DENSITY MATRIX OF THE UNIVERSE AND THE CFT DRIVEN COSMOLOGY

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Initial quantum state

vs

eternal inflation, self-reproduction and multiverse picture

Two known prescriptions for a *pure* initial state:

no-boundary wavefunction (Hartle-Hawking);

$$\Psi[\,g_{ij},\varphi\,] = \int D[\,g_{\mu\nu},\phi\,]\; e^{-S_E[\,g_{\mu\nu},\phi\,]}$$

“tunneling” wavefunction (Linde, Vilenkin, Rubakov, Zeldovich-Starobinsky, ...)

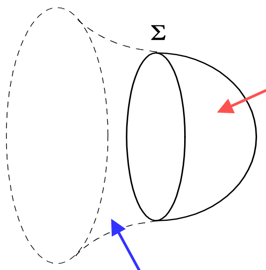
semiclassical solution of the minisuperspace Wheeler-DeWitt equation,
outgoing wave prescription,
etc.

Semiclassical properties of the no-boundary/tunneling states

Hyperbolic nature of the Wheeler-DeWitt equation

$$\left|\Psi_\pm\big(\varphi, \Phi(\mathbf{x})\big)\right| = \exp\left(\mp\frac{1}{2}S_E(\varphi)\right)\left|\Psi_{\mathrm{matter}}\big(\varphi, \Phi(\mathbf{x})\big)\right|$$

inflaton   other fields



Euclidean spacetime

no tunneling, really: "birth from nothing"

Euclidean action of quasi-de Sitter instanton with the effective Λ (slow roll):

$$\Lambda \simeq \frac{V(\varphi)}{M_P^2}$$

Euclidean FRW

$$a_0(\tau) = \frac{1}{H}\sin(H\tau), \quad H = \sqrt{\frac{\Lambda}{3}}$$

$$S_E(\varphi) \simeq -\frac{24\pi^2 M_P^4}{V(\varphi)} < 0$$

Analytic continuation – Lorentzian signature dS geometry:

$$\tau = \pi/2H + it$$

$$a_L(t) = \frac{1}{H}\cosh(Ht)$$

$\Psi_{\mathrm{matter}}\big(\varphi, \Phi(\mathbf{x})\big)$ **-- de Sitter invariant (Euclidean) vacuum**

No-boundary (+): probability maximum at the mininmum of the inflaton potential → infrared catastrophe no inflation

$$|\Psi(\varphi)| \to \infty,$$
$$V(\varphi) \to 0$$

vs

Tunneling (-): probability maximum at the maximum of the inflaton potential

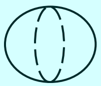
Beyond tree level: inflaton probability distribution:

$$\rho_{\text{no-boundary/tunnel}}(\varphi) = \int d\Big[\Phi(\mathbf{x})\Big]\,\Big|\Psi_{\text{no-boundary/tunnel}}\big(\varphi,\Phi(\mathbf{x})\big)\Big|^2$$

$$= \exp\left(\mp S_E(\varphi) - S_E^{1-\text{loop}}(\varphi)\right)$$

contradicts renormalization theory for + sign

Eucidean effective action on S^4



Both no-boundary (EQG path integral) and tunneling (WKB approximation) do not have a clear operator interpretation

Why should the cosmological state be the dS-invariant vacuum?

Why should it be a vacuum?

Why the cosmological state should be pure?

We suggest a unified framework for the no-boundary and tunneling states in the form of the *path integral of the microcanonical ensemble* in quantum cosmology

A.B. & A.Kamenshchik,
JCAP, 09, 014 (2006)
Phys. Rev. D74, 121502 (2006);
A.B., Phys. Rev. Lett.
99, 071301 (2007)

We apply it to the Universe dominated by:

i) massless matter conformally coupled to gravity →

- new (thermal) status of the no-boundary state;
- bounded range of the primordial Λ with band structure;
- dynamical elimination of vacuum no-boundary and tunneling states;
- inflation and thermal corrections to CMB power spectrum

ii) heavy massive fields →

- initial conditions for SM Higgs inflation with strong non-minimal curvature coupling

Plan

Cosmological initial conditions – density matrix of the Universe:

- microcanonical ensemble in cosmology
- initial conditions for the Universe via EQG statistical sum

CFT driven cosmology:

- constraining landscape of Λ
- suppression of vacuum no-boundary and tunneling states
- inflation and generation of thermally corrected CMB spectrum
- holographic duality – “DGP/CFT correspondence”
- Xtradimensional modifications of gravity -- Big Boost scenario of cosmological acceleration

Microcanonical ensemble in cosmology and EQG path integral

$H_\mu = 0$ **constraints on initial value data – corner stone of any diffeomorphism invariant theory.**

Microcanonical density matrix – projector onto subspace of quantum gravitational constraints

$$\hat\rho \sim \prod_\mu \delta(\hat H_\mu)$$

A.O.B., Phys.Rev.Lett. 99, 071301 (2007)

$\mu = (\perp\mathbf{x},\, i\mathbf{x})$
$\mathbf{x}$ – spatial coordinates

$$\hat H_\mu \equiv \hat H_\perp(\mathbf{x}),\ \hat H_i(\mathbf{x})$$

operators of the Wheeler-DeWitt equations

$$\hat H_\mu\left(q, \frac{\partial}{i\partial q}\right)\ \rho(q,q') = 0$$

Functional coordinate representation of 3-metric and matter fields *q* and conjugated momenta *p*:

$$q = \big(g_{ij}(\mathbf{x}), \varphi(\mathbf{x})\big),\ \ \rho(q,q') = \langle q\,|\,\hat\rho\,|\,q'\rangle$$

$$\hat p = \frac{\partial}{i\partial q} = \left(\frac{\delta}{i\delta g_{ij}(\mathbf{x})}, \frac{\delta}{i\delta\varphi(\mathbf{x})}\right)$$

Motivation:

A simple analogy --- an unconstrained system with a conserved Hamiltonian in $\hat{H}$ the microcanonical state with a fixed energy E

$$\hat{\rho} \sim \delta(\hat{H} - E)$$

Spatially closed cosmology does not have *freely specifiable* constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ, all having a particular value --- zero

⇩

The microcanonical ensemble with

$$\hat{\rho} \sim \left(\prod_\mu \delta(\hat{H}_\mu) \right)$$

is a natural candidate for the quantum state of the closed Universe – ultimate equipartition in the physical phase space of the theory --- *Sum over Everything*.

$$\rho(q_+, q_-) \sim \langle\, q_+ |\, \prod_\mu \delta(\hat{H}_\mu)\, |\, q_- \rangle$$

BRST/BFV construction with FP gauge fixing

Integral representation of this matrix element is the canonical (phase-space or ADM) path integral in Lorentzian theory:

$$\rho(q_+, q_-) = e^{\Gamma} \int_{q(t_\pm)=q_\pm} D[\,q, p, N\,]\; \exp\left[\, i \int_{t_-}^{t_+} dt\, (p\,\dot{q} - N^\mu H_\mu) \right]$$

lapse and shift functions (N^μ)

constraints $H_\mu = H_\mu(q, p)$

Range of integration over real N^μ: $-\infty < N^\mu < \infty \;\Rightarrow\; \delta(H_\mu)$

t is the ordering parameter for non-commuting $\hat{H}_\mu$, no dependence on $t_\pm$

Integrate over momenta → Lagrangian path integral over 4-metrics and matter fields

Lorentzian path integral = Euclidean Quantum Gravity (EQG) path integral with the imaginary lapse integration contour:

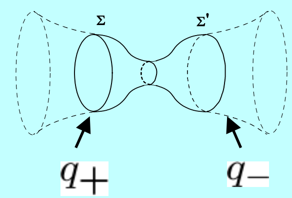
$$x^0 = x^4 \equiv \tau, \quad N_{\text{Lorentzian}} = -iN_{\text{Euclidean}}$$

$$-i\infty < N < i\infty, \quad g^{44} = +N^2$$

Euclidean metric

Euclidean action

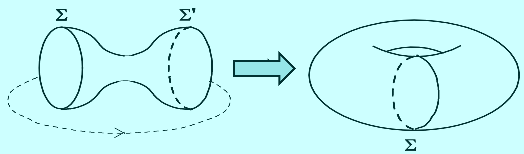
$$\rho(q_+, q_-) = e^{\Gamma} \int_{q(t_\pm)=q_\pm} D[g_{\mu\nu}, \phi]\, e^{-S_E[g_{\mu\nu},\phi]}$$



EQG density matrix
D.Page (1986)

Transition to the statistical sum:

$$\mathrm{tr}\,\hat{\rho} = 1$$

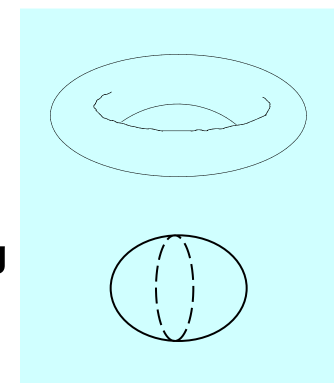


$$e^{-\Gamma} = \int_{\text{periodic}} D[\,g_{\mu\nu}, \phi\,]\; e^{-S_E[\,g_{\mu\nu},\phi\,]}$$

on *S*

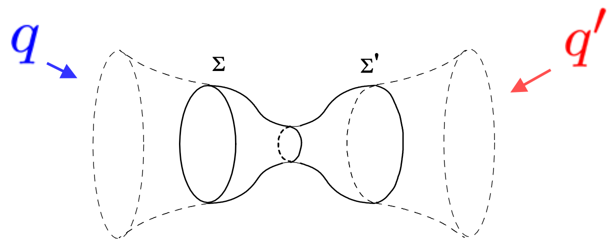
on S^3£ S^1 (thermal)

including as a limiting (vacuum) case S^4



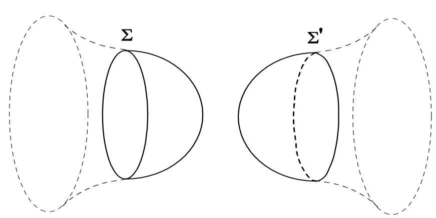
Hartle-Hawking state as a vacuum member of the microcanonical ensemble:

$$\hat{\rho}_{\text{mixed}} = \rho_{\text{mixed}}(q, q')$$

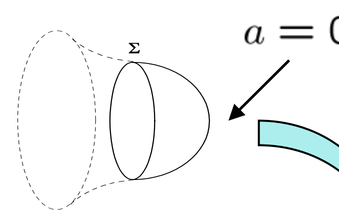


pinching a tubular spacetime

$$|\Psi_{HH}\rangle\langle\Psi_{HH}| = \rho_{HH}(q, q')$$



$$|\Psi_{HH}\rangle = \Psi_{HH}(q)$$



density matrix representation of a pure Hartle-Hawking state – vacuum state of zero temperature T~$1/\eta$:

$$\eta = \int_{a=0} d\tau \, \frac{N}{a} \to \infty$$

(see below)

Path integral calculation:

Disentangling the minisuperspace sector

Euclidean FRW metric

3-sphere of a unit size

$$ds^2 = N^2\, d\tau^2 + a^2\, d^2\Omega^{(3)}$$

lapse **scale factor**

$$[\,g,\phi\,] = [\,a(\tau), N(\tau); \Phi(x)\,]$$

minisuperspace background

quantum "matter" – cosmological perturbations:

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), ...)$$

Decomposition of the statistical sum path integral:

$$e^{-\Gamma} = \int_{\text{periodic}} D[\,a, N\,]\; e^{-S_{\text{eff}}[\,a, N\,]}$$

$$e^{-S_{\text{eff}}[\,a, N]} = \int_{\text{periodic}} D\Phi(x)\; e^{-S_E[\,a, N; \Phi(x)\,]}$$

quantum effective action of Φ on minisuperspace background

Semiclassical expansion and saddle points: no-boundary/tunneling contributions

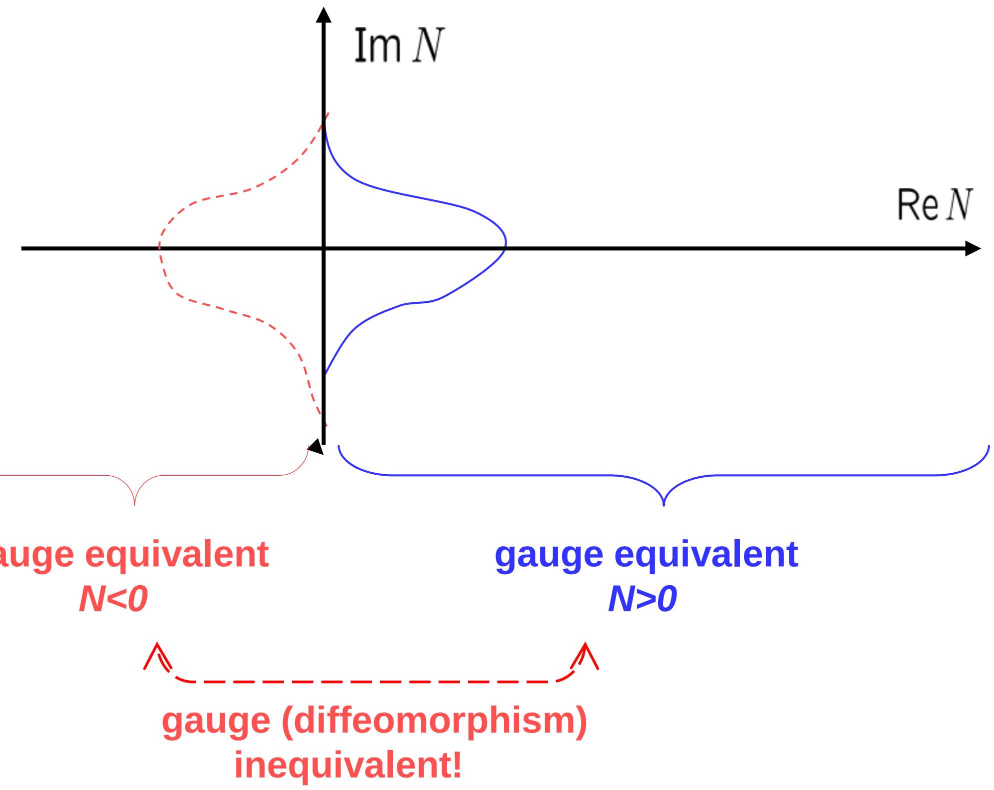
$$\Gamma_0 = S_{\text{eff}}[\, a_0(\tau), N_0(\tau)\,], \qquad \frac{\delta S_{\text{eff}}[\, a_0, N_0\,]}{\delta N_0(\tau)} = 0$$

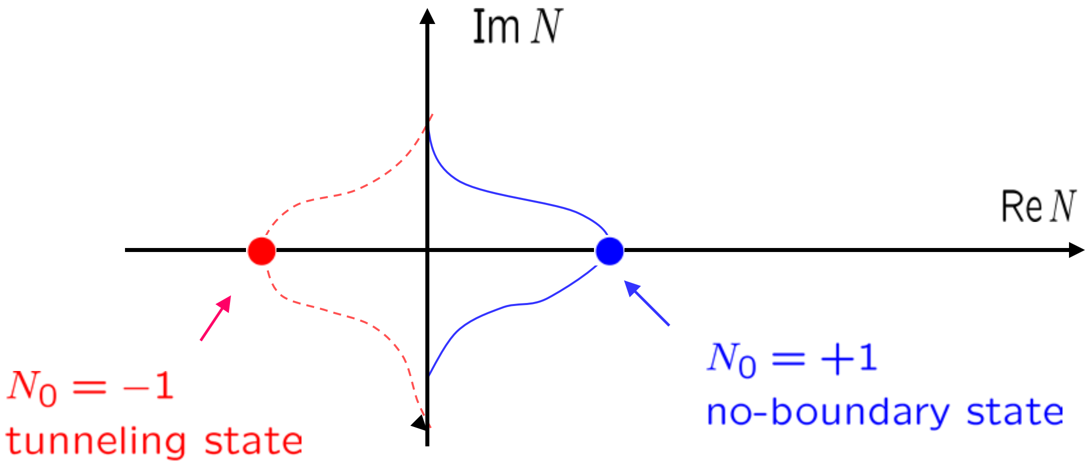
No periodic solutions of effective equations with imaginary Euclidean lapse *N* (Lorentzian spacetime geometry). Saddle points exist for real *N* (Euclidean geometry):

Deformation of the original contour of integration

$$-i\infty < N < i\infty$$

into the complex plane to pass through the saddle point with real *N>0* or *N<0*





$$\Gamma_{\text{no-boundary/tunnel}} = S_{\text{eff}}[\,a_0(\tau), \pm 1\,]$$

$$e^{-\Gamma} = \text{Tr} \prod_{\mu} \delta(\hat{H}_\mu) = e^{-\Gamma_{\text{no-boundary}}} + e^{-\Gamma_{\text{tunnel}}}$$

sum over Everything (all solutions)

will be suppressed dynamically!

Application to the CFT driven cosmology

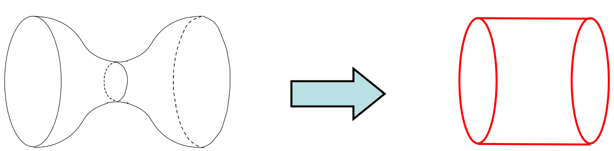
$$S_E[g_{\mu\nu},\phi] = -\frac{1}{16\pi G}\int d^4x\, g^{1/2}\left(R-2\Lambda\right) + S_{CFT}[g_{\mu\nu},\phi]$$

$\Lambda=3H^2$ -- primordial cosmological constant

N_sÀ 1 conformal fields of spin s=0,1,1/2

Conformal invariance → exact calculation of S_{eff}

Assumption of N_{cdf} conformally invariant, N_{cdf} À 1, quantum fields and recovery of the action from the conformal anomaly and the action on a static Einstein Universe



A.A.Starobinsky (1980); Fischetty,Hartle,Hu; Riegert; Tseytlin; Antoniadis, Mazur & Mottola; ……

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}) \quad \Longrightarrow \quad d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}$$

conformal time

spin-dependent coefficients

$$g_{\mu\nu}\frac{\delta\Gamma}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2}g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

Gauss-Bonnet term

Weyl term

$$\beta = \frac{1}{360}(2N_0 + 11N_{1/2} + 124N_1)$$

N_s # of fields of spin s

$$S_{\rm eff} = \text{classical part} + \Gamma_A + \Gamma_{EU}$$

anomaly contribution

Einstein universe contribution

Exactly solvable model in the leading order of $1/N_{cdf}$ - expansion

Contribution of conformal anomaly under the conformal transform

Riegert (1984)
Tseytlin
Antoniadis,
Mazur, Mottola
.....

$$g_{\mu\nu} = e^{\sigma}\bar{g}_{\mu\nu}$$

$$e^{\sigma} = a^2(\tau)$$

$$\Gamma_A[g] \equiv \Gamma[g] - \Gamma[\bar{g}]$$

$$= \frac{1}{32\pi^2}\int d^4x \bar{g}^{1/2}\left\{\frac{1}{2}\left[\gamma \bar{C}^2_{\mu\nu\alpha\beta} + \beta\left(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma + \frac{\beta}{2}\sigma\bar{\mathcal{D}}\sigma\right\}$$

$$-\frac{1}{32\pi^2}\left(\frac{\alpha}{12} + \frac{\beta}{18}\right)\int d^4x \left(g^{1/2}R^2(g) - \bar{g}^{1/2}R^2(\bar{g})\right),$$

$$\mathcal{D} \equiv \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$$

# of degrees of freedom preserving renormalization by the local counterterm » R^2 $(\alpha \to 0)$

⇓

$$\Gamma_A[a, N] = B\int d\tau\, N\left(\frac{a'^2}{a} - \frac{a'^4}{6a}\right) \qquad a' \equiv \frac{1}{N}\frac{da}{d\tau}$$

$$B = \frac{3\beta}{4m_P^2}$$ **-- coefficient of the Gauss-Bonnet term in the conformal anomaly**

Contribution of a static Einstein universe -- typical thermal statistical sum:

$$\Gamma_{EU}[a,N] = F(\eta) + \Gamma_{\text{Casimir}}(\eta)$$

Free energy contribution

$$F(\eta) = \pm \sum_{\omega} \ln\left(1 \mp e^{-\omega\eta}\right),$$

energies of field oscillators on a 3-sphere

$$\eta = \oint d\tau \, \frac{N}{a}$$

instanton period in units of conformal time --- inverse temperature

Casimir energy contribution

$$\Gamma_{\text{Casimir}}(\eta) = \pm\eta \sum_{\omega} \frac{\omega}{2} \Bigg|^{\text{renorm}} = \eta \, \frac{B}{2}$$

same constant as above

Brown & Cassidy (1977) ; Antoniadis, Mazur & Mottola

UV renormalization preserving # of degrees of freedom

Full quantum effective action on FRW background

nonlocal (thermal) part

$$S_{\rm eff}[\,a,N\,] = \int d\tau\, N\mathcal{L}(a,a') + F(\eta)$$

$$\mathcal{L}(a,a') = -aa'^2 - a + \frac{\Lambda}{3}a^3 + B\left(\frac{a'^2}{a} - \frac{a'^4}{6a} + \frac{1}{2a}\right)$$

classical part

conformal anomaly part

vacuum (Casimir) energy – from static EU

$$B = \frac{3\beta}{4m_P^2}$$ **-- coefficient of the Gauss-Bonnet term in the conformal anomaly**

$$\eta = \oint d\tau\, \frac{N}{a}$$

$$a' \equiv \frac{1}{N}\frac{da}{d\tau}$$

-- time reparameterization invariance (1D diffeomorphism)

Effective Friedmann equation for saddle points of the path integral:

$$\frac{\delta S_{\rm eff}[\,a,N\,]}{\delta N(\tau)} = 0$$

amount of radiation constant

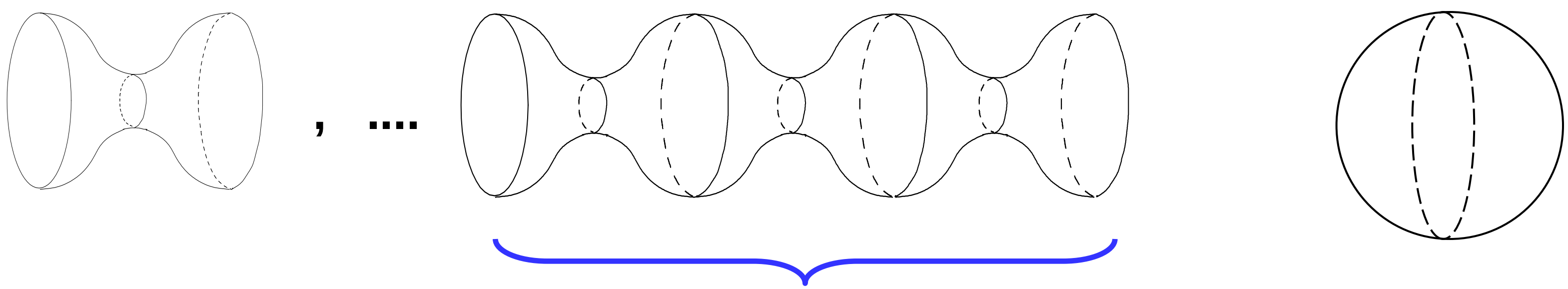
“bootstrap” equation:
η=η[a(τ)]

$$\frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{B}{2}\left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4}, \quad \mathcal{C} = \sum_\omega \frac{\omega}{e^{\omega\eta} \pm 1}$$

$$B = \frac{3\beta}{4m_P^2}$$

-- coefficient of the Gauss-Bonnet term in the conformal anomaly

Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor ($S^1 \times S^3$) and vacuum Hartle-Hawking instantons (S^4)

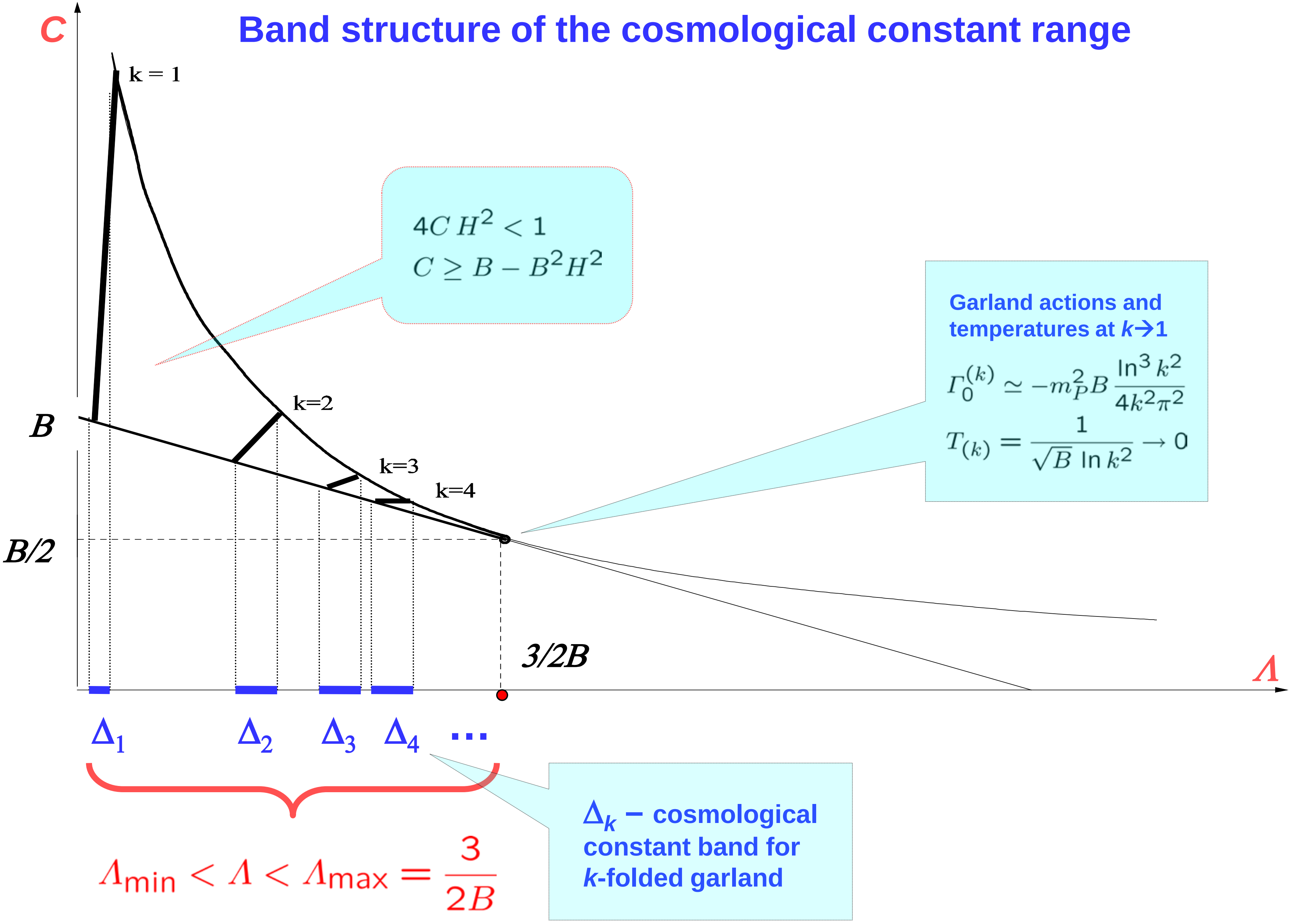


1- fold, *k*=1

***k*- folded garland, *k*=1,2,3,…**

S^4

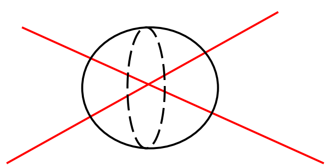
Band structure of the cosmological constant range



- $\Lambda_{\rm min} < \Lambda < \Lambda_{\rm max} = \frac{3}{2B}$ **bounded range of the cosmological constant** (***band structure of Λ - spectrum inside this range***)

  ↑ **new QG scale**

- **For any $\Lambda > 0$ exist S^4 instanton solutions**



elimination of the vacuum no-boundary state:

$$e^{-\Gamma_{\rm no-boundary}} = 0$$

$$\Gamma_{\rm no-boundary} \sim B \int\limits_{a=0} d\tau \left(\frac{a'^2}{a} + ...\right) = +\infty$$

- **Dynamical elimination of tunneling states:**

$$\Gamma_{\text{tunnel}} = S_{\text{eff}}[\,a_0(\tau), N_0 = -1\,]$$

N → -*N*, *η* → -*η*, only vacuum S^4 instantons

$$S_{\text{eff}}[a,-N] = -\text{classical part} - \Gamma_A - \eta\frac{B}{2} + F(-\eta)$$

$$F(-\eta) = \sum_\omega \ln\left(1 - e^{+\omega\eta}\right) = 2\left(\eta\sum_\omega \frac{\omega}{2}\right) + F(\eta) + \text{phase}$$

UV divergent

2 X $$\eta\sum_\omega \frac{\omega}{2} + \text{counterterm} = \eta\sum_\omega \frac{\omega}{2}\Bigg|^{\text{renorm}} + \Gamma_A = \eta\frac{B}{2} + \Gamma_A$$

$$\Gamma_{\text{tunnel}} \sim +\Gamma_A + \ldots$$
$$\sim B\int_{a=0} d\tau\left(\frac{a'^2}{a} + \ldots\right) = +\infty$$

- **Scaling with respect to # of conformal fields:** $\mathbb{N} \gg 1;\ \Lambda_{\text{min}}, \Lambda_{\text{max}} \to \frac{\Lambda_{\text{min}}}{\mathbb{N}}, \frac{\Lambda_{\text{max}}}{\mathbb{N}}$

Large number of conformal species: $\mathbb{N} \propto \beta \gg 1$

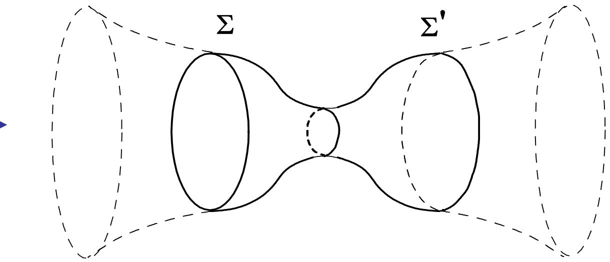
$$\Rightarrow \quad \Lambda = 3H^2 \simeq \frac{2m_P^2}{\beta} \ll m_P^2,$$
$$T \simeq (2/3\beta)^{1/3} \ll m_P,$$
$$\frac{\Lambda}{T^2} \sim \frac{1}{\beta^{1/3}} \to 0, \quad \beta \to \infty$$

Semiclassical limit is OK!

Inflationary evolution

Lorentzian Universe with initial conditions set by the saddle-point instanton. Analytic continuation of the instanton solutions:

$$\tau = it, \;\; a(t) = a_E(it)$$



Expansion and quick dilution of primordial radiation

Inflation via Λ as a composite operator – inflaton potential and a slow roll

$$\Lambda \simeq \frac{V(\varphi)}{M_P^2}$$

Decay of a composite Λ, exit from inflation and particle creation of conformally non-invariant matter:

$$\frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4} \Rightarrow \frac{8\pi G}{3}\,\varepsilon$$

matter energy density

Primordial CMB spectrum with thermal corrections:

$$\langle vac\,|\,\hat\phi_\omega(t)\hat\phi_\omega(t)\,|\,vac\rangle \to \langle\,\hat\phi_\omega(t)\hat\phi_\omega(t)\,\rangle_T$$

thermal contribution

$$\sim \langle\,\hat a^\dagger_\omega \hat a_\omega + \hat a_\omega \hat a^\dagger_\omega\,\rangle_T\,|u_\omega(t)|^2 = |u_\omega(t)|^2\Big(1 + 2n_\omega(T)\Big),$$

$$n_\omega(T) = \frac{1}{e^{k_\omega/T}-1}, \qquad \omega\eta = \omega \oint \frac{d\tau}{a} \simeq \frac{k_\omega}{T}, \qquad k_\omega = \frac{\omega}{a}$$ ← **physical scale**

$$\omega \leftrightarrow l \;\Rightarrow\; C_l^2 \to C_l^2\left(1 + \frac{2}{e^{k_l/T}-1}\right)$$

additional reddening of the CMB spectrum

$$\frac{k_l}{T} \simeq \#\frac{l}{\beta^{1/6}\sqrt{\Omega_0 - 1}}, \qquad \beta \gg 1$$

$$\frac{\Lambda}{T^2} \sim \frac{1}{\beta^{1/3}} \ll 1$$

$\beta \sim 10^6$ $\Rightarrow$ **Thermal contribution to n_s becomes visible with low l ! Unfortunately this is unnaturally high number of conformal species.**

$$\beta = \frac{1}{360}(2N_0 + 11N_{1/2} + 124N_1)$$

Conformal anomaly coefficient

Holographic duality – "DGP/CFT correspondence"

4D CFT cosmology

$B \sim G_4 \beta, \ \Lambda_4, \ \mathcal{C}$

⟺

Brane induced gravity in 5D Schwarzschild-deSitter bulk (5D black hole in dS_5)

$G_5, \ \Lambda_5, \ R_S$

Schwarzschild radius of bulk BH

4D radiation is imitated by the BH mass $\mathcal{C} \sim G_5 M = R_S^2$

But (!):

No SUSY

De Sitter, $\Lambda_5 > 0$

No AdS, no group-theoretical arguments

Holds only in the sector of cosmological variables

CFT cosmology vs DGP model

Dvali, Gabdadze & Porrati (2000)

Generalized cosmological DGP model with Λ_5 , bulk black hole of the mass » C and matter vacuum on the brane

Euclidean action

$$S_{DGP}[G_{AB}(X)] = -\frac{1}{16\pi G_5}\int_{\text{Bulk}} d^5X\, G^{1/2}\left(R^{(5)}(G_{AB}) - 2\Lambda_5\right) - \int_{\text{brane}} d^4x\, g^{1/2}\left(\frac{1}{8\pi G_5}[K] + \frac{1}{16\pi G_4}R(g_{\mu\nu})\right).$$

5D Schwarzschild-dS solution with a bulk black hole of the mass » R_s^2/G_5

$$ds^2_{(5)} = f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2_{(3)}$$

$$f(R) = 1 - \frac{\Lambda_5}{6}R^2 - \frac{R_S^2}{R^2}$$

embedding

$$ds^2_{(4)} = d\tau^2 + a^2(\tau)d\Omega^2_{(3)}$$

$$R = a(\tau)$$

$$T = T(\tau)$$

$$T'(\tau) = \frac{\sqrt{f(a) - a'^2}}{f(a)}$$

DGP crossover scale between 4D and 5D phases

$$r_c = \frac{G_5}{2G_4}$$

5D DGP side

$$r_c^2\left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{\Lambda_5}{6} - \frac{R_S^2}{a^4}$$

$$\left\{ \begin{array}{l} B = 2r_c^2, \quad \Lambda_4 = \dfrac{\Lambda_5}{2} \\ \mathcal{C} = R_S^2 \end{array} \right.$$

4D CFT cosmology

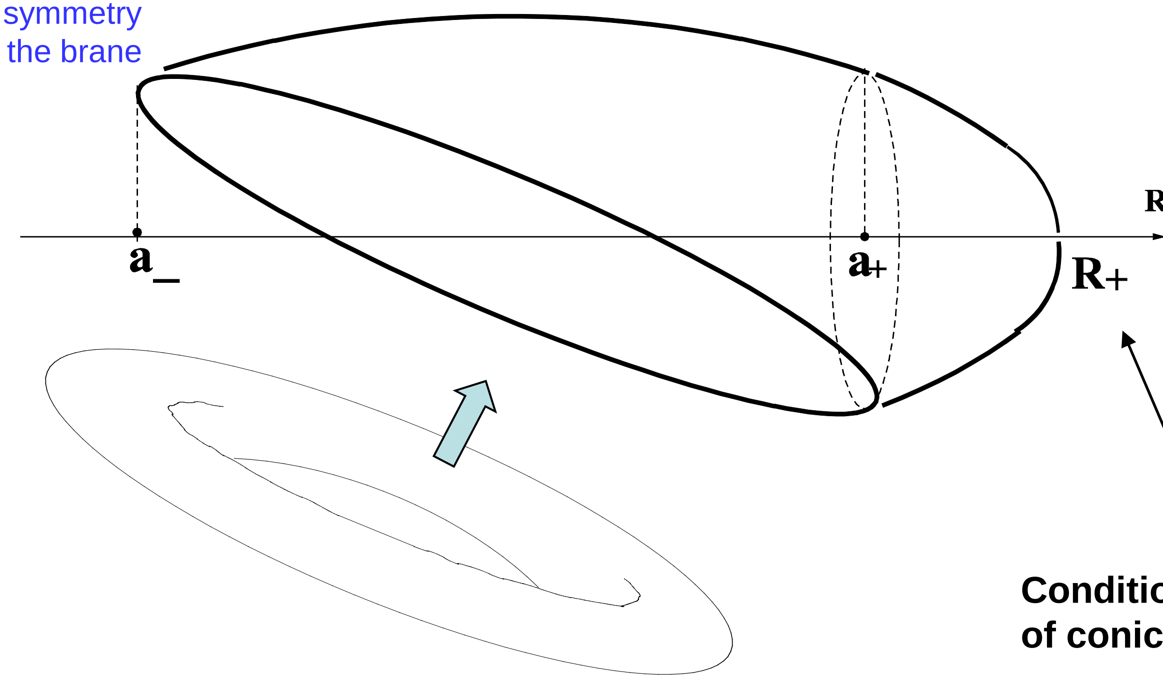
$$\frac{B}{2}\left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{\Lambda_4}{3} - \frac{\mathcal{C}}{a^4}$$

C » mass of the 5D black hole

Dynamical equations coincide, but is there a bootstrap equation for the amount of radiation *C* ?
Yes, there is --- from the absence of conical singularities in the bulk.

Euclidean Schwarzschild-dS "cigar" instanton:

Z_2 symmetry on the brane



$$f(R) \geq 0, \quad R_- \leq R \leq R_+$$

$$R_\pm^2 = \frac{3}{\Lambda_5}\left(1 \pm \sqrt{1 - 2\Lambda_5 R_S^2/3}\right)$$

$$\underbrace{R_- < a_- \leq a(\tau) \leq a_+ < R_+}$$

4D instanton domain

Condition of absence of conical singularity:

S^3£ S^1

$$\oint d\tau\, T'(\tau) \equiv 2k \int_{a_-}^{a_+} da \frac{\sqrt{f(a) - a'^2}}{a' f(a)} = \frac{4\pi}{|df(R_+)/dR_+|}$$

This gives the equation alternative to the CFT bootstrap:

⇕

$$\mathcal{C} = \sum_\omega \frac{\omega}{e^{\omega\eta} \pm 1}$$

Does this duality extend beyond cosmological model?
Partly and sometimes yes:

Israel junction condition in DGP model →

$$\begin{cases} K_{\mu\nu} = -r_c\left(R_{\mu\nu} - \frac{1}{6}\, g_{\mu\nu}\, R\right), \\ K_{\mu\nu}^2 - K^2 = r_c^2\left(R_{\mu\nu}^2 - \frac{1}{3}\, R^2\right) \end{cases}$$

Einstein theory in the bulk – constraint equation:

$$R^{(4)} + K_{\mu\nu}^2 - K^2 - 2\Lambda_5 = 0$$

⇩

$$R^{(4)} + r_c^2\left(R_{\mu\nu}^2 - \frac{1}{3}\, R^2\right) - 2\Lambda_5 = 0$$

DGP side

Trace equation $$g_{\mu\nu}\frac{\delta\Gamma}{\delta g_{\mu\nu}} = 0$$

$$E = C^2_{\mu\nu\alpha\beta} - 2\left(R^2_{\mu\nu} - \frac{1}{3}R^2\right)$$

vanishes on FRW

CFT side $$R^{(4)} + \frac{\beta G_4}{2\pi}\left(R^2_{\mu\nu} - \frac{1}{3}R^2\right) - 4\Lambda_4 - \underbrace{\frac{\beta+\gamma}{4\pi}}_{=\,0} G_4\, C^2_{\mu\nu\alpha\beta} = 0$$

SUSY

$D = 4$ $SU(\mathbb{N})$ $\mathcal{N} = 4$ SYM

$(N_0, N_{1/2}, N_1) = (6\mathbb{N}^2, 4\mathbb{N}^2, \mathbb{N}^2)$

$B = 3\mathbb{N}^2/8m_P^2$;

$\mathcal{N} = 4$ and $\mathcal{N} = 8$ SUGRA

DGP side $$R^{(4)} + r_c^2\left(R^2_{\mu\nu} - \frac{1}{3}R^2\right) - 2\Lambda_5 = 0$$

Xtra dimensional modifications of gravity -- Big Boost scenario of cosmological acceleration

Friedmann equation modified by a conformal anomaly

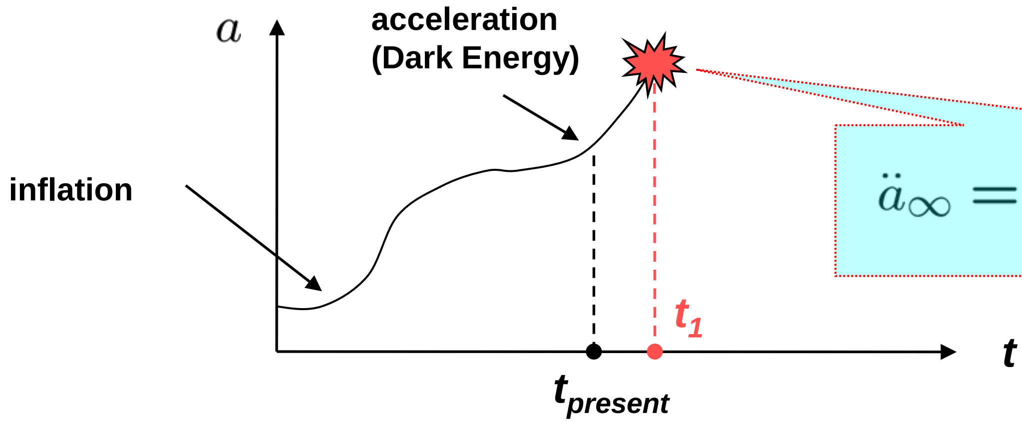
$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{\pi}{\beta G}\left\{1 - \sqrt{1 - \frac{16G^2}{3}\beta\varepsilon}\right\}$$

GR limit $\beta\varepsilon \to 0$

β – a moduli parameter depending on extra dimension (???):

A.B, C.Deffayet and A.Kamenshchik, JCAP 05 (2008) 020

Cosmological acceleration and Big Boost singularity:



$$\ddot{a}_\infty = \infty, \qquad H^2_\infty = \frac{\pi}{\beta G} < \infty$$

Conclusions

Microcanonical density matrix of the Universe

Euclidean quantum gravity path integral – unifying framework for the no-boundary and tunneling states

Application to the CFT driven cosmology with a large # of quantum species – thermal version of the no-boundary state

Dynamical elimination of vacuum no-boundary and tunneling states

Initial conditions for inflation with a limited range of Λ --- cosmological landscape and generation of the thermal CMB spectrum

Dual 5D description of the CFT cosmology via the DGP model with $\Lambda_5>0$ and 5D BH imitating radiation on the brane