Cosmic Acceleration and Modified Gravity

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Outline

- Falsifiablility of ΛCDM and Smooth Dark Energy

  Distance-Redshift vs Structure Growth

- Modified gravity

  Formal equivalence of dark energy and modified gravity

  Nonlinear screening mechanism to return GR locally

  Chameleon and Vainshtein signatures

- Toy model examples: $f(R)$, DGP, massive gravity

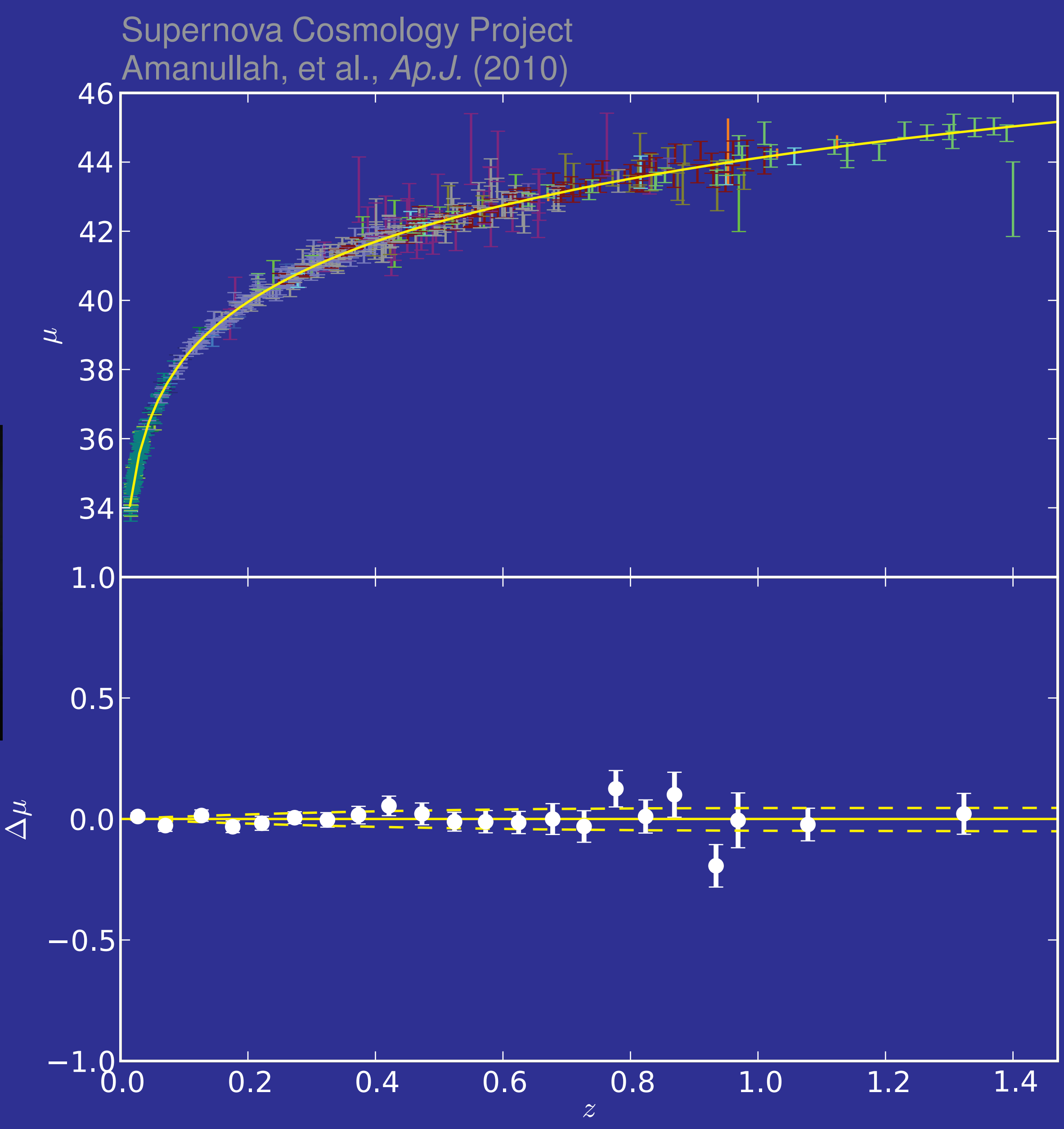
- Collaborators on the Market

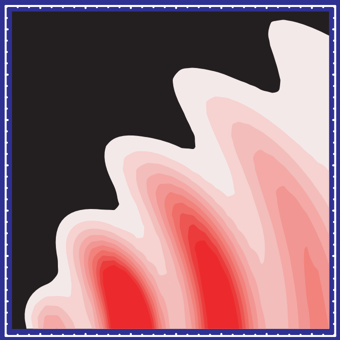
  Lucas Lombriser   Iggy Sawicki
  Amol Upadhye   Mark Wyman
  Ali Vanderveld

Cosmic Acceleration

- Geometric measures of distance redshift from SN, CMB, BAO

Standard(izable)
Candle
Supernovae
Luminosity v Flux

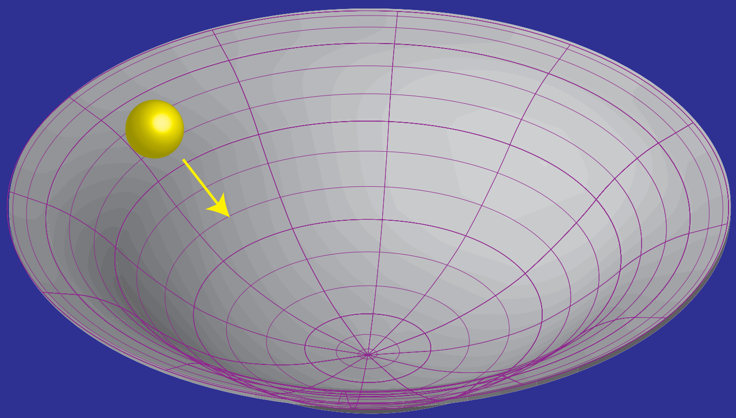




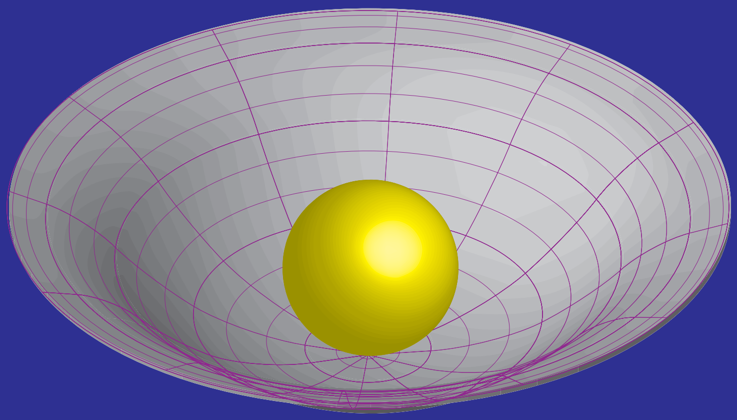
Standard Ruler
Sound Horizon
v CMB, BAO angular
and redshift separation

Mercury or Pluto?

- General relativity says Gravity = Geometry



- And Geometry = Matter-Energy



- Could the missing energy required by acceleration be an incomplete description of how matter determines geometry?

Two Potentials

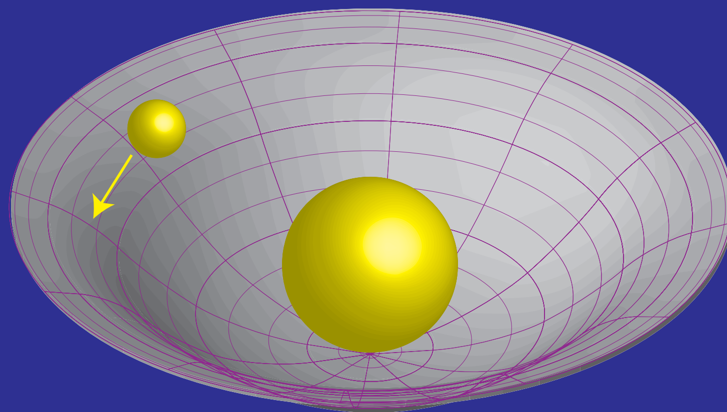
- Line Element

$$ds^2 \;=\; -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2$$

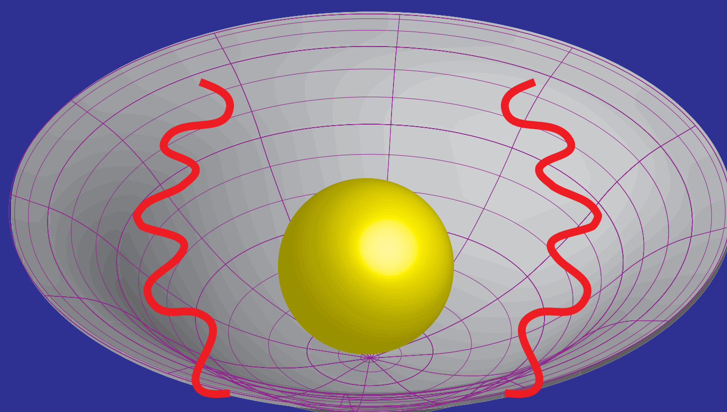
- Newtonian dynamical potential Ψ
- Space curvature potential Φ
- As in the parameterized post Newtonian approach, cosmological tests of the Φ/Ψ
- Space curvature per unit dynamical mass
- Given parameterized metric, matter falls on geodesics

Dynamical vs Lensing Mass

- Newtonian potential: $\Psi=\delta g_{00}/2g_{00}$ which non-relativistic particles feel



- Space curvature: $\Phi=\delta g_{ii}/2g_{ii}$ which also deflects photons



- Tests of space curvature per unit dynamical mass are the least model dependent

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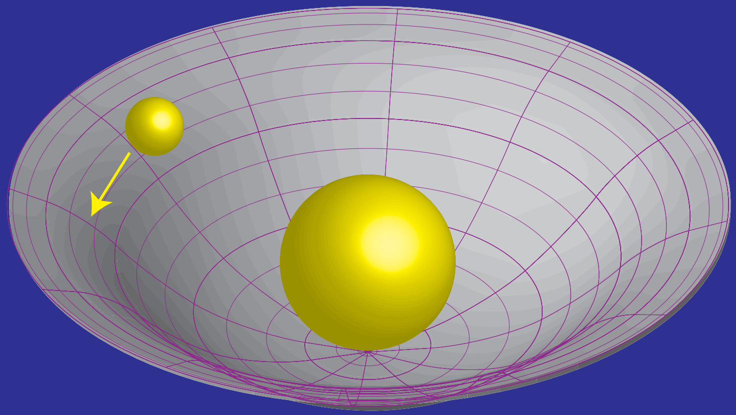

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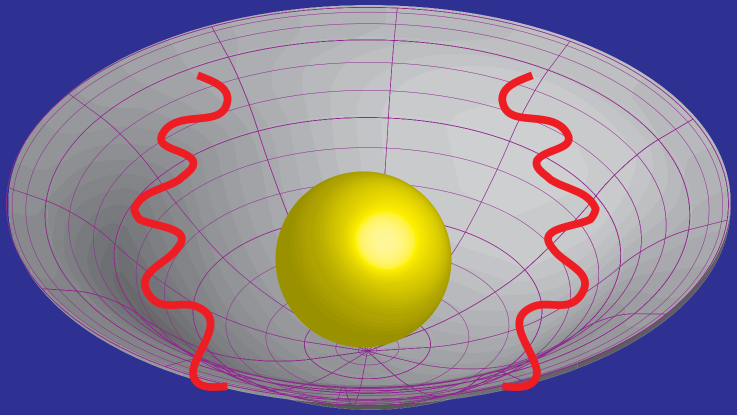


Solar system: sun
Cosmology: unknown
dark sector

- Tests of space curvature per unit dynamical mass are the least model dependent, but one suffices cosmologically combined with distance

Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content
- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy

$$
\begin{aligned}
F(g_{\mu\nu}) + G_{\mu\nu} &= 8\pi G T^{\rm M}_{\mu\nu} \qquad\qquad - F(g_{\mu\nu}) = 8\pi G T^{\rm DE}_{\mu\nu} \\
G_{\mu\nu} &= 8\pi G [T^{\rm M}_{\mu\nu} + T^{\rm DE}_{\mu\nu}]
\end{aligned}
$$

  and the Bianchi identity guarantees $\nabla^\mu T^{\rm DE}_{\mu\nu} = 0$
- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor
- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress

Smooth Dark Energy

- Scalar field dark energy has $\delta p = \delta\rho$ (in constant field gauge) – relativistic sound speed, no anisotropic stress
- Jeans stability implies that its energy density is spatially smooth compared with the matter below the sound horizon

$$
\begin{aligned}
ds^2 &= -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2 \\
\nabla^2\Phi &\propto \text{matter density fluctuation}
\end{aligned}
$$

- Anisotropic stress changes the amount of space curvature per unit dynamical mass: negligible for both matter and smooth dark energy

$$\nabla^2(\Phi+\Psi) \propto \text{anisotropic stress approx } 0$$

in contrast to modified gravity or force-law models

Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate
- Hence geometric measurements of the expansion rate predict the growth of structure
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations
- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)

Falsify Standard Model

- Anomalous events for highly predictive smooth dark energy model

standard model
for appearance of Japanese women

Falsify Standard Model

- Anomalous events for highly predictive smooth dark energy model

anomalous
event

Falsify Standard Model

- Anomalous events for highly predictive smooth dark energy model

Harajuku:
sampling bias, trials factor

Falsify Standard Model

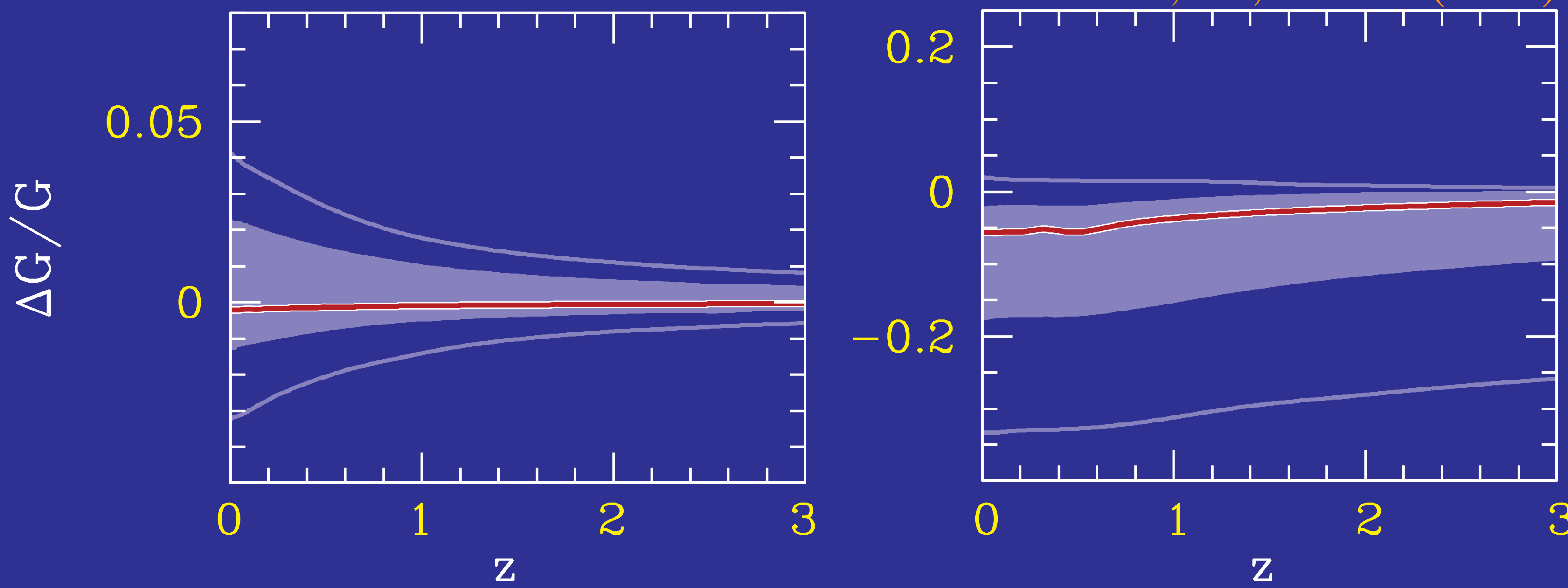
- Anomalous events for highly predictive smooth dark energy model

systematic error!
high redshift interloper

Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)



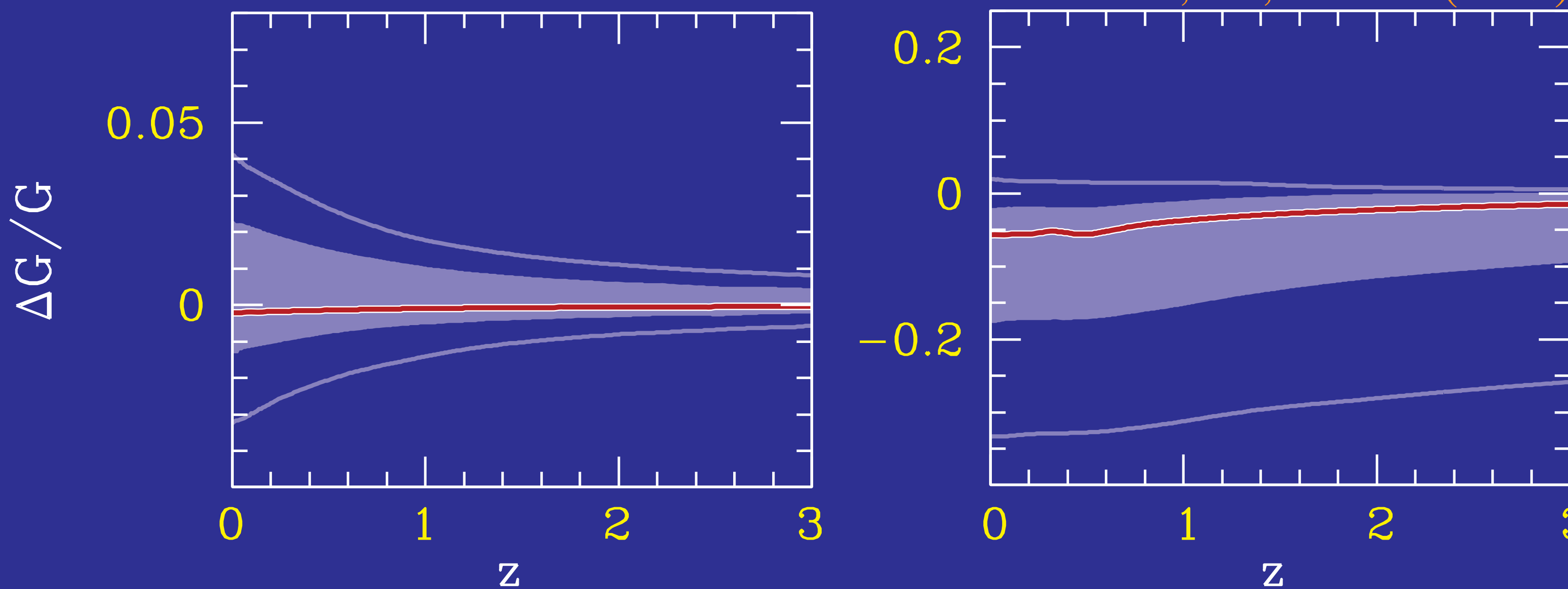
Cosmological Constant Quintessence

- Deviation significantly >2% rules out Λ with or without curvature
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in H_0]

Dynamical Tests of Acceleration

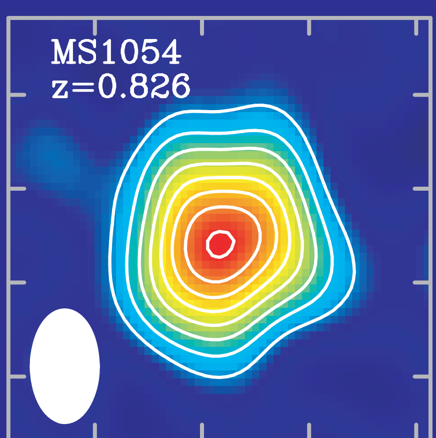
- Dark energy slows growth of structure in highly predictive way

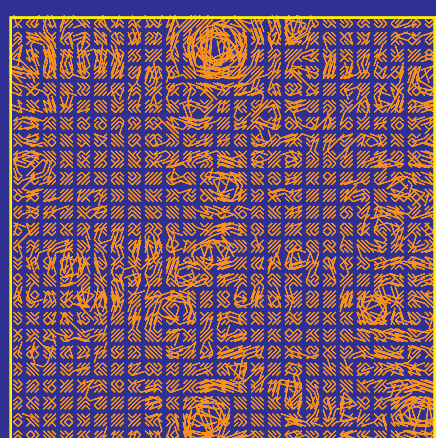
Mortonson, Hu, Huterer (2009)

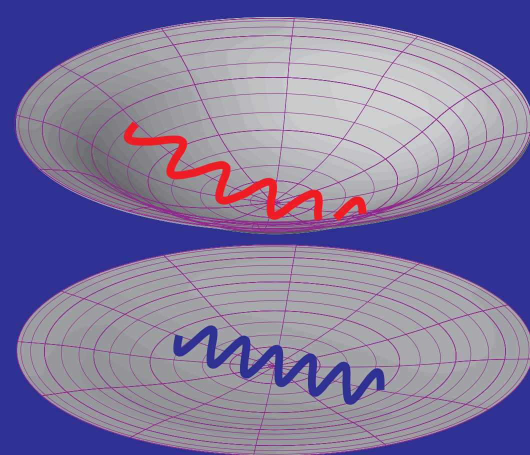


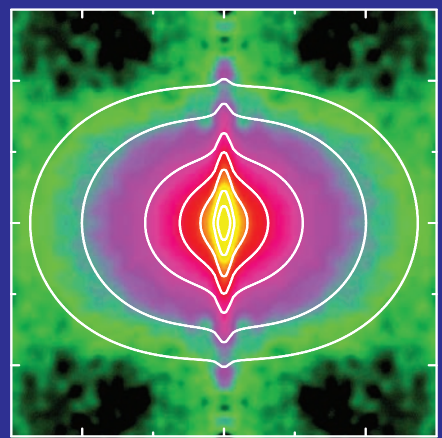
Cosmological Constant

Quintessence





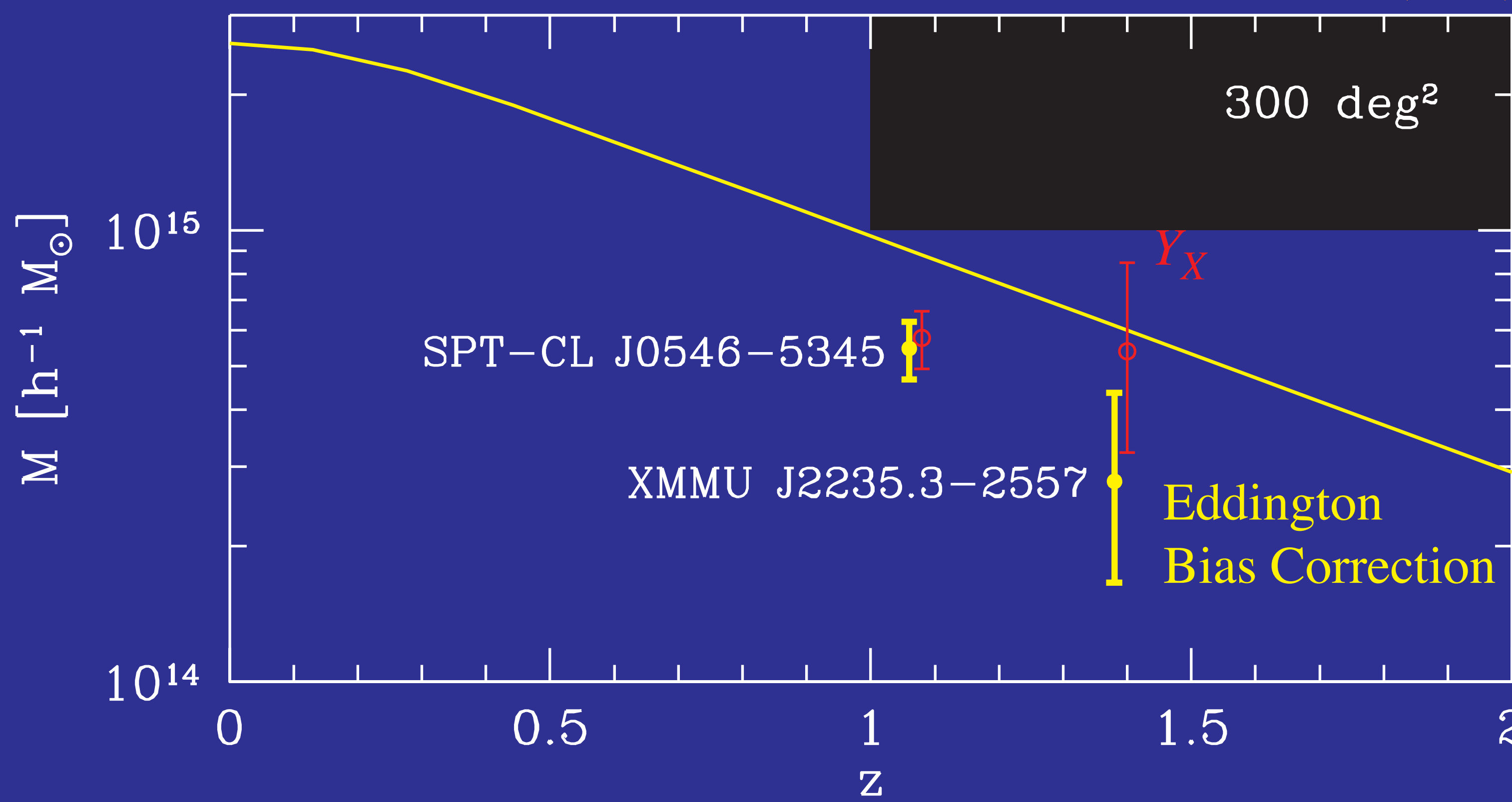




ΛCDM Falsified?

- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area $>M(>z)$

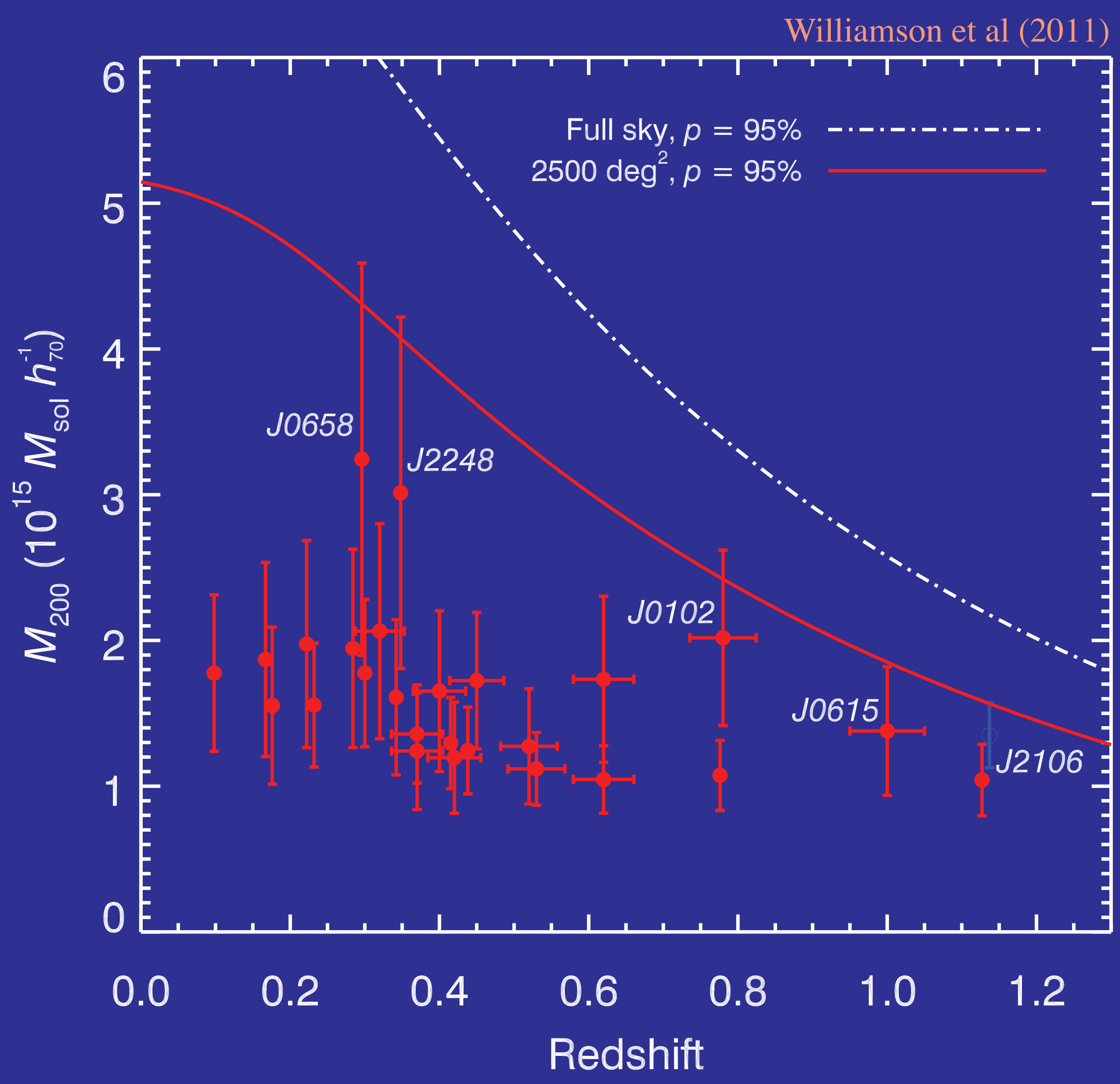
Mortonson, Hu, Huterer (2010)



Threshold Bias: Hotchkiss (2011)

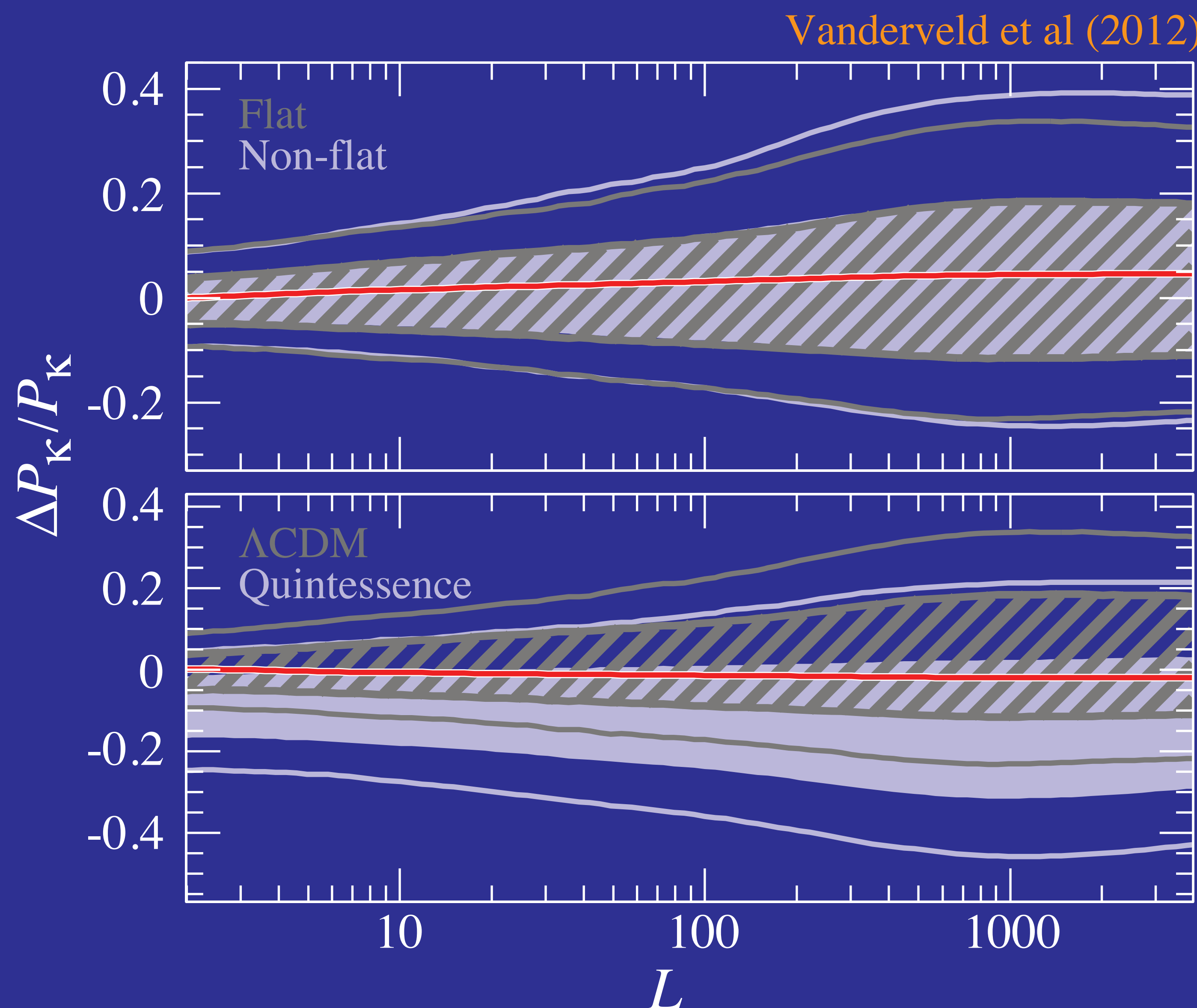
Pink Elephant Parade

- SPT catalogue on 2500 sq degrees



Cosmic Shear Tests

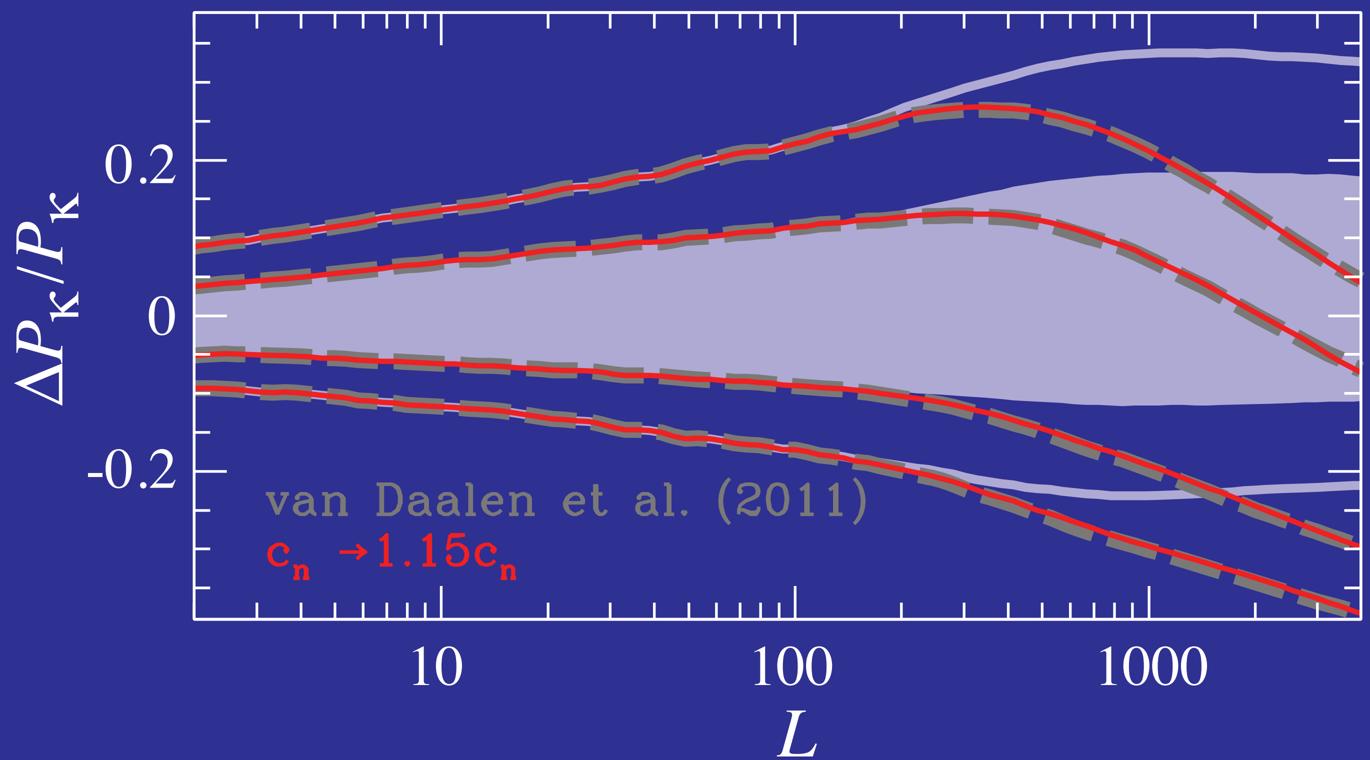
- Convergence power spectrum of CFHLT-like survey; currently consistent with ΛCDM



Cosmic Shear Tests

- Systematics from baryonic feedback (e.g. AGN, cooling, star formation in clusters) comparable to statistical errors
- Calibration must be improved
- Residual uncertainties characterized by variations in Halofit parameters

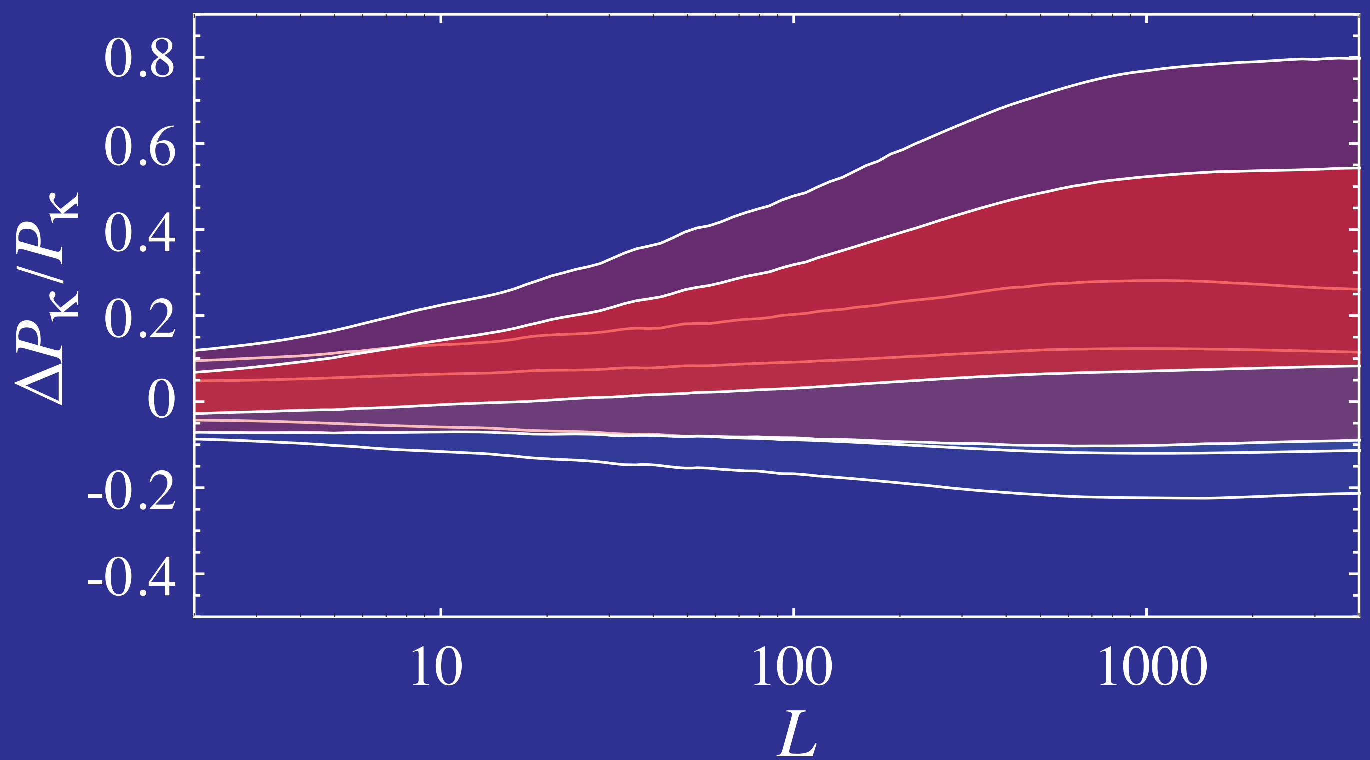
Vanderveld et al (2012)



Neutrinos

- New dark-sector physics not necessarily dark energy
- Sterile neutrinos change CMB inferences and allow more small scale power through tilt
- Accidental degeneracy will soon be resolved by Planck

Vanderveld & Hu (2012)



Falsify in Favor of?

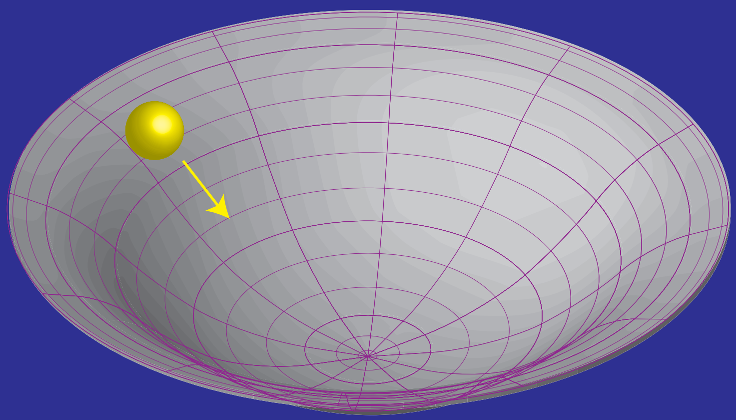
- Parameterize ignorance:
  $\Phi/\Psi(\mathbf{x},t)$ - not constant not k
- Develop and study toy models derivable from Lagrangian
  screening models, nonlinear tests

Hordes of parameters (people, telecons...)

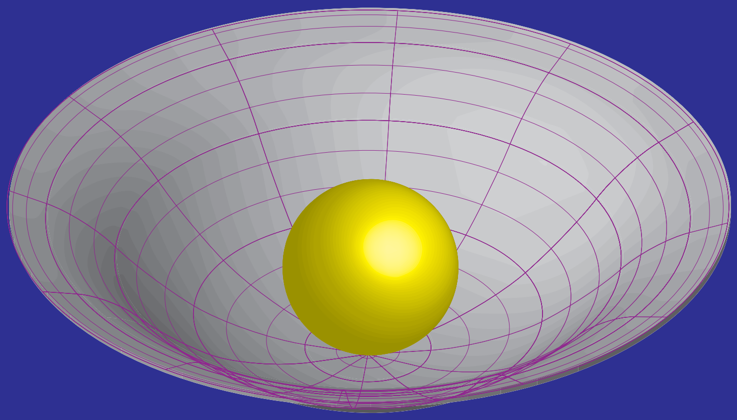
Something fishy (kill what you work on...)

Mercury or Pluto?

- Excess power could be explained by changing force law
  Keep Gravity as Geometry (microscopic equivalence principle)



- But modify how Geometry = Matter-Energy



Nonlinearly Screened DOFs

- Modifications of gravity will introduce new propagating degrees of freedom (Weinberg)
- These DOFs mediate fifth forces and may lead to ghost and tachyon instabilities
- Even attempts to modify gravity on cosmological scales (IR) will have consequences for small scales (e.g. vDVZ discontinuity)
- Fifth forces are highly constrained in the solar system and lab
- Must be screened by a nonlinear mechanism in the presence of matter source: chameleon, symmetron, Vainshtein...
- Realization in models: $f(R)$, DGP, galileon, massive gravity
- $f(R)$, DGP examples solved from horizon scales through to dark matter halo scales with N-body simulations

Cast of $f(R)$ Characters

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Nojiri & Odintsov 2003; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_{\rm m} \right]$$

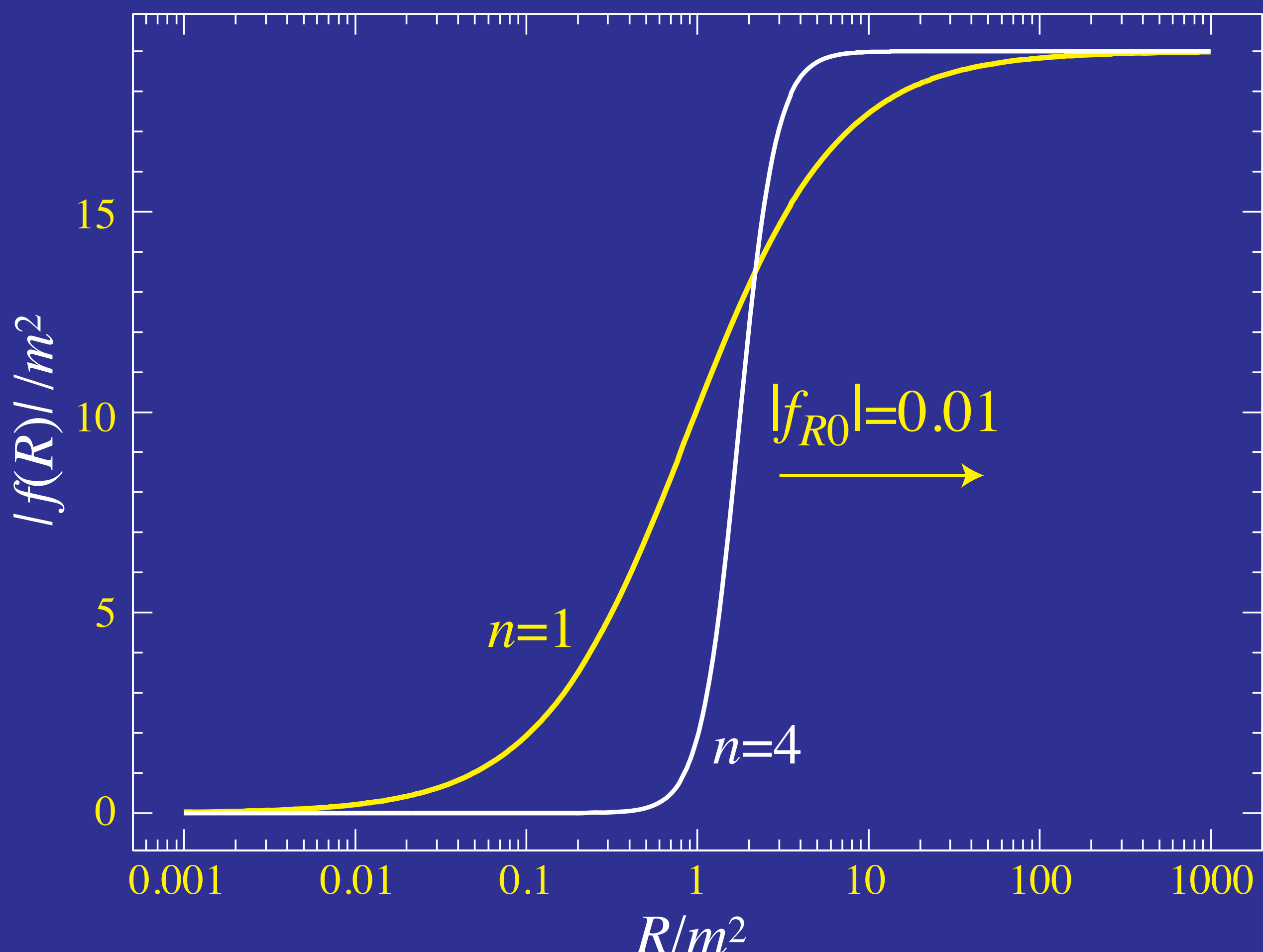
- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B: Compton wavelength of f_R squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate

Form of $f(R)$ Models

- Transition from zero to constant across an adjustable curvature scale
- Slope n controls the rapidity of transition, field amplitude f_{R0} position
- Background curvature stops declining during acceleration epoch and thereafter behaves like cosmological constant



Hu & Sawicki (2007)

Three Regimes

- Fully worked $f(R)$ example show 3 regimes
- Superhorizon regime: constant comoving curvature, $g(a)$
- Linear regime - closure $\leftrightarrow$ "smooth" dark energy density:

$$
\begin{aligned}
k^2(\Phi - \Psi)/2 &= 4\pi G a^2 \Delta\rho \\
(\Phi + \Psi)/(\Phi - \Psi) &= g(a, k)
\end{aligned}
$$

In principle $G(a)$ but conformal invariance: deviations order f_R

- Non-linear regime, scalar f_R:

$$
\begin{aligned}
\nabla^2(\Phi - \Psi)/2 &= -4\pi G a^2 \Delta\rho \\
\nabla^2\Psi &= 4\pi G a^2 \Delta\rho + \frac{1}{2}\nabla^2 f_R
\end{aligned}
$$

with non-linearity in the field equation

$$\nabla^2 f_R = g_{\rm lin}(a) a^2 \left(8\pi G \Delta\rho - N[f_R]\right)$$

Non-Linear Chameleon

- For $f(R)$ the field equation

$$\nabla^2 f_R \approx \frac{1}{3}(\delta R(f_R) - 8\pi G\delta\rho)$$

  is the non-linear equation that returns general relativity

- High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$

- Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if

$$\Delta f_R \leq \frac{2}{3}\Phi\,,$$

  else required field gradients too large despite $\delta R = 8\pi G\delta\rho$ being the local minimum of effective potential

Non-Linear Dynamics

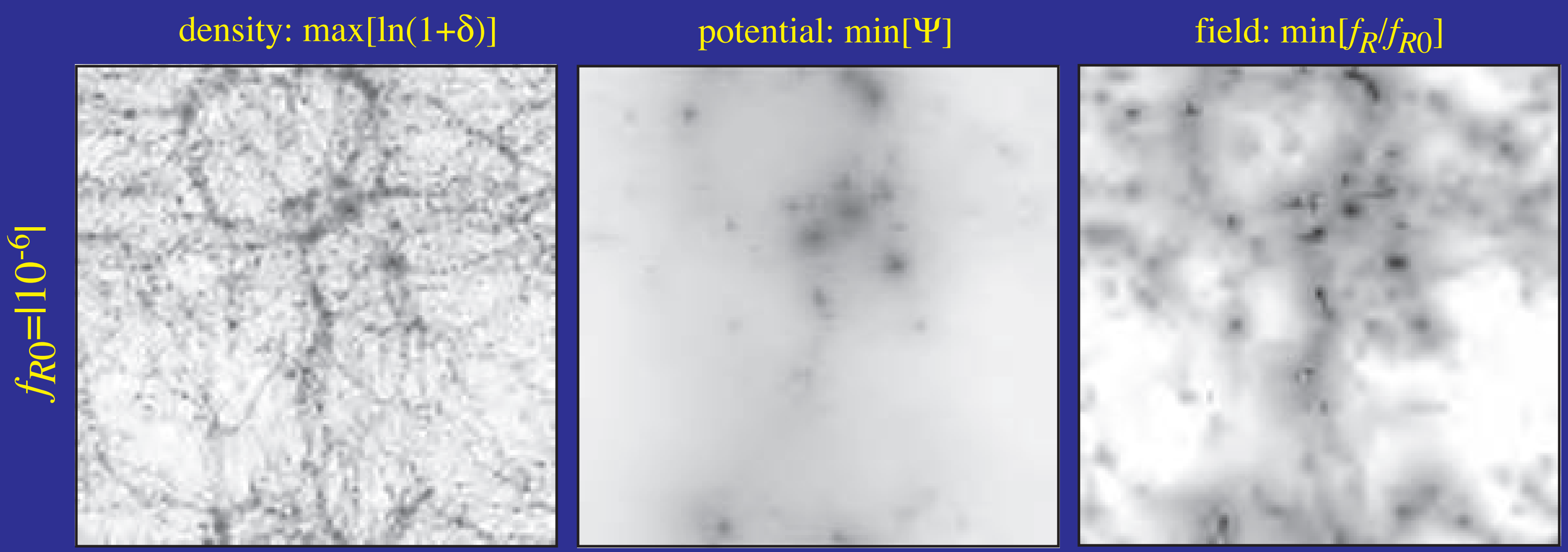
- Supplement that with the modified Poisson equation

$$\nabla^2\Psi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R(f_R)$$

- Matter evolution given metric unchanged: usual motion of matter in a gravitational potential Ψ
- Prescription for N-body code
- Particle Mesh (PM) for the Poisson equation
- Field equation is a non-linear Poisson equation: relaxation method for f_R
- Initial conditions set to GR at high redshift

Environment Dependent Force

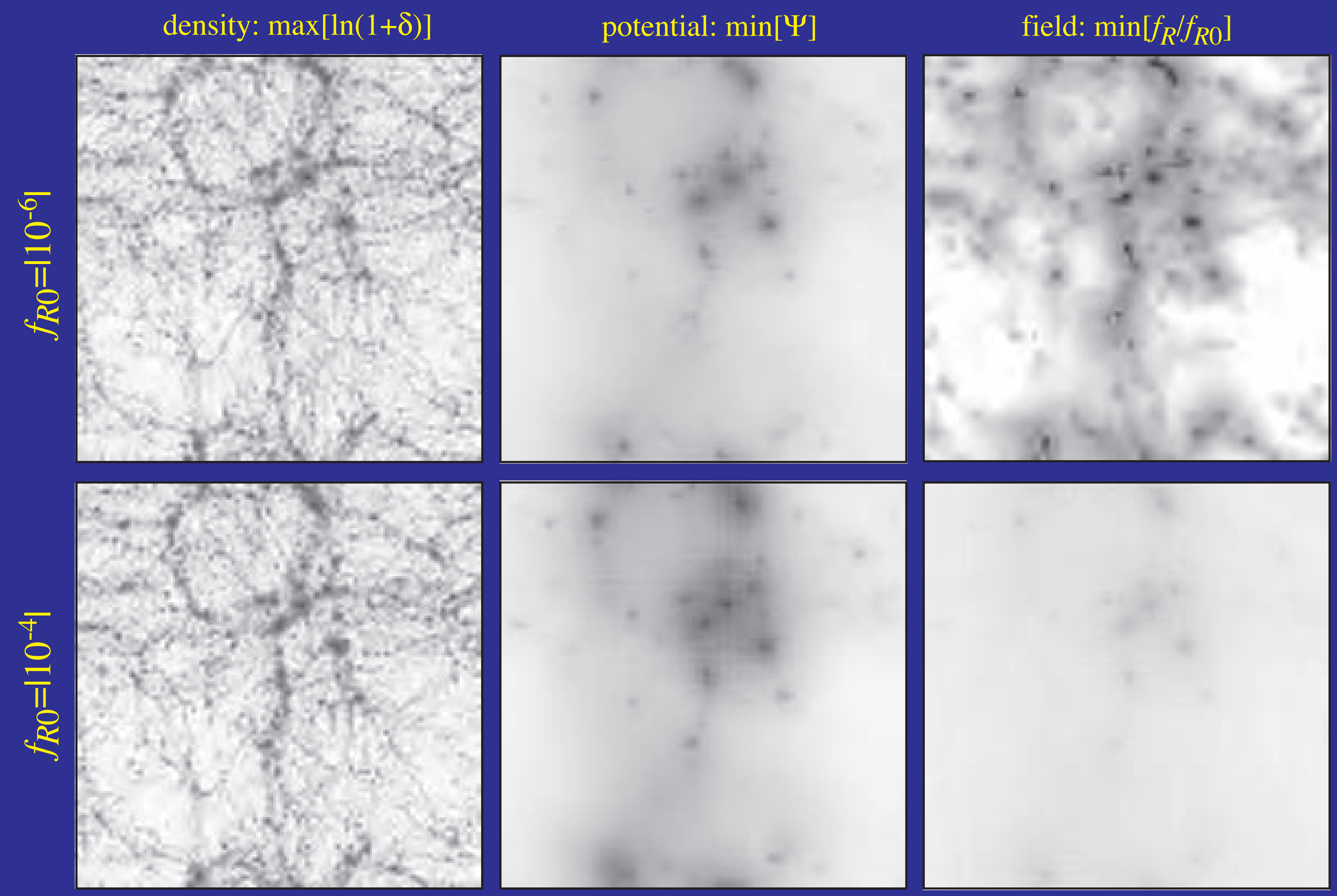
- Chameleon suppresses extra force (scalar field) in high density, deep potential regions



Oyaizu, Lima, Hu (2008)

Environment Dependent Force

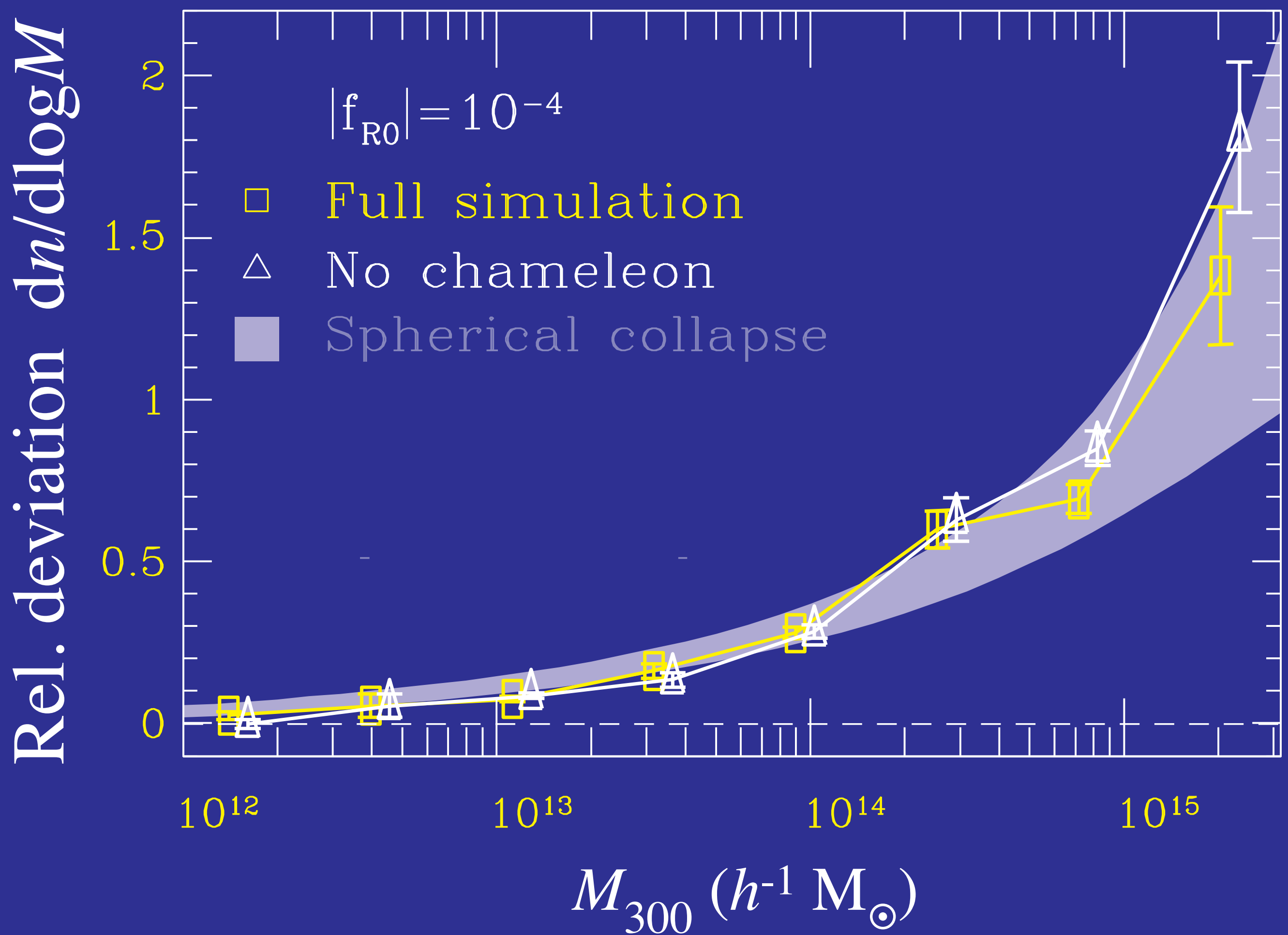
- For large background field, gradients in the scalar prevent the chameleon from appearing



Oyaizu, Lima, Hu (2008) [AMR high resolution: Zhao, Li, Koyama]

Cluster Abundance

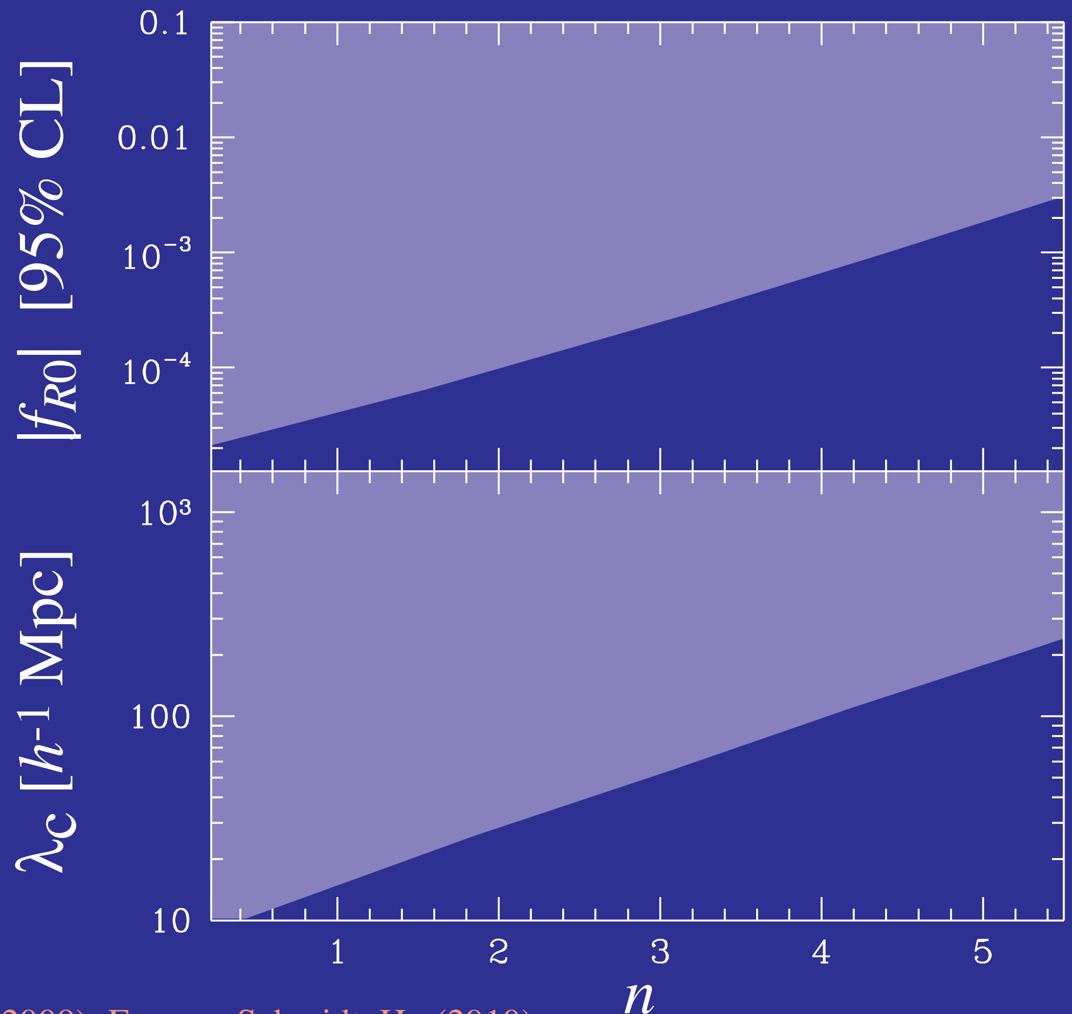
- Enhanced abundance of rare dark matter halos (clusters) with extra force



Lima, Schmidt, Oyaizu, Hu (2008)

Cluster $f(R)$ Constraints

- Clusters provide best current cosmological constraints on $f(R)$ models
- Spherical collapse rescaling to place constraints on full range of inverse power law models of index n



field amplitude

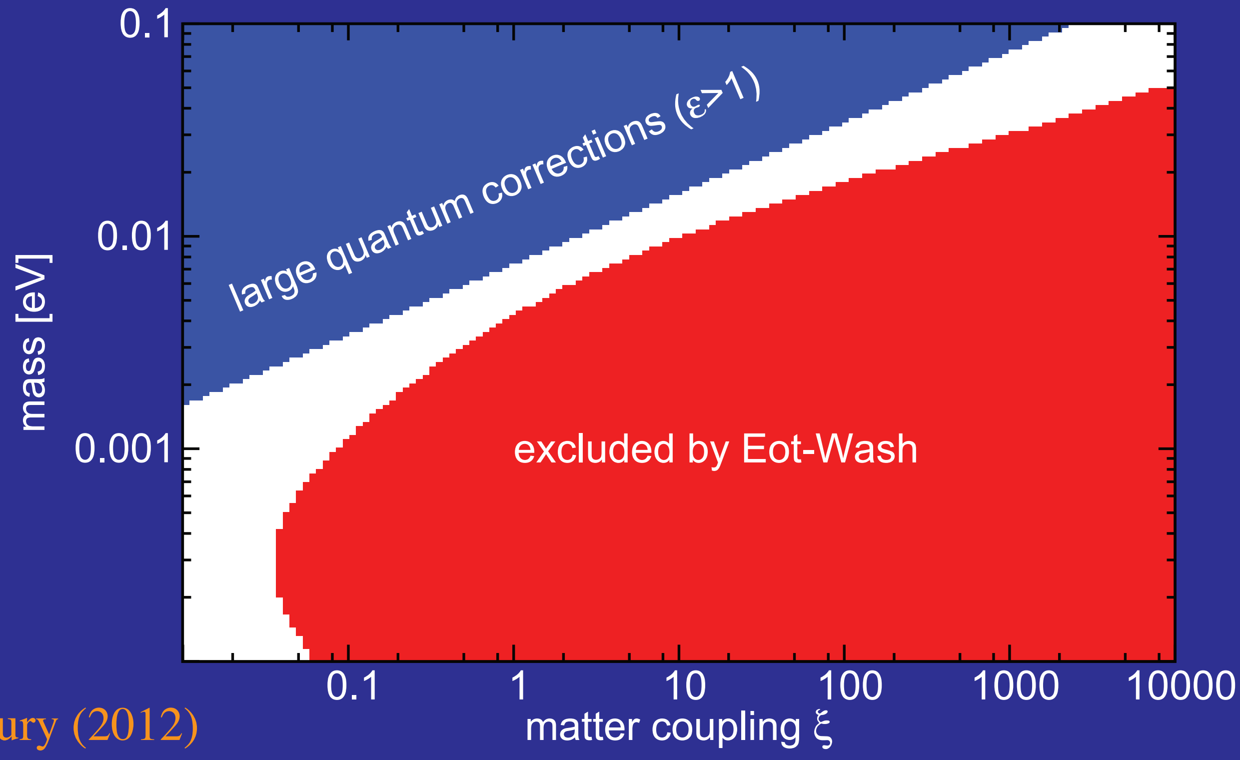
range of force

Schmidt, Vikhlinin, Hu (2009); Ferraro, Schmidt, Hu (2010)

Solar System & Lab

- Strictly valid for solar system / lab or are beyond effective theory?
- If former, solar system $f(R)$ tests of more powerful by at least 10
  (Hu & Sawicki 2009; exosolar tests: Jain et al., Davis et al.)
- Laboratory tests: within factor of 2 of ruling out all gravitational strength chameleon models [$m < 0.0073(\xi\rho/10\mathrm{g\ cm}^3)^{1/3}\mathrm{eV}$]

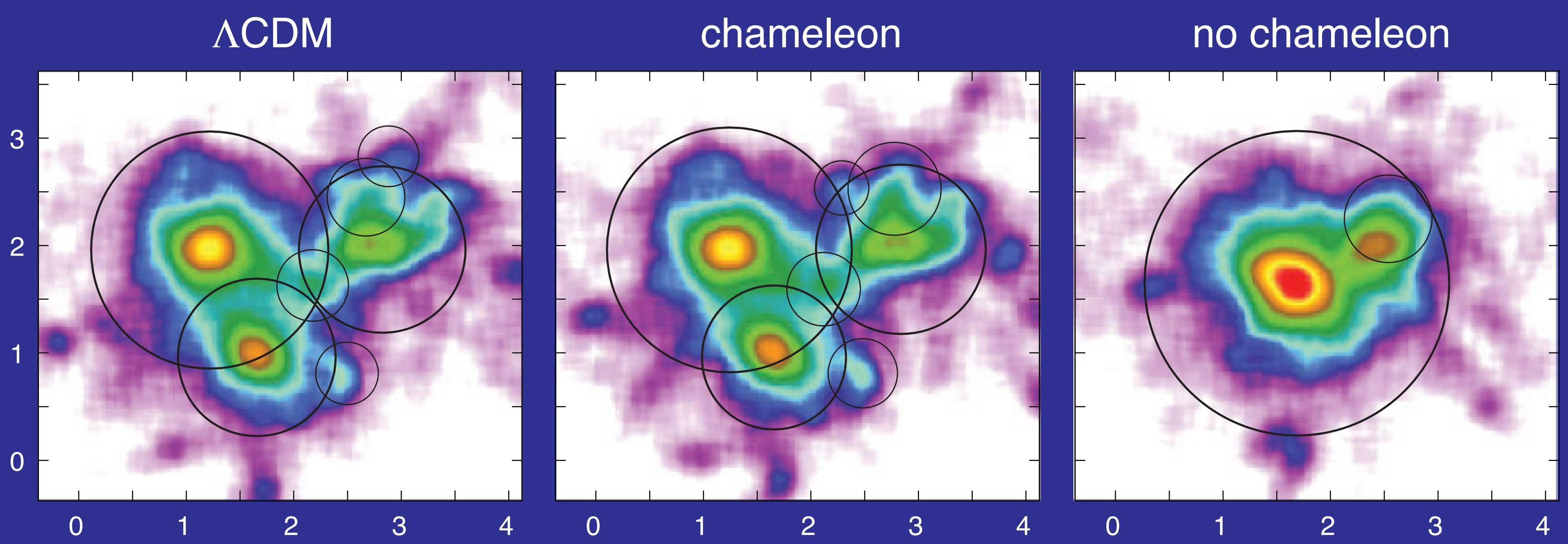
  Already exceeded the vacuum scale (1000km) and earth (1cm) of Vainshtein models (Nicolis & Rattazzi 2004)



Upadhye, Hu, Khoury (2012)

Chameleon Pile-Up

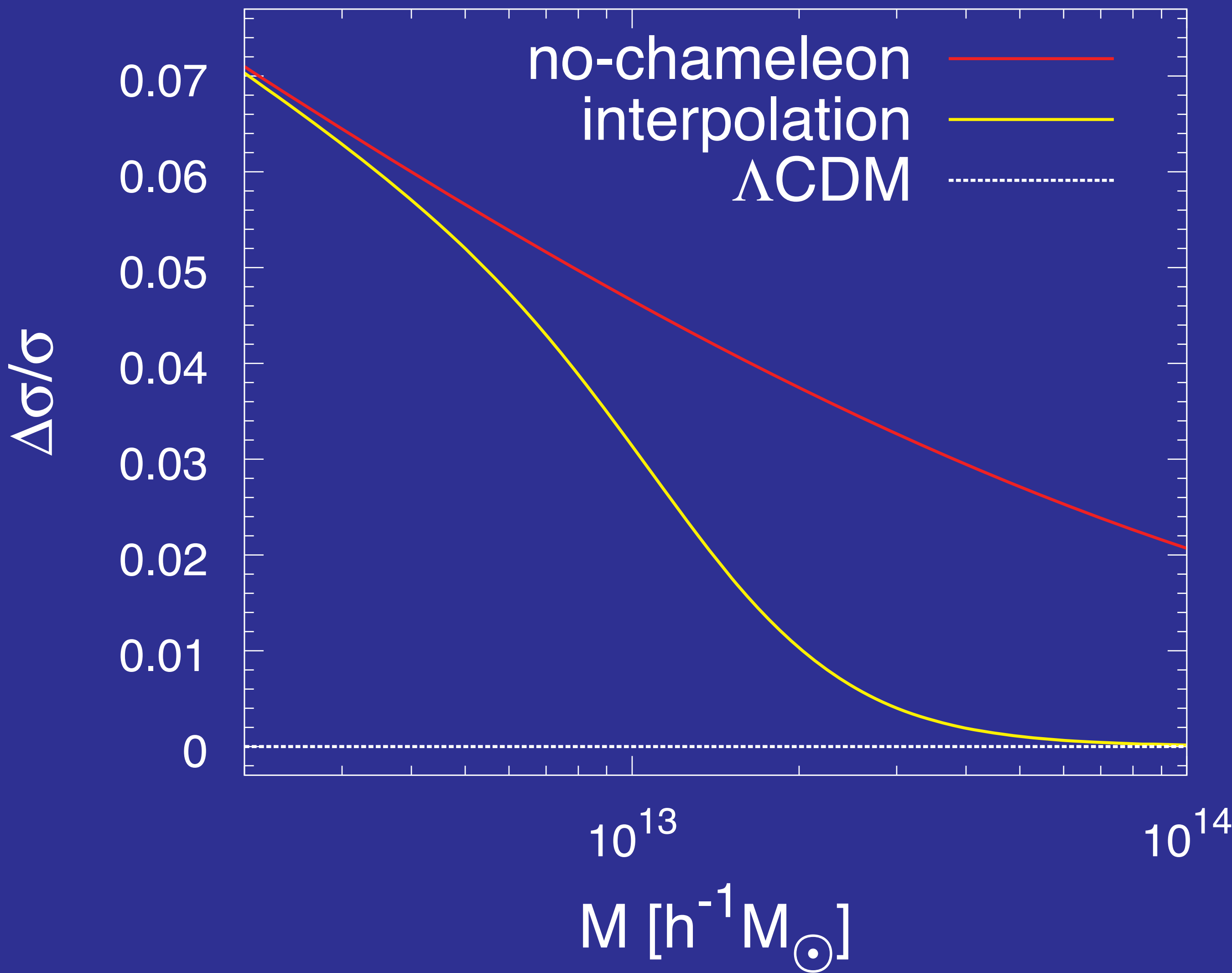
- Chameleon threshold at intermediate masses ($10^{13}\ h^{-1}\ M_\odot$)
- Mergers from smaller masses continues, to higher masses stops
- Pile up of halos at threshold



Li & Hu (2011)

PPF Parameterization

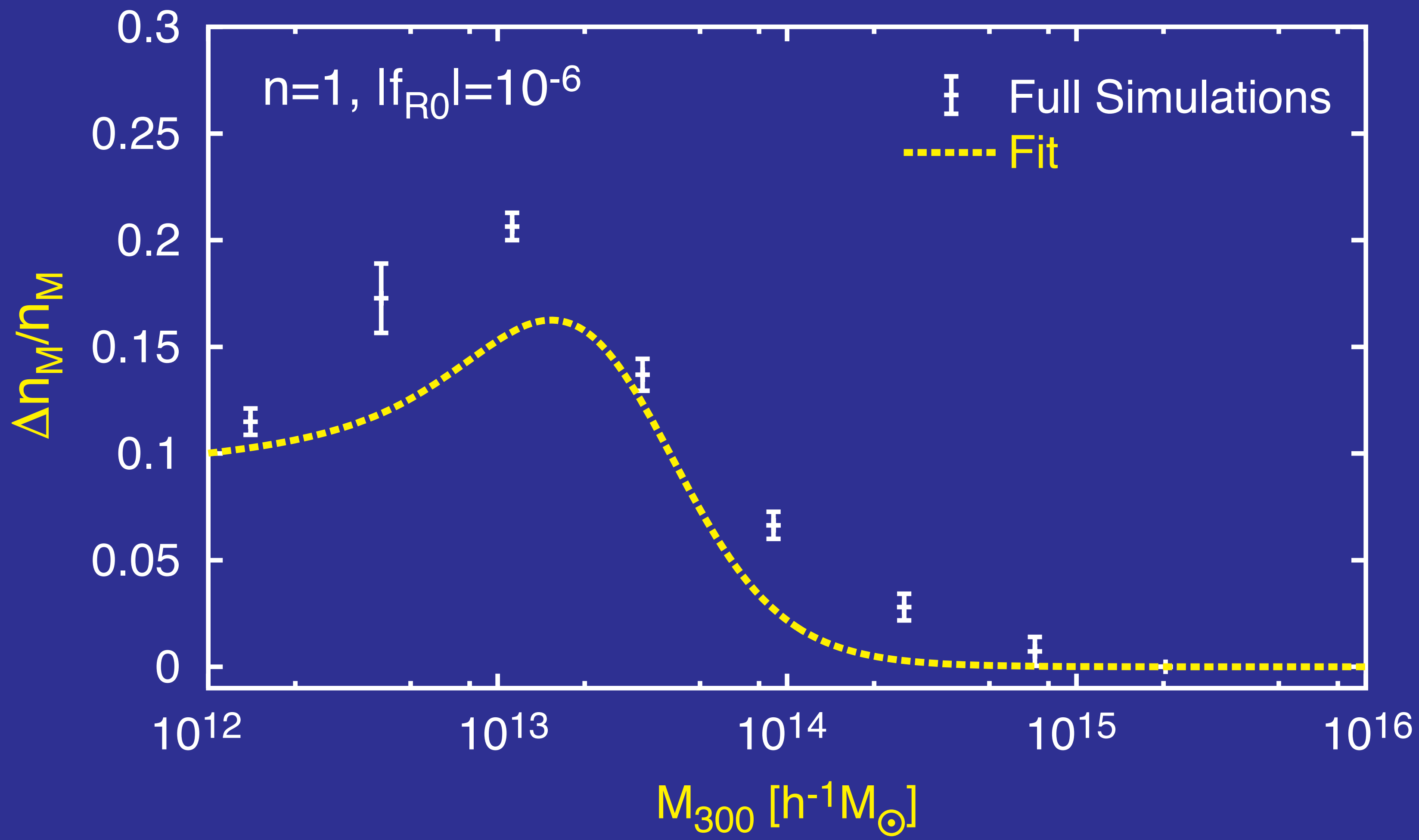
- Interpolate between linear $f(R)$ enhanced $\sigma(M)$ and ordinary gravity



Li & Hu (2011)

Chameleon Mass Function

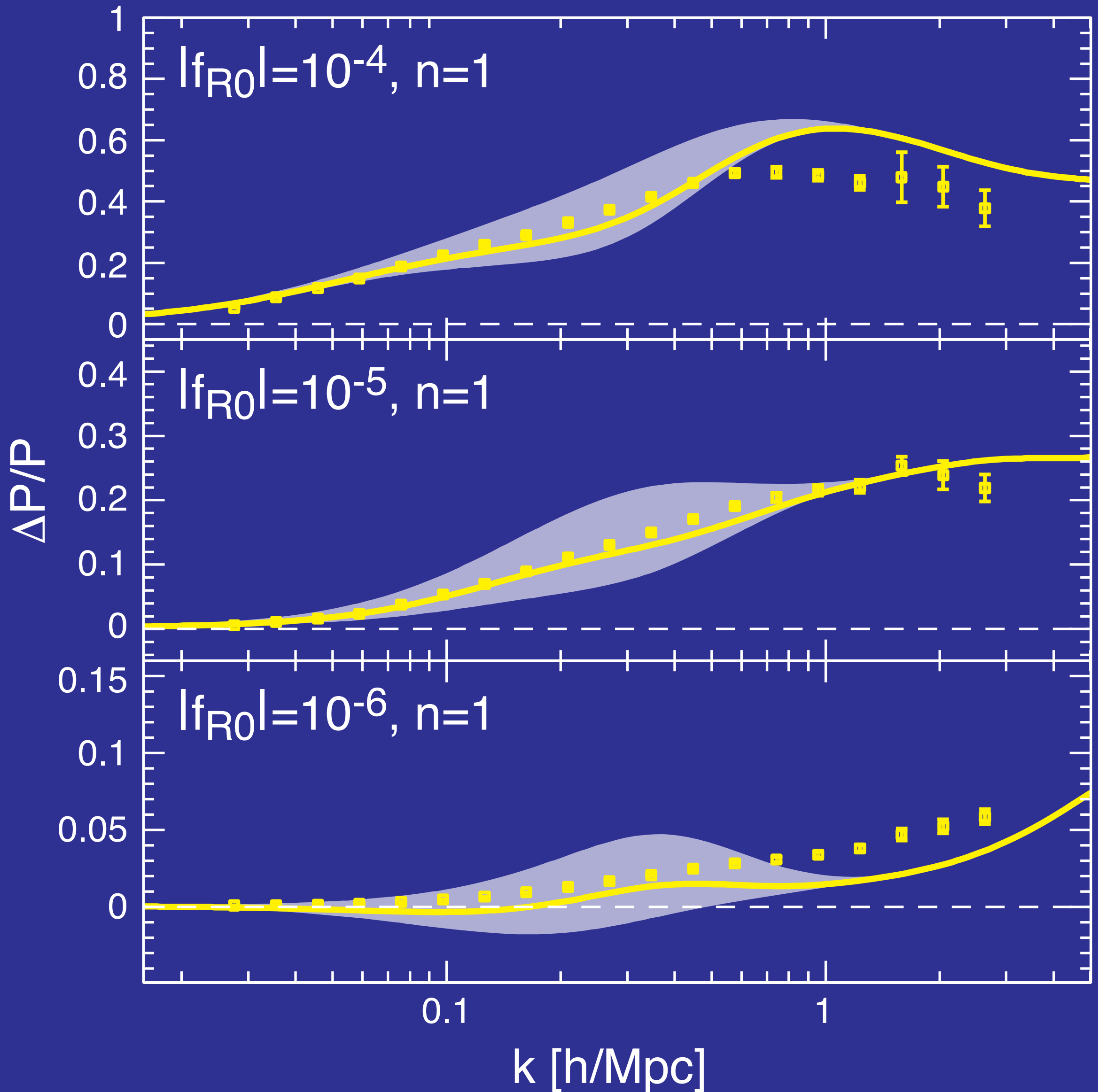
- Simple single parameter extention covers variety of models
- Basis of a halo model based post Friedmann parameterization of chameleon



Li & Hu (2011)

Power Spectrum

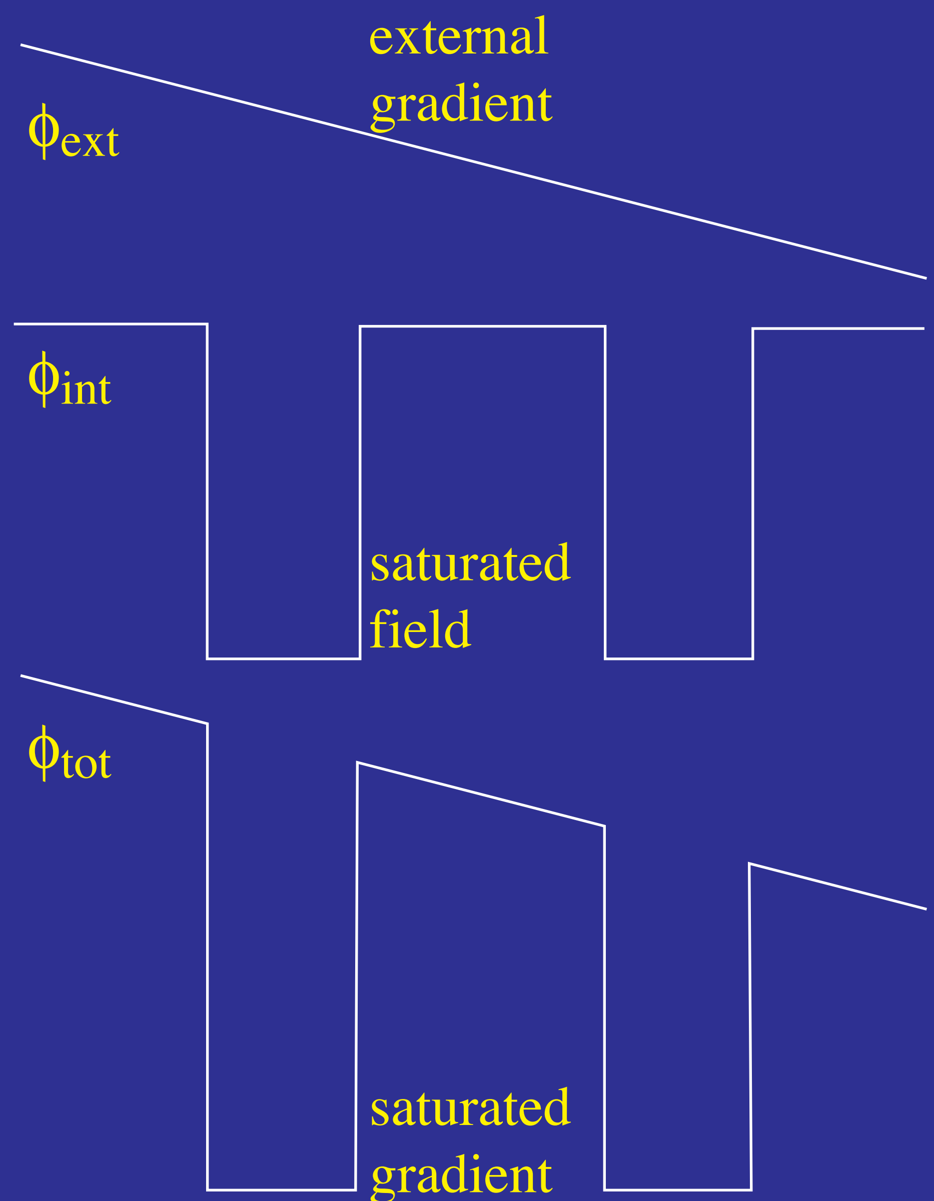
- Connect to linear regime with interpolation of HaloFit



Li & Hu (2011)

Motion: Environment & Object

- Self-field of a "test mass" can saturate an external field (for $f(R)$ in the gradient, for DGP in the second derivatives)



Hui, Nicolis, Stubbs (2009)
Jain & Vanderplas (2011)
Zhao, Li, Koyama (2011)

DGP Braneworld Acceleration

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

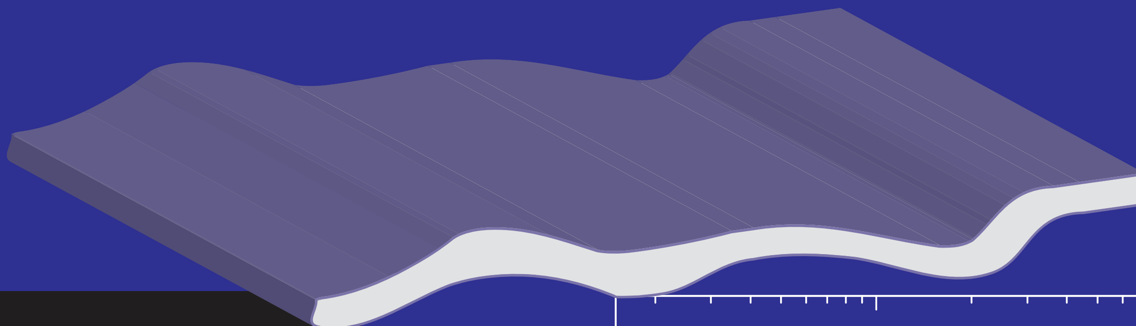
$$S = \int d^5x \sqrt{-g} \left[\frac{^{(5)}R}{2\kappa^2} + \delta(\chi) \left(\frac{^{(4)}R}{2\mu^2} + \mathcal{L}_m \right) \right]$$

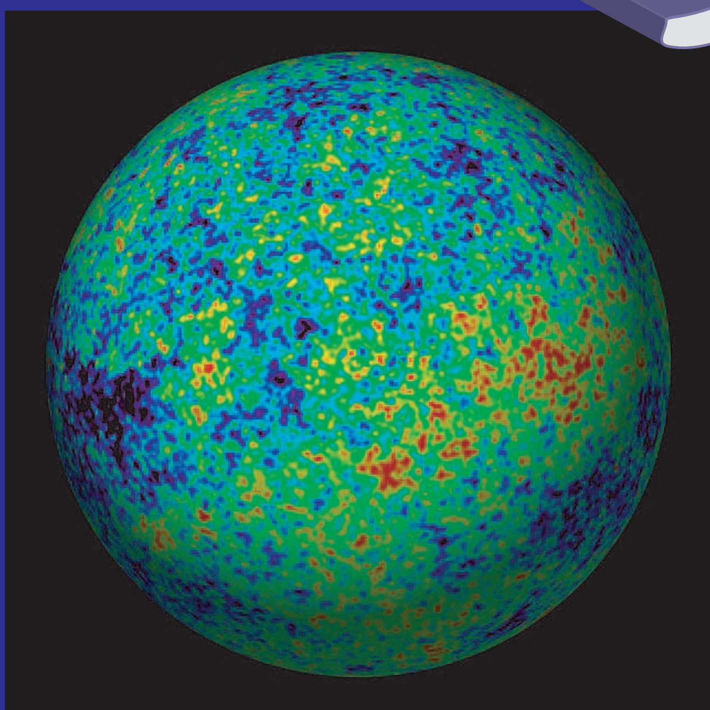
  with crossover scale $r_c = \kappa^2/2\mu^2$

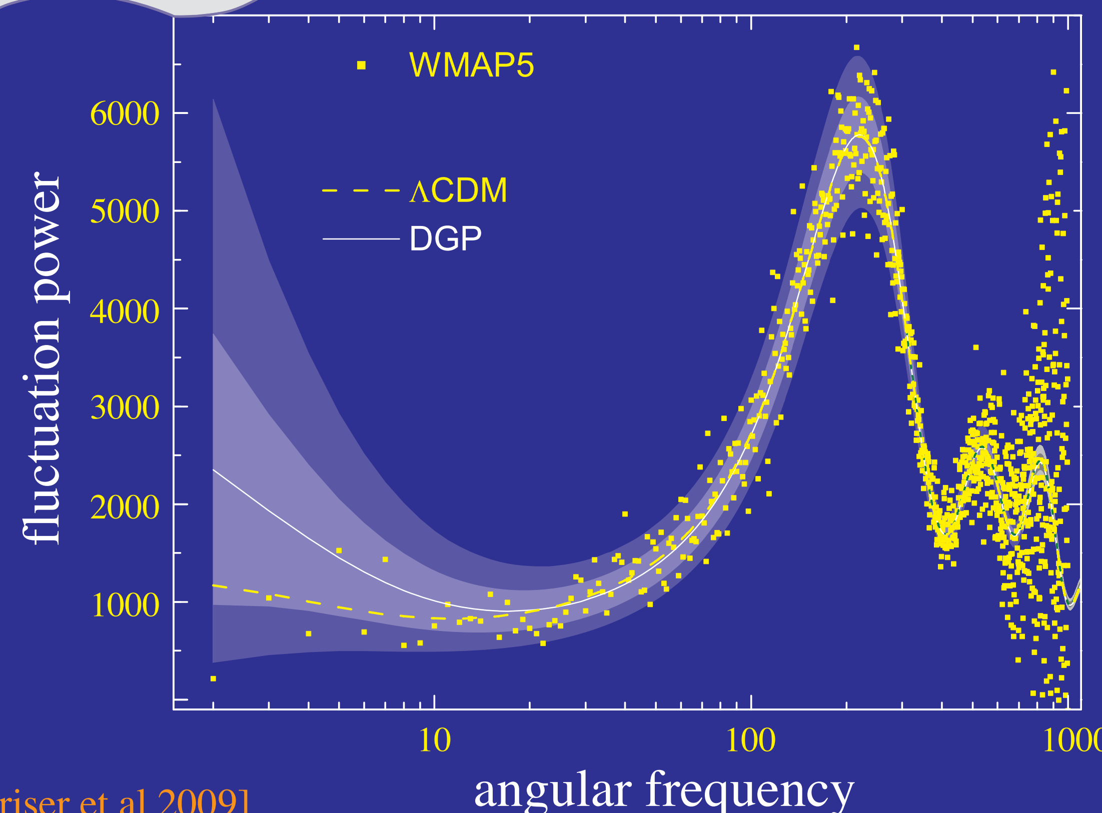
- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)
- Matter still minimally coupled and conserved
- Exhibits the 3 regimes of modified gravity
- Weyl tensor anisotropy dominated conserved curvature regime $r > r_c$ (Sawicki, Song, Hu 2006; Cardoso et al 2007)
- Brane bending scalar tensor regime $r_* < r < r_c$ (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)
- Strong coupling General Relativistic regime $r < r_* = (r_c^2 r_g)^{1/3}$ where $r_g = 2GM$ (Dvali 2006)

DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background







Fang et al (2008) [Seahra & Hu 2010; Lombriser et al 2009]

Massive Gravity

- DGP model motivated re-examination of massive gravity models
- Nonlinearly complete Fierz-Pauli action: Vainshtein strong coupling (restoring vDVZ continuity), no Boulware Deser ghost, effective theory out to Λ_3 Arkani-Hamed, Georgi, Schwartz (2003)
- Massive gravity action [de Rham, Gabadadze, Tolley et al, Hassan & Rosen, ... (2010-2012)]

$$S = \frac{M_p}{2} \int d^4x \sqrt{-g} \left[R - \frac{m^2}{4} \sum_{n=0}^{4} \alpha_n S_n(\sqrt{\mathbf{g}^{-1}\boldsymbol{\eta}}) \right]$$

where $\boldsymbol{\eta}$ is a fiducial (Minkowski) metric

- Diffeomorphism invariance can be restored by introducing Stückelberg fields

$$\mathbf{g}^{-1}\boldsymbol{\eta} \to g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}$$

which carry transformation from unitary to arbitrary gauge

Self Acceleration

- Graviton mass $\sim H_0$ provides self-acceleration
- Generalizing results de Rham et al, Koyama et al, Mukohyama et al... for any isotropic metric a cosmological constant stress-energy is an exact solution Gratia, Hu, Wyman (2012); Motohashi & Suyama (2012)

$$\rho_m = -p_m = \frac{m^2 M_p^2}{2} P_0$$

where P_0 constant given α_n

- Cosmic acceleration if $m \sim H_0$, remains constant for arbitrarily large radial matter perturbations
- Stückelberg fields are inhomogeneous in isotropic coordinates d'Amico et al (2011)
- Stress-energy depends only on spatial Stückelberg fields, leaving a set of solutions that differ in ϕ_0 or the choice of unitary time

Self Acceleration

- Self-accelerating solution approached from arbitrary initial conditions? classically and quantum-mechanically stable?
- Field fluctuations again decouple with spatial Stückelberg field obeying first order closed equation
- Stable to radial field perturbations Wyman, Hu, Gratia (2012)

$$\delta p/\delta\rho = a\ddot{a}/3\dot{a}^2$$

  e.g. de Sitter $\delta p/\delta\rho = 1/3$
- Stückelberg dynamics determined by unitary time: special cases with no dynamics, no stress energy perturbations Gumrukcuoglu et al
- Stability to anisotropic perturbations and higher order terms in action? Koyama et al; de Felice et al; d'Amico
- Effective theory to 1000km in vacuum, on earth 1cm or 1km? Burrage, Kaloper, Padilla (2012)

Nonlinear Interaction

Nonlinearity in field equation recovers linear theory if $N[\phi] \to 0$

$$\nabla^2 \phi = g_{\rm lin}(a) a^2 \left(8\pi G \Delta\rho - N[\phi] \right)$$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

  a nonlinear function of the field

  Linked to gravitational potential

- For DGP, ϕ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4} \left[(\nabla^2\phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$

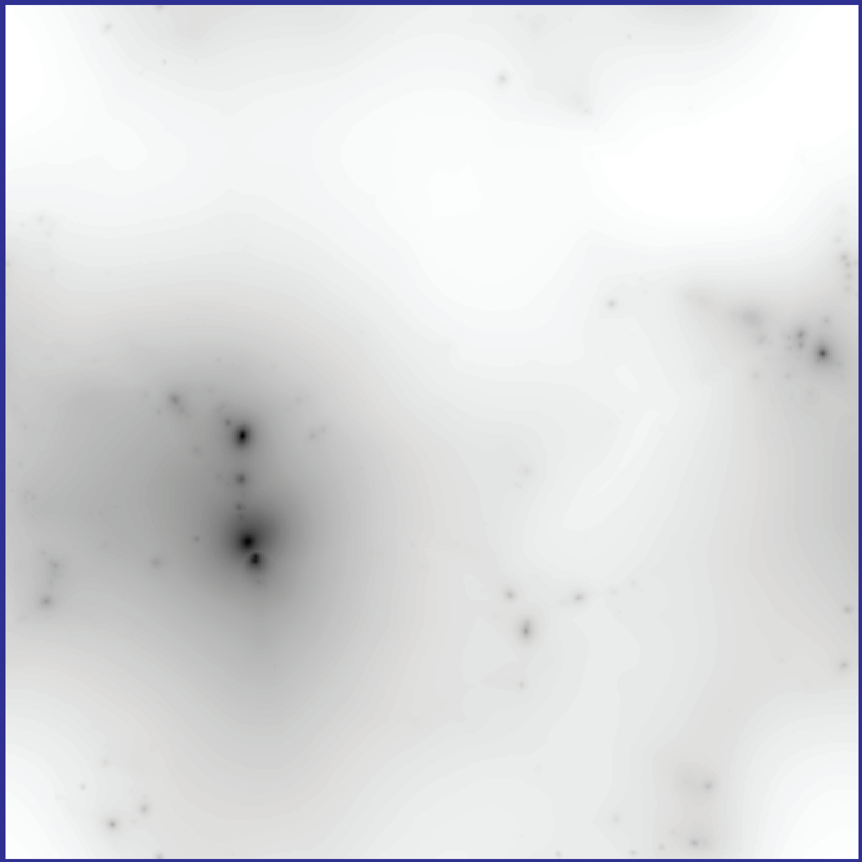
  a nonlinear function of second derivatives of the field

  Linked to density fluctuation - Galileon invariance - no self-shielding of external forces

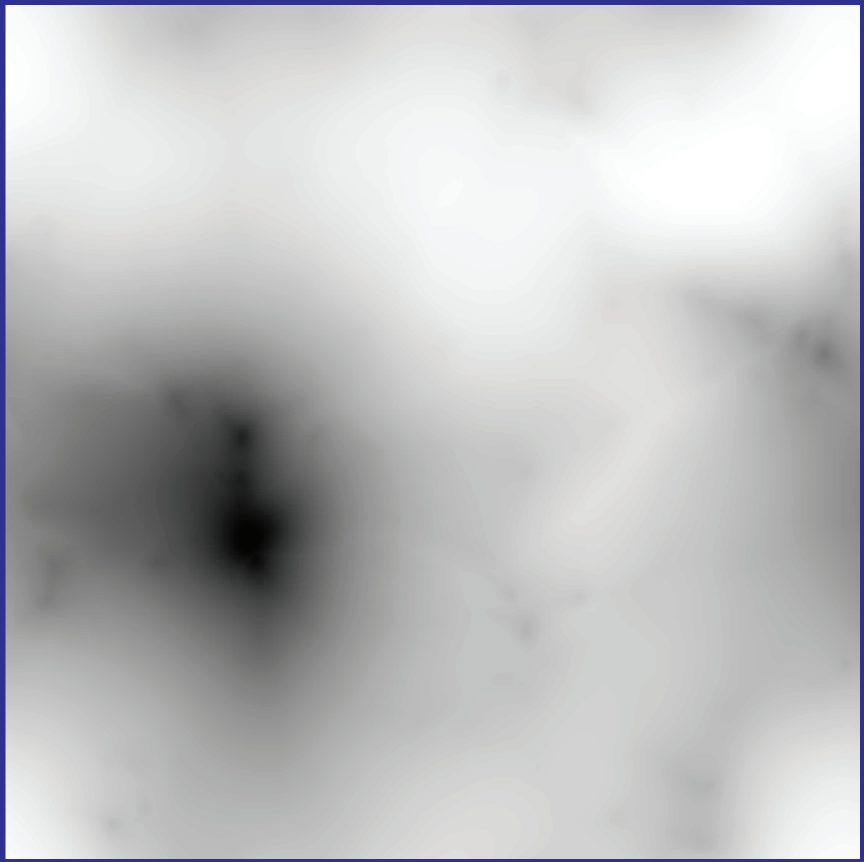
DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential



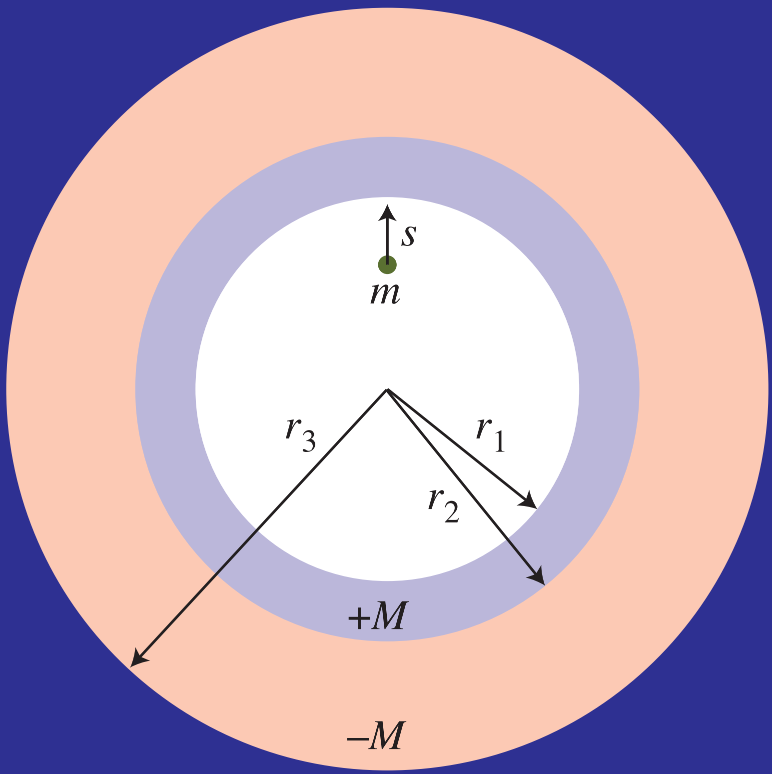
Brane Bending Mode

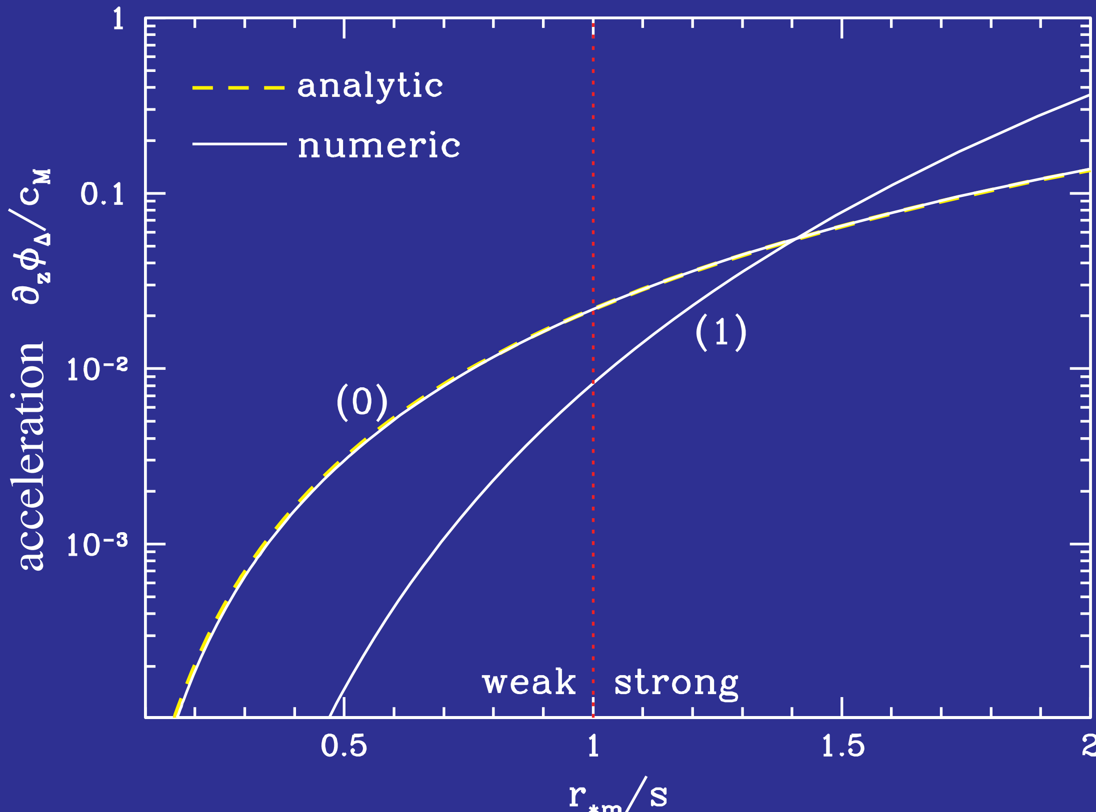


Schmidt (2009); Chan & Scoccimarro (2009) (cf. Khoury & Wyman 2009)

Weak Vainshtein Screening

- Screening occurs when objects are separated by a Vainshtein radius
- Vainshtein radius depends on mass $m^{1/3}$
- Halos in compensated voids experience acceleration toward the center proportional to m

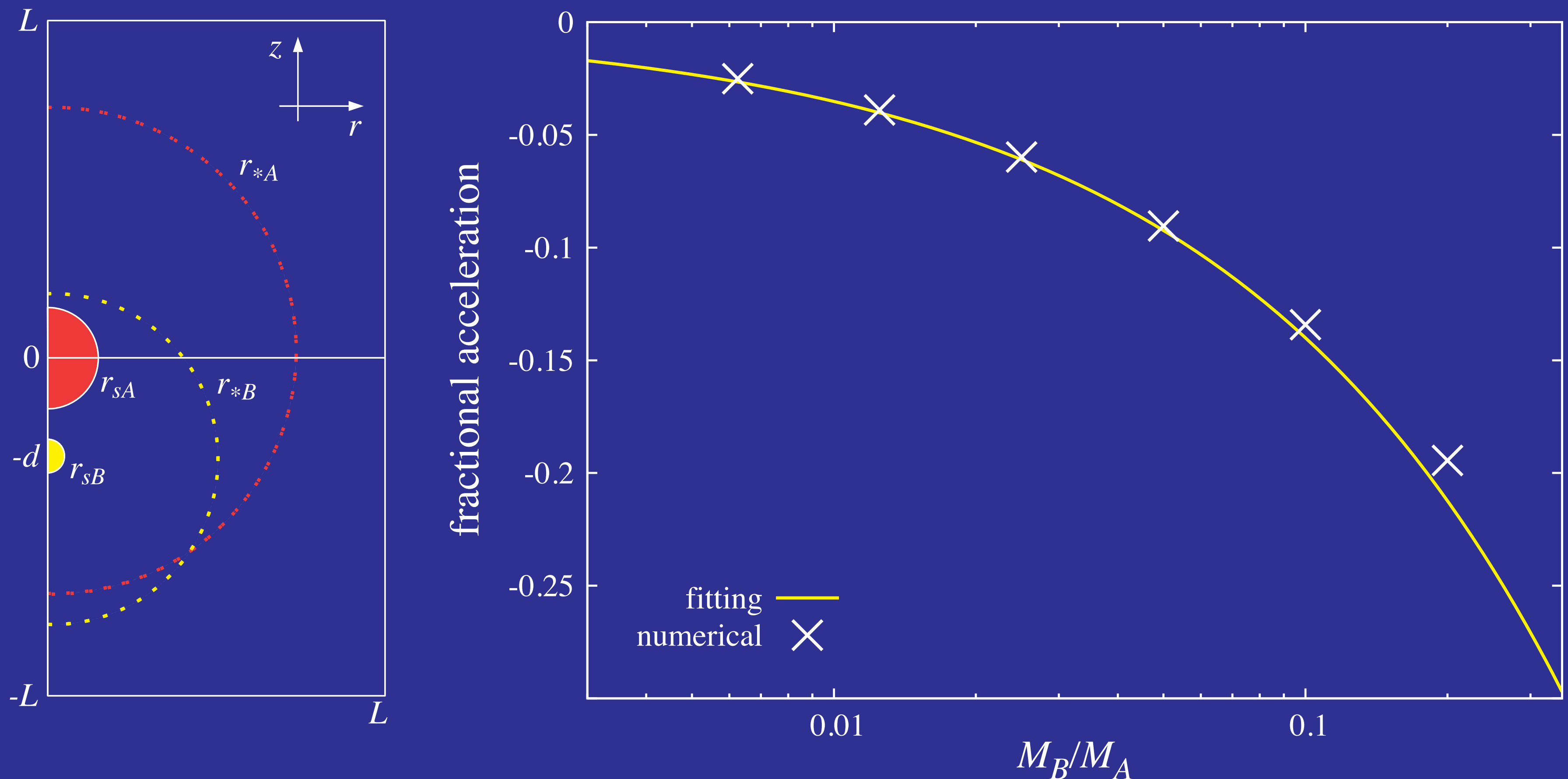




Belikov & Hu (2012)

Strong Vainshtein Screening

- Objects separated by much less than Vainshtein radius
- Screened acceleration also mass dependent due to nonlinearity
- Universal precession rate is not universal: corrections scale as $(M_B/M_A)^{3/5}$



Hiramatsu, Hu, Koyama, Schmidt (2012)

Summary

- Formal equivalence between dark energy and modified gravity
- Practical inequivalence of smooth dark energy and extra propagating scalar fifth force
- Appears as difference between dynamical mass and lensing mass or dark energy anisotropic stress
- Smooth dark energy (e.g. quintessence) highly falsifiable
- Three regimes of modified gravity
- Nonlinear screening in field equations return to ordinary gravity

  Chameleon/symmetron: deep potential well

  Vainshtein: high local density

  manifest in the $f(R)$ model and DGP/galileon/massive gravity
- Characteristic signatures of different screening mechanisms