Lorentz invariant CPT violation and neutrino antineutrino mass splitting in the Standard Model

Kazuo Fujikawa
RIKEN Nishina Center, Japan

PACS numbers:

M. Chaichian, K. Fujikawa and A. Tureanu,
"*Lorentz invariant CPT violation: Particle and antiparticle mass splitting*", Phys. Lett. B712 (2012)115; "*On neutrino masses via CPT violating Higgs interaction in the Standard Model*", Phys. Lett. B 718 (2012) 178; "*Electromagnetic interaction in theory with Lorentz invariant CPT violation*", arXiv:1210.0208 [hep-th] (to appear in PLB).

CPT Theorem:

W. Pauli, *Niels Bohr and the Development of Physics*, W. Pauli (ed.), Pergamon Press, New York, 1955;

G. Lüders, *On the Equivalence of Invariance under Time-Reversal and under Particle-Antiparticle Conjugation for Relativistic Field Theories*, Det. Kong. Danske Videnskabernes Selskab, Mat.-fys. Medd. **28** (5) (1954).

Warming up: C, P, T and CPT

Convention of γ-matrices:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \quad \gamma_\mu^\dagger = \gamma^\mu, \quad \gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_0 = \gamma_5^\dagger,$$
$$\gamma_1^T = -\gamma_1, \quad \gamma_2^T = \gamma_2, \quad \gamma_3^T = -\gamma_3, \qquad \gamma_0^T = \gamma_0, \quad \gamma_5^T = \gamma_5.$$
$$\gamma_1^\star = \gamma_1, \qquad \gamma_2^\star = -\gamma_2, \quad \gamma_3^\star = \gamma_3, \qquad \gamma_0^\star = \gamma_0, \quad \gamma_5^\star = \gamma_5.$$

C: Charge conjugation is defined by

$$\psi(x) \to -C^{-1}\overline{\psi}^T(x), \quad \overline{\psi}(x) \to \psi^T(x)C,$$

where the charge conjugation matrix $C = i\gamma_2\gamma_0$ satisfies

$$C^\dagger C = 1, \qquad C^T = -C, \quad C\gamma_\mu C^{-1} = -\gamma_\mu^T, \quad C\gamma_5 C^{-1} = \gamma_5^T.$$

P: Parity transformation is defined by

$$\psi(t,\vec{x}) \to \gamma_0\psi(t,-\vec{x}), \quad \overline{\psi}(t,\vec{x}) \to \overline{\psi}(t,-\vec{x})\gamma_0,$$

CP transformation is defined as

$$\psi(t,\vec{x}) \to -W^{-1}\overline{\psi}^T(t,-\vec{x}), \quad \overline{\psi}(t,\vec{x}) \to \psi^T(t,-\vec{x})W$$

where

$$W = i\gamma_2, \qquad W^\dagger W = 1,$$

and

$$\begin{aligned} W\gamma_\mu W^{-1} &= \gamma_k^T\,, \quad\ \ \mu = k, \\ &= -\gamma_0^T\,,\ \mu = 0, \\ W\gamma_5 W^{-1} &= -\gamma_5^T\,. \end{aligned}$$

$\mathcal{T}$:**(Anti-unitary) time reversal** is defined by

$$\mathcal{T}\psi_\alpha(t,\vec{x})\mathcal{T}^{-1} = T_{\alpha\beta}\psi_\beta(-t,\vec{x}),$$
$$\mathcal{T}\psi^\dagger_\alpha(t,\vec{x})\mathcal{T}^{-1} = \psi^\dagger_\beta(-t,\vec{x})(T^{-1})_{\beta\alpha},$$

with

$$T = i\gamma^1\gamma^3, \qquad T\gamma_\mu T^{-1} = \gamma_\mu^T = (\gamma^\mu)^\star,$$
$$T = T^\dagger = T^{-1} = -T^\star$$

For example,

$$\begin{aligned} &\mathcal{T} i\mu\bar{\psi}(x^0,\vec{x})\psi(y^0,\vec{y})\mathcal{T}^{-1} \\ &= -i\mu\psi^\dagger(-x^0,\vec{x})T^{-1}(\gamma_0)^\star T\psi(-y^0,\vec{y}) \\ &= -i\mu\bar{\psi}(-x^0,\vec{x})\psi(-y^0,\vec{y}). \end{aligned}$$

Anti-unitary CPT is defined by

$$\mathcal{CPT}\psi_\alpha(t,\vec{x})(\mathcal{CPT})^{-1} = i\gamma^5_{\alpha\beta}\psi^\dagger_\beta(-t,-\vec{x}),$$
$$\mathcal{CPT}\bar{\psi}_\alpha(t,\vec{x})(\mathcal{CPT})^{-1} = -i\psi_\beta(-t,-\vec{x})(\gamma^5\gamma_0)_{\beta\alpha}.$$

For example,

$$\begin{aligned} & \mathcal{CPT}i\mu\bar{\psi}(x^0,\vec{x})\psi(y^0,\vec{y})(\mathcal{CPT})^{-1} \\ = & (-i\mu)\psi_\beta(-x^0,-\vec{x})(\gamma^5\gamma_0)_{\beta\alpha}\gamma^5_{\alpha\delta}\psi^\dagger_\delta(-y^0,-\vec{y}) \\ = & -i\mu\bar{\psi}(-y^0,-\vec{y})\psi(-x^0,-\vec{x}). \end{aligned}$$

where we used the *spin-statistics theorem.*

Possible CPT violation

Conventional Argument: O.W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002).

CPT violation $\Rightarrow$ Lorentz symmetry violation

Counter Example: M. Chaichian, A.D. Dolgov, V.A. Novikov and A. Tureanu, Phys. Lett.B**699**, 177 (2011).

$$\begin{aligned}\mathcal{L} = &\bar{\psi}(x)[i\gamma^\mu\partial_\mu - M]\psi(x) + \frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m^2\phi^2(x)\\ &+g\bar{\psi}(x)\psi(x)\phi(x) - V(\phi)\\ &+g_1\bar{\psi}(x)\psi(x)\int d^4y\theta(x^0-y^0)\theta((x-y)^2)\phi(y)\end{aligned}$$

but C and CP are preserved.

Yukawa-type model of CPT violation:
arXiv:1205.0152[hep-th].

$$\begin{aligned}\mathcal{L} = \bar{\psi}(x)[i\gamma^{\mu}\partial_{\mu} - M]\psi(x) + \frac{1}{2}\partial_{\mu}\phi(x)\partial^{\mu}\phi(x) - \frac{1}{2}m^2\phi^2(x)\\ +g\bar{\psi}(x)\psi(x)\phi(x) - V(\phi)\\ +g_1\bar{\psi}(x)\psi(x)\int d^4y\theta(x^0-y^0)\delta((x-y)^2-l^2)\phi(y).\end{aligned}$$

This Lagrangian is formally Hermitian and the term with a small real g_1 and the step function $\theta(x^0-y^0)$ stands for the CPT and T violating interaction; l is a real constant parameter.

Present CPT violation is based on the extra form factor[9] in momentum space as

$$g_1 \int d^4x \bar{\psi}(x)\psi(x) \int d^4y \theta(x^0 - y^0)\delta((x-y)^2 - l^2)\phi(y)$$
$$= g_1 \int dp_1 dp_2 dq \int d^4x \bar{\psi}(p_1) e^{-ip_1 x} \psi(p_2) e^{-ip_2 x}$$
$$\times \int d^4y \theta(x^0 - y^0)\delta((x-y)^2 - l^2)\phi(q) e^{-iqy}$$
$$= g_1 \int dp_1 dp_2 dq (2\pi)^4 \delta^4(p_1 + p_2 + q)\bar{\psi}(p_1)\psi(p_2) f(q)\phi(q),$$

where $f(q) \equiv \int d^4z \theta(z^0)\delta(z^2 - l^2) e^{iqz}$.

The ordinary local field theory is characterized by $\delta(z)$ and $f(q) = 1$. The above form factor is infrared divergent, and it is quadratically divergent in the present example. This infrared divergence arises from the fact that we cannot divide Minkowski space into (time-like) domains with finite *4-dimensional* volumes in a Lorentz invariant manner. The Minkowski space is hyperbolic rather than elliptic.

It is convenient to define the form factors

$$f_{\pm}(p) = \int d^4 z_1 e^{\pm i p z_1} \theta(z_1^0) \delta((z_1)^2 - l^2),$$

which are inequivalent for the time-like p due to the factor $\theta(z_1^0)$. For the *time-like* momentum p, one may choose a suitable Lorentz frame such that $\vec{p} = 0$ and

$$f_{\pm}(p^0) = 2\pi \int_0^{\infty} dz \frac{z^2 e^{\pm i p^0 \sqrt{z^2 + l^2}}}{\sqrt{z^2 + l^2}},$$

and for the *space-like* momentum p one may choose a suitable Lorentz frame such that $p^0 = 0$ and

$$f_{\pm}(\vec{p}) = \frac{2\pi}{|p|^2} \int_0^{\infty} dz\, z \frac{\sin z}{\sqrt{z^2 + (|p| l)^2}},$$

which is analogous to the Fourier transform of the Coulomb potential and *real*.

The expression $f_\pm(p)$ is mathematically related to the formula of the two-point Wightman function (for a free scalar field), which suggests that $f_\pm(p)$ is mathematically well-defined for $p \neq 0$ at least in the sense of distribution.

Two-point Wightman function for a free scalar

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} 2\pi\delta(k^2 - m^2)\theta(k^0).$$

Quantization:

No canonical quantization of theory non-local in time.

1. Yang-Feldman formulation ($\longleftarrow$equations of motion)

2. Path integral by integrating the formal equations of motion by means of Schwinger's action principle.
K. Fujikawa, Phys. Rev. D**70**, 085006 (2004).

Cf., **Spin-statistical theorem** in path integral
K. Fujikawa, Int.J. Mod. Phys. A16(2001) 4025 [hep-th/0107076].

The generating functional

$$\langle 0, +\infty | 0, -\infty \rangle_J = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\phi \exp\{\frac{i}{\hbar} \int d^4x [\mathcal{L} + \mathcal{L}_J]\}$$

with $\mathcal{L}_J = \bar{\psi}(x)\eta(x) + \bar{\eta}(x)\psi(x) + \phi(x)J(x)$, and one may generate Green's functions in a power series expansion of perturbation such as

$$(i)^n \langle T^\star \phi(x_1)...\phi(x_N) \int d^4y_1 \mathcal{L}_I(y_1).... \int d^4y_n \mathcal{L}_I(y_n) \rangle,$$

We use the *covariant* $T^\star$-product which is essential to make the path integral on the basis of Schwinger's action principle consistent.

Lagrangian model of fermion mass splitting

In the present nonlocal formulation, we have a new possibility which is absent in a smooth nonlocal extension of the CPT-even local field theory. One can consider Hermitian combination

$$\int d^4x d^4y[\theta(x^0-y^0)-\theta(y^0-x^0)] \times\delta((x-y)^2-l^2)[i\mu\bar{\psi}(x)\psi(y)],$$

which is non-vanishing.

We have the following transformation property of the operator part

$$\begin{aligned} &\mathrm{C}:\ i\mu\bar{\psi}(x)\psi(y) \rightarrow i\mu\bar{\psi}(y)\psi(x),\\ &\mathrm{P}:\ i\mu\bar{\psi}(x^0,\vec{x})\psi(y^0,\vec{y}) \rightarrow i\mu\bar{\psi}(x^0,-\vec{x})\psi(y^0,-\vec{y}),\\ &\mathrm{T}:\ i\mu\bar{\psi}(x^0,\vec{x})\psi(y^0,\vec{y}) \rightarrow -i\mu\bar{\psi}(-x^0,\vec{x})\psi(-y^0,\vec{y}), \end{aligned}$$

and thus the overall transformation property is C=−1, P=1, T=1.

Namely, C=CP=CPT=−1.

It is thus interesting to examine a new action

$$
\begin{aligned}
S = &\int d^4x\{\bar{\psi}(x)i\gamma^{\mu}\partial_{\mu}\psi(x) - m\bar{\psi}(x)\psi(x) \\
&- \int d^4y[\theta(x^0 - y^0) - \theta(y^0 - x^0)]\delta((x-y)^2 - l^2) \\
&\times[i\mu\bar{\psi}(x)\psi(y)]\},
\end{aligned}
$$

which is Lorentz invariant and Hermitian. For the real parameter μ, the third term has C=CP=CPT=-1 and no symmetry to ensure the equality of particle and antiparticle masses.

The Dirac equation is replaced by

$$i\gamma^\mu \partial_\mu \psi(x) = m\psi(x)$$
$$+i\mu \int d^4y[\theta(x^0 - y^0) - \theta(y^0 - x^0)]\delta((x-y)^2 - l^2)\psi(y).$$

By inserting an ansatz $\psi(x) = e^{-ipx}U(p)$,

$$\begin{aligned} \not{p}U(p) &= mU(p) \\ &+ i\mu \int d^4y[\theta(x^0 - y^0) - \theta(y^0 - x^0)] \\ &\times \delta((x-y)^2 - l^2)e^{-ip(y-x)}U(p) \\ &= mU(p) + i\mu[f_+(p) - f_-(p)]U(p), \end{aligned}$$

where $f_\pm(p)$ is the Lorentz invariant form factor.

The (off-shell) propagator is defined by

$$\int d^4x e^{ip(x-y)}\langle T^\star\psi(x)\bar{\psi}(y)\rangle$$
$$= \frac{i}{\not p - m + i\epsilon - i\mu[f_+(p) - f_-(p)]},$$

which is manifestly Lorentz covariant. Note that we use the $T^\star$-product for the path integral in accord with Schwinger's action principle, which is based on the equation of motion.

The$T^\star$-product is quite different from the canonical T-product in the present nonlocal theory, and in fact the canonical quantization is not defined in the present theory.

For the *space-like* p, the extra term with μ in the denominator of the propagator vanishes since $f_+(p) = f_-(p)$ for $p = (0, \vec{p})$. Thus the propagator has poles only at the time-like momentum, and in this sense the present Hermitian action does not allow a tachyon.

For *time-like* p, we go to the frame where $\vec{p} = 0$. Then the eigenvalue equation is given by

$$p_0 = \gamma_0 \{m + i\mu[f_+(p_0) - f_-(p_0)]\},$$

namely,

$$p_0 = \gamma_0 \left[m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right]. \quad (1)$$

The solution p_0 of this equation determines the possible mass eigenvalues.

This eigenvalue equation under $p_0 \to -p_0$ becomes:

$$-p_0 = \gamma_0 \left[m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}} \right].$$

By sandwiching this equation by γ_5, we have

$$p_0 = \gamma_0 \left[m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}} \right], \quad (2)$$

which is not identical to the original equation (1).

In other words, if p_0 is the solution of the original equation, $-p_0$ cannot be the solution of the original equation except for $\mu = 0$. The last term in our Lagrangian with C=CP=CPT=-1 splits the particle and antiparticle masses.

As a crude estimate of the mass splitting, one may assume $\mu \ll m$ and solve these equations iteratively.

$$p_0 \simeq m \mp 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m\sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}},$$

where we used the upper component of γ_0.

It is possible to assign a finite value to the mass splitting for $p_0 \neq 0$ by using the formal relation,

$$\int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}} = -\frac{\partial^2}{\partial p_0^2}\int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{[z^2+l^2]^{3/2}}.$$

Neutrino antineutrino mass splitting:

H. Murayama, T. Yanagida, Phys. Lett. B 520 (2001) 263;
G. Barenboim, L. Borissov, J.D. Lykken, A.Y. Smirnov, JHEP 0210 (2002) 001;
G. Barenboim, L. Borissov, J. Lykken, Phys. Lett. B 534 (2002) 106;
S.M. Bilenky, M. Freund, M. Lindner, T. Ohlsson, W. Winter, Phys. Rev. D 65 (2002) 073024;
G. Barenboim, J.D. Lykken, Phys. Rev. D 80 (2009) 113008.

G. Altarelli, The mystery of neutrino mixings, arXiv:1111.6421 [hep-ph].

P. Adamson, et al., MINOS Collaboration, Phys. Rev. Lett. 107 (2011) 181802;
P. Adamson, et al., MINOS Collaboration, Phys. Rev. Lett. 108 (2012) 191801;
P. Adamson, et al., MINOS Collaboration, Phys. Rev. D 85 (2012) 031101.

We consider a minimal extension of the Standard Model by incorporating the right-handed neutrino:

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \psi_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

and the part of the Standard Model Lagrangian relevant to our discussion is given by

$$\begin{aligned}\mathcal{L} = & \overline{\psi}_L i\gamma^\mu(\partial_\mu - igT^a W^a_\mu - i\frac{1}{2}g' Y_L B_\mu)\psi_L \\ & + \bar{e}_R i\gamma^\mu(\partial_\mu + ig' B_\mu)e_R + \overline{\nu}_R i\gamma^\mu \partial_\mu \nu_R \\ & - \left[\frac{\sqrt{2}m_e}{v}\bar{e}_R \phi^\dagger \psi_L + \frac{\sqrt{2}m_D}{v}\overline{\nu}_R \phi_c^\dagger \psi_L + \frac{m_R}{2}\nu_R^T C \nu_R\right] + h.c.\end{aligned}$$

with $Y_L = -1$, and the Higgs doublet and its $SU(2)$ conjugate:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad \phi_c \equiv i\tau_2 \phi^\star = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}.$$

The operator C stands for the charge-conjugation matrix for spinors. The term with m_R in the above Lagrangian is the Majorana mass term for the right-handed neutrino.

We take the above Lagrangian as a *low-energy effective theory* and apply to it the naturalness argument of 't Hooft. We first argue that the choice $m_D^2 \gg m_R^2$ is natural, since by setting $m_R = 0$ one recovers an enhanced fermion number symmetry. We then argue that $m_e \gg m_D$ is also natural, since by setting $m_D = m_R = 0$ one finds an enhanced symmetry $\nu_R(x) \to \nu_R(x) + \xi_R$, with constant ξ_R. Thus, our basic assumption is $m_e \gg m_D \gg m_R$, namely, the so-called *pseudo-Dirac scenario*, and in the explicit analysis below we adopt the Dirac limit $m_R = 0$ for simplicity.

Our next observation is that the combination

$$\phi_c^\dagger(x)\psi_L(x)$$

is invariant under the full $SU(2)_L \times U(1)$ gauge symmetry. One may thus add a hermitian non-local Higgs coupling to the Lagrangian,

$$\mathcal{L}_{CPT}(x) = -i\frac{2\sqrt{2}\mu}{v}\int d^4y\delta((x-y)^2 - l^2)\theta(x^0 - y^0) \\ \left\{\bar{\nu}_R(x)\left(\phi_c^\dagger(y)\psi_L(y)\right) - \left(\bar{\psi}_L(y)\phi_c(y)\right)\nu_R(x)\right\},$$

without spoiling the basic $SU(2)_L \times U(1)$ gauge symmetry.

In the unitary gauge, $\phi^{\pm}(x)=0$ and $\phi^0(x)\to(v+\varphi(x))/\sqrt{2}$, the neutrino mass term (with $m_R=0$)

$$
\begin{aligned}
S_{\nu\mathrm{mass}} = \int d^4x\Big\{ & -m_D\bar{\nu}(x)\nu(x)\left(1+\frac{\varphi(x)}{v}\right) \\
& -i\mu\int d^4y\delta((x-y)^2-l^2)\theta(x^0-y^0) \\
& \times[\bar{\nu}(x)\left(1+\frac{\varphi(y)}{v}\right)(1-\gamma_5)\nu(y) \\
& \quad -\bar{\nu}(y)\left(1+\frac{\varphi(y)}{v}\right)(1+\gamma_5)\nu(x)]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
S_{\nu\mathrm{mass}} = \int d^4x \Big\{ & - m_D \bar{\nu}(x)\nu(x) \left(1 + \frac{\varphi(x)}{v}\right) \\
& -i\mu \int d^4y \delta((x-y)^2 - l^2)[\theta(x^0 - y^0) \\
& \qquad\qquad -\theta(y^0 - x^0)]\bar{\nu}(x)\nu(y) \\
& +i\mu \int d^4y \delta((x-y)^2 - l^2)\bar{\nu}(x)\gamma_5\nu(y) \\
& -i\frac{\mu}{v} \int d^4y \delta((x-y)^2 - l^2)\theta(x^0 - y^0) \\
& \times[\bar{\nu}(x)(1-\gamma_5)\nu(y) - \bar{\nu}(y)(1+\gamma_5)\nu(x)]\varphi(y)\Big\}
\end{aligned}
$$

When one looks at the mass terms without the Higgs ϕ coupling, the first two terms are identical to our simple model but an extra parity-violating non-local mass term appears, which adds an extra term $-i\mu\gamma_5 g(p^2)$ to m in the mass eigenvalue equations; here

$$g(p^2) = \int d^4 z_1 e^{ipz_1} \delta((z_1)^2 - l^2).$$

This extra term is C and CPT preserving and does not contribute to the mass splitting.

Since we are assuming that CPT breaking terms are very small, we may solve the mass eigenvalue equations iteratively. We then obtain

$$m_{\pm} \simeq m_D - i\mu\gamma_5 g(m_D^2) \pm 4\pi\mu \int_0^{\infty} dz \frac{z^2 \sin[m_D\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}},$$

The parity violating mass $-i\mu\gamma_5 g(m_D^2)$ is now transformed away by a suitable global chiral transformation without modifying the last term to the order linear in the small parameter μ.

In this way, the neutrino and antineutrino mass splitting is incorporated in the Standard Model by a Lorentz invariant non-local CPT breaking mechanism, without spoiling the $SU(2)_L \times U(1)$ gauge symmetry.

The Higgs particle φ itself has a tiny C-, CP- and CPT-violating coupling.

If the neutrino–antineutrino mass splitting is confirmed by experiments, it would imply that neutrinos are Dirac-type particles rather than Majorana-type particles. Also, our identification of the neutrino mass terms as the origin of the possible CPT breaking may be natural if one recalls that the mass terms of the neutrinos are the known origin of new physics beyond the original Standard Model.

The remaining couplings of the Standard Model are very tightly controlled by the $SU(2)_L \times U(1)$ gauge symmetry, and only the neutrino mass terms allow the present non-local gauge invariant couplings without introducing Wilson-line type gauge interactions.

Some basic field theoretical issues:

As for the quantization of the theory non-local in time, the path integral on the basis of Schwinger's action principle.

It is also well-known that a theory non-local in time generally spoils unitarity. We have the neutrino propagator

$$\langle T^\star \nu(x)\bar{\nu}(y)\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \times \frac{i}{\not{p} - m_D + i\epsilon + i\mu\gamma_5 g(p^2) - i\mu[f_+(p) - f_-(p)]}$$

which gives mass formula in the pole approximation.

The breaking of unitarity is somewhat analogous to the use of a dimension 5 operator for the neutrino mass term which also breaks unitarity in general.

Otherwise,
It is very gratifying that the basic $SU(2)_L \times U(1)$ gauge symmetry together with Lorentz symmetry are exactly preserved by our non-local CPT violation.
We can thus avoid the appearance of negative norm in the gauge sector if one applies gauge invariant and Lorentz invariant regularization.

Charged particles in the Standard Model: modified QED

$$S = \int d^4x \Big\{ \bar{\psi}(x) i\gamma^\mu D_\mu \psi(x) - m\bar{\psi}(x)\psi(x)$$
$$- \int d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \delta((x-y)^2 - l^2)$$
$$\times i\mu\bar{\psi}(x) \exp\left[ie \int_y^x A_\mu(z) dz^\mu] \psi(y) \right] \Big\}$$
$$-\frac{1}{4} \int d^4x F_{\mu\nu}(x) F^{\mu\nu}(x),$$

with

$$D_\mu = \partial_\mu - ieA_\mu(x).$$

Electromagnetic pair production in the presence of mass splitting was analyzed.

"*Electromagnetic interaction in theory with Lorentz invariant CPT violation*", arXiv:1210.0208 [hep-th] (to appear in PLB).

Possible modification of Sakharov conditions on baryon asymmetry in the presence of CPT violation and particle-antiparticle mass splitting was noted.