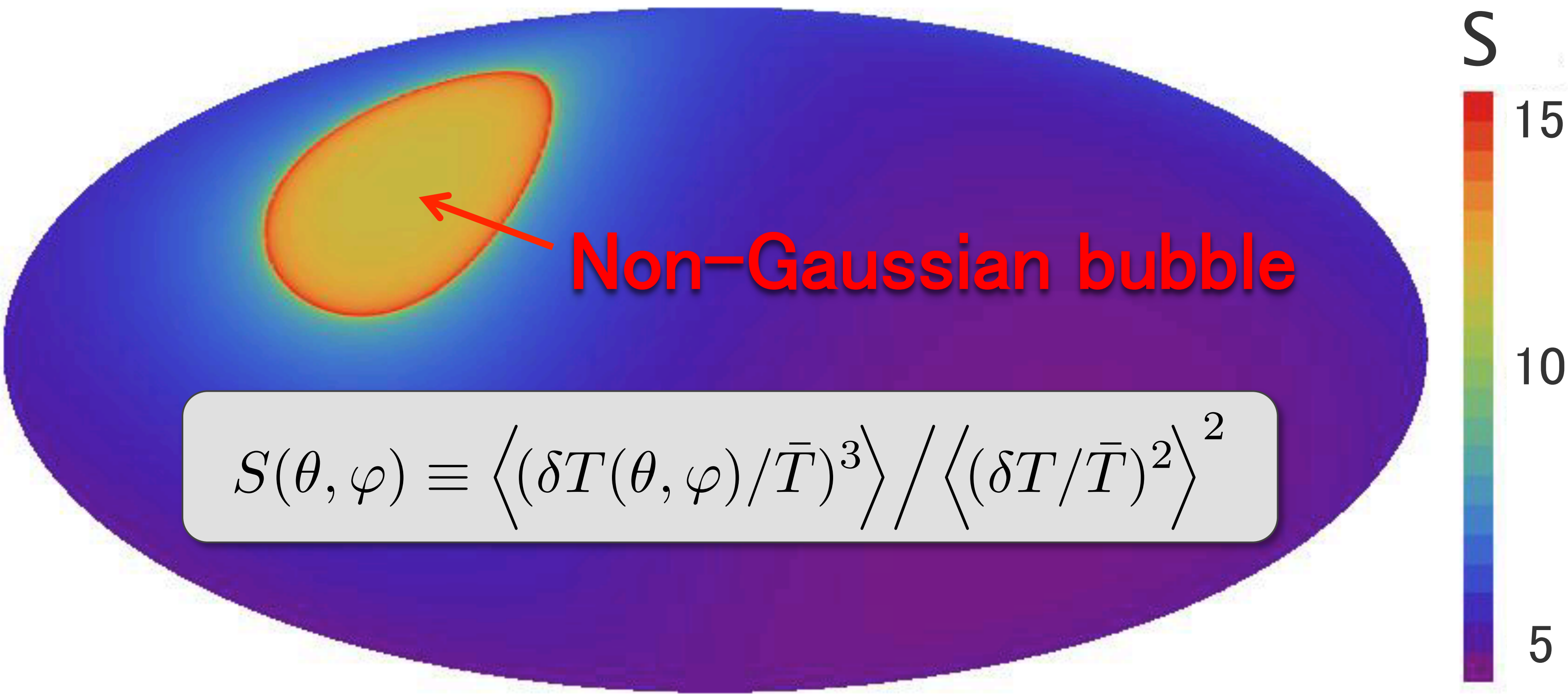
Quantum tunneling in the inflationary era and its observational consequences



$$S(\theta,\varphi) \equiv \left\langle (\delta T(\theta,\varphi)/\bar{T})^3 \right\rangle \Big/ \left\langle (\delta T/\bar{T})^2 \right\rangle^2$$

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D. Yamauchi and M. Sasaki, (EPL 100 (2012) 29004)

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- Quantum tunneling during inflation
- in-in formalism on tunneling background
- Toy model & Non-Gaussian bubble
- Conclusion

String landscape

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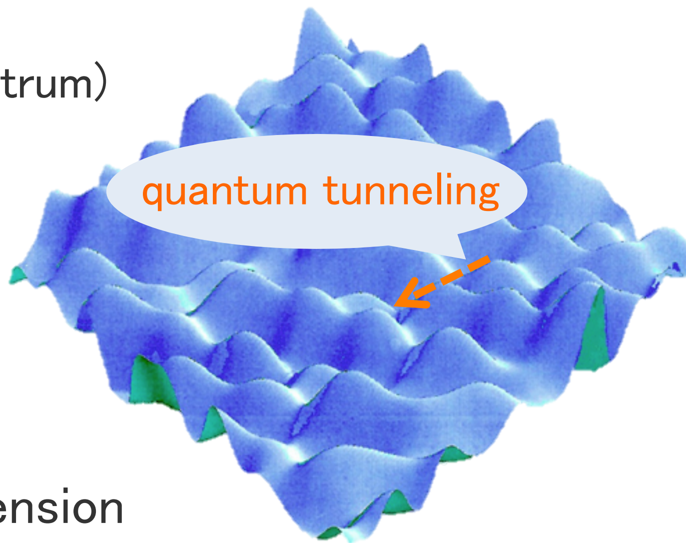

potential for scalar fields

Inflation

(flatness, homogeneity, power spectrum)

- good consistency with observation
- What is the physical origin of inflation?

String landscape (Susskind, 2003)

- degrees of freedom in the shape of extra-dimension
- many scalar fields & local potential minima

Two ways to study string landscape

I. Trying to obtain the potential from the first principle (string theory side)

☆ II. Assuming string landscape and studying its consequences (cosmology side)

We focus on non-Gaussianity

Non-Gaussianity (NG)

- Gaussian variable: ϕ_G ← the simplest inflation model predicts almost Gaussian fluctuations

$$\left\langle \varphi_G(\mathbf{k}_1)\varphi_G(\mathbf{k}_2)\right\rangle = (2\pi)^3\,\delta(\mathbf{k}_1+\mathbf{k}_2)P(k_1)$$

power-spectrum (2pt function) ⇒ all statistical property of Φ_G

Non-Gaussianity (NG)

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power-spectrum (2pt function) ⇨ all statistical property of Φ_G

- Non-Gaussian variable: Φ

$$\left\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3) \right\rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \boxed{B(k_1, k_2, k_3)}$$

power-spectrum (2pt func.), bi-spectrum (3pt func.), tri-spectrum (4pt func.), ・・・

⇨ all statistical property of Φ

Non-Gaussianity (NG)

- Gaussian variable: ϕ_G ← the simplest inflation model predicts almost Gaussian fluctuations

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power-spectrum (2pt func.), bi-spectrum (3pt func.), tri-spectrum (4pt func.), ・・・

⇒ all statistical property of Φ

- Observational constraint on NG using template

Ex) local-type NG for grav. potential $\Phi(x)$

$$\Phi(\mathbf{x}) \sim \varphi_G(\mathbf{x}) + f_{NL}^{(\mathrm{local})}\varphi_G^2(\mathbf{x}) + \cdots$$

$f_{NL}^{(\mathrm{local})}$ = 2.7 ± 5.8 (68% C.L., Planck2013)

⇒ $$B(k_1,k_2,k_3) = f_{NL}^{(\mathrm{local})}\left(P(k_1)P(k_2)+P(k_2)P(k_3)+P(k_3)P(k_1)\right)$$

Note: Use of template increases statistical significance, but each type of NG requires its own optimal template.

Quantum tunneling = Bubble nucleation

- Local minima in the string landscape

we focus on quantum tunneling during inflation

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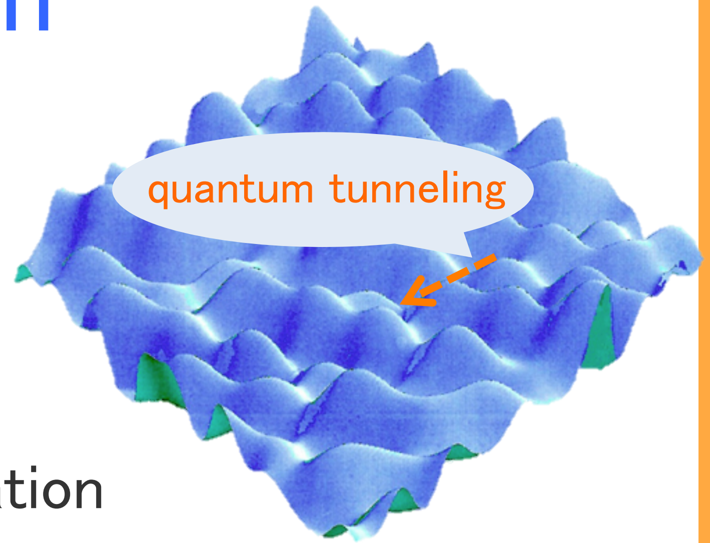
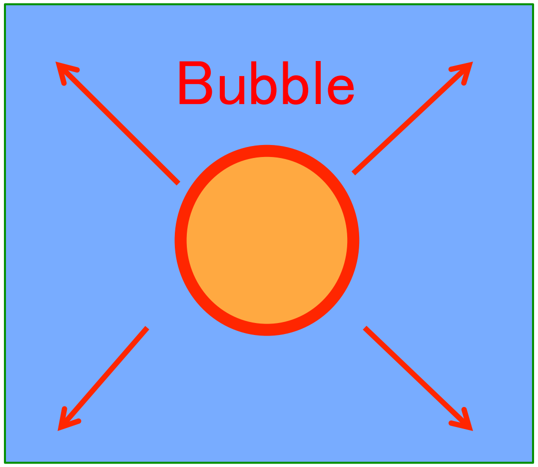


potential for scalar fields

- Bubble nucleation

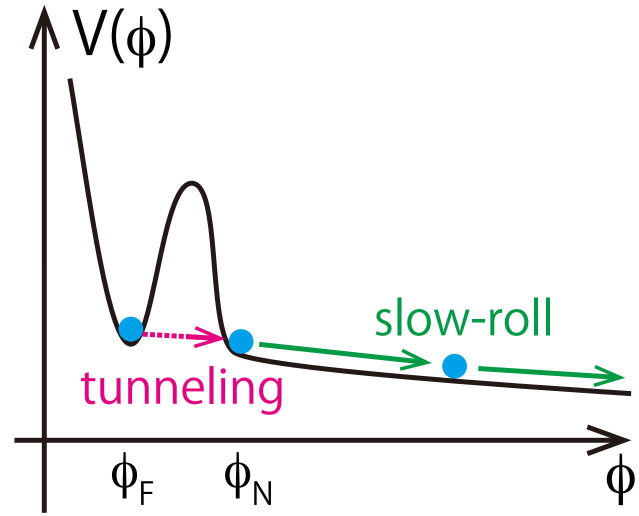
  - quantum tunneling is 1st order phase transition



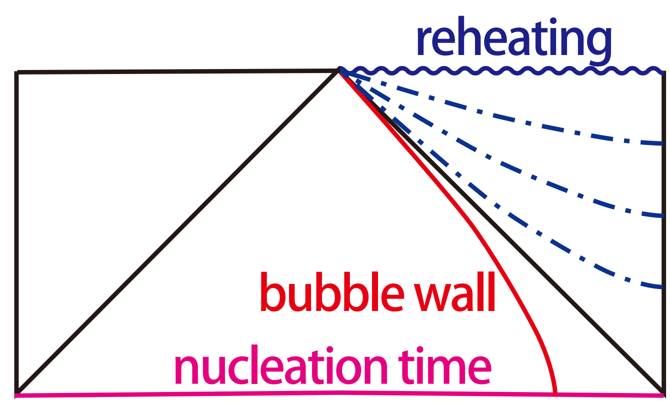
Two types of quantum tunneling during inflation

I. Open inflation

- tunneling triggers slow-roll inflation
- slow-roll inflation proceeds only inside the nucleated bubble



- inflaton constant surface (blue dot-dashed) is 3-dim hyperboloid
- If duration of slow-roll inflation after tunneling is long enough (e-folds $\gtrsim 60$), there is no contradiction to observations ($|\Omega_K| \lesssim 0.01$)



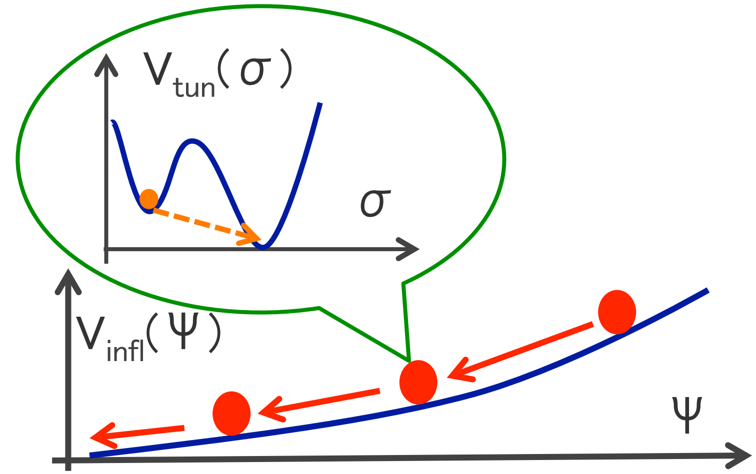
Penrose diagram

(bi-spectrum in open inflation [KS and E. Komatsu in preparation])

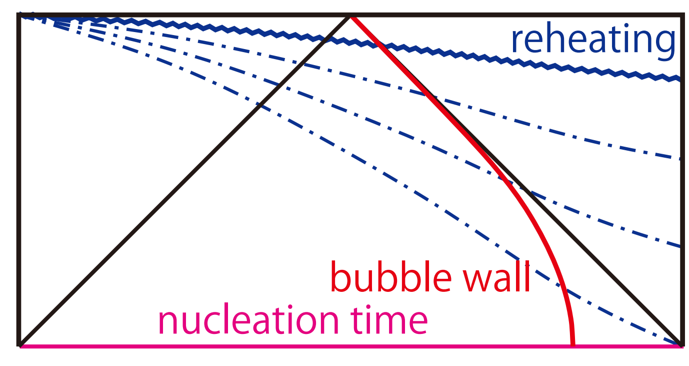
Two types of quantum tunneling during inflation (cont.)

☆ II. Low energy bubble nucleation

- potential $V_{tun}(\sigma)$ for tunneling field σ is much smaller than $V_{infl}(\Psi)$ for inflaton Ψ
- slow-roll inflation by Ψ is not affected by tunneling of σ at some time of inflation



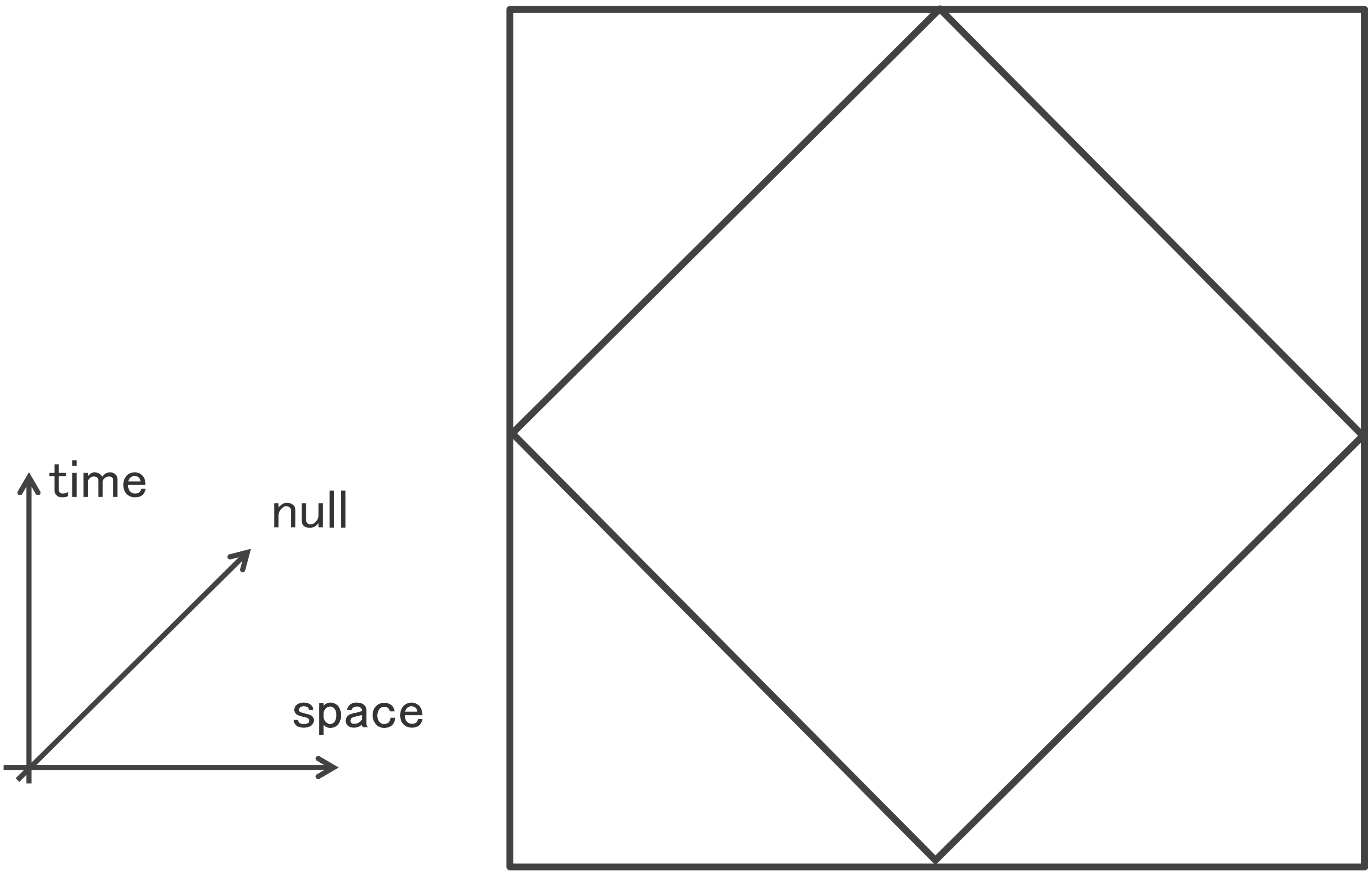
- inflaton constant surface (blue dot-dashed) is flat as usual
- finite size bubble is on that surface



Penrose diagram

Bubble-shaped observational feature may appear as distinct signature of quantum tunneling during inflation!!

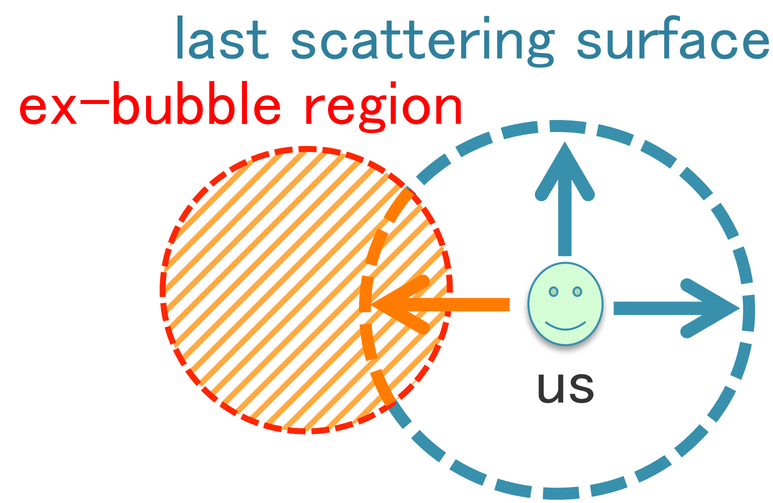
Penrose diagram of de Sitter spacetime



Non-Gaussian Bubbles

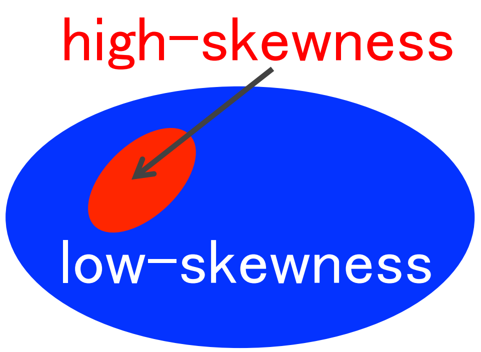
Bubble remnant

- all scalar fields are supposed to decay into radiation after inflation
- However, bubble-shaped feature may be left in the sky



Non-Gaussian bubbles

- low energy bubble nucleation case
- bubble-shaped high skewness region may be created
- I will make explicit calculation for a simple toy model



CMB sky map

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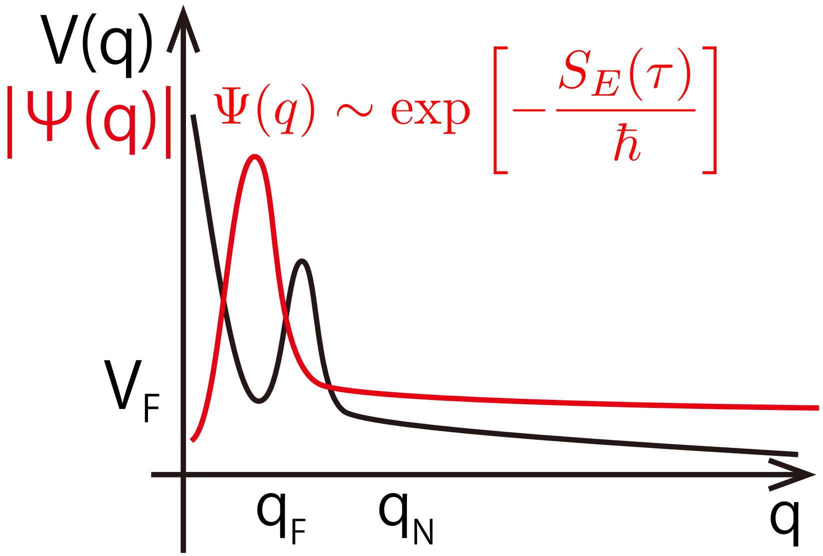
Method of instanton

- 1-dim quantum mechanics example

wave function inside the barrier

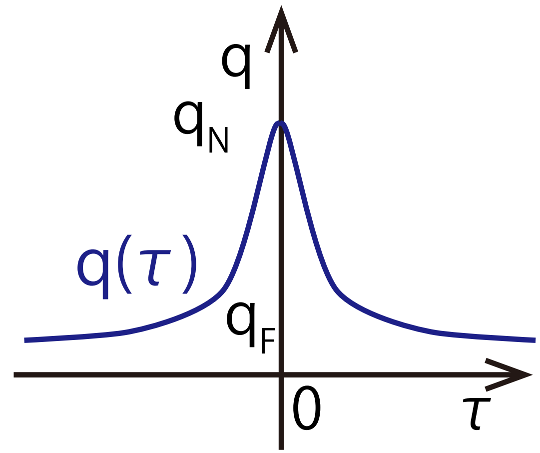


- Euclidean Action

$$S_E(\tau) = \int_{\infty}^{\tau} d\tau' \left(\frac{(dq/d\tau)^2}{2} + V(q) - V_F \right)$$

$$= \int_{q_F}^{q(\tau)} dq \sqrt{V(q) - V_F}$$

instanton



- Euclidean EOM

$$\frac{d^2 q}{d\tau^2}(\tau) = \frac{dV(q)}{dq}$$

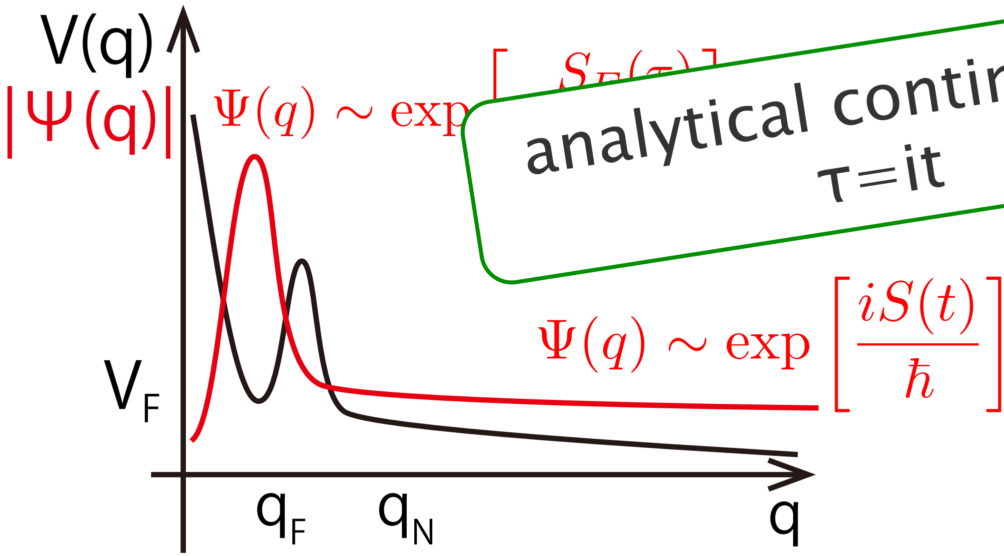
- boundary conditions

$$q(\pm\infty) = q_F \qquad q(0) = q_N$$

Method of instanton

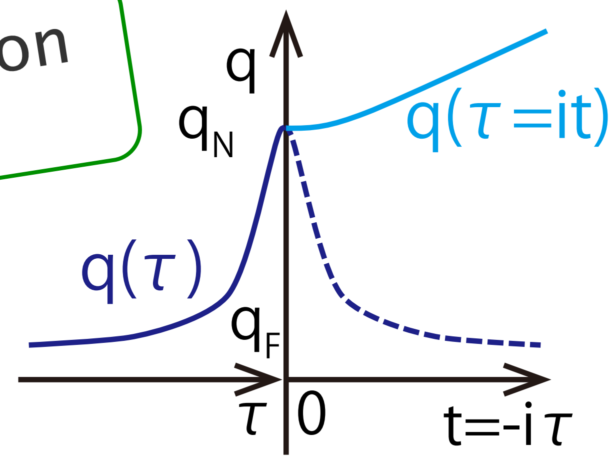
- 1-dim quantum mechanics example

wave function outside the barrier



analytical continuation
τ=it

classical solution



- Lorentzian action

$$iS(t) \equiv -S_E(\tau = it)$$

$$= i \int_{q_F}^{q(\tau=it)} dq \sqrt{V_F - V(q)}$$

- Lorentzian EOM

$$\frac{d^2q}{dt^2}(\tau = it) = -\frac{dV(q)}{dq}$$

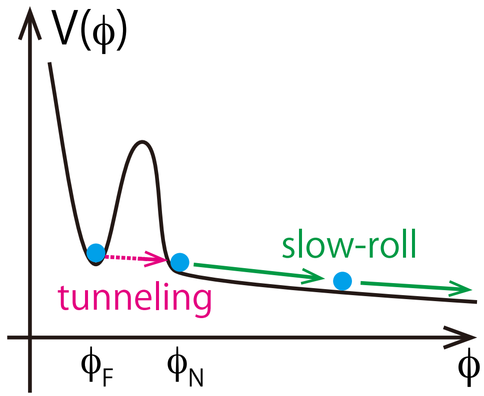
- initial conditions

$$q(0) = q_N \qquad \frac{dq}{dt}(0) = 0$$

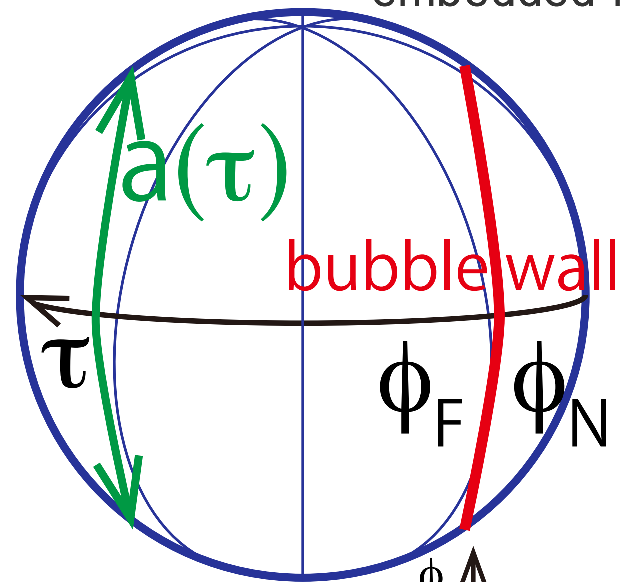
Coleman- De Luccia instanton

(Coleman and De Luccia(CDL), 1980)

- Tunneling of a scalar field with gravity



4dim Euclidean sphere
embedded in 5dim



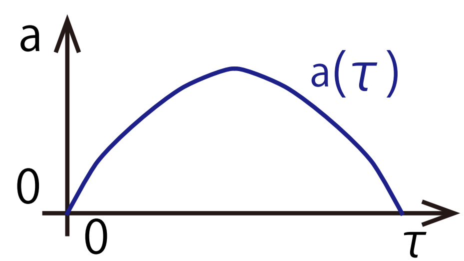
- Corresponding instanton
  - Euclidean metric (O(4)–symmetric)

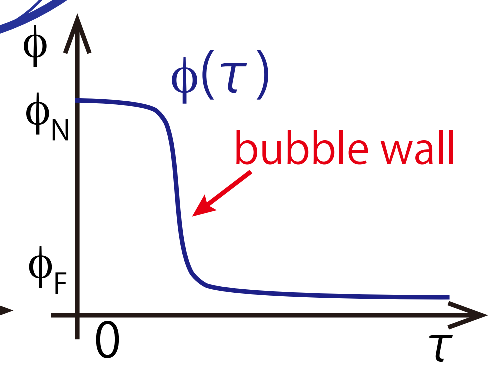
$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3^2$$

  - Euclidean EOM

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{8\pi G}{3}\left(\left(\frac{d\phi}{d\tau}\right)^2 + V(\phi)\right)$$

$$\frac{d^2\phi}{d\tau^2} + \frac{3}{a}\frac{da}{d\tau}\frac{d\phi}{d\tau} - \frac{dV(\phi)}{d\phi} = 0$$





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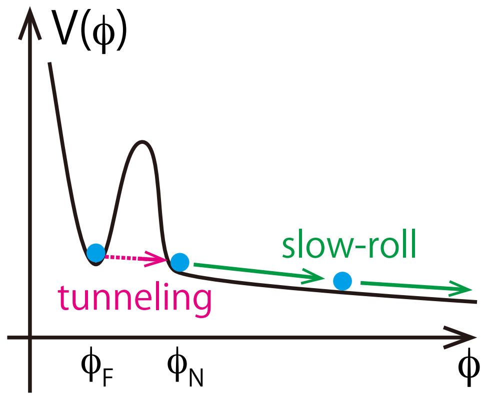
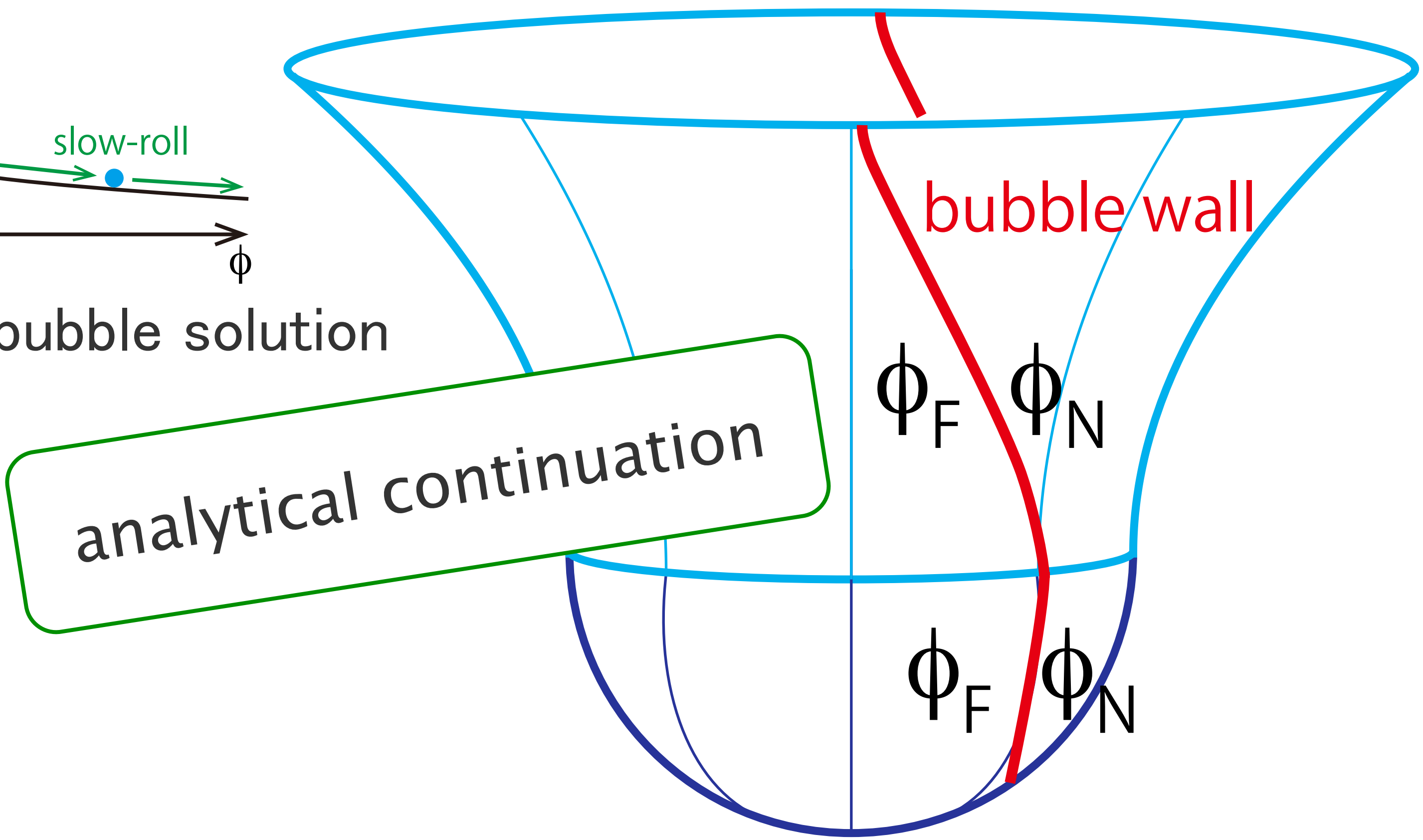


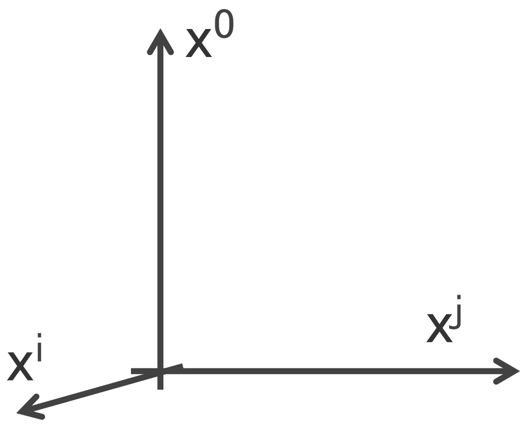
- Expanding bubble solution

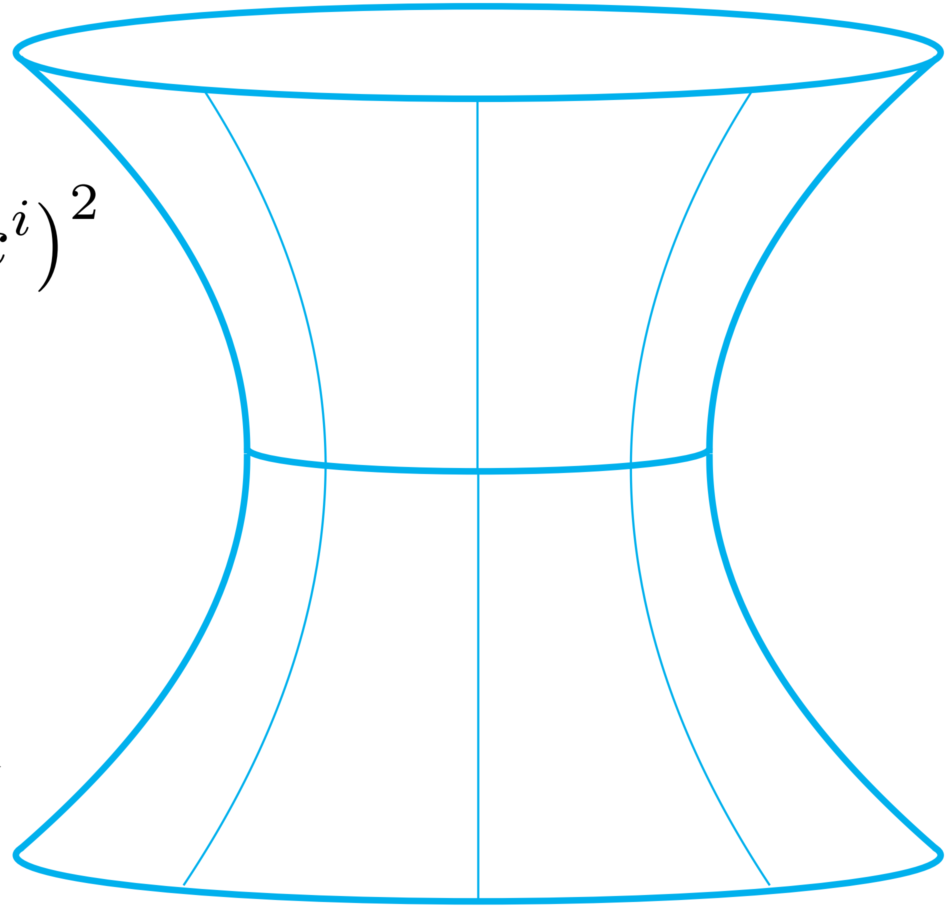


4-dim de Sitter spacetime embedded in 5-dim Minkowski spacetime

$$-\left(x^0\right)^2+\sum_{i=1}^{4}\left(x^i\right)^2=\frac{1}{H^2}$$

$$ds^2=-\left(dx^0\right)^2+\sum_{i=1}^{4}\left(dx^i\right)^2$$





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In-in formalism (standard case)

- Separating Lagrangian to free and interaction parts

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$$

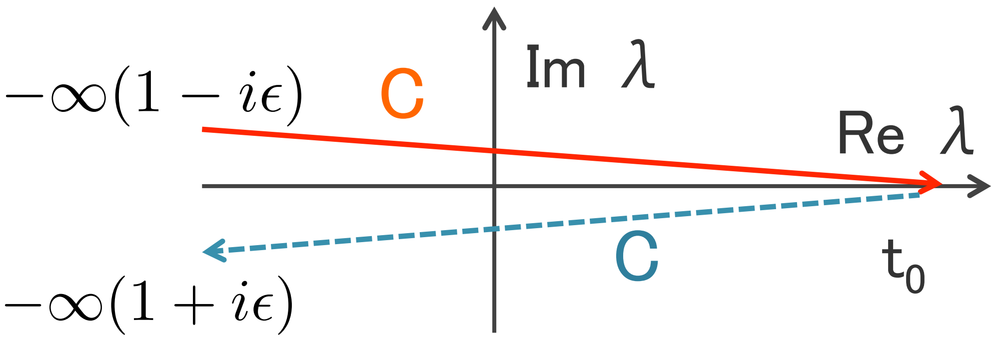
- N-ptfunction formula (in-in formalism)

(all points are at time t_0)

P: path ordering operator

$$\left\langle \varphi(x_1)\varphi(x_2)\cdots\varphi(x_N) \right\rangle = \frac{\left\langle 0 \left| P\,\varphi(x_1)\varphi(x_2)\cdots\varphi(x_N) e^{i\int_{C\times\Sigma_\lambda} d\lambda d^3\mathbf{x}\,\mathcal{L}_{int}(x)} \right| 0 \right\rangle}{\left\langle 0 \left| P e^{i\int_{C\times\Sigma_\lambda} d\lambda d^3\mathbf{x}\,\mathcal{L}_{int}(x)} \right| 0 \right\rangle}$$

- C: λ -integration path (Maldacena 2002)
- Σ_λ: x-integration path



3-dim flat hypersurface

E_3

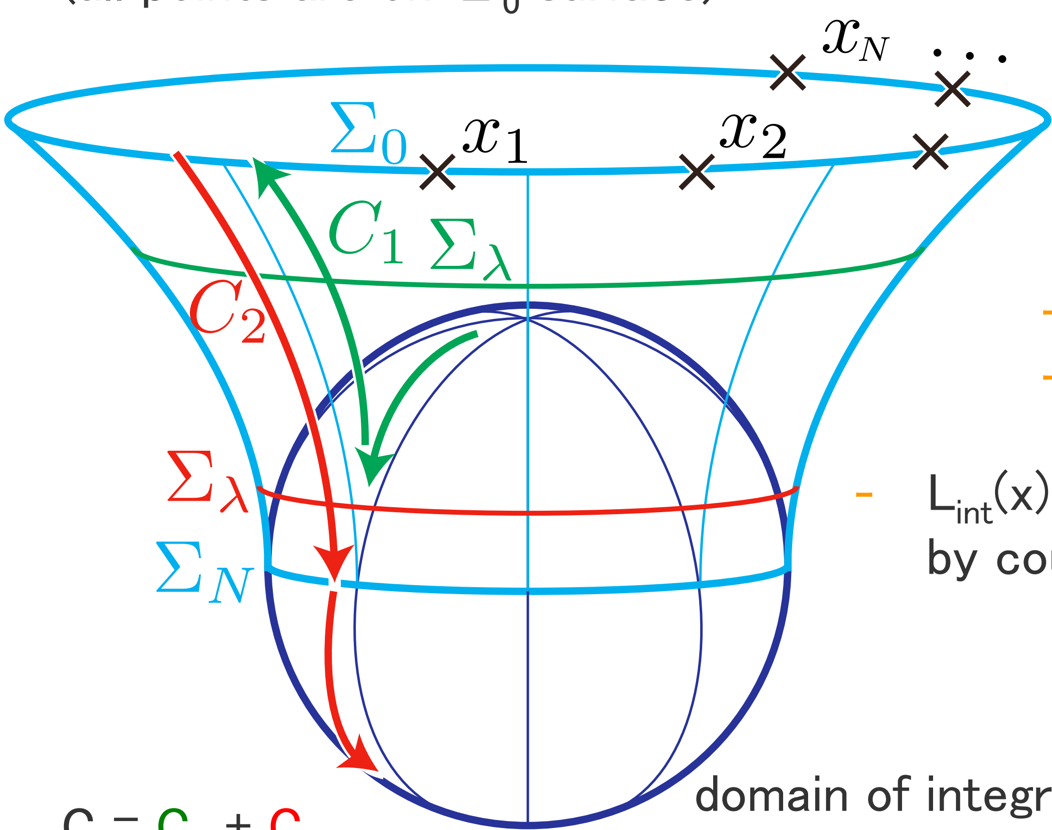
the domain of integral covers the whole spacetime twice

In-in formalism on tunneling background

derivation paper in preparation

$$\left\langle \varphi(x_1)\varphi(x_2)\cdots\varphi(x_N)\right\rangle = \frac{\left\langle 0\left|P\,\varphi(x_1)\varphi(x_2)\cdots\varphi(x_N)e^{i\int_{C\times\Sigma_\lambda} d\lambda d^3\mathbf{x}\,\mathcal{L}_{int}(x)}\right|0\right\rangle}{\left\langle 0\left|Pe^{i\int_{C\times\Sigma_\lambda} d\lambda d^3\mathbf{x}\,\mathcal{L}_{int}(x)}\right|0\right\rangle}$$

(all points are on Σ_0 surface)



$C = C_1 + C_2$

- same expression as the standard case, but different domain of integral

- test field ϕ
- background tunneling field σ

- $L_{int}(x)$ can have explicit x-dependence by coupling between ϕ and σ

  Ex) $L_{int}(x) = \lambda(\sigma(x))\,\phi^3(x)$

domain of integral covers the Lorentzian region twice, and north/south Euclidean hemisphere once

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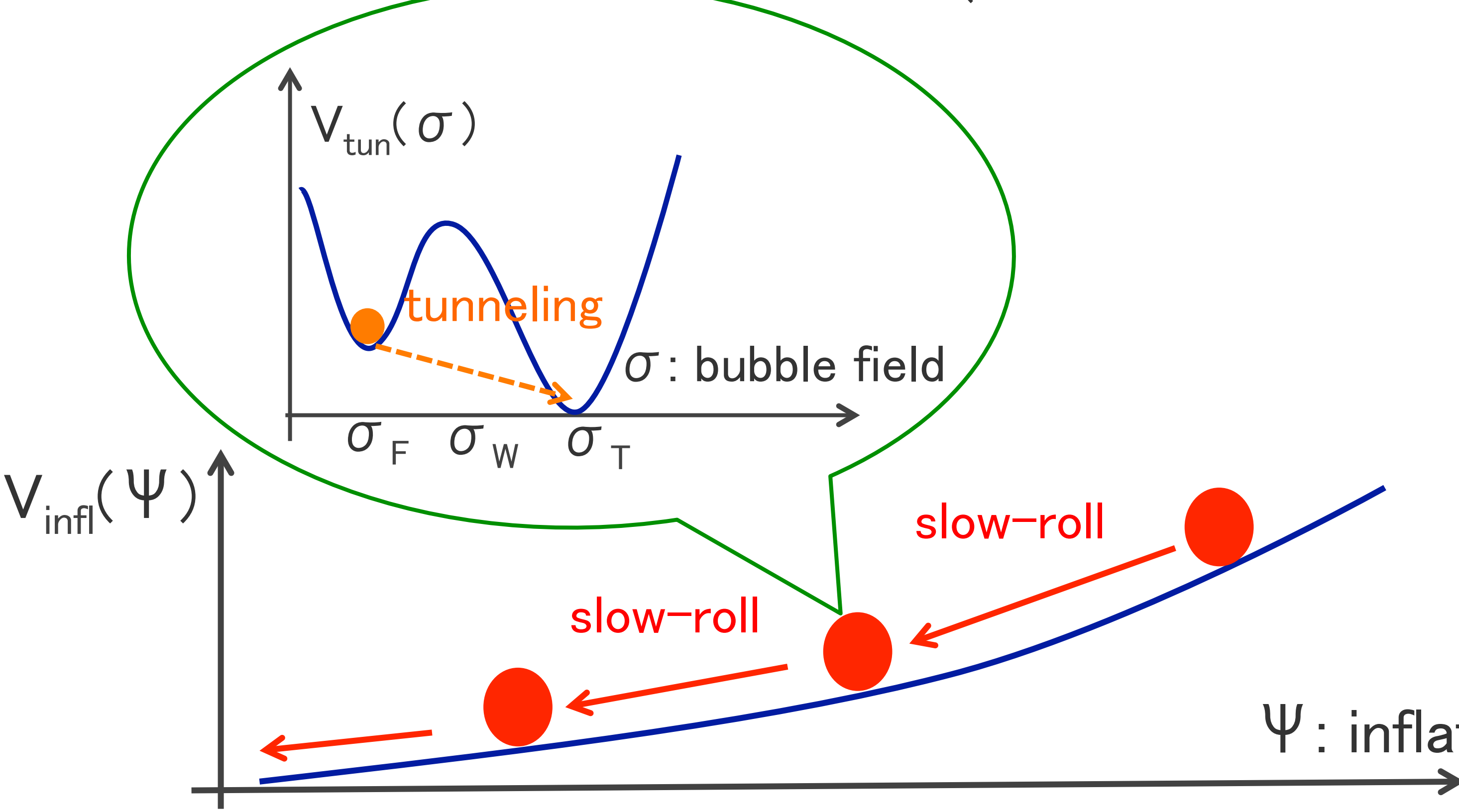
Bubble nucleation during slow-roll inflation

Ψ : inflaton

σ : bubble field

- Slow-roll inflation by inflaton Ψ
- Quantum tunneling of bubble field σ at some instant

(we assume small energy bubble)



Bubble nucleation during slow-roll inflation

Ψ : inflaton

σ : bubble field

- Slow-roll inflation by inflaton Ψ
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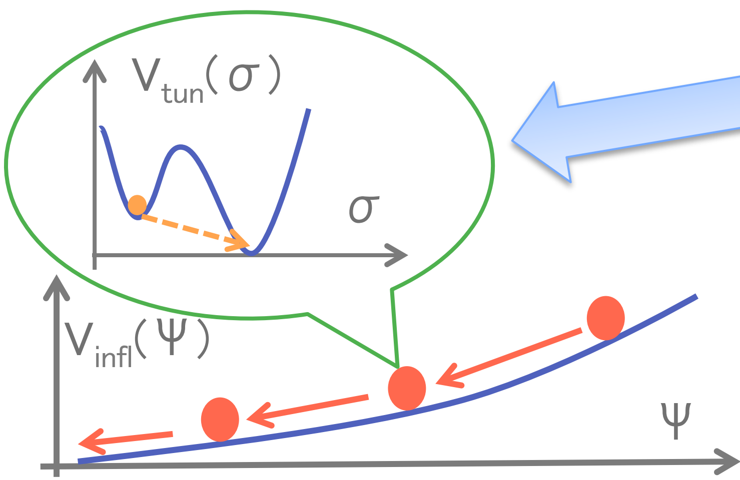
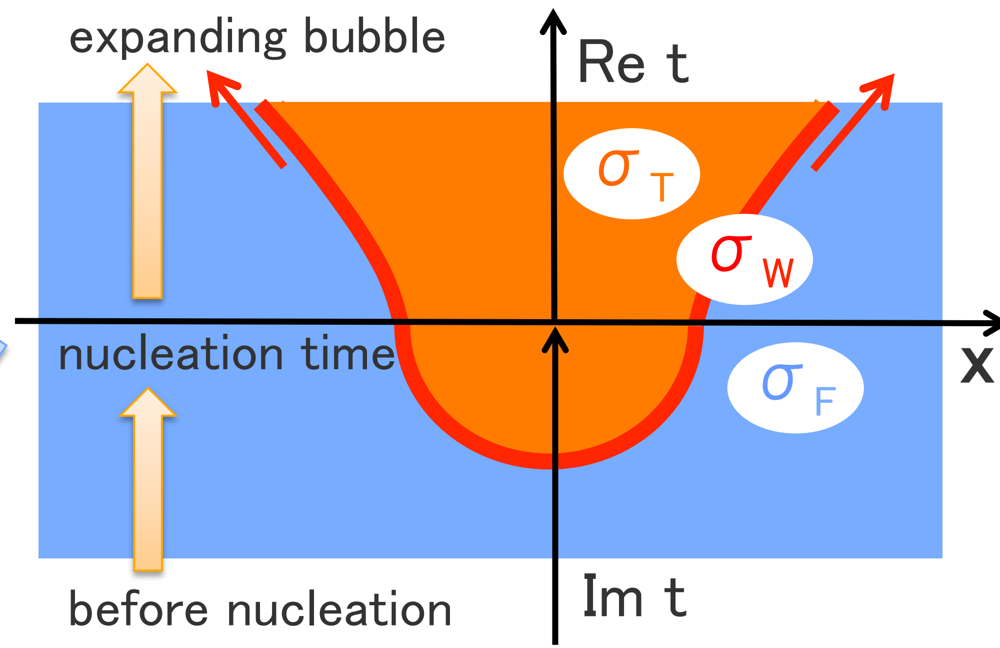
Bubble nucleation

(described by an instanton)





Curvaton evolution in a bubble nucleating universe

Ψ : inflaton
σ : bubble field
ϕ : curvaton

- Potential for curvaton ϕ

$$V(\phi) = \frac{m^2}{2}\phi^2 + \underline{V_{\rm int}(\sigma, \phi)}$$

effective potential

background bubble

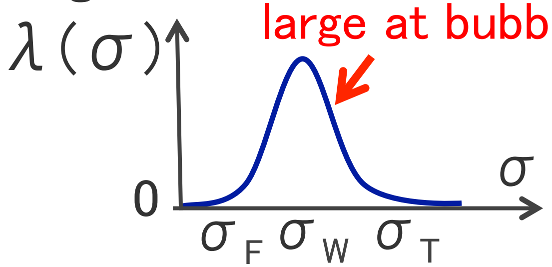
$$\underline{V_{\rm int}^{(\rm eff)}(\phi; x)} := V_{\rm int}(\bar{\sigma}(x), \phi)$$

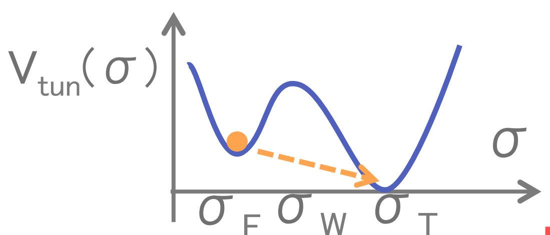
- σ -dependent self-interaction

$$V_{\rm int}(\sigma, \phi) = \lambda(\sigma)\phi^3$$

coupling const.

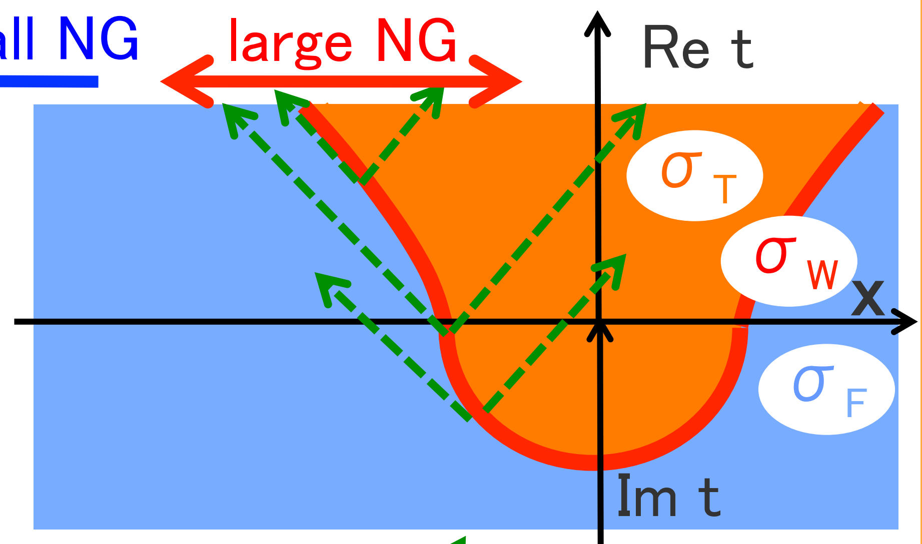
large at bubble wall





small NG

large NG

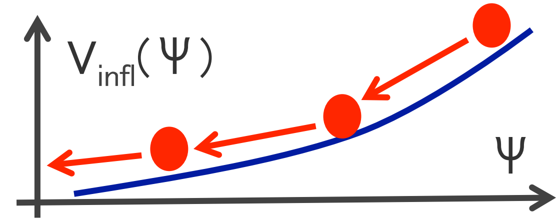


··· NG generation

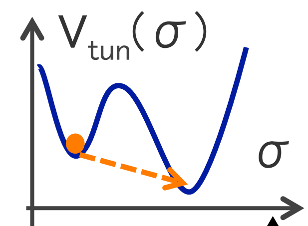
Bubble-shaped non-Gaussianity is generated!!

Summary of our toy model

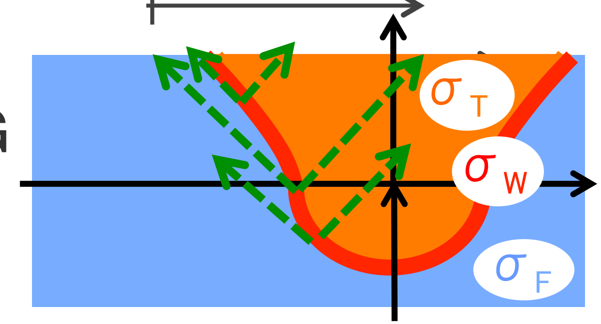
- 3 scalar fields model
  - model gets complicated
  - analysis becomes easier



- Inflaton: Ψ → slow-roll inflation



- Bubble field: σ → bubble nucelation



- Curvaton: ϕ → bubble-shaped NG

(Lyth, Ungarelli & Wands; Enqvist & Sloth; Moroi & Takahashi)

Curvaton mechanism: the energy of Curvaton field is negligible during inflation. But it can creates curvature pert. when it decays later than other scalar fields.

From curvaton fluctuation to δT

Evolution of fluctuations

Curvaton mechanism		Sachs–Wolfe effect	

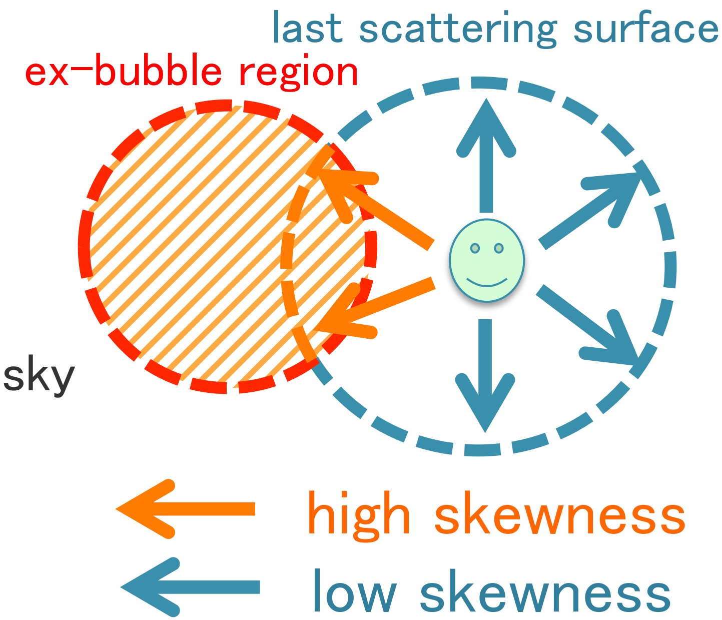
$$\phi(x) \;\Rightarrow\; \zeta(x) \;\Rightarrow\; \delta T(\theta,\varphi)$$

curvaton flucutuations → curvature pert. → CMB anisotropies

CMB skewness: $\left\langle \delta T^3(\theta,\varphi) \right\rangle$

- skewness of ϕ can be calculated using in–in formalism on tunneling background
- observer sees high skewness spot in the sky

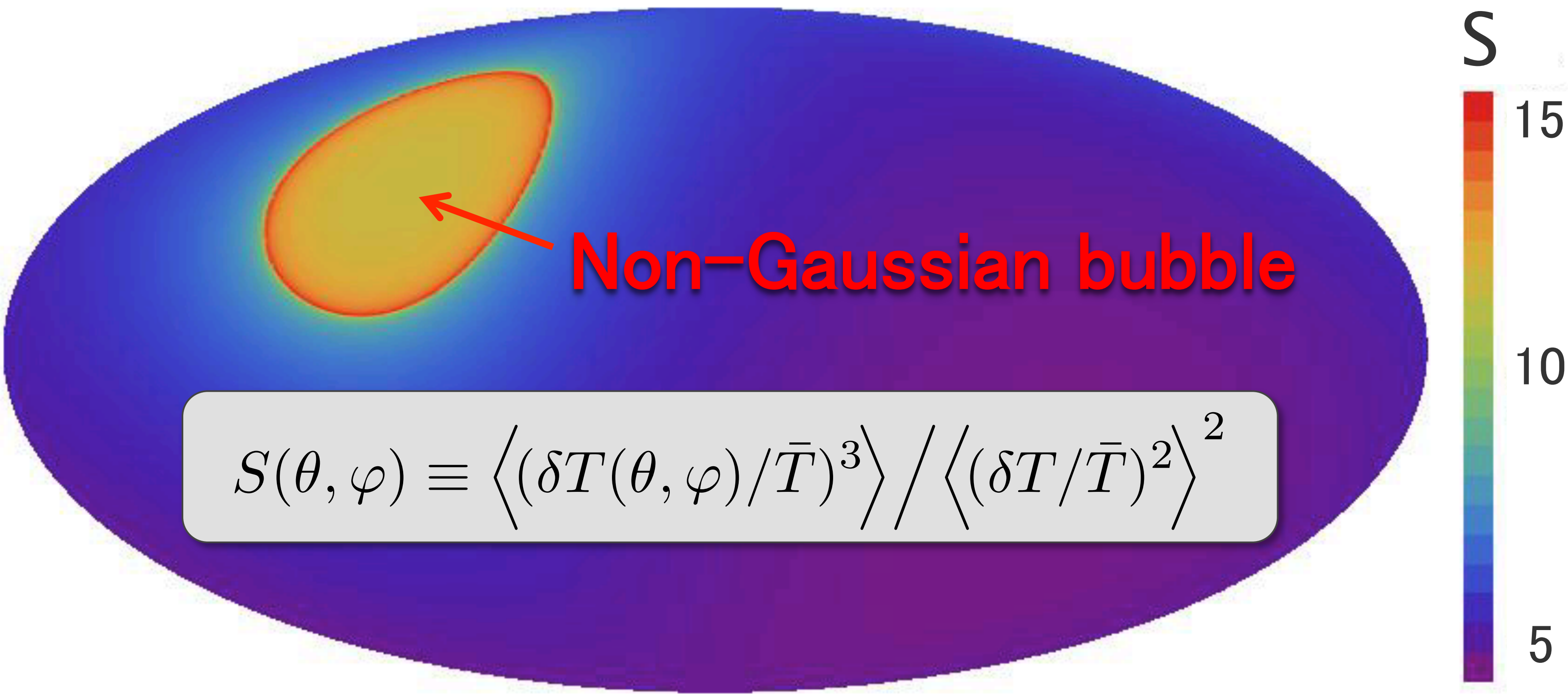
(= Non–Gaussian bubble)



Result

(amplitude of skewness is proportional to the strength of coupling λ)

- calculated skewness with a certain parameter set



$$S(\theta,\varphi) \equiv \left\langle (\delta T(\theta,\varphi)/\bar{T})^3 \right\rangle \Big/ \left\langle (\delta T/\bar{T})^2 \right\rangle^2$$

corresponding Planck's sensitivity
(estimate for homogeneous non-Gaussianity)

$$\Delta S \sim O(5)$$

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Conclusions

- We have studied the observational consequences from a string landscape, focusing on quantum tunneling during inflation
- In-in formalism on tunneling background has been discussed
- Non-Gaussianity has been calculated in a toy model where a low energy bubble is nucleated
- We have shown that a bubble-shaped high-skewness spot in CMB anisotropies may be generated

→ Non-Gaussian bubble

On-going work (with Ely Kovetz @UT Austin)

- Usual analysis using statistically homogeneous templates will miss non-Gaussian bubbles even if they exist.

- We have started a project to search non-Gaussian bubbles by making special analysis targeting them.

- If you find non-Gaussian bubbles in near-future observations, it might be the first observational signature of string theory!