CP and Flavor Symmetries: Ideas and Models

C. Hagedorn

University of Padua and SISSA, Italy

Feruglio/H/Ziegler: 1211.5560 [hep-ph], 1303.7178 [hep-ph]

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Outline

- Lepton mixing: parametrization and experimental results
- Lepton mixing from non-trivial breaking of G_f & CP

  (Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12))
- Model with S_4 and CP for leptons ***(Feruglio et al. ('12,'13))***
- Conclusions

Lepton mixing

- charged lepton mass terms and Majorana neutrino mass terms

$$e^c_a\, m_{e,ab}\, l_b \quad \text{and} \quad \nu_a\, m_{\nu,ab}\, \nu_b$$

  cannot be diagonalized simultaneously

- going to the mass basis

$$U_e^\dagger m_e^\dagger m_e U_e = \text{diag}(m_e^2, m_\mu^2, m_\tau^2) \quad \text{and} \quad U_\nu^T m_\nu U_\nu = \text{diag}(m_1, m_2, m_3)$$

  leads to non-diagonal charged current interactions

$$\bar{l} \not{W}^- U_{PMNS}\, \nu \quad \text{with} \quad U_{PMNS} = U_e^\dagger U_\nu$$

Parametrization of lepton mixing

Parametrization ***(PDG)***

$$U_{PMNS} = \tilde{U}\ \mathrm{diag}(1, e^{i\alpha/2}, e^{i(\beta/2+\delta)})$$

with

$$\tilde{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

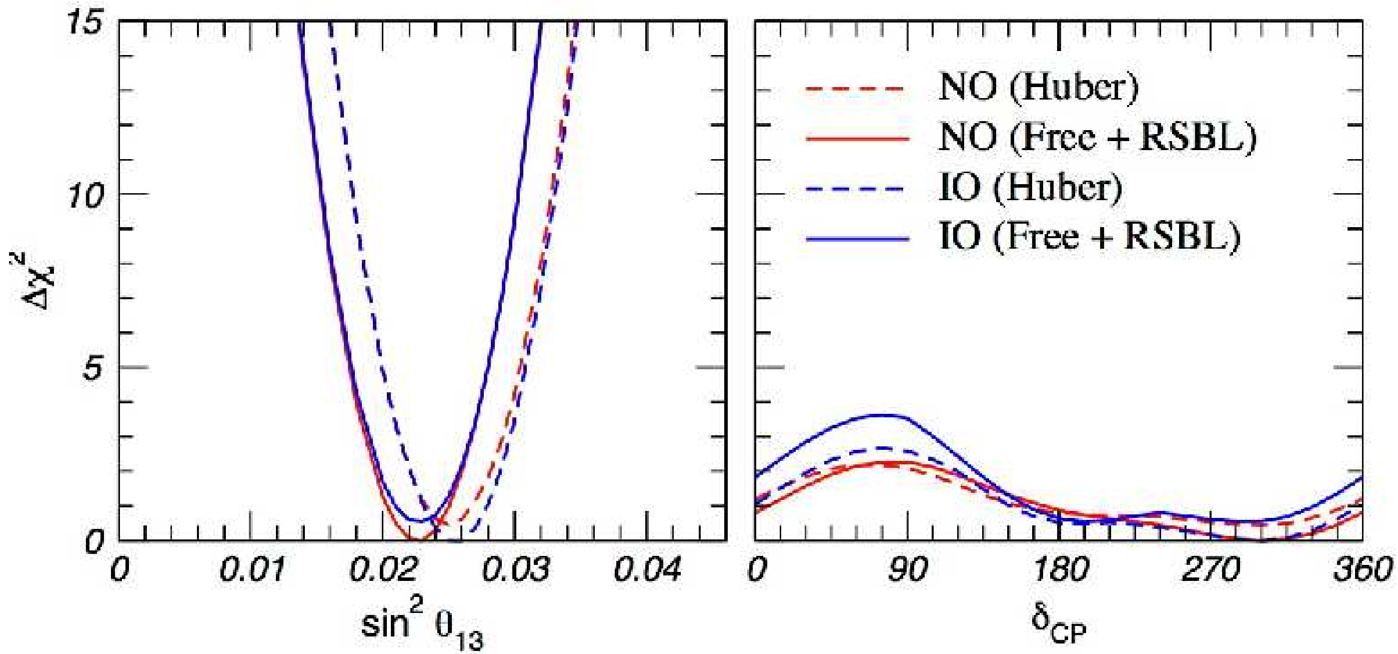
and $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Jarlskog invariant J_{CP}

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left[U_{PMNS,11}U^*_{PMNS,13}U^*_{PMNS,31}U_{PMNS,33}\right] \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta \end{aligned}$$

Experimental results on lepton mixing

Latest global fits ***(Gonzalez-Garcia et al. ('12))***



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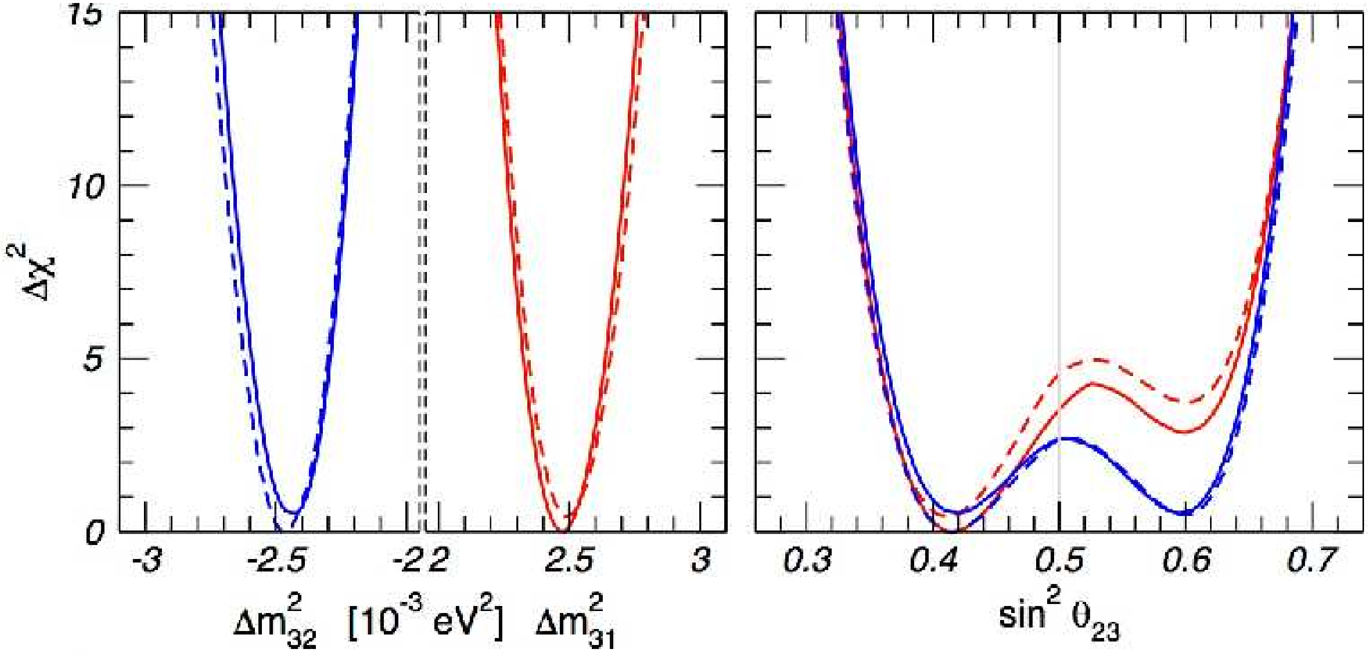

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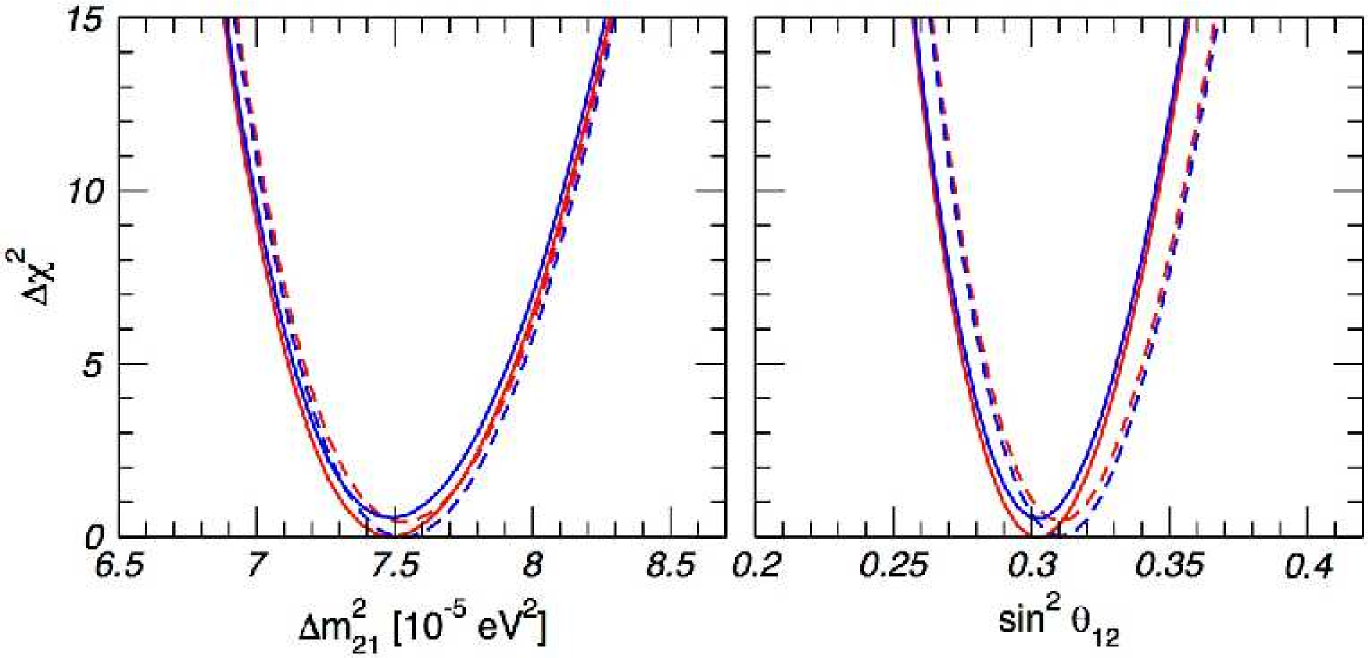

Experimental results on lepton mixing

Latest global fits ***(Gonzalez-Garcia et al. ('12))***

best fit and $1\,\sigma$ error	$3\,\sigma$ range
$\sin^2\theta_{13} = 0.0227^{+0.0023}_{-0.0024}$	$0.0156 \leq \sin^2\theta_{13} \leq 0.0299$
$\sin^2\theta_{12} = 0.302^{+0.013}_{-0.012}$	$0.267 \leq \sin^2\theta_{12} \leq 0.344$
$\sin^2\theta_{23} = \begin{cases} 0.413^{+0.037}_{-0.025} \\ 0.594^{+0.021}_{-0.022} \end{cases}$	$0.342 \leq \sin^2\theta_{23} \leq 0.667$
$\delta = 300^\circ{}^{+\ 66^\circ}_{-138^\circ}$	$0^\circ \leq \delta \leq 360^\circ$
$\alpha\ ,\ \ \beta$	unconstrained

Experimental results on lepton mixing

Latest global fits ***(Gonzalez-Garcia et al. ('12))***

$$||U_{PMNS}|| \approx \begin{pmatrix} 0.83 & 0.54 & 0.15 \\ 0.50 & 0.59 & 0.64 \\ 0.26 & 0.60 & 0.76 \end{pmatrix}$$

and no information on the phases

$$\Downarrow$$

Mismatch in lepton flavor space is large!

Origin of lepton mixing

- interpret this mismatch in lepton flavor space as mismatch of flavor symmetries G_ν and G_e
- if we want to predict lepton mixing, we have to derive this mismatch
- let us assume that there is a symmetry, broken to G_ν and G_e
- this symmetry is in the following a combination of a

discrete non-abelian symmetry G_f and CP

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Non-trivial breaking of G_f and CP

Idea:

Relate lepton mixing to how G_f and CP are broken
Interpretation as mismatch of embedding of different sub-groups G_ν and G_e into G_f and CP

(Feruglio et al. ('12,'13), Holthausen et al. ('12), Grimus/Rebelo ('95))

$$
\begin{array}{ccc}
 & G_f \text{ \& CP} & \\
\swarrow & & \searrow \\
\text{neutrinos} & & \text{charged leptons} \\
G_\nu & & G_e
\end{array}
$$

Non-trivial breaking of G_f and CP

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Relate lepton mixing to how G_f and CP are broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

G_f & CP

$\swarrow \qquad\qquad \searrow$

neutrinos	charged leptons
assume 3 generations of Majorana neutrinos	distinguish 3 generations

Non-trivial breaking of $\boldsymbol{G_f}$ and $\boldsymbol{CP}$

Idea:

Relate lepton mixing to how G_f and CP are broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

$$
\begin{array}{ccc}
 & G_f \text{ \& CP} & \\
\swarrow & & \searrow \\
\text{neutrinos} & & \text{charged leptons} \\
G_\nu = Z_2 \times \text{CP} & & G_e = Z_N \quad \text{with} \quad N \geq 3
\end{array}
$$

An example: $\mu\tau$ reflection symmetry ***(Harrison/Scott ('02,'04), Grimus/Lavoura ('03))***

Non-trivial breaking of G_f and CP

G_f & CP

$\swarrow \qquad\qquad\qquad \searrow$

neutrinos	charged leptons
$G_\nu = Z_2 \times$ CP	$G_e = Z_N$ with $N \geq 3$

Further requirements

- two/three non-trivial mixing angles $\Rightarrow$ irred 3-dim rep of G_f
- "maximize" predictability of approach

Non-trivial breaking of G_f and CP

Consistency conditions have to be fulfilled:

- definition of generalized CP transformation ***(see e.g. Branco et al. ('11))***

$$\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star$$

- for X unitary and symmetric

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{\text{CP}} XX^\star\phi = \phi$$

Non-trivial breaking of G_f and CP

Consistency conditions have to be fulfilled:

- definition of generalized CP transformation ***(see e.g. Branco et al. ('11))***

$$\phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^\star$$

- we have to consistently combine G_f and CP

$$\Downarrow$$

"closure" relations have to hold

Non-trivial breaking of G_f and CP

Consistency conditions have to be fulfilled:

- definition of generalized CP transformation ***(see e.g. Branco et al. ('11))***

$$\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star$$

- assume ϕ transforms as 3-dim rep of G_f, then

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{G_f} XA^\star\phi^\star \xrightarrow{\text{CP}} XA^\star X^\star\phi = \left(X^\star A X\right)^\star \phi$$

$$\Downarrow$$

$$\left(X^\star A X\right)^\star = A' \quad \text{with in general} \quad A \neq A' \quad \text{and} \quad A,\, A' \in G_f$$

Non-trivial breaking of G_f and CP

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$$\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star$$

- "closure" relations

$$(X^\star A X)^\star = A' \quad \text{with in general} \quad A \neq A' \quad \text{and} \quad A,\, A' \in G_f$$

- realize direct product of $Z_2 \subset G_f$ and CP

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- realize direct product of $Z_2 \subset G_f$ and CP; Z generates Z_2

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{Z_2} XZ^\star\phi^\star \quad \text{and} \quad \phi \xrightarrow{Z_2} Z\phi \xrightarrow{\text{CP}} ZX\phi^\star$$

Non-trivial breaking of G_f and CP

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- definition of generalized CP transformation ***(see e.g. Branco et al. ('11))***

$$\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star$$

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$$(X^\star A X)^\star = A' \quad \text{with in general} \quad A \neq A' \quad \text{and} \quad A,\, A' \in G_f$$

- realize direct product of $Z_2 \subset G_f$ and CP; Z generates Z_2

$$X Z^\star - Z X = 0$$

Non-trivial breaking of G_f and CP

$$\begin{array}{ccc} & G_f \text{ \& CP} & \\ \swarrow & & \searrow \\ \text{neutrinos} & & \text{charged leptons} \\ G_\nu = Z_2 \times \text{CP} & & G_e = Z_N \quad \text{with} \quad N \geq 3 \end{array}$$

Further requirements

- two/three non-trivial mixing angles $\Rightarrow$ irred 3-dim rep of G_f
- "maximize" predictability of approach

Non-trivial breaking of $\boldsymbol{G_f}$ and $\boldsymbol{CP}$

- neutrino sector: $Z_2 \times$ CP preserved

  neutrino mass term $\nu_a \, m_{\nu,ab} \, \nu_b$
  is invariant under $\nu_\alpha \to Z_{\alpha\beta} \, \nu_\beta$
  is invariant under generalized CP transformation $\nu_\alpha \to X_{\alpha\beta} \, \nu_\beta^\star$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  charged lepton mass term $e_a^c \, m_{e,ab} \, l_b$
  is invariant under $l_\alpha \to Q_{e,\alpha\beta} \, l_\beta$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

  $$Z^T m_\nu Z = m_\nu \quad \text{and} \quad X m_\nu X = m_\nu^\star$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  charged lepton mass term $e^c_a \, m_{e,ab} \, l_b$
  is invariant under $l_\alpha \rightarrow Q_{e,\alpha\beta} \, l_\beta$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved and generated by ($\nu = \Omega_\nu \, \nu'$)

$$X = \Omega_\nu \Omega_\nu^T \quad \text{and} \quad Z = \Omega_\nu Z^{diag} \Omega_\nu^\dagger$$
$$Z^{diag} = \text{diag}\,(-1, 1, -1) \quad \text{and} \quad \Omega_\nu \quad \text{unitary}$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

charged lepton mass term $e^c_a \, m_{e,ab} \, l_b$
is invariant under $l_\alpha \to Q_{e,\alpha\beta} \, l_\beta$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

$\rightarrow$ neutrino mass matrix m_ν fulfills

$$Z^{diag}[\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^\star$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

charged lepton mass term $e_a^c \, m_{e,ab} \, l_b$
is invariant under $l_\alpha \rightarrow Q_{e,\alpha\beta} \, l_\beta$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν is diagonalized by

  $$\Omega_\nu(X, Z) R(\theta) K_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  charged lepton mass term $e^c_a \, m_{e,ab} \, l_b$
  is invariant under $l_\alpha \rightarrow Q_{e,\alpha\beta} \, l_\beta$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν is diagonalized by

$$\Omega_\nu(X, Z) R(\theta) K_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν is diagonalized by

$$\Omega_\nu(X, Z) R(\theta) K_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved and generated by

$$Q_e = \Omega_e Q_e^{diag} \Omega_e^\dagger \quad \text{with} \quad \Omega_e \quad \text{unitary}$$

$$Q_e^{diag} = \text{diag}\,(\omega_N^{n_e}, \omega_N^{n_\mu}, \omega_N^{n_\tau})$$

$$\text{and} \quad n_e \neq n_\mu \neq n_\tau \quad \text{and} \quad \omega_N = e^{2\pi i/N}$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν is diagonalized by

  $$\Omega_\nu(X, Z) R(\theta) K_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

  $$\Omega_e^\dagger(Q_e) m_e^\dagger m_e \Omega_e(Q_e) \quad \text{is diagonal}$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν is diagonalized by

$$\Omega_\nu(X, Z) R(\theta) K_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

$$\Omega_e^\dagger(Q_e) m_e^\dagger m_e \Omega_e(Q_e) \quad \text{is diagonal}$$

- conclusion: PMNS mixing matrix reads

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu \quad \text{in} \quad \bar{l} \not{W}^- U_{PMNS} \, \nu$$

Non-trivial breaking of G_f and CP

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- U_{PMNS} contains one parameter θ
- permutations of rows and columns of U_{PMNS} possible

$$\Downarrow$$

Predictions:

Mixing angles and CP phases are predicted
in terms of one parameter θ only,
up to permutations of rows/columns

Study of S_4 and CP

Generators in rep. $\mathbf{3}'$: $\left(\omega = e^{2\pi i/3}\right)$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \; , \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \; , \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfill

$$S^2 = \mathbb{1} \; , \;\; T^3 = \mathbb{1} \; , \;\; U^2 = \mathbb{1} \; ,$$
$$(ST)^3 = \mathbb{1} \; , \;\; (SU)^2 = \mathbb{1} \; , \;\; (TU)^2 = \mathbb{1} \; , \;\; (STU)^4 = \mathbb{1}$$

Study of $\boldsymbol{S_4}$ and $\boldsymbol{CP}$

A transformation X in rep. $\mathbf{3}'$ for $Z = S$:

$$X_{\mathbf{3}'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfills

$$XX^\dagger = XX^\star = \mathbb{1}$$

$$(X^\star A X)^\star = A' \quad , \quad XZ^\star - ZX = 0$$

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and our X

(Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12,'13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

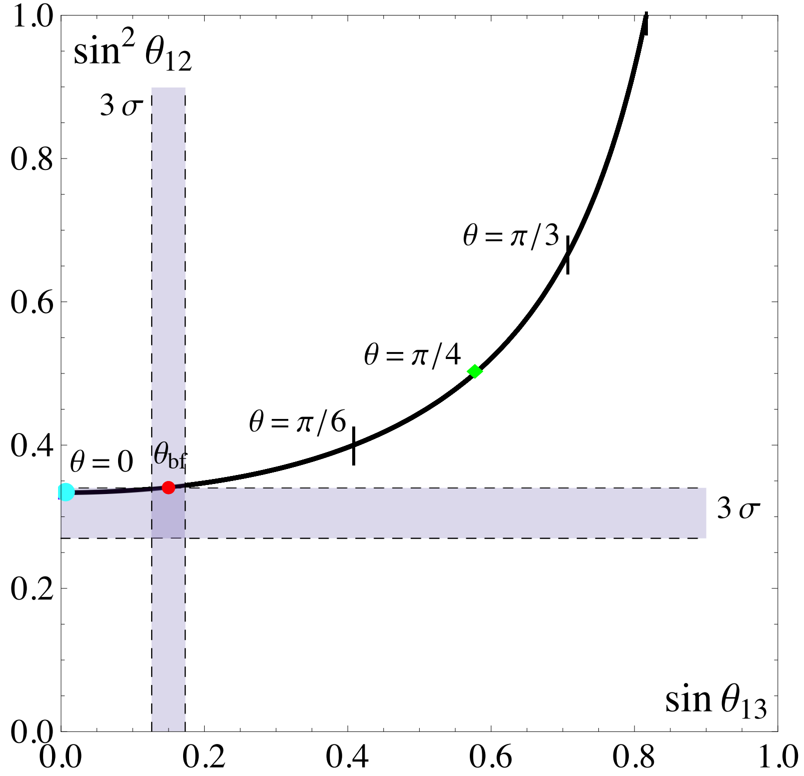
$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \;, \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} \;, \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1 \;\;, \quad |J_{CP}| = \frac{|\sin 2\theta|}{6\sqrt{3}} \;\;, \quad \sin\alpha = 0 \;\;, \quad \sin\beta = 0$$

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and our X



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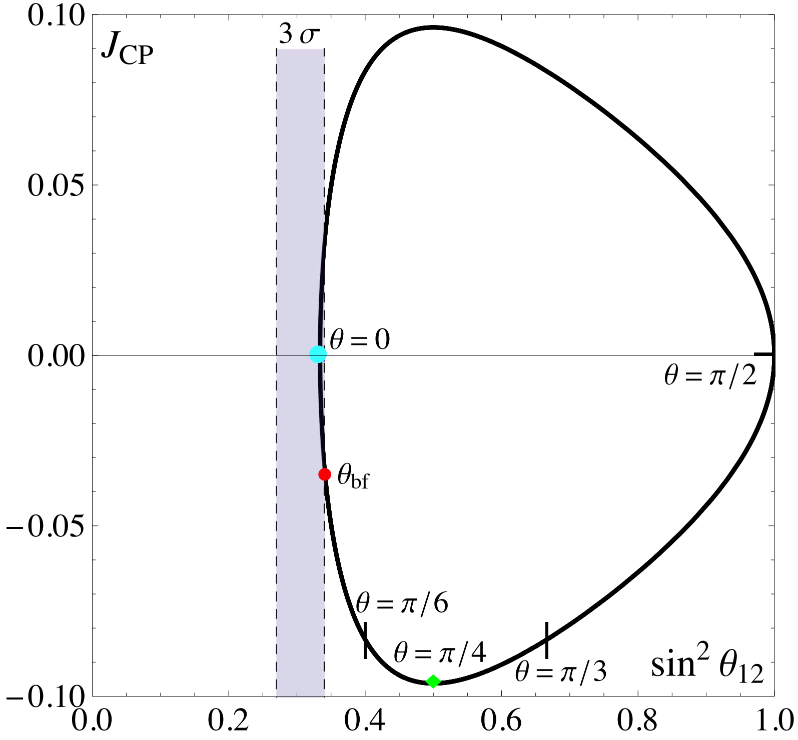

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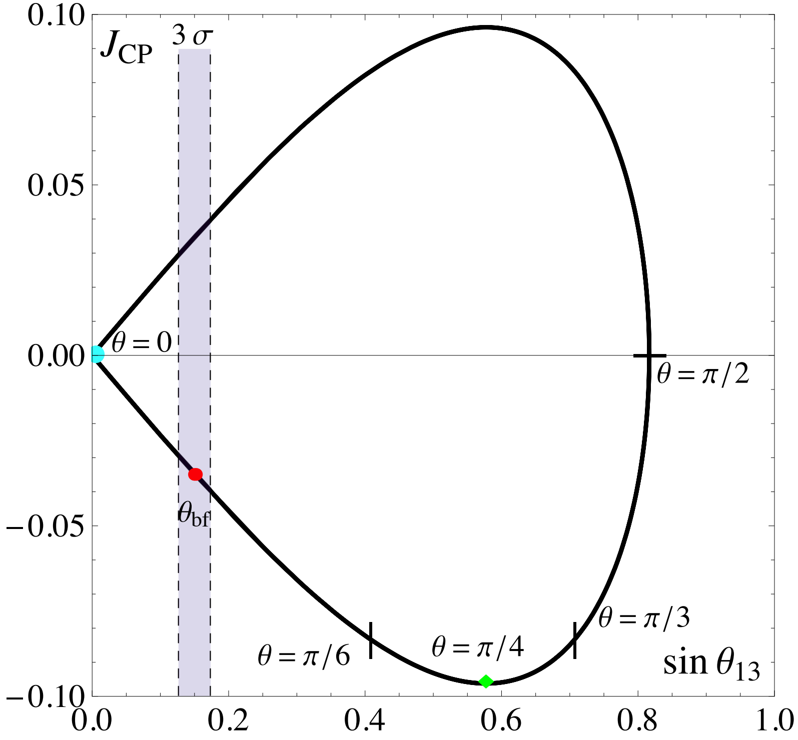
Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and our X



Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and our X

$$\theta_{\text{bf}} \approx 0.185 \;\; , \quad \chi^2_{\text{min}} \approx 18.4 \quad \text{for} \quad \theta_{23} < \pi/4$$

$$\sin^2\theta_{13}(\theta_{\text{bf}}) \approx 0.023 \;\; , \quad \sin^2\theta_{12}(\theta_{\text{bf}}) \approx 0.341 \;\; ,$$
$$|J_{CP}(\theta_{\text{bf}})| \approx 0.0348$$

[Study of S_4 and CP

Non-trivial Majorana phases from $G_e = Z_3$, $Z = S$ and another X

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}\, e^{-i\theta}\,(i\sqrt{3} + e^{2i\theta}) & 1 & \frac{1}{2}\, e^{-i\theta}\,(-i\sqrt{3} + e^{2i\theta}) \\ -e^{i\theta} & 1 & -e^{i\theta} \\ \frac{1}{2}\, e^{-i\theta}\,(-i\sqrt{3} + e^{2i\theta}) & 1 & \frac{1}{2}\, e^{-i\theta}\,(i\sqrt{3} + e^{2i\theta}) \end{pmatrix} K_\nu$$

$$|\sin\delta| = \left| \frac{(4+\sqrt{3}\sin 2\theta)\cos 2\theta \sqrt{4 - 2\sqrt{3}\sin 2\theta}}{5 + 3\cos 4\theta} \right| \quad , \quad |J_{CP}| = \frac{|\cos 2\theta|}{6\sqrt{3}} \quad ,$$

$$|\sin\alpha| = \left| \frac{\sqrt{3} + 2\sin 2\theta}{2 + \sqrt{3}\sin 2\theta} \right| \quad , \quad |\sin\beta| = \left| \frac{4\sqrt{3}\cos 2\theta}{5 + 3\cos 4\theta} \right|$$

Study of S_4 and CP

Non-trivial Majorana phases from $G_e = Z_3$, $Z = S$ and another X

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}\, e^{-i\theta}\,(i\sqrt{3} + e^{2i\theta}) & 1 & \frac{1}{2}\, e^{-i\theta}\,(-i\sqrt{3} + e^{2i\theta}) \\ -e^{i\theta} & 1 & -e^{i\theta} \\ \frac{1}{2}\, e^{-i\theta}\,(-i\sqrt{3} + e^{2i\theta}) & 1 & \frac{1}{2}\, e^{-i\theta}\,(i\sqrt{3} + e^{2i\theta}) \end{pmatrix} K_\nu$$

$$\sin^2\theta_{13} = \frac{1}{3}\left(1 - \frac{\sqrt{3}}{2}\sin 2\theta\right) \,, \quad \sin^2\theta_{12} = \frac{2}{4 + \sqrt{3}\sin 2\theta} \,,$$

$$\sin^2\theta_{23} = \begin{cases} \sin^2\theta_{12} \\ 1 - \sin^2\theta_{12} \end{cases}$$

Study of S_4 and CP

Non-trivial Majorana phases from $G_e = Z_3$, $Z = S$ and another X

$$\theta_{\text{bf}} \approx \pi/4 \ , \quad \chi^2_{\text{min}} \gtrsim 100$$

$$\sin^2\theta_{13}(\theta_{\text{bf}}) = 0.045 \ , \quad \sin^2\theta_{12}(\theta_{\text{bf}}) = 0.349 \ ,$$

$$\sin^2\theta_{23}(\theta_{\text{bf}}) = \begin{cases} 0.349 \\ 0.651 \end{cases}$$

$$\sin\delta(\theta_{\text{bf}}) = 0 \ , \quad |\sin\alpha|(\theta_{\text{bf}}) = 1 \ , \quad \sin\beta(\theta_{\text{bf}}) = 0$$

Study of S_4 and CP

Trivial CP phases from $G_e = Z_3$, $Z = S$ and again another X

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta - \sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + \sqrt{3}\cos\theta \\ -\cos\theta + \sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - \sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \ , \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} \ , \quad \sin^2\theta_{23} = \frac{1}{2}\left(1 - \frac{\sqrt{3}\sin 2\theta}{2+\cos 2\theta}\right)$$

and

$$\sin\delta = 0 \ , \quad \sin\alpha = 0 \ , \quad \sin\beta = 0$$

Study of S_4 and CP

Trivial CP phases from $G_e = Z_3$, $Z = S$ and again another X

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta - \sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + \sqrt{3}\cos\theta \\ -\cos\theta + \sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - \sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

$$\theta_{\text{bf}} \approx \begin{cases} 0.184, & \theta_{23} < \pi/4 \\ 2.958, & \theta_{23} > \pi/4 \end{cases} \quad , \quad \chi^2_{\text{min}} \approx 10 \quad : \quad \sin^2\theta_{13}(\theta_{\text{bf}}) = 0.022 \quad ,$$

$$\sin^2\theta_{12}(\theta_{\text{bf}}) = 0.341 \quad , \quad \sin^2\theta_{23}(\theta_{\text{bf}}) = \begin{cases} 0.394 \\ 0.606 \end{cases} \quad]$$

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

- left-handed leptons l are unified in $\mathbf{3}'$ of S_4

SUSY model with S_4 and CP for leptons

- left-handed leptons l are unified in $\mathbf{3}'$ of S_4
- right-handed charged leptons are singlets under S_4; add Z_3 in order to distinguish them: 1, ω, ω^2

SUSY model with S_4 and CP for leptons

- left-handed leptons l are unified in $\mathbf{3}'$ of S_4
- right-handed charged leptons are singlets under S_4; add Z_3 in order to distinguish them: 1, ω, ω^2
- deviation in breaking pattern: $G_e = Z_3^{(D)}$: diagonal subgroup of $Z_3 \subset S_4$ (generated by T) and add. Z_3

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

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- (not shown an auxiliary symmetry Z_{16} and $U(1)_R$ for vacuum alignment)

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	l	e^c	μ^c	τ^c	h_u	h_d
S_4	$\mathbf{3}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_3	1	1	ω	ω^2	1	1

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	χ_E	φ_E	ξ_N	χ_N	φ_N	ξ'_N
S_4	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}'$
Z_3	ω	ω	1	1	1	1

SUSY model with S_4 and CP for leptons

Remember S, T and U for $\mathbf{3}'$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \;\; , \;\; T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \;\; , \;\; U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

and our chosen $X_{\mathbf{3}'}$

$$X_{\mathbf{3}'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

SUSY model with S_4 and CP for leptons

For the other representations the generators S, T and U and X are

$$\mathbf{1}: \quad S = 1\,, \qquad T = 1\,, \qquad U = 1\,,$$

$$\mathbf{1}': \quad S = 1\,, \qquad T = 1\,, \qquad U = -1\,,$$

$$\mathbf{2}: \quad S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\,, \qquad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}\,, \qquad U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,,$$

$$\mathbf{3}: \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}\,, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}\,, \quad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

and $X_{\mathbf{1}} = 1\,,\ X_{\mathbf{1}'} = -1\,,\ X_{\mathbf{2}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,,\ X_{\mathbf{3}} = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

SUSY model with S_4 and CP for leptons

Charged lepton sector

- breaking to $G_e = Z_3^{(D)}$ via

$$\langle \chi_E \rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad , \quad \langle \varphi_E \rangle \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- lowest order couplings

$$y_\tau (l \varphi_E) \tau^c h_d / \Lambda + y_{\mu,1} (l \varphi_E^2) \mu^c h_d / \Lambda^2 + y_{\mu,2} (l \chi_E \varphi_E) \mu^c h_d / \Lambda^2$$

SUSY model with S_4 and CP for leptons

Charged lepton sector

- lead to non-zero muon and tau lepton mass

$$m_\mu = \left| (2\, y_{\mu,1} v_{\varphi_E} + y_{\mu,2} v_{\chi_E}) \, \frac{v_{\varphi_E}}{\Lambda^2} \right| \langle h_d \rangle \quad m_\tau = \left| y_\tau \frac{v_{\varphi_E}}{\Lambda} \right| \langle h_d \rangle$$

for $\quad \langle \chi_E \rangle \, , \, \langle \varphi_E \rangle \sim \lambda^2 \, \Lambda \quad \Rightarrow \quad m_\mu / m_\tau \approx \lambda^2 \, , \quad m_\tau \approx \lambda^2 \, \langle h_d \rangle$

with $\quad \lambda \approx 0.2$

- electron mass vanishes; generated by higher order terms
- charged lepton mass matrix is diagonal $\quad \Rightarrow \quad \Omega_e = \mathbb{1}$

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

Neutrino sector

- breaking to $Z_2 \times Z_2 (\times \text{CP})$ via

$$\langle \xi_N \rangle = v_{\xi_N} \ , \quad \langle \chi_N \rangle = v_{\chi_N} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \ , \quad \langle \varphi_N \rangle = v_{\varphi_N} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

  and $v_{\xi_N} \ , \ v_{\chi_N} \ , \ v_{\varphi_N}$ have same phase ($\pm\pi$)

- with $\langle \xi'_N \rangle \in i\mathbb{R}$ breaking to $Z_2 \times$ CP
- lowest order couplings

$$y_{\nu,1} (ll) \xi_N h_u^2/\Lambda^2 + y_{\nu,2} (ll \varphi_N) h_u^2/\Lambda^2 + y_{\nu,3} (ll \chi_N \xi'_N) h_u^2/\Lambda^3$$

SUSY model with S_4 and CP for leptons

Neutrino sector

- neutrino mass matrix contains three real parameters $t_\nu \propto v_{\xi_N}/\Lambda$, $u_\nu \propto v_{\varphi_N}/\Lambda$ and $x_\nu \propto \langle \xi'_N \rangle v_{\chi_N}/\Lambda^2$
- they have order: $t_\nu\,,\, u_\nu \sim \lambda$ and $x_\nu \sim \lambda^2$ for $\langle \Phi_N \rangle \sim \lambda\,\Lambda$, $\lambda \approx 0.2$
- form of neutrino mass matrix m_ν

$$m_\nu = \begin{pmatrix} t_\nu + 2\,u_\nu & -u_\nu - i x_\nu & -u_\nu + i x_\nu \\ -u_\nu - i x_\nu & 2\,u_\nu + i x_\nu & t_\nu - u_\nu \\ -u_\nu + i x_\nu & t_\nu - u_\nu & 2\,u_\nu - i x_\nu \end{pmatrix} \frac{\langle h_u \rangle^2}{\Lambda}$$

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

Neutrino sector

- PMNS mixing matrix from neutrino sector only and is of form

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

- the parameter θ is

$$\tan 2\theta = \frac{x_\nu}{\sqrt{3}\, u_\nu} \sim \lambda$$

- and the lepton mixing angles read

$$\sin^2\theta_{13} \approx \tfrac{2}{3}\lambda^2 \ , \ \ \sin^2\theta_{12} \approx \tfrac{1}{3} + \tfrac{2}{9}\lambda^2 \ , \ \ \sin^2\theta_{23} = \tfrac{1}{2}$$

SUSY model with S_4 and CP for leptons

Neutrino sector

- neutrino mass spectrum is normally ordered;
  masses depend on two parameters t_ν and $\tilde{u}_\nu = f(u_\nu, x_\nu)$
  (in units $\langle h_u \rangle^2/\Lambda$)

$$m_1 \propto |t_\nu + \tilde{u}_\nu| \ , \ \ m_2 \propto |t_\nu| \ , \ \ m_3 \propto |t_\nu - \tilde{u}_\nu|$$

- for best fit values of $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$

$$m_1 \approx 0.016\,\mathrm{eV} \ , \ \ m_2 \approx 0.018\,\mathrm{eV} \ , \ \ m_3 \approx 0.052\,\mathrm{eV}$$

$$0.003\,\mathrm{eV} \lesssim m_{ee} \lesssim 0.018\,\mathrm{eV} \quad \text{and} \quad m_\beta \approx 0.018\,\mathrm{eV}$$

SUSY model with S_4 and CP for leptons

NLO contributions for charged leptons

- electron mass is generated
  - ... through operators with five fields Φ_N,
    e.g. $l e^c \xi_N^3 \chi_N \varphi_N h_d/\Lambda^5$, if we consider shifts in $\langle \Phi_N \rangle$

SUSY model with S_4 and CP for leptons

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  - ... through operators with six fields Φ_N,
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SUSY model with S_4 and CP for leptons

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  - ... through operators with six fields Φ_N, e.g. $le^c\xi_N^4\xi'_N\varphi_N h_d/\Lambda^6$ with LO VEVs
  - $\langle\Phi_N\rangle \sim \lambda\,\Lambda$ and $\delta\langle\chi_N\rangle \sim \lambda\,\langle\chi_N\rangle \quad\Rightarrow\quad m_e \approx \lambda^6\,\langle h_d\rangle$

SUSY model with S_4 and CP for leptons

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  - ... through operators with six fields Φ_N, e.g. $le^c\xi_N^4\xi'_N\varphi_N h_d/\Lambda^6$ with LO VEVs
  - $\langle\Phi_N\rangle \sim \lambda\,\Lambda$ and $\delta\langle\chi_N\rangle \sim \lambda\,\langle\chi_N\rangle \quad\Rightarrow\quad m_e \approx \lambda^6\,\langle h_d\rangle$
- charged lepton mass matrix is non-diagonal at NLO; however, induced mixing angles are very small: $\theta^l_{ij} \sim \lambda^4$

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

NLO contributions for neutrinos

- largest correction from VEV shift of field χ_N

$$\langle \chi_N \rangle = v_{\chi_N} \begin{pmatrix} 1 + i\,\alpha\,\lambda \\ 1 - i\,\alpha\,\lambda \end{pmatrix} \quad , \quad \alpha \in \mathbb{R}$$

which still preserves $Z_2 \times$ CP $\quad \Rightarrow \quad$ parameter $p_\nu \sim \lambda^3$:

SUSY model with S_4 and CP for leptons

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which still preserves $Z_2 \times$ CP $\Rightarrow$ parameter $p_\nu \sim \lambda^3$:

$$m_\nu^{\rm NLO} = \begin{pmatrix} t_\nu + 2u_\nu & -u_\nu - ix_\nu + p_\nu & -u_\nu + ix_\nu + p_\nu \\ -u_\nu - ix_\nu + p_\nu & 2u_\nu + ix_\nu + p_\nu & t_\nu - u_\nu \\ -u_\nu + ix_\nu + p_\nu & t_\nu - u_\nu & 2u_\nu - ix_\nu + p_\nu \end{pmatrix} \frac{\langle h_u \rangle^2}{\Lambda}$$

SUSY model with S_4 and CP for leptons

NLO contributions for neutrinos

- largest correction from VEV shift of field χ_N

$$\langle \chi_N \rangle = v_{\chi_N} \left(\begin{array}{c} 1 + i\,\alpha\,\lambda \\ 1 - i\,\alpha\,\lambda \end{array} \right) \quad , \quad \alpha \in \mathbb{R}$$

which still preserves $Z_2 \times$ CP $\quad \Rightarrow \quad$ parameter $p_\nu \sim \lambda^3$:
no effect on mixing; small shift in neutrino masses

SUSY model with S_4 and CP for leptons

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  which still preserves $Z_2 \times$ CP $\Rightarrow$ parameter $p_\nu \sim \lambda^3$: no effect on mixing; small shift in neutrino masses

- other corrections to m_ν are max. order λ^6 in units $\langle h_u \rangle^2/\Lambda$ ... negligible

SUSY model with S_4 and CP for leptons

Flavon superpotential: main ingredients

- a $U(1)_R$ symmetry
- so-called driving fields with R charge $+2$

	$\xi_E^0\,,\,\tilde{\xi}_E^0$	χ_E^0	ξ_N^0	χ_N^0	φ_N^0	$\tilde{\xi}_N^0$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}$
Z_3	ω	ω	1	1	1	1

SUSY model with S_4 and CP for leptons

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SUSY model with S_4 and CP for leptons

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SUSY model with S_4 and CP for leptons

Flavon superpotential: main ingredients

- a $U(1)_R$ symmetry
- so-called driving fields with R charge $+2$
- flavons have R charge 0
- flavor and CP symmetries broken at high energies, SUSY still intact
- F-terms of driving fields need to be set to zero

$$\frac{\partial w}{\partial \Phi^0} = 0$$

- these equations determine the flavons' VEVs

SUSY model with S_4 and CP for leptons

Flavon superpotential:

- a $U(1)_R$ symmetry
- so-called driving fields with R charge $+2$
- flavons have R charge 0
- typical terms in superpotential are of the form

$$M^2 \Phi^0 + \tilde{M} \Phi^0 \Psi + a \, \Phi^0 \Psi \Sigma$$

- form of F-terms is particularly simple

$$M^2 + \tilde{M} \Psi + a \, \Psi \Sigma = 0$$

SUSY model with S_4 and CP for leptons

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- flavons have R charge 0
- typical terms in superpotential are of the form

$$M^2\Phi^0 + \tilde{M}\Phi^0\Psi + a\,\Phi^0\Psi\Sigma$$

- here flavon superpotential can be divided into three parts

$$w_{fl} = w_{fl,e} + w_{fl,\nu} + w_{fl,\xi}$$

SUSY model with S_4 and CP for leptons

Flavon superpotential:

- a $U(1)_R$ symmetry
- so-called driving fields

	$\xi_E^0\,,\,\tilde{\xi}_E^0$	χ_E^0	ξ_N^0	χ_N^0	φ_N^0	$\tilde{\xi}_N^0$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3'}$	$\mathbf{1}$
Z_3	ω	ω	1	1	1	1

- flavon superpotential can be divided into three parts

$$w_{fl} = w_{fl,e} + w_{fl,\nu} + w_{fl,\xi}$$

SUSY model with S_4 and CP for leptons

Flavon superpotential:

- a $U(1)_R$ symmetry
- so-called driving fields

	ξ_E^0 , $\tilde{\xi}_E^0$	χ_E^0	ξ_N^0	χ_N^0	φ_N^0	$\tilde{\xi}_N^0$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}$
Z_3	ω	ω	1	1	1	1

- part I

$$w_{fl,e} = a_e\, \xi_E^0 (\chi_E \chi_E) + \tilde{a}_e\, \tilde{\xi}_E^0 (\varphi_E \varphi_E) + b_e\, (\chi_E^0 \chi_E \chi_E) + c_e\, (\chi_E^0 \varphi_E \varphi_E)$$

SUSY model with S_4 and CP for leptons

Flavon superpotential:

- a $U(1)_R$ symmetry
- so-called driving fields

	$\xi_E^0\,,\ \tilde{\xi}_E^0$	χ_E^0	ξ_N^0	χ_N^0	φ_N^0	$\tilde{\xi}_N^0$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}$
Z_3	ω	ω	1	1	1	1

- part I|

$$\begin{aligned} w_{fl,\nu} \quad = \quad & a_\nu\, \xi_N^0 \xi_N^2 + \tilde{a}_\nu\, \xi_N^0 \tilde{\xi}_N^2 + \bar{a}_\nu\, \xi_N^0 \xi_N \tilde{\xi}_N + b_\nu\, \xi_N^0 (\chi_N \chi_N) + c_\nu\, \xi_N^0 (\varphi_N \varphi_N) \\ & + d_\nu\, (\chi_N^0 \chi_N \chi_N) + e_\nu\, (\chi_N^0 \varphi_N \varphi_N) + f_\nu\, \tilde{\xi}_N (\varphi_N^0 \varphi_N) + g_\nu\, (\varphi_N^0 \varphi_N \varphi_N) \end{aligned}$$

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

Flavon superpotential:

- a $U(1)_R$ symmetry
- so-called driving fields

	$\xi_E^0\,,\,\tilde{\xi}_E^0$	χ_E^0	ξ_N^0	χ_N^0	φ_N^0	$\tilde{\xi}_N^0$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}$
Z_3	ω	ω	1	1	1	1

- part I||

$$w_{fl,\xi} = \tilde{\xi}_N^0 M^2 + a_\xi \tilde{\xi}_N^0 (\xi_N' \xi_N')$$

SUSY model with S_4 and CP for leptons

Flavon superpotential: dominant NLO terms

- are

$$s_{\nu,1}\,\xi_N\,(\chi_N^0\xi_N'\chi_N)/\Lambda + \tilde{s}_{\nu,1}\,\tilde{\xi}_N\,(\chi_N^0\xi_N'\chi_N)/\Lambda + s_{\nu,2}\,(\varphi_N^0\xi_N'\chi_N\varphi_N)/\Lambda$$

- correct the VEVs of the fields $\chi_{N,i}$

$$\langle\chi_N\rangle = v_{\chi_N}\begin{pmatrix} 1 + i\,\alpha\,\lambda \\ 1 - i\,\alpha\,\lambda \end{pmatrix} \quad , \quad \alpha \in \mathbb{R}$$

- other NLO terms give much smaller contributions; thus rest of shifts in VEVs is of relative order λ^4

Conclusions

- Scenarios with G_f and CP predict mixing angles and CP phases in terms of one parameter θ
- Explicit model with S_4 and CP ...
  - predicts θ_{23} and δ maximal as well as α and β trivial due to symmetry breaking pattern

Conclusions

- Scenarios with G_f and CP predict mixing angles and CP phases in terms of one parameter θ
- Explicit model with S_4 and CP ...
  - explains θ_{13} small via naturally small θ,
  - leads to normally ordered light neutrino masses,
  - predicts absolute neutrino mass scale,
  - generates charged lepton masses of correct order

Conclusions

- Scenarios with G_f and CP predict mixing angles and CP phases in terms of one parameter θ
- Explicit model with S_4 and CP constructed
- Consider models with other G_f? Extension to quarks?

Thank you for your attention.

Back up

Study of S_4 and CP –Flavon superpotential

SUSY model with S_4 and CP for leptons

Flavon superpotential: F-terms of driving fields with index E

$$
\begin{aligned}
\frac{\partial w_{fl}}{\partial \xi_E^0} &= 2a_e\, \chi_{E,1}\chi_{E,2} \\
\frac{\partial w_{fl}}{\partial \tilde{\xi}_E^0} &= \tilde{a}_e\, (\varphi_{E,1}^2 + 2\varphi_{E,2}\varphi_{E,3}) \\
\frac{\partial w_{fl}}{\partial \chi_{E,1}^0} &= b_e\, \chi_{E,1}^2 + c_e\, (\varphi_{E,3}^2 + 2\varphi_{E,1}\varphi_{E,2}) \\
\frac{\partial w_{fl}}{\partial \chi_{E,2}^0} &= b_e\, \chi_{E,2}^2 + c_e\, (\varphi_{E,2}^2 + 2\varphi_{E,1}\varphi_{E,3})
\end{aligned}
$$

SUSY model with S_4 and CP for leptons

Flavon superpotential: F-terms of the driving fields ξ_N^0, χ_N^0 and φ_N^0

$$\begin{aligned}
\frac{\partial w_{fl}}{\partial \xi_N^0} &= a_\nu\, \xi_N^2 + \tilde{a}_\nu\, \tilde{\xi}_N^2 + \bar{a}_\nu\, \xi_N \tilde{\xi}_N + 2\, b_\nu\, \chi_{N,1}\chi_{N,2} + c_\nu\, (\varphi_{N,1}^2 + 2\varphi_{N,2}\varphi_{N,3}) \\
\frac{\partial w_{fl}}{\partial \chi_{N,1}^0} &= d_\nu\, \chi_{N,1}^2 + e_\nu\, (\varphi_{N,3}^2 + 2\varphi_{N,1}\varphi_{N,2}) \\
\frac{\partial w_{fl}}{\partial \chi_{N,2}^0} &= d_\nu\, \chi_{N,2}^2 + e_\nu\, (\varphi_{N,2}^2 + 2\varphi_{N,1}\varphi_{N,3}) \\
\frac{\partial w_{fl}}{\partial \varphi_{N,1}^0} &= f_\nu\, \tilde{\xi}_N \varphi_{N,1} + 2\, g_\nu\, (\varphi_{N,1}^2 - \varphi_{N,2}\varphi_{N,3}) \\
\frac{\partial w_{fl}}{\partial \varphi_{N,2}^0} &= f_\nu\, \tilde{\xi}_N \varphi_{N,3} + 2\, g_\nu\, (\varphi_{N,2}^2 - \varphi_{N,1}\varphi_{N,3}) \\
\frac{\partial w_{fl}}{\partial \varphi_{N,3}^0} &= f_\nu\, \tilde{\xi}_N \varphi_{N,2} + 2\, g_\nu\, (\varphi_{N,3}^2 - \varphi_{N,1}\varphi_{N,2})
\end{aligned}$$