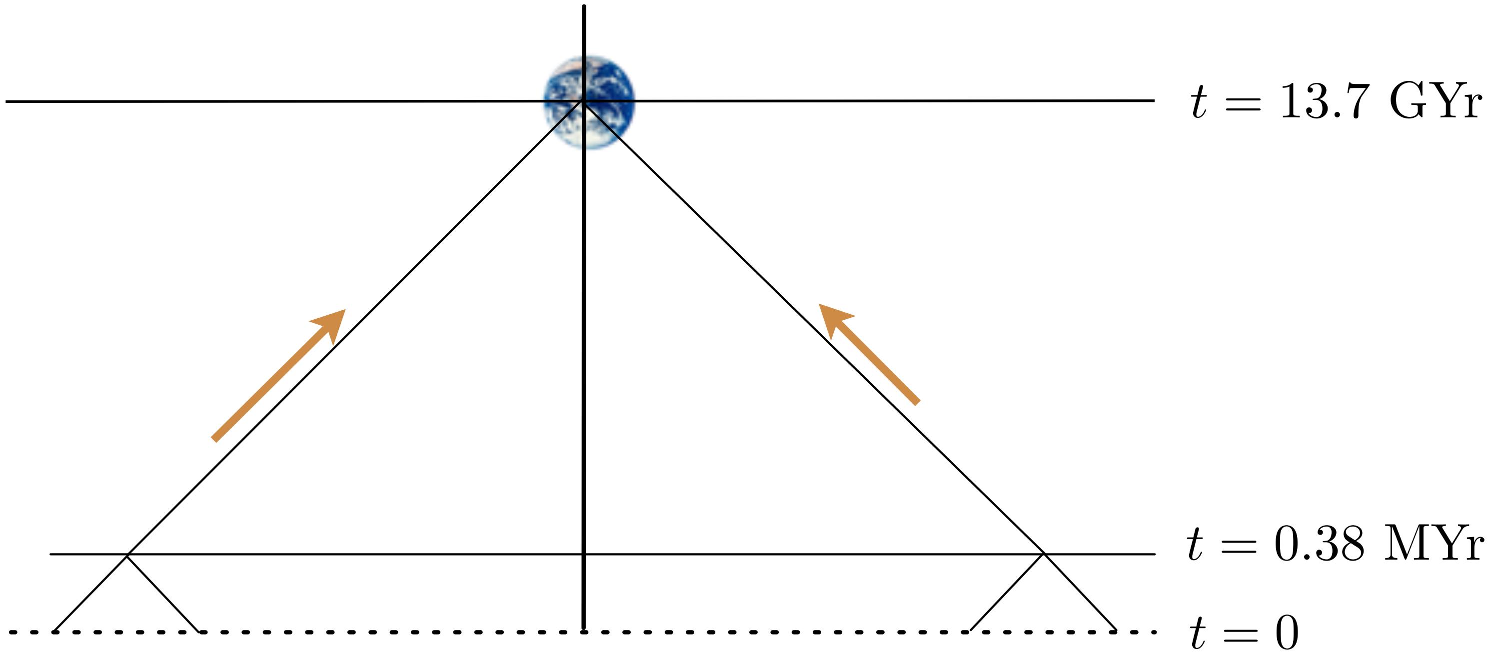


- Data tells us the perturbations imprinted at last scattering are described by nearly scale invariant and gaussian statistics.



Inflation

Inflation is the leading paradigm for explaining the early universe:



- Solves the horizon problem
- Solves the flatness problem, monopole problem (empties out the universe)
- Explains the scale-invariant gaussian perturbations as quantum fluctuations of a primordial field

Inflation is rooted in symmetries

Inflation

Inflation ≈ exponential expansion ≈ de Sitter space

$$ds^2 = -dt^2 + a(t)^2 dx^2 \ , \qquad a(t) = e^{Ht}$$

Driven by vacuum energy with $w =$ -1 , $\rho \sim a^{-3(1+w)}$

$$3M_P^2 H^2 = \rho_V \longleftarrow \text{constant}$$

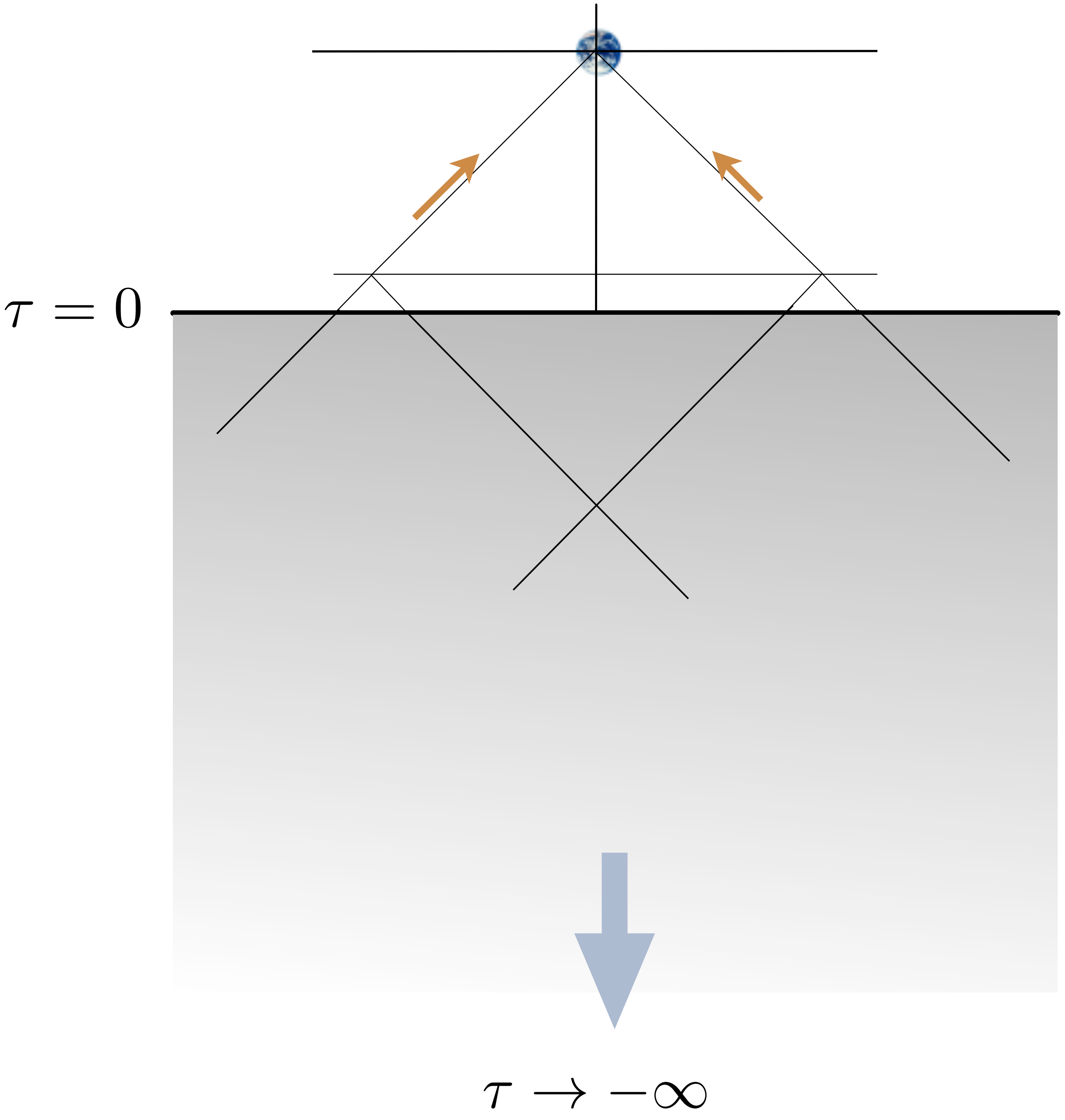
Smoothness, flatness, monopole problems:

Other possible components with $w >$ -1 are emptied out

$$3M_P^2 H^2 = \underset{\underset{\text{constant}}{\uparrow}}{\rho_V} - \underbrace{\overset{\overset{\text{curvature}}{\downarrow}}{\frac{3M_P^2 k}{a^2}} + \overset{\overset{\text{matter}}{\downarrow}}{\frac{\rho_{m,0}}{a^3}} + \overset{\overset{\text{radiation}}{\downarrow}}{\frac{\rho_{r,0}}{a^4}} + \overset{\overset{\text{anisotropies}}{\swarrow}}{\frac{\rho_{A,0}}{a^6}}}_{\underset{\to 0}{\uparrow}}$$

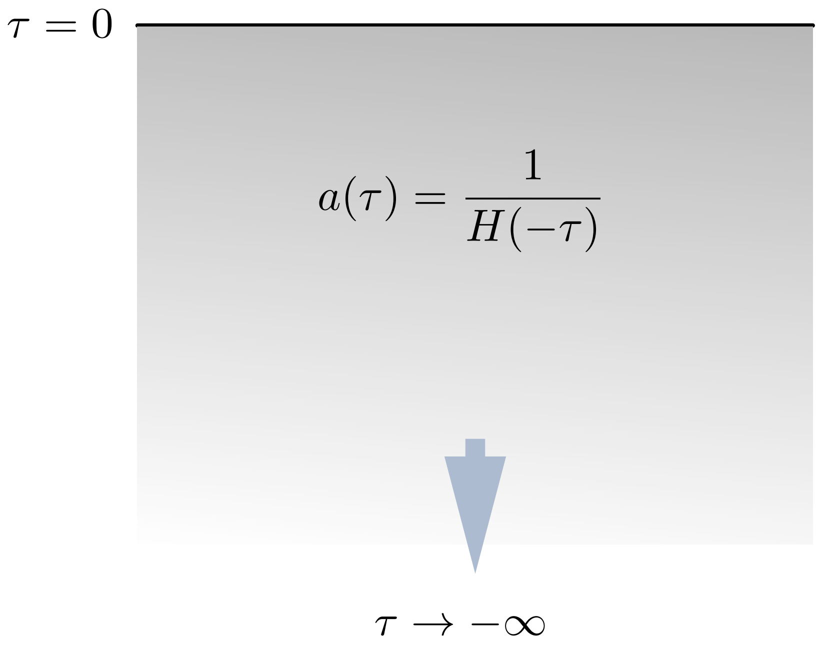
Inflation: horizon problem

Use conformal time: $\tau = -\frac{1}{H}e^{-Ht}, \quad \tau \in (-\infty, 0) \quad , \qquad a(\tau) = \frac{1}{H(-\tau)}$



Inflation is rooted in symmetries

de Sitter space is maximally symmetric:
There are 10 Killing vectors:



- 3 spatial translations and 3 spatial rotations, forming *iso(3)*
- Plus a dilation and 3 special generators:

$$D = \tau\partial_\tau + x^i\partial_i$$

$$K_i = 2x_i\tau\partial_\tau - \left(-\tau^2 + x^2\right)\partial_i + 2x_ix^j\partial_j$$

Forms the de Sitter algebra: *so(4,1)*

Scalar field on de Sitter

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\right] = \int d\tau d^3x\ \frac{1}{2H^2\tau^2}\left[\phi'^2 - (\nabla\phi)^2 - m^2\phi^2\right]$$

de Sitter symmetries act linearly on the field: $\delta_D\phi = D\phi, \quad \delta_{K_i}\phi = K_i\phi$

We are interested in the fields at late times $\tau \to 0$
when all modes are outside the horizon

The isometries act as generators of conformal symmetries of the euclidean boundary at $\tau = 0$, same algebra so(4,1)

$D \to x^i\partial_i, \quad K_i \to x^2\partial_i + 2x_i x^j\partial_j$

Fields at the boundary transform as 3-d conformal fields:

$$\delta_D\phi = -(x^i\partial_i + \Delta)\phi$$

$$\delta_{K_i}\phi = \left(-2x^ix^j\partial_j + x^2\partial_i - 2\Delta x^i\right)\phi$$

$$\Delta = \frac{3}{2}\left(1 - \sqrt{1 - \frac{4m^2}{9H^2}}\right)$$

Conformal symmetry of dS inflation

Constraints of conformal symmetry on correlation functions $\langle\phi_1(x_1)\cdots\phi_N(x_N)\rangle$

$$\sum_{a=1}^{N}\left(\Delta_a + x_a^i\frac{\partial}{\partial x_a^i}\right)\langle\phi_1(x_1)\cdots\phi_N(x_N)\rangle = 0$$

Dilation invariance determines the form of the power spectrum

$$\int d^3x d^3x' e^{ik\cdot x}e^{ik'\cdot x'}\langle\phi(x)\phi(x')\rangle = (2\pi)^3\delta^3(k+k')P(k)$$

$$\left(2\Delta - 3 - k\cdot\frac{\partial}{\partial k}\right)P(k) = 0 \ \Rightarrow\ P(k)\sim\frac{1}{k^{3-2\Delta}}$$

Weight zero fields ($m = 0$) get a scale invariant spectrum:
(Shift symmetry $\phi\to\phi + c$ can enforce $m = 0$)

$$P(k)\sim\frac{1}{k^3},\quad \Delta = 0$$

Similar constraints on higher point functions (non-gaussianity). No constraints relating higher point functions to lower point functions.

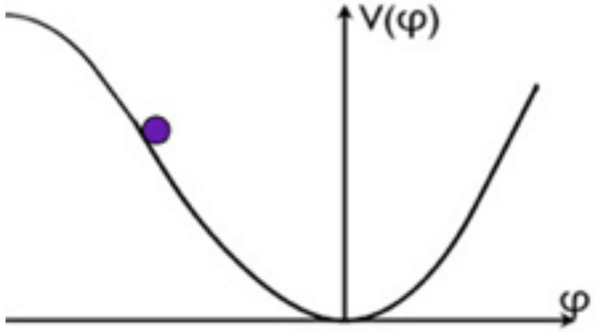
Symmetry algebra is unbroken: $so(4,1)$

Slow-roll inflation

Driven by a scalar field:

$$\int d^4x\sqrt{-G}\left[\frac{M_P^2}{2}R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right] \quad , \qquad 3M_p^2H^2=\frac{1}{2}\dot{\phi}^2+V, \quad \ddot{\phi}+3H\dot{\phi}+V'=0$$

Satisfying slow-roll conditions: $\dot{\phi}^2 \ll M_P^2H^2, \quad \ddot{\phi} \ll H\dot{\phi}$



$$\epsilon \equiv \frac{\dot{\phi}^2}{2M_p^2H^2} = -\frac{\dot{H}}{H^2} \ll 1$$

Not exact de Sitter: deviation measured by slow roll parameters.

Fluctuations around the background are responsible for CMB spectrum.
Scalar fluctuations in co-moving gauge:

$$\phi=\bar{\phi}, \qquad g_{ij}=a^2e^{2\zeta}\delta_{ij}$$

Quadratic action for fluctuations leads to power spectrum:

$$S_2=M_P^2\int \mathrm{d}^3x\mathrm{d}t\; a^3\epsilon\left[\dot{\zeta}^2-a^{-2}(\vec{\partial}\zeta)^2\right]$$

$$P_\zeta(k)=\frac{1}{4k^3}\frac{H^2}{M_P^2}\frac{1}{\epsilon}\bigg|_{t_*} \sim k^{-3+(n_S-1)} \quad , \qquad n_S-1=k\frac{d}{dk}\ln\left(k^3P(k)\right)$$

$\longleftarrow$ horizon crossing time $k=H(t_*)a(t_*)$

symmetries of slow-roll inflation

KH, Lam Hui & Justin Khoury, 1203.6351

Co-moving gauge choice leaves some redundancy:

$$\phi = \bar{\phi}, \qquad g_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

Any transformation on zeta which looks like a coordinate transformation of the spatial metric will be a symmetry:

$$\delta \left(a^2(t) e^{2\zeta(\vec{x},t)} \delta_{ij} \right) = \mathcal{L}_{\vec{\xi}} \left(a^2(t) e^{2\zeta(\vec{x},t)} \delta_{ij} \right)$$

We can do a conformal transformation of the coordinates, compensated by a shift in zeta:

$$\begin{aligned} \delta\zeta &= 1 + \vec{x} \cdot \vec{\nabla}\zeta \\ \delta_{\vec{b}}\zeta &= 2\vec{b} \cdot \vec{x} + \left(2\vec{b} \cdot \vec{x}\, x^i - \vec{x}^2 b^i \right) \partial_i \zeta \end{aligned}$$

- These are global symmetries of the (gauge fixed) action for scalar modes.
- Present for any FRW background and any theory with a single scalar (slow roll condition not required)

symmetries of slow-roll inflation

Ordinary spatial translations and rotations are linearly realized.

$$\delta_i \zeta = -\partial_i \zeta, \quad \delta_{ij}\zeta = (x_i \partial_j - x_j \partial_i)\zeta$$

Ordinary spatial translations and rotations are linearly realized.
These new symmetries are non-linearly realized.

$$\delta\zeta = 1 + \vec{x} \cdot \vec{\nabla}\zeta$$
$$\delta_{\vec{b}}\zeta = 2\vec{b} \cdot \vec{x} + \left(2\vec{b} \cdot \vec{x}\, x^i - \vec{x}^2 b^i\right) \partial_i \zeta$$

They close to form the de Sitter algebra $so(4{,}1)$

Symmetry breaking pattern for slow roll inflation is: $so(4{,}1) \to iso(3)$

symmetries of slow-roll inflation

Broken symmetries lead to Ward identities relating n point functions and n-1 point functions: *consistency relations*

Maldacena (2002); Creminelli & Zaldarriaga (2004);
Cheung et al. (2007)
Assassi, Baumann, Green (2012)
KH, Lam Hui & Justin Khoury (to appear)

$$\lim_{\vec{k}_1 \to 0} \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = -(2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)(n_s - 1) P_{k_1} P_{k_3}$$

Consistency relations for higher-point functions:

KH, Lam Hui & Justin Khoury (to appear)

$$\lim_{k \to 0} \langle \zeta(k)\zeta(k_1)\cdots\zeta(k_N)\rangle = -\delta^3\left(\sum_a k_a\right) P(k)\left[3(N-1) + \sum_a k_a \cdot \frac{\partial}{\partial k_a}\right]\langle\zeta(k_1)\cdots\zeta(k_N)\rangle$$

Novel special conformal consistency relations:

Creminelli, Norena & Simonovic, 1203.4595;
Hinterbichler, Hui & Khoury (to appear)

$$\lim_{k \to 0} \langle \zeta(k)\zeta(k_1)\cdots\zeta(k_N)\rangle = -\frac{1}{2}\delta^3\left(\sum_a k_a\right) P(k) k^i \sum_a \left[6\frac{\partial}{\partial k_a^i} - k_a^i \frac{\partial^2}{\partial k_a \cdot \partial k_a} + 2k_a \cdot \frac{\partial}{\partial k_a}\frac{\partial}{\partial k_a^i}\right]\langle\zeta(k_1)\cdots\zeta(k_N)\rangle$$

Other relations from tensor symmetries...

Hinterbichler, Hui & Khoury (to appear)

Alternatives to inflation?

Inflation is so compelling, why search for alternatives?

- It is good science: science thrives on competition
- The existence of a compelling alternative can provide further non-trivial tests for inflation
- The observed predictions of inflation are pretty generic
- Alternatives may look ugly at first: all problems can't be solved at once. (Inflation has been around and has been developed since the '80s.)
- Alternatives should have a compelling starting point: rooted in symmetries.

Pseudo-conformal scenario

- Non-inflationary scenario
- Gravity is relatively unimportant: spacetime is approximately flat
- More symmetric than inflation: so(4,2)
- Spontaneously broken: so(4,2) → so(4,1)
- Essential physics is fixed by the symmetry breaking pattern, independently of the specific realization or microphysics
- Many possible realizations:
  - Rubakov's U(1) model — V. Rubakov, 0906.3693; 1007.3417; 1007.4949; 1105.6230
  - Galilean Genesis — Creminelli, Nicolis & Trincherini, 1007.0027
  - ϕ^4 model — KH, Justin Khoury arXiv:1106.1428
  - $\vdots$

Simplest example: negative quartic

KH, Justin Khoury arXiv:1106.1428

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4}\phi^4$$

Classically, this is a conformal field theory, with a field of weight $\Delta = 1$

$$\begin{aligned} \delta_{P_\mu}\phi &= -\partial_\mu\phi\,, & \delta_{J^{\mu\nu}}\phi = (x^\mu\partial^\nu - x^\nu\partial^\mu)\phi\,, \\ \delta_D\phi &= -(\Delta + x^\mu\partial_\mu)\phi\,, & \delta_{K_\mu}\phi_I = \left(-2x_\mu\Delta - 2x_\mu x^\nu\partial_\nu + x^2\partial_\mu\right)\phi\,. \end{aligned}$$

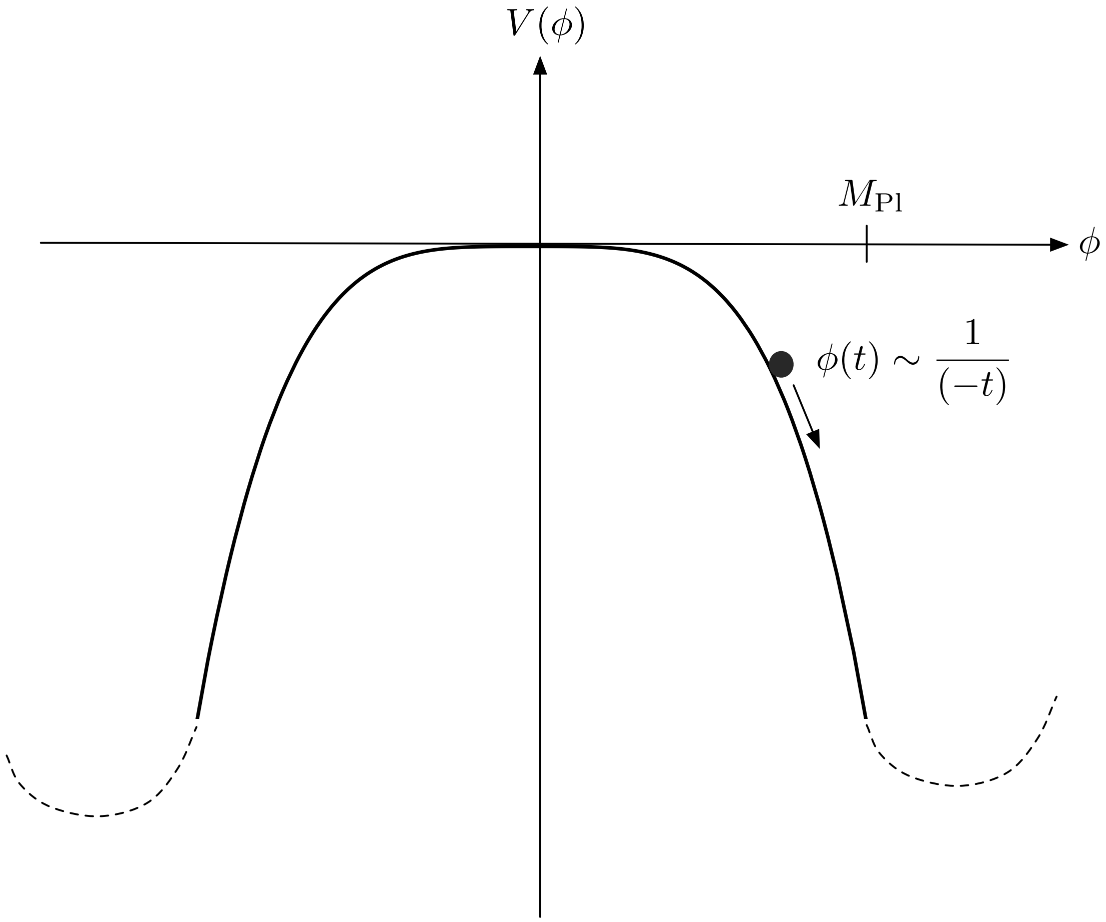
Symmetry algebra $so(4{,}2)$ realized linearly on the field.

$$\begin{aligned} \left[\delta_D, \delta_{P_\mu}\right] &= -\delta_{P_\mu}\,, & \left[\delta_D, \delta_{K_\mu}\right] = \delta_{K_\mu}\,, \quad \left[\delta_{K_\mu}, \delta_{P_\nu}\right] = 2\left(\delta_{J_{\mu\nu}} - \eta_{\mu\nu}\delta_D\right) \\ \left[\delta_{J_{\mu\nu}}, \delta_{K_\lambda}\right] &= \eta_{\lambda\mu}\delta_{K_\nu} - \eta_{\lambda\nu}\delta_{K_\mu}\,, & \left[\delta_{J_{\mu\nu}}, \delta_{P_\sigma}\right] = \eta_{\mu\sigma}\delta_{P_\nu} - \eta_{\nu\sigma}\delta_{P_\mu}\,, \\ \left[\delta_{J_{\mu\nu}}, \delta_{J_{\sigma\rho}}\right] &= \eta_{\mu\sigma}\delta_{J_{\nu\rho}} - \eta_{\nu\sigma}\delta_{J_{\mu\rho}} + \eta_{\nu\rho}\delta_{J_{\mu\sigma}} - \eta_{\mu\rho}\delta_{J_{\nu\sigma}}\,, \end{aligned}$$

Solution $\phi = 0$ is $so(4{,}2)$ invariant.

Simplest example: negative quartic

There is a solution where the field rolls down a negative quartic potential:



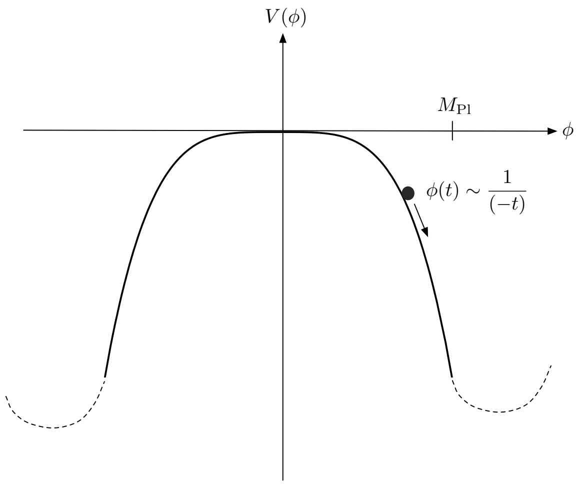
$$\bar{\phi} = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}, \quad t \in (-\infty, 0)$$

- This solution has zero energy $\rho_\phi = 0$

- Pressure is non-zero and positive (satisfies the NEC, infinite equation of state) $p_\phi = \frac{2}{\lambda t^4}, \quad w = \infty$

- Solution is an attractor

Simplest example: negative quartic



$$\bar{\phi} = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}, \quad t \in (-\infty, 0)$$

Rolling solution preserves an *so*(4,1) subgroup:

$$\delta_{P_i}\bar{\phi} = 0\,, \quad \delta_D\bar{\phi} = 0\,, \quad \delta_{J_{ij}}\bar{\phi} = 0\,, \quad \delta_{K_i}\bar{\phi} = 0\,, \quad i = 1, 2, 3$$

$$\delta_{P_0}\bar{\phi} = \frac{\Delta\bar{\phi}}{t}\,, \qquad \delta_{J^{0i}}\bar{\phi} = -\frac{\Delta x^i}{t}\bar{\phi}\,, \qquad \delta_{K_0}\bar{\phi} = -\frac{\Delta x_\mu x^\mu}{t}\bar{\phi}\,.$$

so(4,1) generators are realized linearly on fluctuations around the solution: $\phi = \bar{\phi} + \varphi$

$$\begin{aligned} \delta_{P_i}\varphi &= -\partial_i\varphi\,, & \delta_{J^{ij}}\varphi = \left(x^i\partial^j - x^j\partial^i\right)\varphi\,, \\ \delta_D\varphi &= -\left(\Delta + x^\mu\partial_\mu\right)\varphi\,, & \delta_{K_i}\varphi = \left(-2x_i\Delta - 2x_ix^\nu\partial_\nu + x^2\partial_i\right)\varphi\,, \end{aligned}$$

The rest are realized non-linearly

$$\begin{aligned} \delta_{P_0}\varphi &= \frac{\Delta}{t}\bar{\phi} - \dot{\varphi}\,, \qquad \delta_{J^{0i}}\varphi = -\frac{\Delta x^i}{t}\bar{\phi} + \left(t\partial_i + x^i\partial_t\right)\varphi\,, \\ \delta_{K_0}\varphi &= -\frac{\Delta x^2}{t}\bar{\phi} + \left(2t\Delta + 2tx^\nu\partial_\nu + x^2\partial_t\right)\varphi\,. \end{aligned}$$

Symmetry breaking pattern: *so*(4,2) → *so*(4,1)

negative quartic: fluctuations

Quadratic action for fluctuations $\phi = \bar{\phi} + \varphi$

$$S_{\text{quad}} \sim -\frac{1}{2}\int \mathrm{d}^4x \left(\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{6}{t^2}\varphi^2\right).$$

- Modes propagate exactly at the speed of light (no superluminality issue)
- Power spectrum at late times is red-tilted

$$\ddot{\phi}_k + \left(k^2 - \frac{6}{t^2}\right)\phi_k = 0$$

$$\phi_k \sim \sqrt{-t}H^{(1)}_{5/2}(-kt) \underset{k|t|\ll 1}{\Rightarrow} P_\varphi(k) \sim \frac{1}{k^5 t^4}$$

Getting scale invariant fluctuations

Shift invariant, conformally invariant coupling

Couple in a weight zero field $\Delta_\chi = 0$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4}\phi^4 - \frac{1}{2}\phi^2(\partial\chi)^2$$

The weight zero field has a constant background profile

$$\bar{\phi} = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}, \quad \bar{\chi} = const.$$

Quadratic action for chi fluctuations looks like a massless scalar on a “fake” de Sitter

$$S^{\chi}_{\text{quad}} \sim -\frac{1}{2}\int \mathrm{d}^4x \, \frac{1}{t^2}\eta^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \,,$$

Scale invariant spectrum:

$$v \equiv (-t)\chi, \quad \ddot{v}_k + \left(k^2 - \frac{2}{t^2}\right)v_k = 0$$

$$v_k \sim \frac{e^{-ikt}}{\sqrt{2k}}\left(1 - \frac{i}{kt}\right) \underset{k|t|\ll 1}{\Rightarrow} \quad P_\chi(k) \sim \frac{1}{k^3}$$

Adding gravity

Couple minimally to gravity (breaks conformal invariance at $1/M_P$ level):

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{\lambda}{4}\phi^4\right)$$

Solution has zero energy ⇒ spacetime approximately flat

Solve the Friedman equations in powers of $1/M_P$

$$\phi(t) \approx \frac{\sqrt{2}}{\sqrt{\lambda}(-t)} \quad , \quad H(t) \approx \frac{1}{3\lambda t^3 M_{\rm P}^2} \quad , \quad a(t) \approx 1 - \frac{1}{6\lambda t^2 M_{\rm P}^2}$$

Solution is a slowly contracting universe

Approximation is valid in the range $-\infty < t < t_{\rm end}$

$$t_{\rm end} \sim -\frac{1}{\sqrt{\lambda}M_{\rm Pl}}\,, \quad \phi_{\rm end} \sim M_{\rm Pl}\,.$$

The field forms a very stiff fluid

$$\rho_\phi \approx \frac{1}{3\lambda^2 t^6 M_{\rm P}^2}\,, \quad p_\phi \approx \frac{2}{\lambda t^4}\,, \quad w \approx 6\lambda t^2 M_{\rm P}^2$$

w goes from ≫1 to O(1) as t ranges from -∞ to $t_{\rm end}$

Solution to flatness, smoothness problems

There is now a scalar field component with extremely stiff equation of state $w \gg 1$

$$3M_P^2H^2 = \underbrace{-\frac{3M_P^2 k}{a^2} + \frac{\rho_{m,0}}{a^3} + \frac{\rho_{r,0}}{a^4} + \frac{\rho_{A,0}}{a^6}}_{\uparrow \atop \approx\ \text{constant}} + \underset{\uparrow \atop \text{rapidly } \textit{increasing} \ \sim \frac{1}{t^6}}{\rho_\phi}$$

Homogeneous energy density of the scalar washes out everything else

Similar to ekpyrotic cosmology (contracting universe with $w >> 1$)

Khoury, Ovrut, Steinhardt, Turok (2001);
Gratton, Khoury, Steinhardt, Turok (2003);
Erickson, Wesley, Steinhardt, Turok (2004).

Why this is not inflation

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R + \mathcal{L}_{\mathrm{CFT}}\left[g_{\mu\nu}\right]\right)$$

Can’t we just do a Weyl transformation to bring the background metric to de Sitter?

$$g_{\mu\nu}^{\mathrm{eff}} = \phi^2 g_{\mu\nu}$$

Action in “Jordan frame”

$$S = \int \mathrm{d}^4x\sqrt{-g_{\mathrm{eff}}}\left(\boxed{\frac{M_{\mathrm{Pl}}^2}{2\phi^2}}R_{\mathrm{eff}} + \frac{3M_{\mathrm{Pl}}^2}{\phi^4}g_{\mathrm{eff}}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{\phi^4}\mathcal{L}_{\mathrm{CFT}}\left[\phi^{-2}g_{\mu\nu}^{\mathrm{eff}}\right]\right)$$

↑

Acceleration now results from the non-minimal coupling to the CFT.

Effective Planck mass $M_{\mathrm{Pl}}^{\mathrm{eff}} \sim 1/\phi$ varies by order one in a Hubble time.

General framework

KH, Justin Khoury arXiv:1106.1428

Start with any CFT with scalar primary operators:

$$\phi_I\,, \quad I = 1, \ldots, N. \quad \text{conformal weight } \Delta_I$$

$$\begin{aligned} \delta_{P_\mu}\phi_I &= -\partial_\mu \phi_I\,, & \delta_{J^{\mu\nu}}\phi_I = (x^\mu\partial^\nu - x^\nu\partial^\mu)\phi_I\,, \\ \delta_D\phi_I &= -(\Delta_I + x^\mu\partial_\mu)\phi_I\,, & \delta_{K_\mu}\phi_I = \left(-2x_\mu\Delta_I - 2x_\mu x^\nu\partial_\nu + x^2\partial_\mu\right)\phi_I\,. \end{aligned}$$

These need not be fundamental fields or degrees of freedom, and a conformal invariant stable ground state need not exist.

Dynamics must be such that the operators get a VEV:

$$\bar\phi_I(t) = \frac{c_I}{(-t)^{\Delta_I}}\,,$$

VEV preserves an *so*(4,1) subgroup of *so(4,2):*

$$\delta_{P_i}\bar\phi_I = 0\,, \quad \delta_D\bar\phi_I = 0\,, \quad \delta_{J_{ij}}\bar\phi_I = 0\,, \quad \delta_{K_i}\bar\phi_I = 0\,,$$

$$\delta_{P_0}\bar\phi_I = \frac{\Delta_I\bar\phi_I}{t} \neq 0\,, \qquad \delta_{J^{0i}}\bar\phi_I = -\frac{\Delta_I x^i}{t}\bar\phi_I \neq 0\,, \qquad \delta_{K_0}\bar\phi_I = -\frac{\Delta_I x_\mu x^\mu}{t}\bar\phi_I \neq 0\,.$$

Pseudo-conformal symmetry breaking pattern

$so(4{,}1)$ generators are realized linearly on fluctuations around the VEV: $\phi_I = \bar{\phi}_I + \varphi_I$

$$\begin{aligned} \delta_{P_i}\varphi_I &= -\partial_i\varphi_I\,, & \delta_{J^{ij}}\varphi_I &= \left(x^i\partial^j - x^j\partial^i\right)\varphi_I\,, \\ \delta_D\varphi_I &= -\left(\Delta_I + x^\mu\partial_\mu\right)\varphi_I\,, & \delta_{K_i}\varphi_I &= \left(-2x_i\Delta_I - 2x_ix^\nu\partial_\nu + x^2\partial_i\right)\varphi_I\,, \end{aligned}$$

The rest are realized non-linearly:

$$\begin{aligned} \delta_{P_0}\varphi_I &= \frac{\Delta_I}{t}\bar{\phi}_I - \dot{\varphi}_I\,, \qquad \delta_{J^{0i}}\varphi_I = -\frac{\Delta_I x^i}{t}\bar{\phi}_I + \left(t\partial_i + x^i\partial_t\right)\varphi_I\,, \\ \delta_{K_0}\varphi_I &= -\frac{\Delta_I x^2}{t}\bar{\phi}_I + \left(2t\Delta_I + 2tx^\nu\partial_\nu + x^2\partial_t\right)\varphi_I\,. \end{aligned}$$

Symmetry breaking pattern for pseudo-conformal scenario is: $so(4{,}2) \to so(4{,}1)$

General form of the quadratic action for fluctuations

KH, Justin Khoury arXiv:1106.1428

The broken and unbroken symmetries fix the form of the quadratic fluctuations $\phi_I = \bar{\phi}_I + \varphi_I$

$$\mathcal{L}_{\rm quad} \sim -\frac{1}{2}(-t)^{2(\Delta-1)}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}(-t)^{2(\Delta-2)}(\Delta+1)(\Delta-4)\varphi^2$$

Weight ≠0 fields always red-tilted: $v \equiv (-t)^{\Delta-1}\varphi,$

$$\ddot{v}_k + \left(k^2 - \frac{6}{t^2}\right)v_k = 0\,, \qquad v_k \sim \sqrt{-t}H^{(1)}_{5/2}(-kt) \underset{k|t|\ll 1}{\Rightarrow} P_\varphi(k) \sim \frac{1}{k^5 t^{2(\Delta+1)}}$$

Weight zero fields always get a scale invariant spectrum:

$$S^\chi_{\rm quad} \sim -\frac{1}{2}\int {\rm d}^4x\,\frac{1}{t^2}\eta^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\,,$$

$$v \equiv (-t)\chi, \quad \ddot{v}_k + \left(k^2 - \frac{2}{t^2}\right)v_k = 0 \quad , \qquad v_k \sim \frac{e^{-ikt}}{\sqrt{2k}}\left(1 - \frac{i}{kt}\right) \underset{k|t|\ll 1}{\Rightarrow} P_\chi(k) \sim \frac{1}{k^3}$$

Actions for fields and fluctuations can be constructed using coset techniques.

KH, Justin Khoury, Austin Joyce arXiv:1202.6056

Another example: galileon genesis

Creminelli, Nicolis & Trincherini, 2010

$$\mathcal{L} = -\frac{1}{2}c_2(\partial\phi)^2 + c_3\left[-\frac{1}{2}\frac{(\partial\phi)^2\Box\phi}{\phi^3} + \frac{1}{4}\frac{(\partial\phi)^4}{\phi^4}\right] + \cdots$$

Ghost-free conformal galileon

There is a $so(4{,}1)$ invariant solution $\bar\phi = \frac{\alpha}{(-t)}$ when: $\alpha c_2 - \dfrac{3c_3}{2\alpha} = 0$

- Violates the NEC (negative pressure)
- Slowly *expanding* spacetime (no bounce required)
- Superluminal propagation around nearby solutions (see however Creminelli, KH, Khoury, Nicolis, Trincherini, arXiv:1209.3768)

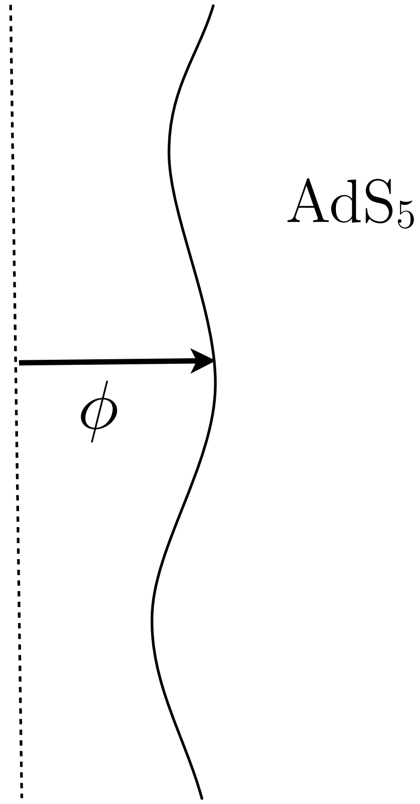
Lagrangian is that appearing as the dilaton in the a-theorem proof

Komargodski, Schwimmer, 2011

DBI realization

KH, Justin Khoury, Austin Joyce, Godfrey Miller arXiv: 1209.5742

$$\mathcal{L} = \phi^4 \left[1 + \frac{\lambda}{4} - \sqrt{1 + \frac{(\partial\phi)^2}{\phi^4}} \right]$$



Different realization of $so(4{,}2)$, inherited from bulk AdS$_5$ isometries

$$\begin{aligned} \delta_D \phi &= -\left(\Delta_\phi + x^\nu \partial_\nu\right) \phi \, , \\ \delta_{K_\mu} \phi &= -2 x_\mu \left(\Delta_\phi + x^\nu \partial_\nu\right) \phi + x^2 \partial_\mu \phi + \frac{1}{\phi^2} \partial_\mu \phi \, , \end{aligned}$$

Rolling solution:

$$\bar{\phi} = \frac{\alpha}{(-t)} \quad , \qquad \bar{\gamma}(\alpha) = 1 + \frac{\lambda}{4} \quad , \qquad \bar{\gamma}(\alpha) = \frac{1}{\sqrt{1 - 1/\alpha^2}} > 1$$

Symmetry breaking pattern is still $so(4{,}2) \rightarrow so(4{,}1)$:

$$\delta_{P_i} \bar{\phi} = 0 \, , \quad \delta_D \bar{\phi} = 0 \, , \quad \delta_{J_{ij}} \bar{\phi} = 0 \, , \quad \delta_{K_i} \bar{\phi} = 0 \, , \quad i = 1, 2, 3$$

$$\delta_{P_0} \bar{\phi} = \frac{\bar{\phi}}{t} \, ; \qquad \delta_{J_{0i}} \bar{\phi} = \frac{x^i \bar{\phi}}{t} \, ; \qquad \delta_{K_0} \bar{\phi} = -\left(x^2 + \frac{1}{\bar{\phi}^2} \right) \frac{\bar{\phi}}{t}$$

DBI realization: quadratic fluctuations

KH, Justin Khoury, Austin Joyce, Godfrey Miller arXiv: 1209.5742

For perturbations about the background scaling solution, the unbroken $so(4{,}1)$ subalgebra action starts at linear order:

$$\delta_{P_i}\varphi = -\partial_i\varphi \qquad \delta_{J_{ij}}\varphi = \left(x^i\partial_j - x^j\partial_i\right)\varphi$$

$$\delta_D\varphi = -\left(1 + x^\mu\partial_\mu\right)\varphi \qquad \delta_{K_i}\varphi = -2x^i\varphi - 2x^ix^\lambda\partial_\lambda\varphi + \left(x^2 + \frac{1}{\bar\phi^2}\right)\partial_i\varphi + \mathcal{O}(\varphi^2)$$

while the broken generators start at zeroth order:

$$\delta_{P_0}\varphi = \frac{\bar\phi}{t} + \mathcal{O}(\varphi)\,, \qquad \delta_{J_{0i}}\varphi = x_i\frac{\bar\phi}{t} + \mathcal{O}(\varphi)\,, \qquad \delta_{K_0}\varphi = -\left(x^2 + \frac{1}{\bar\phi^2}\right)\frac{\bar\phi}{t} + \mathcal{O}(\varphi)$$

Quadratic action is fixed up to overall normalization by these symmetries:

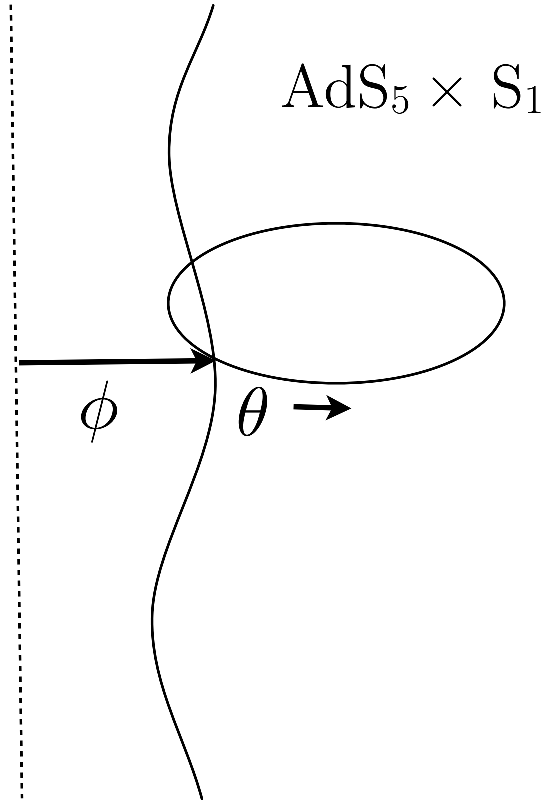
$$S = \frac{1}{2}\bar\gamma^3\int \mathrm{d}^4x\left(\dot\varphi^2 - \frac{1}{\bar\gamma^2}(\partial_i\varphi)^2 + \frac{6}{t^2}\varphi^2\right)$$

Speed of fluctuations is strictly less than 1 $\quad c_s = \frac{1}{\bar\gamma} < 1$

Power spectrum still red-tilted:

$$P_\varphi(k) = \frac{9}{2}\frac{\bar\gamma^2}{k^5(-t)^4}$$

DBI realization: weight 0 fields



Weight 0 fields have a natural interpretation as the brane moving in additional internal directions with isometries.

Example: AdS$_5 \times$ S$_1$ weight 0 field is the S$_1$ direction, shift symmetry comes from isometry of S$_1$

$$S_{\phi\theta} = \int \mathrm{d}^4x\, \phi^4 \left(1 + \frac{\lambda}{4} - \sqrt{1 + \frac{(\partial\phi)^2}{\phi^4} + \frac{(\partial\theta)^2}{\phi^2} + \frac{(\partial\phi)^2(\partial\theta)^2 - (\partial\phi\cdot\partial\theta)^2}{\phi^6}} \right)$$

Quadratic action gives scale invariant fluctuations,

$$\frac{1}{2}\bar{\gamma} \int \mathrm{d}^4x\, \bar{\phi}^2(t) \left(\dot{\vartheta}^2 - \frac{1}{\bar{\gamma}^2}(\partial_i\vartheta)^2 \right) \quad , \qquad P_\vartheta(k) = \frac{\bar{\gamma}^2 - 1}{2}\frac{1}{k^3}$$

This kind of setup is common in attempts to realize inflation in string theory (DBI inflation).

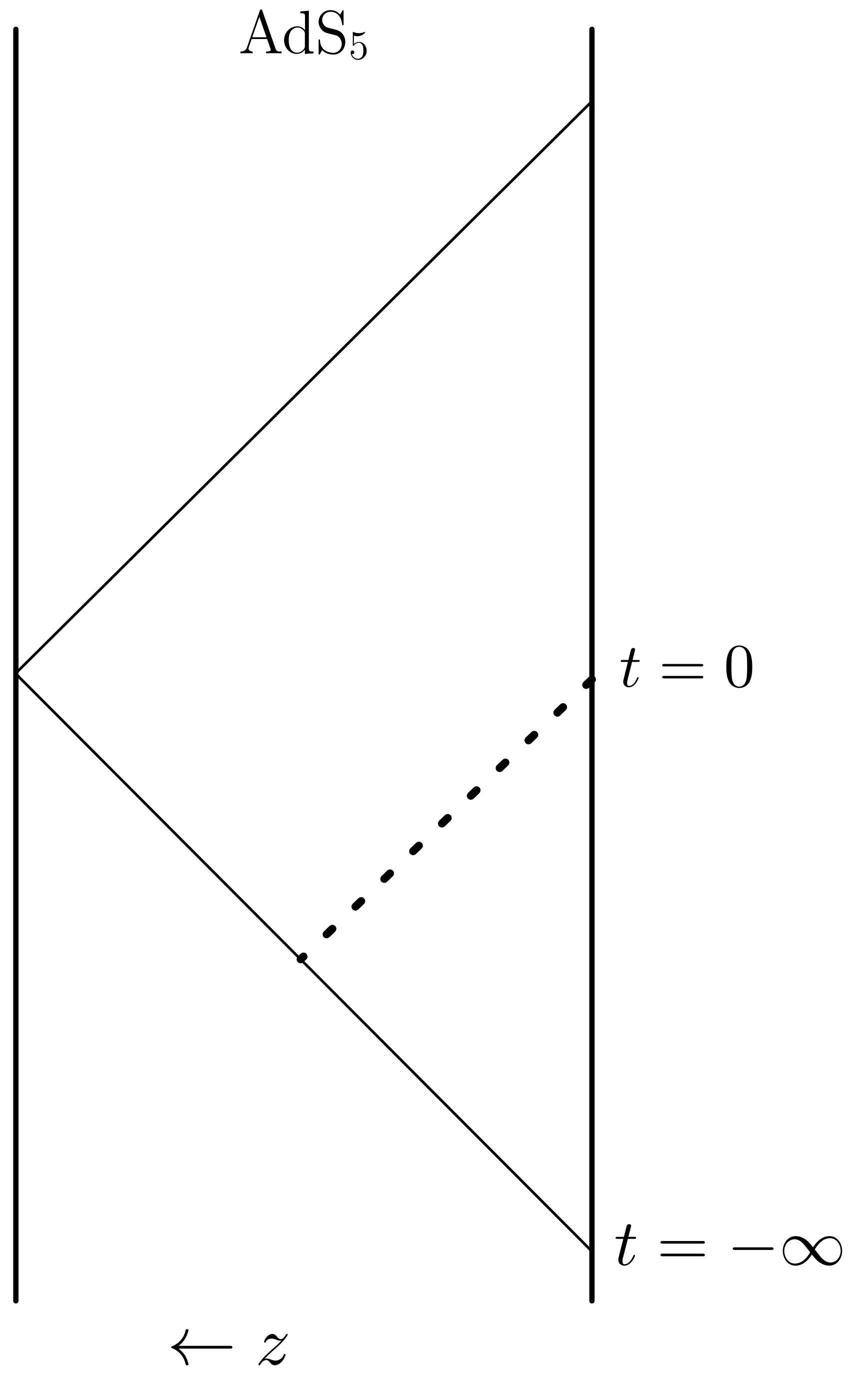
AdS/CFT realization?

- Realization in a truly quantum strongly coupled CFT.

- Cosmological application of AdS/CFT where we are interested in the boundary.

- Dual AdS should be a configuration which is constant on a foliation of AdS_5 by dS_4 leaves:

$$\phi_{\mathrm{Bulk}} = \phi\left(\frac{z}{(-t)}\right)$$



Distinguishable from inflation?

- Detailed predictions (spectral index, non-gaussianity, etc...) will depend on the realization.
- Pseudo-conformal scenario is more symmetric than inflation
- Symmetries → Ward identities → constraints/relations on correlators

Symmetry breaking patterns:

Inflation (spectator)	$so(4{,}1) \rightarrow so(4{,}1)$
Inflation (inflaton)	$so(4{,}1) \rightarrow \mathrm{i}so(3)$
Pseudo-conformal	$so(4{,}2) \rightarrow so(4{,}1)$

Challenges

- Requires matching onto a standard radiation dominated cosmology. Brandenberger, Davis, Perreault 1105.5649

- Seems to require NEC violation at some stage:

  Models which satisfy NEC have contracting spacetime → Bounce required to match → NEC violation during matching.
  Models which have expanding spacetime violate NEC.

- How are the scale invariant perturbations of the weight zero fields imprinted onto the adiabatic mode? Brandenberger, Wang 1206.4309

- Gravity waves?

- Non-gaussianities?

- Solid realization within a stable quantum mechanically conformal CFT (AdS/CFT?)

Conclusions

- The conformal scenario is an alternative to inflation.
- Generic scenario: early universe is a CFT in ≈ flat spacetime, with specific time-dependent VEVs: $\phi_\Delta \sim 1/(-t)^\Delta$
- Symmetry breaking pattern: $so(4{,}2) \to so(4{,}1)$, more symmetric than inflation.
- Requires matching to a standard radiation dominated phase (probably requires some NEC violation).