# Primordial Spikes from Wrapped Brane Inflation

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based on arXiv:1210.4427 w/ J. Yokoyama

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# Primordial Density Perturbations from Cosmic Inflation



image: NASA/WMAP Science Team

# Power Spectrum of Perturbations

#### almost scale-invariant, with amplitude ~10<sup>-9</sup>



image : NASA / WMAP Science Team



image : M. Blanton and SDSS

# A Closer Look at the Primordial Power Spectrum

fine structures in the power
 spectrum can tell us much
 about early universe physics



• e.g. oscillations as "standard clocks" Chen '11

Wrapped brane inflation produces spikes that are tied to properties of the extra dimensional space.

#### Brane Inflation

Dvali, Tye '98

# Branes moving along the extra dimensions can drive cosmic inflation.



### Inflation with Wrapped Branes

TK, Mukohyama, Kinoshita '07 Becker, Leblond, Shandera '07 Silverstein, Westphal '10

p-branes with p > 3 wrap the internal manifold

brane's oscillation modes (i.e. KK modes)
 excited during inflation by localized sources/features

### Inflation with Wrapped Branes

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 excited during inflation by localized sources/features

Excited KK modes produce spikes!

### Effective 4-Dim. Theory

- inflaton (= zero mode position of brane) couples to heavy oscillating fields (= KK modes)
- resonant features in the pert. spectrum are localized to narrow  $\Delta k$ 
  - (in contrast to most previous works)
- brane's Nambu-Goto action sources strong resonance

#### Outline

- wrapped brane inflation setup
- effects on curv. pert. from oscillating KK modes
  - weak resonance sourcing oscillations

strong resonance sourcing spikes



 $ds^{2} = g_{\mu\nu}dx^{\mu}dx^{\nu} + A^{2}dr^{2} + B^{2}r^{2}d\lambda^{2}$ 

Nambu-Goto action  $S = -T_4 \int d^5 \xi \sqrt{-\det(G_{MN})}$ 

cf. D-brane monodromy model by Silverstein & Westphal '08



in the absence of KK modes :

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{V(\phi)}{\gamma} \simeq -V(\phi) - \frac{1}{2}(\partial\phi)^2$$

-1/2

$$V(\phi) = \mu^{10/3} \phi^{2/3}$$

drives large-field inflation

$$\mu \equiv \left(\frac{3\pi p T_4 B}{A}\right)^{1/5} \qquad \gamma = \left(1 + \frac{(\partial \phi)^2}{V}\right)$$

p: winding number



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Effective 4-Dim. Theory  

$$\begin{array}{c}
\hline
\psi_n & & & \\
\hline
\end{pmatrix}
ds_{int}^2 = A^2 dr^2 + B^2 r^2 d\lambda^2$$
full Lagrangian:  

$$\begin{array}{c}
\frac{\mathcal{L}}{\sqrt{-g}} = -V \left(\frac{1}{\gamma} + 2\gamma \sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2}\right) - \gamma \sum_{n \neq 0} \left\{\frac{1}{2}(\partial \psi_n)^2 - \frac{1}{6}\frac{\psi_n^2}{\phi^2}(\partial \phi)^2 + \frac{1}{3}\frac{\psi_n}{\phi}(\partial \phi \cdot \partial \psi_n)\right\}$$

$$+ \frac{\gamma^3}{2V} \sum_{n \neq 0} \left\{(\partial \phi \cdot \partial \psi_n)^2 + \frac{1}{9}\frac{\psi_n^2}{\phi^2}\left((\partial \phi)^2\right)^2 - \frac{2}{3}\frac{\psi_n}{\phi}(\partial \phi)^2(\partial \phi \cdot \partial \psi_n)\right\} + \mathcal{O}(\psi_n^3)$$

$$V(\phi) = \mu^{10/3}\phi^{2/3} \quad \mu \equiv \left(\frac{3\pi pT_4 B}{A}\right)^{1/5} \quad \gamma = \left(1 + \frac{(\partial \phi)^2}{V}\right)^{-1/2} \quad \alpha_n^2 \equiv \frac{1}{9}\frac{A^2}{B^2}\frac{n^2}{p^2}$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = -V\left(\frac{1}{\gamma} + 2\gamma \sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2}\right) - \gamma \sum_{n \neq 0} \left\{ \frac{1}{2} (\partial \psi_n)^2 - \frac{1}{6} \frac{\psi_n^2}{\phi^2} (\partial \phi)^2 + \frac{1}{3} \frac{\psi_n}{\phi} (\partial \phi \cdot \partial \psi_n) \right\}$$
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$$V(\phi) = \mu^{10/3} \phi^{2/3} \qquad \gamma = \left( 1 + \frac{(\partial \phi)^2}{V} \right)^{-1/2}$$

inflaton  $\phi$  : drives slow-roll inflation while  $\phi > M_p$ 

#### effective KK mass

 $rac{\mathcal{L}}{\sqrt{-g}}$ 

$$= -V\left(\frac{1}{\gamma} + 2\gamma \sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2}\right) - \gamma \sum_{n \neq 0} \left\{ \frac{1}{2} (\partial \psi_n)^2 - \frac{1}{6} \frac{\psi_n^2}{\phi^2} (\partial \phi)^2 + \frac{1}{3} \frac{\psi_n}{\phi} (\partial \phi \cdot \partial \psi_n) \right\}$$
$$+ \frac{\gamma^3}{2V} \sum_{n \neq 0} \left\{ (\partial \phi \cdot \partial \psi_n)^2 + \frac{1}{9} \frac{\psi_n^2}{\phi^2} \left( (\partial \phi)^2 \right)^2 - \frac{2}{3} \frac{\psi_n}{\phi} (\partial \phi)^2 (\partial \phi \cdot \partial \psi_n) \right\} + \mathcal{O}(\psi_n^3)$$
$$V(\phi) = \mu^{10/3} \phi^{2/3} \qquad \gamma = \left( 1 + \frac{(\partial \phi)^2}{V} \right)^{-1/2}$$

inflaton  $\phi$  : drives slow-roll inflation while  $\phi > M_p$ 

#### effective KK mass

$$\frac{1}{2} = -V\left(\frac{1}{\gamma} + \frac{2\gamma\sum_{n\neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2}}{\phi^2}\right) - \gamma \sum_{n\neq 0} \left\{\frac{1}{2}(\partial\psi_n)^2 + \frac{1}{6}\frac{\psi_n^2}{\phi^2}(\partial\phi)^2 + \frac{1}{3}\frac{\psi_n}{\phi}(\partial\phi\cdot\partial\psi_n)\right\} + \frac{\gamma^3}{2V}\sum_{n\neq 0} \left\{(\partial\phi/\partial\psi_n)^2 + \frac{1}{9}\frac{\psi_n^2}{\phi^2}\left((\partial\phi)^2\right)^2 - \frac{2}{3}\frac{\psi_n}{\phi}(\partial\phi)^2(\partial\phi\cdot\partial\psi_n)\right\} + \mathcal{O}(\psi_n^3) + V(\phi) = \mu^{10/3}\phi^{2/3} \qquad \gamma = \left(1 + \frac{(\partial\phi)^2}{V}\right)^{-1/2}$$

inflaton  $\phi$  : drives slow-roll inflation while  $\phi > M_p$ KK modes  $\psi_n$ : oscillates with  $m_{KK} \sim \frac{n}{(\text{wrapped volume})}$ 

#### effective KK mass

$$\frac{2}{-g} = -V\left(\frac{1}{\gamma} + 2\gamma \sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2}\right) - \gamma \sum_{n \neq 0} \left\{\frac{1}{2}(\partial\psi_n)^2 + \frac{1}{6}\frac{\psi_n^2}{\phi^2}(\partial\phi)^2 + \frac{1}{3}\frac{\psi_n}{\phi}(\partial\phi \cdot \partial\psi_n)\right\}$$
$$+ \frac{\gamma^3}{2V} \sum_{n \neq 0} \left\{(\partial\phi / \partial\psi_n)^2 + \frac{1}{9}\frac{\psi_n^2}{\phi^2}\left((\partial\phi)^2\right)^2 - \frac{2}{3}\frac{\psi_n}{\phi}(\partial\phi)^2(\partial\phi \cdot \partial\psi_n)\right\} + \mathcal{O}(\psi_n^3)$$
$$V(\phi) = \mu^{10/3}\phi^{2/3} \qquad \gamma = \left(1 + \frac{(\partial\phi)^2}{V}\right)^{-1/2}$$

inflaton  $\phi$ : drives slow-roll inflation while  $\phi > M_p$ KK modes  $\psi_n$ : oscillates with  $m_{\rm KK} \sim \frac{n}{({\rm wrapped volume})}$ 

coupled through potential & kinetic terms

#### Inhomogeneous Fluctuations

 $\phi \to \phi + Q$ 

$$S_2 = \int dt d^3x \, a^3 \left[ \frac{B}{2} \dot{Q}^2 - \frac{G}{2} \frac{(\partial_i Q)^2}{a^2} + \frac{1}{2} \left\{ M - \frac{(a^3 C)}{a^3} \right\} Q^2 \right]$$
  
on flat hypersurfaces

$$B = 1 - 2\sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2} + \frac{3}{2} \sum_{n \neq 0} \frac{\psi_n^2}{V} + \cdots \\ G = 1 - 2\sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2} + \frac{1}{2} \sum_{n \neq 0} \frac{\psi_n^2}{V} + \cdots$$
 socillating b.g.

•

 $Q_k \rightarrow \zeta_k$  via the  $\delta N$ -formalism











# Two Kinds of Parametric Resonance

 Weak Resonance from small KK excitations, sourcing oscillations

 Strong Resonance from large KK excitations, sourcing spikes









$$\begin{aligned} & \mathcal{L}_{\sqrt{-g}} = -\mu^{10/3} \phi^{2/3} \left( \frac{1}{\gamma} + 2\gamma \alpha_1^2 \frac{\psi_1^2}{\phi^2} \right) - \frac{\gamma}{2} (\partial \psi_1)^2 + \cdots \\ & \text{one KK mode is excited} \\ \phi_{\text{exc}} \approx 8.2M_p \quad \mu \approx 0.0016M_p \quad \alpha_1 = 100 \quad (m_{\text{KK}}/H \sim 40) \\ \phi_{\text{exc}} \approx 8.2M_p \quad \psi_1 \exp \left( \frac{M_p}{1000} \right) \\ \phi_{1 \exp} = \frac{M_p}{1000} \\ \phi_{1 \exp} = \frac{M_p}{$$

#### Weak Resonance

resonant signals  $\propto \psi_{\rm exc}^2$ 









Resonant effects depend non-linearly on  $\psi_{\mathrm{exc}}$  .

$$0 = \ddot{u_{k}} + \left\{ \frac{G}{B} \frac{k^{2}}{a^{2}} - \frac{M}{B} + \frac{\left(a^{3}C\right)^{\cdot}}{a^{3}B} - \frac{\left(a^{3/2}B^{1/2}\right)^{\cdot}}{a^{3/2}B^{1/2}} \right\} u_{k}$$

$$B = 1 - 2\sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2} + \frac{3}{2} \sum_{n \neq 0} \frac{\dot{\psi}_n^2}{V} + \cdots$$
$$G = 1 - 2\sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2} + \frac{1}{2} \sum_{n \neq 0} \frac{\dot{\psi}_n^2}{V} + \cdots$$
$$\vdots$$

$$0 = \ddot{u_{k}} + \left\{ \frac{G}{B} \frac{k^{2}}{a^{2}} - \frac{M}{B} + \frac{(a^{3}C)}{a^{3}B} - \frac{(a^{3/2}B^{1/2})}{a^{3/2}B^{1/2}} \right\} u_{k}$$

$$=f_{k\,\mathrm{eff}}^2$$

$$B = 1 - 2\sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2} + \frac{3}{2} \sum_{n \neq 0} \frac{\dot{\psi}_n^2}{V} + \cdots$$
  
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$$= f_{k\,\text{eff}}^{2} \left( \simeq \frac{k^{2}}{a^{2}} \quad \text{when inside the horizon} \right)$$

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$$I.5$$

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$$0.5$$

$$0.5$$

$$\frac{1}{2}$$

$$\frac{1}{4}$$

$$\frac{1}{6}$$

$$\frac{m_{\rm KK}}{2\pi}$$

$$0 = \vec{u_k} + \left\{ \frac{G}{B} \frac{k^2}{a^2} - \frac{M}{B} + \frac{(a^3 C)}{a^3 B} - \frac{(a^{3/2} B^{1/2})}{a^{3/2} B^{1/2}} \right\} u_k$$

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$$0.5$$



normalized fluctuation :  $u_{\mathbf{k}} = (a^3 B)^{1/2} Q_{\mathbf{k}}$ 

$$0 = \ddot{u_{k}} + \left\{ \frac{G}{B} \frac{k^{2}}{a^{2}} - \frac{M}{B} + \frac{\left(a^{3}C\right)^{\cdot}}{a^{3}B} - \frac{\left(a^{3/2}B^{1/2}\right)^{\cdot}}{a^{3/2}B^{1/2}} \right\} u_{k}$$

strong resonant amplification when  $f_{k\,\mathrm{eff}}^2$  wildly oscillates around  $m_{\mathrm{KK}}^2$ 



#### Generation of Spikes

- strong resonant amplification is highly sensitive to the KK amplitude
- expansion of the universe quickly damps away strong resonant effects, leaving spikes in the

pert. spectrum

 $\begin{array}{c|c} \mathcal{P}_{\zeta}(k) \\ 6.\times 10^{-8} \\ 5.\times 10^{-8} \\ 4.\times 10^{-8} \\ 3.\times 10^{-8} \\ 2.\times 10^{-8} \\ 1.\times 10^{-8} \end{array}$ 

What Causes Resonance?  

$$\frac{\mathcal{L}}{\sqrt{-g}} = -V\left(\frac{1}{\gamma} + 2\gamma \sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2}\right) - \gamma \sum_{n \neq 0} \left\{\frac{1}{2}(\partial \psi_n)^2 - \frac{1}{6}\frac{\psi_n^2}{\phi^2}(\partial \phi)^2 + \frac{1}{3}\frac{\psi_n}{\phi}(\partial \phi \cdot \partial \psi_n)\right\}$$

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most significantly oscillates fk eff

Couplings with  $\partial \phi$  source weak/strong resonance.

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most significantly oscillates fk eff

Couplings with  $\partial \phi$  source weak/strong resonance. General Lesson : Kinetic couplings efficiently produce sharp resonant features in the perturbation spectrum.

$$\begin{aligned} & \frac{\mathcal{L}}{\sqrt{-g}} = -V\left(\frac{1}{\gamma} + 2\gamma \sum_{n \neq 0} \alpha_n^2 \frac{\psi_n^2}{\phi^2}\right) - \gamma \sum_{n \neq 0} \left\{\frac{1}{2}(\partial \psi_n)^2 - \frac{1}{6}\frac{\psi_n^2}{\phi^2}(\partial \phi)^2 + \frac{1}{3}\frac{\psi_n}{\phi}(\partial \phi \cdot \partial \psi_n)\right\} \\ & + \frac{\gamma^3}{2V} \sum_{n \neq 0} \left\{(\partial \phi \cdot \partial \psi_n)^2 + \frac{1}{9}\frac{\psi_n^2}{\phi^2}\left((\partial \phi)^2\right)^2 - \frac{2}{3}\frac{\psi_n}{\phi}(\partial \phi)^2(\partial \phi \cdot \partial \psi_n)\right\} + \mathcal{O}(\psi_n^3) \end{aligned}$$

Cubic and higher order KK interactions become important for very large KK excitations.

Even stronger resonance?

# Summary

- Wrapped brane inflation models possess KK degrees of freedom that can be excited during inflation.
- Brane's Nambu-Goto action gives kinetic couplings efficient in producing resonant signals in the perturbation spectrum.
- Weak resonance sources oscillations, and strong resonance sources sharp spikes.
- Resonant signals can be used to probe extra dim.

#### **Future Directions**

observational consequences of resonant signals

• STRONGLY resonant non-Gaussianity Flauger et al. '09

explicit example

