The Integrated Perturbation Theory for the Large-scale Structure of the Universe

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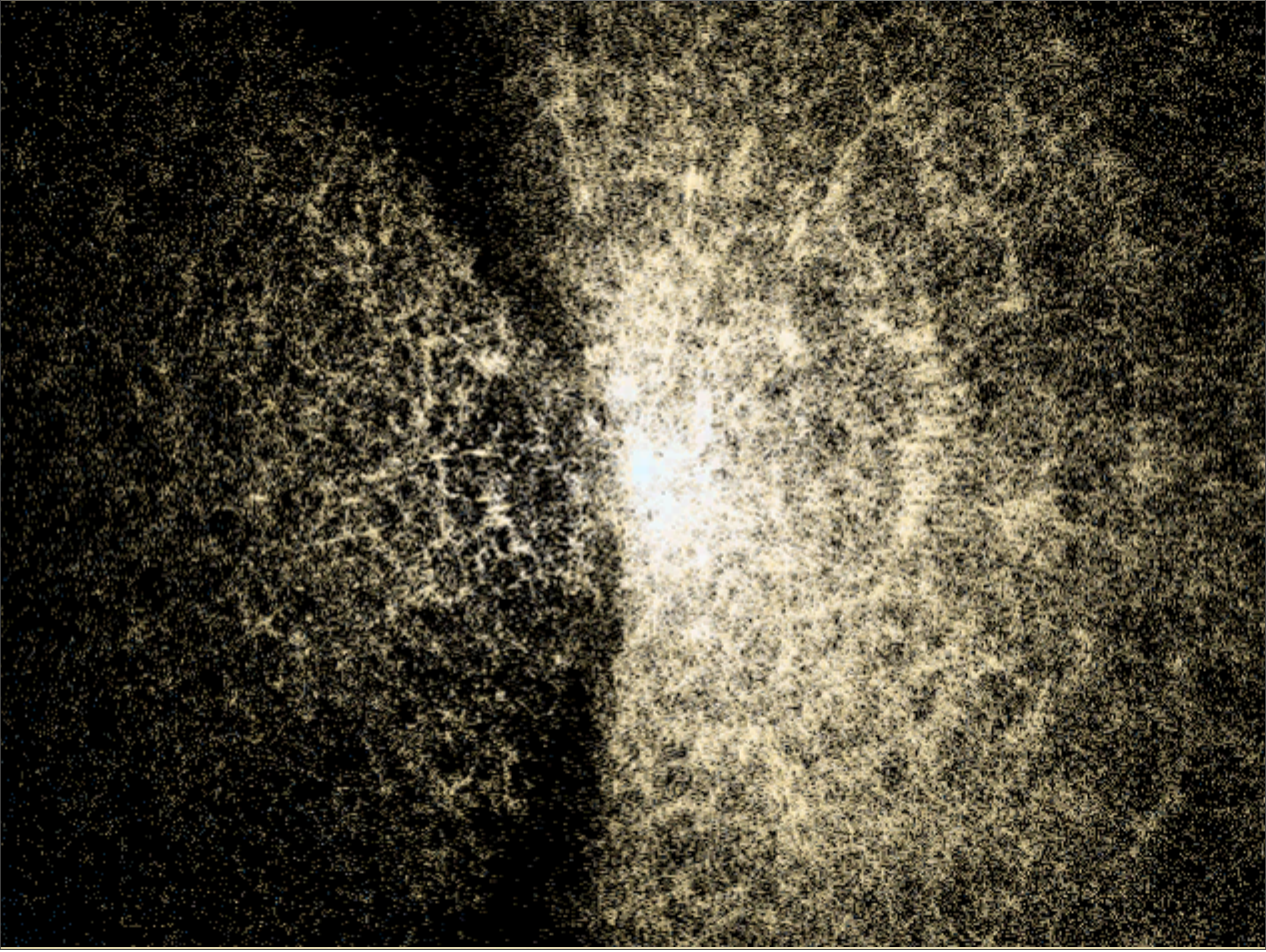

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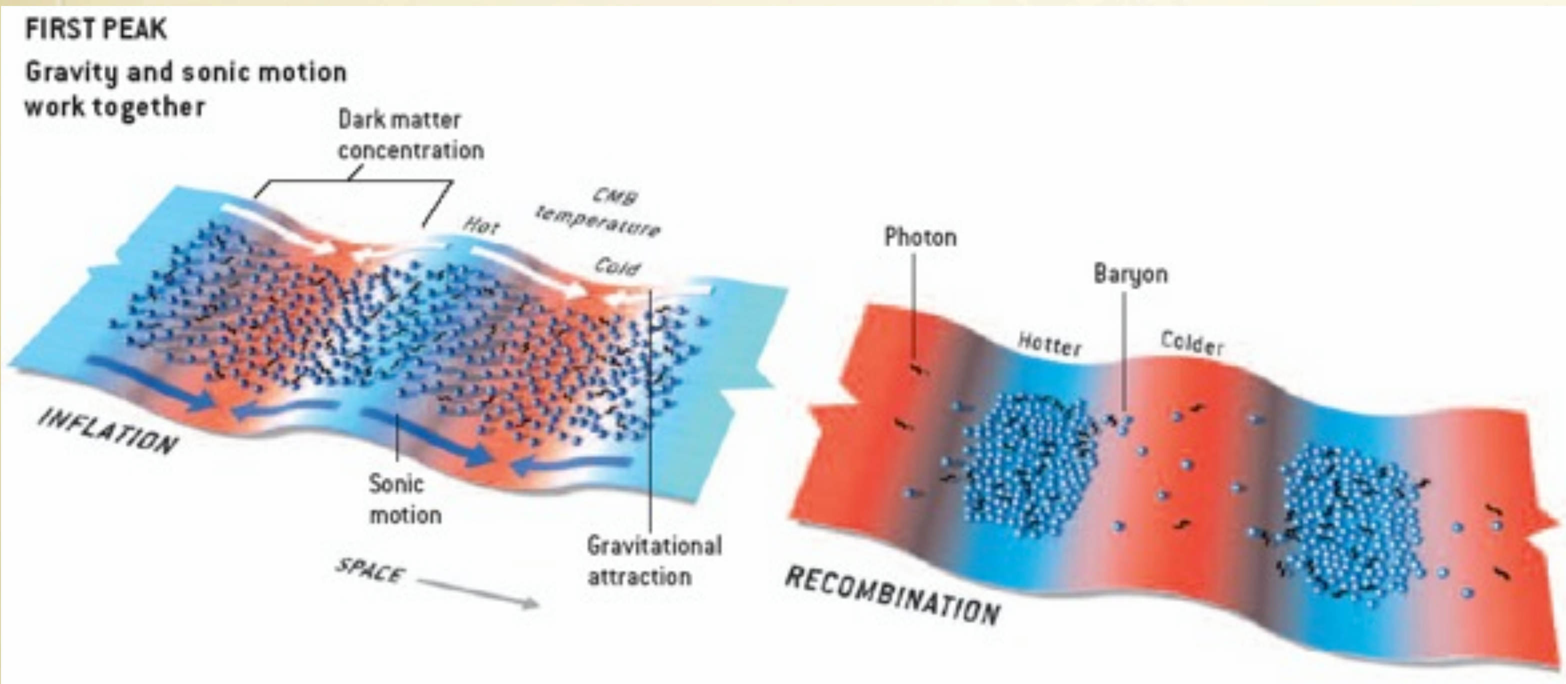
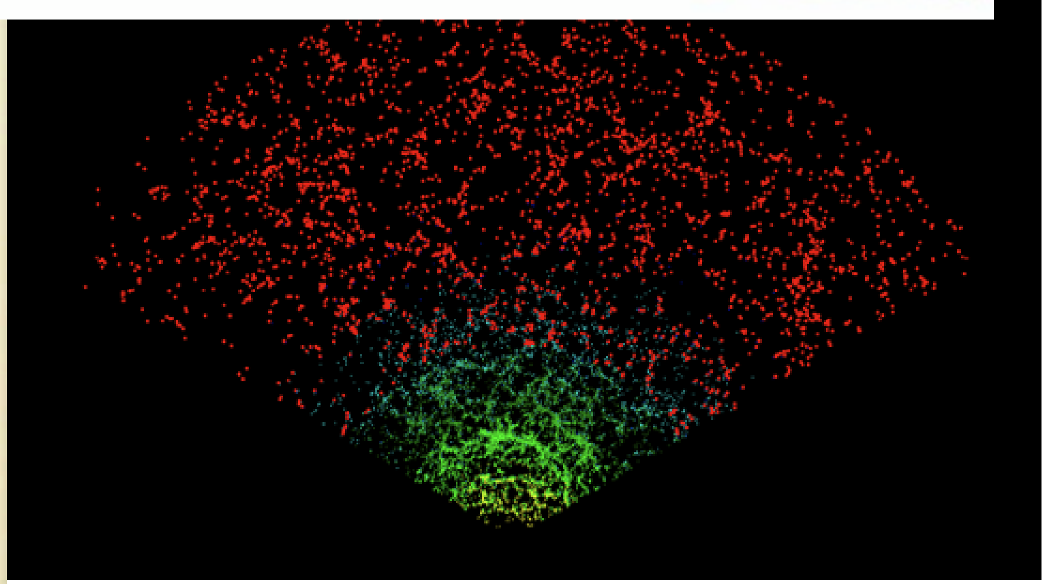
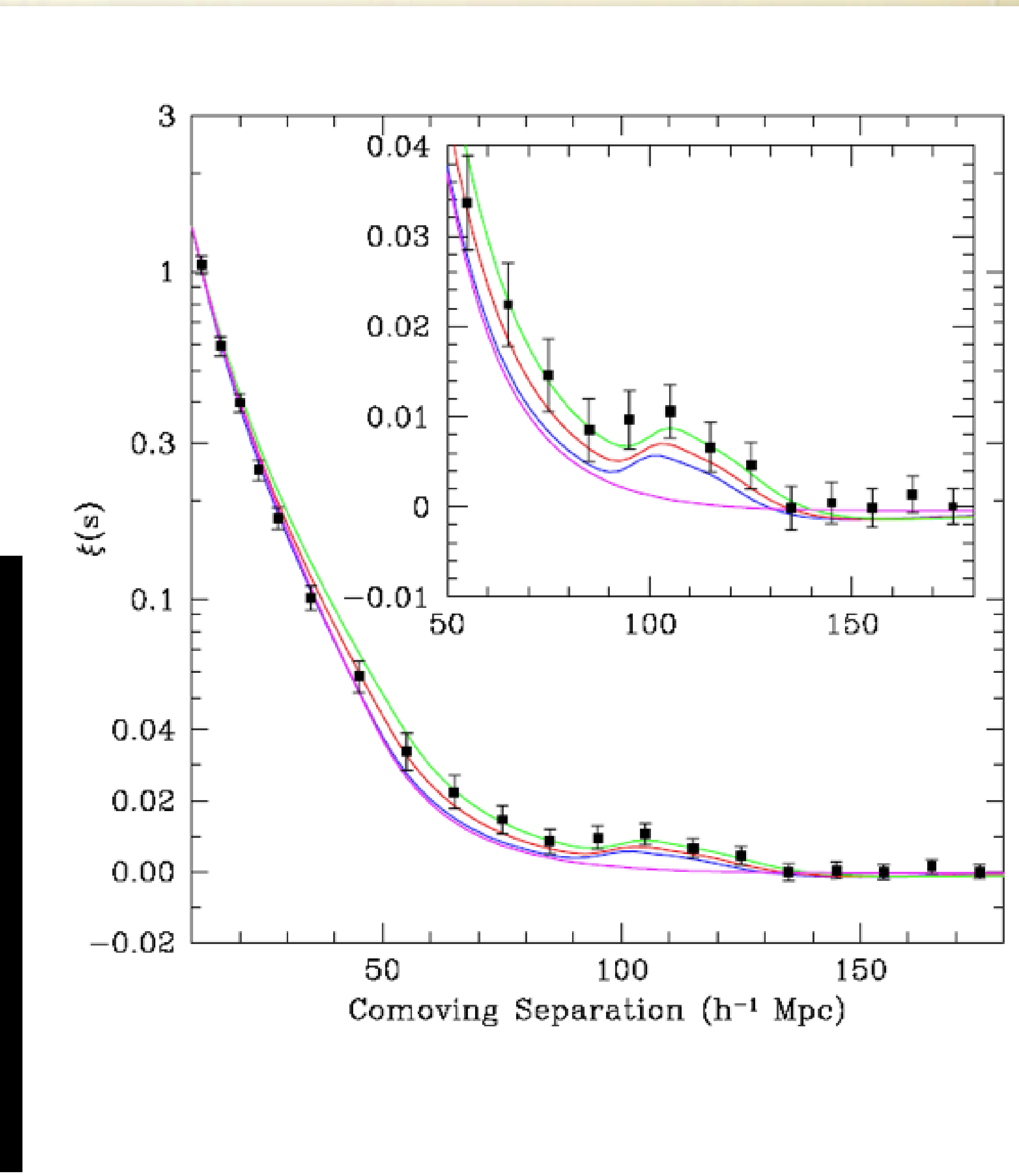
BAO

- **Baryon Acoustic Oscillations (BAO)**



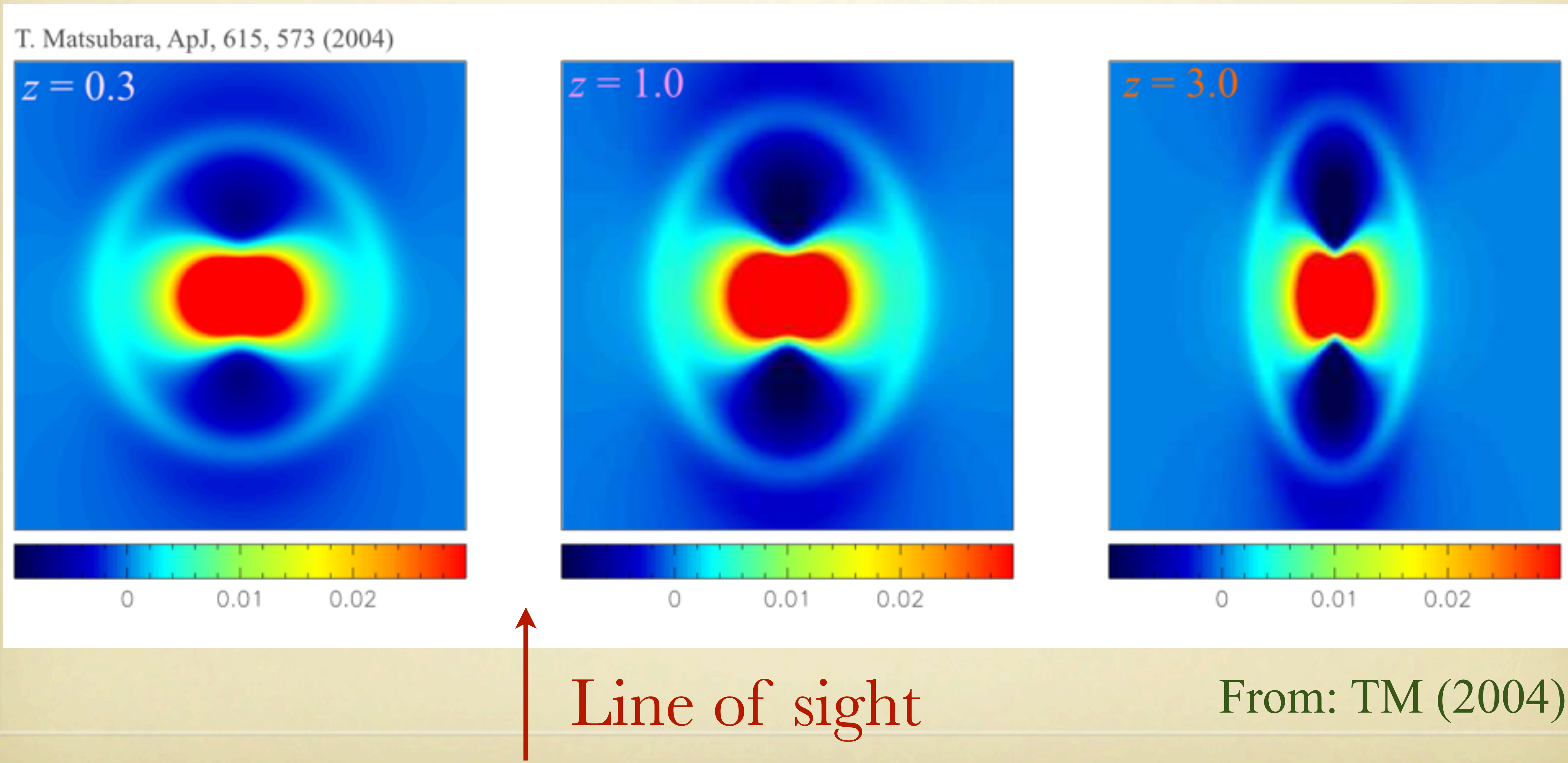




From: Hu & White (2004); Eisenstein+ (2005)

BAO ring

- In 2D, "BAO ring" is a double ruler
  - $H(z)$, $D_A(z)$: sensitive to the dark energy



From: TM (2004)

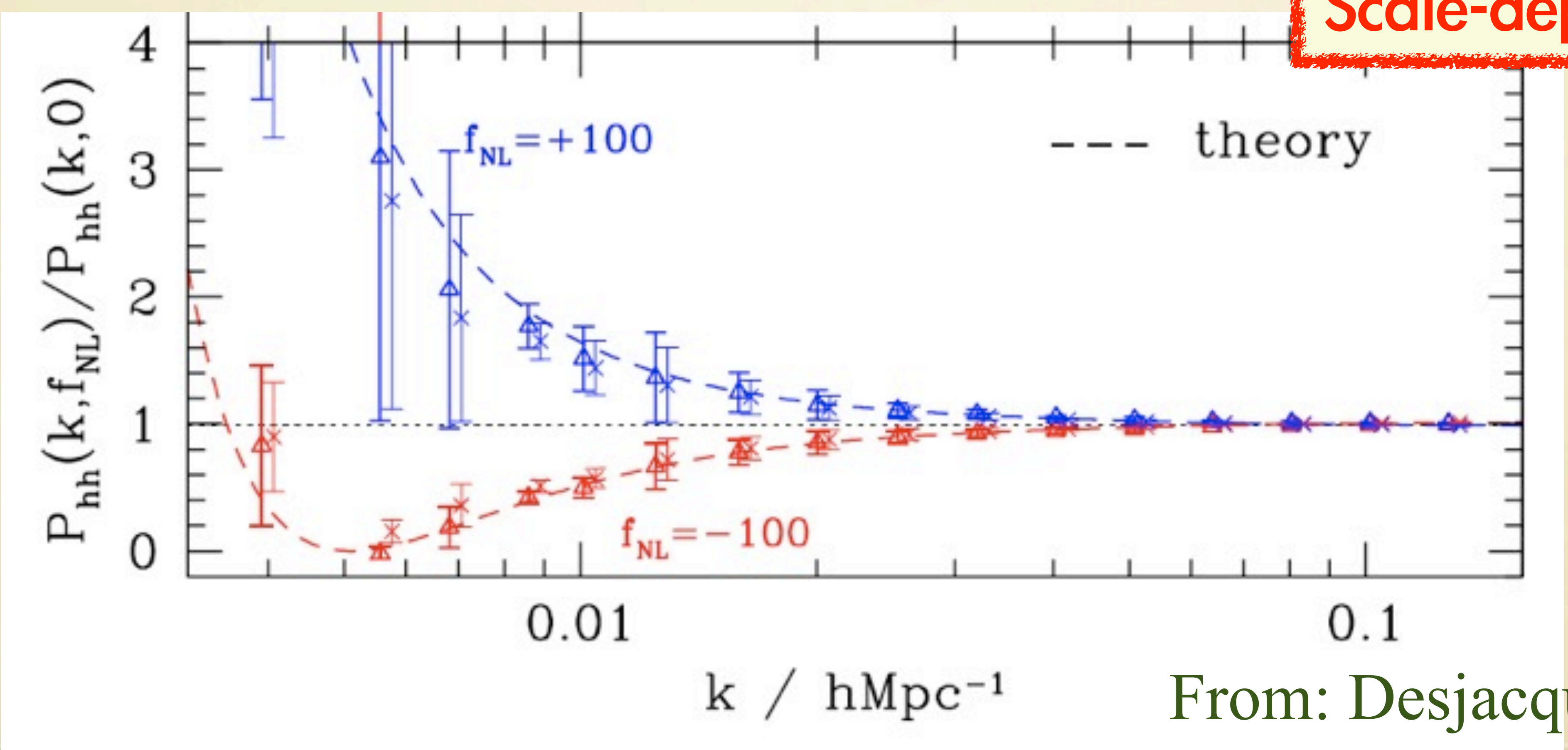
Primordial non-Gaussianity in LSS

- Primordial non-Gaussianity increases the power spectrum of galaxies on VERY LARGE SCALES

$$\Phi(\mathbf{r}) = \Phi_L(\mathbf{r}) + f_{\rm NL}\left(\Phi_L^2(\mathbf{r}) - \langle\Phi_L^2(\mathbf{r})\rangle\right)$$

$$\Delta b(M,k) = 3 f_{\rm NL}(b-1)\delta_c \frac{\Omega_m}{k^2 T(k) D(z)}\left(\frac{H_0}{c}\right)^2$$

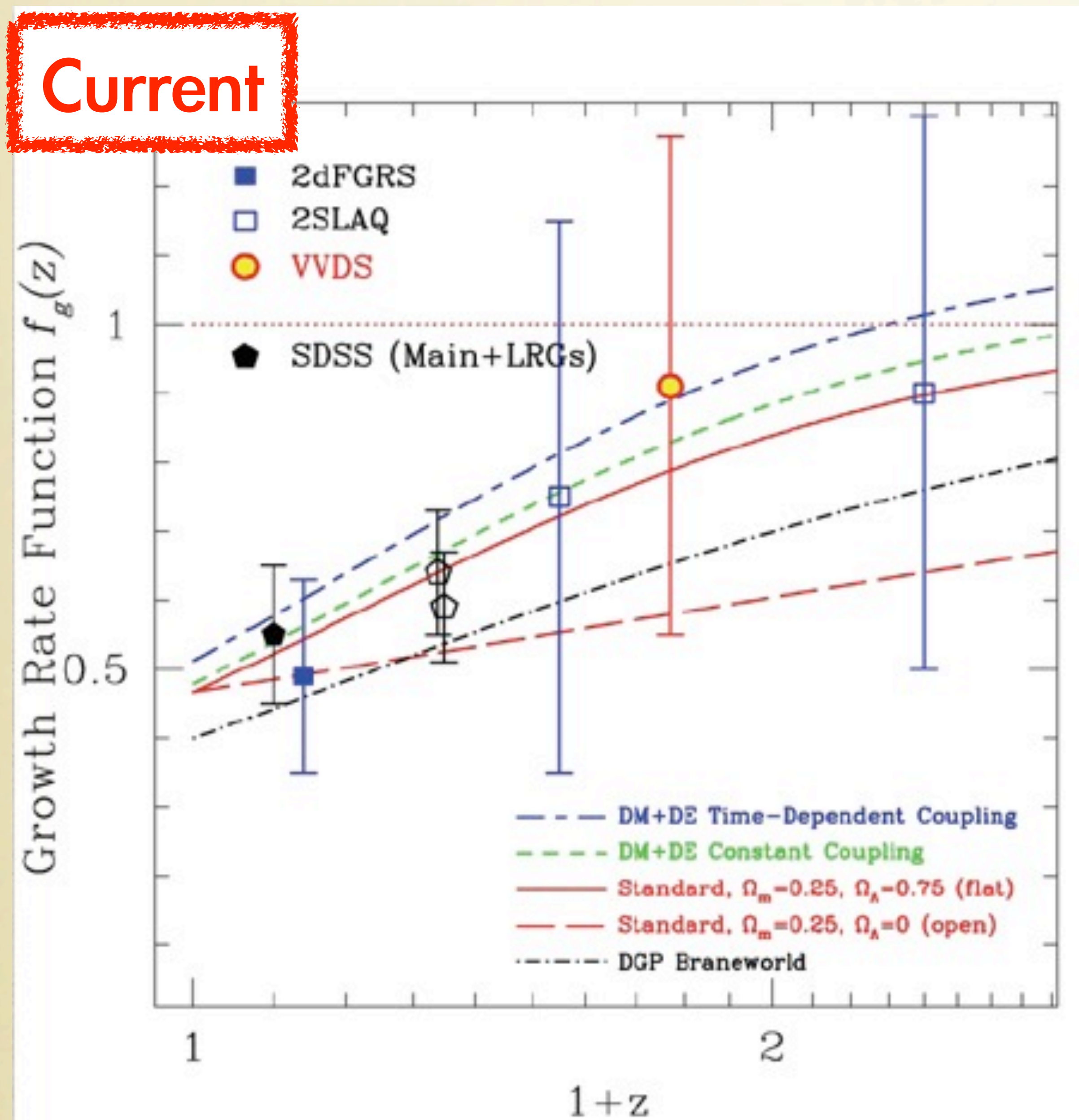
Scale-dependent bias

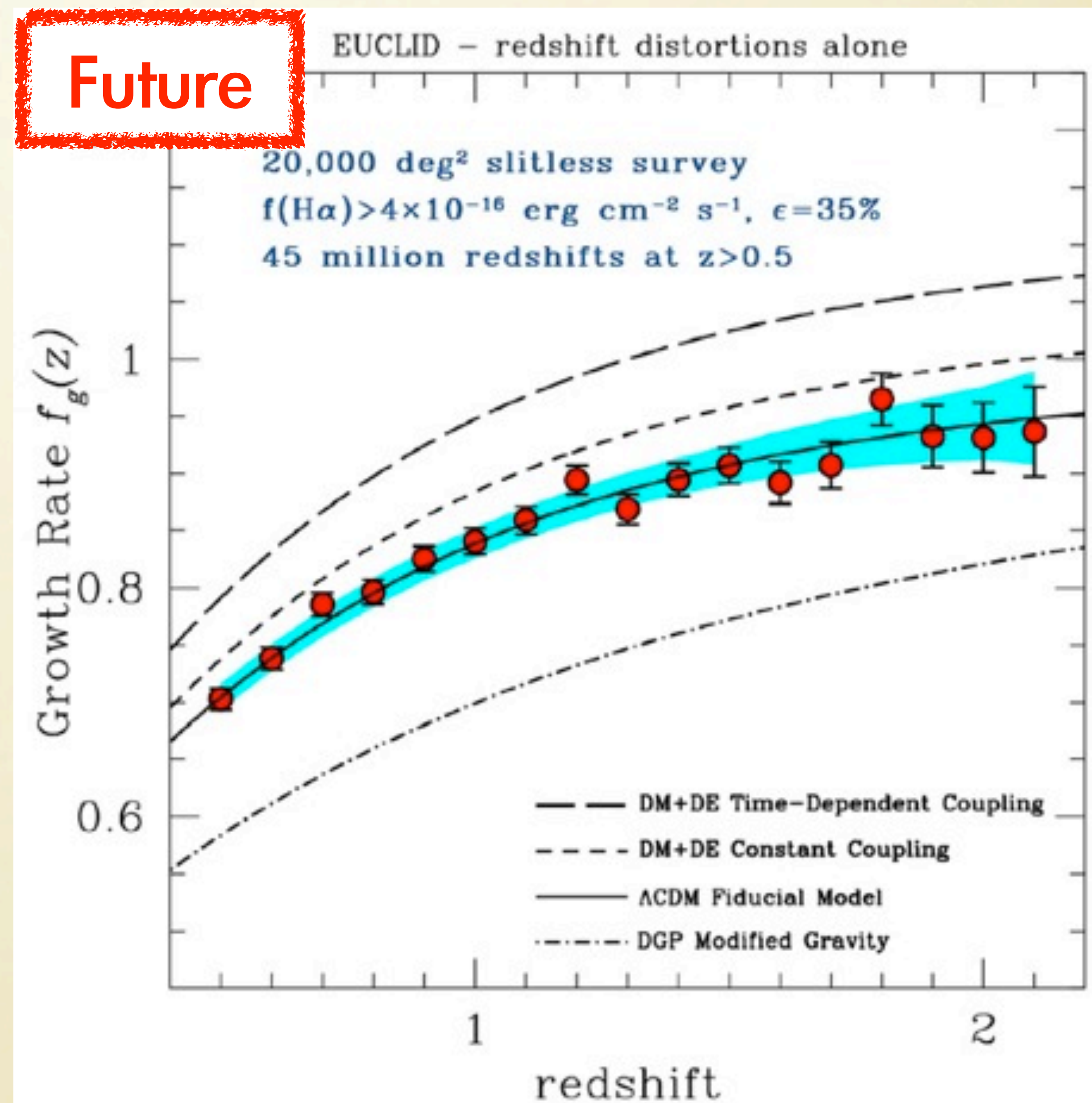


From: Desjacques+ (2009)

Constraining modified gravity

- **RSD measures:** $$f_g(z) = \frac{d\ln D}{d\ln a} \simeq \Omega_{\mathrm{M}}^{\gamma}$$





From: Guzzo+ (2008); Euclid Yellow Book (2011)

Complexities in LSS

- Nonlinear evolutions
  - analytically hard problem
  - important even on large-scales (for precision cosmology)
- Redshift-space distortions (RSD)
  - peculiar velocities of galaxy displace the position in redshift space
- Biasing
  - galaxies are not unbiased tracers of matter

Perturbation theory approach

- **Nonlinear evolutions can be solved by the perturbation theory (on large scales)**

Tomita (1967), Juszkiewicz (1981), Vishniac (1983), Goroff et al. (1986), Makino et al. (1992), ...

Continuity: $$\frac{\partial \delta}{\partial t} + \frac{1}{a}\boldsymbol{\nabla} \cdot [(1+\delta)\boldsymbol{v}] = 0$$

Euler: $$\frac{\partial \boldsymbol{v}}{\partial t} + \frac{\dot{a}}{a}\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = -\frac{1}{a}\boldsymbol{\nabla}\Phi$$

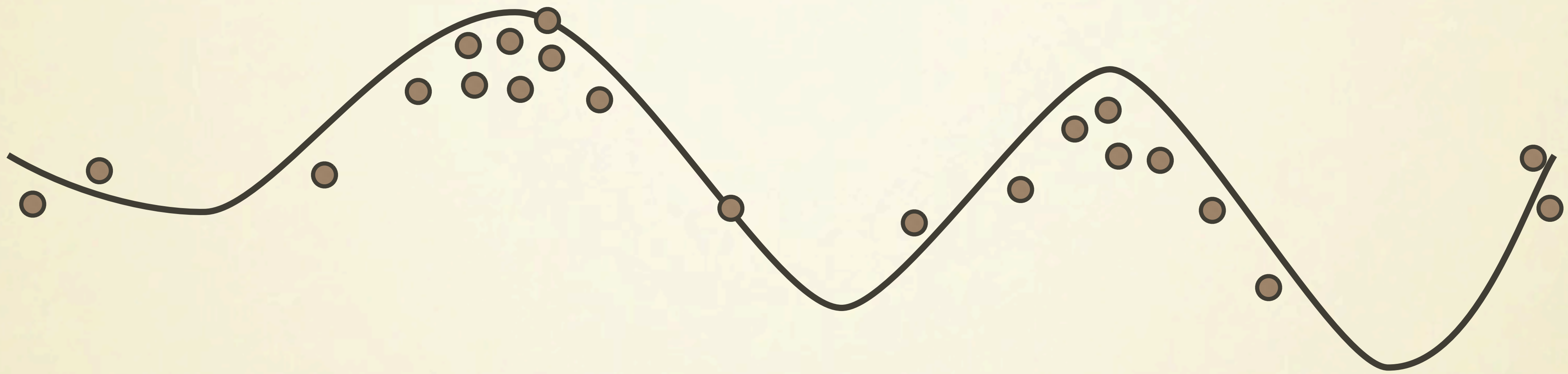
Poisson: $$\triangle \Phi = 4\pi G a^2 \bar{\varrho} \delta$$

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots$$

Integrated perturbation theory (iPT)

- Integration of Four “Non-’s”
  - nonlinear perturbation theory
  - nonlocal bias
  - nonlinear redshift-space distortions
  - non-Gaussianity of primordial density fields

Bias between mass and objects



- Mass density ≠ number density (in general)
- Densities of both mass and astronomical objects are determined by initial density field
  - There should be a relation

$$\delta_{\mathrm{m}}(\boldsymbol{x}) = \frac{\rho_{\mathrm{m}}(\boldsymbol{x})}{\bar{\rho}_{\mathrm{m}}} - 1 \quad \Leftrightarrow \quad \delta_{\mathrm{X}}(\boldsymbol{x}) = \frac{\rho_{\mathrm{X}}(\boldsymbol{x})}{\bar{\rho}_{\mathrm{X}}} - 1$$

Eulerian Local Bias model

- Local bias: a simple model usually adopted in the nonlinear perturbation theory
  - The number density is assumed to be locally determined by (smoothed) mass density

$$\delta_{\mathrm{X}}(\boldsymbol{x}) = F_{\mathrm{X}}(\delta_{\mathrm{m}}(\boldsymbol{x}))$$

  - Apply a Taylor expansion

$$\delta_{\mathrm{X}}(\boldsymbol{x}) = b_0 + b_1 \delta_{\mathrm{m}}(\boldsymbol{x}) + \frac{b_2}{2!}\delta_{\mathrm{m}}^2(\boldsymbol{x}) + \cdots$$

  - PHENOMENOLOGICAL model, just for simplicity, divergences in loop corrections

Nonlocal bias

- "Functional" instead of function

$$\delta_{\mathrm{m}} = \mathcal{F}_{\mathrm{m}}[\delta_{\mathrm{L}}], \quad \delta_{\mathrm{X}} = \mathcal{F}_{\mathrm{X}}[\delta_{\mathrm{L}}]$$

- For a single streaming fluid (quasi-nonlinear regime)

$$\delta_{\mathrm{L}} = \mathcal{F}_{\mathrm{m}}^{-1}[\delta_{\mathrm{m}}] \quad \Rightarrow \quad \delta_{\mathrm{X}} = \mathcal{F}_{\mathrm{X}}[\mathcal{F}_{\mathrm{m}}^{-1}[\delta_{\mathrm{m}}]]$$

- Taylor expansion of the functional

$$\delta_{\mathrm{X}}(\boldsymbol{x}) = \int d^3x_1 b_1(\boldsymbol{x} - \boldsymbol{x}_1)\delta_{\mathrm{m}}(\boldsymbol{x}_1) + \frac{1}{2!}\int d^3x_1 d^3x_2 b_2(\boldsymbol{x} - \boldsymbol{x}_1, \boldsymbol{x} - \boldsymbol{x}_2)\delta_{\mathrm{m}}(\boldsymbol{x}_1)\delta_{\mathrm{m}}(\boldsymbol{x}_2) + \cdots$$

Nonlocal bias

- Perturbative expansions in Fourier space:
  - nonlocal bias:

$$\delta_{\mathrm{X}}(\boldsymbol{k}) = b_1(\boldsymbol{k})\delta_{\mathrm{m}}(\boldsymbol{k}) + \frac{1}{2!}\int \frac{d^3k'}{(2\pi)^3} b_2(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}')\delta_{\mathrm{m}}(\boldsymbol{k}')\delta_{\mathrm{m}}(\boldsymbol{k}-\boldsymbol{k}') + \cdots$$

  - nonlinear dynamics

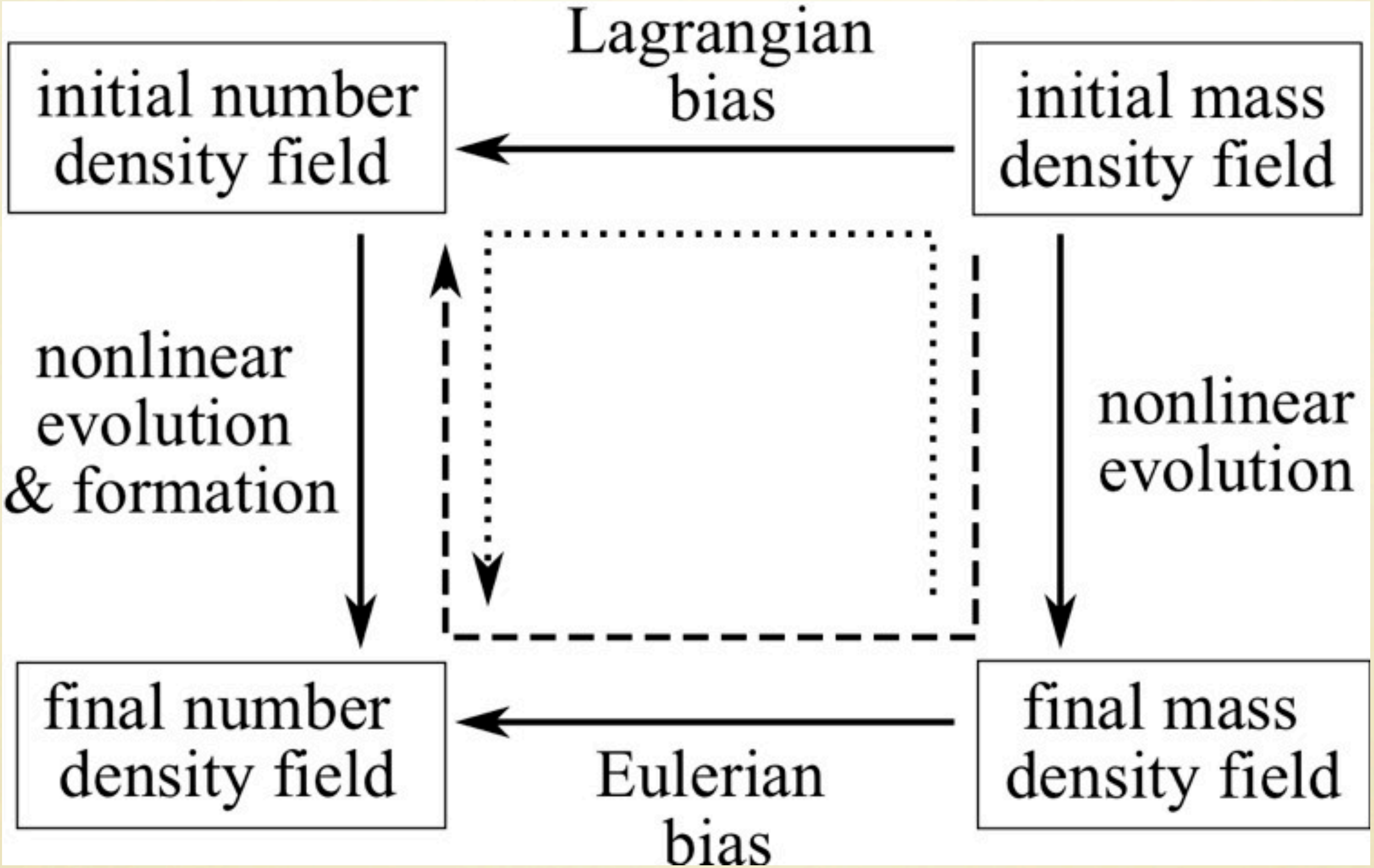
$$\delta_{\mathrm{m}}(\boldsymbol{k}) = \delta_{\mathrm{L}}(\boldsymbol{k}) + \frac{1}{2!}\int \frac{d^3k'}{(2\pi)^3} F_2(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}')\delta_{\mathrm{L}}(\boldsymbol{k}')\delta_{\mathrm{L}}(\boldsymbol{k}-\boldsymbol{k}') + \cdots$$

  - It is straightforward to calculate observables, such as power spectrum, bispectrum, etc.

The problem with Eulerian bias

- Eulerian bias is not physically motivated
  - Phenomenological & simple model
- Physical models of bias known so far is provided in Lagrangian space
  - e.g., Halo bias model, Peak bias model,...
- Relation between Eulerian bias and Lagrangian bias?

Eulerian & Lagrangian bias



Nonlinear dynamics is nonlocal, so is nonlinear bias

Eulerian & Lagrangian bias

- Eulerian and Lagrangian biases are compatible in a nonlocal formalism
  - The relations can be explicitly derived in perturbation theory:

$$
\begin{aligned}
b_1(\boldsymbol{k}) &= b_1^{\mathrm{L}}(\boldsymbol{k}) + 1, \\
b_2(\boldsymbol{k}_1, \boldsymbol{k}_2) &= b_2^{\mathrm{L}}(\boldsymbol{k}_1, \boldsymbol{k}_2) - b_1^{\mathrm{L}}(\boldsymbol{k}_1 + \boldsymbol{k}_2) F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) \\
&\qquad + \left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_1^2}\right) b_1^{\mathrm{L}}(\boldsymbol{k}_2) + \left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_2^2}\right) b_1^{\mathrm{L}}(\boldsymbol{k}_1),
\end{aligned}
$$

etc.

Local biases are incompatible

Lagrangian perturbation theory with Lagrangian (nonlocal) bias

- The relation between Eulerian density fluctuations and Lagrangian variables

$$1+\delta_{\rm X}(\boldsymbol{x}) = \int d^3q \left[1+\delta_{\rm X}^{\rm L}(\boldsymbol{q})\right] \delta_{\rm D}^3[\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{\Psi}(\boldsymbol{q})]$$

Eulerian density field

Biased field in Lagrangian space

displacement (& redshift distortions)

- Perturbative expansion in Fourier space

Kernel of the Lagrangian bias

$$\delta_{\rm X}^{\rm L}(\boldsymbol{k}) = \sum_{n=1}^{\infty} \frac{1}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta_{\rm D}^3(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k}) b_n^{\rm L}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) \delta_{\rm L}(\boldsymbol{k}_1)\cdots\delta_{\rm L}(\boldsymbol{k}_n)$$

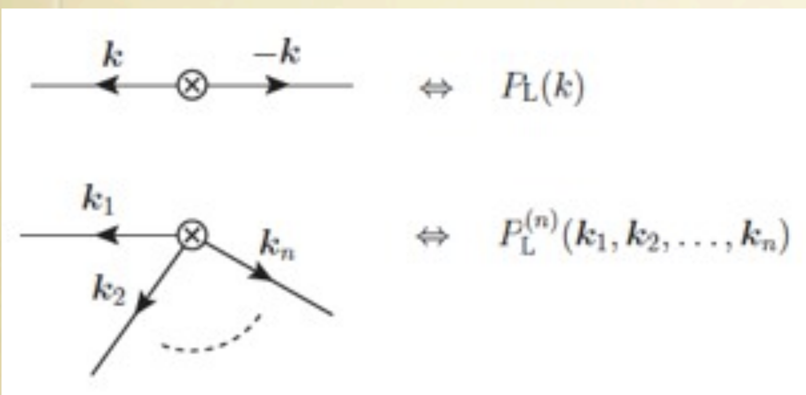
$$\tilde{\boldsymbol{\Psi}}(\boldsymbol{k}) = \sum_{n=1}^{\infty} \frac{i}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta_{\rm D}^3(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k}) \boldsymbol{L}_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) \delta_{\rm L}(\boldsymbol{k}_1)\cdots\delta_{\rm L}(\boldsymbol{k}_n)$$

$$\boldsymbol{k}_{1\cdots n} \equiv \boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n$$

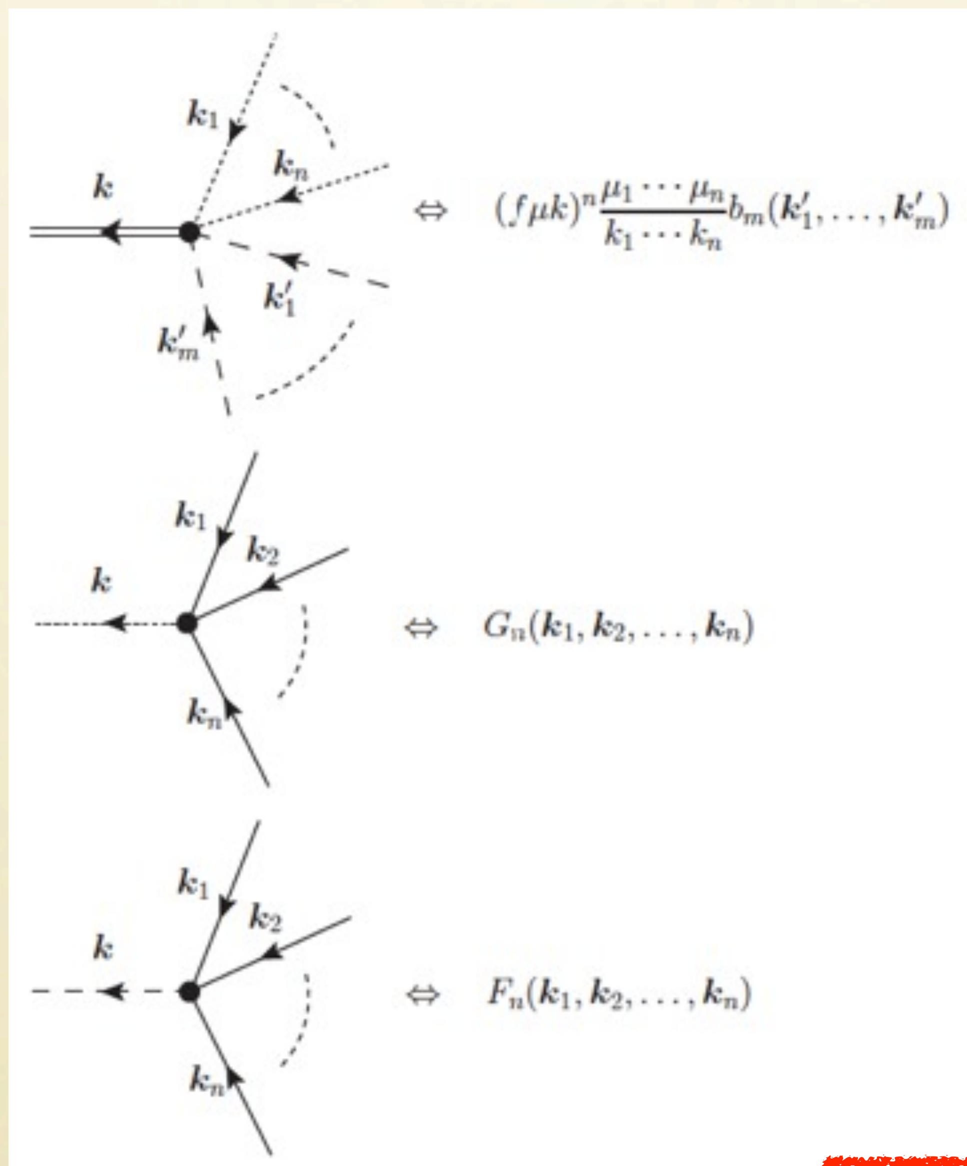
Kernel of the displacement field (& redshift distortions)

Diagrams

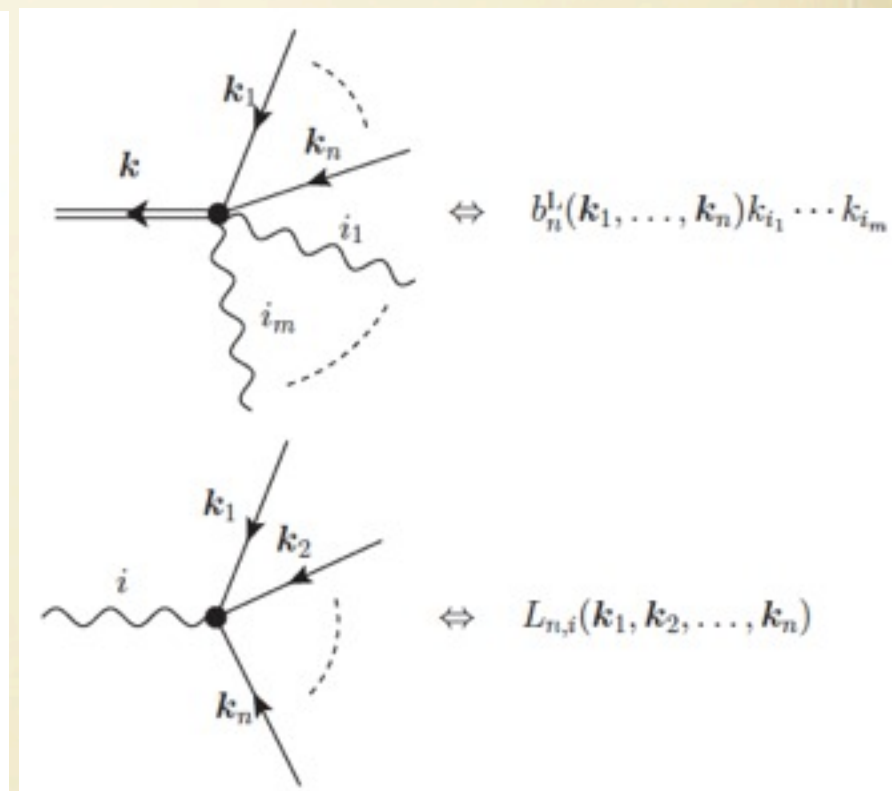
Primordial spectra



Vertices in Eulerian PT



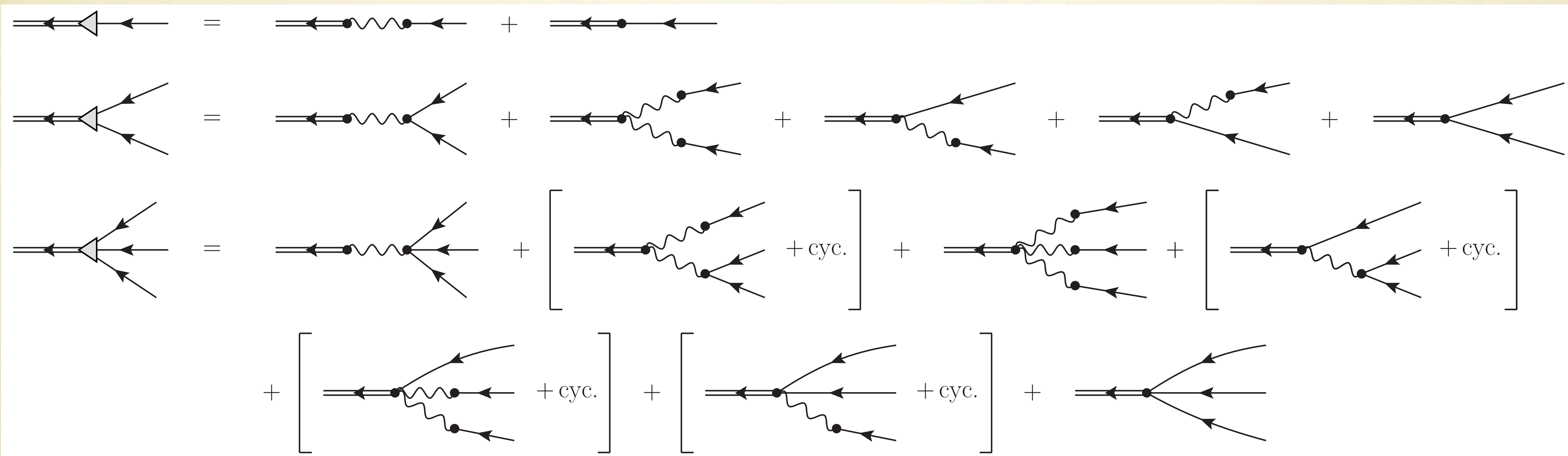
Vertices in Lagrangian PT



RSD and nG included

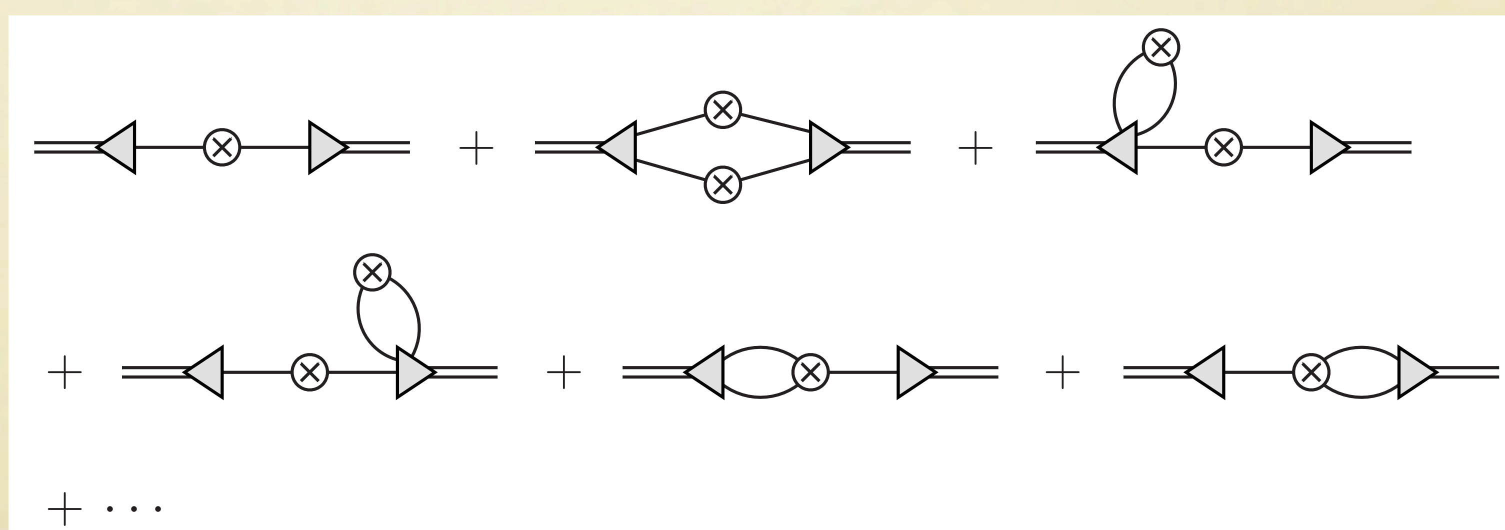
Vertex shrunk

- Shrunk vertices



- Ex.)

$$P_{\mathrm{X}}(k) =$$

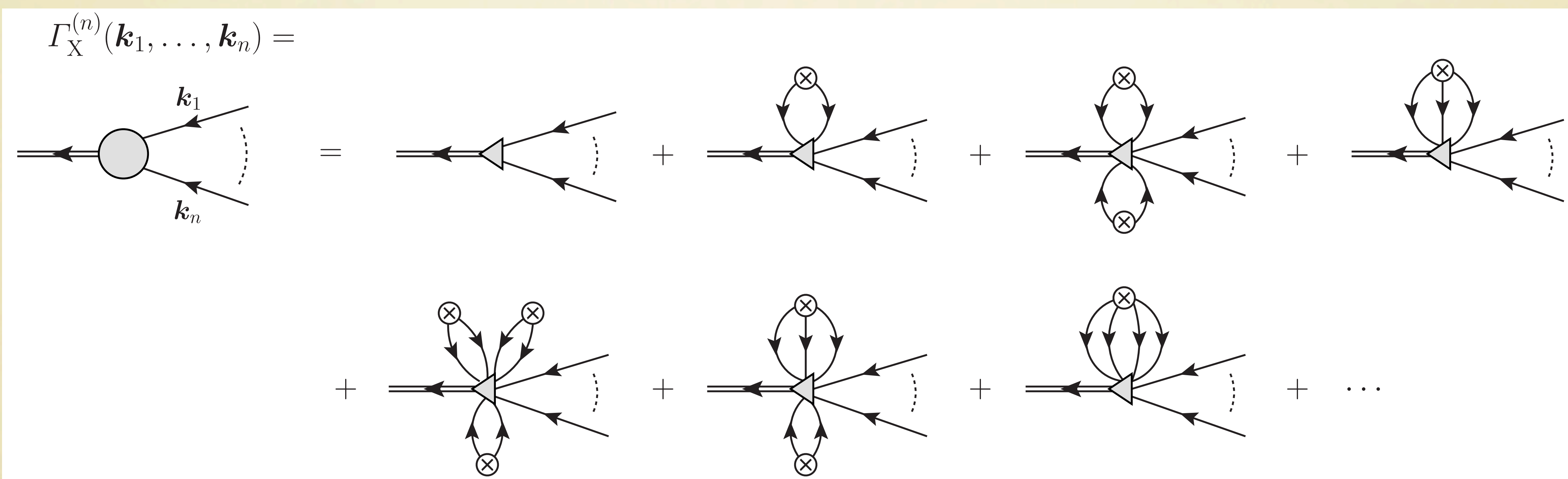


Multi-point propagator

TM (1995); Crocce & Scoccimarro (2006), Bernardeau et al. (2008)

- **Density sector of multi-point propagator with nonlocal bias and RSD**

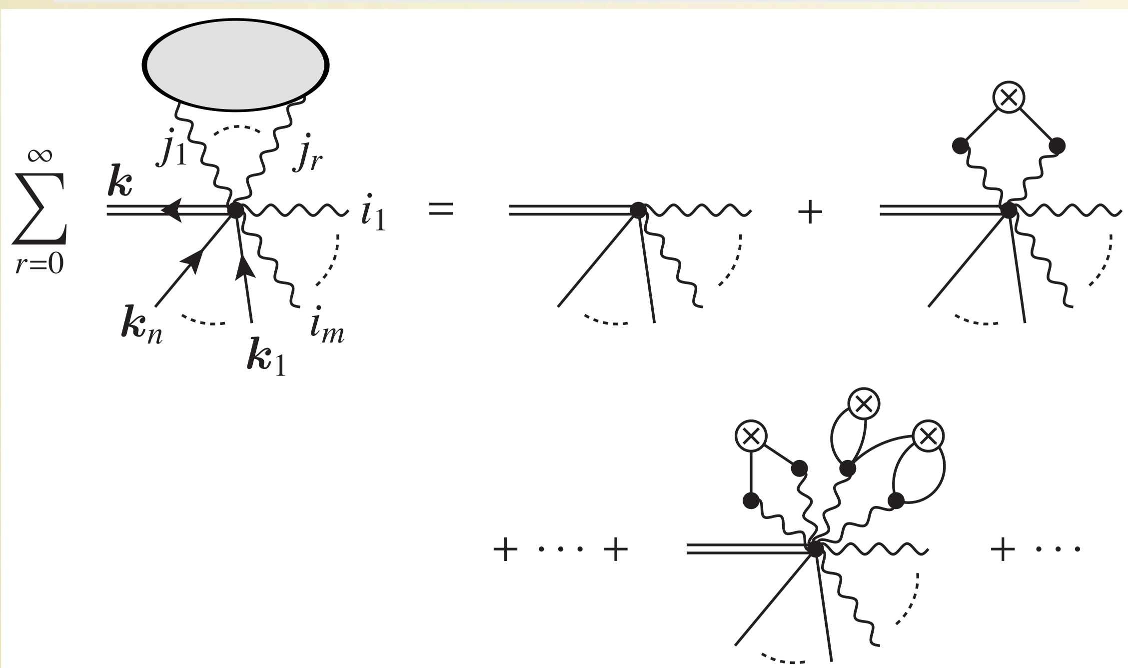
$$\left\langle \frac{\delta^n \delta_{\rm X}(\boldsymbol{k})}{\delta\delta_{\rm L}(\boldsymbol{k}_1)\cdots\delta\delta_{\rm L}(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta_{\rm D}^3(\boldsymbol{k}-\boldsymbol{k}_{1\cdots n})\Gamma_{\rm X}^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$$



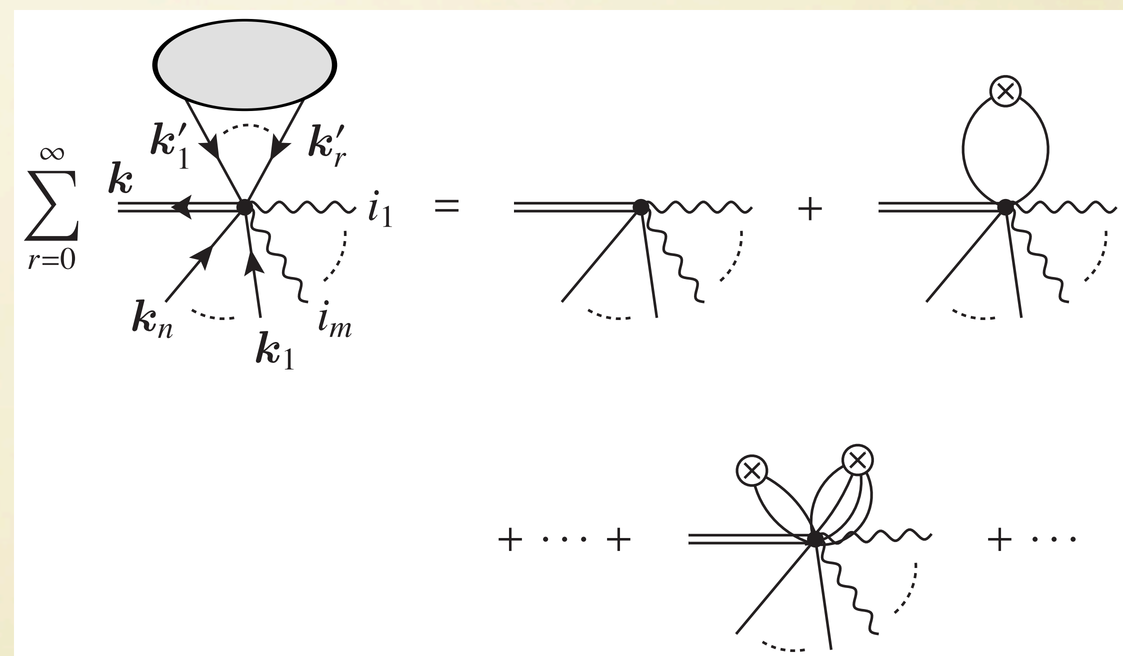
Multi-point propagator

- Full evaluations of MP propagator are difficult
- Partial resummations in the Lagrangian PT

Lagrangian vertex resummation



Lagrangian bias renormalization



$$\begin{aligned}\Pi(\boldsymbol{k}) &= \left\langle e^{-i\boldsymbol{k}\cdot\boldsymbol{\Psi}}\right\rangle \\ &= \exp\left[\sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\langle(\boldsymbol{k}\cdot\boldsymbol{\Psi})^n\rangle_{\mathrm{c}}\right]\end{aligned}$$

$$b_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n}\int\frac{d^3k'}{(2\pi)^3}\left.\frac{\delta^n\delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)}\right|_{\delta_{\mathrm{L}}=0}$$

$$\Rightarrow\ c_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n}\int\frac{d^3k'}{(2\pi)^3}\left\langle\frac{\delta^n\delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)}\right\rangle$$

Renormalized bias functions

- **Introduction of the "renormalized bias functions" is essential in iPT**

$$b_n^{\rm L}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n}\int\frac{d^3k'}{(2\pi)^3}\left.\frac{\delta^n\delta_{\rm X}^{\rm L}(\boldsymbol{k}')}{\delta\delta_{\rm L}(\boldsymbol{k}_1)\cdots\delta\delta_{\rm L}(\boldsymbol{k}_n)}\right|_{\delta_{\rm L}=0}$$

$$\Rightarrow\ c_n^{\rm L}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n}\int\frac{d^3k'}{(2\pi)^3}\left\langle\frac{\delta^n\boxed{\delta_{\rm X}^{\rm L}(\boldsymbol{k}')}}{\delta\delta_{\rm L}(\boldsymbol{k}_1)\cdots\delta\delta_{\rm L}(\boldsymbol{k}_n)}\right\rangle$$

"Renormalized bias functions"

Biased field in Lagrangian space

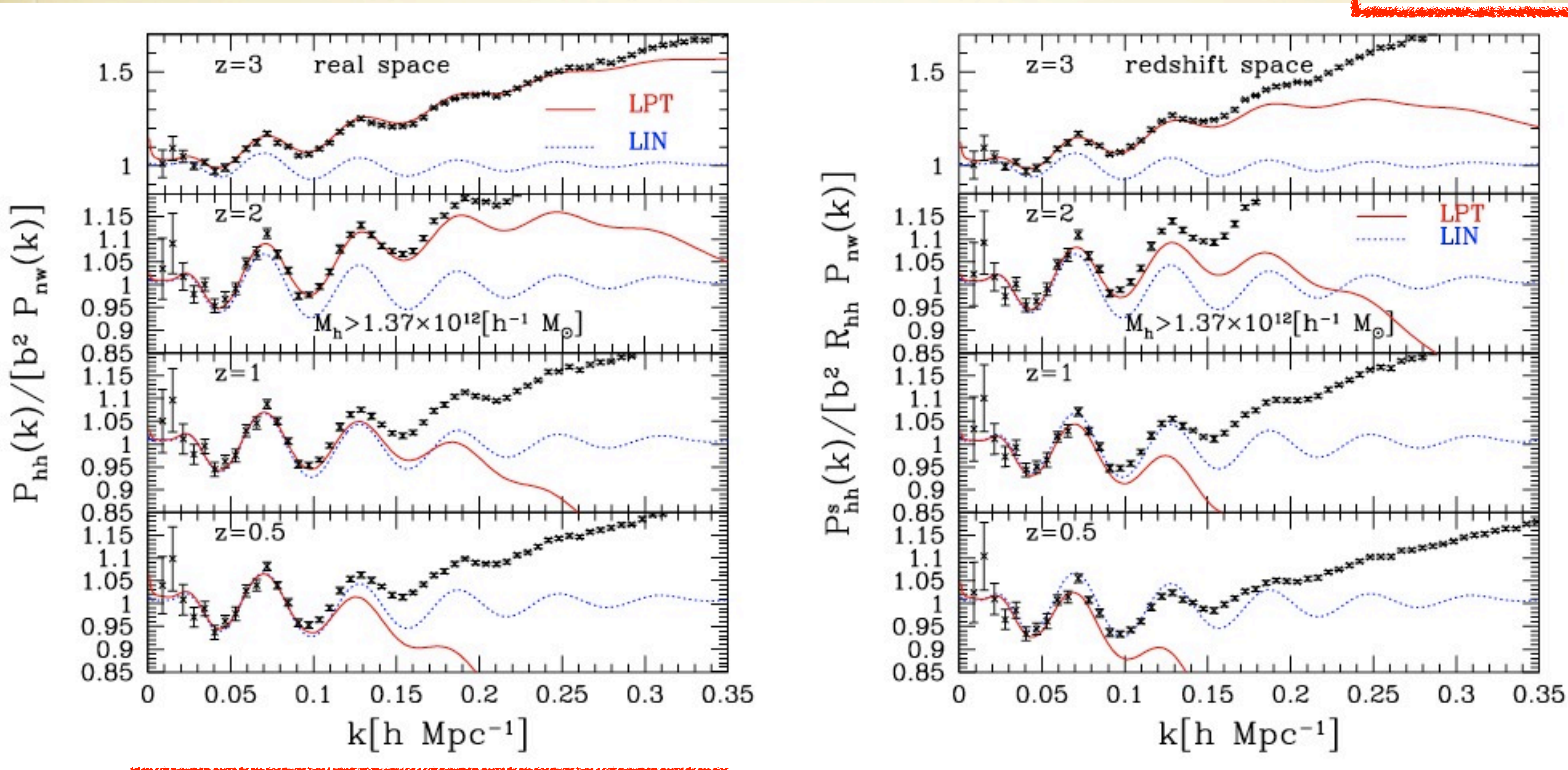
$$\Gamma_{\rm X}^{(1)}(\boldsymbol{k}) = 1 + c_1^{\rm L}(k)$$

$$\Gamma_{\rm X}^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) = F_2(\boldsymbol{k}_1,\boldsymbol{k}_2) + \left(1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{{k_1}^2}\right)c_1^{\rm L}(k_2) + \left(1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{{k_2}^2}\right)c_1^{\rm L}(k_1) + c_2^{\rm L}(\boldsymbol{k}_1,\boldsymbol{k}_2)$$

(eg.lowest-order)

Halo clustering: Lagrangian resummation & N-body

One-loop

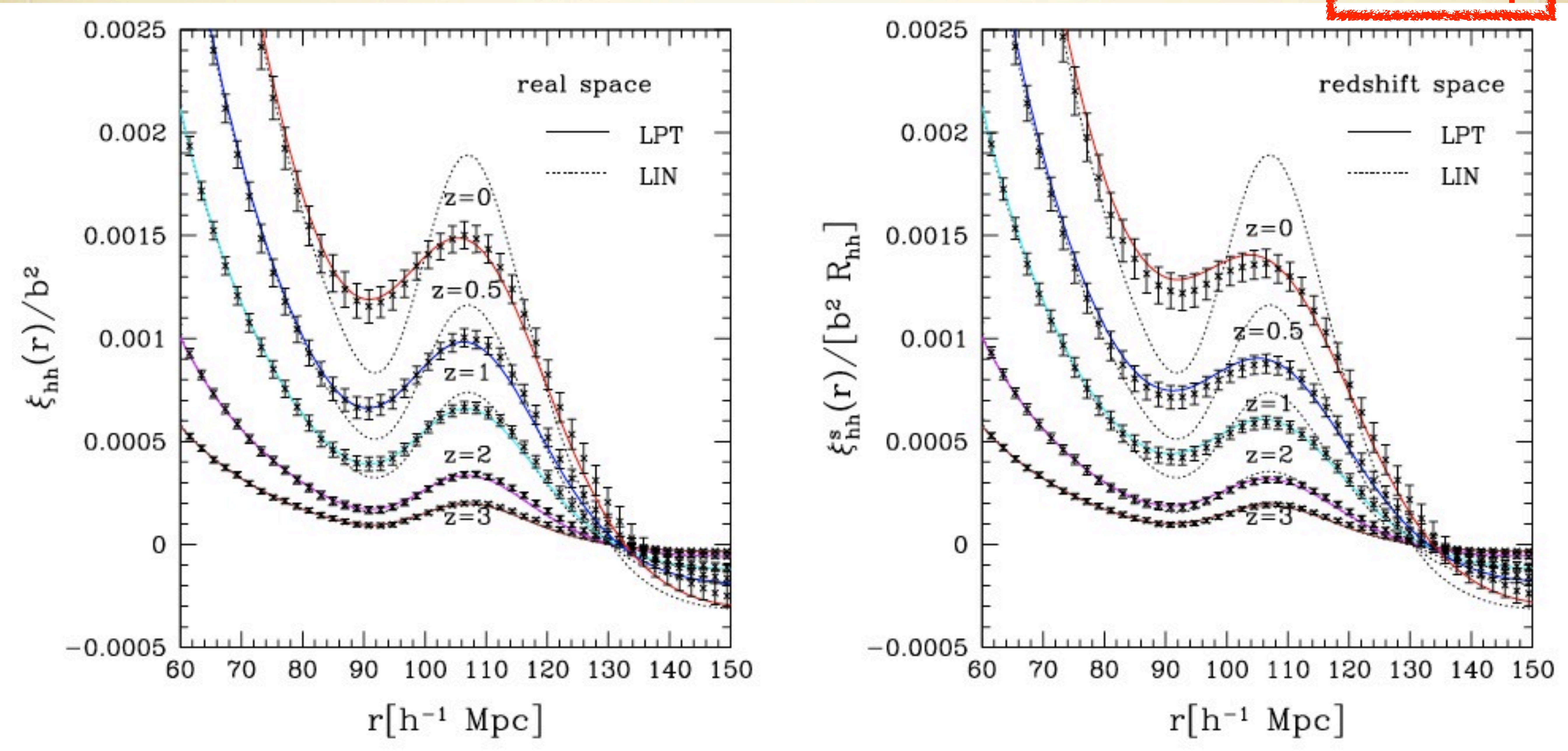


No fitting parameter

Sato & TM (2011)

Halo clustering: Comparison with N-body simulations

One-loop



No fitting parameter

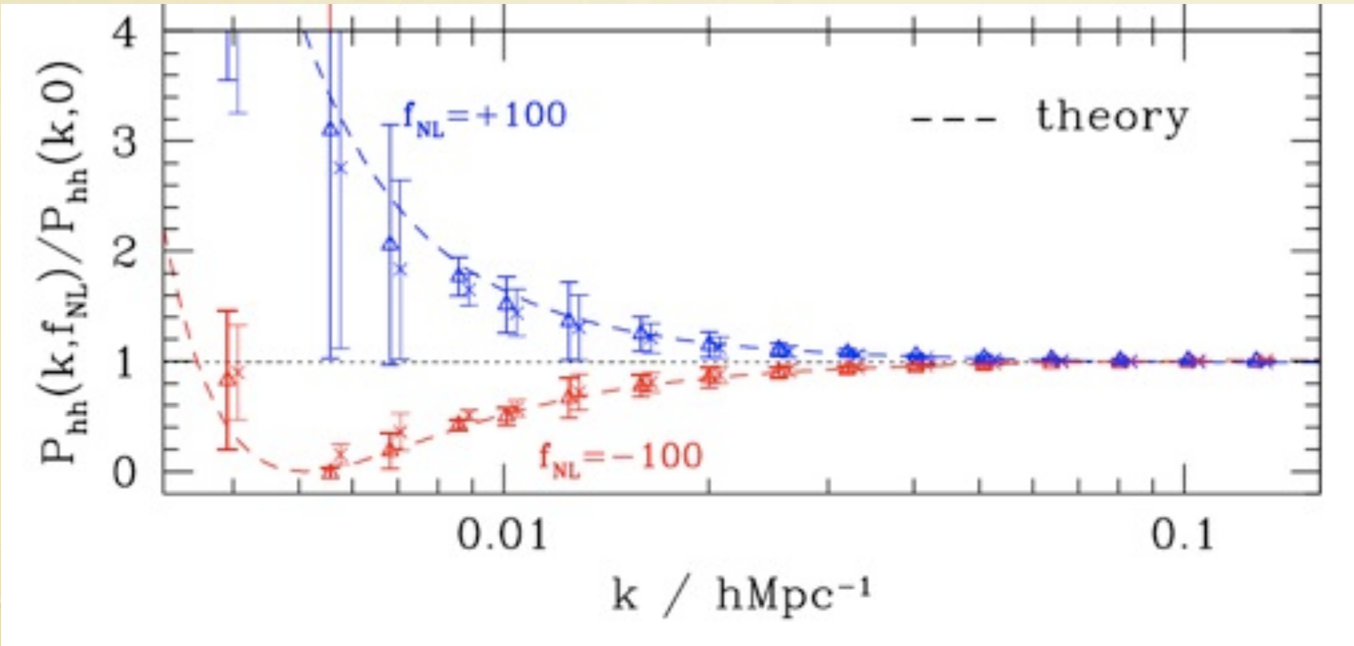
Sato & TM (2011)

Bias and primordial non-Gaussianity

- Primordial non-Gaussianity
  - A way to observationally discriminates theories of the early universe
- Scale-dependent bias and the primordial non-Gaussianity
  - Local-type non-Gaussianity (Dalal et al. 2008; Matarrese & Verde 2008; Slosar et al. 2008)

$$\Phi(\mathbf{r}) = \Phi_L(\mathbf{r}) + f_{\rm NL}\left(\Phi_L^2(\mathbf{r}) - \langle\Phi_L^2(\mathbf{r})\rangle\right)$$

$$\Delta b(M,k) = 3 f_{\rm NL}(b-1)\delta_c \frac{\Omega_m}{k^2 T(k) D(z)}\left(\frac{H_0}{c}\right)^2$$



Desjacques, Seljak & Iliev (2009)

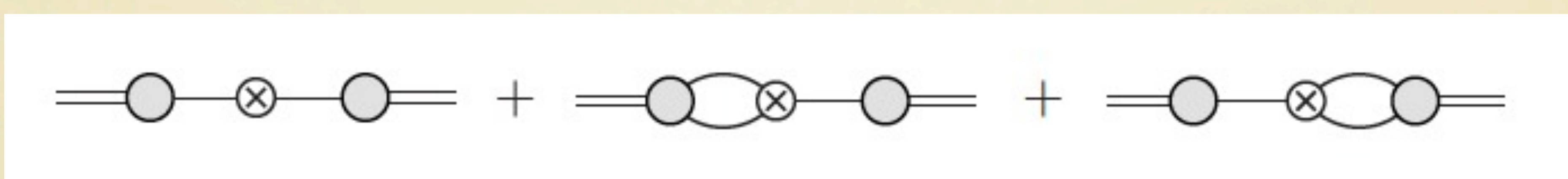
More general formulas

- There are several derivations for the scale-dependent bias with arbitrary non-Gaussianity
  - Halo bias + peak-background split
    - Dalal+ ’08; Slosar+ ’08; Schmidt & Kamionkowski ’10; Desjacques+ ’11, Scoccimarro+ ’11
  - High-peaks limit of threshold regions
    - Matarrese & Verde ’08; Verde & Matarrese ’09; Jeong & Komatsu ’09; Desjacques+ ’11
  - Local bias model
    - McDonald ’08; Taruya+ ’08

Scale-dependent bias and iPT

- iPT most accurately predicts scale-dep. bias

$$
\begin{aligned}
P_{\mathrm{X}}(k) = {} & [\Gamma_{\mathrm{X}}^{(1)}(\boldsymbol{k})]^2 P_{\mathrm{L}}(k) \\
& + \Gamma_{\mathrm{X}}^{(1)}(\boldsymbol{k}) \int \frac{d^3k'}{(2\pi)^3} \Gamma_{\mathrm{X}}^{(2)}(\boldsymbol{k}', \boldsymbol{k} - \boldsymbol{k}') B_{\mathrm{L}}(k, k', |\boldsymbol{k} - \boldsymbol{k}'|) \\
& + \cdots , \qquad (2)
\end{aligned}
$$



General results

- dominant term on large scales:

$$\Delta b(k) \approx \frac{1}{2P_{\mathrm{L}}(k)} \int \frac{d^3k'}{(2\pi)^3} c_2^{\mathrm{L}}(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}') B_{\mathrm{L}}(k, k', \boldsymbol{k}-\boldsymbol{k}')$$

- scalings do not depend on details of bias

$$\Delta b^{\mathrm{loc.}} \propto k^{-2},$$
$$\Delta b^{\mathrm{eql.}} \propto k^{0},$$
$$\Delta b^{\mathrm{fol.}} \propto k^{-1},$$
$$\Delta b^{\mathrm{ort.}} \propto k^{-1}$$

- amplitudes depend on details of bias

Application to the Halo bias

- **Global mass function (a simple model)**

$$n(M)MdM = 2\bar{\rho}\left[P_{>\delta_c}(M) - P_{>\delta_c}(M+dM)\right] = -2\bar{\rho}\frac{dP_{>\delta_c}}{dM}dM$$

- **Localized mass function**

$$n(\boldsymbol{x}, M)MdM = 2\bar{\rho}\left\{\Theta[\delta_M(\boldsymbol{x}) - \delta_c] - \Theta[\delta_{M+dM}(\boldsymbol{x}) - \delta_c]\right\}$$
$$= -2\bar{\rho}\delta_D[\delta_M(\boldsymbol{x}) - \delta_c]\frac{d\delta_M(\boldsymbol{x})}{dM}dM$$

- **Derived renormalized bias functions**

$$c_n^{\mathrm{L}}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) = \frac{(2\pi)^{3n}}{n(M)}\left\langle \frac{\delta^n n(\boldsymbol{x} = \boldsymbol{0}, M)}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)}\right\rangle$$
$$= -\frac{2\bar{\rho}}{n(M)M}\left(-\frac{\partial}{\partial\delta_c}\right)^n \frac{\partial}{\partial M}\left[\underline{\langle\Theta(\delta_M - \delta_c)\rangle} W(k_1R)\cdots W(k_nR)\right]$$

Renormalized bias functions

- As a result,

$$c_2^{\mathrm{L}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = b_2^{\mathrm{L}} W(k_1 R) W(k_2 R) + \frac{\delta_{\mathrm{c}} b_1^{\mathrm{L}} + 1}{\delta_{\mathrm{c}}^2} \frac{\partial}{\partial \ln \sigma_M} \left[W(k_1 R) W(k_2 R) \right]$$

$$= \frac{\delta_{\mathrm{c}}^2 b_2^{\mathrm{L}} + 2\delta_{\mathrm{c}} b_1^{\mathrm{L}} + 1}{\delta_{\mathrm{c}}^2} W(k_1 R) W(k_2 R)$$

bias nonlocality

$$+ \frac{\delta_{\mathrm{c}} b_1^{\mathrm{L}} + 1}{\delta_{\mathrm{c}}^2} \sigma_M^2 \frac{\partial}{\partial \ln \sigma_M} \left[\frac{W(k_1 R) W(k_2 R)}{\sigma_M^2} \right]$$

- In the case of PS mass fn.: $b_1^{\mathrm{L}} = \dfrac{\nu^2 - 1}{\delta_{\mathrm{c}}}, \quad b_2^{\mathrm{L}} = \dfrac{\nu^4 - 3\nu^2}{\delta_{\mathrm{c}}^2}$
- Cancellation of b2 in c2

$(\nu \equiv \delta_{\mathrm{c}} / \sigma_M)$

$$c_2^{\mathrm{L}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{\delta_{\mathrm{c}} b_1^{\mathrm{L}}}{\sigma_M^2} W(k_1 R) W(k_2 R) + \frac{\partial}{\partial \ln \sigma_M} \left[\frac{W(k_1 R) W(k_2 R)}{\sigma_M^2} \right]$$

Notable findings

- No need to introduce auxiliary Gaussian field (c.f., issue of kernel choice in PBS)
- PBS results are reproduced when PS mass function is assumed

$$\Delta b(k) \approx \frac{1}{2}\delta_c b_1^{\mathrm{L}} \mathcal{I}(k) + \frac{1}{2}\frac{\partial \mathcal{I}(k)}{\partial \ln \sigma_M}.$$

  - (Exactly matches with the results of Desjacques+ '11)

$$\begin{aligned}\mathcal{I}(k) &\equiv \frac{\mathcal{I}_2(k)}{\sigma_M{}^2 P_{\mathrm{L}}(k)} \\ &\approx \frac{1}{\sigma_M{}^2 P_{\mathrm{L}}(k)} \int \frac{d^3k'}{(2\pi)^3} W^2(k'R) B_{\mathrm{L}}(k, k', |\boldsymbol{k} - \boldsymbol{k}'|).\end{aligned}$$

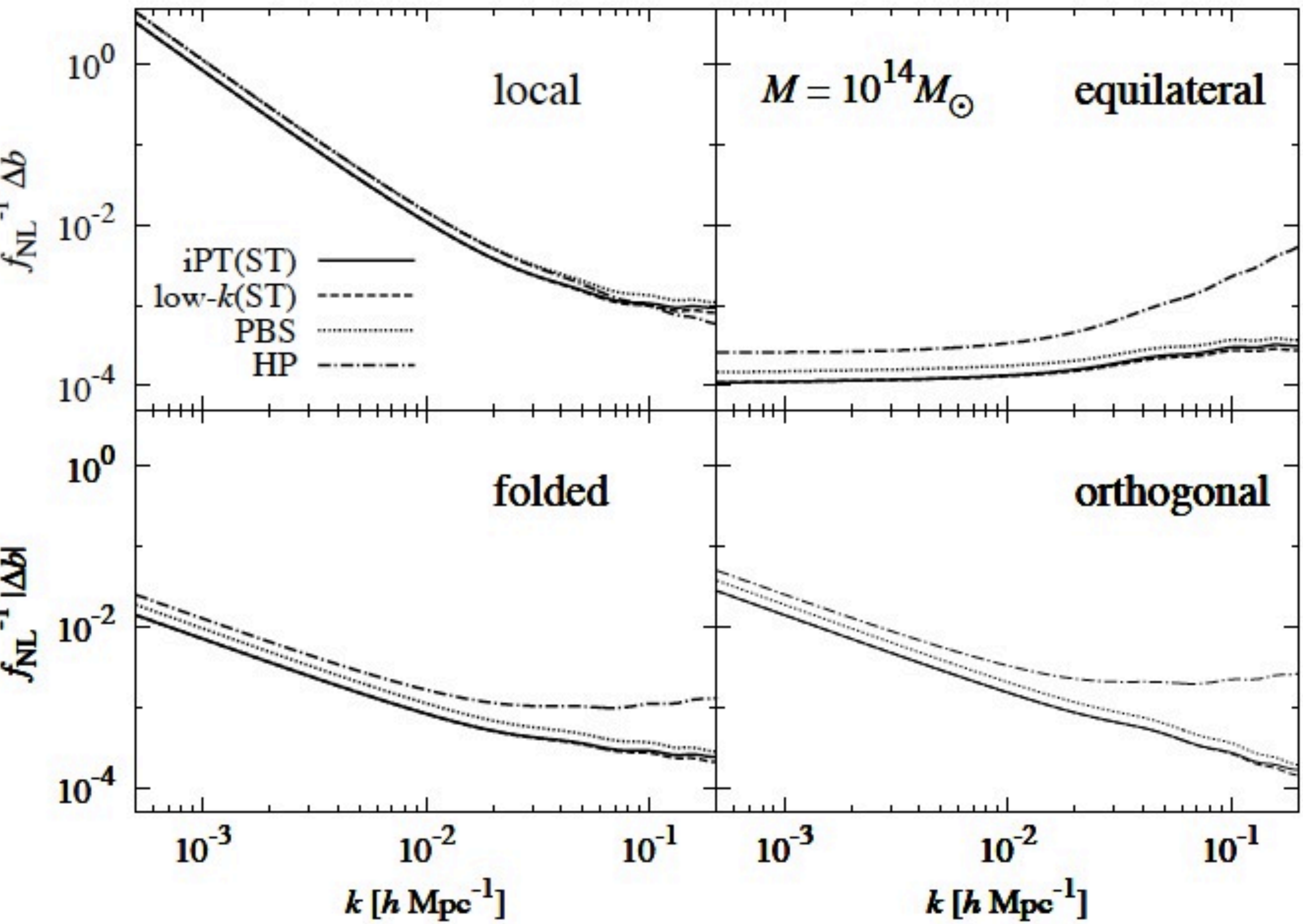
New general formula

- When the mass function is arbitrary, we have a new formula

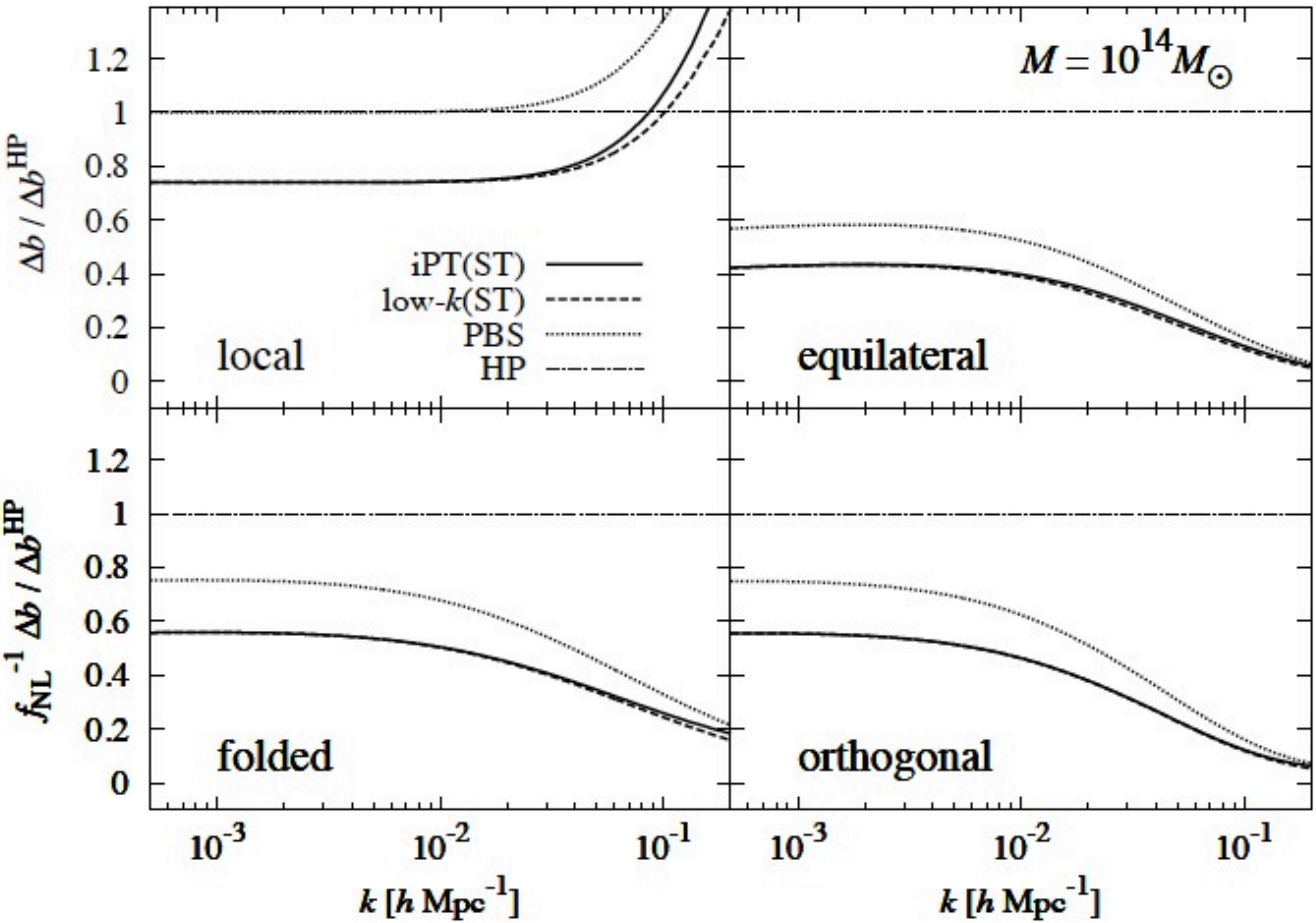
$$\Delta b(k) \approx \frac{{\sigma_M}^2}{2{\delta_c}^2}\left[\left(2 + 2\delta_c b_1^{\mathrm{L}} + {\delta_c}^2 b_2^{\mathrm{L}}\right)\mathcal{I}(k) + \left(1 + \delta_c b_1^{\mathrm{L}}\right)\frac{d\mathcal{I}(k)}{d\ln\sigma_M}\right].$$

  - E.g., Sheth-Tormen mass function:

$$\Delta b(k) \approx \left[\frac{q\delta_c b_1^{\mathrm{L}}}{2} + \frac{1}{\nu^2}\frac{p(q\nu^2 + 2p + 1)}{1 + (q\nu^2)^p}\right]\mathcal{I}(k) + \left[\frac{q}{2} + \frac{1}{\nu^2}\frac{p}{1 + (q\nu^2)^p}\right]\frac{d\mathcal{I}(k)}{d\ln\sigma_M}.$$



local
$M = 10^{14} M_\odot$
equilateral
folded
orthogonal
iPT(ST)
low-k(ST)
PBS
HP
$f_{NL}^{-1}\,\Delta b$
$f_{NL}^{-1}\,|\Delta b|$
$k\ [h\ \mathrm{Mpc}^{-1}]$


$M = 10^{14} M_\odot$
$\Delta b \,/\, \Delta b^{\mathrm{HP}}$
$f_{\mathrm{NL}}^{-1}\, \Delta b \,/\, \Delta b^{\mathrm{HP}}$
iPT(ST)
low-k(ST)
PBS
HP
local
equilateral
folded
orthogonal
k [h Mpc^{-1}]

Redshift-space distortions

- **Redshift-space distortions are straightforwardly included in iPT**

$$\Delta p_0(k) \approx \left(\frac{f}{3} + b_1\right) Q_2(k), \quad \Delta p_2(k) \approx \frac{2f}{3} Q_2(k),$$

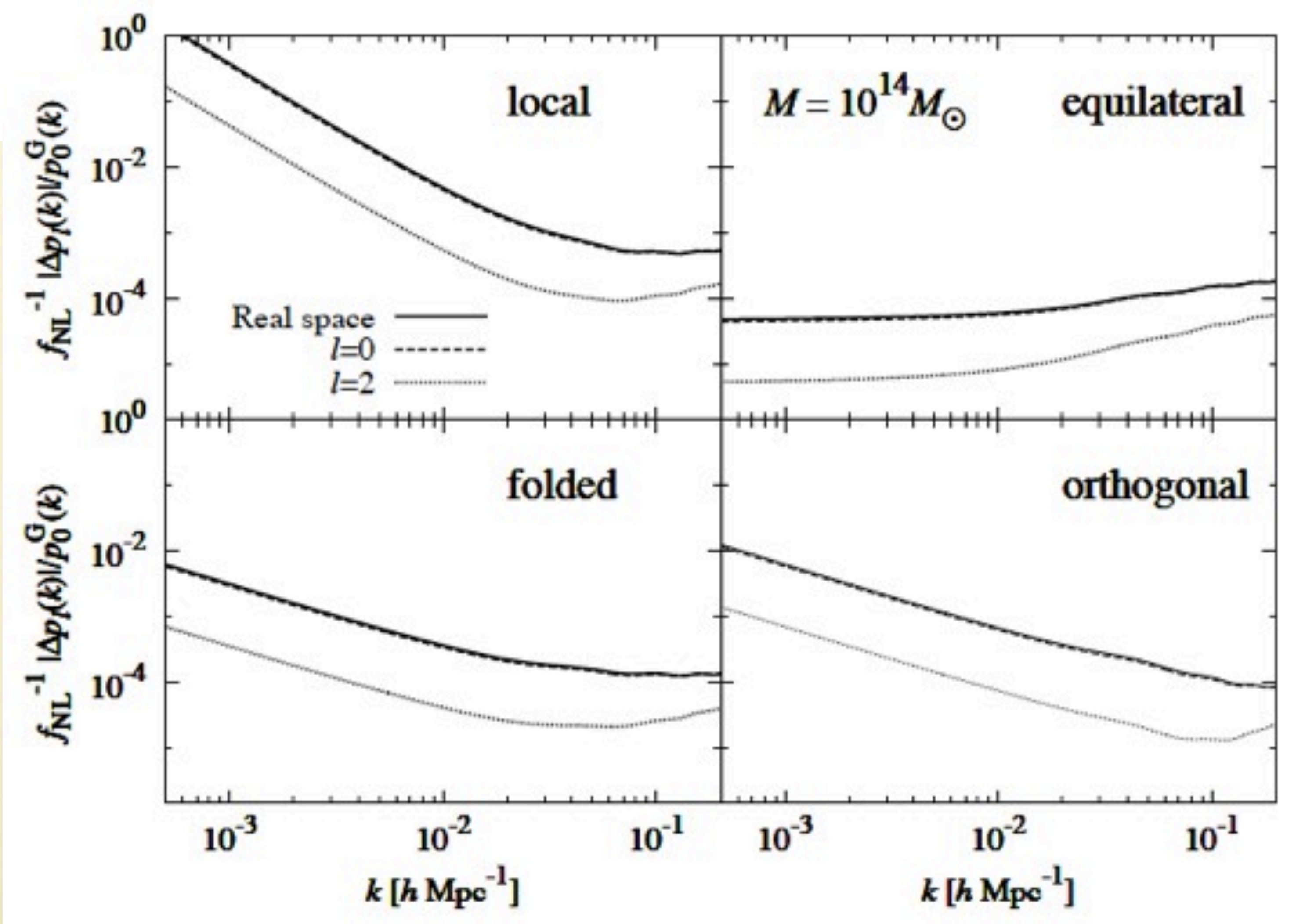
$$\Delta p_4(k), \Delta p_6(k) \ll \Delta p_0(k), \Delta p_2(k),$$

$$\begin{aligned}
\Delta p_0(k) = {} & 2\left[\frac{f}{3} + \frac{2f^2}{5} + \frac{f^3}{7} + \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right) b_1(k)\right] R_1(k) \\
& - 2\left[\frac{2f}{21} + \frac{4f^2}{35} + \frac{3f^3}{35} + \left(\frac{2}{7} + \frac{4f}{21} + \frac{2f^2}{15}\right) b_1(k)\right] R_2(k) \\
& + \left[\frac{f}{3} + \frac{f^2}{5} + \left(1 + \frac{f}{3}\right) b_1(k)\right] Q_1(k) \\
& + \left[\frac{f}{3} + b_1(k)\right] Q_2(k), \qquad (147)
\end{aligned}$$

$$\begin{aligned}
\Delta p_2(k) = {} & 4f\left[\frac{1}{3} + \frac{4f}{7} + \frac{5f^2}{21} + \left(\frac{2}{3} + \frac{2f}{7}\right) b_1(k)\right] R_1(k) \\
& - f\left[\frac{8}{21} + \frac{32f}{49} + \frac{11f^2}{21} + \left(\frac{16}{21} + \frac{13f}{21}\right) b_1(k)\right] R_2(k) \\
& + 2f\left[\frac{1}{3} + \frac{2f}{7} + \frac{1}{3} b_1(k)\right] Q_1(k) + \frac{2f}{3} Q_2(k), \qquad (148)
\end{aligned}$$

$$\begin{aligned}
\Delta p_4(k) = {} & 16f^2\left[\frac{2}{35} + \frac{3f}{77} + \frac{1}{35} b_1(k)\right] R_1(k) \\
& - \frac{4f^2}{35}\left[\frac{16}{7} + \frac{26f}{11} + b_1(k)\right] R_2(k) + \frac{8f^2}{35} Q_1(k), \qquad (149)
\end{aligned}$$

$$\Delta p_6(k) = \frac{32f^3}{231} R_1(k) - \frac{8f^3}{231} R_2(k). \qquad (150)$$



Summary

- Integrated perturbation theory
  - a consistent formulation of nonlinear perturbation theory, generally including nonlocal bias, RSD and nG
  - vertex resummations and bias renormalizations
- The most general formula of the scale-dependent bias from iPT
  - Previous formulas are re-derived from the new formula by taking appropriate limits