Singular points and confinement in SQCD

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QCD and confinement

Strong interactions are described by an $SU(3)$ gauge theory (QCD). Its elementary fields are:

- Gluons: A^a_μ, $a = 1, \ldots, 8$.
- Quarks: q^a_i, $a = 1, 2, 3$, $i = 1, \ldots, N_f$.

This theory is confining: we see only gauge invariant objects.

Mesons: $q^a_i \bar{q}^j_a$; **Baryons:** $\epsilon_{abc} q^a_i q^b_j q^c_k$.

We still don't know the underlying mechanism!

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't Hooft-Polyakov monopoles

We consider an $SO(3)$ gauge theory with a field ϕ in the adjoint representation:

$$W_\mu = W_\mu^a T_a; \quad \phi = \phi^a T_a; \quad V(\phi) = \frac{\lambda}{4}(\phi^2 - b^2)^2.$$

The space of vacua is a sphere of radius b and in each vacuum the gauge group is broken to $U(1)$ (electromagnetism). The corresponding gauge field is $A_\mu = \phi^a_{vac} W_{\mu a}$.

There are static, finite energy solutions of the EoM's such that

$$B_i = \epsilon_{ijk}\partial^j A^k \longrightarrow \frac{\nu}{e}\frac{x_i}{r^3} \quad (\text{for} \;\; r \to \infty) \Longrightarrow g = \frac{4\pi\nu}{e}.$$

ν is the winding number of the map $\phi_{vac} : S^2_{r\to\infty} \to S^2_{vac}$.

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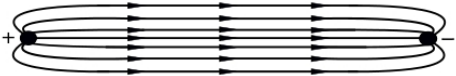
The dual superconductor picture

In a superconductor the condensation of Cooper pairs (electron-electron boundstates) breaks the $U(1)$ gauge symmetry leading to confinement of magnetic charge.



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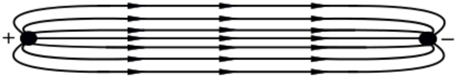

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The $\mathcal{N} = 2$ vectormultiplet describes the fields $(\phi, \psi, \lambda, A_\mu)$.

The lagrangian contains the scalar potential

$$V = -\frac{1}{2g^2} \mathrm{Tr}([\phi, \phi^\dagger]^2).$$

The vacuum solutions are $\phi_{vac} = \mathrm{Diag}(a_1, \ldots, a_n)$ $(\sum_i a_i = 0)$.

The set of vacua in this theory is called **moduli space** and at each point the gauge group is broken to $U(1)^{n-1}$.

These models have 't Hooft-Polyakov magnetic monopoles in their spectrum.

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Effective action for $SU(2)$ SYM

The moduli space is parametrized by $u = \langle \mathrm{Tr}\,\phi^2 \rangle$.

$\mathcal{N} = 2$ susy imposes strong constraints on the effective action:

$$\mathcal{L} = \frac{1}{8\pi} \mathrm{Im} \left(\int d^2\theta \mathcal{F}''(\Phi) W^\alpha W_\alpha + 2 \int d^4\theta \Phi^\dagger \mathcal{F}'(\Phi) \right).$$

The holomorphic function $\mathcal{F}$ is called **prepotential.**

Seiberg-Witten solution: N. Seiberg, E. Witten '94.

$$a = \frac{\sqrt{2}}{\pi} \int_{-1}^{1} \frac{\sqrt{x-u}}{\sqrt{x^2-1}} dx, \quad \frac{\partial \mathcal{F}}{\partial a} = a_D = \frac{\sqrt{2}}{\pi} \int_{1}^{u} \frac{\sqrt{x-u}}{\sqrt{x^2-1}} dx.$$

The prepotential is enceded in a family of elliptic curves:

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Confinement in Seiberg-Witten model

At $u = \pm 1$ monopoles become massless due to strong quantum effects. If I give mass μ to (ϕ, ψ) only these two vacua remain.

The low energy effective action has the superpotential

$$\mathcal{W} = \sqrt{2}\widetilde{M}A_D M + \mu U, \quad U = \langle \mathrm{Tr}\,\Phi^2 \rangle.$$

From the equations of motion we find

$$\langle \widetilde{M}M \rangle = \frac{\mu}{\sqrt{2}} \frac{\partial U}{\partial A_D} \neq 0.$$

The monopole condensate breaks $U(1)$, giving mass gap and confinement via the 't Hooft-Mandelstam mechanism.

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Outlook of Part 2

- The SW solution alone is not always enough to understand the dynamics (e.g. $SO(N)$, $USp(2N)$ $\mathcal{N}=2$ SQCD).
- This problem can be approached using the recent developments in $\mathcal{N}=2$ theories (Argyres-Seiberg duality, 6d constructions...).
- I will explain how one can understand confinement and chiral symmetry breaking (for particular choices of N_f), finding an "unusual" realization of 't Hooft-mandelstam mechanism.

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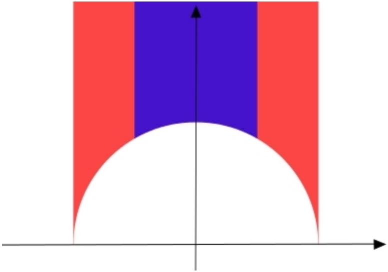
PART 2

Based on: SG and K. Konishi, arXiv:1301.0420 [hep-th]; SG, arXiv:1207.4037 [hep-th].

Scale invariance and infinite coupling



- $SU(2)$ SQCD with $N_f = 4$ has $SL(2,\mathbb{Z})$ S-duality:

$$\tau \to \tau + 1;\ \tau \to -\frac{1}{\tau};\quad \tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

- Higher rank scale invariant SQCD has $\Gamma^0(2)$ S-duality:

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New perspective: Argyres-Seiberg duality

Scale invariant $\mathcal{N} = 2$ $SU(N)$ SQCD admits in the infinite coupling limit a dual description involving two sectors (weakly) coupled by a gauge multiplet: P. Argyres, N. Seiberg '07.

- One sector is free and describes two massless hypermultiplets. It has $SU(2)$ flavor symmetry.
- The other sector is a SCFT with (at least) $SU(2) \times SU(N_f)$ flavor symmetry.
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One can analyze this duality realizing the four dimensional theory as the compactification of 6d $\mathcal{N} = (2, 0)$ (A_n or D_n) theory on a surface with punctures. D. Gaiotto '09; Y. Tachikawa '09.

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Softly broken $SU(N_c)$ $\mathcal{N}=2$ SQCD

In the vacua surviving the $\mathcal{N}=1$ perturbation the SW curve

$$y^2 = P_{N_c}^2(x, u_i) - \Lambda^{2N_c - N_f} \prod_i (x + m_i).$$

factorizes as follows: P. Argyres, R. Plesser, N. Seiberg '96.

$$y^2 = (x+m)^{2r}(x-a)(x-b)Q^2(x), \quad r \leq N_f/2.$$

At each r vacuum the effective theory has $U(r) \times U(1)^{N_c - r - 1}$ gauge group and N_f (magnetic) multiplets $q_{\alpha i}$ of $U(r)$.

After the $\mathcal{N}=1$ perturbation, the pattern of flavor symmetry breaking is $U(N_f) \to U(r) \times U(N_f - r)$ $\forall r$.

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$$\langle \tilde{Q}_i Q_j \rangle = \begin{pmatrix} c 1_r & 0 \\ 0 & c' 1_{N_f - r} \end{pmatrix}; \quad \langle q_{\alpha i} \rangle \propto \begin{pmatrix} 1_r & 0 \end{pmatrix}.$$

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For $m = 0$ the global symmetry is enhanced from $U(N_f)$ to $USp(2N_f)$ and $SO(2N_f)$ respectively. All the r vacua merge in this limit, giving an interacting fixed point (**Chebyshev point**).

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After the $\mathcal{N} = 1$ perturbation the flavor group is broken to $U(N_f)$:

$$\langle M_{ij} \rangle \propto 1_{N_f} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Singular points in $USp(2N)$ theory

The SW curve for the $USp(2N)$ theory is

$$xy^2 = [xP_N(x) + 2\Lambda^{2N-N_f+2}\prod_i m_i]^2 - 4\Lambda^{4N-2N_f+4}\prod_i (x - m_i^2).$$

In the limit $m \to 0$ the effective theory at the $r = N_f/2$ vacuum goes to infinite coupling!

The collision of r vacua (Chebyshev point) produces a singularity of the form

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Scaling dimensions of chiral operators

Let us analyze a "neighbourhood" of the fixed point in the moduli space:

$$y^2 = x^k + \sum_i u_i x^{k-i}, \quad \lambda \approx \frac{y}{x^{N_f/2}} dx.$$

One can determine the scaling dimensions of chiral operators imposing T. Eguchi, K. Hori, K. Ito, S. Yang '96.

$$[\lambda] = 1 \;\; (2[y] = 2 + (N_f - 2)[x]); \qquad 2[y] = k[x].$$

When the theory has a nonAbelian global symmetry there is another constraint: P. Argyres, M. Douglas, N. Seiberg, E. Witten '96.

$$\prod_i (x - m_i^2) = x^{N_f} + \sum_i c_{2i} x^{N_f - i}; \quad [c_i] = 2i.$$

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Starting from the previous curve

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A possible solution is to introduce two sectors, with a different scaling of x. D. Gaiotto, N. Seiberg, Y. Tachikawa '10.

Let us rewrite the curve as

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We now introduce two scales $\epsilon_A, \epsilon_B \ll 1$.

- In one sector ($x \sim \epsilon_A$) we impose $[x] = 2$, so $\tilde{y}^2 \sim \epsilon_A$.
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Identifying the two sectors

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  This curve describes the $D_{N+2-N_f/2}$ AD theory, which has (at least) $SU(2)$ flavor symmetry. S. Cecotti, C. Vafa '11.

- In the A sector ($x \sim \epsilon_A$) the curve is

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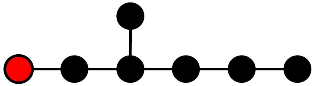
- For $N_f = 6$ the flavor symmetry of the A sector is enhanced from $SU(2) \times SO(12)$ to E_7. Y. Tachikawa '09.



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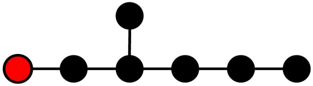

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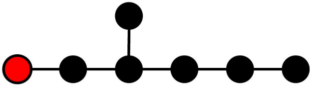

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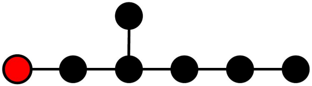

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Collision of r vacua: the $N_f = 4$ case

When $N_f = 4$ the effective action at the singular point includes the superpotential

$$\mathcal{W} = \tilde{Q}_0 A_D Q^0 + \tilde{Q}_0 \phi Q^0 + \sum_{i=1}^{4} \tilde{Q}_i \phi Q^i .$$

This effective theory has to reproduce the semiclassical results:

- The pattern of flavor SB after the $\mathcal{N} = 1$ perturbation is

$$SO(8) \to U(4).$$

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Breaking to $\mathcal{N}=1$ in the effective theory

Singular points and confinement in SQCD

Simone Giacomelli

Scuola Normale Superiore, INFN Pisa

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The equations of motion become:

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The Q_0 and Q_1 condensates break the $SU(2) \times U(1)$ gauge symmetry and induce confinement.

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These match precisely the expected 12 Goldstone multiplets from the breaking $SO(8) \to U(4)$. These fields are the SUSY analog of pions.

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$SO(N)$ SQCD

We can repeat this analysis for $SO(N)$ gauge theories.

We find again at the Chebyshev vacua the effective description

$$\boxed{1} \Longleftarrow SU(2) \Longrightarrow \boxed{\text{SCFT SECTOR}}.$$

- For $SO(2N)$ theory the SCFT sector can be constructed compactifying on a three-punctured sphere the D_N 6d theory.
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$SO(2N+1)$ theory with 1 flavor

$SO(2N+1)$ **SQCD with** $N_f = 1$**:** The SW curve is $y^2 \approx x^4$. The low energy theory is a $SU(2) \times U(1)$ model with one fundamental and one adjoint.

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There are two solutions for $m \neq 0$ ($\Phi = a\tau_3$ with $a = im/\sqrt{2}$).

For $m = 0$ a diagonal combination of the cartans of

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$SO(2N)$ **SQCD with** $N_f = 2$**:** The SW curve is $y^2 \approx x^6$. The IR description a $U(1) \times SU(2) \times SU(2)$ theory with two bifundamentals.

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From the equations of motion we find

$$\tilde{Q} Q = -\mu\Lambda; \quad \Phi = \Phi_3 \tau_3; \quad \Psi = \Psi_3 \tau_3.$$

$$\begin{pmatrix} (m_i + \frac{\Phi_3}{\sqrt{2}} + \frac{\Psi_3}{\sqrt{2}}) a_i & (m_i + \frac{\Phi_3}{\sqrt{2}} - \frac{\Psi_3}{\sqrt{2}}) b_i \\ (m_i - \frac{\Phi_3}{\sqrt{2}} + \frac{\Psi_3}{\sqrt{2}}) c_i & (m_i - \frac{\Phi_3}{\sqrt{2}} - \frac{\Psi_3}{\sqrt{2}}) d_i \end{pmatrix} = 0$$

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- In supersymmetric QCD confinement and "chiral" symmetry breaking have the same origin: the condensation of magnetic monopoles ('t Hooft-Mandelstam mechanism).
- There are singular points in the moduli space where the low energy physics can be described in terms of two scale-invariant sectors, coupled together by a gauge multiplet. The UV and IR Dofs are different.
- In special cases both sectors are free (or at least lagrangian) and we find an effective description which reproduces the expected pattern of symmetry breaking and multiplicity of vacua. This model includes NA monopoles and abelian confining strings.

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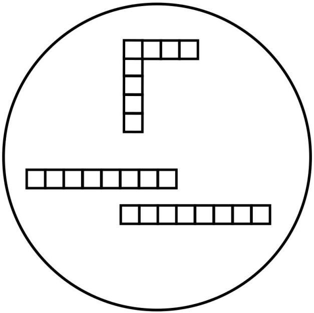
Thank you for your attention!

The A sector



$$\lambda^{2N} = \sum_{k=1}^{N} \lambda^{2N-2k} \phi_{2k}(z); \quad \lambda = xdz.$$

The order of the poles at the punctures are:

$$\{1, 2, \ldots, 2; 1\}; \quad \{1, \ldots, 2N-3; N-1\}.$$