# Spherically symmetric analysis on open FLRW solution in non-linear massive gravity

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# Contents

# (Theoretical) Motivation

・General relativity

theory for massless spin-2

↓ Can spin-2 particle have mass

・Massive gravity

theory for massive spin-2

What is it?
How is theory defined?

# (Observational) Motivation

## Accelerated Universe

▪Dark Energy??

or

▪Modification of Gravity??

⇩

Massive gravity is one of possibilities

Present Hubble scale

Because gravity is suppressed

⇩

beyond graviton mass scale,

universe looks accelerated.

# Sporadic progress of massive gravity

1930’ ・Linear theory of massive gravity

1970’ ・Observational trouble of linear theory
・Non-linear properties

2010’ ・Ghost-free nonlinear theory of massive gravity
・Cosmology

# Linear theory of Massive Gravity

(Fierz and Pauli 1939)

$$L = L_{EH}^{lin}[h] + m_g^2(a_1 h_{\mu\nu}h^{\mu\nu} + a_2 h^2)$$

$$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$$

$$h \equiv h_\mu{}^\mu$$

- Broken diffeomorphism ⇒ Additional 4 D.o.F

6 D.o.F ⇒ 5 D.o.F of massive spin-2

1 D.o.F of spin-0 ⇐ ghost

- Unique linear theory without ghosts

$a_1 + a_2 = 0$ ⇒ Spin-0 mode disappears

# van Dam, Veltman, Zakharov discontinuity

(1970)

Massless limit of linear massive theory
does not correspond to linear massless theory

Propagator of massless spin-2 (2 D.o.F)

$$D_{\alpha\beta,\sigma\lambda} = \frac{-i}{p^2}\Big[\frac{1}{2}(\eta_{\alpha\sigma}\eta_{\beta\lambda} + \eta_{\alpha\lambda}\eta_{\beta\sigma}) - \frac{1}{2}\eta_{\alpha\beta}\eta_{\sigma\lambda}\Big]$$

**mismatch**

Propagator of massive spin-2 (5 D.o.F)

$$D_{\alpha\beta,\sigma\lambda} = \frac{-i}{p^2+m^2}\Big[\frac{1}{2}(\eta_{\alpha\sigma}\eta_{\beta\lambda} + \eta_{\alpha\lambda}\eta_{\beta\sigma}) - \frac{1}{3}\eta_{\alpha\beta}\eta_{\sigma\lambda}\Big]$$

This mismatch leads to 25% off of light bending.

# Vainshtein effects (1972)

Can non-linear effect resolve vDVZ discontinuity?

$$L = L_{EH}[h] + m_g^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

Non-linear terms

Spherical symmetry $ds^2 = -e^{\nu}dt^2 + e^{\sigma}dr^2 + r^2 e^{\mu}(d\theta^2 + \sin^2\theta d\phi^2)$

Week field expansion

$$r_m \equiv 2G_N M$$

$$\nu = -\frac{r_m}{r}\left[1 + O\left(\frac{r_m}{m^4 r^5}\right)\right],$$

$$\lambda = \frac{1}{2}\frac{r_m}{r}\left[1 + O\left(\frac{r_m}{m^4 r^5}\right)\right],$$

$$\mu = \frac{1}{2}\frac{r_m}{m^2 r^3}\left[1 + O\left(\frac{r_m}{m^4 r^5}\right)\right],$$

Non-linear effect becomes dominant for

$$r < r_V \equiv \left(\frac{r_m}{m^4}\right)^{\frac{1}{5}} \longrightarrow \infty$$

$$m \longrightarrow 0$$

# Vainshtein effects (1972)

Can non-linear effect resolve vDVZ discontinuity?

$$L = L_{EH}[h] + m_g^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

Non-linear terms

Spherical symmetry $ds^2 = -e^{\nu}dt^2 + e^{\sigma}dr^2 + r^2 e^{\mu}(d\theta^2 + \sin^2\theta d\phi^2)$

Expansion w.r.t graviton mass

$$r_m \equiv 2G_N M$$

$$\nu = -\frac{r_m}{r}\left[1 + O\left(\sqrt{\frac{m^4 r^5}{r_m}}\right)\right],$$

$$\lambda = \frac{r_m}{r}\left[1 + O\left(\sqrt{\frac{m^4 r^5}{r_m}}\right)\right],$$

$$\mu = \sqrt{\frac{8r_m}{13r}}\left[1 + O\left(\sqrt{\frac{m^4 r^5}{r_m}}\right)\right],$$

correspond to leading order of Schwarzschild

This expansion is valid for

$$r < r_V \equiv \left(\frac{r_m}{m^4}\right)^{\frac{1}{5}}$$

# Boulware Deser (BD) ghost (1972)

$$L = L_{EH}[h] + m_g^2(a_1 h_{\mu\nu}h^{\mu\nu} + a_2 h^2)$$

lapse N and shift Ni are quadratic in the action

Equations from deviation w.r.t lapse and shift are

not like constraints for 3-dimensional metric ⇒ 6 D.o.F

but fixed lapse and shift themselves

$a_1 + a_2 = 0$ ⇐ lapse N becomes linear in linearized action

⇓ Non-linear

lapse N becomes quadratic or higher order

⇒ Spin-0 can survive

⇕

BD ghost (unbounded Hamiltonian)

# Stuckelberg fields

Arkani-Hamed, Georgi & Schwarz (2003)

Covariant form of mass term

Stuckelberg scalar fields $\phi^a \ (a = 0, 1, 2, 3)$

$$g_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b + H_{\mu\nu} \qquad \phi^a = x^a + \bar{A}^a$$

fiducial metric (gauge fixing $\bar{A}^a = 0 \quad \rightarrow \quad H_{\mu\nu} = h_{\mu\nu}$)

spin-0 mode of $\bar{A}^a$ ⇔ BD ghost

transverse ↓

extract the spin-0 mode $\pi$ ⇒ $\eta_{ab}\bar{A}^b = \partial_a\pi + A_a$

$$H_{\mu\nu} = h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A\mu - \partial_\mu A^\alpha\partial_\nu A_\alpha$$
$$+ 2\partial_\mu\partial_\nu\pi - \partial_\mu A^\alpha\partial_\nu\partial_\alpha\pi - \partial_\nu A^\alpha\partial_\mu\partial_\alpha\pi - \partial_\mu\partial^\alpha\pi\partial_\nu\partial_\alpha\pi$$

$\pi$ always appears with the second derivatives

↳ origin of BD ghost

# Avoidance of BD ghost

In order to avoid BD ghost, $\pi$ should not appear ➡ $\pi$ appears only in the form of total derivatives

Concentrate only on $\pi$

$$\Pi_{\mu\nu} \equiv \partial_\mu \partial_\nu \pi$$

Three possible forms

$$L_2 = [\Pi]^2 - [\Pi^2]$$
$$L_3 = [\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]$$
$$L_4 = [\Pi]^4 - 6[\Pi^2][\Pi^2] + 8[\Pi^3][\Pi] + 3[\Pi^2]^2 - 6[\Pi^4]$$

Bracket $[T]$ means trace of $T$

$$([\Pi] \equiv g^{\mu\nu}\Pi_{\mu\nu})$$

# Nonlinear massive gravity de Rham, Gabadadze 2010

- Relation between $\Pi_{\mu\nu}$ and $H_{\mu\nu}$

For $h_{\mu\nu} = 0, A_\mu = 0$ ⇨ $H_{\mu\nu} = 2\Pi_{\mu\nu} - \Pi_{\mu\alpha} g^{\alpha\beta} \Pi_{\beta\nu}$

⇨ $g^{\mu\alpha}\Pi_{\alpha\nu} = \delta^\mu_\nu - \left(\sqrt{g^{-1}(g-H)}\right)^\mu{}_\nu$

- Replace $\Pi_{\mu\nu}$ with $K^\mu{}_\nu \equiv \delta^\mu_\nu - \left(\sqrt{g^{-1}(g-H)}\right)^\mu{}_\nu$

$$L_2 = [K]^2 - [K^2]$$

$$L_3 = [K]^3 - 3[K][K^2] + 2[K^3]$$

$$L_4 = [K]^4 - 6[K^2][K^2] + 8[K^3][K] + 3[K^2]^2 - 6[K^4]$$

No BD ghost even for $h_{\mu\nu} \neq 0, A_\mu \neq 0$ (Hassan&Rosen)

$$S_{mass} = M_{pl}^2 m_g^2 \int d^4x \sqrt{-g}\,(L_2 + \alpha_3 L_3 + \alpha_4 L_4)$$

# FLRW universe

- No flat FLRW solution D'Amico, et al. (2011)
- No closed FLRW solution
- Open FLRW solution can exist

Gumrukcuoglu, Lin, Mukohyama (2011)

$Z_{\mu\nu} \equiv \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$ : fiducial metric

$$Z_{\mu\nu}dx^\mu dx^\nu = -\dot{f}(t)^2dt^2 + |K|f(t)^2\Omega_{ij}dx^idx^j$$

$$g_{\mu\nu}dx^\mu dx^\nu = -N(t)^2dt^2 + a(t)^2\Omega_{ij}dx^idx^j$$

$$\begin{cases} \phi^0 = f(t)\sqrt{1+|K|(x^2+y^2+z^2)} \\ \phi^1 = \sqrt{|K|}\,f(t)x \\ \phi^2 = \sqrt{|K|}\,f(t)y \\ \phi^3 = \sqrt{|K|}\,f(t)z \end{cases}$$

$$\Omega_{ij}dx^idx^j = dx^2+dy^2+dz^2 - \frac{|K|(xdx+ydy+zdz)^2}{1+|K|(x^2+y^2+z^2)}$$

E.o.M for metric

$$3H^2 + \frac{3K}{a^2} = \Lambda_\pm + M_{pl}^{-2}\rho$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\Big[(1+\alpha_3)(2+\alpha_3+2\alpha_3^2-3\alpha_4) \pm 2(1+\alpha_3+\alpha_3^2-\alpha_4)^{3/2}\Big]$$

# Cosmological perturbation

Gumrukcuoglu, Lin, Mukohyama (2011)

Gauge invariant variables in GR

$$S^{(2)} = \tilde{S}^{(2)}[Q_I, \Phi, \Psi, B_i, \gamma_{ij}] + \tilde{S}^{(2)}_{mass}[\psi^\pi, E^\pi, F^\pi_i, \gamma_{ij}]$$

$$\tilde{S} \equiv S_{EH+\tilde{\Lambda}}[g_{\mu\nu}] + S_{matter}[g_{\mu\nu}, \sigma_I] \qquad \tilde{\Lambda} \equiv \Lambda + \Lambda_\pm$$

$$\tilde{S}^{(2)}_{mass} \equiv M^2_{pl} \int d^4x N a^3 \sqrt{\Omega}\, M^2_{GW}(t) \times \left[3(\psi^\pi)^2 - \frac{1}{12} E^\pi \Delta(\Delta + 3K) E^\pi + \frac{1}{16} F^\pi_i (\Delta + 2K) F^\pi_i - \frac{1}{8}\gamma^2_{ij}\right]$$

$$M^2_{GW}(t) \equiv \pm(r-1) m_g^2 X^2_\pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4} \qquad r \equiv \frac{na}{N\alpha}$$

Integrate out, $\psi^\pi, E^\pi$ and $F^\pi_i$ ⇒ $S^{(2)}_{s,v} = S^{(2)}_{GR\ s,v}$

Only massive helicity-2 mode of graviton can propagate

# Spherically symmetric analysis on open FLRW solution in non-linear massive gravity

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# Motivation

Massive graviton on Minkowski 5 D.o.F

Mismatch

Massive graviton on open FLRW 2 D.o.F

Where do 3 D.o.F disappear to?

2 Helicity-1 modes
1 Helicity-0 mode

In order to catch these 3 D.o.F., we look the detail.

considering only spherically symmetric case
without matter (vacuum)

# Metrics and gauge fixing

Gauge fixing

No perturbation of Stuckelberg fields $\delta\phi^a = 0$

⇩

fiducial metric

$$Z_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2 dt^2 + \frac{|K|(f(t))^2}{1-Kr^2}dr^2 + |K|(f(t))^2 r^2 d\Omega_{(2)}^2,$$

Physical metric

$$ds^2 = -e^{2\Phi}dt^2 + \frac{a^2}{1-Kr^2}e^{2\Psi}\left(dr + \beta dt\right)^2 + a^2 r^2 e^{2E} d\Omega_{(2)}^2,$$

# Result ①

$$ds^2 = -e^{2\Phi}dt^2 + \frac{a^2}{1-Kr^2}e^{2\Psi}\left(dr+\beta dt\right)^2 + a^2r^2e^{2E}d\Omega_{(2)}^2,$$

Non-linear form of effective energy tensor $T^{eff}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_{mass}}{\delta g^{\mu\nu}}$

$$T^{\rm eff}_{mn} = -2M_{Pl}^2m_g^2\Bigg(\Bigg[\Big\{-3+3X_\pm e^{-E}-\frac{1}{2}X_\pm^2e^{-2E} + \alpha_3\left(-2+3X_\pm e^{-E}-X_\pm^2e^{-2E}\right)-\frac{\alpha_4}{2}\left(1-X_\pm e^{-E}\right)^2\Big\} + \mathbb{W}_T\left\{\left(\frac{3}{2}-X_\pm e^{-E}\right)+\alpha_3\left(\frac{3}{2}-2X_\pm e^{-E}+\frac{1}{2}X_\pm^2e^{-2E}\right)+\frac{\alpha_4}{2}\left(1-X_\pm e^{-E}\right)^2\right\}\Bigg]g_{mn} + \left\{\left(-3+2X_\pm e^{-E}\right)+\alpha_3\left(-3+4X_\pm e^{-E}-X_\pm^2e^{-2E}\right)-\alpha_4\left(1-X_\pm e^{-E}\right)^2\right\}\mathcal{W}_{mn}\Bigg), \quad (3.13)$$

$$T^{\rm eff}_{ij} = -2M_{Pl}^2m_g^2\Bigg[\left\{-3+\frac{3}{2}X_\pm e^{-E}+\alpha_3\left(-2+\frac{3}{2}X_\pm e^{-E}\right)+\frac{\alpha_4}{2}\left(-1+X_\pm e^{-E}\right)\right\} + \mathbb{W}_T\left\{\frac{3}{2}-\frac{1}{2}X_\pm e^{-E}+\alpha_3\left(\frac{3}{2}-X_\pm e^{-E}\right)+\frac{\alpha_4}{2}\left(1-X_\pm e^{-E}\right)\right\} + \sqrt{\det(\mathbb{Z})}\left\{-\frac{1}{2}+\alpha_3\left(-1+\frac{1}{2}X_\pm e^{-E}\right)+\frac{\alpha_4}{2}\left(-1+X_\pm e^{-E}\right)\right\}\Bigg]g_{ij}, \quad (3.14)$$

Only if E=0, $T^{eff}_{\mu\nu} = -M_{pl}^2\Lambda_\pm g_{\mu\nu}$

# Result ②

$$ds^2 = -e^{2\Phi}dt^2 + \frac{a^2}{1-Kr^2}e^{2\Psi}\left(dr + \beta dt\right)^2 + a^2r^2e^{2E}d\Omega_{(2)}^2,$$

Even in linear, we can see the difference from in GR

Linearized E.o.M

$$\frac{2(1-Kr^2)}{a^2r^2}A + \frac{2(1-Kr^2)}{a^2r}A' + 2H^2B = 0,$$

$$\frac{2}{r}\left(\frac{\dot{H}}{H}A + HB\right) = 0,$$

$$\frac{2}{r^2}\left[\left(\frac{\dot{H}}{H^2} - 1\right)A - \frac{1}{H}\left(\dot{A} - HB\right)\right] = -2m_g^2C_\pm\left(\frac{a^2}{1-Kr^2}\right)\left(1 - \frac{aH}{\sqrt{|K|}}\right)E,$$

$$r(1-Kr^2)B' - a^2r^2(\dot{H}B + H\dot{B}) - (2Kr^2 + 3a^2r^2H^2)B - \frac{r(1-Kr^2)}{H}\dot{A}' + \frac{Kr^2}{H}\dot{A} - r(1-Kr^2)\left(1 - \frac{\dot{H}}{H^2}\right)A' = -m_g^2C_\pm a^2r^2\left(1 - \frac{aH}{\sqrt{|K|}}\right)(\Psi + E).$$

$A, B$

Gauge invariant combination in GR

eliminate $B, \Psi$

$$A' = -\frac{(1-3Kr^2)}{r(1-Kr^2)}A.$$

$$A = \frac{1}{8\pi M_{Pl}^2}\frac{M(t)}{ar(1-Kr^2)}$$

$$-\frac{2}{H^2r^2}\left(H^2A + H\dot{A}\right) = -2m_g^2C_\pm\left(\frac{a^2}{1-Kr^2}\right)\left(1 - \frac{aH}{\sqrt{|K|}}\right)E,$$

While in GR ( $m_g = 0$ ) we can fix M(t), in MG we cannot fix M(t)

# Result ③ & Discussion

$$A = \frac{1}{8\pi M_{Pl}^2}\frac{M(t)}{ar(1-Kr^2)},$$

$$B = -\frac{1}{8\pi M_{Pl}^2}\frac{K}{a^2H^2}\frac{M(t)}{ar(1-Kr^2)},$$

Gauge invariant variables in GR

$$E = \frac{1}{8\pi M_{Pl}^2}\frac{1}{m_g^2 C_\pm}\left(1-\frac{aH}{\sqrt{|K|}}\right)^{-1}\frac{\dot{M}(t)}{a^3r^3},$$

$$\Psi = -\frac{1}{8\pi M_{Pl}^2}\frac{2}{m_g^2 C_\pm}\left(1-\frac{aH}{\sqrt{|K|}}\right)^{-1}\frac{\dot{M}(t)}{a^3r^3},$$

Non-gauge-invariant variables in GR

- Only when $\dot{M}=0 \;\; (E=0)$
  solution becomes the same as that in GR
- D.o.F. of M(t) is probably related to helicity-0 mode
- M(t) depends only on time ➡ Infinite speed of propagation??

# Other works

Non-linear instability of open FLRW solution

DeFelice, Gumrukcuoglu, Mukohyama (2012)

Solutions with less symmetric fiducial metric

Anisotropic fiducial metric

Gumrukcuoglu, Lin, Mukohyama (2012)

Spherically symmetric fiducialmetric

Kobayashi, Siino, Yamaguchi, Yoshida (2012)

# Summary

- We have analyzed spherically symmetric configuration on open FLRW solution in Massive gravity theory.

- Only when E=0, GR is recovered.

- While in GR mass should be conserved, on open FLRW solution in massive gravity we have time-dependent mass.

    ⬇

    - Infinite speed of propagation of helicity-0 mode??
    - This give a possibility to distinguish massive gravity from GR