Squashed group manifolds in String Theory

brane realization and classical integrability

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Outline

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Squashed geometries

- String theory provides a dual description of the same physics: **worldsheet** and **target space**
- In very few cases we can access both and learn from both sides
- Most of these cases (flat spacetime, group manifolds, plane waves) have a **high degree of symmetry**
- More symmetry means **more structure** (and simpler analysis)
- Can we break part of this symmetry, at the same time preserving the "nice" structures?

Three dimensional gravity

- **Three–dimensional gravity** provides a simple laboratory for quantum gravity
- There are no propagating gravitons
- There are non-trivial solutions (BTZ black holes)
- Pure gravity solutions are always **locally** AdS_3
- Anti–de Sitter spaces appear in **near-horizon geometries** of various D–brane configurations
- AdS_3 appears as an **exact string theory background** (Wess–Zumino–Witten model)

TMG

- Three–dimensional AdS spaces also appear as solutions of **topologically massive gravity** (TMG)

$$S_{tmg}(g) = \frac{1}{16\pi G}\int d^3x\,\sqrt{-g}\left(R + \frac{2}{\ell^2}\right) + \mu\, S_{CS}$$

- for $\mu \neq 1$ the anti–de Sitter solution is unstable, but there are **two warped solutions** with metric of the type

$$ds^2[WAdS_3] = R^2\left[d\omega^2 - \cosh^2\omega\, d\tau^2 + \frac{1}{\cosh^2\Theta_w}\left(d\beta + \sinh\omega\, d\tau\right)^2\right]$$

where Θ_w is a deformation parameter:

  - for $\Theta_w = 0$ this is AdS_3
  - for $\Theta_w \to \infty$ this is $AdS_2 \times S^1$

Today's talk

- **Geometry** of squashed groups in general (and AdS_3 in particular)
- **T–duality** acts on principal fibrations
- **Type II** solutions with squashed AdS_3 and S^3
- **Integrability** (without RR fields)

Outline

The Geometry – Group Manifold

- Consider a **group manifold** G (e.g. AdS_3 or S^3)
- There is a bi-invariant metric such that the isometry group is $G \times G$
- The **metric** can be written **in terms of the currents** J_a that generate half of the symmetry

$$\mathrm{d}s^2[G] = \sum_{a=1}^{\dim G} J_a \otimes J_a$$

- If $H \subset G$ is compact, G is the total space of a **Hopf fibration** over G/H:

$$\begin{array}{ccc} H & \longrightarrow & G \\ & & \downarrow \\ & & G/H \end{array}$$

- Today $H = U(1)$ and $G/H = S^2, \mathrm{AdS}_2$.

The Geometry – Squashed group

- SqG is a deformation of G described by the same currents
- The symmetry group is $G(\mathbb{R}) \times U(1)$ (only left-invariance)
- The metric can be written in terms of the **same J_a currents**. Fix $\dim G = 3$

$$ds^2[\mathrm{SqG}] = J_1 \otimes J_1 + J_2 \otimes J_2 + \frac{1}{\cosh^2 \Theta_w} J_3 \otimes J_3$$

- SqG is the base space for a fibration that has $G \times S^1$ total space

$$\begin{array}{ccc} S^1 \times S^1 & \longrightarrow & G \times S^1 \\ & & \downarrow \\ & & G/U(1) \end{array} \quad \xrightarrow{\text{choose embedding}} \quad \begin{array}{ccc} S^1 & \longrightarrow & G \times S^1 \\ & & \downarrow \\ & & \mathrm{SqG} \end{array}$$

the **embedding of** $S^1 \hookrightarrow S^1 \times S^1$ is described by Θ_w

T-duality to "undo" fibrations

- Construct NLSM on squashed groups via T-duality from principal chiral models with group manifold target space G
- We are not considering here conformal models: we start with the **metric only** (a B-field will appear)
- Main observation: if the space has a S^1 fibration structure

$$\begin{array}{ccc} S^1 & \longrightarrow & M \\ & & \downarrow \\ & & N \end{array}$$

- T-duality along the fiber will **"undo" the fibration** and give a direct product

$$\widetilde{M} = N \times S^1$$

T-duality on the worldsheet

- If there is a S^1 fibration, the action can be written as

$$S[u^i, z] = \int_\Sigma G_{ij}(u)\, du^i \wedge *du^j + \left(dz + f_i(u)\, du^i\right) \wedge *\left(dz + f_j(u)\, du^j\right) ,$$

which is to say, the metric has a block form

$$\left(\begin{array}{c|c} G_{ij}(u) + f_i(u) f_j(u) & f_i(u) \\ \hline f_i(u) & 1 \end{array}\right) .$$

- we want to T-dualize the S^1 described by z.

T-duality on the worldsheet

- introduce a gauge field A and a Lagrange multiplier

$$S[u^i, A, \tilde{z}] = \int_\Sigma G_{ij}(u)\, du^i \wedge *du^j + \left(A + f_i(u)\, du^i\right) \wedge *\left(A + f_i(u)\, du^i\right) - 2\tilde{z}\, dA\,.$$

- The EOM for $\tilde{z}$ give

$$dA = 0 \quad \Rightarrow \quad A = dz\,,$$

which leads back to the original action

- The EOM for A gives

$$*d\tilde{z} = A + f_i(u)\, du^i = dz + f_i(u)\, du^i\,.$$

T-duality on the worldsheet

- The resulting action describes a **direct product metric** plus a B **field**

$$S[u^i, \tilde{z}] = \int_\Sigma G_{ij}(u)\, du^i \wedge *du^j + d\tilde{z} \wedge *d\tilde{z} - 2\, d\tilde{z} \wedge f_i(u)\, du^i$$

- as promised

$$\begin{array}{ccc} S^1 & \longrightarrow & M \\ & & \downarrow \\ & & N \end{array} \qquad \xrightarrow[\text{T-duality}]{} \qquad \widetilde{M} = N \times S^1$$

Squashed groups

- start with

$$
\begin{array}{ccc}
U(1) & \longrightarrow & G \times U(1) \\
 & & \downarrow \\
 & & \mathrm{SqG}
\end{array}
$$

  the fiber is a linear combination of the U(1) and one direction in the Cartan of G

- The metric on SqG is

$$\mathrm{d}s^2[\mathrm{SqG}] = \mathrm{d}s^2[G] + \tanh^2 \Theta \, j_C \otimes j_C \,,$$

  where Θ measures the combination of the two U(1):
  - for $\Theta \to 0$, $\mathrm{SqG} = G$
  - for $\Theta \to \infty$, $\mathrm{SqG} = G/\mathrm{U}(1) \times \mathrm{U}(1)$.

Outline

Initial setup

- Consider the superposition of a D_1/D_5 system with a **magnetic monopole** and a **plane wave**
- This is the T–dual of the $D = 4$ extremal dyonic black string.
- The field content is

$$ds^2 = H_1^{1/2} H_5^{1/2} \left(H_1^{-1} H_5^{-1} \left(du\, dv + K\, du^2 \right) + H_5^{-1} \left(dy_1^2 + \dots dy_4^2 \right) + \right.$$
$$\left. + V^{-1} \left(d\psi + A_i\, dx^i \right)^2 + V \left(dx_1^2 + \dots dx_3^2 \right) \right)$$

$$e^{2\varphi} = H_1^{-1} H_5 \,, \qquad F_{[3]} = H_1^{-1}\, dt \wedge du \wedge dv - B_i\, dx^i \wedge d\psi$$

where $H_1(x), H_5(x), K(x), V(x), A_i(x), B_i(x)$ are harmonic functions of the transverse coordinates $x_i, i = 1, 2, 3$ and

$$dB = - * dH_5 \,, \qquad dA = - * dV$$

Initial setup

- Consider the near–horizon limit

$$ds^2 = Q_m Q_1^{1/2} Q_5^{1/2} \left(- d\tau^2 + d\omega^2 + Q_w d\sigma^2 + 2 Q_w{}^{1/2} \sinh\omega \, d\sigma \, d\tau \right) + Q_m Q_1^{1/2} Q_5^{1/2} \left(d\theta^2 + d\varphi^2 + d\psi^2 + 2\cos\theta \, d\psi \, d\varphi \right) + Q_1^{1/2} Q_5^{-1/2} \left(dy_1^2 + \cdots + dy_4^2 \right) ,$$

$$F_{[3]} = Q_m Q_1^{1/2} Q_5^{1/2} \left(\cosh\omega \, d\tau \wedge d\omega \wedge d\sigma + \sin\theta \, d\varphi \wedge d\psi \wedge d\theta \right) .$$

- The geometry is $AdS_3 \times S^3 \times T^4$
- The radii are fixed by the monopole charge Q_m (quantized)
- The plane wave charge appears in the AdS_3 part

Hopf–T–duality

- We want to use the fact that

$$\begin{array}{ccc} S^1 & \longrightarrow & AdS_3 \times S^1 \\ & & \downarrow \\ & & WAdS_3 \end{array}$$

- Single out a $AdS_3 \times S^1$ part from the ten-dimensional geometry
- Implement **Hopf–T–duality**.
  If the geometry is the total space for a S^1 fibration and there are **only Ramond–Ramond fields**, the **T–dual along the fiber has geometry** $B \times S^1$.

$$\begin{array}{ccc} S^1 & \longrightarrow & E \\ & & \downarrow + \text{RR fields} \quad \xrightarrow{\text{T–dual}} \quad \left(B \times S^1\right) + \text{RR and NS fields} \\ & & B \end{array}$$

Hopf–T–duality

- Starting from $AdS_3 \times S^1$:

$$ds^2 = R^2 \left(- d\tau^2 + d\omega^2 + Q_w d\sigma^2 + 2\sqrt{Q_w} \sinh\omega \, d\sigma \, d\tau \right) + \sqrt{\frac{Q_1}{Q_5}} dy_1^2$$

$$F_{[3]} = R^2 \cosh\omega \, d\tau \wedge d\omega \wedge d\sigma$$

T–duality

- we obtain the metric we want: $WAdS_3 \times S^1$

$$ds^2 = R^2 \left[d\omega^2 - \cosh^2\omega \, d\tau^2 + \frac{1}{\cosh^2\Theta_w} \left(d\beta + \sinh\omega \, d\tau \right)^2 \right] + d\zeta_w^2$$

$$F_4 = \frac{R^2}{\cosh^2\Theta_w} \cosh\omega \, d\omega \wedge d\tau \wedge d\beta \wedge d\zeta_w ,$$

$$F_2 = R \tanh\Theta_w \cosh\omega \, d\omega \wedge d\tau ,$$

$$H_3 = R \tanh\Theta_w \cosh\omega \, d\omega \wedge d\tau \wedge d\zeta_w .$$

Observations

- Having started from a pure RR background, the geometry is **globally $WAdS_3$**. We can get **orbifolds by adding NS fluxes** in the initial background
- The **parameters** of the solution are understood **in terms of charges**:

$$R^2 = Q_m \sqrt{Q_1 Q_5} \qquad \sinh^2 \Theta_w = 4 Q_w Q_m Q_5$$

- the quantization of the deformation corresponds to the quantization of the linear combination of the fibers.
- The group $SL_2(\mathbb{R})$ has **three different types of generators**. Each can be chosen for this construction and lead to **different geometries**.
- The same construction can be used to describe other squashed groups (simplest case: squashed S^3)

Outline

Currents and EOM

- Consider the PCM for a group G

$$S = -\frac{1}{2} \int_{\Sigma} \mathrm{Tr}[\mathrm{d}g(x,t) \wedge *\mathrm{d}g^{-1}(x,t)] ,$$

where g is a map from the worldsheet to the group $g : \Sigma \to G$

- The equations of motion are

$$\mathrm{d}*(g^{-1}\,\mathrm{d}g) = \mathrm{d}*(\mathrm{d}g\,g^{-1}) = 0 .$$

- these are the conservation laws for two currents

$$j = g^{-1}\,\mathrm{d}g , \qquad \bar{j} = -\,\mathrm{d}g\,g^{-1} .$$

- the currents are flat and thus fulfill the *Maurer–Cartan* (MC) equations:

$$\mathrm{d}j + j \wedge j = 0 , \qquad \mathrm{d}\bar{j} + \bar{j} \wedge \bar{j} = 0 .$$

- **Conservation and flatness are the reasons for the integrability.**

Lax current

- We will consider only the left currents. The right side works in the same way
- Introduce the **one-parameter family** of currents

$$J(x,t;\zeta) = -\frac{\zeta}{1-\zeta^2}\left(\zeta\, j(x,t) + *j(x,t)\right)$$

where $\zeta \in \mathbb{C}$ is the *spectral parameter*.

- The flatness of J is an equation for the components (J_x, J_t), the so-called **Lax equations**:

$$\partial_t J_x - \partial_x J_t + [J_t, J_x] = 0$$

this is a **Lax Pair**.

Lax current

- The flatness of J and $\bar{J}$ implies both the EOM and the MC equations.
- Conversely, imposing the EOM and MC equations results in the flatness of the currents.
- This can be easily verified by observing that

$$\mathrm{d}J(\zeta) + J(\zeta) \wedge J(\zeta) = \frac{\zeta}{\zeta^2 - 1} \left(\mathrm{d}{*}j + \zeta \left(\mathrm{d}j + j \wedge j \right) \right) .$$

- We started with a conserved current. Its flatness implies the **existence of a one-parameter family of flat currents**.
- Algebraically we passed from $j \in \mathfrak{g}$ to the **loop algebra** $J \in \mathfrak{g} \otimes \mathbb{C}[\zeta, \zeta^{-1}]$
- We literally have **infinitely more currents** (after Fourier transform).

Wilson line

- We have constructed infinite currents. Where are the **conserved charges**?
- Since $J(\zeta)$ is flat we can introduce a **Wilson line** as the path-ordered exponential

$$W(x,t|x_0,t_0;\zeta) = \mathrm{P}\left\{\exp\left[\int_{\mathcal{C}:(x_0,t_0)\to(x,t)} J(\xi,\tau;\zeta)\right]\right\},$$

and

$$J(x,t;\zeta) = W^{-1}(x,t;\zeta)\,\mathrm{d}W(x,t;\zeta)\,.$$

- This generalizes the relation $j = g^{-1}\,\mathrm{d}g$ to the loop algebra.
- For spin chain experts: this is the **transfer matrix**.

Wilson loop and conserved charges

- We can now define a **one-parameter family of conserved charges**:

$$Q(t;\zeta) = W(\infty, t|-\infty, t;\zeta) = \mathrm{P}\left\{\exp\left[\int_{-\infty}^{\infty} J_x(x,t;\zeta)\,\mathrm{d}x\right]\right\}.$$

- note that Q goes "all around" the worldsheet. This is a **Wilson loop**
- Key point: using the **Lax equations** and with appropriate BC, the **one-parameter charge $Q(t;\zeta)$ is conserved**

$$\frac{\mathrm{d}}{\mathrm{d}t} Q(t;\zeta) = 0$$

- Expand on ζ and find an **infinite set of conserved charges Q_n**

$$Q(t;\zeta) = 1 + \sum_{n=0}^{\infty} \zeta^{n+1} Q^{(n)}(t).$$

for which

$$\frac{\mathrm{d}}{\mathrm{d}t} Q^{(n)}(t) = 0, \qquad \forall n = 0, 1, \dots .$$

Integrability of the PCM (in short)

- The principal chiral model has two **conserved currents** corresponding to the $G \times G$ **symmetry** of the action
- These currents are also **flat**
- Out of these one can construct two **one-parameter families of flat currents**
- The Wilson loops of these currents are time-independent
- The Fourier development gives **infinite conserved charges**
- These charges close under an infinite-dimensional algebra (**Yangian** or affine)
- the $\mathfrak{g} \oplus \mathfrak{g}$ symmetry of the action is the zero mode of the $\widehat{\mathfrak{g}} \oplus \widehat{\mathfrak{g}}$ symmetry of the equations of motion.

Integrability and squashed groups

- How much of this structure remains **after T–duality**?
- We have a **linear transformation of the current** components $J(\zeta) \mapsto \widetilde{J}(\zeta)$ that leaves the (on-shell) flatness conditions invariant:

$$d\widetilde{J} + \widetilde{J} \wedge \widetilde{J} = 0 \,,$$

- concretely we define T–dual Lax currents $\widetilde{J}(\zeta)$ by imposing the condition

$$* d\tilde{z} = dz + f_i(u) \, du^i \,.$$

- **Flatness is the key**. This is preserved: the system is **still integrable**.
- The condition is not local (mix time and space derivatives). The resulting **charges are all non-local** and **do dot correspond to isometries**.

A technical remark

- The current that we use for T–duality does not commute with the others
- some of the components of J depend explicitly on z, when we have an equation for dz.
- We need to perform a **gauge transformation**

$$J' = h^{-1} J h + h^{-1} \mathrm{d}h \,,$$

- after the transformation, the new current has a zero–mode (in the ζ expansion)

$$\widetilde{J}'(\zeta) = h^{-1} \mathrm{d}h - \Lambda(\zeta) h^{-1} j h \Big|_{\mathrm{d}z = *\mathrm{d}\tilde{z} - f_i(u)\, \mathrm{d}u^i} = \widetilde{J}'^{(0)} - \Lambda(\zeta) \tilde{j} \,.$$

- for the experts: in the hierarchies we will have to covariantize w.r.t. $\widetilde{J}'^{(0)}$.

The simplest example: Squashed three-sphere

$$\mathcal{J}^1(\zeta) = -\imath \Lambda(\zeta) \sin\theta \, d\varphi \,,$$

$$\mathcal{J}^2(\zeta) = -\imath \Lambda(\zeta) \, d\theta \,,$$

$$\mathcal{J}^3(\zeta) = -\imath \left[(1 + \Lambda(\zeta)) \left(\frac{d\alpha + \cos\theta \, d\varphi}{\cosh^2\Theta} + \tanh\Theta \, * d\tilde{z} \right) - \cos\theta \, d\varphi \right] ,$$

$$\mathcal{J}^4(\zeta) = -\imath \tanh\Theta \, \Lambda(\zeta) \left(* d\tilde{z} - \tanh\Theta \, (d\alpha + \cos\theta \, d\varphi) \right) \,,$$

and

$$\tilde{\mathcal{J}}^1(\zeta) = -\imath \Lambda(\zeta) \left[\frac{1}{\cosh^2\Theta} \cos\varphi \sin\theta \, d\alpha - \sin\varphi \, d\theta + \tanh^2\Theta \cos\varphi \sin\theta \cos\theta \, d\varphi + \tanh\Theta \cos\varphi \sin\theta \, * d\tilde{z} \right]$$

$$\tilde{\mathcal{J}}^2(\zeta) = \imath \Lambda(\zeta) \left[\frac{1}{\cosh^2\Theta} \sin\varphi \sin\theta \, d\alpha + \cos\varphi \, d\theta - \tanh^2\Theta \sin\varphi \sin\theta \cos\theta \, d\varphi + \tanh\Theta \sin\varphi \sin\theta \, * d\tilde{z} \right] ,$$

$$\tilde{\mathcal{J}}^3(\zeta) = \imath \Lambda(\zeta) \left[\frac{1}{\cosh^2\Theta} \cos\theta \, d\alpha + \left(1 - \tanh^2\Theta \cos^2\theta \right) d\varphi + \tanh\Theta \cos\theta \, * d\tilde{z} \right] ,$$

$$\tilde{\mathcal{J}}^4(\zeta) = \imath \tanh\Theta \, \Lambda(\zeta) \left(* d\tilde{z} - \tanh\Theta \, (d\alpha + \cos\theta \, d\varphi) \right) .$$

- we recover $SU(2) \times SU(2) \times U(1)$ currents even if **the isometry is** $SU(2) \times U(1) \times U(1)$
- this is promoted to affine when looking at the **non-local charges**

Symmetries of the squashed group model

- The action with squashed group target space is obtained via T–duality
- T–duality **preserves the integrable structure** of the PCM
- The **full** $\widehat{\mathfrak{g}} \oplus \widehat{\mathfrak{g}}$ symmetry is preserved
- Only **part of the zero-modes are realized as isometries** $\mathfrak{g} \oplus \mathfrak{u}(1)$
- The other zero modes are non-local
- **Adding RR fluxes** does not change the overall picture:
  - NS and R sectors are separated under T–duality
  - We already know (from the previous section) the expressions for the RR fields
  - The PCM + RR fields is integrable and has an infinite symmetry
  - This infinite symmetry will be preserved

The Green–Schwarz action

$$
\begin{aligned}
\mathcal{L}_{NS}^{SqS^3} = & -i\bar\theta^{1}\left(\sqrt{h}h^{ij}-\varepsilon^{ij}\Gamma_{11}\right)\Big\{\sum_{a=0}^{5} e_i{}^{a}\gamma_a\Big[\partial_j-\frac{1}{2R}\gamma^{5[4}e_j{}^{3]}\cos 2\varpi+\frac{1}{4R}\gamma^{34}\Big(e_j{}^{5}-2\cot\theta\, e_j{}^{4}\Big)\Big] \\
& \sum_{m=6}^{9} e_i{}^{m}\gamma_m\Big[\partial_j-\frac{1}{2R}\gamma^{9[4}e_j{}^{3]}\sin 2\varpi\Big]\Big\}\theta^{1} \\
& +i\bar\theta^{2}\left(\sqrt{h}h^{ij}-\varepsilon^{ij}\Gamma_{11}\right)\Big\{\sum_{a=0}^{z} e_i{}^{a}\gamma_a\Big[\partial_j+\frac{1}{2R}\gamma^{z[4}e_j{}^{3]}\cos 2\varpi-\frac{1}{4R}\gamma^{34}\Big(e_j{}^{z}-2\cot\theta\, e_j{}^{4}\Big)\Big] \\
& +\sum_{m=6}^{\bar z} e_i{}^{m}\gamma_m\Big[\partial_j-\frac{1}{2R}\gamma^{\bar z[4}e_j{}^{3]}\sin 2\varpi\Big]\Big\}\theta^{2} \\
& -i\bar\theta^{1}\left(\sqrt{h}h^{ij}-\varepsilon^{ij}\Gamma_{11}\right)\Big\{\sum_{a=0}^{5} e_i{}^{a}\gamma_a\Big[\partial_j+\frac{1}{4R}\left(\gamma^{5[4}e_j{}^{3]}-\gamma^{z[4}e_j{}^{3]}\right)-\frac{1}{4R}\gamma^{34}\Big(\frac{1-\gamma^{5z}}{2}\Big)\Big(e_j{}^{z}-2\cot\theta\, e_j{}^{4}\Big)\Big] \\
& +\sum_{m=6}^{9} e_i{}^{m}\gamma_a\Big[\partial_j+\frac{1}{4R}\left(\gamma^{5[4}e_j{}^{3]}+\gamma^{z[4}e_j{}^{3]}\right)-\frac{1}{4R}\gamma^{34}\Big(\frac{1+\gamma^{9\bar z}}{2}\Big)\Big(e_j{}^{z}-2\cot\theta\, e_j{}^{4}\Big)\Big]\Big\}\theta^{2} \\
& +i\bar\theta^{2}\left(\sqrt{h}h^{ij}-\varepsilon^{ij}\Gamma_{11}\right)\Big\{\sum_{a=0}^{z} e_i{}^{a}\gamma_a\Big[\partial_j+\frac{1}{4R}\left(\gamma^{5[4}e_j{}^{3]}-\gamma^{z[4}e_j{}^{3]}\right)+\frac{1}{4R}\gamma^{34}\Big(\frac{1+\gamma^{9\bar z}}{2}\Big)\Big(e_j{}^{5}-2\cot\theta\, e_j{}^{4}\Big)\Big] \\
& +\sum_{m=6}^{\bar z} e_i{}^{a}\gamma_a\Big[\partial_j-\frac{1}{4R}\left(\gamma^{5[4}e_j{}^{3]}+\gamma^{z[4}e_j{}^{3]}\right)+\frac{1}{4R}\gamma^{34}\Big(\frac{1-\gamma^{5z}}{2}\Big)\Big(e_j{}^{5}-2\cot\theta\, e_j{}^{4}\Big)\Big]\Big\}\theta^{1}
\end{aligned}
$$

Integrability for the full superstring

- The Green–Schwarz superstring on $\mathrm{AdS}_3 \times S^3$ can be understood in terms of the sigma model on the supergroup $PSU(1,1|2)$
- there is a $\mathbb{Z}_4$ grading, *i.e.* the Lie algebra decomposes into the form

$$\mathfrak{g} = \bigoplus_{n=0}^{3} \mathfrak{g}_n$$

- the decomposition works for the Noether currents and is needed to impose a flatness condition
- the same decomposition (and flatness condition) is preserved by T-duality precisely in the same way as before
- we obtain a set of (non-local) currents for the squashed group that still generate $\mathfrak{psu}(1,1|2)$.

Outline

Summary

- Squashed group manifolds have met renewed attention during the last years
  - Topologically massive gravity
  - Schrödinger spacetimes
- They can be understood as natural deformations of group manifolds
- Using the **Hopf fibration** structure we can construct **type II backgrounds**.
- Using the **Lie algebra** structure we can construct exact **heterotic backgrounds**.
- Using both structures we can prove their classical integrability

The end

Thank you

for your attention