Testing local isotropy with weak lensing

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Cosmological models

Theoretical physics → Cosmology ← Astrophysics

Principles
Local law of nature

Phenomena

Extrapolations → models ← Constraints

Gravity=GR
Matter=SU(3)XSU(2)xU(1)

Compatible with local physics
Save the apperances

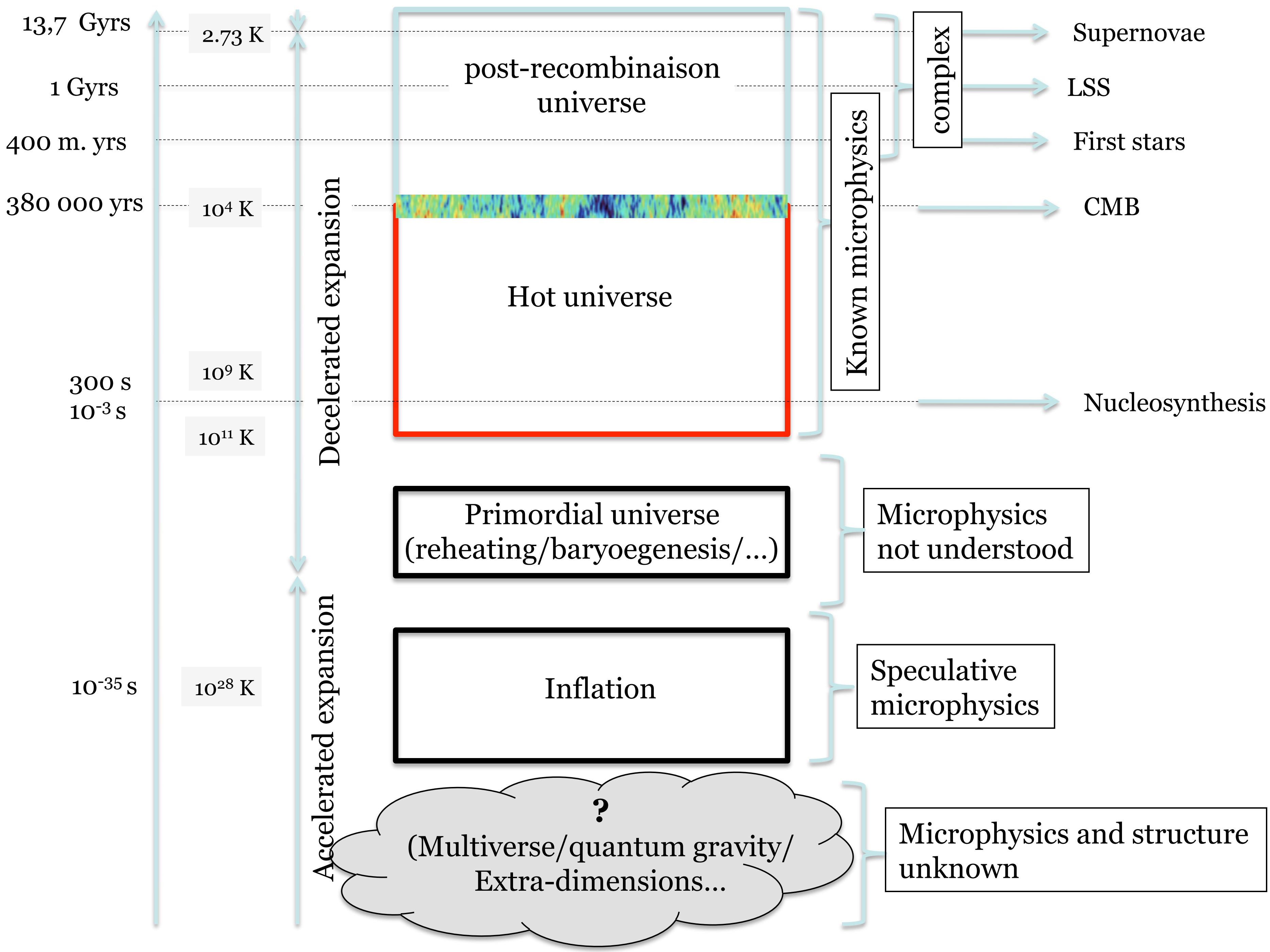
Its construction relies on 4 hypothesis

1. Theory of gravity [General relativity]
2. Matter [Standard model fields + CDM + Λ]
3. Symmetry hypothesis [Copernican Principle]
4. Global structure [Topology of space is trivial]

→ New physics with simple cosmological solution

→ Standard physics with more involved solution

In agreement with all the data.


13,7 Gyrs
2.73 K
1 Gyrs
400 m. yrs
380 000 yrs
10⁴ K
300 s
10⁹ K
10⁻³ s
10¹¹ K
10⁻³⁵ s
10²⁸ K
Decelerated expansion
Accelerated expansion
post-recombinaison universe
Hot universe
Primordial universe (reheating/baryoegenesis/...)
Inflation
?
(Multiverse/quantum gravity/ Extra-dimensions...
Known microphysics
complex
Supernovae
LSS
First stars
CMB
Nucleosynthesis
Microphysics not understood
Speculative microphysics
Microphysics and structure unknown

Needs to test these hypothesis

This model is in agreement with all existing observational data at the expense of the introduction of a **dark sector**:

- *dark matter*: galaxy rotation curves, properties of the large scale structure of the universe.

candidate: Weakly interacting massive particles (many candidates, e.g. from supersymmetry)
or a modification of general relativity in the low acceleration regime

- *dark energy*: acceleration of the cosmic expansion

candidate: cosmological constant
or more exotic models (quintessence, modification of GR in the IR etc….)

This reveals the need for new degrees of freedom in our cosmological model:

- physical
- geometrical

Motivations
to test
the Copernican principle

Cosmological models

Its construction relies on 4 hypothesis

1. Theory of gravity [General relativity]
2. Matter [Standard model fields + CDM + Λ]
3. Symmetry hypothesis [Copernican Principle]
4. Global structure [Topology of space is trivial]

Testing these hypothesis has been a part of my scientific activity over the past years:

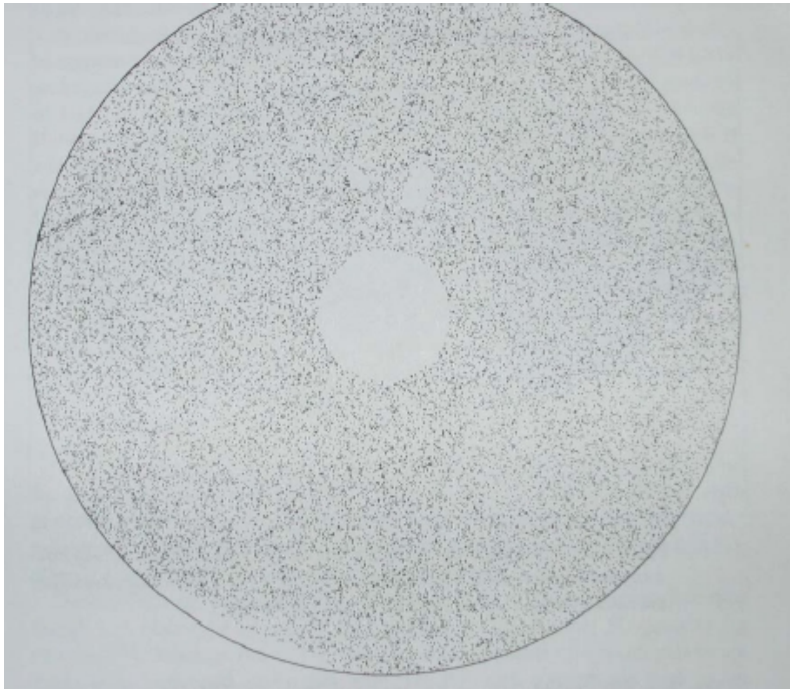
- testing GR on astrophysical scales
  . Equivalence principle [jpu (2000,2011)]
  . Large scale structure [jpu & Bernardeau (2001)]

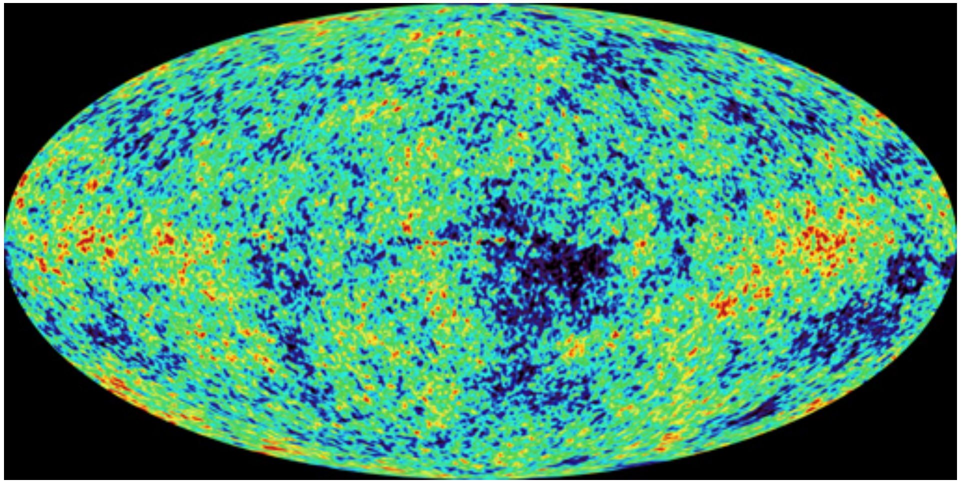
- testing for spatial topology [jpu, lehoucq, luminet, riazuelo, weeks (2003)]

- testing for the Copernican principle [jpu, clarckson, Ellis (2008)]

Isotropy: motivation

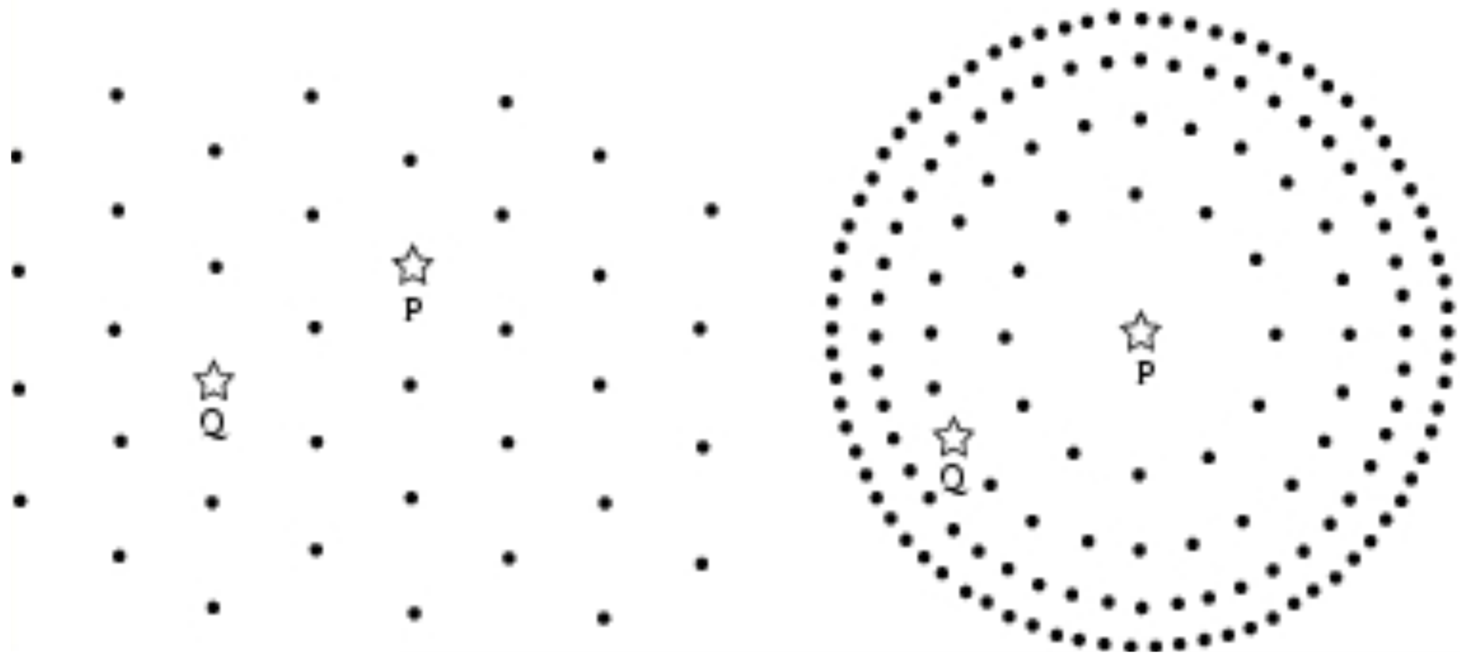
Observationally, the universe seems very isotropic around us.





Uniformity principle

Two possibilities to achieve this:



Spatially homogeneous & isotropic

Spherically symmetric
Universe has a center

Copernican Principle: we do not occupy a particular spatial location in the universe

Geometrical implication

There exists a priviledged class of observers for which the spatial sections look isotropic and homogeneous.

The spatial sections are constant curvature hypersurface.

The spacetime is of the Friedmann-Lemaître type with metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}(x^k)\mathrm{d}x^i\mathrm{d}x^j$$

$$\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = \mathrm{d}\chi^2 + f_K^2(\chi)\mathrm{d}\Omega^2 \qquad f_K(\chi) = \begin{cases} K^{-1/2}\sin\left(\sqrt{K}\chi\right) & K>0 \\ \chi & K=0 \\ (-K)^{-1/2}\sinh\left(\sqrt{-K}\chi\right) & K<0 \end{cases}$$

<u>Consequences</u>:

1- The dynamics of the universe reduces to the one of the scale factor

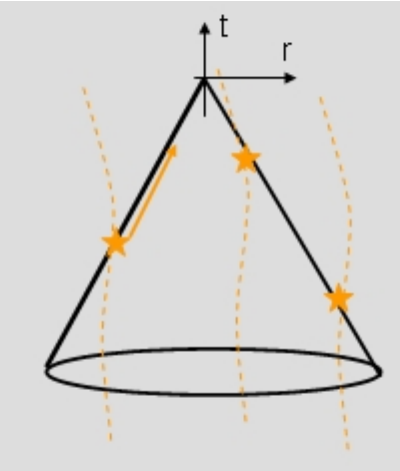
2- It is dictated by the Friedmann equations

$$3\left(H^2 + \frac{K}{a^2}\right) = 8\pi G\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

Implications of the Copernican principle

Independently of any theory (**H1, H3**), the Copernican principle implies that the geometry of the universe reduces to *a(t)*.



Consequences:

- $$1 + z = \frac{E_{rec}}{E_{em}} \overset{\text{H2}}{=} \frac{a_0}{a(t)}$$

- $$a(t) = a_0 \left[1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots \right]$$

  so that

$$H^2(z)/H_0^2 = 1 + (q_0 + 1)z + \mathcal{O}(z^2)$$

$$q_0 = \Omega_{m0}/2$$

- **Hubble diagram** gives
  - H_o at small z
  - q_o

Supernovae data (1998+) show

$$\boxed{q_0 < 0}$$ ⟷ The expansion is now **accelerating**

No hypothesis on gravity at this stage.

Motivations to test for isotropy

The Copernican principle has thus important implications for our understanding of the origin of the acceleration of the cosmic expansion.

Among other competing explanations, void models have been proposed.

At early time, the isotropisation of the universe is efficient only if inflation lasts long enough.

What is the dynamical effect of an early anisotropy?
Can it be constrained observationally?

*Thiago Pereira, Cyril Pitrou, J.-P. U., JCAP **09** (2007) 006*
*Cyril Pitrou, Thiago Pereira, J.-P. U., JCAP **04** (2008) 004*

At late time:

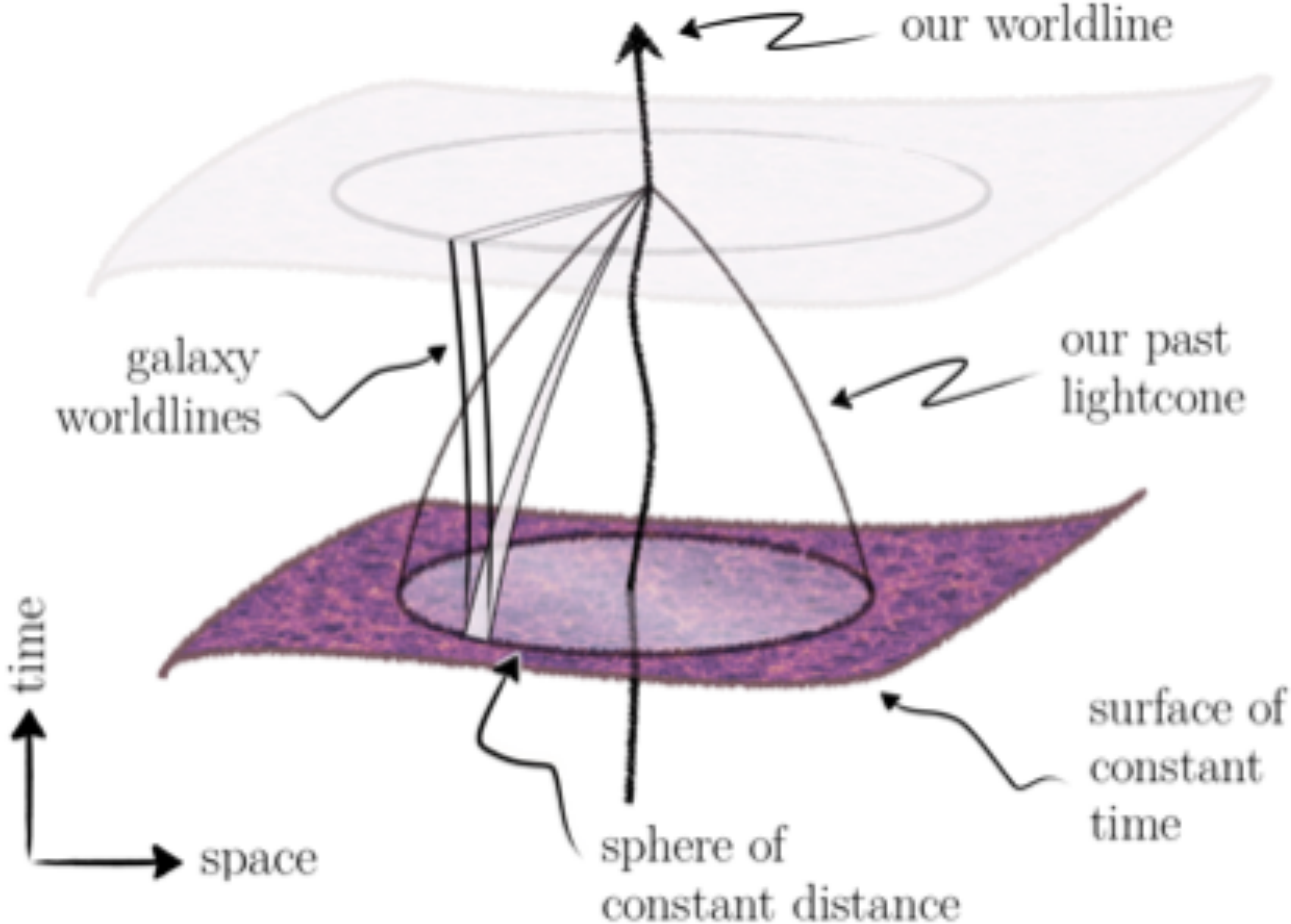
- any model of dark energy involving vector fields and some models of modification of gravity have a non-vanishing anisotropic stress tensor;
- it has also been shown that in the approach in which the acceleration is explained by backreaction effect, the Hubble flow becomes anisotropic at late time

[Marozzi, JPU, arXiv:1206.4887]

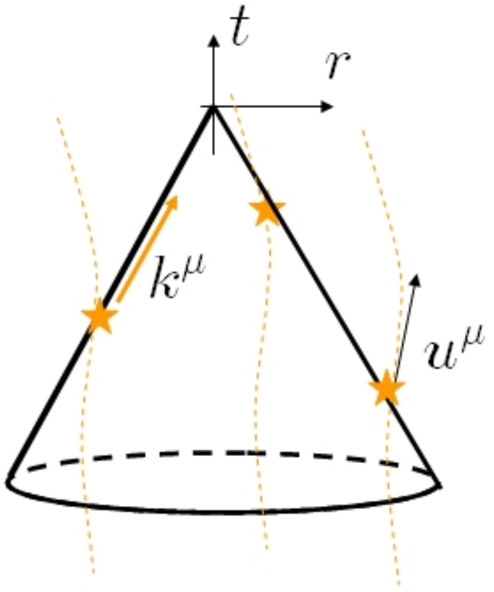
How can we constrain a late time anisotropy?

Test of homogeneity

This is difficult



Test of homogeneity



Homogeneous and isotropic universe

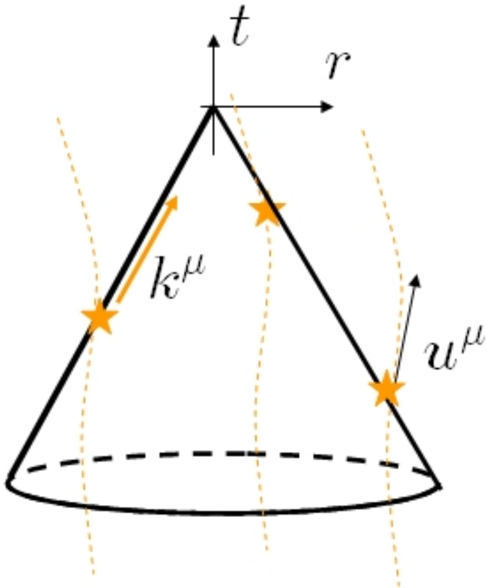
$$\dot{z} = H_0(1+z) - H(z)$$

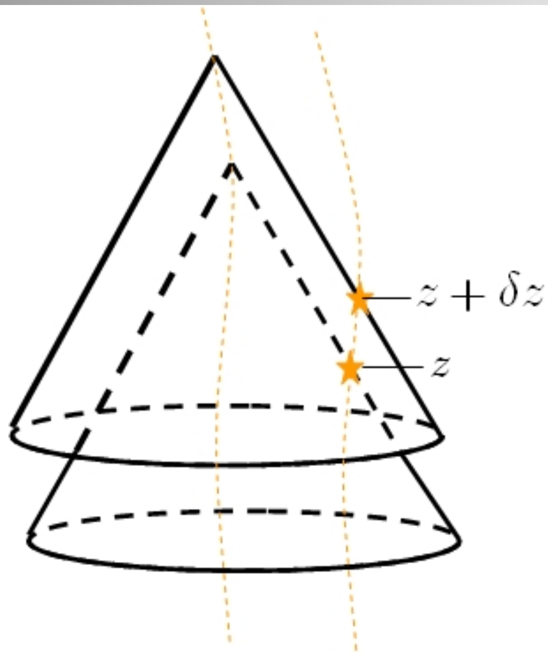
[Sandage1962, McVittie 1962]

Typical order of magnitude (z~4)

$$\delta z \sim -5 \times 10^{-10} \qquad \text{on} \qquad \delta t \sim 10\,\mathrm{yr}$$

Test of homogeneity





Homogeneous and isotropic universe

$$\dot z = H_0(1+z) - H(z)$$

[Sandage1962, McVittie 1962]

Inhomogeneous universe

$$\dot z = (1+z)H_0 - H_\perp(z)$$

[JPU, Clarkson, Ellis, PRL (2008)]

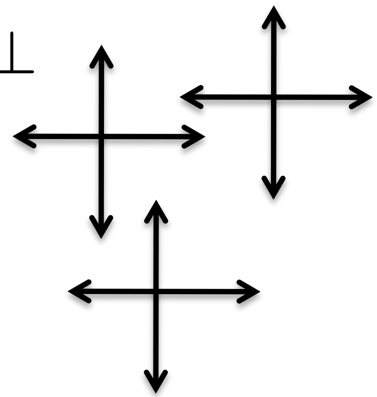
Typical order of magnitude (z~4)

$$\delta z \sim -\,5\times 10^{-10} \qquad \text{on} \qquad \delta t \sim 10\,\mathrm{yr}$$

Time drift and homogeneity

Friedmann-Lemaître

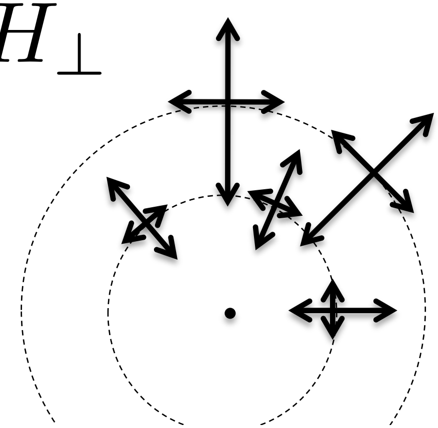
$$H_{||} = H_{\perp}$$



$$\dot{z} = H_0(1+z) - H(z)$$

LTB (spherically symmetric universe)

$$H_{||} \neq H_{\perp}$$



$$\dot{z} = (1+z)H_0 - H_{\perp}(z)$$

By combining distance measurements (D_A or D_L), one can test whether

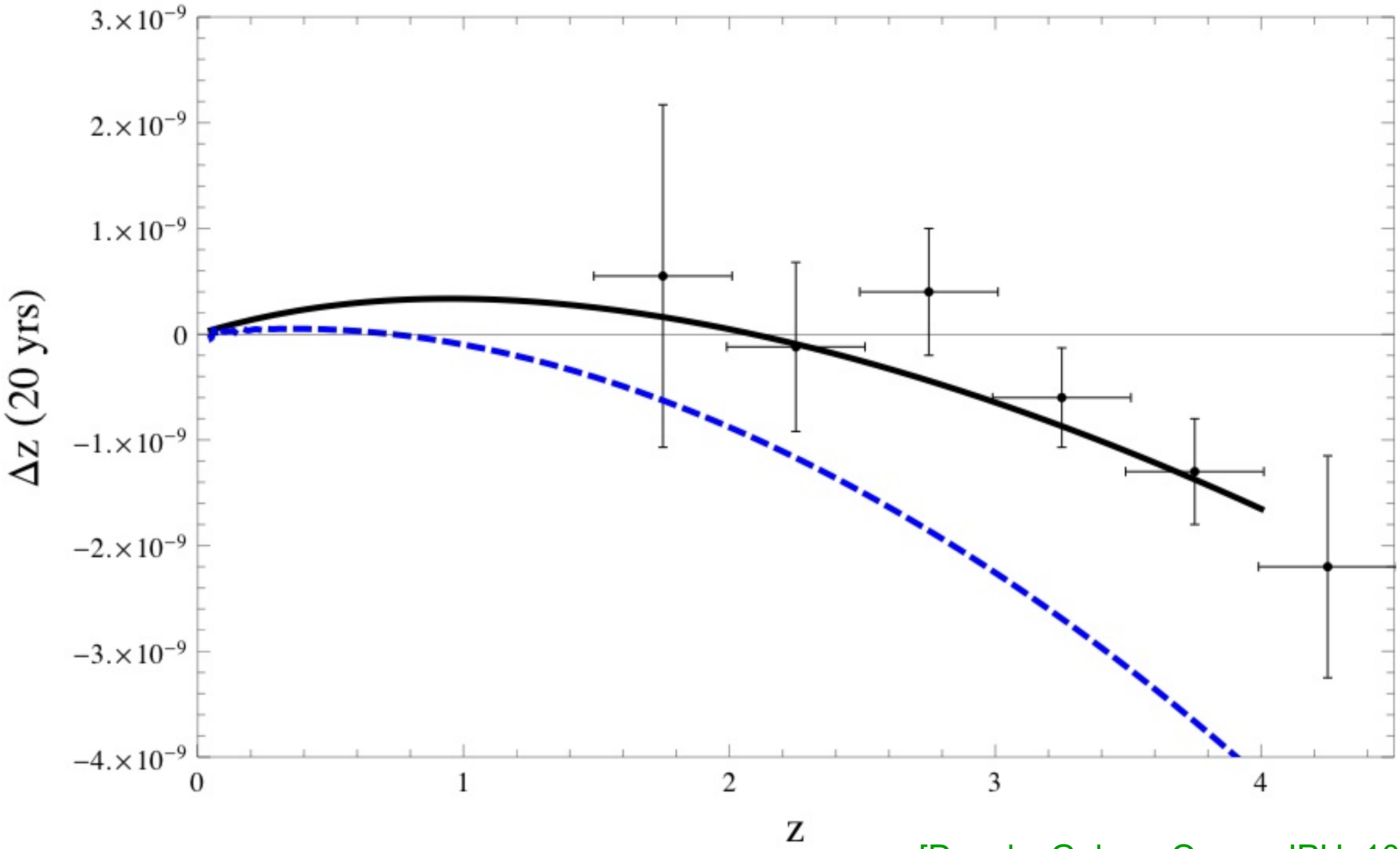
$$H_{||} = H_{\perp}$$

We have information off the past light-cone.

How sensitive can such a test be?

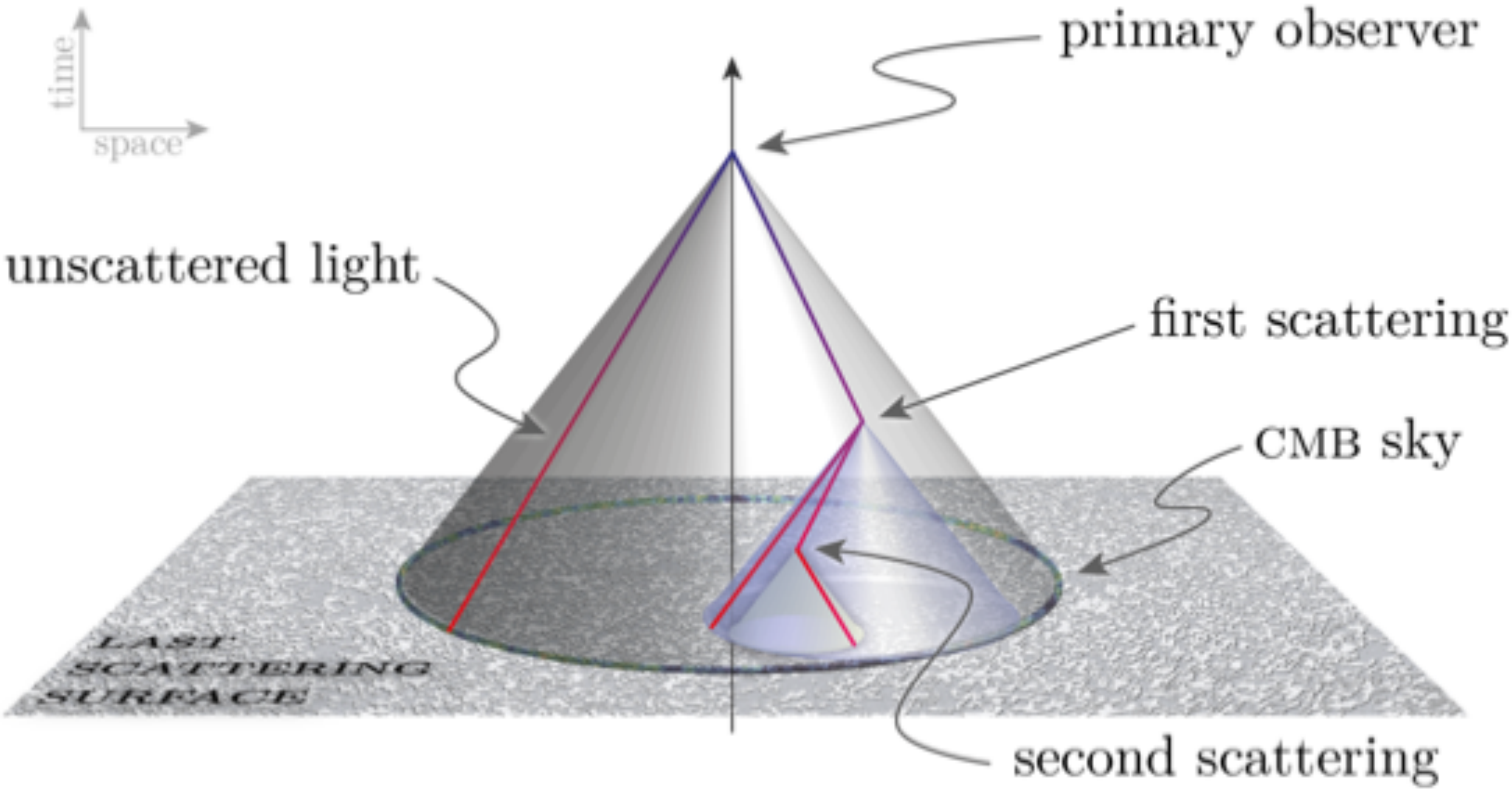
We assume that $8\pi G\rho(z) = 8\pi G\rho_{FL}(z) = 3\Omega_{\mathrm{m}0}H_0^2(1+z)^3$

i.e. same $D_L(z)$ & same matter profile BUT NO cosmological constant



[Dunsby,Goheer,Osano,JPU, 1002.2397]

Other methods



Interaction of CMB photon with hot gas of galaxy clusters
CMB photons are scattered into our line of sight.

Gives information on the **inside** of our past lightcone

Other methods

SZ effect – Temperature of scattered CMB photons:

- It induces spectral distortions:
    - Thermal SZ (due to thermal motion of electrons) reflects the monopole seen by the cluster
    - If the blackbody T from different direction are different, then huge distortion.

    - Motion of the cluster induces kinetic SZ that reflects the CMB dipole

    - Applied to a class of LTB model. Impressive constraints (even if do not rule out all the models)

[Goodman, astro-ph/9506068; Caldwell & Stebbins, arXiv:0711.3459; Zhang & Stebbins, arXiv:1009.3967]

SZ effect – Polarisation of scattered CMB photons:

- Cluster bulk tranverse velocity & CMB monopole, quadrupole and octopole induce modifications in CMB polarisation through scattering off cluster gas.

In principle, gives access to monopole-octopole of the CMB as *seen by the cluster*.

Beyond Friedmann-Lemaître spacetimes

Classifying spacetimes

The solutions of Einstein equations can be classified according to their symmetries. The symmetries are characterized by the set of Killing vectors:

$$\pounds_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0.$$

The Killing vectors satisfy $[\xi_a, \xi_b] = C^c_{ab}\xi_c$:

Isometry group	=	isotropy group « rotation »	+	transitivity group « translations »
$r \leq \dfrac{n(n+1)}{2}$		$q \leq \dfrac{n(n-1)}{2}$		$s \leq n$

One thus needs to specify (s, q, r) & C^a_{bc}

Simplest solutions

In $n=4$ dimensions, $r=q+s \leq 10$ so that

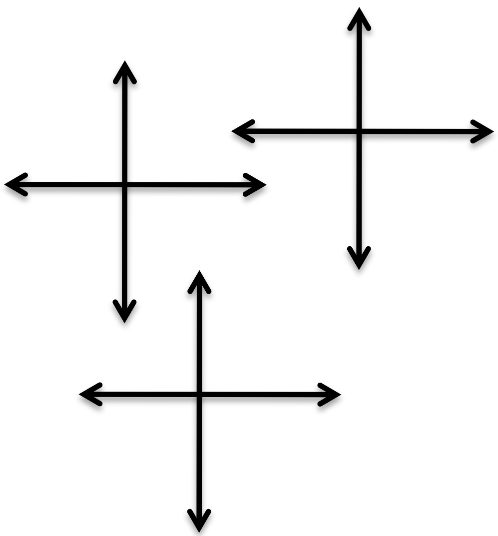
s can run between 0 and 4.
s=4 corresponds to static spacetimes

For $s<4$, the possibilities are then
$q=3$ (isotropic)
$q=2$ is impossible [no subgroup of O(3) of dimension 2]
$q=1$ (locally invariant under rotation)
$q=0$ (anisotropic)

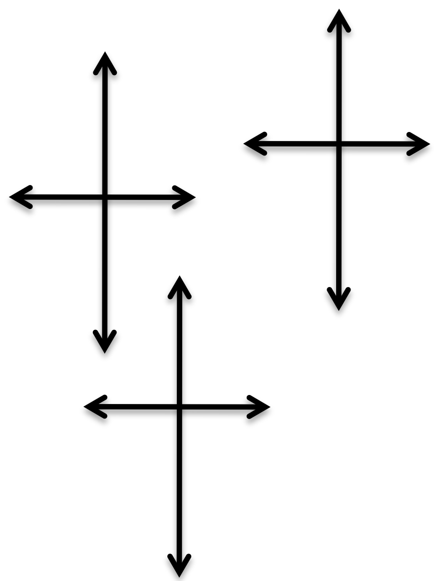
	$s=4$ space-time homogeneous	$s=3$ spatially homogeneous	$s=2$ inhomogeneous
$q=6$	Minkowski, de Sitter anti-de Sitter	none	none
$q=3$	Einstein static	Friedmann–Lemaître	none
$q=1$		Bianchi Kantowski–Sachs	Lemaître–Tolman–Bondi
$q=0$		Bianchi	

Indeed, the real universe has $r=0$!

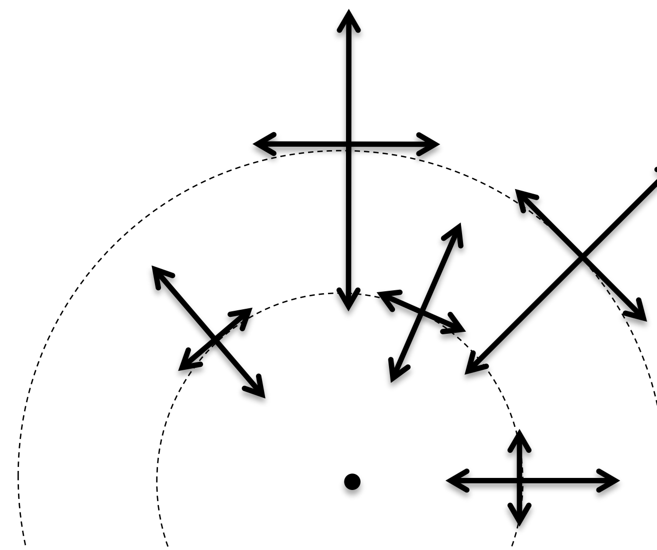
Examples



Friedmann-Lemaître
homogène-isotrope
$r = 6$
$s = 3 \qquad q = 3$



Bianchi I
homogène-anisotrope
$r = 3$
$s = 3 \qquad q = 0$



Lemaître-Tolman-Bondi
inhomogène-loc. isotrope
$r = 3$
$s = 2 \qquad q = 1$
(centre) $s = 0 \qquad q = 3$

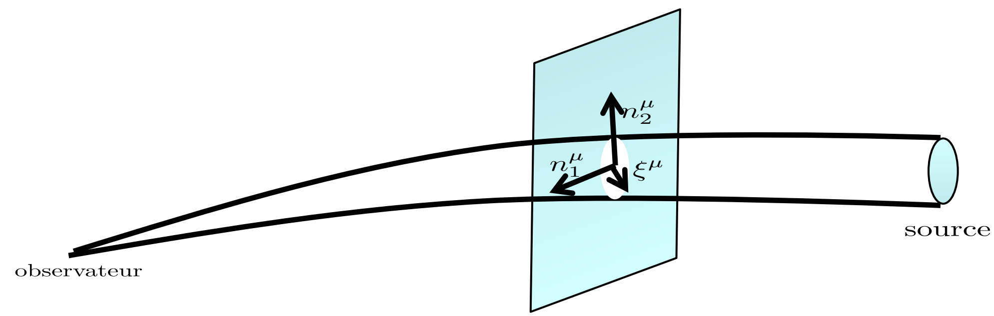
Testing local isotropy with weak lensing

[Pitrou, JPU, Pereira, arXiv:1203.4069]

Description of the geodesic bundle

Let us consider a bundle of null geodesics, $x^\mu(v,s)$.

v is an affine parameter along the geodesic and we choose $v=0$ at the observer.



s labels the different geodesics of the bundle.

We define: k^μ as the tangent vector of the nll-geodesic
u^μ as the 4-velocity of the observer

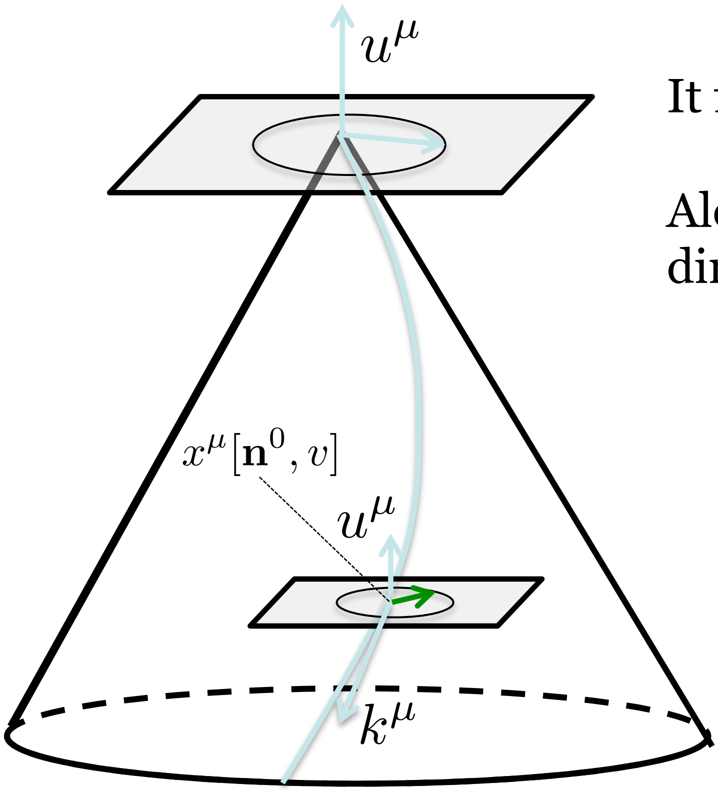
Then the redshift is given by: $$1+z(v) \equiv \frac{(k_\mu u^\mu)_v}{(k_\mu u^\mu)_0}$$

so that the energy of the incoming photon is $U = U_0(1+z)$

We then introduce n^μ, the spacelike unit vector pointing along the line of sight

$$\hat{k}^\mu \equiv U^{-1} k^\mu = -u^\mu + n^\mu \qquad u^\mu n_\mu = 0\,, \quad n_\mu n^\mu = 1$$

Screen description



The integration of the geodesic equation gives *$x^\mu[\boldsymbol{n}^o, v]$*.

It follows that *$\boldsymbol{n}[\boldsymbol{n}^o, v]$*.

Along the geodesic, we can define a basis of the 2-dimensional orthogonal to $\boldsymbol{n}$.

$$n_a^\mu n_{b\mu} = \delta_{ab}\,, \quad n_a^\mu u_\mu = n_a^\mu n_\mu = 0\,, \quad (a = 1, 2)$$

This basis can be parallely transported along the geodesic

$$S_{\mu\sigma} k^\nu \nabla_\nu n_a^\sigma = 0.$$

where we have introduced the screen projector

$$S_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu - n_\mu n_\nu$$

From this basis, we can define the helicity basis by $\quad \mathbf{n}_\pm = \frac{1}{\sqrt{2}}(\mathbf{n}_1 \pm \mathbf{n}_2)$

Sachs equation

The deformation of the geodesic bunble can be obtained from the geodesic deviation equation

$$\frac{\mathrm{d}^2\eta^\mu}{\mathrm{d}s^2} = R^\mu{}_{\nu\alpha\beta}k^\nu k^\alpha \eta^\beta$$

We can then decompose the connecting vector on the basis of the screen to obtain

$$\frac{\mathrm{d}^2\eta_a}{\mathrm{d}v^2} = \mathcal{R}_{ab}\eta^b \qquad \text{with} \qquad \mathcal{R}_{ab} \equiv R_{\mu\nu\alpha\beta}k^\nu k^\alpha n_a^\mu n_b^\beta$$

The linearity of this equation implies that $\eta^a(v) = \mathcal{D}^a_b(v)\left(\mathrm{d}\eta^b/\mathrm{d}v\right)|_{v=0}$

Sachs equation	Initial condition
$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\mathcal{D}^a_b = \mathcal{R}^a_c\mathcal{D}^c_b$	$\mathcal{D}^a_b(0) = 0\,, \quad \frac{\mathrm{d}\mathcal{D}^a_b}{\mathrm{d}v}(0) = \delta^a_b$

Program

Sachs equation	Initial condition
$$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\mathcal{D}^a_b = \mathcal{R}^a_c\mathcal{D}^c_b$$	$$\mathcal{D}^a_b(0) = 0\,, \quad \frac{\mathrm{d}\mathcal{D}^a_b}{\mathrm{d}v}(0) = \delta^a_b$$

1- Express the projected Riemann tensor
in terms of Ricci / eletric and magnetic part of the Weyl

2- Decompose the Jacobi matrix
in terms of a convergence/shear/rotation

3- Derive an equation of propagation for these components

4- Perform an harmonic decomposition.

Expression of the projected Riemann tensor

It is convenient to change the affine parameter to $\mathrm{d}/\mathrm{d}\hat{v} \equiv \hat{k}^\mu \nabla_\mu$

It is related to v by $\mathrm{d}\hat{v}/\mathrm{d}v = U$ and we have $\dfrac{\mathrm{d}\ln U}{\mathrm{d}\hat{v}} = H_{||}(\boldsymbol{n}^o, \hat{v})$

$$\begin{aligned} H_{||}(\boldsymbol{n}^o, \hat{v}) &\equiv \hat{k}^\mu \hat{k}^\nu \nabla_\mu u_\nu \\ &= \frac{1}{3}\Theta + \hat{\sigma}_{\mu\nu} n^\mu n^\nu + A_\mu n^\mu \end{aligned}$$

The projected Ricci tensor then takes the form

$$\mathcal{R}_{ab} = U^2 \left(\mathcal{R} I_{ab} + \mathcal{W}_{ab} \right)$$

Ricci

Weyl

$$\mathcal{R} \equiv -\frac{1}{2} R_{\mu\nu} \hat{k}^\mu \hat{k}^\nu \,, \quad \mathcal{W}_{ab} \equiv C_{\mu\rho\sigma\nu} \hat{k}^\rho \hat{k}^\sigma n_a^\mu n_b^\nu$$

The he Weyl tensor can be expressed as $\mathcal{W}_{ab}(\boldsymbol{n}^o, v) = -2 n^\mu_{\langle a} n^\nu_{b\rangle} \left[\mathcal{E}_{\mu\nu} + \mathcal{B}_\mu{}^\sigma \epsilon_{\sigma\nu}(\boldsymbol{n}) \right]\Big|_{\substack{x^\mu(\boldsymbol{n}^o, v) \\ \boldsymbol{n}(\boldsymbol{n}^o, v)}}$,

with $\mathcal{E}_{\mu\nu} \equiv C_{\mu\rho\nu\sigma} u^\rho u^\sigma \,, \quad \mathcal{B}_{\mu\nu} \equiv \dfrac{1}{2} \varepsilon_{\mu\alpha\beta\sigma} u^\sigma C_{\nu\rho}{}^{\alpha\beta} u^\rho$

Expression of the projected Riemann tensor

The projected Ricci tensor then takes the form

$$\mathcal{R}_{ab} = U^2\left(\mathcal{R} I_{ab} + \mathcal{W}_{ab}\right)$$

Ricci

Weyl

$$\mathcal{R} \equiv -\frac{1}{2} R_{\mu\nu}\hat{k}^{\mu}\hat{k}^{\nu}\,, \quad \mathcal{W}_{ab} \equiv C_{\mu\rho\sigma\nu}\hat{k}^{\rho}\hat{k}^{\sigma} n_a^{\mu} n_b^{\nu}$$

The Ricci term is a scalar.

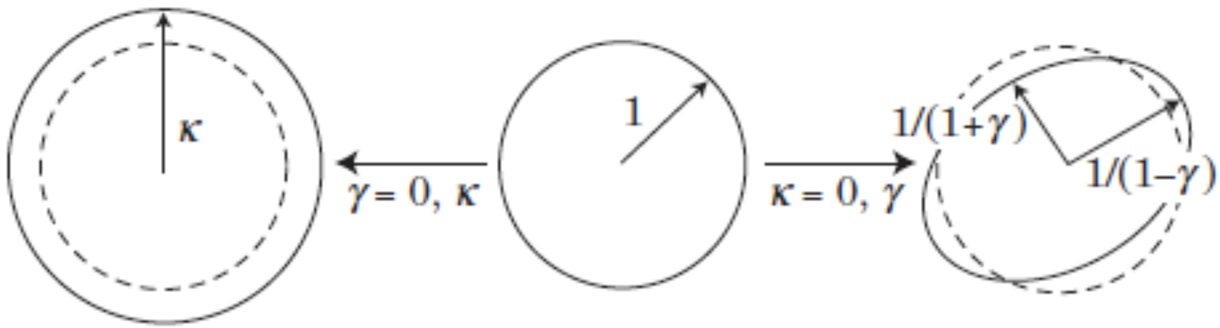
The Weyl term is a spin-2. $\mathcal{W}_{ab}(\boldsymbol{n}^o, v) \equiv -2\sum_{\lambda=\pm}\mathcal{W}^{\lambda}(\boldsymbol{n}^o, v) n_a^{\lambda} n_b^{\lambda}.$

It can be expressed as $\mathcal{W}_{ab}(\boldsymbol{n}^o, v) = -2 n^{\mu}_{\langle a} n^{\nu}_{b\rangle}\left[\mathcal{E}_{\mu\nu} + \mathcal{B}_{\mu}{}^{\sigma}\epsilon_{\sigma\nu}(\boldsymbol{n})\right]\Big|_{\substack{x^{\mu}(\boldsymbol{n}^o, v)\\ \boldsymbol{n}(\boldsymbol{n}^o, v)}},$

with $\mathcal{E}_{\mu\nu} \equiv C_{\mu\rho\nu\sigma}u^{\rho}u^{\sigma}\,, \quad \mathcal{B}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\alpha\beta\sigma}u^{\sigma}C_{\nu\rho}{}^{\alpha\beta}u^{\rho}$

Decomposition of the Jacobi matrix

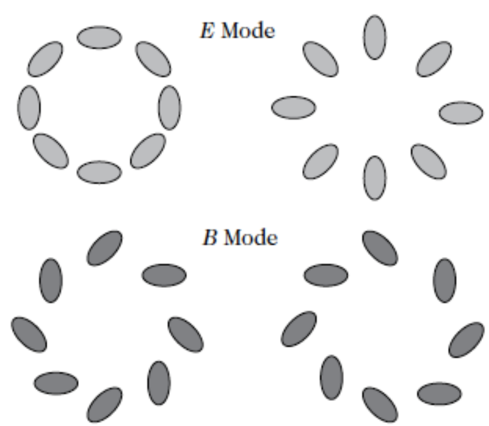
$$\mathcal{D}_{ab} \equiv \kappa I_{ab} + V\epsilon_{ab} + \gamma_{ab} \qquad \epsilon_{ab} = 2\mathrm{i} n^-_{[a} n^+_{b]}$$



The shear can be shown to be a spin-2 and can thus be decomposed as

$$\gamma_{ab}(\boldsymbol{n}^o, v) \equiv \sum_{\lambda=\pm} \gamma^\lambda(\boldsymbol{n}^o, v) n^\lambda_a n^\lambda_b$$

$$\gamma^\pm(\boldsymbol{n}^o, \hat{v}) = \sum_{\ell,m} \left[E_{\ell m}(\hat{v}) \pm \mathrm{i} B_{\ell m}(\hat{v})\right] Y^{\pm 2}_{\ell m}(\boldsymbol{n}^o)$$



Propagation of the degrees of freedom

The sachs equation gives the equation of propagation for the components of the Jacobi matrix

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}\hat{v}^2} + H_{\parallel}\frac{\mathrm{d}}{\mathrm{d}\hat{v}} - \mathcal{R}\right)\begin{pmatrix}\kappa \\ \mathrm{i}V \\ \gamma^{\pm}\end{pmatrix} = -2\begin{pmatrix}\mathcal{W}^{(-}\gamma^{+)} \\ \mathcal{W}^{[-}\gamma^{+]} \\ \mathcal{W}^{\pm}(\kappa \pm \mathrm{i}V)\end{pmatrix}$$

The integration of this system requires:

- to determine the light-cone structure

  $\boldsymbol{n}^a(\boldsymbol{n}^o,v)$, $H_{//}(\boldsymbol{n}^o,v)$,....

  in this sense, this equation is non-local.

  in general $\boldsymbol{n}^a(\boldsymbol{n}^o,v) \neq \boldsymbol{n}^o$

- the extraction of the component +/- also depends on $\boldsymbol{n}^a(\boldsymbol{n}^o,v)$

  $\boldsymbol{n}^o$ is the direction of observation.

To go further we insert the multipolar decomposition in this equation.

Multipolar decomposition

	Scalars	Spin-2
Jacobi matrix *[shape of the bundle]*	$\kappa(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m}\kappa_{\ell m}(\hat{v})Y_{\ell m}(\boldsymbol{n}^o)$ $V(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m}V_{\ell m}(\hat{v})Y_{\ell m}(\boldsymbol{n}^o)$	$\gamma^{\pm}(\boldsymbol{n}^o,\hat{v})$
Source terms *[Properties of the spacetime]*	$\mathcal{R}(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m}\mathcal{R}_{\ell m}(\hat{v})Y_{\ell m}(\boldsymbol{n}^o)$ $H_{\parallel}(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m}h_{\ell m}(\hat{v})Y_{\ell m}(\boldsymbol{n}^o)$	$\mathcal{W}^{\pm}(\boldsymbol{n}^o,\hat{v})$

$$\mathcal{W}^{\pm}(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m}\left[\mathcal{E}_{\ell m}(\hat{v}) \pm \mathrm{i}\mathcal{B}_{\ell m}(\hat{v})\right] Y^{\pm 2}_{\ell m}(\boldsymbol{n}^o)$$

$$\gamma^{\pm}(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m}\left[\underset{\displaystyle\downarrow\atop\displaystyle(-1)^{\ell}}{E_{\ell m}(\hat{v})} \pm \mathrm{i}\underset{\displaystyle\downarrow\atop\displaystyle(-1)^{\ell+1}}{B_{\ell m}(\hat{v})}\right] Y^{\pm 2}_{\ell m}(\boldsymbol{n}^o)$$

Multipolar hierachy

I skip the details but it involves decomposing products of spherical (spinned) harmonics, hence the C-coefficients.

$$
\begin{aligned}
\frac{\mathrm{d}^2 E_{\ell m}}{\mathrm{d}\hat{v}^2} &= {}^2C^{m m_1 m_2}_{\ell \ell_1 \ell_2}\Big[\Big(\mathcal{R}_{\ell_1 m_1} - h_{\ell_1 m_1}\frac{\mathrm{d}}{\mathrm{d}\hat{v}}\Big)\left(\delta^+_L E_{\ell_2 m_2} + \mathrm{i}\delta^-_L B_{\ell_2 m_2}\right) - 2\kappa_{\ell_1 m_1}\left(\delta^+_L \mathcal{E}_{\ell_2 m_2} + \mathrm{i}\delta^-_L \mathcal{B}_{\ell_2 m_2}\right) \\
&\quad + 2V_{\ell_1 m_1}\left(-\mathrm{i}\delta^-_L \mathcal{E}_{\ell_2 m_2} + \delta^+_L \mathcal{B}_{\ell_2 m_2}\right)\Big] \\
\frac{\mathrm{d}^2 B_{\ell m}}{\mathrm{d}\hat{v}^2} &= {}^2C^{m m_1 m_2}_{\ell \ell_1 \ell_2}\Big[\Big(\mathcal{R}_{\ell_1 m_1} - h_{\ell_1 m_1}\frac{\mathrm{d}}{\mathrm{d}\hat{v}}\Big)\left(\delta^+_L B_{\ell_2 m_2} - \mathrm{i}\delta^-_L E_{\ell_2 m_2}\right) - 2\kappa_{\ell_1 m_1}\left(\delta^+_L \mathcal{B}_{\ell_2 m_2} - \mathrm{i}\delta^-_L \mathcal{E}_{\ell_2 m_2}\right) \\
&\quad - 2V_{\ell_1 m_1}\left(\delta^-_L \mathrm{i}\mathcal{B}_{\ell_2 m_2} + \delta^+_L \mathcal{E}_{\ell_2 m_2}\right)\Big] \\
\frac{\mathrm{d}^2 \kappa_{\ell m}}{\mathrm{d}\hat{v}^2} &= \Big\{ {}^0C^{m m_1 m_2}_{\ell \ell_1 \ell_2}\left(\mathcal{R}_{\ell_1 m_1}\kappa_{\ell_2 m_2} - h_{\ell_1 m_1}\frac{\mathrm{d}\kappa_{\ell_2 m_2}}{\mathrm{d}\hat{v}}\right) \\
&\quad - 2(-1)^{m_1}\,{}^2C^{-m_2 - m m_1}_{\ell_2\ \ell\ \ell_1}\left[\delta^+_L\left(E_{\ell_1 m_1}\mathcal{E}_{\ell_2 m_2} + B_{\ell_1 m_1}\mathcal{B}_{\ell_2 m_2}\right) + \mathrm{i}\delta^-_L\left(B_{\ell_1 m_1}\mathcal{E}_{\ell_2 m_2} - E_{\ell_1 m_1}\mathcal{B}_{\ell_2 m_2}\right)\right]\Big\} \\
\frac{\mathrm{d}^2 V_{\ell m}}{\mathrm{d}\hat{v}^2} &= \Big\{ {}^0C^{m m_1 m_2}_{\ell \ell_1 \ell_2}\left(\mathcal{R}_{\ell_1 m_1}V_{\ell_2 m_2} - h_{\ell_1 m_1}\frac{\mathrm{d}V_{\ell_2 m_2}}{\mathrm{d}\hat{v}}\right) \\
&\quad + 2(-1)^{m_1}\,{}^2C^{-m_2 - m m_1}_{\ell_2\ \ell\ \ell_1}\left[\delta^-_L \mathrm{i}\left(E_{\ell_1 m_1}\mathcal{E}_{\ell_2 m_2} + B_{\ell_1 m_1}\mathcal{B}_{\ell_2 m_2}\right) - \delta^+_L\left(B_{\ell_1 m_1}\mathcal{E}_{\ell_2 m_2} - E_{\ell_1 m_1}\mathcal{B}_{\ell_2 m_2}\right)\right]\Big\}
\end{aligned}
$$

-Hierarchy similar to CMB Boltzmann hierarchy
- does not depend on the choice of any background spacetime [but needs h_{lm} etc..]
- never been derived and generalized the particular FL case
- non-vanishing Weyl implies E/B modes due to coupling to convergence

Application to FL spacetime

First, we need to show that we recover the standard results.

Background FL spacetime:

- vanishing Weyl so that $\mathcal{E}^{(0)}_{\ell m} = \mathcal{B}^{(0)}_{\ell m} = 0$
- geodesic equation leads to $\boldsymbol{n}(\boldsymbol{n}^o, \hat{v}) = \boldsymbol{n}^o$ and thus h_{lm} are all vanishing but

$$h^{(0)}_{00} = H$$

- space is homogeneous so that the Ricci depends only on time. Only $R^{(o)}{}_{oo}$

κ and V have only the oo components which is non-vanishing
satisfy the same equations but with different initial conditions

$$\mathcal{D}^a_b(0) = 0\,, \quad \frac{\mathrm{d}\mathcal{D}^a_b}{\mathrm{d}v}(0) = \delta^a_b$$

$$\boxed{\begin{gathered} V^{(0)}_{\ell m} = E^{(0)}_{\ell m} = B^{(0)}_{\ell m} = 0 \\ \kappa^{(0)}_{00} = D_A \end{gathered}}$$

Linear perturbations

We work at linear order in perturbation

3 types of modes: S, V, T. Only S are important at low redshift.

$$\mathcal{R}^{(1)}_{ab} = -D_a D_b(\Phi + \Psi)$$

This implies that the Weyl has no magnetic part: $\mathcal{B}^{(1)}_{ab} = 0$

We can work in the Born approximation: $\boldsymbol{n}(\boldsymbol{n}^o, \hat{v}) = \boldsymbol{n}^o$ so that only $h_{oo} \neq 0$

Shear: E and B modes

$$E^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - h^{(0)}_{00}\frac{d}{d\hat{v}}\right) E^{(1)}_{\ell m} - 2\kappa^{(0)}_{00}\,\mathcal{E}^{(1)}_{\ell m}$$

$$B^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - h^{(0)}_{00}\frac{d}{d\hat{v}}\right) B^{(1)}_{\ell m}$$

Only E-modes are sourced.

Linear perturbations

Convergence and rotation:

$$\kappa_{\ell m}^{(1)} \to \left(\mathcal{R}_{00}^{(0)} - h_{00}^{(0)} \frac{d}{d\hat{v}} \right) \kappa_{\ell m}^{(1)} + \mathcal{R}_{\ell m}^{(1)} \kappa_{00}^{(0)}$$

$$V_{\ell m}^{(1)} \to \left(\mathcal{R}_{00}^{(0)} - h_{00}^{(0)} \frac{d}{d\hat{v}} \right) V_{\ell m}^{(1)}$$

Rotation is not sourced.

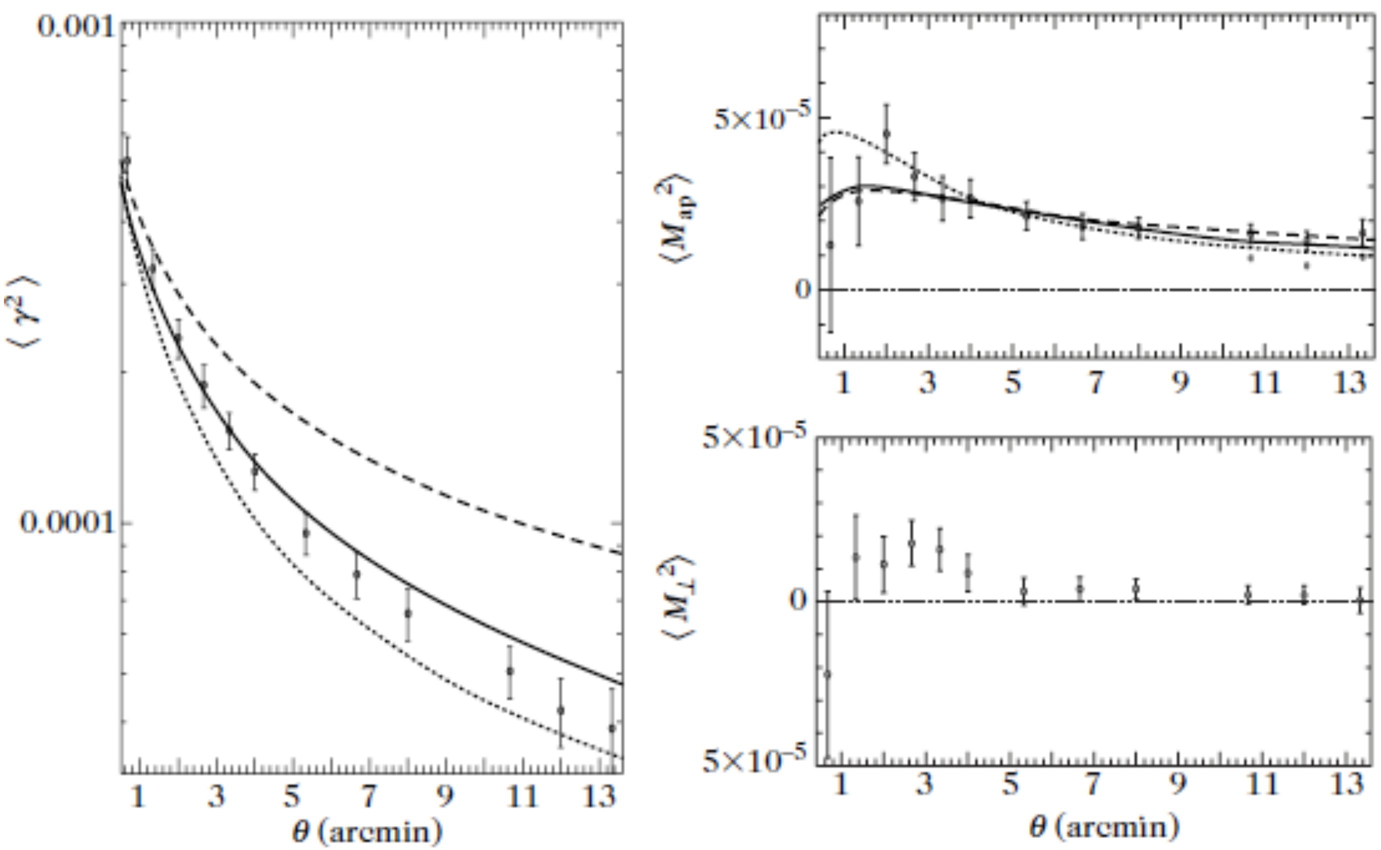
$$\boxed{\begin{gathered} V_{\ell m}^{(1)} = B_{\ell m}^{(1)} = 0 \\ \kappa_{\ell m}^{(1)} \neq 0 \;\&\; E_{\ell m}^{(1)} \neq 0 \end{gathered}}$$

This reproduces the standard lore.

Observational constraints

B-modes:
- V & T modes
- non-linear perturbation (electroc and magntic Weyl / beyond Born approx)
  [see Bonvin et al. PRD 81 (2010) 083002]
- astrophysical effects [intrinsic alignments / lens-lens coupling]
- systematics

[Fu et al., A&A 479 (2008) 9]



Bianchi universes

There are major differences with FL case

1- At background level the Weyl is non vanishing

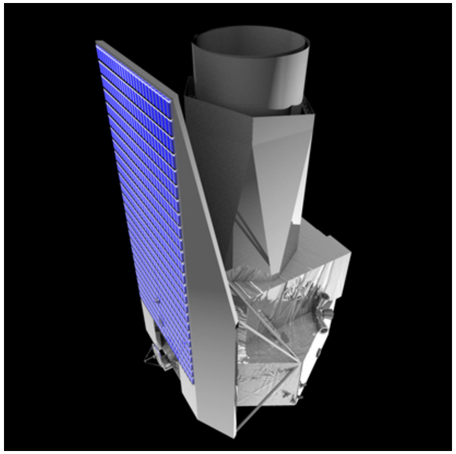
$$\mathcal{E}_{ij}^{(0)} = H\sigma_{ij} + \frac{1}{3}\sigma^2\gamma_{ij} - \sigma_{ik}\sigma_j^k$$
$$\mathcal{B}_{ij}^{(0)} = 0$$

This implies *l*=2 terms $\mathcal{E}_{2\pm 0}^{(0)} = \sqrt{\frac{\pi}{5}}(\mathcal{E}_{xx} - \mathcal{E}_{yy})$ $\mathcal{E}_{20}^{(0)} = \sqrt{\frac{6\pi}{5}}\mathcal{E}_{zz}$

2- The geodesic structure implies that $\mathbf{n}(\mathbf{n}^0, \hat{v}) \neq \mathbf{n}^0$

One can demonstrate that this sources B-modes for all multipole.

What to expect with Euclid



(i) Visible imaging (ii) NIR photometry (iii) NIR spectroscopy.
15,000 square degrees
100 million redshifts, 2 billion images
Median z~1

I will provide:
 - P(k,z) on 15,000 square degrees, 70,000,000 galaxy redshift with 0.5<z<2.
 - weak lensing on 15,000 square degrees, 40 galaxy images per square arcmin with 0.5<z<3.

The error bars on the B-modes should be divided by (10-40) compared to CFHTLS.

Linear regime typically for scales larger than 1 deg. And Euclid will probe scales up to ~40 deg.

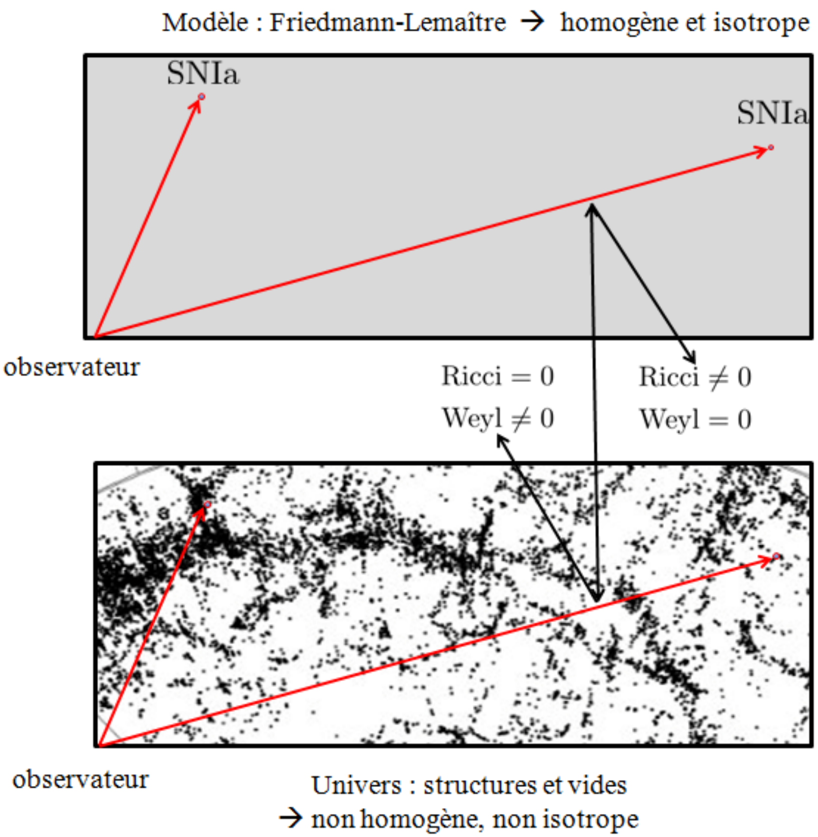
It will probe large scales for which astrophysical effects leading to B-modes are important.

Testing the Copernican principle

Fluid approximation

Extending CP: how homogeneous is homogeneous?

[Clarkson, Ellis, Maartens, Umeh, JPU, arXiv:1109.2484]



$$R_{\mu\nu} \neq 0$$
$$C_{\mu\nu\alpha\beta} = 0$$

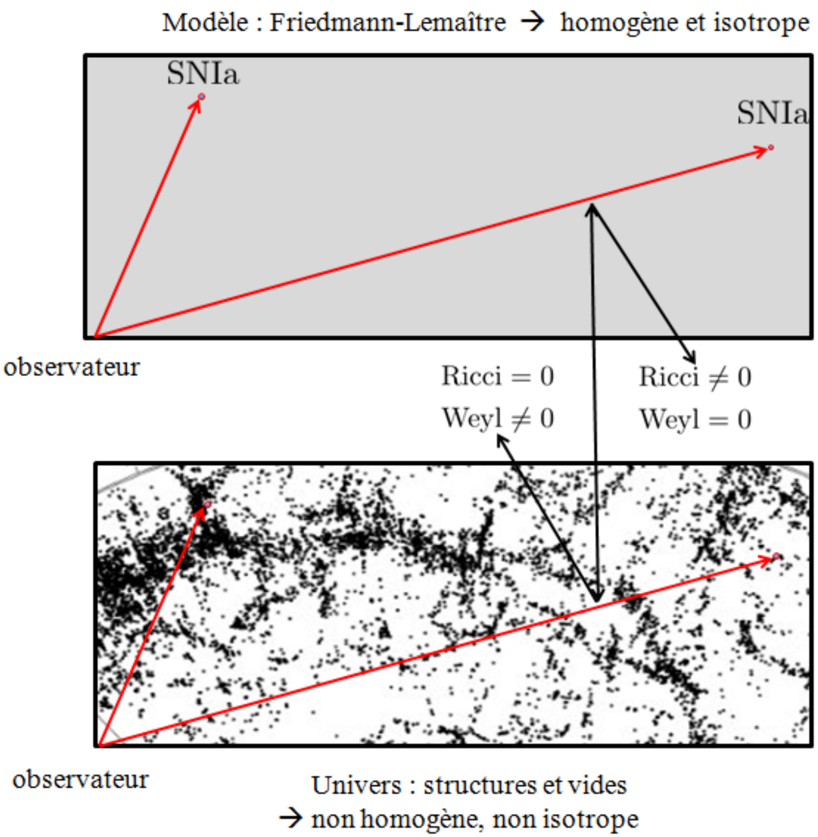
$$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\mathcal{D}^a_b = \mathcal{R}^a_c \mathcal{D}^c_b$$

$$\mathcal{R}_{ab} = U^2 \left(\mathcal{R} I_{ab} + \mathcal{W}_{ab}\right)$$

$$R_{\mu\nu} = 0$$
$$C_{\mu\nu\alpha\beta} \neq 0$$

Extending CP: how homogeneous is homogeneous?

[Clarkson, Ellis, Maartens, Umeh, JPU, arXiv:1109.2484]



$$R_{\mu\nu} \neq 0$$
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$$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\mathcal{D}^a_b = \mathcal{R}^a_c\mathcal{D}^c_b$$

$$\mathcal{R}_{ab} = U^2\left(\mathcal{R}I_{ab} + \mathcal{W}_{ab}\right)$$

$$R_{\mu\nu} = 0$$
$$C_{\mu\nu\alpha\beta} \neq 0$$

SN: - beam is very thin: 1 AU @ z=1 corresponds to 10^{-7} arcsec. Typically smaller than the distance between any massive object.

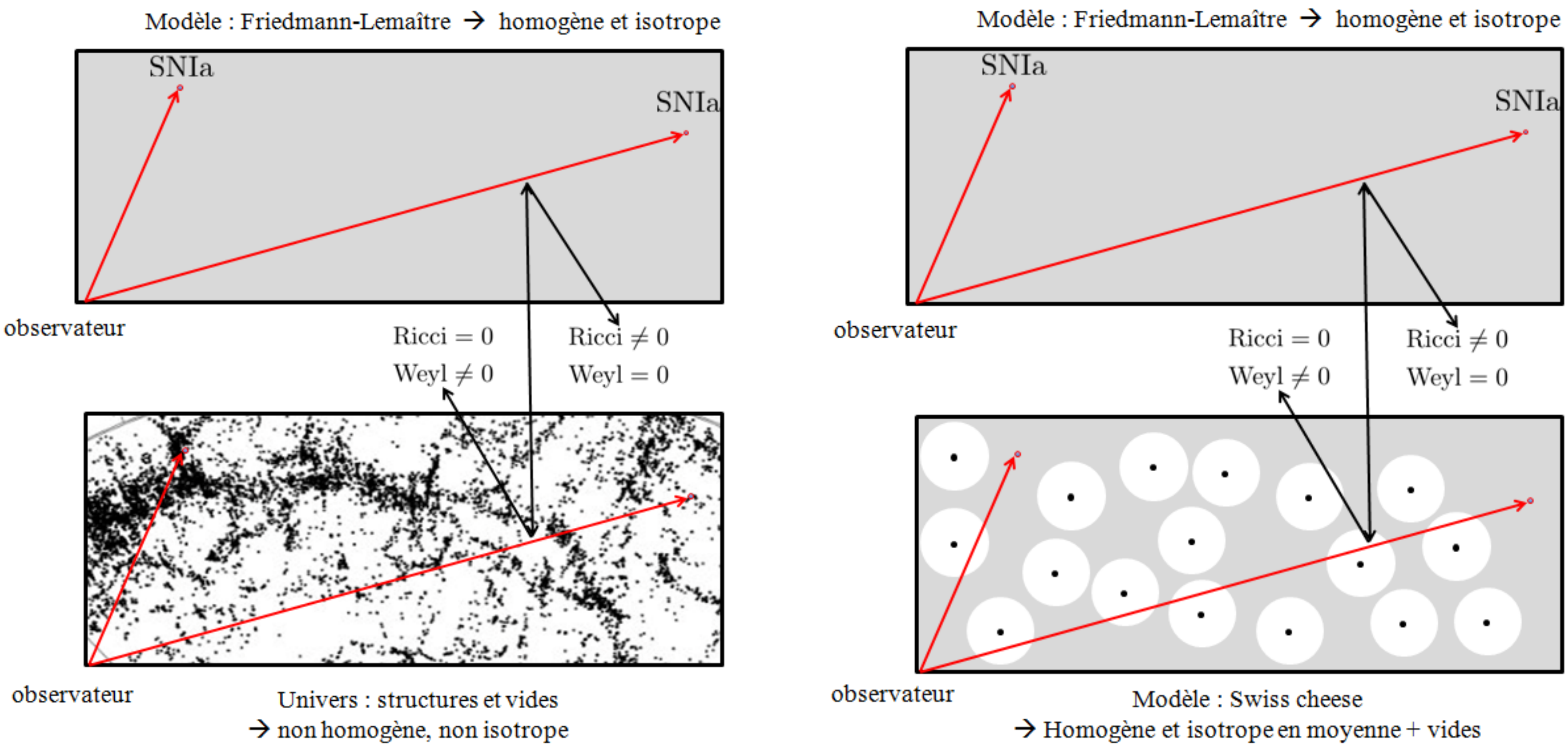
- beam propagates mostly in underdense regions [Zel' dovich, Dyer, Roeder]
- distribution of magnification
- scatter of the m-z diagram allow to constrain the smoothness of the matter distribution.

systematic shift + scatter

On which scale are we allowed to use the fluid approximation?
Different from the backreaction approach.

Extending CP: how homogeneous is homogeneous?

Small scales (e.g. ~AU) cannot be reched by numerical simulations.
Find some approximate exact solutions

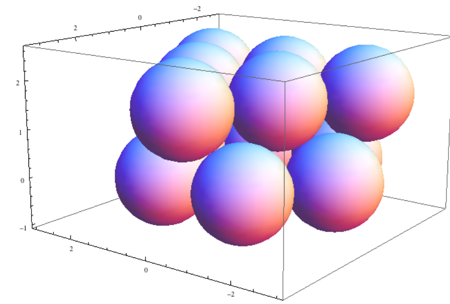


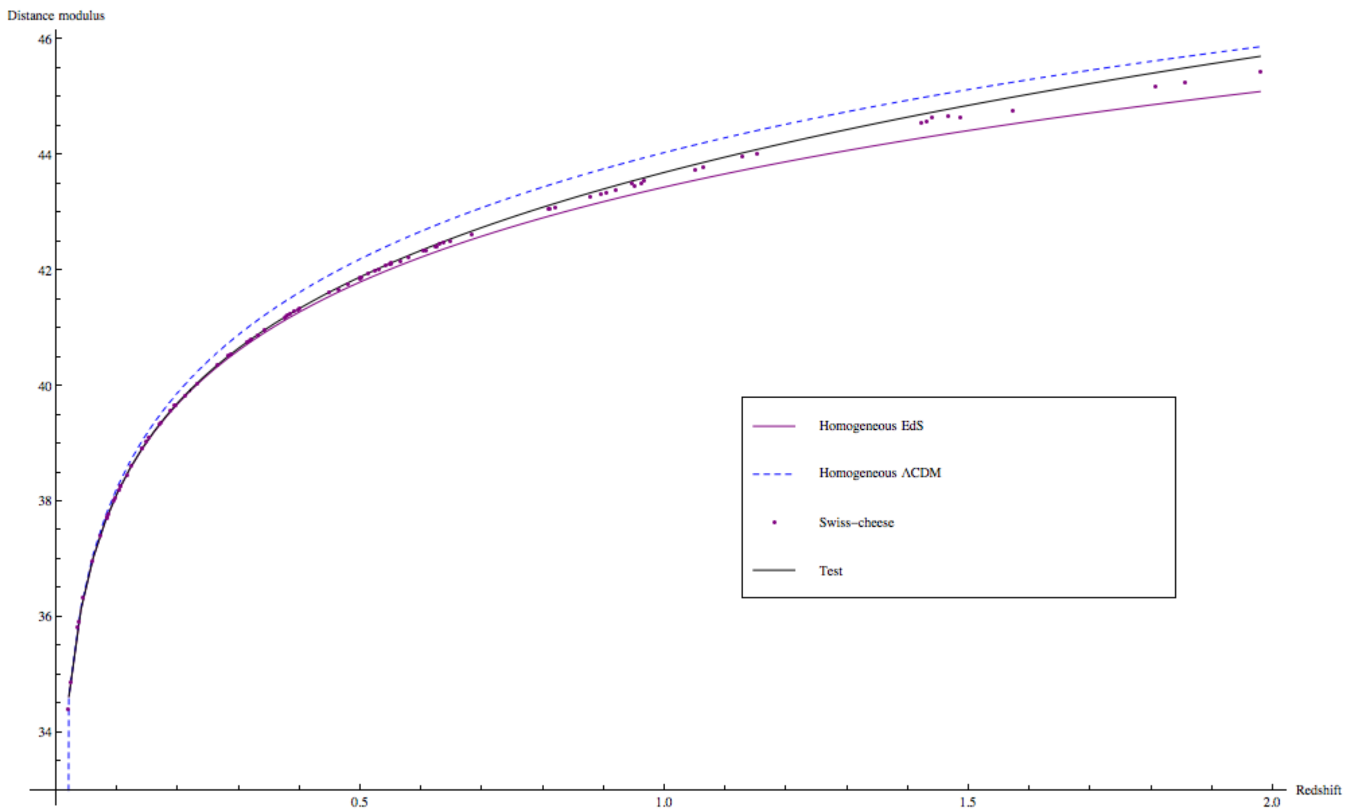
[Fleury, Dupuy, JPU, in preparation]

Extending CP: how homogeneous is homogeneous?

M	Ω_Λ^{SC}	$(\Omega_\Lambda^{FL})_{best}$	$(\Omega_M^{FL})_{best}$	$(\Omega_K^{FL})_{best}$	$\Omega_\Lambda^{SC} - (\Omega_\Lambda^{FL})_{best}$
1	0.5	0.43	0.26	0.31	0.07
1	0.6	0.56	0.21	0.23	0.04
10000	0.4	0.23	0.33	0.44	0.17
10000	0.5	0.36	0.28	0.36	0.14
10000	0.6	0.5	0.23	0.27	0.10

74% matter in point masses
26% matter in homoheneous fluid





[Fleury, Dupuy, JPU, in preparation]

Conclusions

We have demonstrated that a violation of local spatial isotropy implies the existence of non-vanishing B-modes for the cosmic shear.

This is based on a new formalism providing a multipolar hierarchy for weak-lensing independently of the choice of a particular background spacetime.

It recovers the standard lore when the background is FL.

For Bianchi I universe, we have shown that B-modes will be non-vanishing on all multipoles.

The exact amplitude of the expected B/E-modes requires to work out the complete perturbation theory. This is undergoing.

Hopefully, it will allow to set strong constraints from the Euclid observations.

This is important for some dark energy model (with anisotropic stress) and also for backreaction models.