SOME MUTANT FORMS OF QUANTUM MECHANICS

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ln:

- arXiv:1205.4800
- arXiv:1206.0064
- arXiv:1208.5189
- arXiv:1208.5544
- more on the way

Inspired by:

- Y. Nambu, "Field Theory of Galois Fields" (1987)
- D. R. Finkelstein, "Quantum relativity" (1996)
- B. Schumacher and M. D. Westmoreland, *"Modal Quantum Theory,"* arXiv:1010.2929

We don't really "understand" Quantum Mechanics, do we?

- For those who are not shocked when they first come across quantum theory cannot possibly have understood it – Niels Bohr (quoted in Heisenberg, 1971).
- I think I can safely say that nobody understands quantum mechanics Richard Feynman (1965).
- It is often stated that of all the theories proposed in this century, the silliest is quantum theory. In fact, some say that the only thing that quantum theory has going for it is that it is unquestionably correct – Michio Kaku (1995).

What is Quantum Mechanics?

- We have prescriptions for the "quantization" of a physical system, and the "interpretation" of the resulting theory, but beyond that do we really understand what it means?
- QM predicts the probabilities of possible outcomes of a "measurement," but what is "measurement" anyway?
- What is probability? (Especially if something only happens once.)

What is Quantum Mechanics?

- Where should we draw the line between the observer and the observed?
- How does deterministic CM emerge from probabilistic QM?
- Neither QFT nor String Theory challenge the basic tenets of QM.
- More phenomenology will not tell us anything except what we already know – QM works!

Why Quantum Mechanics?

- Is QM inevitable?
- Can it be derived from a few basic physical principles that everyone can agree on, a la Relativity?
- Can the mathematical axioms of QM be derived from those physical principles?
- Will understanding the connection between the physics and the math teach us how to quantize gravity? (And perhaps explain dark energy?)

Geneticist's Approach to Quantum Mechanics:

- How are the Mathematical Genotypes of QM related to its Physical Phenotypes?
 - Genotypes: complex vector space, inner product, normalizable states, hermitian operators, etc.
 - Phenotypes: interference, probabilities that do not allow hidden variable mimics, violation of Bell's inequalities, etc.
- Which "gene" is responsible for which characteristic?
 ⇒Try to create "mutant" versions of QM in which some of the "genes" are knocked out.

Bell's Inequality:



Figure from the Wikipedia

Clauser-Horne-Shimony-Holt (CHSH):

$$\begin{split} \left| \left\langle AB \right\rangle + \left\langle Ab \right\rangle + \left\langle aB \right\rangle - \left\langle ab \right\rangle \right| &\leq 2 \qquad \text{(classical, Bell)} \\ &\leq 2\sqrt{2} \qquad \text{(QM, Cirel'son)} \\ &\leq 4 \qquad \text{(Absolute bound)} \end{split}$$

The Mutation: $C \rightarrow GF(q)$

 Replace vector space over the complex number field C with that over the finite Galois field GF(q), q=pⁿ, p=prime :

$$\begin{split} GF(2) &= Z_2 = \{\underline{0}, \underline{1}\} \\ GF(3) &= Z_3 = \{\underline{0}, \underline{1}, \underline{2}\} \\ GF(4) &= Z_2[\underline{\omega}] = \{\underline{0}, \underline{1}, \underline{\omega}, \underline{\omega}^2\}, \quad \underline{\omega}^2 + \underline{\omega} + \underline{1} = \underline{0} \\ GF(5) &= Z_5 = \{\underline{0}, \underline{1}, \underline{2}, \underline{3}, \underline{4}\} \\ GF(7) &= Z_7 = \{\underline{0}, \underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6}\} \\ GF(8) &= Z_2[\underline{\varepsilon}] = \{\underline{0}, \underline{1}, \underline{\varepsilon}, \underline{1} + \underline{\varepsilon}, \underline{\varepsilon}^2, \underline{1} + \underline{\varepsilon}^2, \underline{\varepsilon} + \underline{\varepsilon}^2, \underline{1} + \underline{\varepsilon} + \underline{\varepsilon}^2\}, \quad \underline{\varepsilon}^3 + \underline{\varepsilon} + \underline{1} = \underline{0} \\ GF(9) &= Z_3[\underline{i}] = \{\underline{0}, \underline{1}, \underline{2}, \underline{i}, \underline{2i}, \underline{1} + \underline{i}, \underline{1} + \underline{2i}, \underline{2} + \underline{i}, \underline{2} + \underline{2i}\}, \quad \underline{i}^2 + \underline{1} = \underline{0} \\ \end{split}$$

 No inner product, no normalizable states, no symmetric/ hermitian operators!

Inner Product :

- The inner product is a map $V \times V \rightarrow K$ such that:
 - Bilinear: $(|u\rangle, a|v\rangle + b|w\rangle) = a(|u\rangle, |v\rangle) + b(|u\rangle, |w\rangle)$ $(a|u\rangle + b|v\rangle, |w\rangle) = a(|u\rangle, |w\rangle) + b(|v\rangle, |w\rangle)$
 - Symmetric:

$$(|u\rangle,|v\rangle) = (|v\rangle,|u\rangle)$$

Positive Definite:

 $(|u\rangle, |u\rangle) \ge 0$, and $(|u\rangle, |u\rangle) = 0$ if and only if $|u\rangle = 0$

• Not to be confused with: $\langle a | u \rangle$, $\langle a | \in V^*$, $| u \rangle \in V$

Mutation 1: States, Outcomes, Observables, and Probabilities:

States :
$$|\psi\rangle \in V = K^N$$
, $K = GF(q)$
Outcomes : $\langle x | \in V^*$, $\langle x | \psi \rangle \in K$

Observables : choice of basis of V^*

Probability:
$$P(x;\psi) = \frac{|\langle x|\psi\rangle|^2}{\sum_{y} |\langle y|\psi\rangle|^2}$$

These relations are identical to canonical QM.

Absolute Value Function:

Define absolute values as:

$$\left| \underline{k} \right| = \begin{cases} 0 & \text{if } \underline{k} = \underline{0} \\ 1 & \text{if } \underline{k} \neq \underline{0} \end{cases} \text{ so that } \left| \underline{k} \underline{l} \right| = \left| \underline{k} \right| \left| \underline{l} \right|$$

- All non-zero elements of the Galois field are mapped to 1.
 - \rightarrow They are all "phases."
 - $\rightarrow |\psi\rangle$ and $\underline{k}|\psi\rangle$ represent the same state. ($\underline{k} \neq \underline{0}$)
 - \rightarrow State space has projective geometry *PG(N-1,q)*:

$$PG(N-1,q) = \frac{K^N \setminus \{\vec{0}\}}{K \setminus \{0\}}$$

Projective Geometry & Projective Group:

$$PG(N-1,q) = \frac{GL(N,q)}{AGL(N,q) \times Z(N,q)}$$

$$PGL(N,q) = \frac{GL(N,q)}{Z(N,q)}$$

cf. *K*=*C* case:

$$CP^{N-1} = \frac{C^N \setminus \{\vec{0}\}}{C \setminus \{0\}} = \frac{U(N)}{U(N-1) \times U(1)}$$
$$SU(N) = \frac{U(N)}{U(1)}$$

Example: *N*=2, *K*=*GF*(2)

Only three states:

$$|a\rangle = \begin{bmatrix} 1\\ 0\\ 0 \end{bmatrix}, |b\rangle = \begin{bmatrix} 0\\ 1\\ 1 \end{bmatrix}, |c\rangle = \begin{bmatrix} 1\\ 1\\ 1 \end{bmatrix}$$

Only three outcomes:

$$\left\langle \overline{a} \right| = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad \left\langle \overline{b} \right| = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \left\langle \overline{c} \right| = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Brackets:

$$\left\langle \overline{r} \middle| s \right\rangle = \begin{cases} \underline{0} & \text{if } r = s \\ \underline{1} & \text{if } r \neq s \end{cases} \longrightarrow \left| \left\langle \overline{r} \middle| s \right\rangle \right| = 1 - \delta_{rs}$$

Observables:

Six observables:

$$A_{ab} = \left\{ \left\langle \overline{a} \right|, \left\langle \overline{b} \right| \right\}, \quad A_{bc} = \left\{ \left\langle \overline{b} \right|, \left\langle \overline{c} \right| \right\}, \quad A_{ca} = \left\{ \left\langle \overline{c} \right|, \left\langle \overline{a} \right| \right\}, \\ A_{ba} = \left\{ \left\langle \overline{b} \right|, \left\langle \overline{a} \right| \right\}, \quad A_{cb} = \left\{ \left\langle \overline{c} \right|, \left\langle \overline{b} \right| \right\}, \quad A_{ac} = \left\{ \left\langle \overline{a} \right|, \left\langle \overline{c} \right| \right\}.$$

• Spins:

$$A_{rs} = \left\{ \begin{array}{c} \left\langle \overline{r} \right| & , \quad \left\langle \overline{s} \right| \end{array} \right\} \qquad A_{rs} = -A_{sr}$$

$$\uparrow \qquad \uparrow \qquad \uparrow \qquad +1 \qquad -1$$

• Three "spin directions":

$$Z = A_{ab} = \left\{ \left\langle \overline{a} \right|, \left\langle \overline{b} \right| \right\}, \quad X = A_{bc} = \left\{ \left\langle \overline{b} \right|, \left\langle \overline{c} \right| \right\}, \quad Y = A_{ca} = \left\{ \left\langle \overline{c} \right|, \left\langle \overline{a} \right| \right\}$$

Rotations: $PGL(2,2)=S_3$



Probabilities:

$$P(A_{rs} = +1;t) = \frac{\left|\left\langle \bar{r} \right| t\right\rangle\right|^{2}}{\left|\left\langle \bar{r} \right| t\right\rangle\right|^{2} + \left|\left\langle \bar{s} \right| t\right\rangle\right|^{2}} = \frac{1 - \delta_{rt}}{2 - (\delta_{rt} + \delta_{st})} = \begin{cases} 0 & \text{if } t = r \\ \frac{1}{2} & \text{if } t \neq r, s \\ 1 & \text{if } t = s \end{cases}$$
$$P(A_{rs} = -1;t) = \frac{\left|\left\langle \bar{s} \right| t\right\rangle\right|^{2}}{\left|\left\langle \bar{r} \right| t\right\rangle\right|^{2} + \left|\left\langle \bar{s} \right| t\right\rangle\right|^{2}} = \frac{1 - \delta_{st}}{2 - (\delta_{rt} + \delta_{st})} = \begin{cases} 1 & \text{if } t = r \\ \frac{1}{2} & \text{if } t \neq r, s \\ 0 & \text{if } t = s \end{cases}$$

Two-Particle Spin Correlations:

- Tensor two vector spaces together.
- The space has 2^4 -1=15 states = 9 product + 6 entangled.

$$|S\rangle = |a\rangle \otimes |a\rangle + |b\rangle \otimes |b\rangle + |c\rangle \otimes |c\rangle$$
$$|(ab)\rangle = |a\rangle \otimes |b\rangle + |b\rangle \otimes |a\rangle + |c\rangle \otimes |c\rangle$$
$$|(bc)\rangle = |a\rangle \otimes |a\rangle + |b\rangle \otimes |c\rangle + |c\rangle \otimes |b\rangle$$
$$|(ca)\rangle = |a\rangle \otimes |c\rangle + |b\rangle \otimes |b\rangle + |c\rangle \otimes |a\rangle$$
$$(abc)\rangle = |a\rangle \otimes |b\rangle + |b\rangle \otimes |c\rangle + |c\rangle \otimes |a\rangle$$
$$(acb)\rangle = |a\rangle \otimes |c\rangle + |c\rangle \otimes |b\rangle + |b\rangle \otimes |a\rangle$$

Product observables:

$$A_{rs}A_{tu} = \left\{ \begin{array}{c} \left\langle \bar{r} \right| \otimes \left\langle \bar{t} \right|, \quad \left\langle \bar{r} \right| \otimes \left\langle \bar{u} \right|, \quad \left\langle \bar{s} \right| \otimes \left\langle \bar{t} \right|, \quad \left\langle \bar{s} \right| \otimes \left\langle \bar{u} \right| \end{array} \right\} \right\}$$

Probabilities and Correlations:

For the state $|S\rangle$:

	++	+ -	-+		Corr.
XX,YY,ZZ	0	$\frac{1}{2}$	$\frac{1}{2}$	0	-1
XY,YZ,ZX	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$
XZ, ZY, YX	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Classical Implications:



No hidden variable mimic is possible.

cf:

Greenberger, Horne, Zeilinger, arXiv:0712.0921v1, Greenberger, Horne, Shimony, Zeilinger, Am. J. Phys. 58, 1131 (1990), Hardy, Phys. Rev. Lett. 71, 1665 (1993).

Clauser-Horne-Shimony-Holt Inequality:

• It is straightforward to show that:

$$\begin{split} \left| \left\langle AB \right\rangle + \left\langle Ab \right\rangle + \left\langle aB \right\rangle - \left\langle ab \right\rangle \right| &\leq 2 \qquad \text{(classical, Bell)} \\ &\leq 2\sqrt{2} \qquad \text{(QM, Cirel'son)} \\ &\leq 2 \qquad \text{(Galois Field QM)} \end{split}$$

 The Galois Field QM bound of 2 applies to all GF(q), not just for the case q=2.

Other *q*:

q	# of states	# of spin directions	PGL(2,q)
2	3	6	S_3
3	4	12	S_4
4	5	20	A_5
5	6	30	S_5





q=4, PGL(2,4)=A₅



q=5, PGL(2,5)=S₅



Mutation 2 : Biorthogonal Quantum Mechanics

Biorthogonal System :

Basis of
$$V = K^N$$
: $\{|1\rangle, |2\rangle, \dots, |N\rangle\}$
Basis of V^* : $\{\langle 1|, \langle 2|, \dots, \langle N|\}\}$ $\{\langle r|s\rangle = \underline{\delta}_{rs}$

Observables: $\hat{A} = \sum_{k=1}^{N} \underline{\alpha}_{k} |k\rangle \langle k|, \quad \underline{\alpha}_{k} \in K$

Expectation Value: $\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle \in K \longrightarrow R$

where $|\psi\rangle$ and $\langle\psi|$ are members of some biorthogonal system and φ is a product preserving map from *K* to *R*.

Example : *N*=2, *K*=*GF*(9)

The only vectors and dual vectors that are members of biorthogonal systems are the following and their scalar multiples:

$$\begin{aligned} |a\rangle = \begin{bmatrix} 1\\ 0\\ \end{bmatrix}, \quad |b\rangle = \begin{bmatrix} 0\\ 1\\ \end{bmatrix}, \quad |c\rangle = \begin{bmatrix} 1\\ 1\\ \end{bmatrix}, \quad |d\rangle = \begin{bmatrix} 1\\ -1\\ \end{bmatrix}, \quad |e\rangle = \begin{bmatrix} 1\\ \frac{1}{i}\\ \end{bmatrix}, \quad |f\rangle = \begin{bmatrix} 1\\ -\frac{1}{i}\\ \end{bmatrix} \\ \langle a| = \begin{bmatrix} 0\\ 1\\ \end{bmatrix}, \quad \langle b| = \begin{bmatrix} 1\\ 0\\ \end{bmatrix}, \quad \langle c| = \begin{bmatrix} -1\\ -1\\ \end{bmatrix}, \quad \langle c| = \begin{bmatrix} -1\\ -1\\ \end{bmatrix} \\ \langle d| = \begin{bmatrix} -1\\ 1\\ \end{bmatrix}, \quad \langle e| = \begin{bmatrix} -1\\ \frac{1}{i}\\ \end{bmatrix}, \quad \langle f| = \begin{bmatrix} -1\\ -\frac{1}{i}\\ \end{bmatrix} \end{aligned}$$

Biorthogonal Systems :

 $\left\{ \left\{ \left\langle a \right|, \left\langle b \right| \right\}, \left\{ \left|a \right\rangle, \left|b \right\rangle \right\} \right\}, \quad \left\{ \left\{ \left\langle c \right|, \left\langle d \right| \right\}, \left\{ \left|c \right\rangle, \left|d \right\rangle \right\} \right\}, \quad \left\{ \left\{ \left\langle e \right|, \left\langle f \right| \right\}, \left\{ \left|e \right\rangle, \left|f \right\rangle \right\} \right\} \right\}$

Observables :

$$\begin{split} \underline{1}|a\rangle\langle a|-\underline{1}|b\rangle\langle b| &= \begin{bmatrix} \underline{1} & \underline{0} \\ \underline{0} & -\underline{1} \end{bmatrix} = \hat{\underline{\sigma}}_{3} \\ \underline{1}|c\rangle\langle c|-\underline{1}|d\rangle\langle d| &= \begin{bmatrix} \underline{0} & \underline{1} \\ \underline{1} & \underline{0} \end{bmatrix} = \hat{\underline{\sigma}}_{1} \\ \underline{1}|e\rangle\langle e|-\underline{1}|f\rangle\langle f| &= \begin{bmatrix} \underline{0} & -\underline{i} \\ \underline{i} & \underline{0} \end{bmatrix} = \hat{\underline{\sigma}}_{2} \end{split}$$

Expectation Values :

 $\begin{aligned} \left\langle a \left| \underline{\hat{\sigma}}_{1} \right| a \right\rangle &= \underline{0} \quad \rightarrow \quad 0 \\ \left\langle b \left| \underline{\hat{\sigma}}_{1} \right| b \right\rangle &= 0 \quad \rightarrow \quad 0 \\ \left\langle c \left| \underline{\hat{\sigma}}_{1} \right| c \right\rangle &= \underline{1} \quad \rightarrow \quad 1 \\ \left\langle d \left| \underline{\hat{\sigma}}_{1} \right| d \right\rangle &= -\underline{1} \quad \rightarrow \quad -1 \\ \left\langle e \left| \underline{\hat{\sigma}}_{1} \right| e \right\rangle &= \underline{0} \quad \rightarrow \quad 0 \\ \left\langle f \left| \underline{\hat{\sigma}}_{1} \right| f \right\rangle &= \underline{0} \quad \rightarrow \quad 0 \end{aligned}$

$$\begin{aligned} &\langle a | \hat{\underline{\sigma}}_2 | a \rangle = \underline{0} \quad \rightarrow \quad 0 \\ &\langle b | \hat{\underline{\sigma}}_2 | b \rangle = \underline{0} \quad \rightarrow \quad 0 \\ &\langle c | \hat{\underline{\sigma}}_2 | c \rangle = \underline{0} \quad \rightarrow \quad 0 \\ &\langle d | \hat{\underline{\sigma}}_2 | d \rangle = \underline{0} \quad \rightarrow \quad 0 \\ &\langle e | \hat{\underline{\sigma}}_2 | e \rangle = \underline{1} \quad \rightarrow \quad 1 \end{aligned}$$

- $\langle f | \hat{\underline{\sigma}}_2 | f \rangle = -\underline{1} \rightarrow -1 \quad \langle f | \hat{\underline{\sigma}}_3 | f \rangle = \underline{0} \rightarrow 0$
- $\langle a | \hat{\underline{\sigma}}_{3} | a \rangle = \underline{1} \rightarrow 1$ $\langle b | \hat{\underline{\sigma}}_{3} | b \rangle = -\underline{1} \rightarrow -1$ $\langle c | \hat{\underline{\sigma}}_{3} | c \rangle = \underline{0} \rightarrow 0$ $\langle d | \hat{\underline{\sigma}}_{3} | d \rangle = \underline{0} \rightarrow 0$ $\langle e | \hat{\underline{\sigma}}_{3} | e \rangle = \underline{0} \rightarrow 0$ $\langle f | \hat{\underline{\sigma}}_{3} | f \rangle = \underline{0} \rightarrow 0$

Two-Spin System :

$$\hat{\underline{\sigma}}_{1} \otimes \hat{\underline{\sigma}}_{1} = \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{0} \\ \underline{0} & \underline{1} & \underline{0} & \underline{0} \\ \underline{1} & \underline{0} & \underline{0} & \underline{0} \end{bmatrix} \qquad \hat{\underline{\sigma}}_{1} \otimes \hat{\underline{\sigma}}_{3} = \begin{bmatrix} \underline{0} & \underline{0} & \underline{1} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & -\underline{1} \\ \underline{1} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & -\underline{1} & \underline{0} & \underline{0} \end{bmatrix}$$
$$\hat{\underline{\sigma}}_{3} \otimes \hat{\underline{\sigma}}_{3} = \begin{bmatrix} \underline{0} & \underline{1} & \underline{0} & \underline{0} \\ \underline{0} & -\underline{1} & \underline{0} & \underline{0} \\ \underline{0} & -\underline{1} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & -\underline{1} & \underline{0} \end{bmatrix} \qquad \hat{\underline{\sigma}}_{3} \otimes \hat{\underline{\sigma}}_{3} = \begin{bmatrix} \underline{1} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & -\underline{1} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & -\underline{1} & \underline{0} \\ \underline{0} & \underline{0} & -\underline{1} & \underline{0} \end{bmatrix}$$
etc.

Correlations :

Consider the two-spin state :

$$|U\rangle = \begin{bmatrix} 1\\ 0\\ 1\\ 1+i \end{bmatrix}, \quad \langle U| = \begin{bmatrix} 1 & 0 & 1 & 1-i \end{bmatrix} \qquad \langle ++|\hat{\underline{\sigma}}_3 \otimes \hat{\underline{\sigma}}_3|U\rangle = 1 \\ \langle +-|\hat{\underline{\sigma}}_3 \otimes \hat{\underline{\sigma}}_3|U\rangle = 0 \\ \langle -+|\hat{\underline{\sigma}}_3 \otimes \hat{\underline{\sigma}}_3|U\rangle = 1 \\ \langle --|\hat{\underline{\sigma}}_3 \otimes \hat{\underline{\sigma}}_3|U\rangle = 1+i \end{bmatrix}$$

$$\langle U | \hat{\underline{\sigma}}_1 \otimes \hat{\underline{\sigma}}_1 | U \rangle = \langle U | \hat{\underline{\sigma}}_1 \otimes \hat{\underline{\sigma}}_3 | U \rangle = \langle U | \hat{\underline{\sigma}}_3 \otimes \hat{\underline{\sigma}}_3 | U \rangle = -\underline{1} \quad \xrightarrow{\varphi} \quad -1$$

$$\langle U | \hat{\underline{\sigma}}_3 \otimes \hat{\underline{\sigma}}_1 | U \rangle = \underline{1} \quad \xrightarrow{\varphi} \quad 1$$

$$\left|\varphi\left(\left\langle\underline{\hat{\sigma}}_{1}\otimes\underline{\hat{\sigma}}_{3}\right\rangle\right)+\varphi\left(\left\langle\underline{\hat{\sigma}}_{3}\otimes\underline{\hat{\sigma}}_{3}\right\rangle\right)+\varphi\left(\left\langle\underline{\hat{\sigma}}_{1}\otimes\underline{\hat{\sigma}}_{1}\right\rangle\right)-\varphi\left(\left\langle\underline{\hat{\sigma}}_{3}\otimes\underline{\hat{\sigma}}_{1}\right\rangle\right)\right|=4$$

The CHSH bound for this mutant is the super-quantum 4!

Probabilities?

$$-1 = \varphi \left(\left\langle U \middle| \underline{\hat{\sigma}}_{3} \otimes \underline{\hat{\sigma}}_{3} \middle| U \right\rangle \right)$$
$$= P(++) + P(--) - P(+-) - P(-+)$$
$$\downarrow$$
$$0 = P(++) + P(--)$$
$$1 = P(+-) + P(-+)$$

Probabilities are indeterminate even though we have definite expectation values.

Conclusions:

- It is possible to construct QM-like theories on a vector space without an inner product, normalizable states, or symmetric/hermitian operators in more than one way.
- The probabilities predicted by mutant #1 cannot be reproduced in any hidden variable theory. Nevertheless, the CHSH bound of the mutant is the "classical" 2.
- The CHSH bound of mutant #2 is the super-quantum 4. The mutant predictions expectation values without going through probabilities. In fact, probabilities in this mutant are indeterminate.
- These mutants serve as existence proofs of theories that are QM-like, but their correlations are quite different.

Work in Progress : Quantum Mechanics on Banach Spaces

Banach Space:

- A complete normed vector space over *R* or *C*.
- Natural generalization of Hilbert spaces.
- Do not have inner products in general.
- Many different kinds with a variety of different properties.
- Could provide a rich supply of mutants.

Conjecture:

- CM can be understood as a mutation of QM.
- Quantum Gravity can be constructed as a mutant of QM.