BPS States in the Duality Web of the Omega deformation

Susanne Reffert
CERN

based on work with with D. Orlando and S. Hellerman
(arXiv:1106.2097, 1108.0644, 1111.4811, 1204.4192, 1210.7805)

Motivation

The Omega background was introduced by Nekrasov as a way of regularizing the 4d instanton partition function and reproducing the results of Seiberg and Witten.

Many applications:

In limit $\epsilon_1 = -\epsilon_2 \propto g_s$, partition fn is the same as for topological string (for $\epsilon_1 \neq \epsilon_2$, refined top. string)

In the Nekrasov Shatashvili limit $\epsilon_1 = 0$, related to quantum integrable models with $\hbar = \epsilon_2$

Compactification of 6D (2,0) theory in Omega BG leads to AGT

All of the above can be understood via string theory by placing branes into a geometrical deformation of the bulk corresponding to the Omega BG!

Overview

Gauge/Bethe correspondence relates susy gauge theories in 2d/4d to (quantum) integrable systems. Find/study string theory realization!

fluxbrane BG

↓ T duality

fluxtrap BG

+ D2, NS5 ↙ ↘ + D4, NS5

2d N=2 gauge theory w. twisted masses (2d Gauge/Bethe corr.)

4d N=2 gauge theory in Omega BG (4d Gauge/Bethe corr.)

↓

M Theory lift

The same string theory background can give rise to different deformations (twisted masses/Omega deformation) depending on how we place branes in it!

Overview

Start from M theory lift of background.

Study different limits of (2,0) theory in 6d.

Why (2,0) theory?

(2,0) in 6d on $\mathbf{M}_4 \times \Sigma$

compactify on Σ

compactify on $\mathbf{M}_4 = S^4$

4d SYM in Omega BG ⟷ Liouville field theory (2d)

AGT

Here:

(2,0) in 6d on $\mathbb{R}^4_\Omega \times T^2$ in fluxtrap

compactify on T^2

compactify on angular directions of $\mathbb{R}^4_\Omega$

4d N=4 SYM in Omega BG

reciprocal field theory (4d)

Theories are realized via a chain of dualities.

Study BPS states in this duality web.

Overview

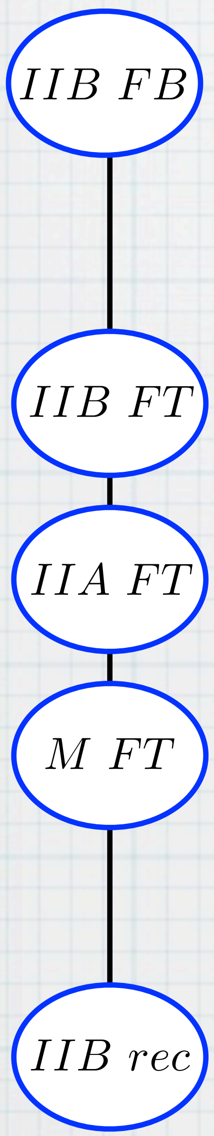
Bulk	Probe	Gauge Theory
type IIB in Melvin space	D5	six-dimensional gauge theory with Wilson line boundary conditions in two directions
	↓ *T–duality in $\tilde{u}_1$ and $\tilde{u}_2$*	
type IIB in complex fluxtrap	D3	Ω–deformed $\mathcal{N} = 4$ SYM
	↓ *T–duality in $\tilde{x}_6$ and lift*	
M–theory fluxtrap	M5	$(2, 0)$ six-dimensional theory
	↓ *reduction in σ_1 and σ_2*	
type IIB in deformed D5/NS5 (reciprocal background)	D3	Reciprocal gauge theory

Outline

- Introduction
- A Chain of Dualities
  - Bulk
  - Branes
- BPS States in the Duality Web
- Summary

A Chain of Dualities

A Chain of Dualities



Bulk	Probe	Gauge Theory
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	reduction in σ_1 and σ_2 ⇩	
type IIB in deformed D5/NS5 (reciprocal background)	D3	Reciprocal gauge theory

IIB FB

IIB FT

IIA FT

M FT

IIB rec

A Chain of Dualities: Bulk

Fluxbrane background with two independent deformation parameters, one purely real, the other purely imaginary

$$\rho_1, \theta_1, \rho_2, \theta_2, \rho_3, \theta_3, \widetilde{x}_6, x_7, \widetilde{u}_1, \widetilde{u}_2$$

$$\theta_1 \simeq \theta_1 + 2\pi\, k_2\,, \qquad \theta_2 \simeq \theta_2 + 2\pi\, k_3$$

$$\widetilde{x}_8 = \widetilde{R}_1\ \widetilde{u}_1\ , \qquad \widetilde{x}_9 = \widetilde{R}_2\ \widetilde{u}_2$$

impose identifications

fluxbrane parameter

$$\begin{cases} \widetilde{u}_1 \simeq \widetilde{u}_1 + 2\pi\ , \\ \theta_1 \simeq \theta_1 + 2\pi\epsilon_1 \widetilde{R}_1\ , \\ \theta_3 \simeq \theta_3 - 2\pi\epsilon_1 \widetilde{R}_1\ , \end{cases} \qquad \begin{cases} \widetilde{u}_2 \simeq \widetilde{u}_2 + 2\pi\ , \\ \theta_2 \simeq \theta_2 + 2\pi\epsilon_2 \widetilde{R}_2\ , \\ \theta_3 \simeq \theta_3 - 2\pi\epsilon_2 \widetilde{R}_2\ , \end{cases}$$

This corresponds to the well known Melvin or fluxbrane background.

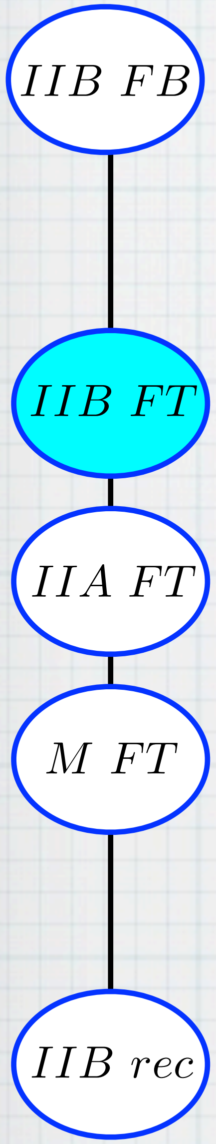
Introduce new angular variables

$$\theta_1 = \phi_1 + R_1\epsilon_1\widetilde{u}_1\ ,$$

$$\theta_2 = \phi_2 + R_2\epsilon_2\widetilde{u}_2\ ,$$

$$\theta_3 = \phi_3 - R_1\epsilon_1\widetilde{u}_1 - R_2\epsilon_2\widetilde{u}_2\ .$$

A Chain of Dualities: Bulk



T dualize $\widetilde{u}_1$ and $\widetilde{u}_2$ into u_1 and u_2 Fluxtrap background

Coordinate change: $\phi_1 + \phi_2 + \phi_3 = \psi$

$$\rho_1, \phi_1, \rho_2, \phi_2, \rho_3, \psi, \widetilde{x}^6, x^7, x_8 = \frac{\alpha'}{\widetilde{R}_1} u_1, x_9 = \frac{\alpha'}{\widetilde{R}_2} u_2$$

Study the limit $\rho_3 << \rho_1, \rho_2$

Before T duality, locally, the metric was still flat, but some of the rotation symmetries were broken globally.

not anymore flat

Bulk fields

$$\mathrm{d}s^2 = \mathrm{d}\rho_1^2 + \frac{\rho_1^2\,\mathrm{d}\phi_1^2 + \mathrm{d}x_8^2}{1+\epsilon_1^2\rho_1^2} + \mathrm{d}\rho_2^2 + \frac{\rho_2^2\,\mathrm{d}\phi_2^2 + \mathrm{d}x_9^2}{1+\epsilon_2^2\rho_2^2} + \mathrm{d}\rho_3^2 + \rho_3^2\,\mathrm{d}\psi^2 + \mathrm{d}\widetilde{x}_6^2 + \mathrm{d}x_7^2\,,$$

B field has appeared

$$B = \epsilon_1 \frac{\rho_1^2}{1+\epsilon_1^2\rho_1^2}\,\mathrm{d}\phi_1 \wedge \mathrm{d}x_8 + \epsilon_2 \frac{\rho_2^2}{1+\epsilon_2^2\rho_2^2}\,\mathrm{d}\phi_2 \wedge \mathrm{d}x_9\,,$$

creates a potential that localizes instantons

$$\mathrm{e}^{-\Phi} = \sqrt{(1+\epsilon_1^2\rho_1^2)\,(1+\epsilon_2^2\rho_2^2)}$$

A quarter of the original supersymmetries are preserved.

IIB FB

IIB FT

IIA FT

M FT

IIB rec

A Chain of Dualities: Bulk

$$\rho_1, \phi_1, \rho_2, \phi_2, \rho_3, \psi, \widetilde{x}^6, x^7, x_8 = \frac{\alpha'}{\widetilde{R}_1} u_1, x_9 = \frac{\alpha'}{\widetilde{R}_2} u_2$$

Space splits into

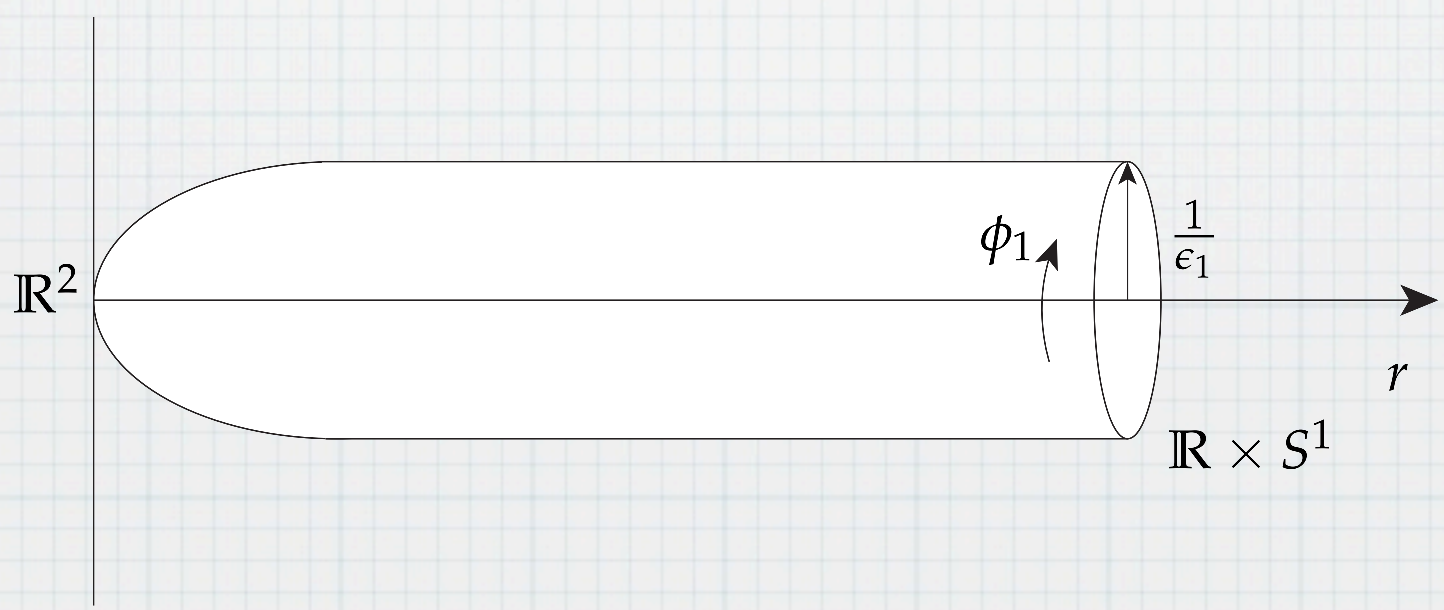
$$M_{10} = M_3(\epsilon_1) \times M_3(\epsilon_2) \times \mathbb{R}^3 \times S^1$$

$(\rho_3, \psi, x_7) \rightarrow \mathbb{R}^3$, $\quad \widetilde{x}_6 \rightarrow S^1$

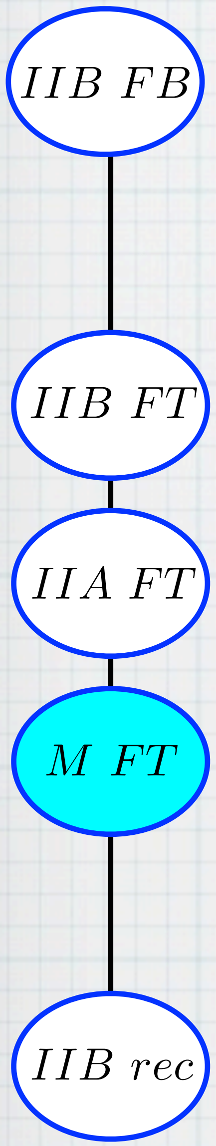
$\mathbb{R}\langle x_8 \rangle \longrightarrow M_3(\epsilon_1)$

$\downarrow$

cigar $\langle \rho_1, \phi_1 \rangle$



A Chain of Dualities: Bulk



Now we want to lift to M theory. First T dualize in $\tilde{x}_6$ to IIA, then lift: $\rho_3 << \rho_1, \rho_2$

$$\mathrm{d}s^2 = (\Delta_1\Delta_2)^{2/3}\left[\mathrm{d}\rho_1^2 + \frac{\epsilon_1^2\rho_1^2}{1+\epsilon_1^2\rho_1^2}\,\mathrm{d}\sigma_1^2 + \frac{\mathrm{d}x_8^2}{1+\epsilon_1^2\rho_1^2} + \mathrm{d}\rho_2^2 + \frac{\epsilon_2^2\rho_2^2}{1+\epsilon_2^2\rho_2^2}\,\mathrm{d}\sigma_2^2 + \frac{\mathrm{d}x_9^2}{1+\epsilon_2^2\rho_2^2}\right.$$
$$\left. + \mathrm{d}\rho_3^2 + \rho_3^2\,\mathrm{d}\psi^2 + \mathrm{d}x_6^2 + \mathrm{d}x_7^2\right] + (\Delta_1\Delta_2)^{-4/3}\,\mathrm{d}x_{10}^2 \ ,$$

$$A_3 = \frac{\epsilon_1^2\rho_1^2}{1+\epsilon_1^2\rho_1^2}\,\mathrm{d}\sigma_1 \wedge \mathrm{d}x_8 \wedge \mathrm{d}x_{10} + \frac{\epsilon_2^2\rho_2^2}{1+\epsilon_2^2\rho_2^2}\,\mathrm{d}\sigma_2 \wedge \mathrm{d}x_9 \wedge \mathrm{d}x_{10}$$

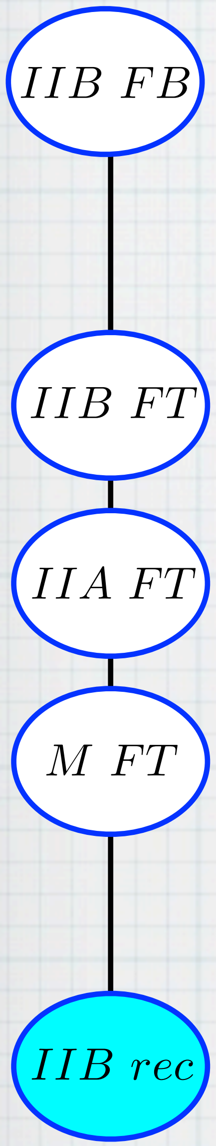
$$\sigma_i = \frac{\phi_i}{\epsilon_i} \qquad \Delta_i^2 = 1 + \epsilon_i^2\rho_i^2 \qquad x_{10} = x_{10} + 2\pi R_{10}$$

Symmetric under exchange $\{\rho_1, \phi_1, x_8, \epsilon_1\} \leftrightarrow \{\rho_2, \phi_2, x_9, \epsilon_2\}$

Origin of S duality covariance in final BG.

Return to IIA by reducing on σ_1: backreaction of the near horizon limit of a D6 brane in the fluxtrap

A Chain of Dualities: Bulk



Last step: T duality in σ_2

$$\mathrm{d}s^2 = \epsilon_1 \rho_1 \sqrt{1+\epsilon_2^2\rho_2^2}\left[\mathrm{d}\rho_1^2 + \mathrm{d}\rho_2^2 + \frac{\mathrm{d}\tilde{\sigma}_2^2}{\epsilon_1^2\rho_1^2\epsilon_2^2\rho_2^2} + \mathrm{d}\rho_3^2 + \rho_3^2\,\mathrm{d}\psi^2 + \mathrm{d}x_6^2 + \mathrm{d}x_7^2 + \right.$$
$$\left. + \frac{\mathrm{d}x_8^2}{1+\epsilon_1^2\rho_1^2} + \frac{\mathrm{d}x_9^2}{1+\epsilon_2^2\rho_2^2} + \frac{\mathrm{d}x_{10}^2}{(1+\epsilon_1^2\rho_1^2)(1+\epsilon_2^2\rho_2^2)}\right] ,$$

$$B = \frac{\epsilon_1^2\rho_1^2}{1+\epsilon_1^2\rho_1^2}\,\mathrm{d}x_8 \wedge \mathrm{d}x_{10} \,,$$ ← from NS5

$$e^{-\Phi} = \frac{\epsilon_2\rho_2}{\epsilon_1\rho_1}\sqrt{\frac{1+\epsilon_1^2\rho_1^2}{1+\epsilon_2^2\rho_2^2}} \,,$$

from D5 →

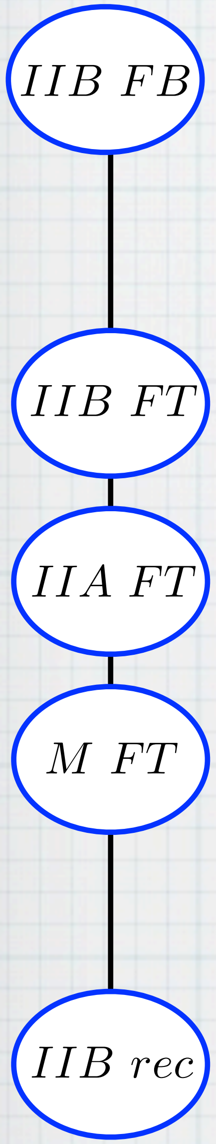
$$C_2 = \frac{\epsilon_2^2\rho_2^2}{1+\epsilon_2^2\rho_2^2}\,\mathrm{d}x_9 \wedge \mathrm{d}x_{10} \qquad \tilde{\sigma}_2 = \tilde{\sigma}_2 + 2\pi\alpha'\epsilon_2$$

The bulk is the backreaction of the near horizon limit of an NS5 and a D5 brane.

String coupling constant: $g_{IIB}^{\mathrm{rec}} = g_{IIA}^{\mathrm{rec}} l_{\mathrm{rec}} \epsilon_2 = \dfrac{\epsilon_2}{\epsilon_1}$

Under S duality, ϵ_1 is exchanged with ϵ_2, which amounts to swapping the D5 with the NS5.

A Chain of Dualities: Branes



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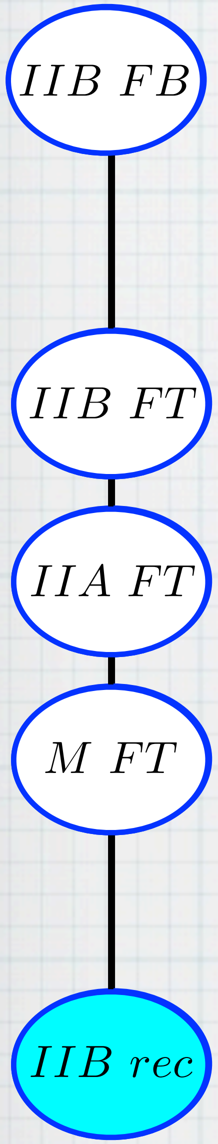


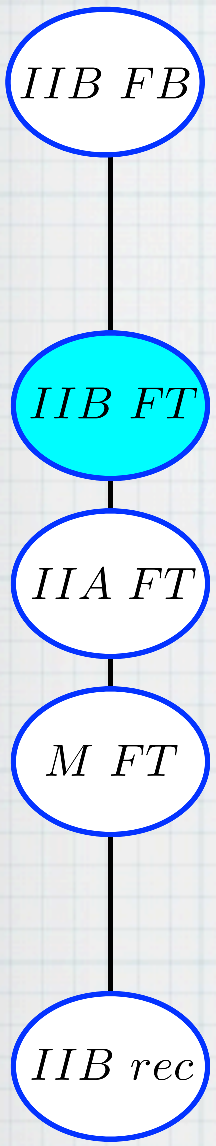
Start from M5 brane extended in $(\rho_1, \rho_2, \sigma_1, \sigma_2, x_6, x_{10})$
$\mathbb{R}^2 \times T^2 \times T^2$

Ω deformed $\mathcal{N} = 4$ SYM:
D3 branes extended in
$(\rho_1, \phi_1, \rho_2, \phi_2)$ in fluxtrap

Reciprocal gauge theory:
D3 branes in reciprocal BG
extended in $(\rho_1, \rho_2, x_6, x_{10})$

Effective 4D gauge theories on D3 branes in different duality frames/bulk backgrounds.

A Chain of Dualities: Branes



Ω deformed $\mathcal{N}=4$ SYM: D3 branes extended in $(\rho_1, \phi_1, \rho_2, \phi_2)$ in fluxtrap bg

3 cplx scalars describing motion of D3:

$$w(\xi) = \frac{\rho_3\, \mathrm{e}^{\mathrm{i}\,\psi}}{\pi\alpha'}\,, \qquad z(\xi) = \frac{x_6 + \mathrm{i}\, x_7}{\pi\alpha'}\,, \qquad \varphi(\xi) = \frac{x_8 + \mathrm{i}\, x_9}{\pi\alpha'}$$

DBI action for one D3 brane:

$$\mathscr{L}_\Omega = \frac{1}{4 g^2_{\mathrm{YM}}} \Big[F_{ij}F^{ij} + \frac{1}{2}\left(\partial^i\varphi + V^k F_k{}^i\right)\left(\partial_i\bar\varphi + \bar V^k F_{ki}\right) - \frac{1}{8}\left(\bar V^i\,\partial_i\varphi - V^i\,\partial_i\bar\varphi + V^k \bar V^l F_{kl}\right)^2$$

$$+ \frac{1}{4}\left(\delta^{ij} + V^i\bar V^j\right)\left(\partial_i z\,\partial_j\bar z + \text{c.c.}\right) + \frac{1}{4}\left(\delta^{ij} + V^i\bar V^j\right)\left(\partial_i w\,\partial_j\bar w + \text{c.c.}\right)$$

$$+ \frac{1}{2\,\mathrm{i}}\left(\epsilon_3 \bar V^i + \bar\epsilon_3 V^i\right)\left(\bar w\,\partial_i w - \text{c.c.}\right) + \frac{1}{2}\left|\epsilon_3\right|^2 w\bar w \Big]$$

new kinetic term

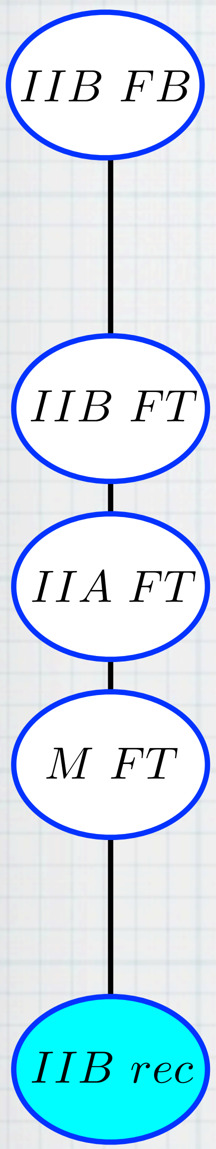
mass term

$$V = \epsilon_1\left(\xi^0\,\partial_1 - \xi^1\,\partial_0\right) + \mathrm{i}\,\epsilon_2\left(\xi^2\,\partial_3 - \xi^3\,\partial_2\right) \qquad g^2_{\mathrm{YM}} = 2\pi g^\Omega_{\mathrm{IIB}}$$

one derivative term breaks Lorentz invariance

A Chain of Dualities: Branes

Reciprocal gauge theory:



D3 branes in reciprocal BG extended in $(\rho_1, \rho_2, x_6, x_{10})$

Supersymmetric non Lorentz invariant gauge theory

Embedding $\rho_1 = y_1 \, , \qquad \rho_2 = y_2 \, , \qquad x_6 = y_3 \, , \qquad x_{10} = y_4$

Dynamics described by

$$U_1 + \mathrm{i}\, U_2 = \frac{\rho_3\, \mathrm{e}^{\mathrm{i}\,\psi}}{2\pi\alpha'} \, , \quad U_3 = \frac{x_7}{2\pi\alpha'} \, , \quad U_4 = \frac{\tilde{\sigma}_2}{2\pi\alpha'} \, , \quad U_5 = \frac{x_8}{2\pi\alpha'} \, , \quad U_6 = \frac{x_9}{2\pi\alpha'}$$

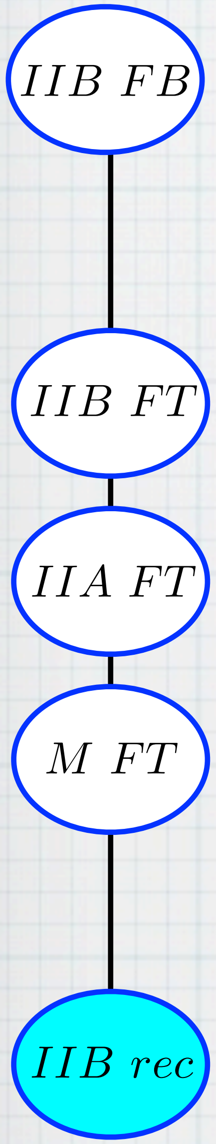
Effective action for D3 brane: from B field, from C field

$$\mathscr{L}_{\text{rec}} = \frac{y_2}{8\pi y_1} F_{kl} F_{kl} + \frac{\epsilon_1^2 y_1 y_2}{4\pi} \Bigg[\sum_{k=1}^{3} \left(F_{k4} - \partial_k U_5\right)^2 + \frac{1}{\Delta_2^2} \sum_{k=1}^{3} \left(\mathrm{i}\, \frac{\epsilon_2}{\epsilon_1} \frac{y_2}{y_1} (*F)_{k4} - \partial_k U_6 \right)^2$$

$$+ \tau^{kl}(\xi) h^{ij}(\xi)\, \partial_k U_i\, \partial_l U_j + \Delta_2^2 (\partial_4 U_5)^2 + \Delta_1^2 (\partial_4 U_6)^2 + \left(y_1^{-2} + y_2^{-2}\right) \left(U_1^2 + U_2^2\right) \Bigg]$$

$$\tau^{kl}(\xi) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & \Delta_1^2 \Delta_2^2 \end{pmatrix} \, , \quad h^{ij}(\xi) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & (\epsilon_1 y_1)^{-2} (\epsilon_2 y_2)^{-2} \end{pmatrix}$$

A Chain of Dualities: Branes



Define gauge kinetic tensor:

$$\mathscr{L}_g = M^{ijkl} F_{ij} F_{kl}$$

Define scalar gauge coupling:

$$\frac{1}{g_{eff}^2} = \sqrt{\frac{2}{3}} ||M||_K = \sqrt{\frac{2}{3} \epsilon_{ijkl} \epsilon_{i'j'k'l'} M^{iji'j'} M^{klk'l'}}$$

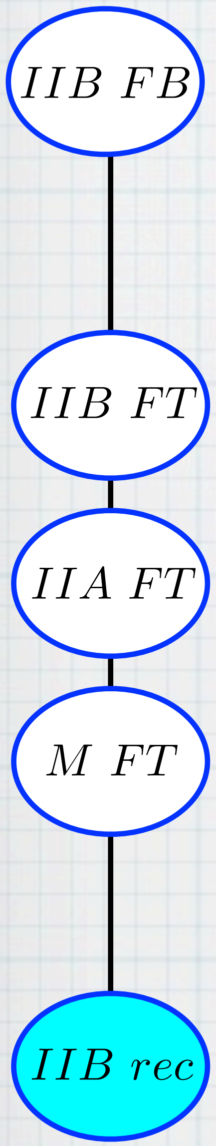
Gauge coupling in reciprocal gauge theory:

$$\frac{1}{g_{rec}^2} = \frac{1}{2\pi} \frac{y_2 \sqrt{1+\epsilon_1^2 y_1^2}}{y_1 \sqrt{1+\epsilon_2^2 y_2^2}}$$

Far away from the branes (large y limit), this reduces to

$$g_{rec}^2 \xrightarrow[y_1, y_2 \to \infty]{} 2\pi \frac{\epsilon_2}{\epsilon_1}$$

A Chain of Dualities: Branes



The S dual of the Lagrangian is defined as follows:

$$\mathscr{L}_{\text{dual}} = \frac{1}{16\pi^2}(M^{-1})^{ijkl}(*F)_{ij}(*F)_{kl}$$

$$\mathscr{L}_{\text{dual}}(\epsilon_1,\epsilon_2) = \frac{y_1}{4\pi y_2}\left[\frac{(*F)_{4k}(*F)_{k4}}{1+\epsilon_1^2 y_1^2} + F_{k4}F_{k4}\left(1+\epsilon_2^2 y_2^2\right)\right]$$

By comparison we find

$$\mathscr{L}_{\text{dual}}(\epsilon_1,\epsilon_2) = \mathscr{L}_g(\epsilon_2,\epsilon_1)$$

Define Liouville parameter as $b^2 = \frac{\epsilon_2}{\epsilon_1}$

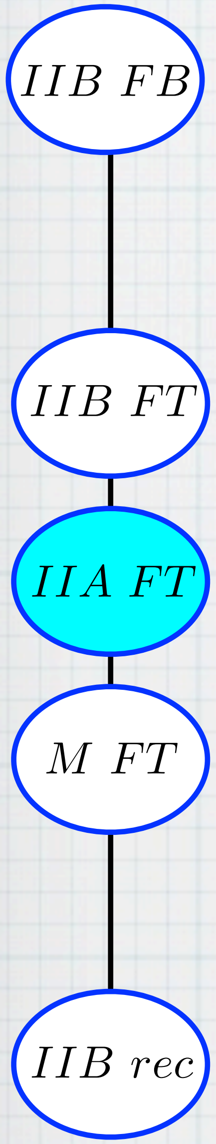
The reciprocal gauge theory displays several features of Liouville field theory:

- Asymptotic coupling constant is proportional to b^2
- S duality exchanges $b \leftrightarrow 1/b$, like Liouville duality which exchanges perturbative and instanton spectrum

BPS States in the Duality Web

BPS States in the Duality Web

In Omega deformed SYM: 2 sets of BPS objects



BPS instanton configurations in IIB and particles in IIA, localized at the origin

oscilloids

$$\mathscr{L} = -\mu_0 e^{-\Phi} = -\frac{1}{g^{\Omega}_{IIA} l_\Omega}\sqrt{\left(1+\epsilon_1^2\rho_1^2\right)\left(1+\epsilon_2^2\rho_2^2\right)}$$

$$E_{D0} = \frac{n_{D0}}{g^{\Omega}_{IIA} l_\Omega}\sqrt{\left(1+\epsilon_1^2\rho_1^2\right)\left(1+\epsilon_2^2\rho_2^2\right)}$$

Give rise to the modular form defining the holomorphic factor of the Liouville partition function

perturbative modes of the fundamental string stretching between two D4 branes carrying angular momentum along the $U(1)\times U(1)$ rotational isometries localized at infinity

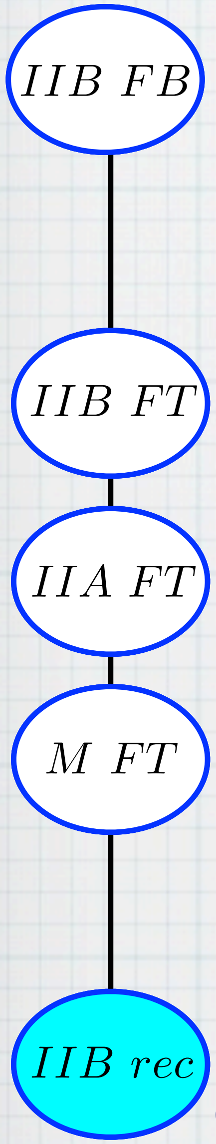
DOZZoids

$$E^2 = \left(\epsilon_2 J_2 - \frac{L_2}{2\pi\alpha'}\right) + \frac{J_2^2}{\rho_2^2}$$

Reproduce the holomorphic DOZZ factors of the Liouville partition function

BPS States in the Duality Web

Follow BPS states along chain of dualities to reciprocal frame.

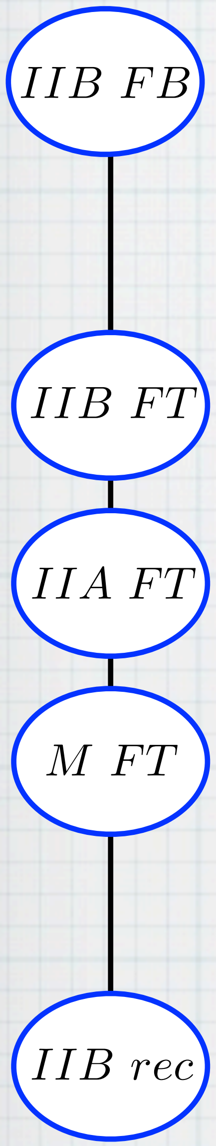


oscilloids		DOZZoids
D0 branes in fluxtrap at origin	type IIA	fundamental string extended in x_9 with momentum in ϕ_2
	↓	
momentum mode in x_{10}	M theory	M2 brane with momentum in ϕ_2
	↓	
momentum mode in x_{10}	reciprocal frame	D3 brane in $(x_6, x_9, x_{10}, \tilde{\sigma}_2)$ with el. flux in $\tilde{\sigma}_2$

Now only one kind of BPS state carrying both types of charge and living at a finite radius.

BPS States in the Duality Web

Follow the BPS states along the chain of dualities:



frame	object	ρ_1	ϕ_1	ρ_2	ϕ_2	ρ_3	ψ	x_6	x_7	x_8	x_9	x_{10}
IIA fluxtrap	D4	$\times$	$\times$	$\times$	$\times$			$\times$			$\times$	$\blacksquare$
	F1				$\circlearrowleft$			$\times$			$\times$	$\blacksquare$
	D0							$\times$				$\blacksquare$
M–theory	M5	$\times$	$\times$	$\times$	$\times$			$\times$			$\times$	
	M2				$\circlearrowleft$			$\times$			$\times$	$\times$
	momentum							$\nearrow$				$\nearrow$
reciprocal frame	D3	$\times$	$\blacksquare$	$\times$				$\times$				$\times$
	D3		$\blacksquare$		$\times$			$\times$			$\times$	$\times$
	momentum		$\blacksquare$					$\nearrow$				$\nearrow$
	D5		$\blacksquare$	$\times$				$\times$	$\times$	$\times$	$\times$	$\times$
	NS5	$\times$	$\blacksquare$					$\times$	$\times$	$\times$	$\times$	$\times$

angular momentum

extended in direction

dynamical branes

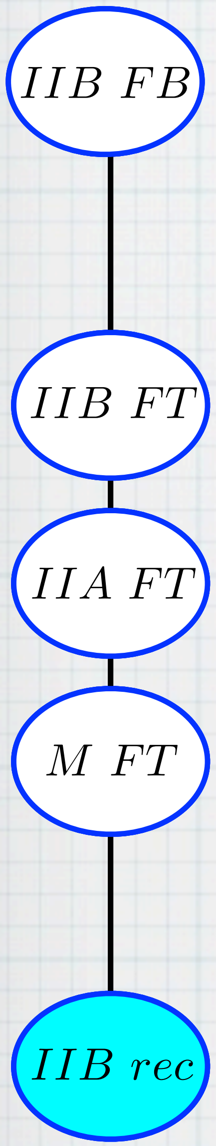
momentum

nongeometrical direction

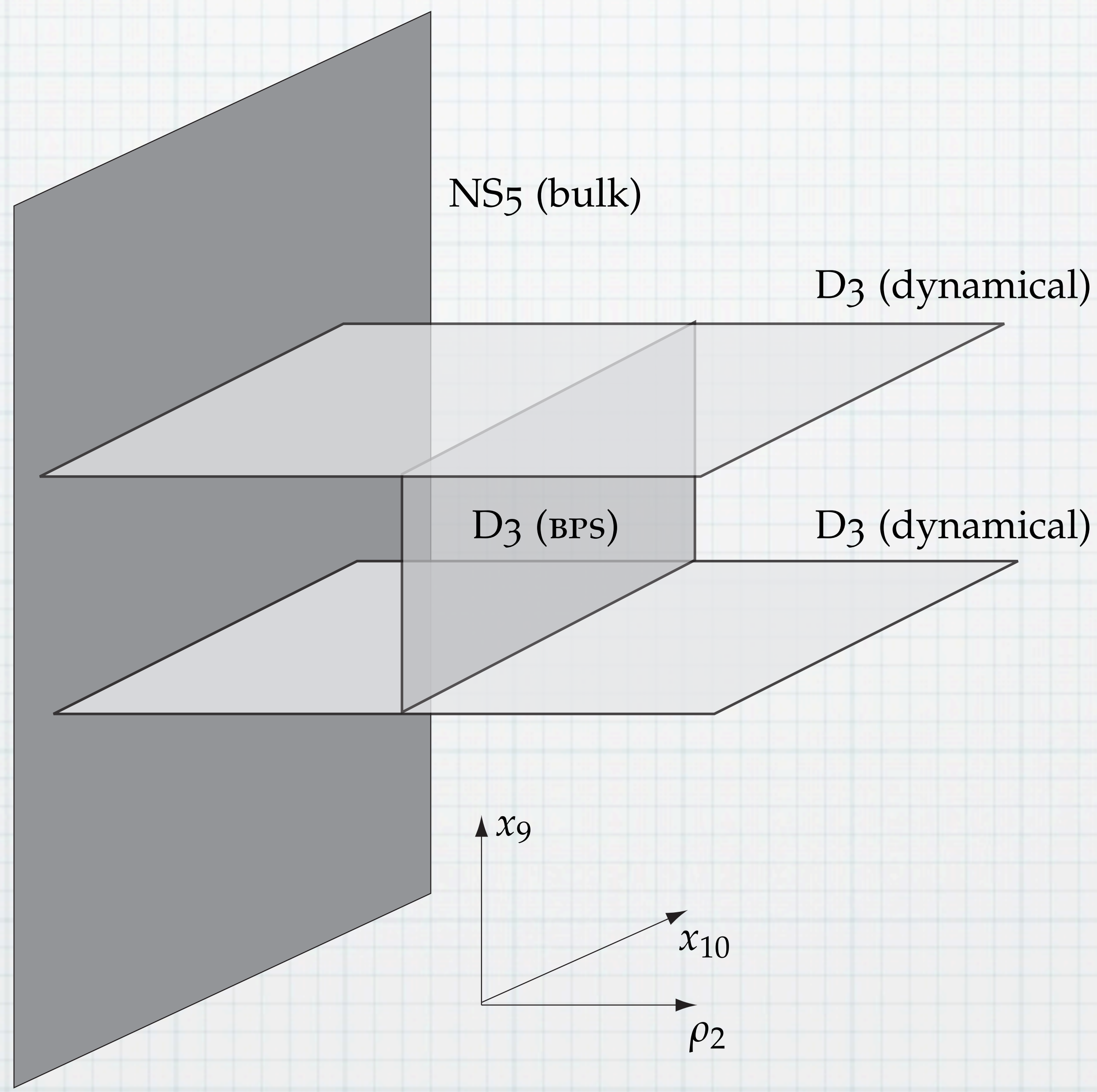
BPS excitations

non dynamical

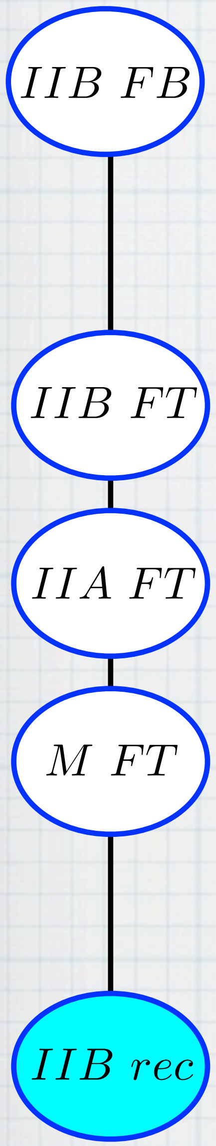
BPS States in the Duality Web



Extended objects in the reciprocal frame:



BPS States in the Duality Web



Introduce a D3 brane extended in $(x_6, x_9, x_{10}, \tilde{\sigma}_2)$ with an electric field in $\tilde{\sigma}_2$, velocity v in x_{10} and another component of the electric field in x_{10}, which is required by the coupling of v in the DBI action.

$$x_6 = i\zeta_0 \ , \qquad x_9 = \frac{L_2}{\pi R_{WS}}\zeta_1 \ , \quad \tilde{\sigma}_2 = 2\pi\alpha'\epsilon_2\frac{\zeta_2}{\kappa} \ , \quad x_{10} = \zeta_3 + v\zeta_0 \ , \quad x_8 = 0 \ ,$$

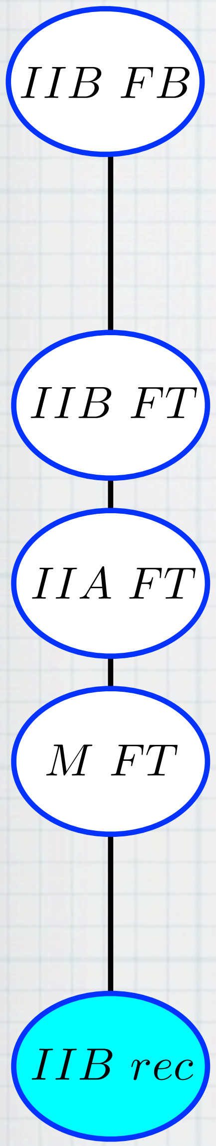
$$\rho_1 = \text{const.} \ , \quad \rho_2 = \text{const.} \ , \qquad \rho_3 = 0 \ , \qquad x_7 = 0$$

$$F_{02} = \frac{1}{\kappa}f_{02} \ , \qquad\qquad F_{03} = \frac{1}{\kappa}f_{03}$$

$$S = -\mu_3\int \mathrm{d}^4\zeta e^{-\Phi}\sqrt{-\det(g + B + 2\pi\alpha' F)} + \mu_3\int \exp(B + 2\pi\alpha' F)\wedge\sum_q C_q$$

$$= \frac{\epsilon_1 L_2 R_{10}}{2\pi\alpha'}\frac{\epsilon_2\rho_2^2\left(f_{02} + v f_{03}\right) - \sqrt{\left(1 + \epsilon_2^2\rho_2^2\right)\left(1 + \rho_2^2 f_{03}^2\left(1 + \epsilon_1^2\rho_1^2\right)\right) - \rho_2^2\left(f_{02} + v f_{03}\right)^2}}{1 + \epsilon_2^2\rho_2^2}$$

$$J_2 = \frac{\delta S}{\delta f_{02}} \ , \qquad D_3 = \frac{\delta S}{\delta f_{03}} \ , \qquad P = \frac{\delta S}{\delta v}$$



BPS States in the Duality Web

From the Hamiltonian, we find

$$E^2 = \left[\left(\frac{\epsilon_1 R_{10}}{2\pi\alpha'}L_2 - \epsilon_2 J_2\right)^2 + \left(\frac{J_2}{\rho_2}\right)^2\right]\left[1 + \rho_2^2\left(1+\epsilon_1^2\rho_1^2\right)\left(\frac{P}{J_2}\right)^2\right].$$

This is minimized for

$$\rho_2^2 = \frac{J_2^2}{P\left(\frac{\epsilon_1 R_{10}}{2\pi\alpha'}L_2 - \epsilon_2 J_2\right)}$$

Energy of the BPS states:

$$E_{BPS} = \left|\frac{\epsilon_1 R_{10}}{2\pi\alpha'}L_2 - \epsilon_2 J_2 + P\right|$$

$L_2 = J_2 = 0$ → oscilloids

$$E_{\rm osc} = P$$

$P = 0$ → DOZZoids

$$E_{\rm DOZZ} = \left|\frac{\epsilon_1 R_{10}}{2\pi\alpha'}L_2 - \epsilon_2 J_2\right|$$

$$E_{BPS} = E_{\rm osc} + E_{\rm DOZZ}$$

IIB FB

IIB FT

IIA FT

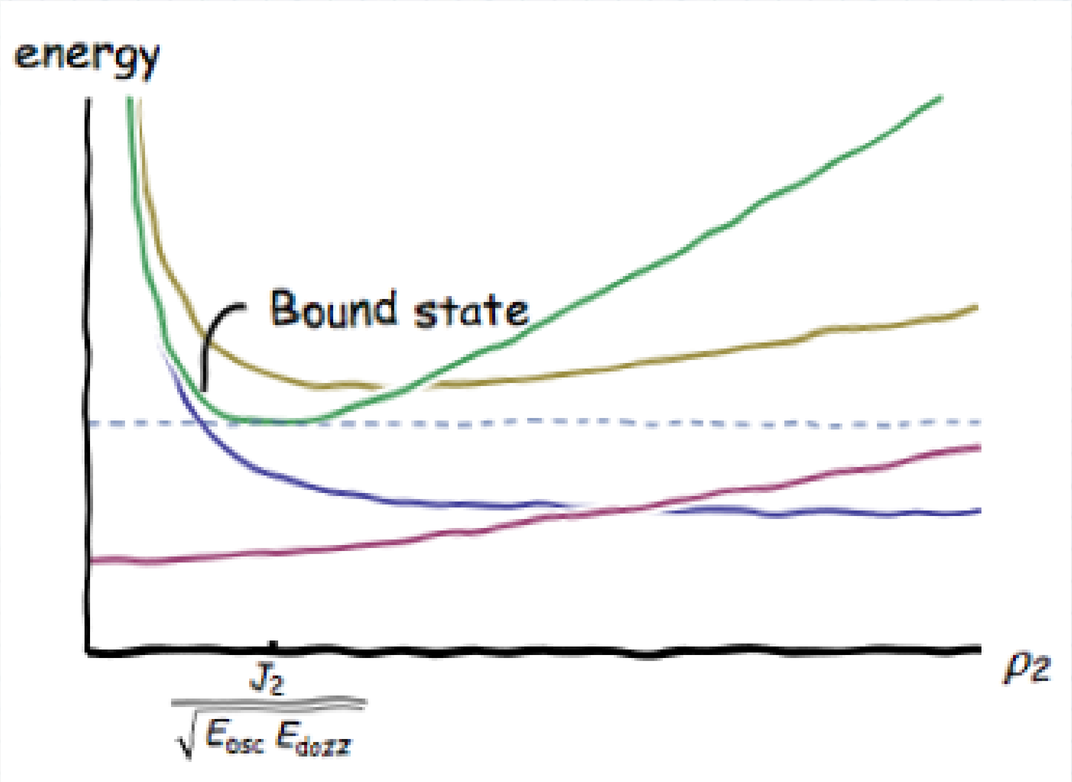
M FT

IIB rec

BPS States in the Duality Web

States are only marginally stable with respect to their decay into separate oscilloids and DOZZoids.

Any decay would have to tunnel over a barrier, since a state that is broken apart locally would have an energy that is strictly larger than the one of the bound state. Any such decay process would have to be nonperturbatively suppressed.



Summary

Summary

We described the Omega BG and its various limits via a fluxtrap BG in string theory.

Allows us to study different gauge theories of interest via string theoretic methods.

Omega deformation and twisted mass deformation have same origin.

Gives a geometrical interpretation for the Omega BG and its properties, such as localization etc.

Understanding of relation between deformation parameters and quantization of spectral curve.

Summary

Progress towards understanding the (2,0) theory in 6D in Omega BG.

Studied two limits of (2,0) theory linked by a chain of dualities.

Compactify on torus: Omega deformed N=4 SYM on $\mathbb{R}^4$

Compactify on angular directions: Reciprocal gauge theory on $\mathbb{R}^2_+ \times T^2$

Reciprocal gauge theory displays properties of Liouville field theory:

- Asymptotic coupling constant is proportional to $b^2 = \frac{\epsilon_2}{\epsilon_1}$
- S duality exchanges $b \leftrightarrow 1/b$, like Liouville duality which exchanges perturbative and instanton spectrum

Important step towards realization of Liouville theory from string theory.

Summary

BPS states appearing on the SYM side of the AGT correspondence: fundamental strings and D0 branes in type IIA fluxtrap

Reciprocal frame: D3 branes carrying both electric fields and momentum in the 10 direction, localized at a finite value of the radial direction

Two distinct types of states localized in different positions (the origin and infinity) in one duality frame.

States carrying both charges localized in one place in the other duality frame.

Presence of new phenomena which are only accessible via a microscopic description.

Outlook

The reciprocal theory is intrinsically fourdimensional.

Aim: construct Liouville field theory as compactification of an M5 brane in M theory fluxtrap background.

Start from a geometry of the type $S^4_\Omega \times \Sigma$ to be able to reduce on the 4d part and realize a 2d theory on the Riemann surface.

Thank you for your attention!