The Charge Radius of the Proton

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1) Richard. J. Hill, GP	PRD	**82**	113005 (2010)	arXiv:1008.4619
2) Richard. J. Hill, GP	PRL	**107**	160402 (2011)	arXiv:1103.4617
3) Bhubanjyoti Bhattacharya, Richard J. Hill, GP	PRD	**84**	073006 (2011)	arXiv:1108.0423
4) Richard J. Hill, Gabriel Lee, GP, Mikhail P. Solon	PRD		(to appear)	arXiv:1212.4508

Outline

- Introduction: a 5σ discrepancy
- Model independent extraction of the proton charge radius from electron scattering
- Interlude: The axial mass of the nucleon, another discrepancy?
- Model independent analysis of proton structure for hydrogenic bound states
- Conclusions and outlook

Introduction: 5σ discrepancy

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)| \sum_q e_q \, \bar{q}\gamma^\mu q |N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu\right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\langle r^2 \rangle_E^p = 6 \frac{dG_E^p}{dq^2}\bigg|_{q^2=0}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p}$

Charge radius from atomic physics

$$\langle p(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|p(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^p(q^2)q^\nu\right]u(p_i)$$

- For a point particle amplitude for $p+\ell \to p+\ell$

$$\mathcal{M} \propto \frac{1}{q^2} \quad \Rightarrow \quad U(r) = -\frac{Z\alpha}{r}$$

- Including q^2 corrections from proton structure

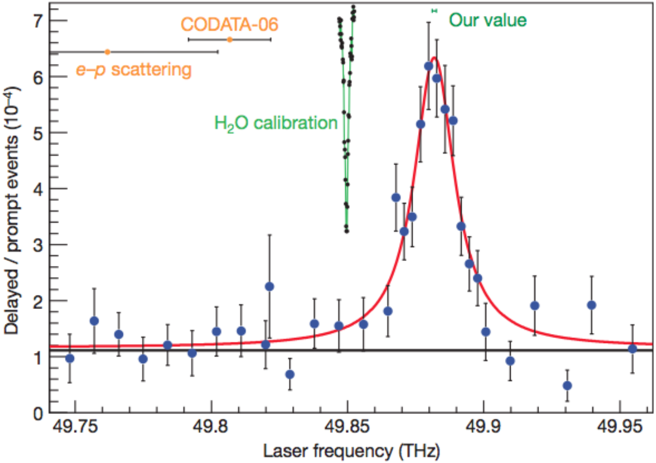
$$\mathcal{M} \propto \frac{1}{q^2}q^2 = 1 \quad \Rightarrow \quad U(r) = \frac{4\pi Z\alpha}{6}\delta^3(r)(r_E^p)^2$$

- Proton structure corrections $\left(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell\right)$

$$\Delta E_{r_E^p} \;=\; \frac{2(Z\alpha)^4}{3n^3}m_r^3(r_E^p)^2\delta_{\ell 0}$$

- **Muonic hydrogen can give the best measurement of $\mathbf{r_E^p}$!**

Charge radius from Muonic Hydrogen



- CREMA Collaboration measured for the **first time** $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ transition in Muonic Hydrogen [Pohl et al. Nature **466**, 213 (2010)]

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**
- We can also extract it from electron-proton scattering data
  What does the PDG say?

What does the PDG say?

K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010)

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	DOCUMENT ID		TECN	COMMENT
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$e p \to e p$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$e p \to e p$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR...	00		$e p$ + Coul. corrections
0.847 ±0.008	MERGELL	96		$e p$ + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz $e p$ data
0.865 ±0.020	MCCORD	91		$e p \to e p$
0.862 ±0.012	SIMON	80		$e p \to e p$
0.880 ±0.030	BORKOWSKI	74		$e p \to e p$
0.810 ±0.020	AKIMOV	72		$e p \to e p$
0.800 ±0.025	FREREJACQ...	66		$e p \to e p$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$e p \to e p$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

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- What does PDG say?
  - $\approx$ 50 years of $e - p$ scattering data
  - r_E^p between $0.8 - 0.9$ fm
  - Different data sets
  - Different extraction methods

"We do not use the following data for averages, fits, limits, etc."

- PDG refuses to say anything...
- What does the Data say?

Model independent extraction of the proton charge radius from electron scattering

Richard J. Hill, GP

PRD **82** 113005 (2010) [arXiv:1008.4619]

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  Is this a problem? why not fit a straight line?

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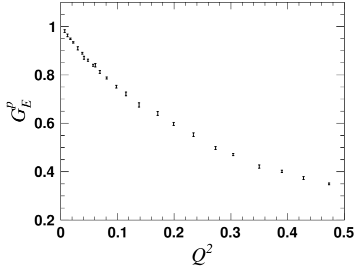

Data from [Arrington et al. PRC **76**, 035205 (2007)]

- We don't know the functional form of G_E^p

How to extract r_E^p?

- How to extract r_E^p from G_E^p? Usually use either

  1) model dependent form for G_E^p, e.g. poles+continuum form

     problem: how to estimate model dependence?

  2) A series expansion

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- There are several possibilities of series expansion

  1) Taylor series

$$G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \dots,$$

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  $$G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \dots,$$
  2) Continued fraction [Sick PLB **576**, 62 (2003)]
  $$G_E^p(q^2) = \frac{1}{1 + \frac{a_1\, q^2}{1 + \frac{a_2\, q^2}{1 + \dots}}}$$

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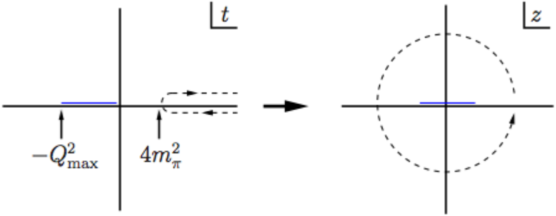
  3) z expansion

z expansion

- Analytic properties of $G_E^p(t)$ are known
  $G_E^p(t)$ is analytic outside a cut $t \in [4m_\pi^2, \infty]$
  $e - p$ scattering data is in $t < 0$ region
- We can map the domain of analyticity onto the unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



- Expand G_E^p in a Taylor series in z: $G_E^p(q^2) = \sum_{k=0}^{\infty} a_k\, z(q^2)^k$

z expansion

- Standard tool in analyzing **meson** transition form factors
  - Bourrely et al. NPB **189**, 157 (1981)
  - Boyd et al. arXiv:hep-ph/9412324
  - Boyd et al. arXiv:hep-ph/9508211
  - Lellouch arXiv:hep-ph/9509358
  - Caprini et al. arXiv:hep-ph/9712417
  - Arnesen et al. arXiv:hep-ph/0504209
  - Becher et al. arXiv:hep-ph/0509090
  - Hill arXiv:hep-ph/0607108
  - Bourrely et al. arXiv:0807.2722 [hep-ph]
  - Bharucha et al. arXiv:1004.3249 [hep-ph]
  - ...
- Not applied to **nucleon** form factors before

Comparison of series expansions

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- Use data sets tabulated by Rosenfelder [arXiv:nucl-th/9912031] with $Q^2 < 0.04\,\mathrm{GeV}^2$, fit the following ($t_{\rm cut} = 4m_\pi^2$)

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$$G_E^p(q^2) = 1 + a_1 \frac{q^2}{t_{\rm cut}} + a_2 \left(\frac{q^2}{t_{\rm cut}}\right)^2 + \dots$$

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$$G_E^p(q^2) = 1 + a_1 z(q^2) + a_2 z^2(q^2) + \ldots$$

4) z expansion with a constraint on a_k: $|a_k| \leq 10$

Comparison of series expansions

r_E^p in $10^{-18}m$

polynomial

continued fraction

z expansion (no bound)

z expansion ($|a_k| \leq 10$)

Comparison of series expansions

r_E^p in $10^{-18}m$

	$k_{\rm max} = 1$
polynomial	836^{+8}_{-9}
continued fraction	882^{+10}_{-10}
z expansion (no bound)	918^{+9}_{-9}
z expansion ($\lvert a_k \rvert \le 10$)	918^{+9}_{-9}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\rm max} = 1$	2
polynomial	836^{+8}_{-9}	867^{+23}_{-24}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}
z expansion ($\lvert a_k \rvert \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}

Comparison of series expansions

r_E^p in $10^{-18}m$

	$k_{\rm max} = 1$	2	3
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}
z expansion ($\lvert a_k\rvert \le 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}

Comparison of series expansions

r_E^p in $10^{-18}m$

	$k_{\max} = 1$	2	3	4
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}	959^{+85}_{-93}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	—	—
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}	1022^{+102}_{-114}
z expansion ($\|a_k\| \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}	880^{+39}_{-61}

Comparison of series expansions

r_E^p in $10^{-18}m$

	$k_{\max} = 1$	2	3	4	5
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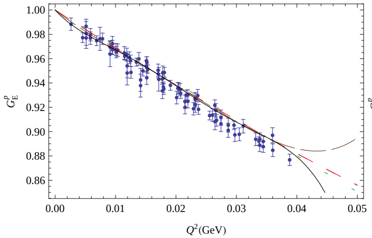
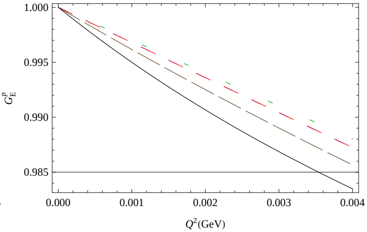
Conclusions:

- Fit with two parameters agree well
- As we increase $k_{\rm max}$ the errors for the first three fits grow
- For the continued fraction fit for $k_{\rm max} > 3$ the slope is not positive
- To get a meaningful answer we must constrain a_k. How?

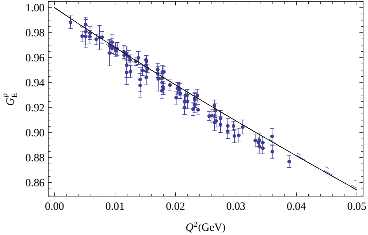
Comparison of Taylor and constrained z fits

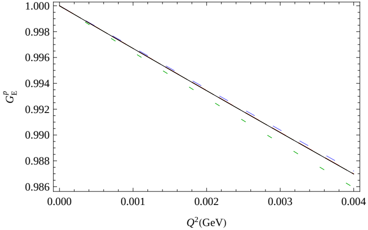
- Taylor fit





- Constrained z fit



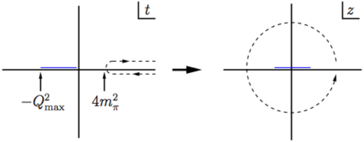


See also:

"Constrained curve fitting" : Lepage et al. Nucl.Phys.Proc.Suppl. 106 (2002) 12-20

Analytic structure and a_k

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$



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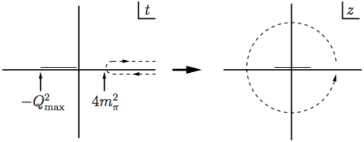

- Analytic structure implies:
  Information about $\mathrm{Im}\, G_E^p(t + i0) \Rightarrow$ information about a_k
- $G(t) = \sum_{k=0}^{\infty} a_k\, z(t)^k$, z^k are orthogonal over $|z| = 1$

$$a_0 = G(t_0)$$
$$a_k = \frac{2}{\pi} \int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0} \sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}}\, \mathrm{Im}\, G(t) \sin[k\theta(t)]\,, \quad k \geq 1$$
$$\sum_k a_k^2 = \frac{1}{\pi} \int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0} \sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}} |G|^2$$

- How to constrain $\mathrm{Im}\, G(t)$?

Size of a_k: Summary

- We study the size of a_k using
  - vector dominance ansatz
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- We extract r_E^p using
  - Low Q^2 proton data
  - Low + High Q^2 proton data
  - proton and neutron data
  - proton, neutron and $\pi\,\pi$ data

Results

- Using proton low: $Q^2 < 0.04\,\mathrm{GeV}^2$ scattering data from Rosenfelder [arXiv:nucl-th/9912031], we find

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

- Rosenfelder gets

$$r_E^p = 0.880 \pm 0.015\,\mathrm{fm}$$

  from the same data!

- Conclusion: not using model independent approach underestimates the error by a factor of two!

Results

- Proton low: $Q^2 < 0.04\,\mathrm{GeV}^2$

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

- Proton high: $Q^2 < 0.5\,\mathrm{GeV}^2$

$$r_E^p = 0.870 \pm 0.023 \pm 0.012\,\mathrm{fm}$$

- Proton and neutron data

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007\,\mathrm{fm}$$

- Proton, neutron and $\pi\,\pi$ data

$$r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$$

Nuclear form factors: Future directions

- Use recent high precision data from
- A1 experiment at Mainz [PRL **105**, 242001 (2010)]
- JLAB [PLB **705**, 59-64 (2011)]

  to improve precision on r_E^p

- Model independent extraction of r_M^p

  [Epstein, GP, Roy in progress]

  PDG 2012:

  $r_M^p = 0.777 \pm 0.16$ [Bernauer et al. 2010]
  $r_M^p = 0.876 \pm 0.19$ [Borisyuk et al. 2010]
  $r_M^p = 0.854 \pm 0.05$ [Belushkin et al. 2007]

- Model independent extraction of r_E^n, r_M^n

Model independent determination of the axial mass parameter in quasielastic neutrino-nucleon scattering

Bhubanjyoti Bhattacharya, Richard J. Hill, GP

PRD **84** 073006 (2011) [arXiv:1108.0423]

The Axial Mass

- The problem is not unique to the vector form factor!
- The axial current gives rise to the the axial form factor

  For axial form factor m_A analogous to r_E^p

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2}q^2 + \ldots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F'_A(0)}}$$

- Charged current quasielastic scattering

$$\nu_\mu + n \to \mu^- + p$$

Another discrepancy?

- Neutrino scattering:

  $m_A^{\rm dipole} = 1.35 \pm 0.17$ GeV

  MiniBooNE Collaboration

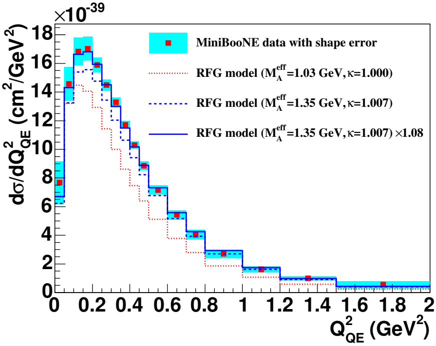
  PRD **81** (2010) 092005

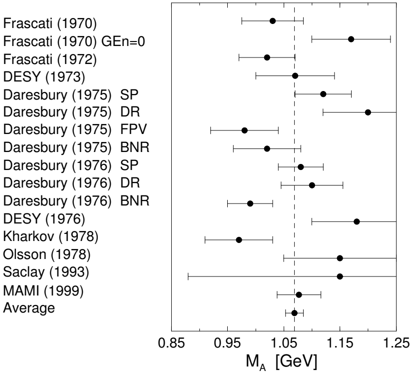
- Pion electro-prodcution:

  $m_A^{\rm dipole} = 1.07 \pm 0.02$ GeV

  Bernard, Elouadrhiri, Meissner

  J. Phys. G 28, R1 (2002)





Both use dipole ansatz for axial form factor

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\rm dipole})^2]^{-2}$$

Another Discrepancy?

- Axial mass $m_A^{\rm dipole} = 1.35 \pm 0.17$ GeV
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
  Similar result from other recent ν experiments
- K2K SciFi: $m_A^{\rm dipole} = 1.20 \pm 0.12$ GeV
  [K2K Collaboration, PRD **74** 052002 (2006)]
- K2K SciBar $m_A^{\rm dipole} = 1.144 \pm 0.077({\rm fit})^{+0.078}_{-0.072}({\rm syst})$ GeV
  Espinal, Sanchez, AIP Conf. Proc. **967**, 117 (2007)
- Minos $m_A^{\rm dipole} = 1.19^{+0.09}_{-0.1}({\rm fit})^{+0.12}_{-0.14}({\rm syst})$ GeV
  [MINOS Collaboration, AIP Conf. Proc. **1189**, 133 (2009)]

- Nomad: $m_A^{\rm dipole} = 1.05 \pm 0.02 \pm 0.06$ GeV
  [NOMAD Collaboration, EPJ C **63**, 355 (2009)]
- Pion electro-prodcution: $m_A^{\rm dipole} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)
- ν experiments before 1990: $m_A^{\rm dipole} = 1.026 \pm 0.021$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)

What could be the source of the discrepancy?

- Theoretical studies focus on nuclear modeling
  For MiniBooNE neutrinos scatter of carbon
  $\Rightarrow$ need behavior of nucleons in nucleus

- MiniBooNE use "Relativistic Fermi Gas" (RFG) model
  [Smith, Moniz, NPB **43**, 605 (1972)]
  Model validity and parameters from quasi-elastic *e*-nuclei scattering
  Moniz, Sick, Whitney, Ficenec, Kephart, Trower, PRL **26**, 445 (1971)

Theoretical studies focus on nuclear modeling

- Modify nuclear model
  [Butkevich, PRC **82**, 055501 (2010); Benhar, Coletti, Meloni, PRL **105**, 132301 (2010); Juszczak, Sobczyk, Zmuda, PRC **82**, 045502 (2010)]

- Include multi-nucleon emission
  [Martini, Ericson, Chanfray, Marteau
  PRC **80**, 065501 (2009), PRC **81**, 045502 (2010);
  Amaro, Barbaro, Caballero, Donnelly, Williamson
  PLB **696**, 151 (2011), PRD **84**, 033004 (2011);
  Nieves, Ruiz Simo, Vicente Vacas
  PRC **83**, 045501 (2011), arXiv:1106.5374]

- Modify G_M for bound nucleons but not G_E or F_A
  [Bodek, Budd, EPJ C **71**, 1726 (2011)]

- **All** use dipole form factor

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\rm dipole})^2]^{-2}$$

m_A^{dipole} is not m_A!

- The physical parameter is

$$m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Everyone extracts m_A^{dipole} from

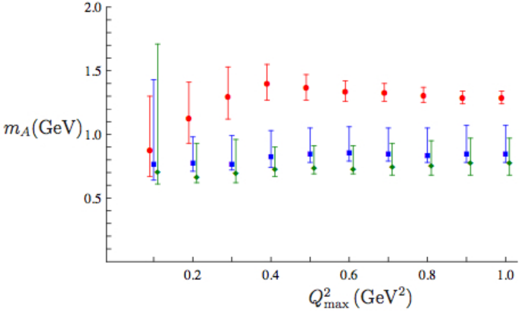
$$F_A = F_A(0)\left[1 - q^2/(m_A^{\text{dipole}})^2\right]^{-2}$$

- When extractions of m_A^{dipole} disagree is it
- A problem of the use of the dipole model?
- Real disagreement between experiments?

- Need to extract m_A in a model independent way!
  [Bhubanjyoti Bhattacharya, Richard J. Hill, GP, PRD **84** 073006 (2011)]

Neutrino: Model independent approach

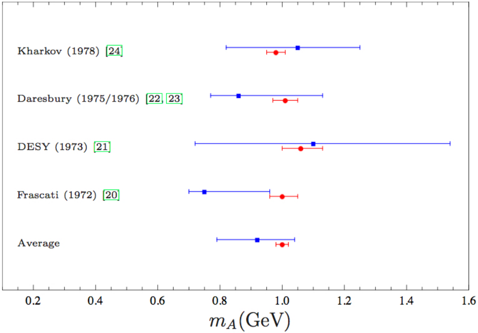
- Our z expansion fit to MiniBooNE data (Assuming RFG):
  Red: dipole, Blue: z, $|a_k| \leq 5$, Green: z, $|a_k| \leq 10$



- Our fit using z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV
  Our fit using dipole model: $m_A^{\text{dipole}} = 1.29 \pm 0.05$ GeV
  MiniBooNE's fit: $m_A^{\text{dipole}} = 1.35 \pm 0.17$ GeV

Pion Electro-production: Model independent approach

- Is there a discrepancy with pion electro-production data?
  Red: dipole, Blue: z, $|a_k| \leq 5$



- Our fit using z expansion: $m_A = 0.92^{+0.12}_{-0.13} \pm 0.08$ GeV
  Our fit using dipole model: $m_A^{\text{dipole}} = 1.00 \pm 0.02$ GeV
  Bernard et. al. fit using dipole model: $m_A^{\text{dipole}} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)

Model independent approach

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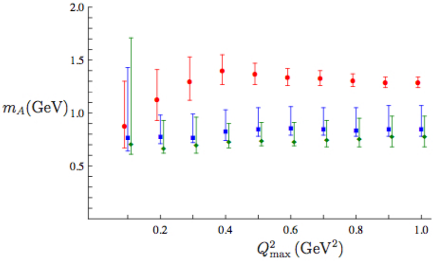

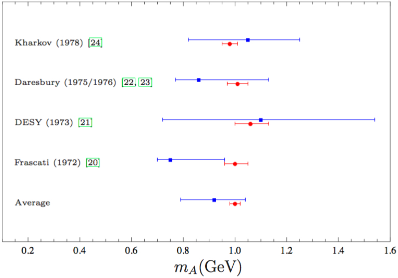

Discrepancy is an artifact of the use of the dipole form factor!

Axial form factor: Future directions

- Extract m_A from other ν experiments, e.g. Minerνa
- Is m_A consistent between experiments?
- m_A from pion electro-production data, extrapolated from soft π limit

  Extract m_A in a model-independent way
- ν experiments need F_A, extract it from another source
- After F_A is under control, discuss nuclear models
- Impact on θ_{13}? [Bhattacharya, Hill, GP in progress]

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- ν experiments need F_A, extract it from another source
- After F_A is under control, discuss nuclear models
- Impact on θ_{13}? [Bhattacharya, Hill, GP in progress]
- But wait, what about the 5σ discrepancy ?

The recent discrepancy

- Based on a model-independent approach
  using scattering data from proton, neutron and $\pi\,\pi$
  [Hill, GP PRD **82** 113005 (2010)]
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- CODATA value (extracted mainly from electronic hydrogen)
  [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm

- Lamb shift in muonic hydrogen
  [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm

- Our results are more consistent with the CODATA value

Lamb shift in muonic hydrogen

- CREMA measured [Pohl et al. Nature **466**, 213 (2010)]

$$\Delta E = 206.2949 \pm 0.0032 \text{ meV}$$

- Comparing to the theoretical expression
  [Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3 \text{ meV}$$

- They got

$$r_E^p = 0.84184(67) \text{ fm}$$

- How reliable is the theoretical prediction?

Model independent analysis of proton structure for hydrogenic bound states

Richard J. Hill, GP

PRL **107** 160402 (2011) [arXiv:1103.4617]

How reliable is the theoretical prediction?

- The theoretical calculation was redone

  Jentschura, Annals Phys. **326**, 500-515 (2011)
  Carlson, Vanderhaeghen PRA **84**, 020102 (2011)

  Confirmed the muonic hydrogen result

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- Inadequate treatment of proton structure effects?
  1) De Rujula, PLB **693**, 555 (2010)
  2) Miller, Thomas, Carroll, Rafelski, PRA **84**, 020101(R) (2011)

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  Ruled out by data:
  1) Electron-proton: Distler, Bernauer, Walcher PLB **696**, 343 (2011)
  2) Compton scattering: Carlson, Vanderhaeghen arXiv:1109.3779

- New Physics?

New Physics?

- New particle that couples to nucleons and μ (but not e or τ)
  [Barger, Chiang, Keung, Marfatia PRL **106** (2011) 153001]
  Assuming same coupling to Υ, η, π rules this out

- New MeV particle that couples to protons (g_p) and muons (g_μ)
  [Tucker-Smith, Yavin PRD **83** (2011) 101702]
  Can explain r_E^p and muon $g-2$ but $g_p \approx g_n$ is problematic

- New $U(1)$ that couples only to right-handed muons
  [Batell, McKeen, Pospelov PRL **107** (2011) 011803]
  Constrained by missing mass in $K \to \mu\nu$ decays
  [Barger, Chiang, Keung, Marfatia, PRL **108** (2012) 081802]

The Theoretical Prediction

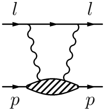
- Is there a problem with the theoretical prediction?

[Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = \quad 209.9779(49) \quad - \quad 5.2262(r_E^p)^2 \quad + \quad 0.0347(r_E^p)^3 \text{ meV}$$

↑	↑	↑
mostly μ QED	already discussed	where does this term come from?

Two-photon amplitude: “standard” calculation



- “Standard” calculation: separate to proton and non-proton
- For proton
  - Insert form factors into vertices: $\mathcal{M} = \int_0^\infty dq^2\, f(G_E, G_M)$
  - Using a “dipole form factor”: $G_i(q^2)/G_i(0) \approx [1 - q^2/\Lambda^2]^{-2}$
  - $\mathcal{M}$ is a function of $\Lambda \Rightarrow (r_E^p)^3$ term
  - $\Lambda^2 = 0.71\,\mathrm{GeV}^2 \Rightarrow \Delta E \approx 0.018$ meV [Pachucki, PRA **53**, 2092 (1996)]
- Need $0.258(90)\,\mathrm{meV}$ (scattering) or $0.311(63)\,\mathrm{meV}$ (spec.) to explain discrepancy
- Look more carefully at the calculation [Richard J. Hill, GP PRL **107** 160402 (2011)]

NRQED

- Model Independent approach: use NRQED
  [Caswell, Lepage PLB **167**, 437 (1986); Kinoshita Nio PRD **53**, 4909 (1996); Manohar PRD **56**, 230 (1997)]

$$\begin{aligned}\mathcal{L}_p \;=\; & \psi_p^\dagger \Big\{ iD_t + \frac{\mathbf{D}^2}{2m_p} + \frac{\mathbf{D}^4}{8m_p^3} + c_F e \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_p} + c_D e \frac{[\boldsymbol{\partial}\cdot\mathbf{E}]}{8m_p^2} \\ & + i c_S e \frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_p^2} + c_{W1} e \frac{\{\mathbf{D}^2, \boldsymbol{\sigma}\cdot\mathbf{B}\}}{8m_p^3} \\ & - c_{W2} e \frac{D^i \boldsymbol{\sigma}\cdot\mathbf{B} D^i}{4m_p^3} + c_{p'p} e \frac{\boldsymbol{\sigma}\cdot\mathbf{D}\mathbf{B}\cdot\mathbf{D} + \mathbf{D}\cdot\mathbf{B}\boldsymbol{\sigma}\cdot\mathbf{D}}{8m_p^3} \\ & + i c_M e \frac{\{\mathbf{D}^i, [\boldsymbol{\partial}\times\mathbf{B}]^i\}}{8m_p^3} + c_{A1} e^2 \frac{\mathbf{B}^2 - \mathbf{E}^2}{8m_p^3} - c_{A2} e^2 \frac{\mathbf{E}^2}{16m_p^3} + \ldots \Big\} \psi_p\end{aligned}$$

- Need also

$$\mathcal{L}_{\text{contact}} = d_1 \frac{\psi_p^\dagger \boldsymbol{\sigma} \psi_p \cdot \psi_l^\dagger \boldsymbol{\sigma} \psi_l}{m_l m_p} + d_2 \frac{\psi_p^\dagger \psi_p \psi_l^\dagger \psi_l}{m_l m_p}$$

NRQED

- From c_i and d_i determine proton structure correction, e.g.

$$\delta E(n,\ell) = \delta_{\ell 0}\frac{m_r^3(Z\alpha)^3}{\pi n^3}\left(\frac{Z\alpha\pi}{2m_p^2}c_D^{\mathrm{proton}} - \frac{d_2}{m_l m_p}\right)$$

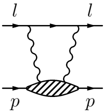
- Matching

- Operators with one photon coupling:
  c_i given by $F_i^{(n)}(0)$

- Operators with only two photon couplings:
  c_{A_i} given by forward and backward Compton scattering

- d_i from two-photon amplitude

Two-photon amplitude: matching



$$\frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \mathbf{k},s|\,T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\mathbf{k},s\rangle$$

$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

- Matching

$$\frac{4\pi m_r}{\lambda^3}-\frac{\pi m_r}{2m_l m_p\lambda}-\frac{2\pi m_r}{m_p^2\lambda}\left[F_2(0)+4m_p^2F_1'(0)\right]$$

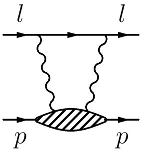
$$-\frac{2}{m_l m_p}\left[\frac{2}{3}+\frac{1}{m_p^2-m_l^2}\left(m_l^2\log\frac{m_p}{\lambda}-m_p^2\log\frac{m_l}{\lambda}\right)\right]+\frac{d_2(Z\alpha)^{-2}}{m_l m_p}$$

$$=-\frac{m_l}{m_p}\int_{-1}^{1}dx\sqrt{1-x^2}\int_0^\infty dQ\,\frac{Q^3}{(Q^2+\lambda^2)^2(Q^2+4m_l^2x^2)}$$

$$\times\left[(1+2x^2)W_1(2im_pQx,Q^2)-(1-x^2)m_p^2W_2(2im_pQx,Q^2)\right]$$

d_2

- In order to determine d_2 need to know W_i

- Im  $\sim$ Im W_i

  can be extracted from on-shell quantities:

  Proton form factors and Inelastic structure functions

- To find W_i from Im W_i, need dispersion relations

Dispersion relation

- Dispersion relations ($\nu = 2k\cdot q,\ Q^2 = -q^2$)

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{{\rm Im}\, W_1(\nu', Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{{\rm Im}\, W_2(\nu', Q^2)}{\nu'^2-\nu^2}$$

- W_1 requires subtraction...

- ${\rm Im}\, W_i^p$ from form factors
- ${\rm Im}\, W_i^c$ from DIS
- What about $W_1(0, Q^2)$?

$W_1(0, Q^2)$

- Can calculate in two limits: [Hill, GP, PRL **107** 160402 (2011)]

- $Q^2 \ll m_p^2$
  The photon sees the proton "almost" like an elementary particle
  Use NRQED to calculate $W_1(0, Q^2)$ upto $\mathcal{O}(Q^2)$ (including)

$$W_1(0, Q^2) = 2(c_F^2 - 1) + 2\frac{Q^2}{4m_p^2}\left(c_{A_1} + c_F^2 - 2c_F c_{W1} + 2c_M\right)$$

- $Q^2 \gg m_p^2$
  The photon sees the quarks inside the proton
  Use OPE to find $W_1(0, Q^2) \sim 1/Q^2$ for large Q^2

- In between you will have to model!
  Current calculation **pretends** there is no model dependence
  How big is the model dependence?

Bound State Energy

1) Proton: $\mathrm{Im}\, W_i^p$ using dipole form factor

$$\Delta E = -0.016 \text{ meV}$$

2) Continuum: $\mathrm{Im}\, W_i^c$ [Carlson, Vanderhaeghen PRA **84** 020102 (2011)]

$$\Delta E = 0.0127(5) \text{ meV}$$

3) What about $W_1(0, Q^2)$?

"Sticking In Form Factors" (SIFF) model

$$W_1^{\text{SIFF}}(0, Q^2) = 2F_2(2F_1 + F_2) \quad F_i \equiv F_i(Q^2)$$

SIFF

- "Sticking In Form Factors" (SIFF) model

$$W_1^{\rm SIFF}(0,Q^2) = 2F_2(2F_1+F_2) \quad F_i \equiv F_i(Q^2)$$

Notice that for large Q^2, $W_1^{\rm SIFF}(0,Q^2) \propto 1/Q^8$

In contradiction to OPE

- There is **no** local Lagrangian that has a Feynman rule

$$\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2(q^2)q_\nu$$

- Numerically using the dipole form factor

$$\Delta E^{\rm SIFF} = 0.034 \text{ meV}$$

Model Dependence

- How big is the model dependence?

$$0.018\,\text{meV} = \underset{\substack{\uparrow \\ \text{Model independent}}}{-0.016\,\text{meV}} + \underset{\substack{\uparrow \\ \text{Model dependent}}}{0.034\,\text{meV}}$$

- The model dependent piece is the dominant one!

- Experimental discrepancy $\sim$ 0.3 meV

- It is possible that the true $W_1(0, Q^2)$ explains (or reduces) the discrepancy

- Can we extract d_2 in a different way?
  The MUon proton Scattering Experiment (MUSE) at PSI
  How to extract d_2? [Gonderinger, GP in progress]

Two photon amplitude: summary

- To determine two photon amplitude need
- $\mathrm{Im}\, W_i$ which can be extracted from data
- $W_1(0, Q^2)$ which currently cannot be extracted from data

- Unlike $\mathrm{Im}\, W_i$, $W_1(0, Q^2)$ **cannot** be written model independently as a sum of "proton" and "non-proton" terms

- Model independent properties of $W_1(0, Q^2)$:
- Low Q^2 via NRQED
- High Q^2 via OPE

  Intermediate region poorly constrained

- Lack of theoretical control over $W_1(0, Q^2)$ introduces theoretical uncertainties not taken into account in the literature

Beyond the 5 σ discrepancy:

- Established NRQED as a tool to analyze nucleon structure effects
- Systematically separate perturbative/non perturbative
- Easy derivation of low energy theorems of Compton scattering

- [Low, Phys. Rev. **96**, 1428 (1954);
  Gell-Mann, Goldberger, Phys. Rev. **96**, 1433 (1954)]
  Carefully expand Green's functions

- NRQED: few tree level diagrams
  [Hill, GP in preparation]

- Bound state energy in the small lepton mass limit
  [Hill, Lee, GP, Solon arXiv:1212.4508, to appear in PRD]

Conclusions and Outlook

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen
- We presented model independent extraction of the charge radius from $e - p$ scattering data using the z expansion
- using scattering data from proton, neutron and $\pi\pi$
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$
- Previous extractions have underestimated the error
  Similar problem for the axial form factor
- Results are compatible with CODATA value of $r_E^p = 0.8768(69)$ fm

Conclusions

- Analyzed proton structure effects in hydrogen-like systems using NRQED
- Isolated model-**dependent** assumptions in previous analyses: $W_1(0, Q^2)$ was calculated by "Sticking In Form Factors" model
- Model **independent** calculation of $W_1(0, Q^2)$: low Q^2 via NRQED, high Q^2 via OPE
- Possibility for a significant new effects in the two-photon amplitude
- Beyond the 5 σ discrepancy: **NRQED as a tool to analyze nucleon structure effects**

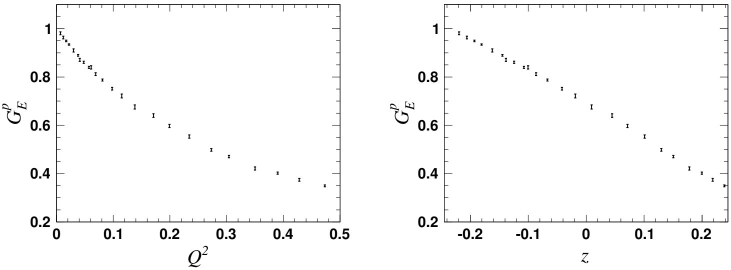
Future Directions

- Applying z expansion to other form-factors
- Analyze spin dependent effects
- Application to deuterium
- Connect muon-proton scattering data to muonic hydrogen
- Resolution of the discrepancy?

Backup

z expansion

- The curvature is smaller in the z variable



Data from [Arrington et al. PRC **76**, 035205 (2007)]

Comparison to the Literature

- Using low $Q^2 < 0.04\,\mathrm{GeV}^2$ data we find

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

- How did [Rosenfelder PLB **479**, 381 (2000) (arXiv:nucl-th/9912031)] get

$$r_E^p = 0.880 \pm 0.015\,\mathrm{fm}$$

from the same data?

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from the same data?

- Rosenfelder used a taylor series

$$G_E^p(q^2) = 1 + a_1\frac{q^2}{t_{\mathrm{cut}}} + a_2\left(\frac{q^2}{t_{\mathrm{cut}}}\right)^2 + \ldots$$

but a_2 was not fitted, instead it was taken from higher Q^2 data [Borkowski et al. NPA **222**, 269 (1974)]: $a_2^{\mathrm{Taylor}}/t_{\mathrm{cut}}^2 = 0.014(4)\,\mathrm{fm}^4$ (similar procedure was used in [Simon et al. NPA **333**, 381 (1980)])

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$$a_2^{\rm Taylor}/t_{\rm cut}^2 = 0.014(4)\,{\rm fm}^4$$

- We find from [Arrington et al. PRC **76**, 035205 (2007)], $Q^2_{\rm max} = 1\,{\rm GeV}^2$

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- Using this value we find

$$r_E^p = 0.878 \pm 0.008^{+0.047}_{-0.039} \quad \textbf{Taylor}$$

errors are from data and $a_2^{\rm Taylor}/t_{\rm cut}^2$ only

- Compatible with

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,{\rm fm} \quad \textbf{z expansion}$$

Comparison to the Literature

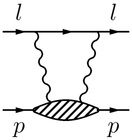
- Using the continued fraction expansion
  Sick and Blunden and Sick have found

  [Sick PLB **576**, 62 (2003)] : $r_E^p = 0.895 \pm 0.010 \pm 0.013\,\mathrm{fm}$
  [Blunden, Sick PRC **72**, 057601 (2005)] $r_E^p = 0.897 \pm 0.018\,\mathrm{fm}$

- Their error estimate relies on model datasets

- We find the expansion becomes unstable
  when including more then 2 parameters

Two-photon amplitude: "standard" calculation



- "standard" calculation: separate to proton and non-proton
- non-proton $\leftrightarrow$ DIS
- For proton
- Insert form factors into vertices

$$\mathcal{M} = \int_0^\infty dq^2\, f(G_E, G_M)$$

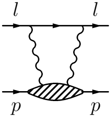
- Using a "dipole form factor"

$$G_i(q^2) \approx G_i(q^2)/G_i(0) \approx [1 - q^2/\Lambda^2]^{-2}$$

- $\mathcal{M}$ is a function of $\Lambda \Rightarrow (r_E^p)^3$ term

- Using, $\Lambda^2 = 0.71\,\mathrm{GeV}^2 \Rightarrow \Delta E \approx 0.018$ meV
  [Pachucki, PRA **53**, 2092 (1996)]

Two-photon amplitude: "standard" calculation



- Why is the insertion of form factors in vertices valid?

- Even if it was, result looks funny
  two-photon amplitude $\Leftrightarrow$ the charge radius
  only for one parameter model for G_E and G_M

- Improvement?
  Treat the "proton" part of two-photon amplitude as a new parameter

- "Zemach" approximation: $m_l, \langle q \rangle \ll m_p$
  [Friar Annals Phys. **122**, 151 (1979),
  Eides et al. Theory of Light Hydrogenic Bound states, Springer]

$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.00913\,\langle r^3 \rangle_{(2)}\ \mathrm{meV}$$

Third Zemach Moment

$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.00913\,\langle r^3\rangle_{(2)}\ \mathrm{meV}$$

- The formula for the Lamb shift has two unknowns
  $\Rightarrow$ use the CODATA value of r_E^p and solve for $\langle r^3\rangle_{(2)}$
- The result [De Rújula PLB **693**, 555 (2010)]

$$[\langle r^3\rangle_{(2)}]^{1/3} = 3.32 \pm 0.21\ \mathrm{fm} \quad \textbf{muonic hydrogen}$$

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- Looks fine until we compare it to $e - p$ scattering data

$$[\langle r^3 \rangle_{(2)}]^{1/3} = 1.39 \pm 0.02\ \mathrm{fm} \quad \textbf{scattering data}$$

[Sick, Friar PRA **72**, 040502(R) (2005)]

Much more than 5σ... there is still a discrepancy

“Standard” Calculation: Summary

- Using
  $r_E^p = 0.871(10)$ fm [Hill, GP PRD **82** 113005 (2010)]
  or
  $r_E^p = 0.8768(69)$ fm [Mohr et al. RMP **80**, 633 (2008)]

- The measured interval in muonic hydrogen lies
  $0.258(90)\,\mathrm{meV}$ or $0.311(63)\,\mathrm{meV}$ above theory.

- Using $\Lambda^2 = 0.71\,\mathrm{GeV}^2$,
  the proton contribution from the two-photon amplitude
  - 0.018 meV to $E(2p) - E(2s)$
  - 0.021 meV to $E(2p) - E(2s)$ in the “Zemach” limit
  [K. Pachucki, PRA **53**, 2092 (1996)]
- Is there a problem with the theoretical prediction?

Zemach?

- "Zemach" approximation: $m_l, \langle q \rangle \ll m_p$
  but for $\Lambda^2 = 0.71\,\mathrm{GeV}^2$
  $\Lambda \approx 0.84$ GeV is not small compared to m_p

- Even worse

- Proton pole term

$$E(2p) - E(2s) = -\,0.016 \text{ meV}$$

- Using the Zemach approximation for proton pole term

$$E(2p) - E(2s) = +\,0.021 \text{ meV}$$

$\Rightarrow$ Thought to be an approximation only because $W_1^{\mathrm{SIFF}}(0, Q^2)$!