

# Ising Model D-Branes from String Field Theory

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(based also on recent work with C. Maccaferri)

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# Outline

- Introduction
- Review of D-branes:
  - in Ising model
  - in  $(\text{Ising})^2 = \text{Free boson in } S^1/\mathbb{Z}_2$
- Numerical solutions in OSFT
  - $1, \varepsilon$  branes from  $\sigma$
  - $\sigma$  brane from  $1, \varepsilon \rightarrow \text{Positive energy solutions !!}$
  - $1 \otimes 1, \varepsilon \otimes \varepsilon, 1 \otimes \varepsilon, \varepsilon \otimes 1, \dots$  branes from  $\sigma \otimes \sigma \rightarrow \text{Fractional branes !!}$
  - **Double branes** in the universal sector
- Conclusion

# Boundary states

- The nicest problems in theoretical physics are ones which are: easy to state, difficult to solve and with many relations to other branches to physics
- One such problem is classification of the boundary states in a given CFT or equivalently admissible open string vacua or D-branes.

# Boundary states

- Describe possible boundary conditions from the closed string channel point of view.
- Conformal boundary states obey:
  - 1) the gluing condition  $(L_n - \bar{L}_{-n})|B\rangle = 0$
  - 2) Cardy condition (modular invariance)
  - 3) sewing relations (factorization constraints)

See e.g. reviews by Gaberdiel or by Cardy

# Boundary states

- The gluing condition is easy to solve:  
For any spin-less primary  $|V_\alpha\rangle$  we can define

$$||V_\alpha\rangle\rangle = \sum_{IJ} M^{IJ}(h_\alpha) L_{-I} \bar{L}_{-J} |V_\alpha\rangle$$

where  $M^{IJ}$  is the inverse of the real symmetric Gram matrix

$$M_{IJ}(h_\alpha) = \langle V^\alpha | L_I L_{-J} | V_\alpha \rangle$$

where  $L_{-X} \equiv L_{-n_k} \dots L_{-n_1}$   
(with possible null states projected out).

# Boundary states

- Explicitly:

$$||V_\alpha\rangle\rangle = \left[ 1 + \frac{1}{2h_\alpha} L_{-1} \bar{L}_{-1} + B(h_\alpha, c) \left( 2(1 + 2h_\alpha) L_{-2} \bar{L}_{-2} - 3(L_{-2} \bar{L}_{-1}^2 + L_{-1}^2 \bar{L}_{-2}) + \frac{8h_\alpha + c}{4h_\alpha} L_{-1}^2 \bar{L}_{-1}^2 \right) + \dots \right] |V_\alpha\rangle$$

$$B(h_\alpha, c) = \frac{1}{2h_\alpha(8h_\alpha - 5) + c(2h_\alpha + 1)}.$$

- The other conditions are much harder to deal with however. Perhaps not even the full set of necessary conditions is known.
- We will show today, how to construct boundary states (appropriate linear combinations of Ishibashi states) from OSFT solutions.

# Boundary states – Cardy's solution

- By demanding that

$$\langle\langle \alpha | q^{\frac{1}{2}(L_0 + \bar{L}_0 - \frac{c}{12})} | \beta \rangle\rangle = \text{Tr}_{\mathcal{H}_{\alpha\beta}^{\text{open}}} \left( \tilde{q}^{L_0 - \frac{c}{24}} \right)$$

and noting that RHS can be expressed as

$$\sum_i n_{\alpha\beta}^i \chi_i(\tilde{q})$$

Cardy derived integrality constraints on the boundary states. Surprisingly, for certain class of rational CFT's he found an elegant solution (relying on Verlinde formula)

$$|\tilde{k}\rangle\rangle = \sum_j \sqrt{\frac{S_k^j}{S_0^j}} |j\rangle\rangle$$

where  $S_k^j$  is the modular matrix.

# Ising model CFT

- Ising model is the simplest of the unitary minimally models with  $c = 1/2$ .
- It has 3 primary operators
  - $1$   $(0,0)$
  - $\varepsilon$   $(1/2, 1/2)$
  - $\sigma$   $(1/16, 1/16)$
- The modular S-matrix takes the form

$$S = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$



# Boundary states – Cardy's solution

- And thus the Ising model conformal boundary states are

$$||\tilde{0}\rangle\rangle = \frac{1}{\sqrt{2}}|0\rangle\rangle + \frac{1}{\sqrt{2}}|\varepsilon\rangle\rangle + \frac{1}{\sqrt[4]{2}}|\sigma\rangle\rangle$$

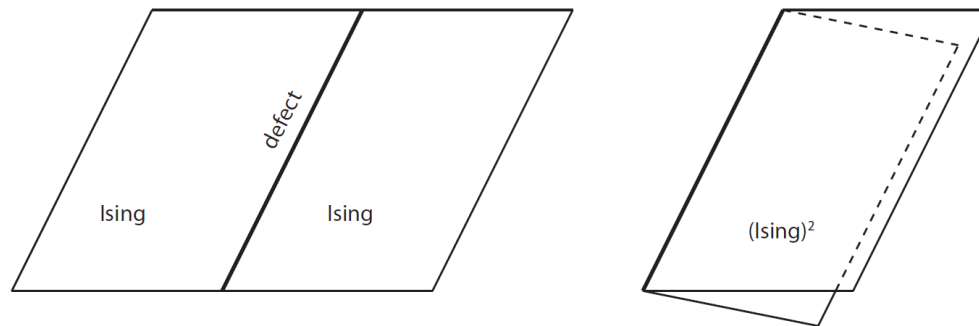
$$||\tilde{\varepsilon}\rangle\rangle = \frac{1}{\sqrt{2}}|0\rangle\rangle + \frac{1}{\sqrt{2}}|\varepsilon\rangle\rangle - \frac{1}{\sqrt[4]{2}}|\sigma\rangle\rangle$$

$$||\tilde{\sigma}\rangle\rangle = |0\rangle\rangle - |\varepsilon\rangle\rangle$$

- The first two boundary states describe fixed (+/-) boundary condition, the last one free boundary condition

# $(\text{Ising})^2$

- This model naturally arises when one considers Ising model on a plane with a defect line and employs the folding trick.

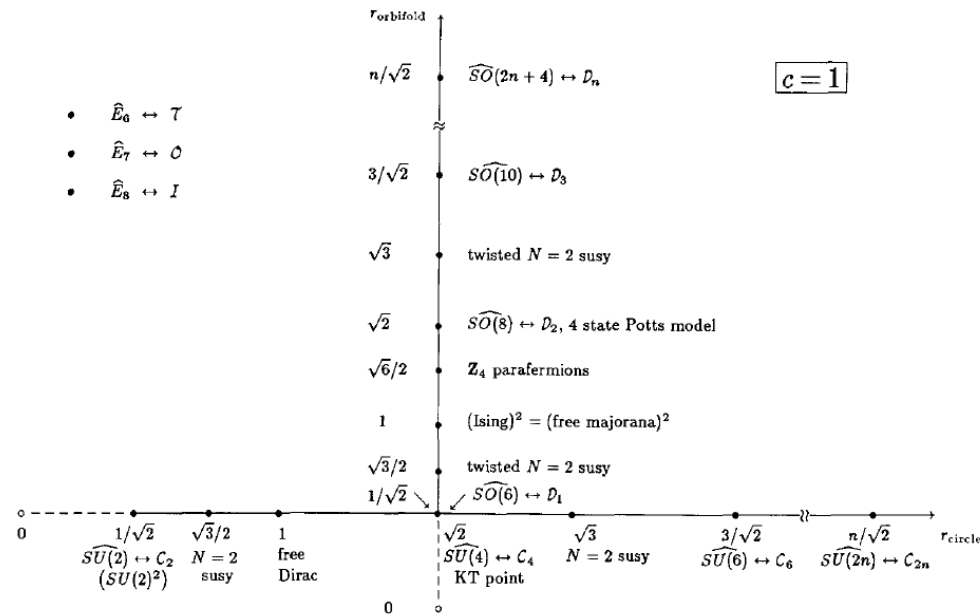


# (Ising)<sup>2</sup>

- (Ising)<sup>2</sup> model is well known point on the orbifold branch of the moduli space of  $c=1$  models

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P. Ginsparg / Curiosities at  $c=1$



# (Ising)<sup>2</sup>

- Even though Ising model itself has only 3 bulk primaries, (Ising)<sup>2</sup> has infinite number of them (Yang 1987)

$\Delta = \bar{\Delta}$	Multiplicity	(Ising) <sup>2</sup> Examples	Orbifold Examples
$n^2 = 0, 1, 4, \dots$	1	$1 \otimes 1, \varepsilon \otimes \varepsilon$	$1, \partial X \bar{\partial} X$
$\frac{(n+1)^2}{2} = \frac{1}{2}, 2, \frac{9}{2}, \dots$	2	$1 \otimes \varepsilon, \varepsilon \otimes 1$	$\cos(\sqrt{2}X), \cos(\sqrt{2}\tilde{X})$
$\frac{(2n+1)^2}{8} = \frac{1}{8}, \frac{9}{8}, \frac{25}{8}, \dots$	1	$\sigma \otimes \sigma$	$\sqrt{2} \cos(\frac{X}{\sqrt{2}})$
$\frac{(2n+1)^2}{16} = \frac{1}{16}, \frac{9}{16}, \frac{25}{16}, \dots$	2	$1 \otimes \sigma, \sigma \otimes 1, \varepsilon \otimes \sigma, \sigma \otimes \varepsilon$	twist fields, excited twist fields

- It is precisely equivalent to a free boson on an orbifold  $S^1/Z_2$  with radius  $R_{orb} = \sqrt{2}$  (in our units  $\alpha' = 1$ .)

# Boundary states in (Ising)<sup>2</sup>

- Some boundary states are readily available
- However, in general the problem of constructing D-branes in tensor product of simple CFT's is rather difficult since we get always **infinitely many new primaries**, and hence potentially many new exotic boundary states.

# Boundary states in (Ising)<sup>2</sup>

- Here is the list found by Affleck and Oshikawa (1996)

(Ising) <sup>2</sup> D-brane	Interpretation	Energy = $\langle 1 \rangle$	$\frac{\langle \partial X \bar{\partial} X \rangle}{\langle 1 \rangle}$	Position
$1 \otimes \varepsilon$	fractional D0	$\frac{1}{2}$	+1	$\pi R$
$\varepsilon \otimes 1$	fractional D0	$\frac{1}{2}$	+1	$\pi R$
$1 \otimes 1$	fractional D0	$\frac{1}{2}$	+1	0
$\varepsilon \otimes \varepsilon$	fractional D0	$\frac{1}{2}$	+1	0
$1 \otimes \sigma$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	—
$\sigma \otimes 1$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	—
$\varepsilon \otimes \sigma$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	—
$\sigma \otimes \varepsilon$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	—
$\sigma \otimes \sigma$	centered bulk D0	1	+1	$\frac{\pi R}{2}$
$\sum_i a_i(\phi)  AT, i\rangle\rangle$	generic bulk D0	1	+1	$\phi R$
$\sum_i b_i(\tilde{\phi})  AT, i\rangle\rangle$	generic bulk D1	$\sqrt{2}$	-1	—

**Now we would like to find all this from OSFT !?!**

# Numerical solutions in OSFT

- To construct new D-branes in a given BCFT with central charge  $c$  using OSFT, we consider strings ‘propagating’ in a background  $\text{BCFT}_c \otimes \text{BCFT}_{26-c}$  and look for solutions which do not excite any primaries in  $\text{BCFT}_{26-c}$ .

# Numerical solutions in OSFT

- To get started with OSFT, we first have to specify the starting BCFT, i.e. we need to know:
  - spectrum of boundary operators
  - their 2pt and 3pt functions
  - bulk-boundary 2pt functions (to extract physics)
- The spectrum for the open string stretched between D-branes  $a$  and  $b$  is given by boundary operators which carry labels of operators which appear in the fusion rules

$$\phi_a \times \phi_b = \sum_c N_{ab}^c \phi_c$$



# Numerical solutions in OSFT

- In the case of Ising the boundary spectrum is particularly simple

D-brane	Energy	Boundary spectrum
$  \tilde{1}\rangle\rangle$	$\frac{1}{\sqrt{2}}$	1
$  \tilde{\varepsilon}\rangle\rangle$	$\frac{1}{\sqrt{2}}$	1
$  \tilde{\sigma}\rangle\rangle$	1	1, $\varepsilon$

# Ellwood conjecture

- Solutions to OSFT e.o.m. are believed to be in 1-1 correspondence with consistent boundary conditions.
- The widely believed (and tested, but unproven) Ellwood conjecture states that for every on-shell  $\mathcal{V}_{cl}$

$$\langle \mathcal{V}_{cl} | c_0^- | B_\Psi \rangle = -4\pi i \langle I | \mathcal{V}_{cl}(i) | \Psi - \Psi_{TV} \rangle ,$$

Here  $\Psi$  is a solution of the e.o.m.,  $\Psi_{TV}$  is the tachyon vacuum and  $|B_\Psi\rangle$  is the boundary state we are looking for.

# Generalized Ellwood invariants

- The restriction to an on-shell state can be bypassed. Any solution built using reference  $\text{BCFT}_0$  can be written as

$$\Psi = \sum_j \sum_{\substack{I = \{n_1, n_2, \dots\} \\ J = \{m_1, m_2, \dots\}}} a_{IJ}^j L_{-I}^{\text{matter}} |\mathcal{V}_j\rangle \otimes L_{-J}^{\text{ghost}} c_1 |0\rangle$$

and uplifted to  $\text{BCFT}_0 \otimes \text{BCFT}_{\text{aux}}$ , where  $\text{BCFT}_{\text{aux}}$  has  $c=0$  and contains free boson  $Y$  with Dirichlet b.c. One can then compute Ellwood invariant with  $\tilde{\mathcal{V}}^\alpha = c\bar{c}V^\alpha e^{2i\sqrt{1-h}Y} w$

Trick inspired by Kawano, Kishimoto  
and Takahashi (2008)

# Generalized Ellwood invariants

- Since  $|B_\Psi\rangle^{\text{CFT}_0 \otimes \text{CFT}_{\text{aux}}} = |B_\Psi\rangle^{\text{CFT}_0} \otimes |B_0\rangle^{\text{CFT}_{\text{aux}}}$

we find

That the lift leads to the factorization is our little assumption !

$$\langle c\bar{c}V^\alpha | c_0^- | B_\Psi \rangle = -4\pi i \langle E[\tilde{\mathcal{V}}^\alpha] | \tilde{\Psi} - \tilde{\Psi}_{TV} \rangle$$

- This is gauge invariant even w.r.t. the gauge symmetry of the original OSFT based on  $\text{BCFT}_0$

$$\text{Lift} \circ (\text{Gauge Transf})_\Lambda = (\text{Gauge Transf})_{\text{Lift}(\Lambda)} \circ \text{Lift}$$

# Boundary state from Ellwood invariants

- The coefficients of the boundary state

$$|B_\Psi\rangle = \sum_{\alpha} n_{\Psi}^{\alpha} ||V_{\alpha}\rangle\rangle$$

can be computed from OSFT solution via

$$n_{\Psi}^{\alpha} = 2\pi i \langle I | \mathcal{V}^{\alpha}(i) | \Psi - \Psi_{\text{TV}} \rangle^{\text{BCFT}_0 \otimes \text{BCFT}_{\text{aux}}}$$

$$\mathcal{V}^{\alpha} = c\bar{c}V^{\alpha} e^{2i\sqrt{1-h_{\alpha}}Y} w$$

See: Kudrna, Maccaferri, M.S. (2012)

Alternative attempt:

Kiermaier, Okawa, Zwiebach (2008)

# Tachyon condensation on the $\sigma$ -brane

- The computation proceeds along the similar line as for (Moeller, Sen, Zwiebach)
- String field truncated to level 2:

$$|\psi\rangle = tc_1|0\rangle + ac_1|\epsilon\rangle + uc_{-1}|0\rangle + vc_1L_{-2}^I|0\rangle + wc_1L_{-2}^R|0\rangle$$

- The action is

$$\begin{aligned}\mathcal{V}(t, a, u, v, w) = & -\frac{1}{2}t^2 - \frac{1}{4}a^2 - \frac{1}{2}u^2 + \frac{1}{8}v^2 + \frac{51}{8}w^2 + \frac{27}{64}va^2 - \frac{255}{64}wa^2 + \frac{11}{16}ua^2 + \\ & - \frac{165\sqrt{3}}{3456}tuv - \frac{8415\sqrt{3}}{3456}tuw + \frac{1049\sqrt{3}}{9216}tv^2 + \frac{256423563\sqrt{3}}{746496}tw^2 + \frac{66\sqrt{3}}{128}ut^2 - \\ & - \frac{15\sqrt{3}}{256}vt^2 - \frac{765\sqrt{3}}{256}wt^2 + \frac{19\sqrt{3}}{192}tu^2 + \frac{27\sqrt{3}}{64}t^3 + \frac{425\sqrt{3}}{1536}tvw + \frac{27}{16}ta^2.\end{aligned}$$

# Tachyon condensation on the $\sigma$ -brane

- Going to higher levels, we should properly take care of the Ising model null-states
- It turns out that we can effectively remove them by considering only Virasoro generators:
  - in the Verma module of 1:

$$L_{-2}, L_{-3}, L_{-4}, L_{-5}, L_{-11}, L_{-12}, L_{-13}, L_{-14}, L_{-18}, L_{-19}, L_{-20}, L_{-21}, \dots$$

- in the Verma module of  $\varepsilon$  :

$$L_{-1}, L_{-4}, L_{-6}, L_{-7}, L_{-9}, L_{-10}, L_{-12}, L_{-15}, L_{-17}, L_{-20}, L_{-20}, L_{-22}, \dots$$

The patterns repeats modulo 16!

- Had we needed Verma module of  $\sigma$  only,  
 $L_{odd}$  would be needed!

# Tachyon condensation on the $\sigma$ -brane

- Already in the lowest truncation levels we see two solution corresponding to 1- and  $\varepsilon$ -branes

Level	0.5	2.0	2.5
$2\pi^2\mathcal{V}(\psi)$	-0.16971	-0.24579	-0.26454
Percentage	57.9 %	83.9 %	90.3 %
$c_1 0\rangle$	0.14815	0.20553	0.21454
$c_1 \epsilon\rangle$	$\pm 0.24348$	$\pm 0.27818$	$\pm 0.29230$
$c_{-1} 0\rangle$		0.07382	0.07305
$c_1 L_{-2}^I 0\rangle$		-0.09006	-0.10418
$c_1 L_{-2}^R 0\rangle$		0.02750	0.02643
$c_{-1} \epsilon\rangle$			$\pm 0.02764$
$c_1 L_{-2}^I \epsilon\rangle$			$\pm 0.02178$
$c_1 L_{-2}^R \epsilon\rangle$			$\pm 0.00915$

Level	$2\pi^2\mathcal{V}(\psi)$	$n_\psi^1$	$n_\psi^\epsilon$	$n_\psi^\sigma$
1	-0.169718	0.767289	-0.767289	0.643203
2	-0.250828	0.733703	0.893387	0.739416
3	-0.261047	0.725226	0.945626	0.76589
4	-0.273442	0.722133	0.487621	0.778236
5	-0.276177	0.719333	0.500237	0.796483
6	-0.280671	0.715848	0.721123	0.801822
7	-0.281747	0.714764	0.730309	0.80727
8	-0.284039	0.714011	0.629844	0.810113
9	-0.284577	0.713460	0.631591	0.814922
10	-0.285964	0.712159	0.704802	0.816787
$\infty$	-0.294334	0.705668	0.700167	0.839425
Expected	-0.292893	0.707106	0.707106	0.840896



# Positive energy solutions on the 1-brane

- On the 1-brane we expect to find the usual tachyon vacuum, but can we find also something else ???

# Positive energy solutions on the 1-brane

- On the 1-brane we expect to find the usual tachyon vacuum, but can we find also something else ???
- Yes !

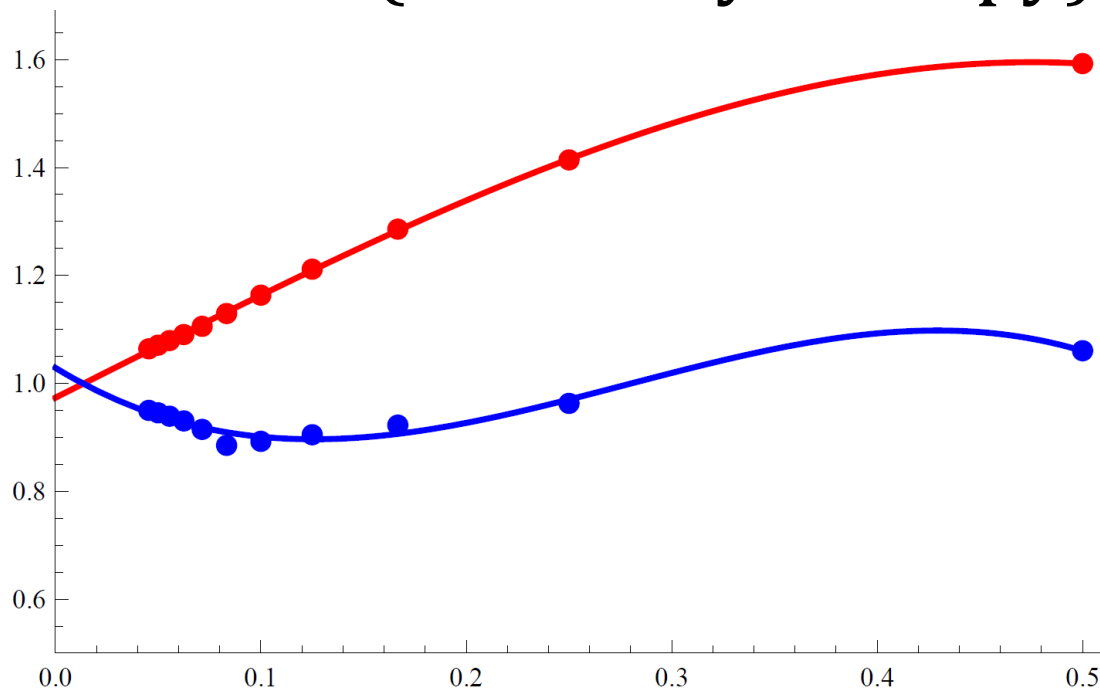
# Positive energy solutions on the 1-brane

- Starting with a complex solution at level 2 we find a real solution at level 14 and higher!

$L$	Energy	$n_{\psi}^1$	$n_{\psi}^{\varepsilon}$	$n_{\psi}^{\sigma}$	Im/Re	Its.	Time(s)
2	$1.59267 + 0.726878i$	$1.06048 - 0.184547i$	$-9.73471 - 5.23904i$	$-0.343579 - 0.970819i$	0.788398	3	0
4	$1.41414 + 0.201521i$	$0.962899 - 0.142672i$	$-0.66854 + 1.99191i$	$-0.369755 - 0.564227i$	0.438384	6	0
6	$1.28579 + 0.0766818i$	$0.922618 - 0.113783i$	$-3.86207 - 0.373757i$	$-0.389334 - 0.394362i$	0.307463	5	0
8	$1.2116 + 0.0305389i$	$0.904803 - 0.0868545i$	$-0.575138 + 0.822662i$	$-0.372165 - 0.281935i$	0.221002	5	0
10	$1.16345 + 0.0100715i$	$0.892563 - 0.0617353i$	$-2.48552 + 0.00261068i$	$-0.376292 - 0.192323i$	0.152223	5	2
12	$1.12943 + 0.00122565i$	$0.885097 - 0.0310941i$	$-0.569512 + 0.245609i$	$-0.368913 - 0.0939861i$	0.0748655	6	8
14	1.10568	0.914693	-1.93951	-0.266065	0	9	92
16	1.09045	0.930444	-0.950873	-0.206326	0	5	326
18	1.07936	0.939178	-1.69824	-0.174965	0	5	4037
20	1.07084	0.945384	-1.04849	-0.15003	0	5	33258
22	1.06405	0.949943	-1.55407	-0.13398	0	4	230589

# Positive energy solutions on the 1-brane

- Cubic extrapolations of **energy** and **Ellwood invariant** (boundary entropy) to infinite level



# Boundary states in $(\text{Ising})^2$ from OSFT

- In  $(\text{Ising})^2$  we have focused so far on the tachyon condensation of the  $\sigma \otimes \sigma$  -brane. To lowest level the string field takes the form

$$|\psi\rangle = tc_1|0\rangle + ac_1|\epsilon^{(1)}\rangle + bc_1|\epsilon^{(2)}\rangle + cc_1|\epsilon^{(1)}\epsilon^{(2)}\rangle$$

and the action to this order

cf. Longton & Karczmarek

$$-\frac{1}{2}t^2 - \frac{1}{4}a^2 - \frac{1}{4}b^2 + \frac{1}{3}K^3t^3 + \frac{4}{3\sqrt{3}}K^3a^2t + \frac{4}{3\sqrt{3}}K^3b^2t + \frac{3\sqrt{3}}{2}abc + \frac{3\sqrt{3}}{4}tc^2$$

There are four interesting solutions :

$ B\rangle_\psi$	$ \mathbb{1}\rangle \otimes  \mathbb{1}\rangle$	$ \mathbb{1}\rangle \otimes  \epsilon\rangle$	$ \epsilon\rangle \otimes  \mathbb{1}\rangle$	$ \epsilon\rangle \otimes  \epsilon\rangle$
$c_1 0\rangle$	0.23926	0.23926	0.23926	0.23926
$c_1 \epsilon^{(1)}\rangle$	-0.16828	0.16828	-0.16828	0.16828
$c_1 \epsilon^{(2)}\rangle$	-0.16828	-0.16828	0.16828	0.16828
$c_1 \epsilon^{(1)}\epsilon^{(2)}\rangle$	-0.11836	0.11836	0.11836	-0.11836

# Boundary states in $(\text{Ising})^2$ from OSFT

- For example the  $1 \otimes 1$  brane solution has the following invariants:

Level	$2\pi^2\mathcal{V}(\psi)$	$n_{\psi}^{11}$	$n_{\psi}^{1\epsilon}$	$n_{\psi}^{1\sigma}$	$n_{\psi}^{\epsilon 1}$	$n_{\psi}^{\epsilon\epsilon}$	$n_{\psi}^{\epsilon\sigma}$	$n_{\psi}^{\sigma 1}$	$n_{\psi}^{\sigma\epsilon}$	$n_{\psi}^{\sigma\sigma}$
1.0	-0.28149	0.62417	-0.62417	0.44455	-0.62417	0.62417	-0.44455	0.44455	-0.44455	0.52585
2.0	-0.39683	0.58024	0.28858	0.48785	0.28858	-1.15741	-0.48785	0.48785	-0.48785	0.61593
2.5	-0.43040	0.54753	0.43339	0.53440	0.43339	-1.41439	0.71872	0.53440	0.71872	0.64164
3.0	-0.43544	0.54367	0.48059	0.53102	0.48059	-1.50484	0.74643	0.53103	0.74643	0.62219
4.0	-0.45553	0.53344	0.26231	0.53601	0.26231	1.54248	0.81851	0.53601	0.81851	0.63243
4.5	-0.46222	0.52735	0.27837	0.55040	0.27837	1.74282	0.29745	0.55040	0.29745	0.63174
5.0	-0.47130	0.52629	0.28106	0.54984	0.28106	1.80851	0.29662	0.54984	0.29662	0.65732
6.0	-0.47130	0.51879	0.41792	0.55168	0.41792	-0.7563	0.29982	0.55168	0.29982	0.66158
6.5	-0.47397	0.51657	0.43234	0.55874	0.43234	-0.84041	0.60779	0.55874	0.60779	0.66244
7.0	-0.47397	0.51614	0.43788	0.55856	0.43788	-0.87424	0.61640	0.55856	0.61640	0.66078
8.0	-0.47476	0.51333	0.37867	0.55949	0.37867	1.25240	0.62768	0.55949	0.62768	0.66279
$\infty$	-0.49473	0.49752	0.44967	0.58356	0.44967	0.50129	0.51098	0.58356	0.51098	0.72564
Expected	-0.5	0.5	0.5	0.59460	0.5	0.5	0.59460	0.59460	0.59460	0.70711

i.e.  $2^{-3/4}$

# Comments on double branes

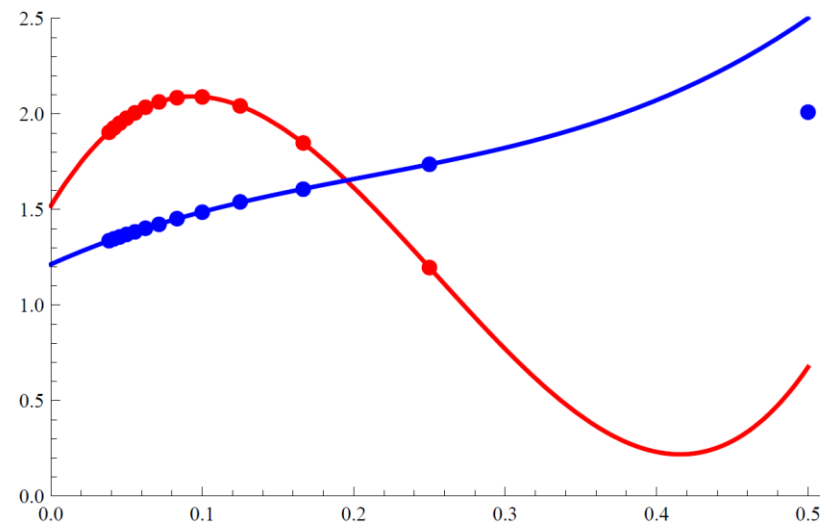
- Since we have developed quite an efficient code for solving e.o.m. in level truncation, it is natural to look for other solutions in the universal basis besides the tachyon vacuum.
- We start with a promising complex solution found easily at level 2 and improve it via the Newton's method to higher levels.

# Comments on double branes

- We found the following dependence on the level:

$L$	$Energy$	$W$
2	$-1.42791 - 3.40442i$	$2.00934 - 0.0545341i$
4	$1.19625 - 2.25966i$	$1.73651 + 0.117637i$
6	$1.84813 - 1.58507i$	$1.60634 + 0.195442i$
8	$2.04207 - 1.14971i$	$1.53973 + 0.217911i$
10	$2.08908 - 0.866428i$	$1.48598 + 0.22801i$
12	$2.08515 - 0.674602i$	$1.4521 + 0.227059i$
14	$2.06302 - 0.53887i$	$1.42232 + 0.224184i$
16	$2.03499 - 0.439057i$	$1.40194 + 0.218266i$
18	$2.00593 - 0.363272i$	$1.38304 + 0.212332i$
20	$1.9778 - 0.304197i$	$1.36942 + 0.205378i$
22	$1.95135 - 0.257139i$	$1.35632 + 0.198765i$
24	$1.92679 - 0.218971i$	$1.34654 + 0.191784i$
26	$1.90411 - 0.187545i$	$1.33691 + 0.185169i$

Real part of **energy** and **W-invariant** as a function of  $1/L$  :

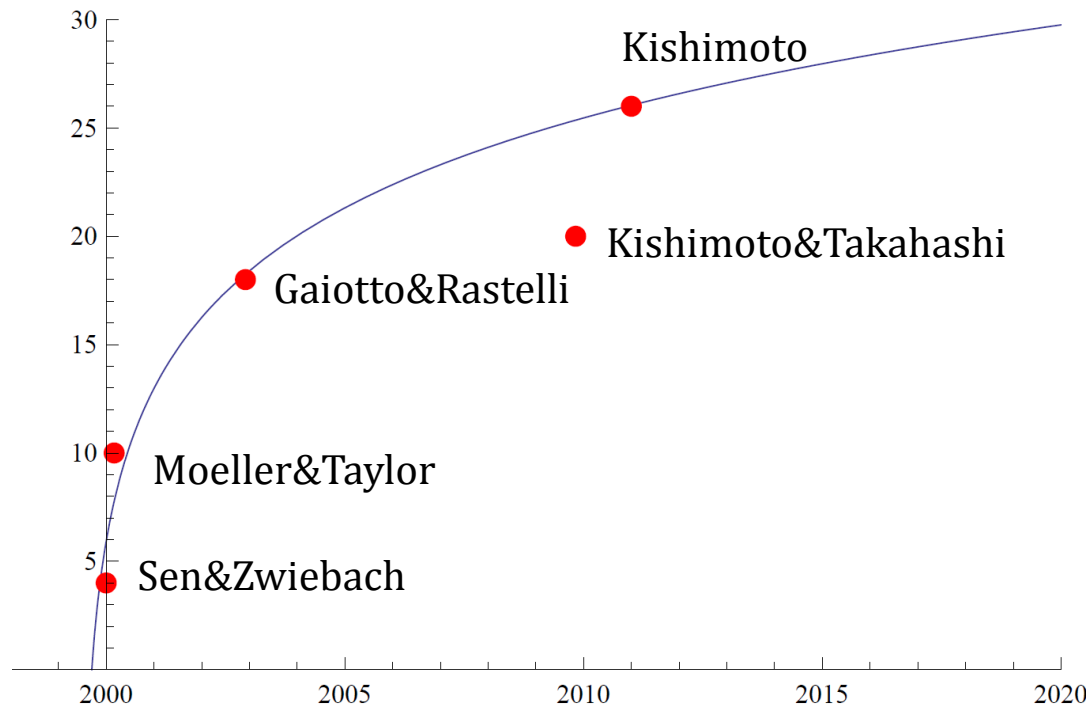


- Should we dismiss the solution, or hope for a oscillation or a cusp at higher levels?



# Future of level truncation

- Hopefully soon level 30 should be reached



## Required tools:

- universal basis
- conservation laws
- C++
- $SU(1,1)$  singlet basis
- Parallelism
- ???

N.B.: Level 30 is interesting, as we should see the oscillation for the tachyon vacuum energy predicted by Gaiotto and Rastelli.

# Conclusions

- High level numerical computations in OSFT have the potential to discover new boundary states (e.g. they could have predicted existence of fractional D-branes) .
- The key tool for physical identification are the generalized Ellwood invariants.
- First well behaved positive energy solution discovered ! (describing  $\sigma$ -brane on a 1-brane, or perhaps double branes)
- We are coming to an era of possible computer exploration of the OSFT landscape – stay tuned!