ACCCOSMICELERATION
Dark energy and *beyond*

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- **MS**, " **A primer on problems and prospects of dark energy** " Curr Sci, V 97, 848 (2009) [arXiv:0904.3445].
- M. Turner, D. Huterer, "**Cosmic Acceleration, Dark Energy and Fundamental Physics** "J.Phys.Soc.Jap.76:111015,2007.
- E. Copeland, **MS**, S. Tsujikawa, " **Dynamic of Dark Energy**", hep-th/0603057.
- Zeldovic and Novikov, Structure and evolution of universe, V 2

BRIEF OVER VIEW

- **Cosmological constant a la Hook's law---Static universe**
  Neumann (1895) and Seeliger (1896)
- **Late time inconsistency of hot big bang: Resolution of age crisis.**
- **Theoretical issues related cosmological constant (Sakharov)**
  Coincidence ; Fine tuning.
- **Quintessence : quintessential inflation; problems.**
- **Modified theories of gravity:**
- **Local physics---Screening mechanisms.**
  Chameleon, Symmetron , Vainshtein mechanism.
  Scope of chameleon/symmetron : Self acceleration.
  Phenomenological extensions of massive gravity.

Homogeneous and isotropic universe

Hubble Law

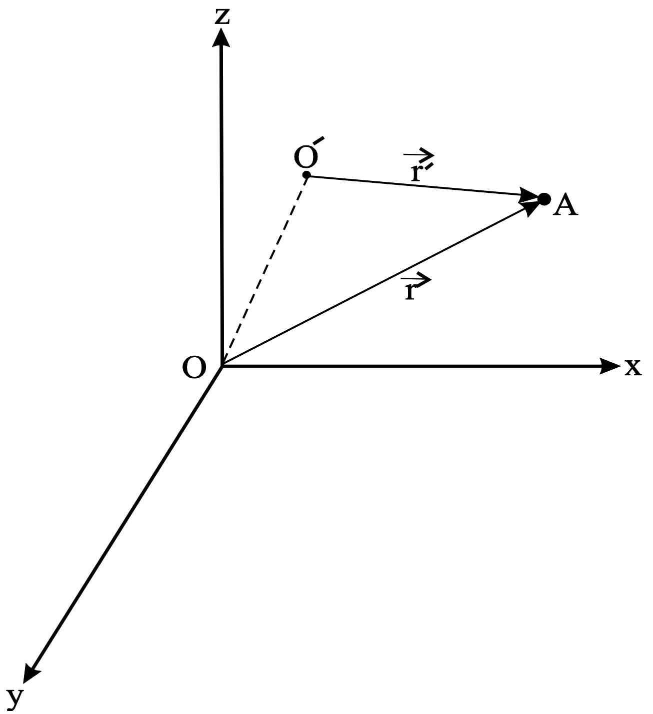
$$\mathbf{v}(\mathbf{r},t) = H(t)\mathbf{r}$$

$$\mathbf{r}'(t) = \mathbf{r}(\mathbf{t}) - \mathbf{r}_{o'}(t)$$

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_{o'} = H\mathbf{r}'$$

$$\mathbf{r}(t) = \mathbf{r}(t=0)e^{\int H(t)dt} \equiv a(t)\mathbf{x}$$

$$\frac{\dot{a}}{a} = H$$



NEWTONIAN COSMOLOGY

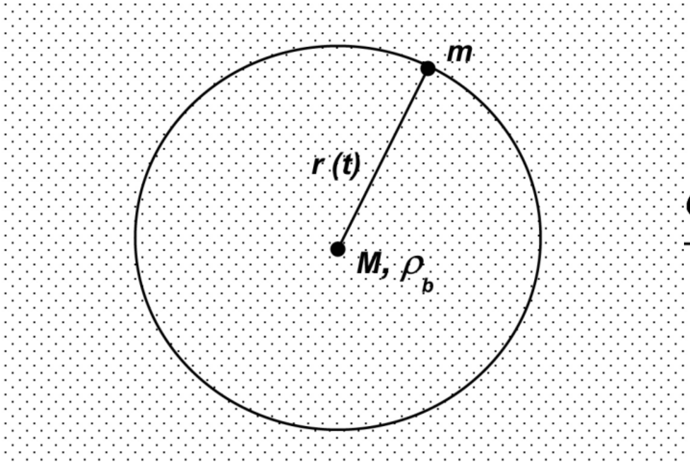
(1895- Neumann, 1896-Seeliger)

MS, arXiv: 0904.3445

$$\mathbf{F} = -\frac{4\pi G}{3}\rho_b(t)\mathbf{r}(t) \Rightarrow \quad \frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\rho_b(t)$$

Co-moving system: $\mathbf{r}(t) \equiv a(t)\mathbf{x}$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2}, \quad K \equiv a_0^2\left(\frac{8\pi G\rho_b^0}{3} - H_0^2\right)$$



$$\frac{\partial \rho_b(t)}{\partial t} + (\nabla.\rho_b\mathbf{v}) = 0$$

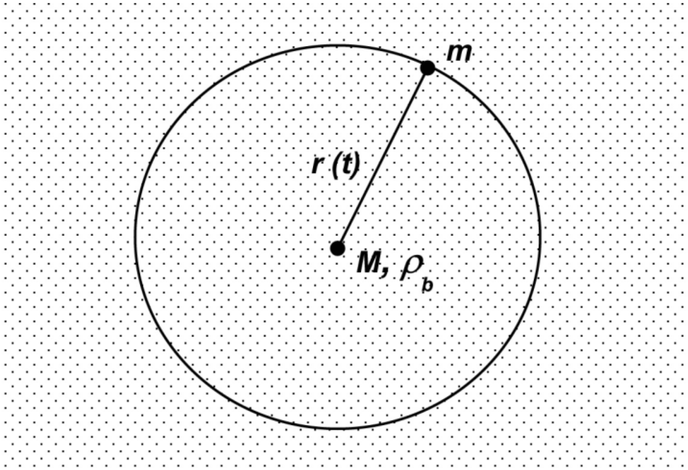
$$\frac{\partial \rho_b(t)}{\partial t} + 3H\rho_b = 0, \quad \rho_b(t) = \rho_b^{(0)}\left(\frac{a_0}{a}\right)^3$$

No static universe: $\dot{a} = \ddot{a} = 0$

Cosmological constant or Hook's law


Static universe
Neumann (1895) and Seeliger(1896)

$$\mathbf{F} = -\frac{4\pi G}{3}\rho_b \mathbf{r} \quad + \frac{1}{3}\Lambda \mathbf{r}$$



Einstein 1917

MS, arXiv: 0904.3445

NEWTONIAN COSMOLOGY

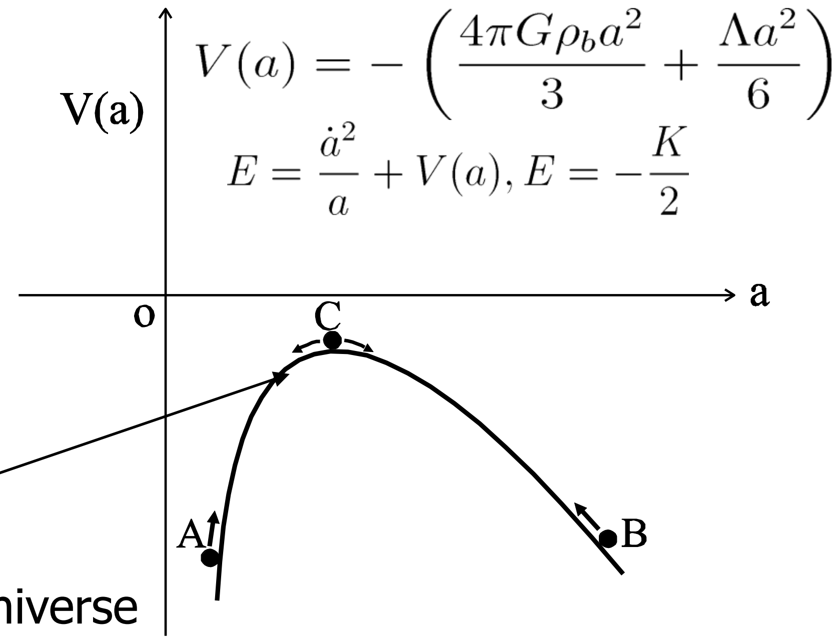
Cosmological constant a la Hook's law

$$\mathbf{F} = -\frac{4\pi G}{3}\rho_b \mathbf{r} + \frac{1}{3}\Lambda \mathbf{r}$$

$$\frac{1}{a}\frac{d^2 a}{dt^2} = -\frac{4\pi G}{3}\rho_b(t) + \frac{\Lambda}{3}$$

$$H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2} + \frac{\Lambda}{3}$$

(K>0) $\Lambda = \Lambda_c = 4\pi G \rho_b^{(0)}$ → Static universe



MS, Curr Sci, V 97, 848 (2009) [arXiv:0904.3445].

If no quasi-static universe then away with cosmological constant!

Inconsistencies of standard model

EARLY TIMES:

Flatness problem

Primordial inhomogeneities **INFLATION**

Horizon problem

LATE TIMES: Age crisis

AGE CRISIS IN HOT BIG BANG

Matter dominated universe:

$$\rho_b(t) = \rho_b^{(0)} \left(\frac{t_0}{t}\right)^2$$

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3} \longrightarrow H(t) = \frac{2}{3}\frac{1}{t} \Rightarrow t_0 = \frac{2}{3}\frac{1}{H_0}$$

$$H_0^{-1} = 9.8h^{-1}Gyr, \quad 0.64 \lesssim h \lesssim 0.8 \rightarrow t_0 = (8-10)Gyr$$

AGE CRISIS IN HOT BIG BANG

$$v = Hr(t)$$

Let us ignore gravity for a moment :

$$v = const \rightarrow \frac{1}{t} = H$$

Present epoch:

$$\frac{1}{t_0} = H_0$$

AGE CRISIS AND ITS RESOLUTION WITH Λ

Open universe (K<0)

$$\Omega_m^{(0)} = \rho_m^{(0)} / \rho_{cr}$$

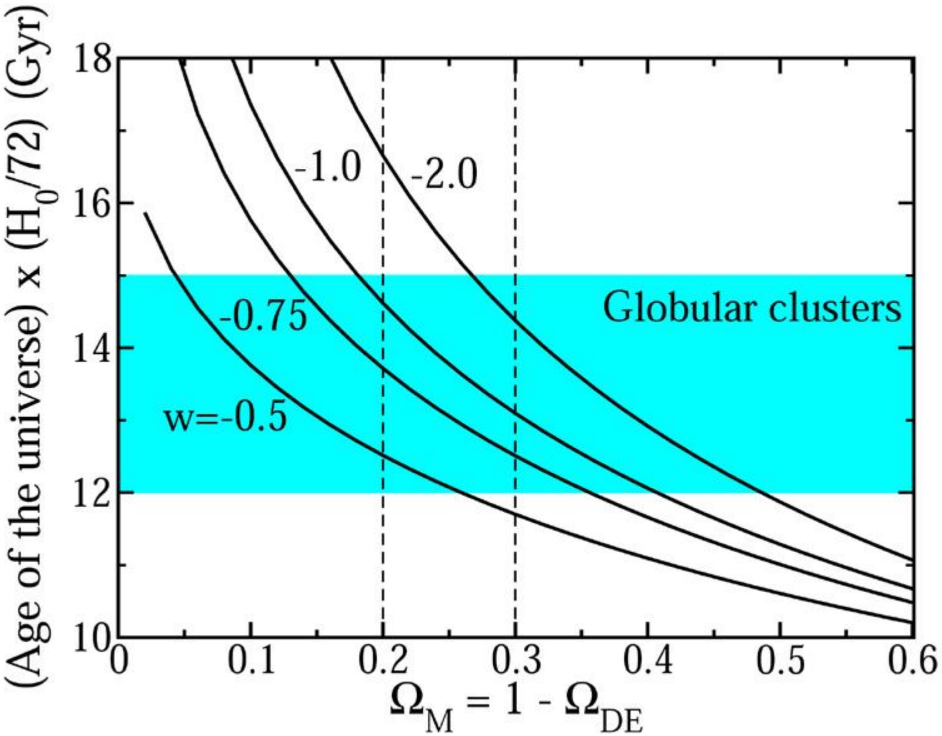
$$H_0 t_0 \to \frac{2}{3} \quad \Omega_m^{(0)} \to 1 \qquad\qquad H_0 t_0 \to 1 \quad \Omega_m^{(0)} \to 0$$

Repulsive effect Λ :

$$t_0 = \frac{1}{H_0} \int_0^\infty \frac{dz}{(1+z)\sqrt{\Omega_m^0 (1+z)^3 + \Omega_\Lambda}}$$

$$t_0 H_0 = \frac{2}{3} \frac{1}{\Omega_\Lambda^{1/2}} \ln \left(\frac{1 + \Omega_\Lambda^{1/2}}{\Omega_m^{1/2}} \right)$$

AGE CRISIS AND ITS RESOLUTION WITH Λ



ENERGY OF VACUUM

SAKHAROV 1968

$$< 0|T_{\mu\nu}|0 >= -\rho_v \eta_{\mu\nu} \ ;$$

$$< 0|T_{\mu\nu}|0 >= -\rho_v g_{\mu\nu} \ ;$$

$$< 0|T_{\mu\nu}|0 >= -V(\varphi_{min}) g_{\mu\nu} \ \rightarrow Scalar \ \ field \ \varphi$$

$$< 0|T_{\mu\nu}|0 >= -\rho_v g_{\mu\nu} \rightarrow Vacuum \ \ fluctuations$$

The effective Cosmological constant

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + g_{\mu\nu} \Lambda_B = T^m_{\mu\nu} + < 0 | T_{\mu\nu} | 0 >$$

$$\Lambda_{eff} = \Lambda_B + \rho_v$$

$$p_v = < 0 | p | 0 >$$

COSMOLOGICAL CONSTANT

$$< T_{\mu\nu} >= -\rho_v g_{\mu\nu} \; ; \quad \rho_v = \frac{1}{2}\int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^2}\sqrt{k^2+m^2}$$

$$\rho_v \sim M_P^4$$

$$\Omega_v + \Omega_m^{(0)} \simeq 1 \to \Lambda_{eff} \lesssim \rho_{cr} \quad (\Omega = \rho/\rho_{cr})$$

$$\Lambda_{eff} \sim 10^{-120} M_P^4 \, (\Lambda_{eff} = \Lambda_B + \rho_v)$$

Old cosmological constant problem

$$\rho_v \sim \rho_m^{(0)}$$

New cosmological constant problem

The correct level of fine tuning

$$\rho_v \simeq \frac{m^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right)$$

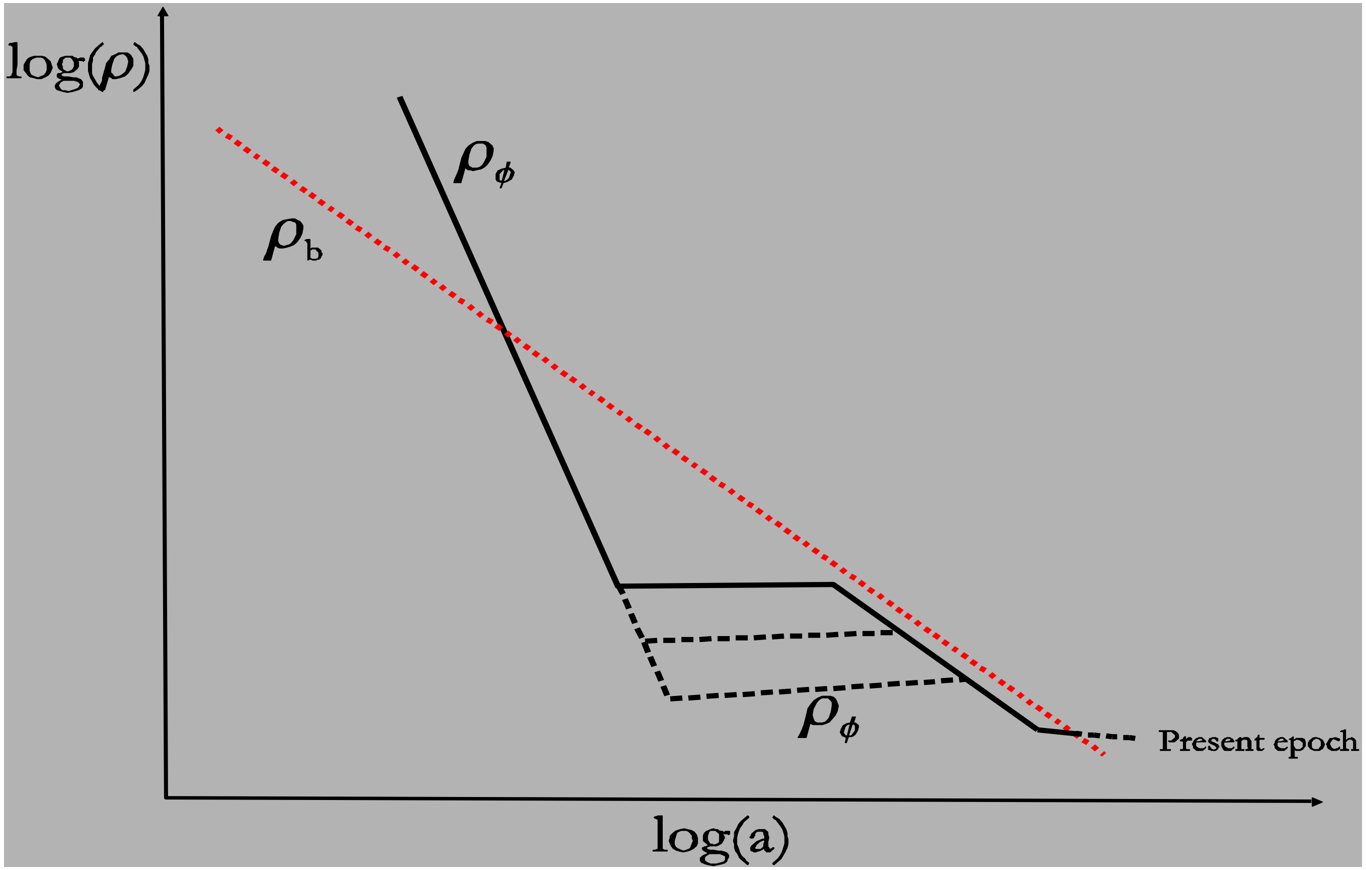
$$m_H \simeq 125 GeV; \quad m_t \simeq 171 GeV, \quad m_{z,w} \simeq 100 GeV$$

$$\mu \sim \sqrt{E_g E_\gamma} \quad E_g \sim H_0 \sim 10^{-41} GeV; \ \lambda \sim 500 nm$$

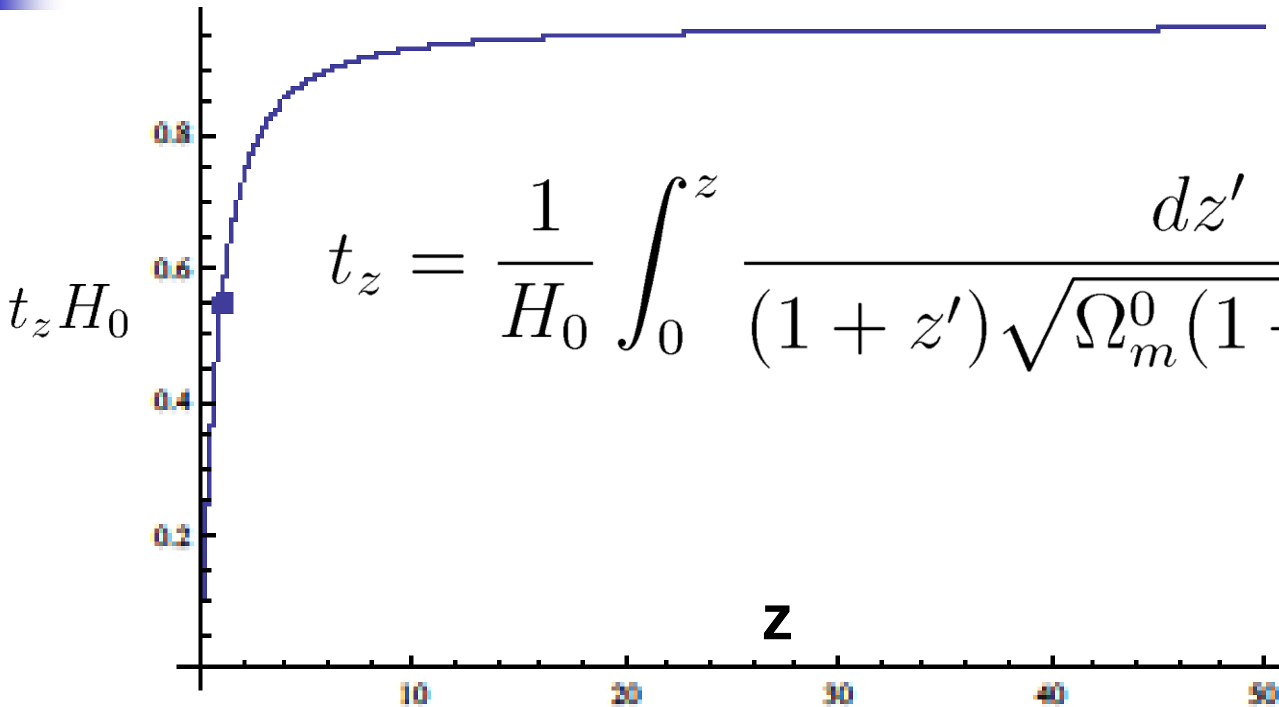
$$\mu \simeq 10^{-25} GeV \rightarrow \quad \rho_v \simeq 10^8 GeV^4$$

$$\Lambda_{eff} \simeq 10^{-48} GeV^4 : \qquad 10^{56}$$

CINCIDENCE PROBLEM ?



WHERE IS THE COINSIDENCE ?



$$t_z = \frac{1}{H_0}\int_0^z \frac{dz'}{(1+z')\sqrt{\Omega_m^0(1+z')^3+\Omega_\Lambda}}$$

SCALAR FIELD AS DARK ENERGY Quintessence

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \ , \qquad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \ P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$H^2 = \frac{8\pi G}{3}\rho_\phi$$

$$\rho_\phi = \rho_\phi^0 \exp\left(-\int 3(1+w(\phi))\frac{da}{a}\right) \ , \quad w(\phi) = P_\phi/\rho_\phi$$

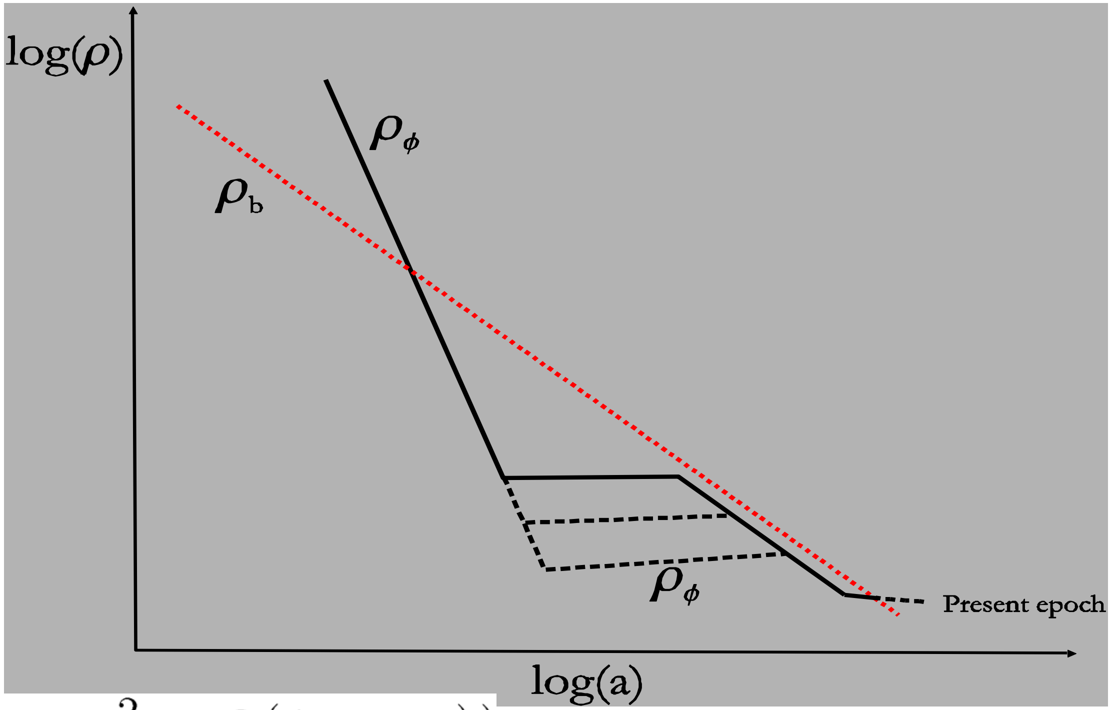
$$\rho_\phi \sim a^{-n}, \ 0 \leq n \leq 6 \ , \qquad \rho_\phi \sim 1/a^6 \longrightarrow$$ **for steep pot.**

Predictive power of scalar fields

For a priori given cosmic history, it is always possible to construct a field potential such that it gives rise to the desired result. Thus the scalar field models should be judged by their generic features.

Scalar Field Dynamics in presence of background matter : Tracker or Freezing Models

$$H^2 = \frac{8\pi G}{3}\left(\rho_\phi + \rho_b\right)$$



Scaling Solution:

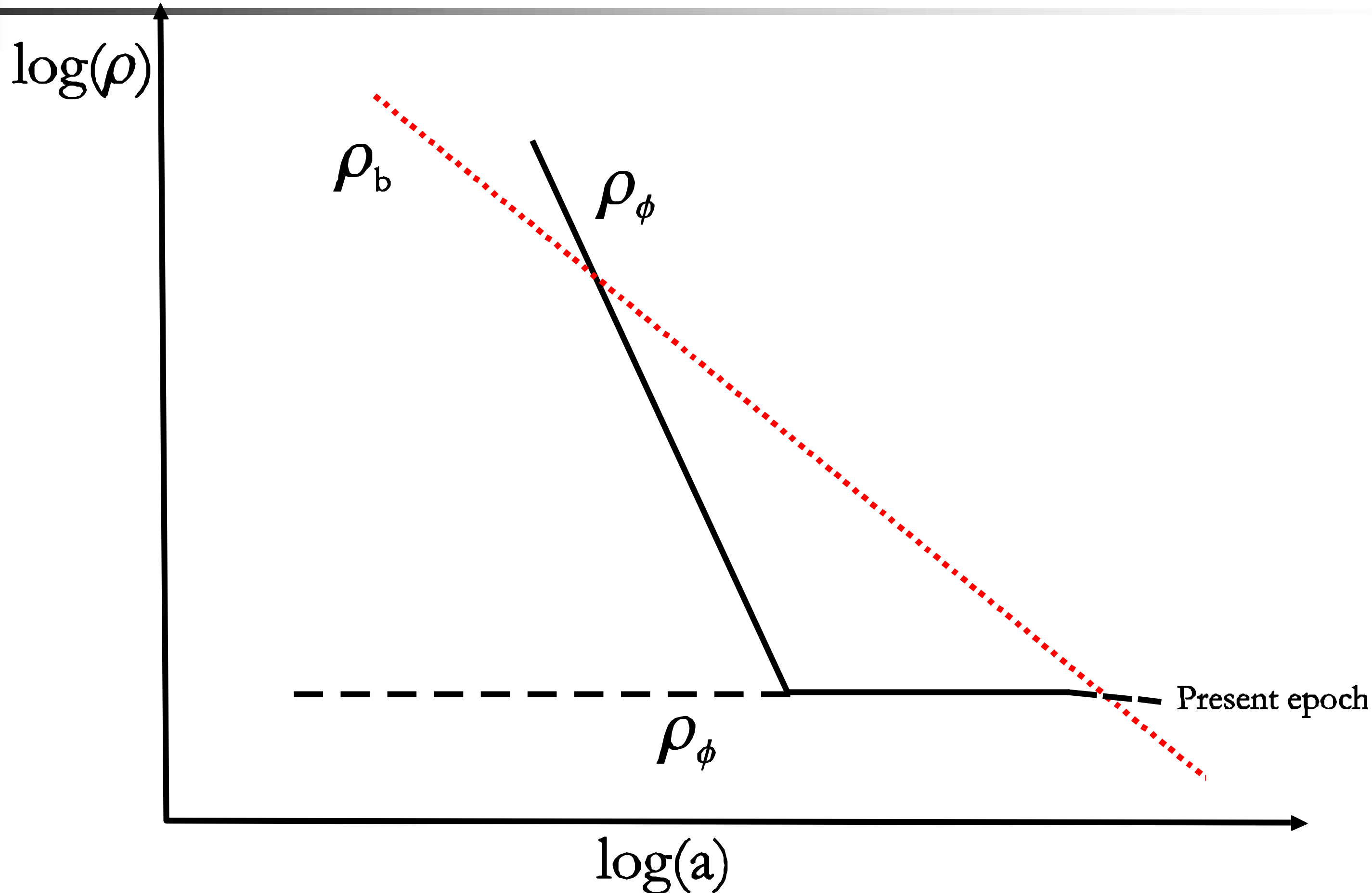
$$\frac{\rho_\phi}{\rho_b} = Const$$

$$V = V_0 e^{\alpha\phi/M_p}(steep - \alpha^2 > 3(1+w_b))$$

$$\Omega_\phi = \frac{3(1+w_b)}{\alpha^2} \lesssim 0.13 \rightarrow \alpha \gtrsim 5$$

MS, **Curr Sci, V 97, 848 (2009) [arXiv:0904.3445]**

Absence of trackers: Thawing Models

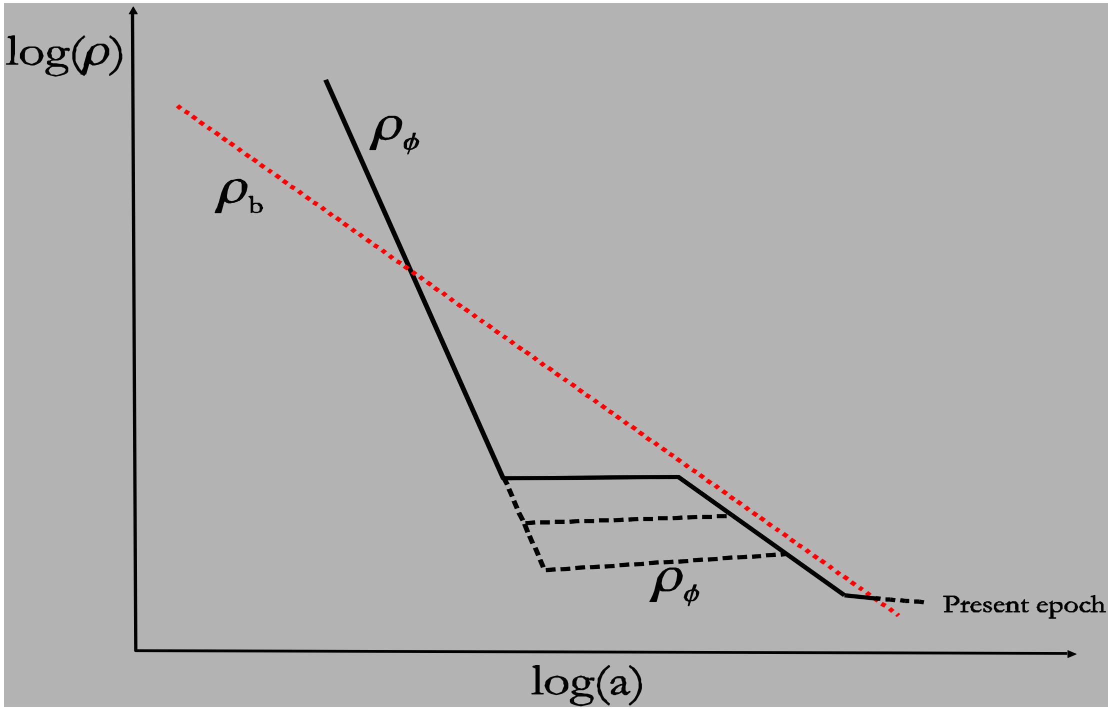


Scalar Field Dynamics in presence of background matter : Tracker or Freezing Models

$$H^2 = \frac{8\pi G}{3}\left(\rho_\phi + \rho_b\right)$$

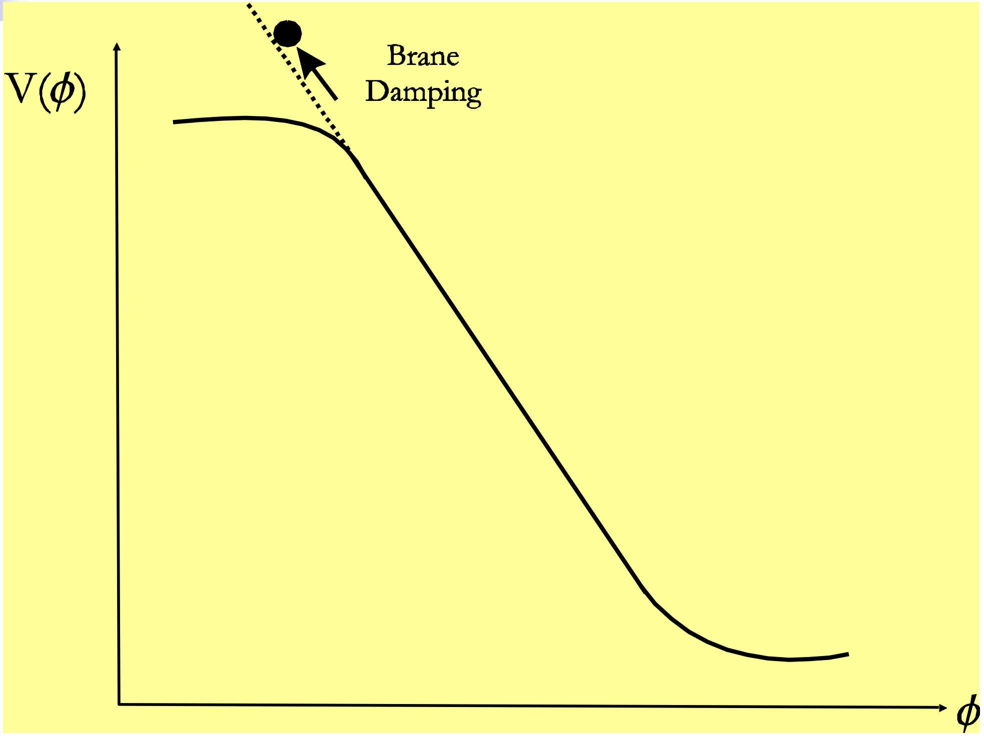
Scaling Solution:

$$\frac{\rho_\phi}{\rho_b} = Const$$



MS, **Curr Sci, V 97, 848 (2009) [arXiv:0904.3445]**

Quintessential inflation-Model Of reincarnation



$$H^2 = \frac{8\pi G}{3}\rho_b\left(1 + \frac{\rho_b}{2\lambda_B}\right)$$

$$\epsilon = \epsilon_{FRW}\frac{1 + V/\lambda_B}{(1 + V/2\lambda_B)^2}$$

$$\eta = \eta_{FRW}(1 + V/2\lambda_B)^{-1}$$

P.J.E. Peebles, A.Vilenkin, PRD59,06350(1999)
MS, V. Sahni, Phys. Rev. D **70, 083513 (2004);**
Phys. Rev. D **65, 023518 (2002); MS, N. Dadhich, T.Shiromizu, PLB568, 118(2003).**

TRACKER POTENTIAL

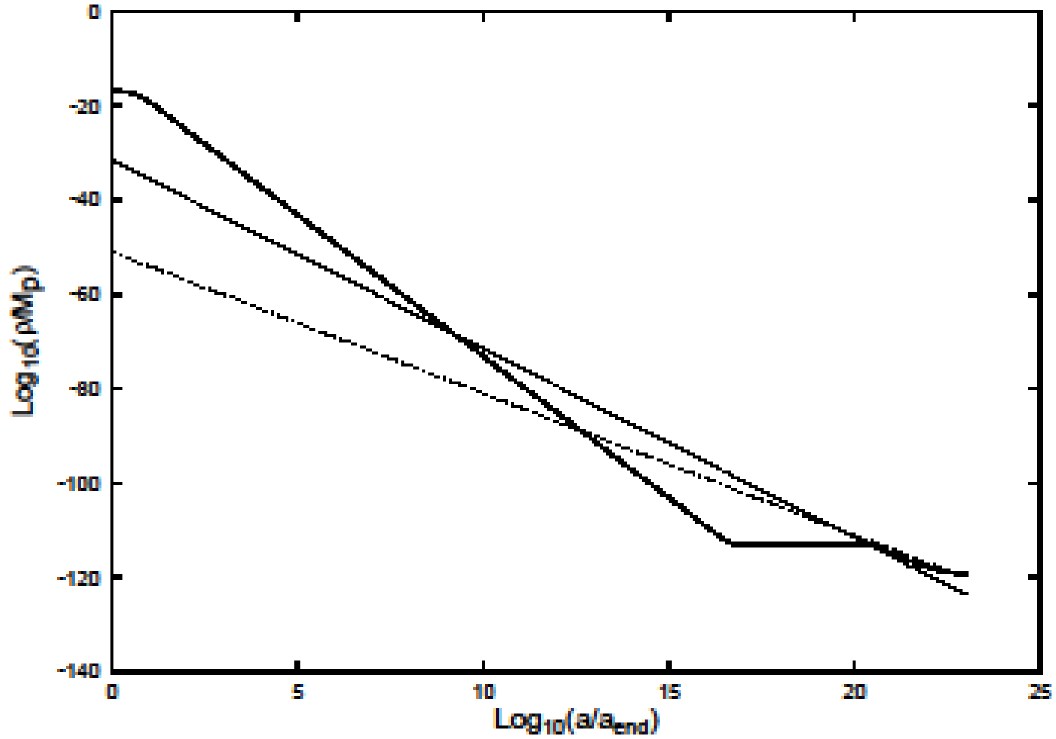
$$V(\phi) = V_0\Big[\cosh(\tilde{\alpha}\phi/M_P) - 1\Big]^p,\ 0 < p < 1/2$$

$$V(\phi) = \frac{V_0}{2^p} e^{\tilde{\alpha}p\phi/M_P}, \quad \tilde{\alpha}|\phi/M_P >> 1$$

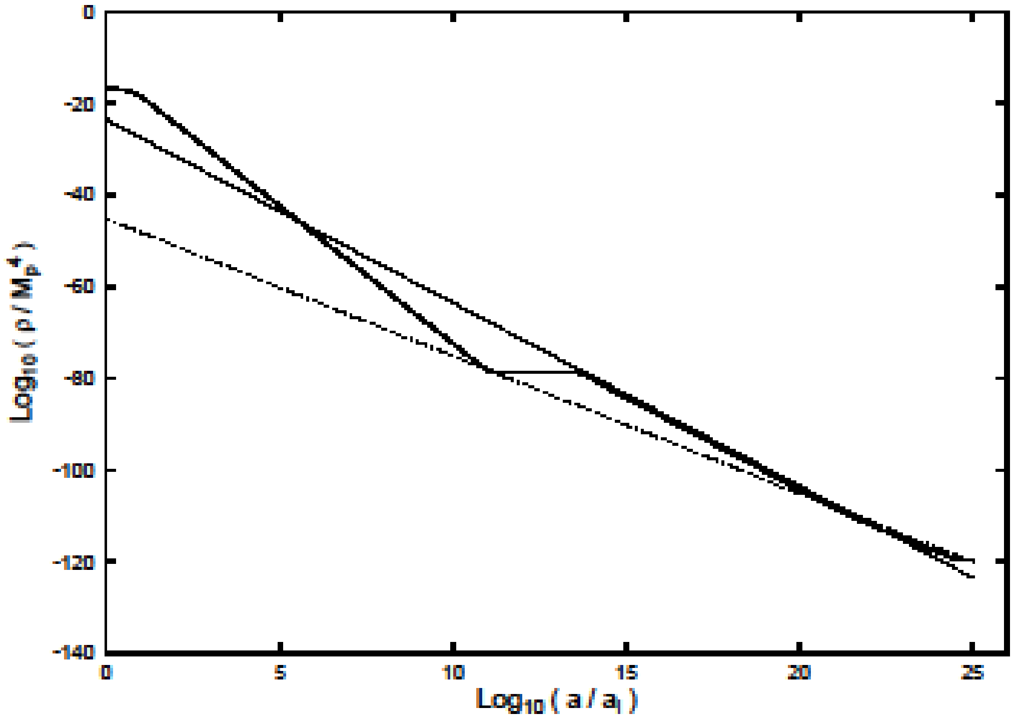
$$V(\phi) = \frac{V_0}{2^p}\Big[\frac{\tilde{\alpha}\phi}{M_P}\Big]^{2p}, \quad \tilde{\alpha}|\phi/M_P << 1; \qquad < w >= \frac{p-1}{p+1}$$

$$\alpha = \tilde{\alpha}p$$

Quintessential Inflation (**gravitational particle production**)



Quintessential Inflation (**instant preheating**)



OBSERVATIONAL CONSTRAINTS

$$n_S = 1 - \frac{4}{N}$$

$$r = \frac{24}{N}$$

$$N = 70, \quad n_S = 0.94, \quad r = 0.34$$

Quintessence problem

$$M_p^2 \left(V'/V\right)^2 << 1$$

C. Kolda and D Lyth, hep-th/9811375

$$M_p^2 V''/V << 1$$

$$m^2 = V'' \simeq \frac{V}{M_p^2} \simeq \frac{H_0^2 M_p^2}{M_p^2} \simeq \left(10^{-33} eV\right)^2$$

Unstable under radiative corrections when coupled to other fields unless severely fine tuned.

MODIFIED THEORIES OF GRAVITY

LARGE SCALE MODIFICATION

- **Chameleon theories.**
- **Galileon modified theories**
- **Massive gravity**

Scalar degree(s) of freedom

Requirements

- **Local physics be intact**
- **Cosmic acceleration**

SCALARON: Requirements

DE:

$$m_\varphi = V_{\varphi\varphi}(\varphi) \simeq H_0 \simeq 10^{-33} eV$$

Solar Physics:

$$m_\varphi >> m_{AU}$$

$$\left(r_{AU} \sim 10^8 Km,\ m_{AU} \sim 10^{-27} GeV\right)$$

Chameleon Field

MASS SCREENING OR FIFTH FORCE SUPPRESSION
(Basic Idea)

$$L = -(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{\alpha}{M_p}\varphi T$$

Localized source: $r = 0, \quad T = -M\delta^3(r)$

$$\varphi = -\frac{\alpha M}{M_p}\frac{e^{-mr}}{4\pi r} \qquad \frac{\alpha}{M_P}\varphi = -2\alpha^2 GM\frac{e^{-mr}}{r}$$

$$(m,\ \alpha) \to m(\rho),\ \alpha(\rho)$$ **Chameleon or Symmetron**

$$Kinetic\ Suppression\ Galileon$$

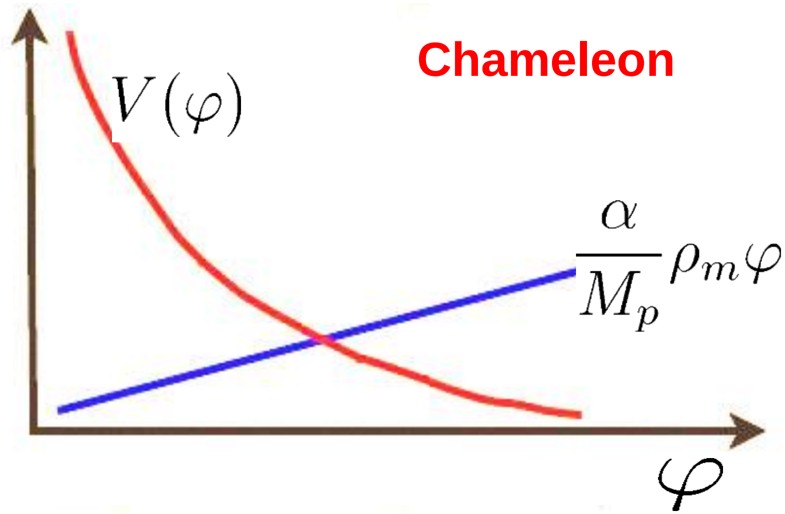
Kinetic suppression

$$\Box\varphi = \left(V(\varphi) + \frac{\alpha}{M_p}\rho_m\varphi\right)_{,\varphi}$$

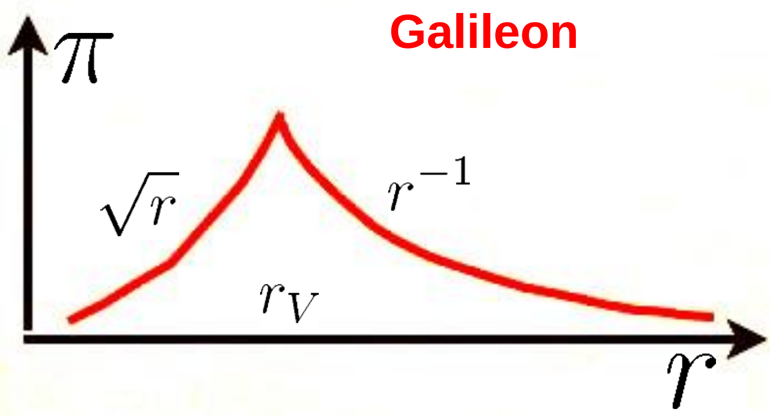
$$\Box\varphi = +\frac{\alpha}{M_p}\rho_m$$

Chameleon versus Galileon(Vainshtein)

$$V_{eff} = V(\varphi) + \frac{\alpha}{M_p}\rho_m\varphi$$



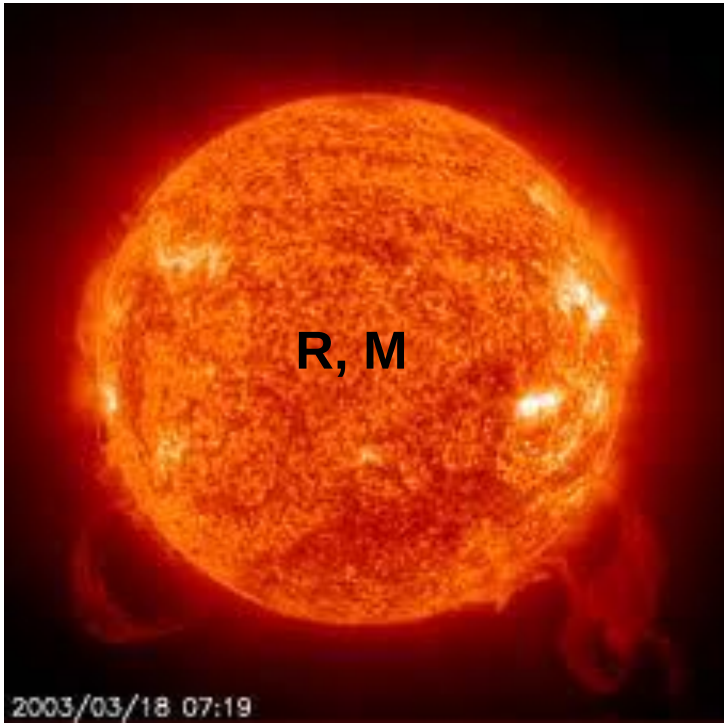
$$\Box\pi + \frac{1}{m^2}[(\Box\pi)^2 - \partial^\mu\partial^\nu\pi\partial_\mu\partial_\nu\pi] = \frac{\alpha}{M_p}\rho_m$$



Small distances: $\pi(r) \sim \sqrt{r};$

Large distances: $\pi(r) \sim r^{-1}$

Chameleon at work



$$V(\varphi_s) = -\frac{GM}{r}\alpha^2\epsilon_{thin}$$

$$\epsilon_{thin} \sim \frac{\Delta R}{R}$$

$$\epsilon_{thin} \propto \frac{\varphi_{out} - \varphi_{in}}{\Phi_s}$$

Vainstein Effect- dynamical suppression of fifth force

$$T^{(m)} = -M\delta(r); \qquad \frac{1}{r^2}\frac{d}{dr}\left(r^3\left[(\pi'/r) + \frac{1}{m^2}(\pi'/r)^2\right]\right) = \alpha M\delta(r)$$

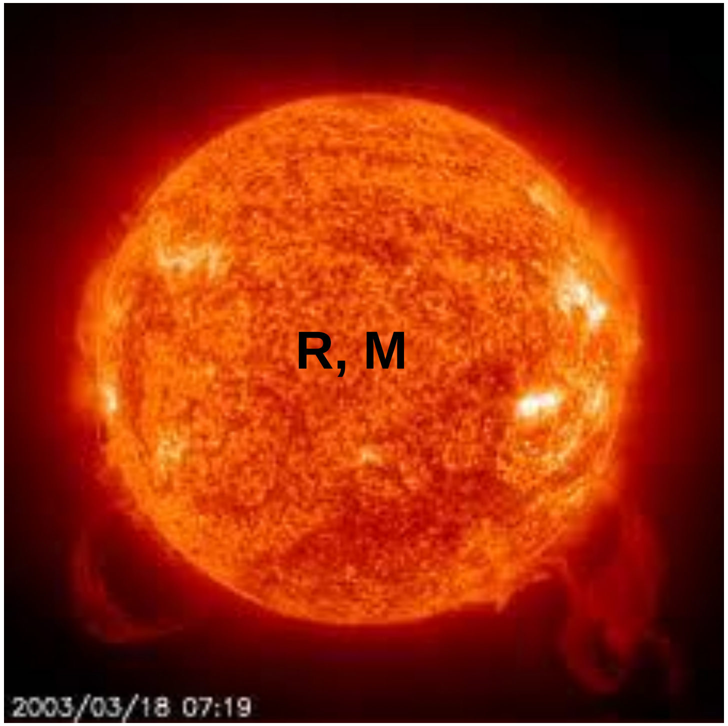
$$\left(\frac{\pi'(r)}{r}\right) + \frac{1}{m^2}\left(\frac{\pi'(r)}{r}\right)^2 = \alpha\frac{r_s}{r^3}$$

Short distances: $\pi' = (r_S \alpha m^2)^{1/2}\frac{1}{r^{1/2}}$

$$\frac{F_\pi}{F_{\rm grav}} = \left(\frac{r}{r_V}\right)^{3/2}, \quad r << r_V, \quad r_V = \left(\frac{r_S}{m^2\alpha}\right)^{1/3}$$

Large distances: $\pi' = r_S\alpha\frac{1}{r^2} \quad \Rightarrow \quad \frac{F_\pi}{F_{\rm grav}} \sim \alpha$

Vainshtein at work



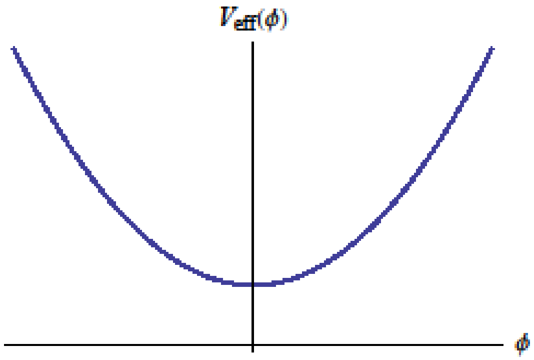
$$r_V = \frac{GM_s}{m^2} = \frac{M_s}{H_0^2 M_p^2} \simeq 100pc$$

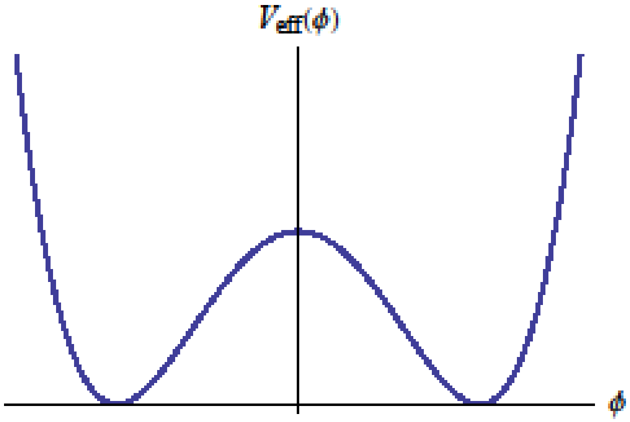
$$r_V(Gal) \simeq 1.2Mpc$$

Symmetron: Dark energy

$$V_{\text{eff}} = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4; \quad m_{eff} = \left(\frac{\rho}{M^2} - \mu^2\right)$$

$$m_{eff} \simeq \frac{\rho}{M^2} \to \phi_0 = 0$$





$$\phi_0 = \pm\sqrt{\frac{\mu^2 - \rho/M^2}{\lambda}};$$

$$m_s = \sqrt{2}\mu;$$

Symmetron: Dark energy

Symmetry should break:

$$\rho \sim \rho_{cr} = H_0^2 M_p^2 \rightarrow m_s \simeq \frac{H_0 M_p}{M}$$

Kurt Hinterbichler, Justin Khoury: arxiV:1001.4525

Local gravity constraints

$$M < 10^{-4} M_p \rightarrow m_s \simeq 10^4 H_0$$

arXiv: 1211.2289: K. Bamba, R. Gannouji, MS

Scalar tensor theories

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\nabla\varphi)^2 - V(\varphi)\right] + \mathcal{S}_m\left[A^2(\varphi)g_{\mu\nu}, \Psi_m\right] .$$

-Einstein frame

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\phi\tilde{R} - \frac{\omega_{BD}(\phi)}{2\phi}(\tilde{\nabla}\phi)^2 - \phi^2 V(\phi)\right] + \mathcal{S}_m\left[\tilde{g}_{\mu\nu}, \Psi_m\right] .$$

$$\phi = A^{-2}(\varphi)$$

-Jordon frame

$$2\omega_{BD} + 3 = \frac{1}{2\alpha^2};\ \ \alpha \equiv M_p\frac{d\ln A(\varphi)}{d\varphi}$$

$$G_{eff} = G_0 A^2\left(1 + 2\alpha^2\right)$$

$$f(R) : \omega_{BD} = 0 \rightarrow \alpha = \frac{1}{\sqrt{6}}$$

Local gravity constraints: Screening Mechanism

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[\varphi R-U(\varphi)\Big]+\int d^4x\sqrt{-g}L_m(g_{\mu,\nu})$$

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[\varphi R-\frac{w_{BD}}{2\varphi}(\Delta\varphi)^2-U(\varphi)\Big]+\int d^4x\sqrt{-g}L_m(g_{\mu,\nu})$$

$$f(R) \quad \Rightarrow \quad w_{BD}=0$$

$$\varphi \; -light \;\; (m_\varphi << m_{AU} > \quad m_\varphi r_{AU} << 1) \Rightarrow \quad \gamma=\frac{1+w_{BD}}{2+w_{BD}}$$

$$Observation: \quad |\gamma-1|\simeq 10^{-5}$$

$$\gamma=\frac{1-e^{-mr}/(2w_{BD}+3)}{1+e^{-mr}/(2w_{BD}+3)}; \qquad m_\varphi >> m_{AU}, \quad \Rightarrow \gamma \to 1$$

$$\varphi \; - \;\; Quintessence \Rightarrow m_\varphi \sim H_0 \;\; \Rightarrow$$ **Chameleon Field**

VIABLE f(R) MODELS

Stability conditions: $G_{eff} = \frac{G}{f'}\ , \quad G'_{eff} = -\frac{f''G}{f'^2}\ ,$

$$R^{(0)}, \rho^{(0} : \quad M^2 \simeq f_{,R}/(3f_{,RR})|_{R=R^{(0)}}$$

Stable de Sitter: $Rf_{,R} = 2f \quad : \ 0 < Rf_{,RR}/f_{,R} < 1$

Local gravity constraints: $f(R) \rightarrow R - 2\Lambda \ \ for \quad R >> R_c$

$$\Delta = \alpha R_c \left(\left[1 + \frac{R^2}{R_c^2} \right]^{-n} - 1 \right); \ \ \alpha, R_c, n > 0$$

Problems of f(R): scalaron mass

$$\mathrm{m}_\phi \sim \sqrt{R_c}\left(\frac{R}{R_c}\right)^{n+1} \quad (n \gtrsim 1)$$

$$\frac{\sqrt{R_c}}{M_p} \sim \frac{H_0}{M_p} = \frac{2.13 \times 0.74 \times 10^{-42} GeV}{1.22 \times 10^{19} GeV} = 1.292 \times 10^{-61},$$

$$\rho/\rho_c \simeq 10^{43}$$

Thongkool, MS, R. Gannouji, S.Jhingan, Phys.Rev.D80:043523,2009 ;

CURING THE SINGULARITY ?

Countering Term: $\dfrac{\mu}{R_c} R^2$

$$m_\phi^2 \simeq \frac{1}{3}\left(\Delta_{,RR}\right) = \frac{R_c}{6\mu}$$

Thongkool, MS, S. Rai Choudhury, arXiv:0908.1663

Thongkool, MS, R. Gannouji, S. Jhingan, Phys.Rev.D80:043523,2009 ;
S A. Appleby,**R. A. Battye, A. Starobinsky, arXiv:0909.1737**

SELF ACCELERATION: SCOPE OF CHAMELEON THEORIES

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] + \mathcal{S}_m\left[A^2(\phi)g_{\mu\nu}, \Psi_m\right]$$

$$a^J(t^J) = A(\phi)a^E(t^E)\,, \quad dt^J = A(\phi)dt^E; \qquad dt = a(t)d\eta$$

$$\ddot{a}^J a^J - \ddot{a}^E a^E = \left(\frac{A'}{A}\right)'; \qquad \frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm Pl}^2}\left((\rho_\phi + 3P_\phi) + \beta_s\rho A(\phi)\right)$$

Self acceleration: $\ddot{a}^E a^E < 0;\ \ \ddot{a}^J a^J > 0 \rightarrow \frac{\Delta A}{A} \geq 1;\ \ \Delta A = \left(\frac{1}{H^J}\frac{dA}{dt^J}\right)$

Screening: $\Delta A << 1, \qquad z = 1 \rightarrow z = 0$

Chameleon/Symmetron theories-

No scope for self acceleration

arXiv:1284612: J. Wang, L. Hui and J. Khoury

arXiv: 1211.2289: K. Bamba, R. Gannouji, **MS**

Massive gravity: possible extensions

$h_{\mu\nu}$ **: (helicity 2 object) + (helicity 1 object)+ (helicity zero object)**

Universally couples to all fields

$$L = L_0[(\partial h)^2] - \frac{1}{2} m_g^2 \left(h^{\mu\nu} h_{\mu\nu} - h^2 \right)$$

a

b

$$(a-b)(\Box\phi)^2$$

Ghost: $m_{ghost} \sim \frac{m_g^2}{a-b}$

Stuckelberging

Massive gravity: possible extensions

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma) - \mathcal{L}_{mass}\Big]$$

$$+\int d^4x\sqrt{-\tilde{g}}\mathcal{L}_m(A^2 g_{\mu\nu})$$

$$\mathcal{L}_m = \frac{m_g^2 M_P^2}{4}A^{-4}\left(-4\mathcal{U}_2(\mathcal{K}) + \alpha_3\mathcal{U}_3(\mathcal{K}) + \alpha_4\mathcal{U}_4(\mathcal{K})\right)$$

$$\tilde{g}_{\mu\nu} = A^2(\sigma)g_{\mu\nu} \quad A^2(\sigma) = e^{-2\beta\sigma/M_P}$$

$$\mathcal{U}_2 = -\frac{1}{2!}\epsilon^{\mu\nu..}\epsilon_{\alpha\beta..}\mathcal{K}^\mu_\alpha\mathcal{K}^\nu_\beta$$

$$\mathcal{U}_3 = -\frac{1}{1!}\epsilon^{\mu\alpha\gamma.}\epsilon_{\nu\beta\delta.}\mathcal{K}^\mu_\nu\mathcal{K}^\alpha_\beta\mathcal{K}^\gamma_\delta$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$$

$$\mathcal{U}_4 = -\frac{1}{0!}\epsilon_{\mu\alpha\gamma\rho}\epsilon^{\nu\beta\delta\sigma}\mathcal{K}^\mu_n u\mathcal{K}^\alpha_\beta\mathcal{K}^\gamma_\delta\mathcal{K}^\rho_\sigma$$

Massive gravity: possible extensions

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} \tilde{R}\Psi - \frac{M_P^2}{2} \frac{\omega(\Psi)}{\Psi} \tilde{g}^{\mu\nu} \left(\partial_\mu \Psi \partial_\nu \Psi \right) - \Psi^2 V(\Psi) - \mathcal{L}_{mass} \right]$$

$$\mathcal{L}_{mass} = \frac{m_g^2 M_P^2}{4} \left(-4\mathcal{U}_2(\tilde{\mathcal{K}}) + \alpha_3 \mathcal{U}_3(\tilde{\mathcal{K}}) + \alpha_4 \mathcal{U}_4(\tilde{\mathcal{K}}) \right)$$

$$\tilde{\mathcal{K}}^\mu_\nu = \delta^\mu_\nu - A(\sigma) \sqrt{g^{\mu\alpha} \partial_\alpha \phi^a \partial_\nu \phi^b \eta_{ab}} \; ; \Psi \equiv A^{-2}$$

$$\omega = \frac{1 - 6\beta^2}{M_P^2}$$

Massive gravity: possible extensions

$$\mathcal{L}_{dc} = \frac{1}{2}h_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}h_{\mu\nu} - \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + 3\pi\Box\pi - \frac{D}{\Lambda_3^3}\eta^{\mu\nu}\pi X^{(2)}_{\mu\nu} + \frac{D^2}{\Lambda_3^6}\partial_\mu\pi\partial_\nu\pi X^{(2)}_{\mu\nu}$$

$$-\beta\sigma\left[-4\frac{\mathcal{U}_2(\Pi)}{\Lambda_3^3} - 4\alpha_4\frac{\mathcal{U}_3(\Pi)}{\Lambda_6^3} + \alpha_4\frac{\mathcal{U}_4(\Pi)}{\Lambda_3^9}\right] + \frac{1}{M_P}\pi T + \frac{1}{M_P}\frac{D}{\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}$$

$$h_{\mu\nu} \to h_{\mu\nu} + \pi\eta_{\mu\nu} + \frac{D}{\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi; \;\; D = -(3\alpha_4 + 1); \quad \alpha_3 = -4\alpha_4$$

$$\lambda_\pi \simeq -\left(\frac{r_v}{r}\right)^{3/5}\left(\frac{1}{192\beta^2\alpha_4^2}\right)^{1/5} \to \frac{F_\pi}{F_N} \propto \left(\frac{r}{r_v}\right)^{12/5}, \;\; r << r_v$$

$$\lambda_\pi = \pi'/\Lambda_3^3 r, \lambda_\sigma = \sigma'/\Lambda_3^3 r \quad r_v = \left(\frac{M}{M_P^2 m^2}\right)^{1/3}$$

Massive gravity: possible extensions

$$\mathcal{S} = \mathcal{S}_{EH} + \mathcal{S}_{mass} + \mathcal{S}_{\sigma} + \mathcal{S}_{m}$$

$$\mathcal{S}_{EH} = \int d^4x\sqrt{-g}F_1(\sigma)R$$

$$\mathcal{S}_{mass} = \int d^4x\sqrt{-g}F_2(\sigma)\Big[-4U_2 + \alpha_3 U_3 + \alpha_4 U 4\Big]$$

$$\mathcal{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - F_3(\sigma)\sqrt{g^{\mu\alpha}\partial_{\alpha}\phi^a\partial_{\mu}\phi^b\eta_{ab}}$$

Massive gravity: possible extensions

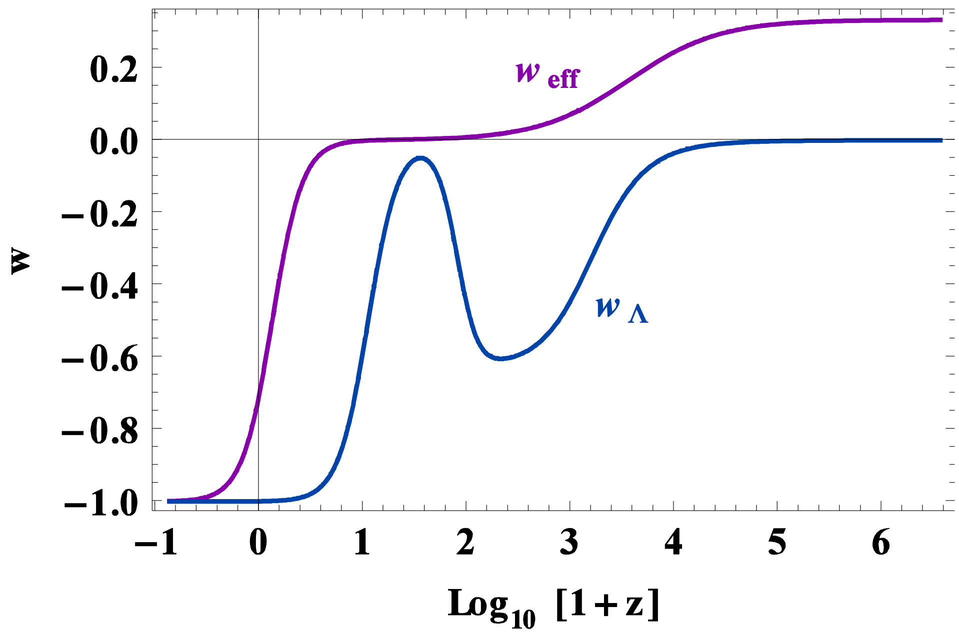
$$ds^2 = -N^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

$$\phi^0 = f(t), \quad \phi^i = x^i$$

$$\mathcal{S}_{EH} = \int dt \Big[-6 \frac{a\dot{a}}{N} \frac{d}{dt}(aF_1) \Big]$$

$$\mathcal{S}_{mass} = 6 \int dt \Big[N G_1(a,\sigma) - \dot{f} G_2(a,\sigma) \Big]$$

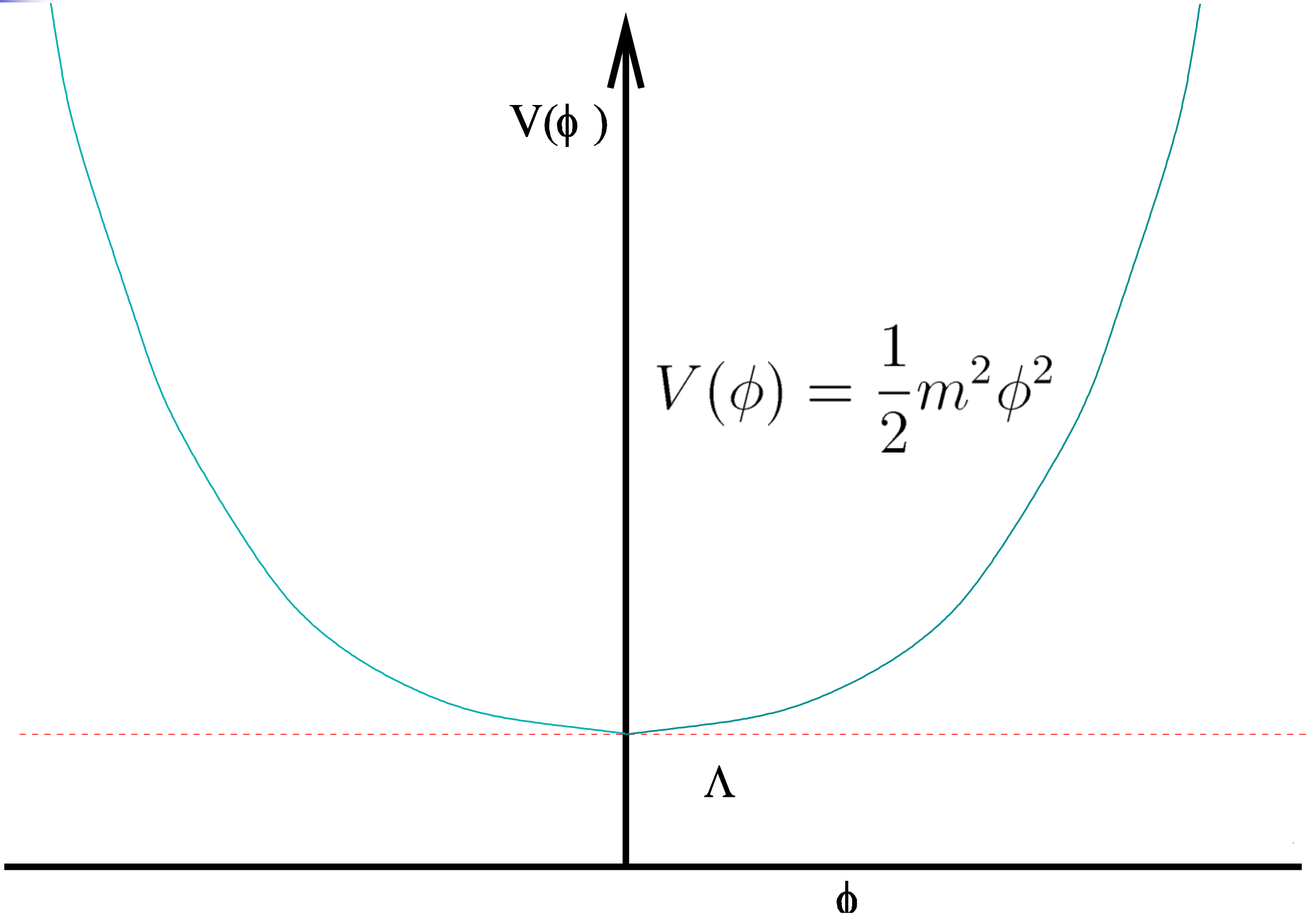
$$H^2 = \frac{\rho_m - 6G_1/a^3 + V}{6F_1 + 6aF_1' - a^2\sigma'^2/2}; \qquad G_2(a,\sigma) = C$$



w eff
w Λ
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
w
−1
0
1
2
3
4
5
6
Log10 [1 + z]

BEST MODEL OF INFLATION AND DARK ENERGY?

BEST MODEL OF INFLATION AND DARK ENERGY?



NATURE OF DARK ENERGY

IT COULD BE ANYTHING

OR

IT COULD BE NOTHING