Natural GUT Scale Mass Ratios

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SISSA
International School for Advanced Studies

Outline

- Introduction
- Fermion Mass Ratios
- Gaugino Mass Ratios
- Summary and Conclusions

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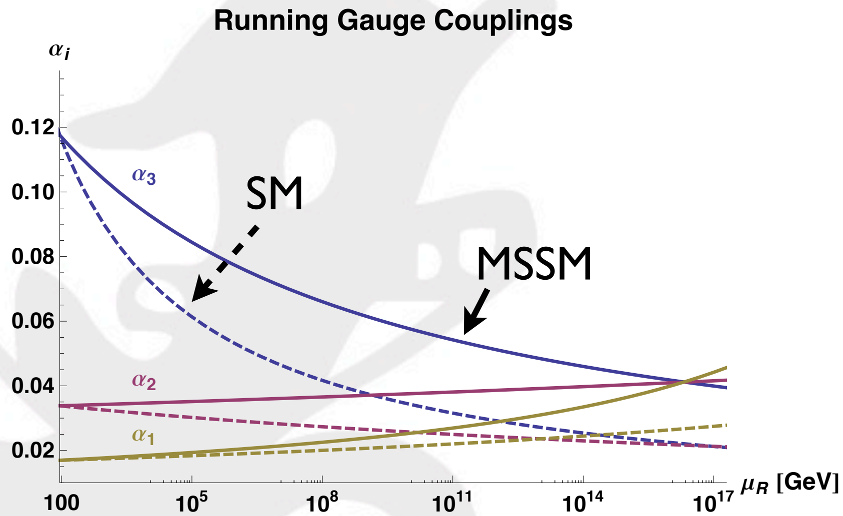
Some Virtues of Grand Unification

- Follows the goal of simplification (SU(3)xSU(2)xU(1) unified into a single group)
- Solves one naturalness problem:
  - ➡ The cancellation of the electron and proton charge in H
- Gives relation for fermion masses (Yukawa couplings)
- Small neutrino masses due to the seesaw

Why SUSY GUTs?

- What is SUSY?

- What is SUSY good for?
  - Gives dark matter candidate
  - Solves the hierarchy problem
  - Gauge coupling unification



Why this talk?

1. Are the GUT predictions for the fermion mass ratios realistic?

2. Does the MSSM really solve the hierarchy problem (little hierarchy)?

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- **Fermion Mass Ratios**
  - Theoretical Preliminaries
  - Comparison to Data
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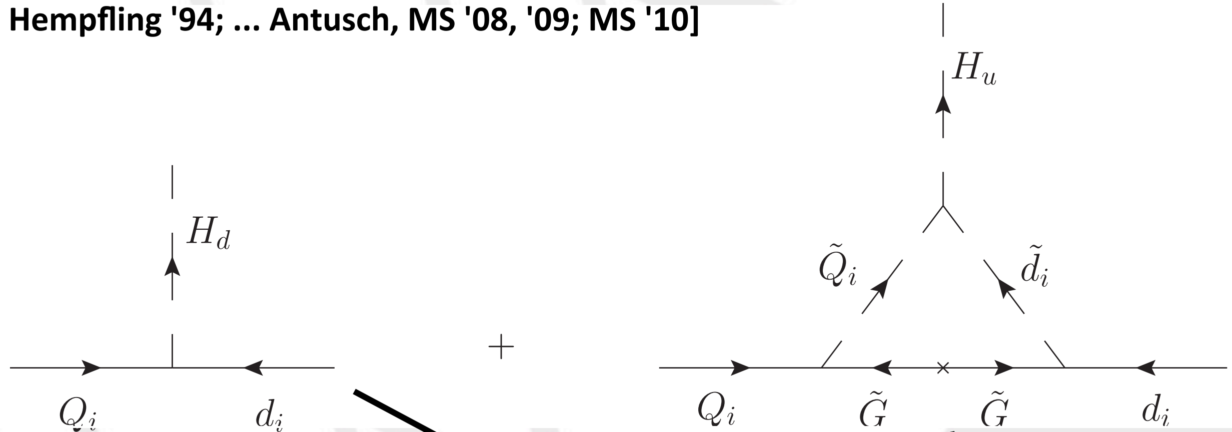
SUSY Threshold Corrections I

[Hall, Rattazzi, Sarid '94; Blazek, Raby, Pokorski '95; Carena, Olechowski, Pokorski, Wagner '94; Hempfling '94; ... Antusch, MS '08, '09; MS '10]



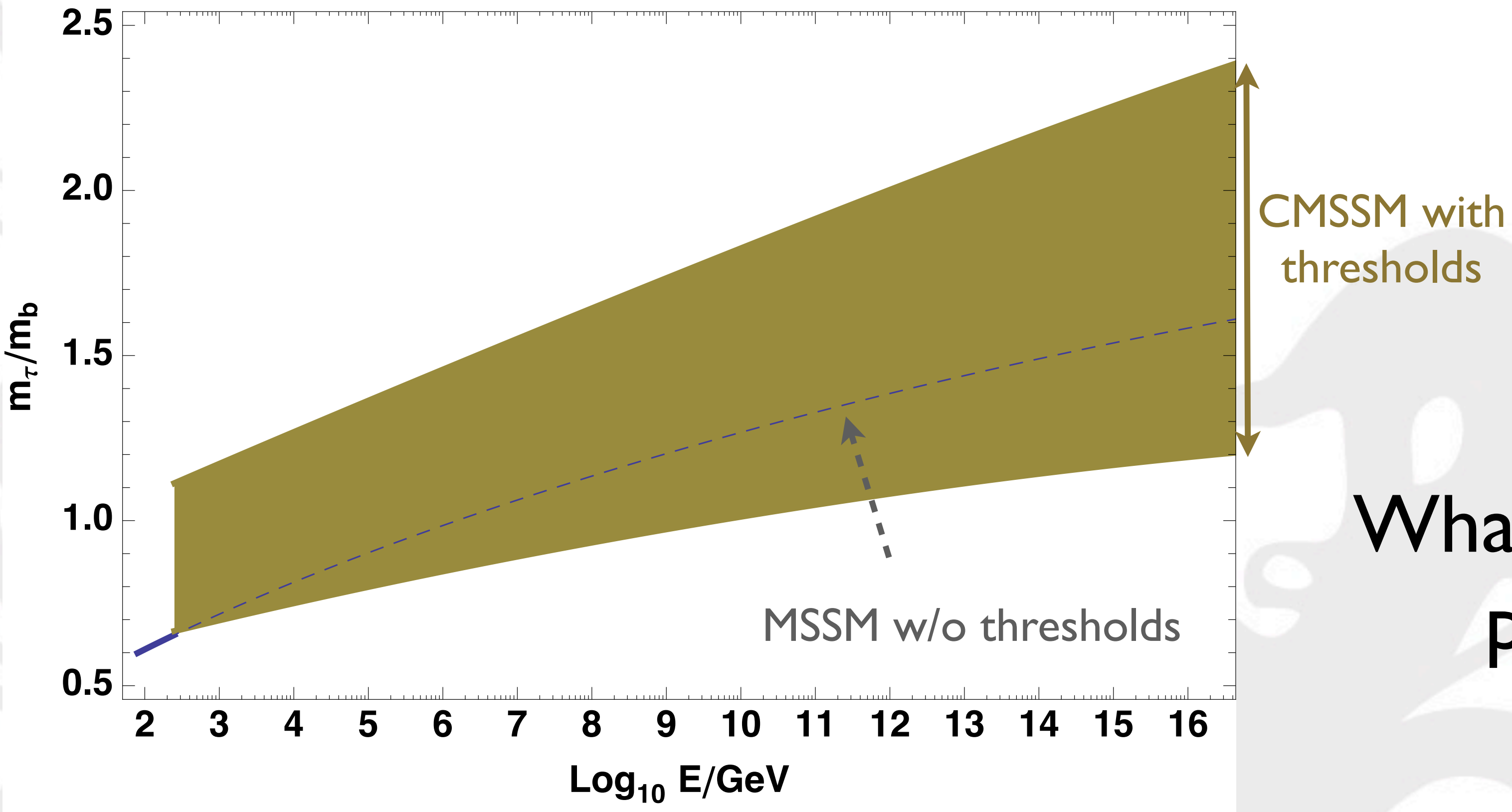
Matching: $\mathcal{L}_{\text{eff}}^{\text{MSSM}} = -y_b^{\text{MSSM}} \left(\bar{b}_R Q_L H_d + \varepsilon_b \bar{b}_R Q_L H_u^c \right) + \text{H.c.}$

to: $\mathcal{L}^{\text{SM}} = -y_b^{\text{SM}} \bar{b}_R Q_L \epsilon H + \text{H.c.}$

gives: $y_b^{\text{SM}} = y_b^{\text{MSSM}} \cos\beta \, (1 + \varepsilon_b \tan\beta)$

tan β enhanced!!!

SUSY Threshold Corrections II



What do GUTs predict?

Alternative Ratios I

Consider first the most minimal case:

$$F_3 T_3 \langle \overline{H}_5 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} b^c_R \\ b^c_B \\ b^c_G \\ \tau \\ -\nu_\tau \end{pmatrix} \begin{pmatrix} 0 & -t^c_G & t^c_B & -t_R & -b_R \\ t^c_G & 0 & -t^c_R & -t_B & -b_B \\ -t^c_B & t^c_R & 0 & -t_G & -b_G \\ t_R & t_B & t_G & 0 & -\tau^c \\ b_R & b_B & b_G & \tau^c & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}$$

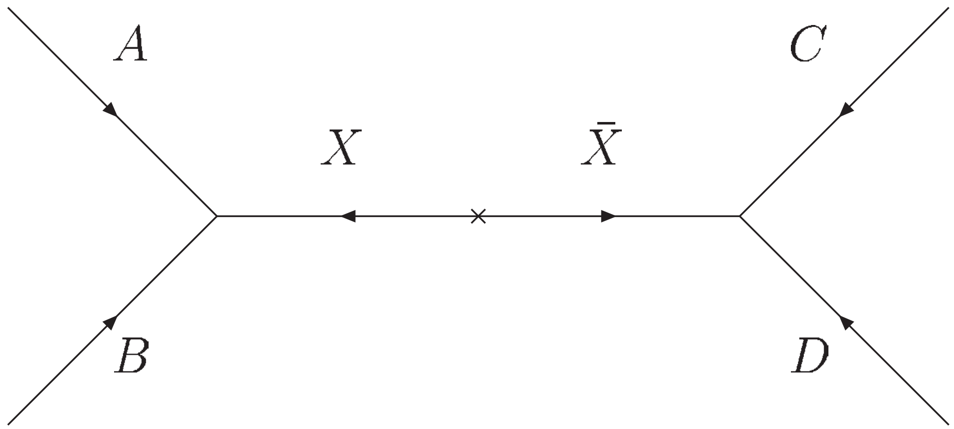
$$\xrightarrow{SSB} -\frac{v}{\sqrt{2}} \left(\bar{b} b + \overline{\tau} \tau \right) \longrightarrow y_\tau / y_b = 1$$

Using a 45-dim. Higgs gives a ratio of -3

[Georgi, Jarlskog '79]

Alternative Ratios II

[Antusch, MS '09; MS '10]



- Take A = F_2, B = H_{24}, C = T_2, D = H_{45} and X = 5
- From $\langle H_{24} \rangle$: $y_\mu / y_s = -3/2$
- From $\langle H_{45} \rangle$: $y_\mu / y_s = -3$
- From both combined: $y_\mu / y_s = 9/2$

Alternative Ratios III

[Antusch, MS '09; MS '10]

Operator Dimension	y_e/y_d
4	1
	-3
5	-1/2
	1
	$\pm 3/2$
	-3
	9/2
	6
	9
	-18

Summary of possible predictions from SU(5)

Operator Dimension	$(y_e/y_d,\ y_u/y_d)$
4	(1,1)
	(-3,1)
5	(1,1)
	(-3,1)
	(9,1)

Summary of possible predictions from Pati-Salam

Which ratios are realistic?

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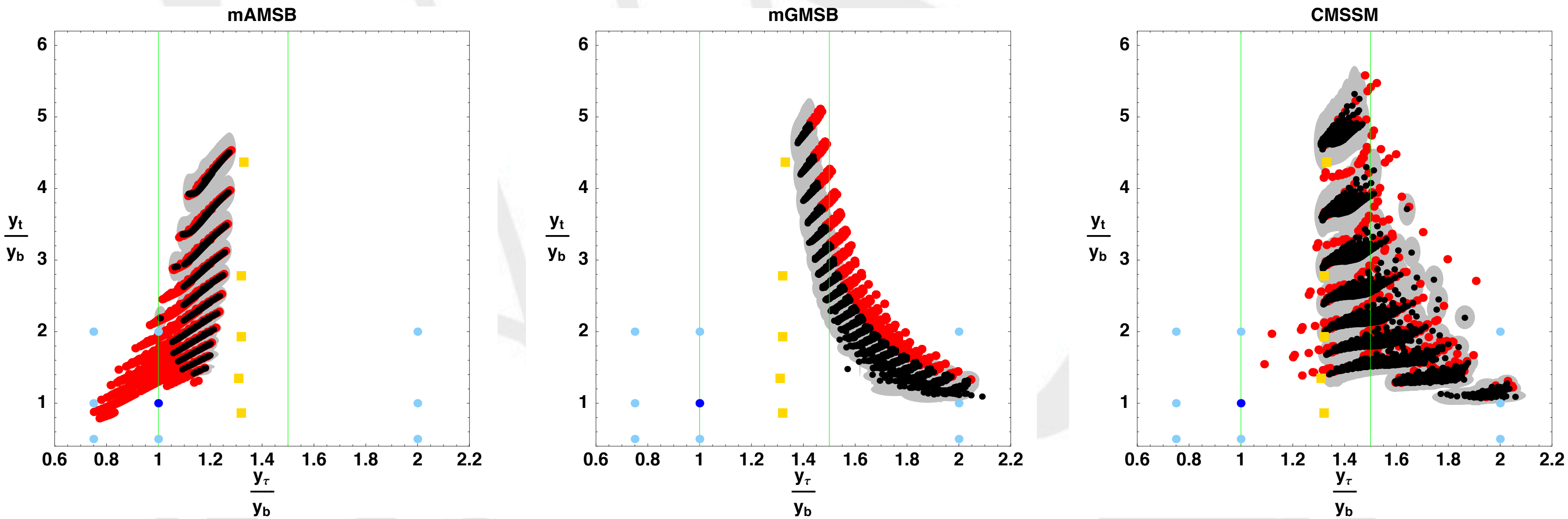
Used Data in the 2009 study

Pre LHC data

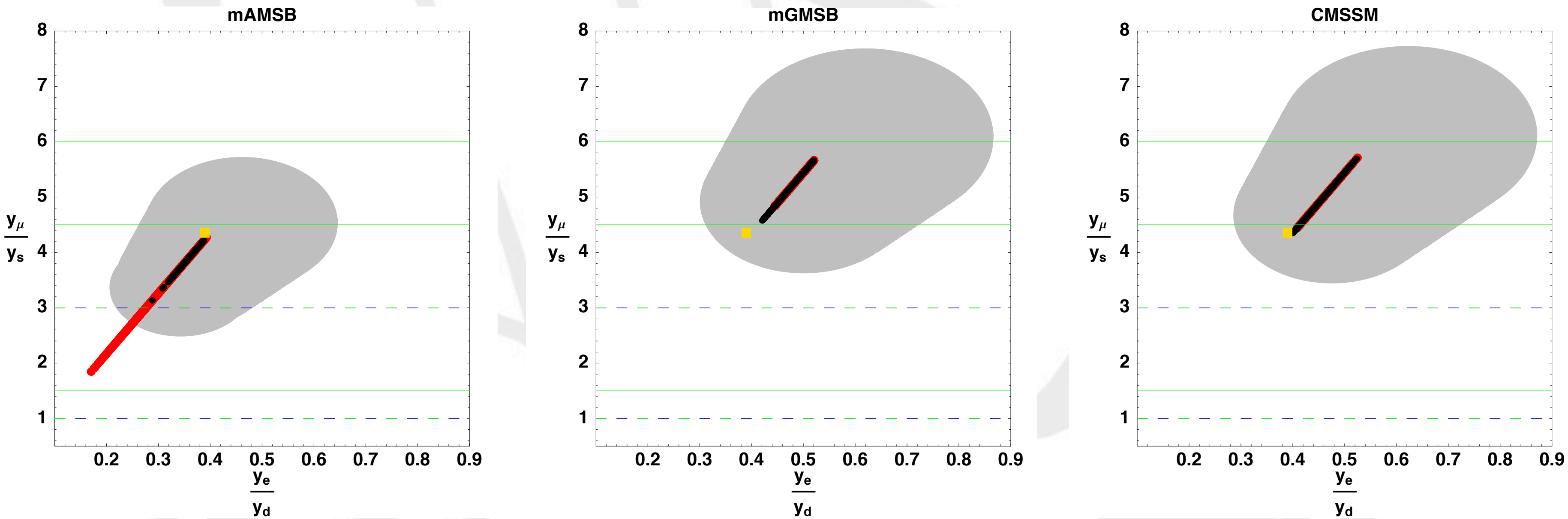
- Direct LEP searches
- Electroweak precision observables
- B-physics:
  - $BR(b \to s\gamma) = (3.55 \pm 0.36) \times 10^{-4}$
  - $BR(B_s \to \mu\mu) \leq 5.8 \times 10^{-8}$
- Anomalous magnetic moment of the muon

Results for the 3rd generation



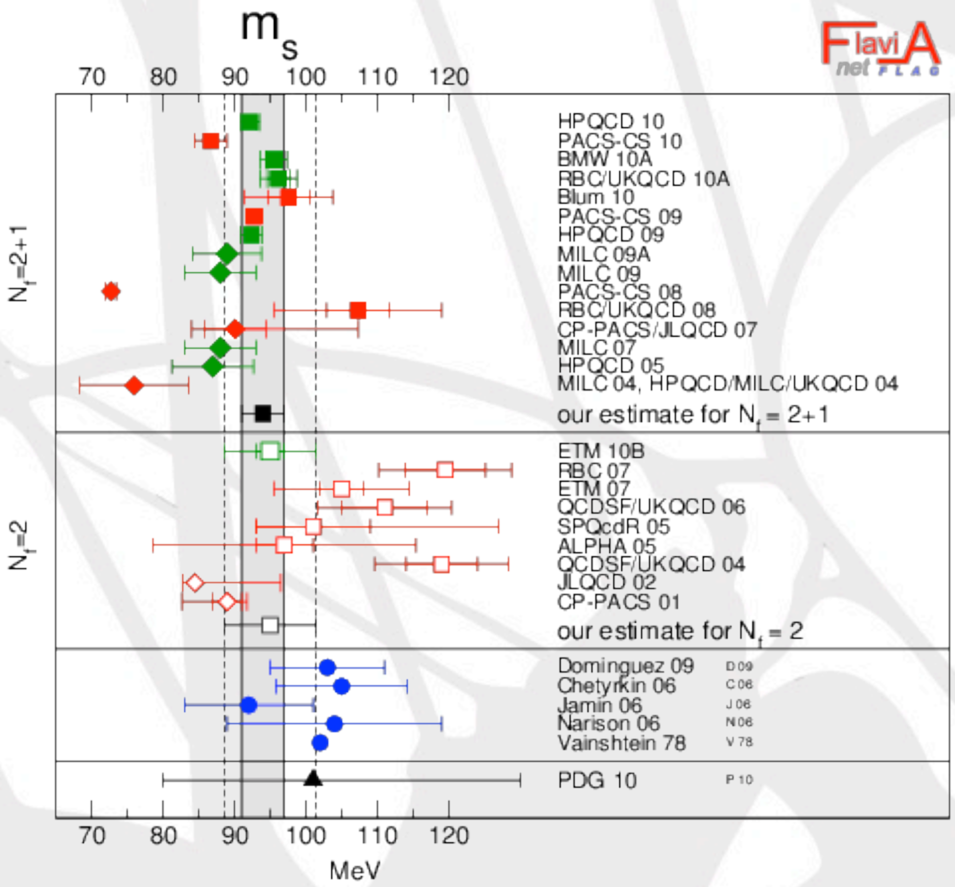
exp. allowed (2009) | exp. disfavored (2009)
mass uncertainties | w/o thresholds
SU(5) predictions | Pati-Salam predictions

Results for the 1st and 2nd generation



exp. allowed (2009)
exp. disfavored (2009)
mass uncertainties
w/o thresholds
SU(5) predictions
Pati-Salam predictions

Regarding mass uncertainties...



we took this number

[taken from http://itpwiki.unibe.ch/flag/index.php/Quark_masses on 14/01/13]

The θ_{13} study - Setup

[Marzocca *et al*. '11, see also Antusch, Maurer '11]

- Assume SU (5) relations:

$$\hat{\lambda}^D_{[12]} = \begin{pmatrix} a & b' \\ b & c \end{pmatrix} \qquad \hat{\lambda}^E_{[12]} = \begin{pmatrix} \alpha a & \beta b \\ \beta' b' & \gamma c \end{pmatrix}$$

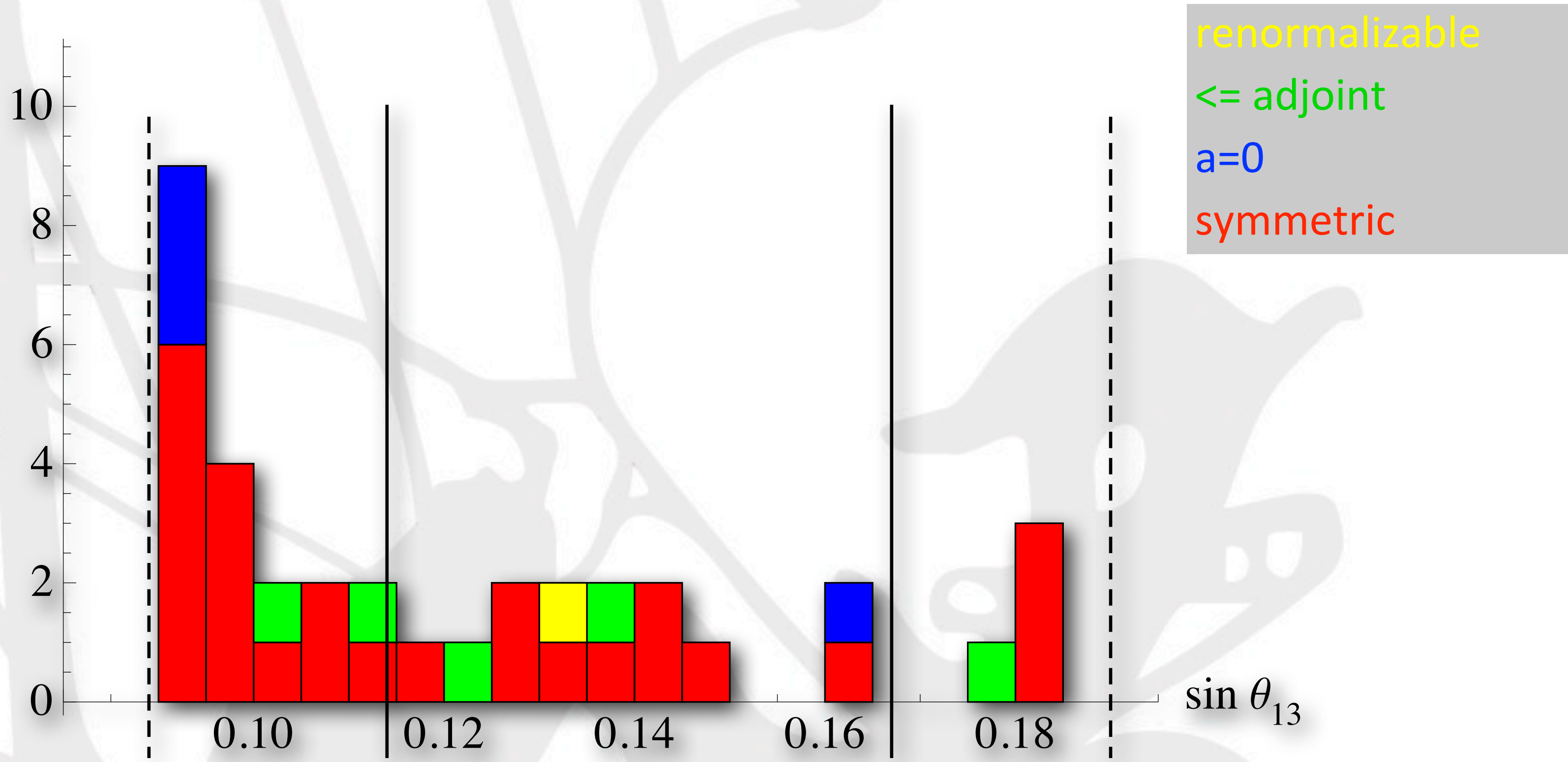
- No 1-3 mixing in the neutrino and only 1-2 mixing in the charged lepton sector:

$$\sin\theta_{13} \approx \sin\theta^e_{12} \sin\theta^\nu_{23} \approx \frac{1}{\sqrt{2}} \frac{\beta'}{\gamma} \frac{b'}{c}$$

- $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\beta'}$, $\boldsymbol{\gamma}$ are SU(5) Clebsch-Gordan coefficients (1, -3/2, -3 , 9/2, 6, ...)

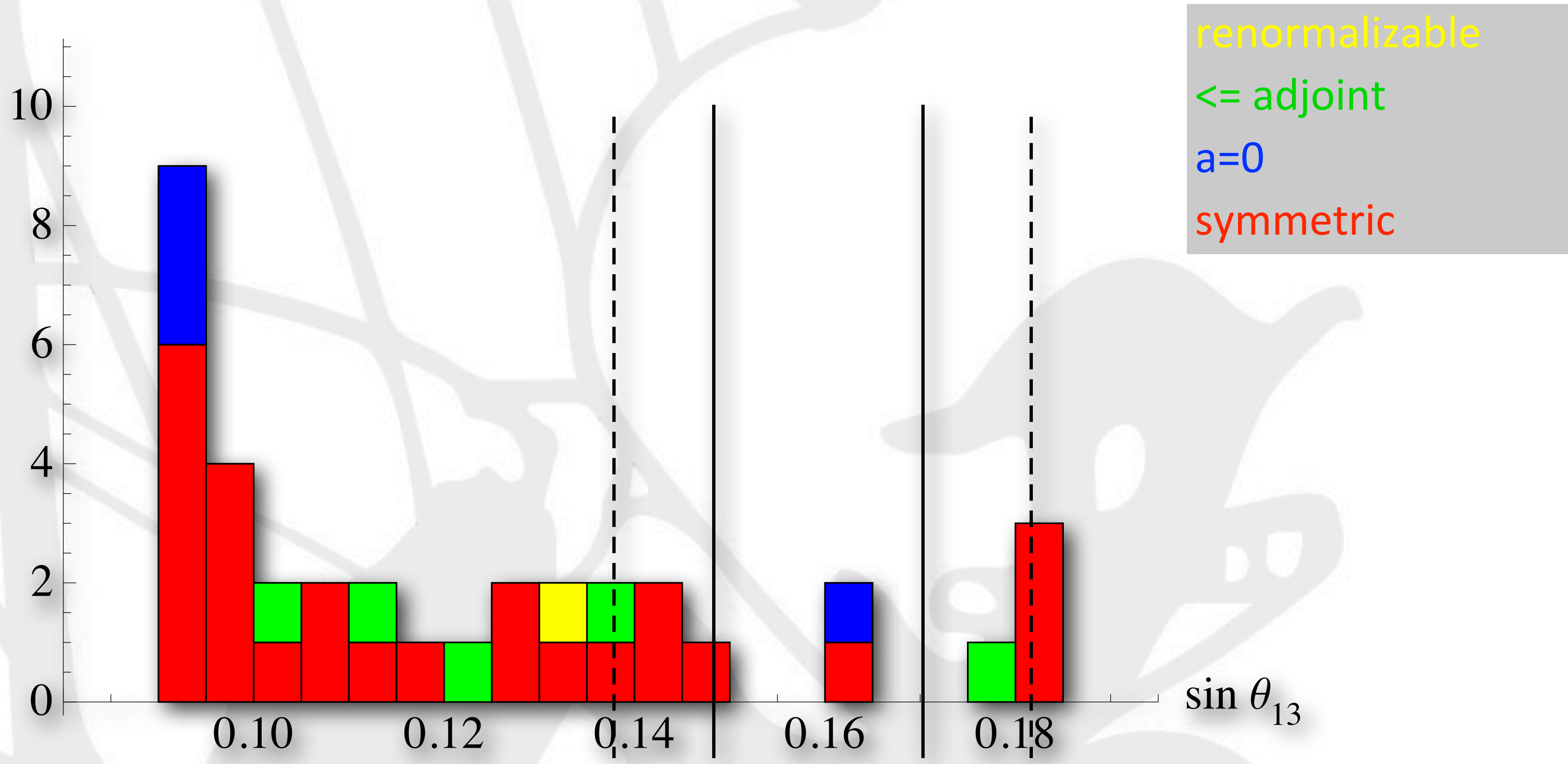
[for a list see Antusch, MS '09]

Scan results with free θ_{13} (June '11)



[Taken from Marzocca *et al.* '11, based on the global fit by Fogli *et al.* '11]

After Fitting to Exp. Data (May '12)



[Based on the global fit by Forero, Tortola, Valle '12, Thanks to D. Marzocca for providing this plot]

The Good Cases

$\{\alpha, \beta, \beta', \gamma\}$	$\sin\theta_{13}$
$\{-, -1/2, 6, 6\}$	0.164 ± 0.013
$\{-3/2, -3, -3, -3\}$	0.164 ± 0.007
$\{-18, 9/2, 9/2, 9/2\}$	0.149 ± 0.003

[Taken from Marzocca *et al.* `11]

For a model implementation see, e.g. [A. Meroni, S. T. Petcov, MS '12]

Corrections to other Mixing Angles

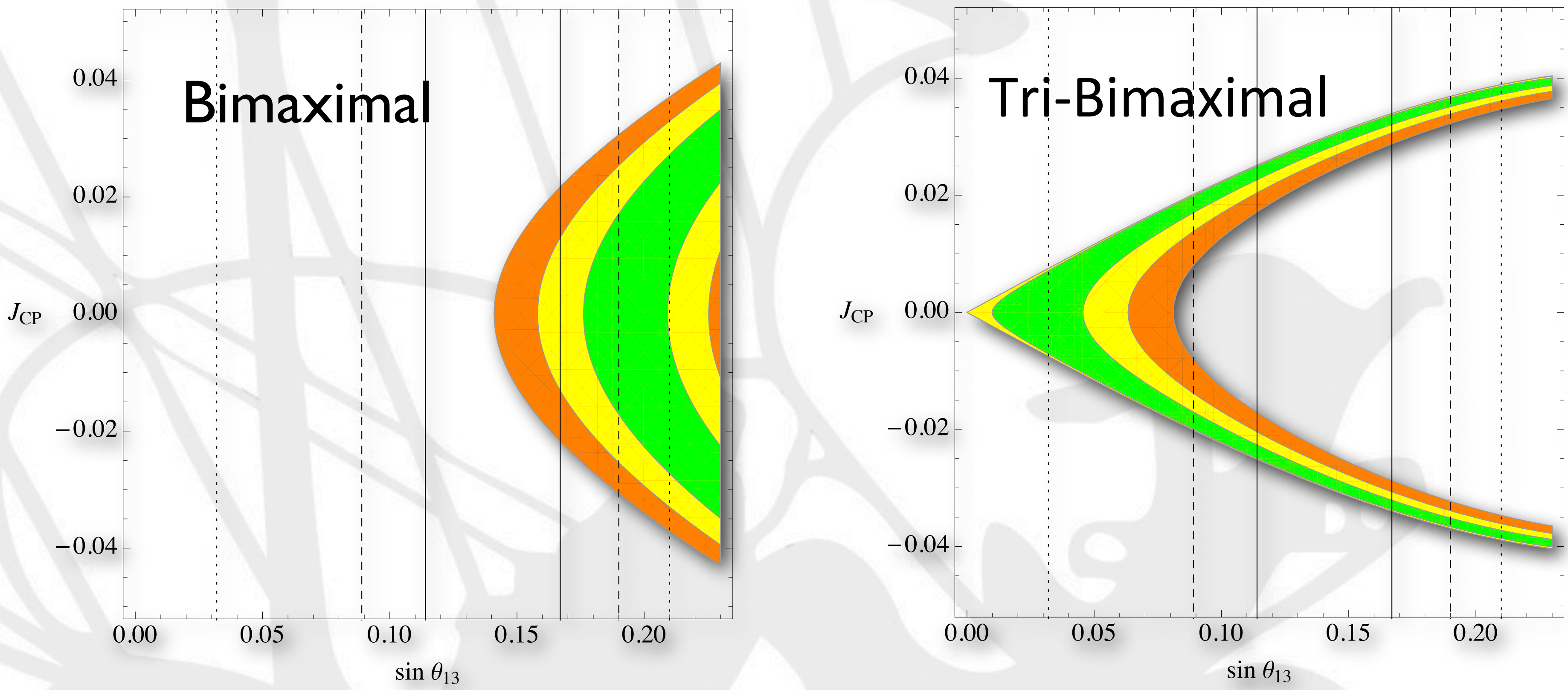
- For Bimaximal mixing:

$$\sin^2\theta_{12} \approx \frac{1}{2} + \sin\theta_{13}\cos\delta$$

- For Tri-Bimaximal mixing:

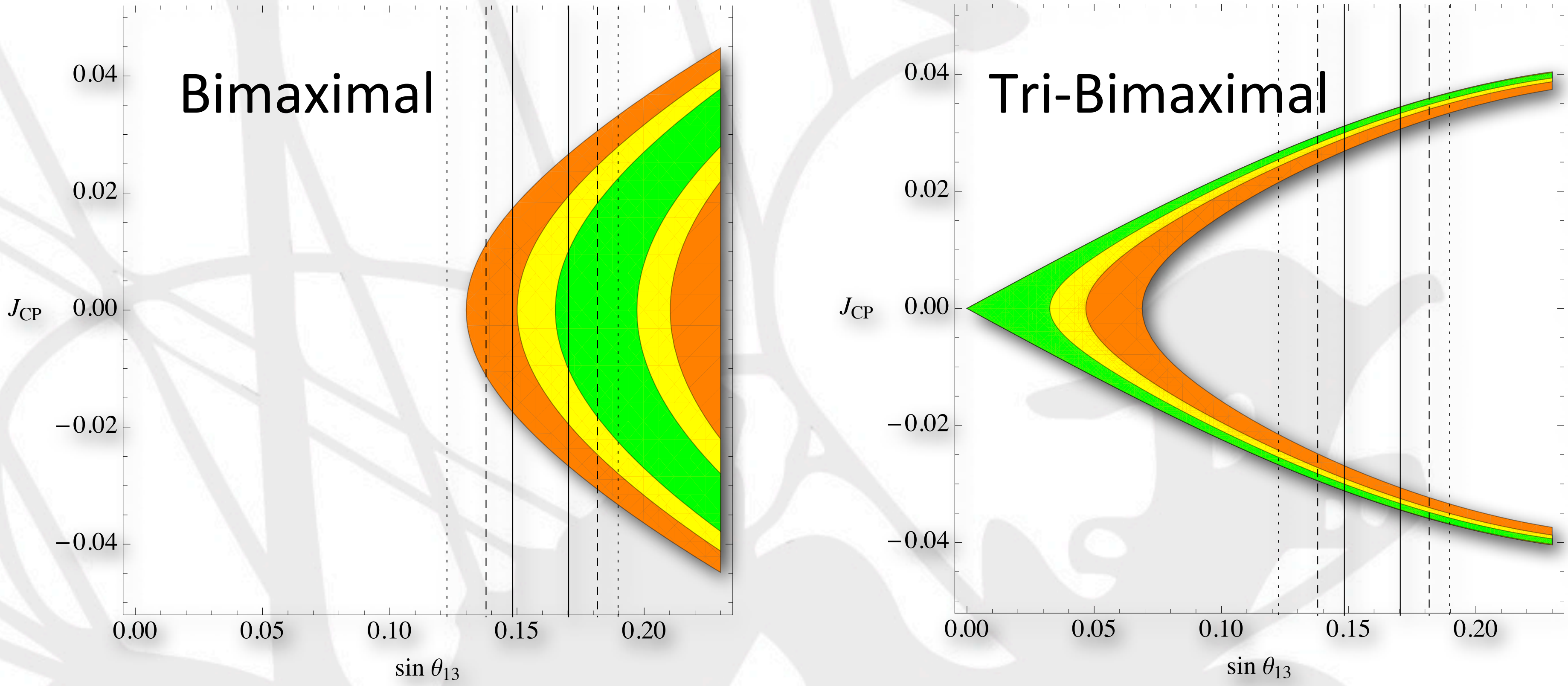
$$\sin^2\theta_{12} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3}\sin\theta_{13}\cos\delta$$

Implications for J_{CP} (June '11)



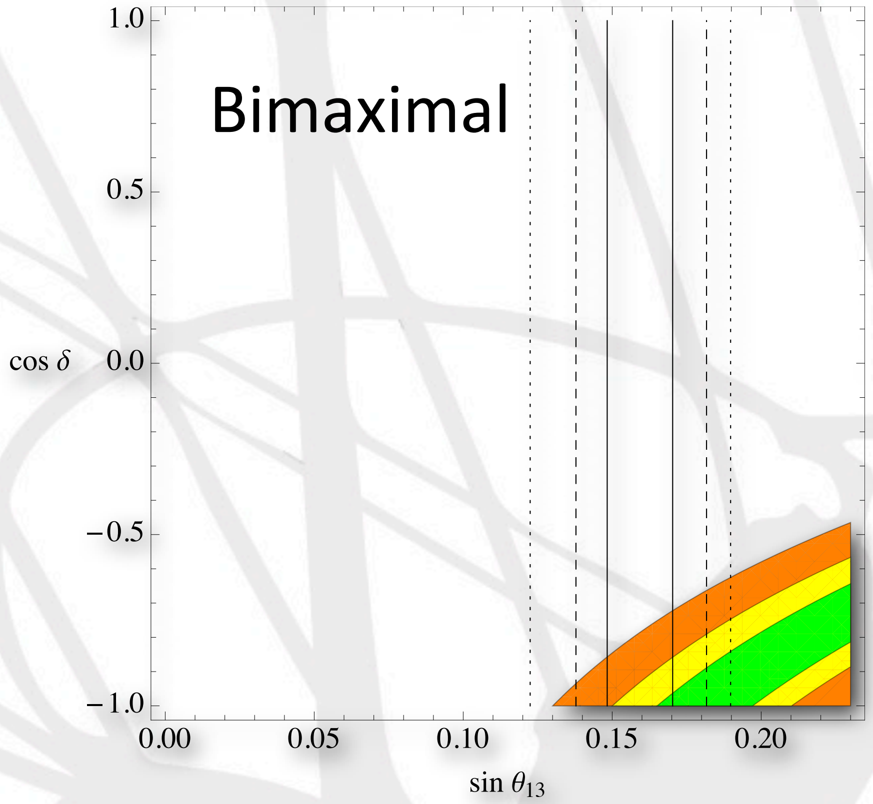
[Taken from Marzocca *et al.* '11, based on the global fit by Fogli *et al.* '11]

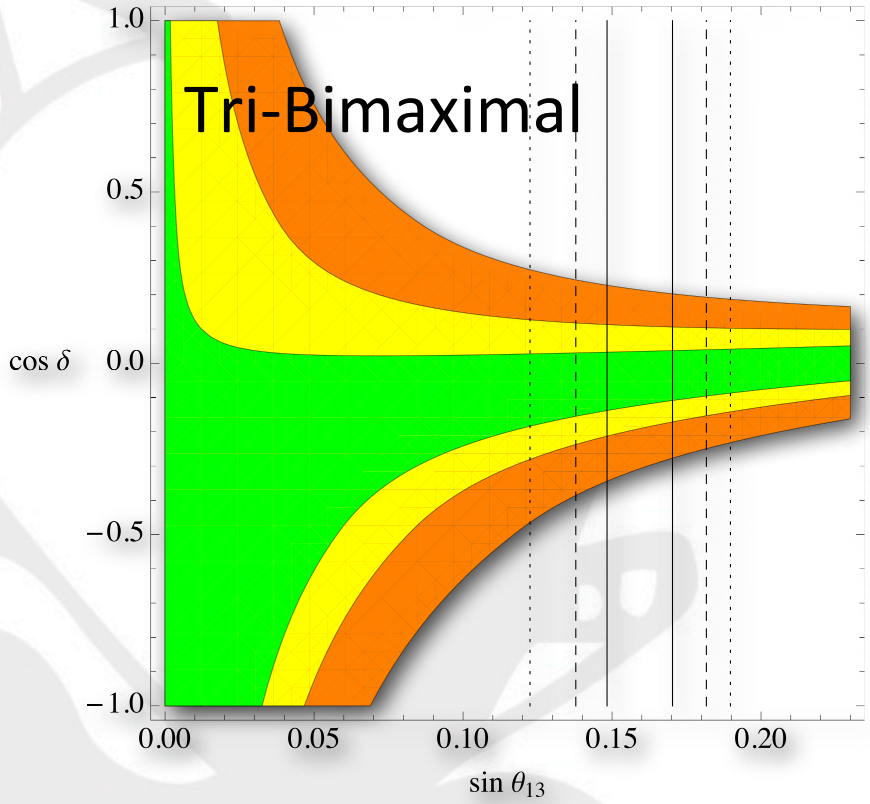
Implications for J_{CP} (May '12)



[Based on the global fit by Forero, Tortola, Valle '12, Thanks to D. Marzocca for providing this update]

Implications for δ (May '12)





[Based on the global fit by Forero, Tortola, Valle '12, Thanks to D. Marzocca for providing this update]

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Is the MSSM still natural?

LHC tells us

- Higgs boson at 125 GeV
- SUSY is above 1 TeV (a little simplified)

Fine-tuning measure (*a* GUT scale parameter):

$$\Delta = \max_a \left| \frac{\partial \log M_Z}{\partial \log a} \right| = \left| \frac{a}{2M_Z^2} \frac{\partial M_Z^2}{\partial a} \right|$$

[Barbieri, Giudice '88, ...; loop corrected: Horton, Ross '10]

What is the minimal fine-tuning in the MSSM?

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What about the scalar mass parameters?

- Recapitulate (for largish tan β):

$$M_Z^2 \approx -2|\mu|^2 - 2m_{H_u}^2$$

- For the scalar mass parameters:

$$\begin{aligned} m_{H_u}^2(M_{\mathrm{SUSY}}) = & -0.0459 m_{\tilde{Q}_1}^2 + 0.0988 m_{\tilde{U}_1}^2 - 0.0469 m_{\tilde{D}_1}^2 + 0.0488 m_{\tilde{L}_1}^2 - 0.0541 m_{\tilde{E}_1}^2 \\ & -0.3347 m_{\tilde{Q}_3}^2 - 0.2500 m_{\tilde{U}_3}^2 - 0.0154 m_{\tilde{D}_3}^2 + 0.0245 m_{\tilde{L}_3}^2 - 0.0236 m_{\tilde{E}_3}^2 \\ & +0.6481 m_{h_u}^2 + 0.0273 m_{h_d}^2 + \ldots \end{aligned}$$

[Antusch, Calibbi, Maurer, Monaco, MS `12]

- Universal boundary conditions less tuned

Gaugino masses and trilinears I

- We find: **[Antusch, Calibbi, Maurer, Monaco, MS '12]**

$$\begin{aligned} m_{H_u}^2(M_{\rm SUSY}) &= -M_3^2(1.2865 - 0.0216\eta_1 - 0.0242\eta_1^2) \\ &\quad - M_3^2(0.0230\eta_2 - 0.2177\eta_2^2 + 0.0813\eta_1\eta_2) \\ &\quad + M_3^2\eta_A(0.2904 + 0.0112\eta_1 + 0.0652\eta_2) \\ &\quad - 0.1131 M_3^2\eta_A^2 + \ldots \\ &\equiv (f_1(\eta_1,\eta_2) + f_2(\eta_1,\eta_2)\eta_A + f_3\eta_A^2)M_3^2 + \ldots \end{aligned}$$

- where we have set ($\eta_{1,2}$ fixed):

$$A_0 = M_3\eta_A \quad , \quad M_{1,2} = M_3\eta_{1,2}$$

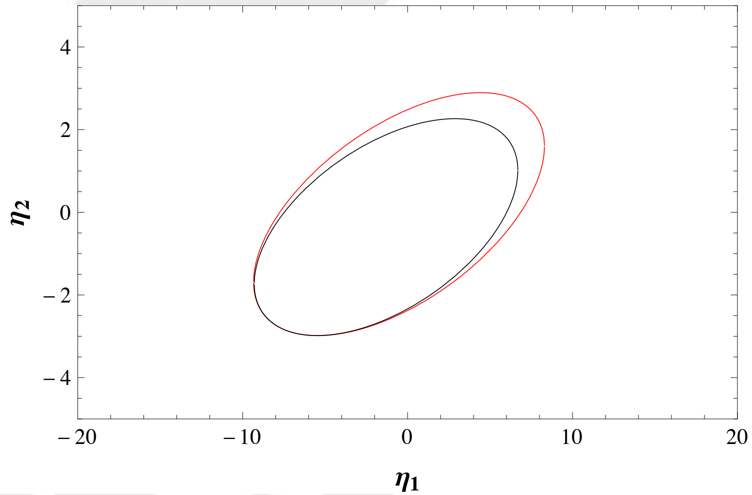
Gaugino masses and trilinears II

- Two solutions with vanishing fine-tuning in this sector

$$1)\ f_1(\eta_1, \eta_2) = 0, \qquad \eta_A = 0$$

$$2)\ f_1(\eta_1, \eta_2) = \frac{f_2(\eta_1, \eta_2)^2}{4 f_3}, \qquad \eta_A = -\frac{f_2(\eta_1, \eta_2)}{2 f_3}$$

- Corresponding to two ellipses in the η_1-η_2 plane



[Antusch, Calibbi, Maurer, Monaco, MS '12]

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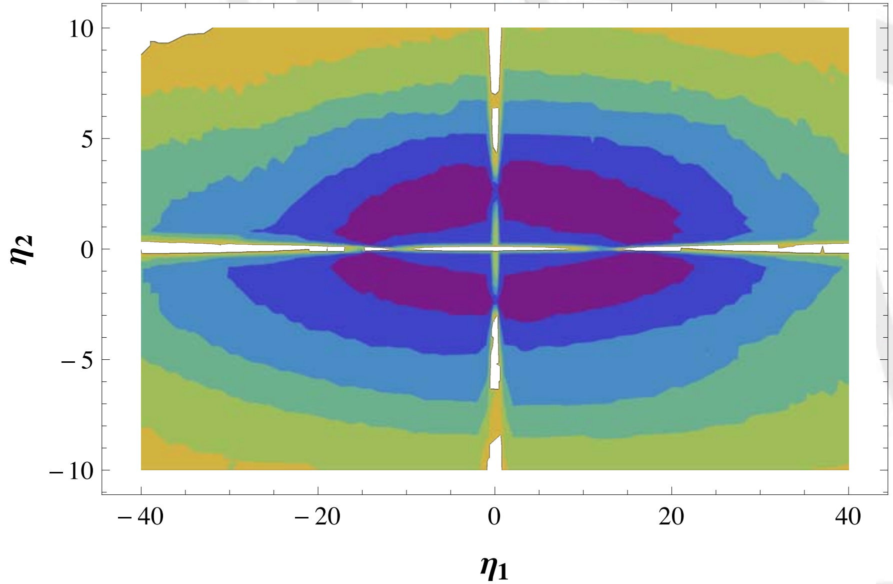
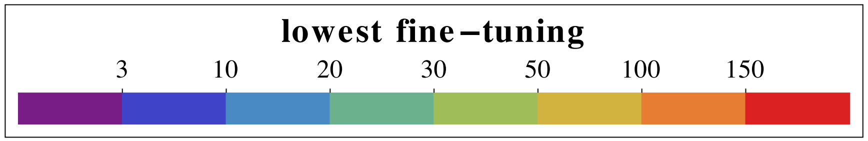
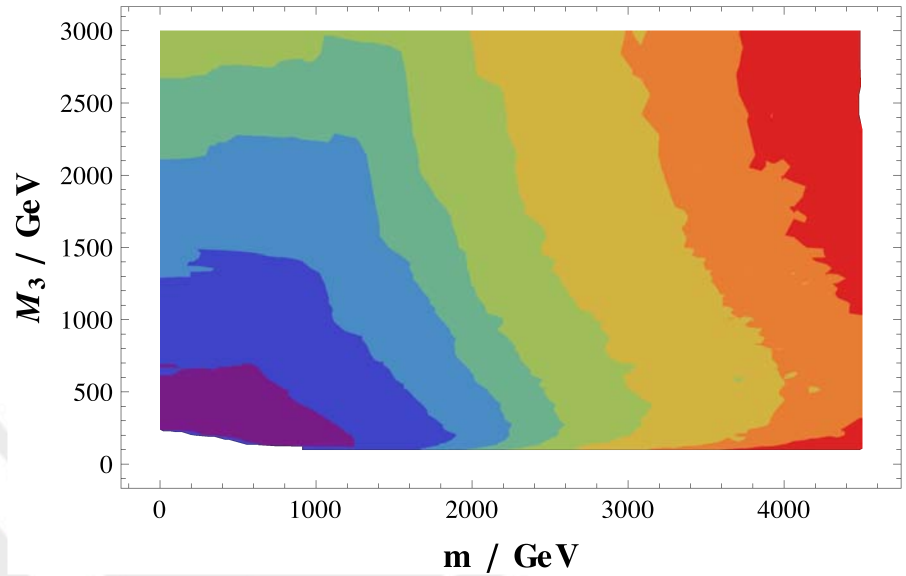
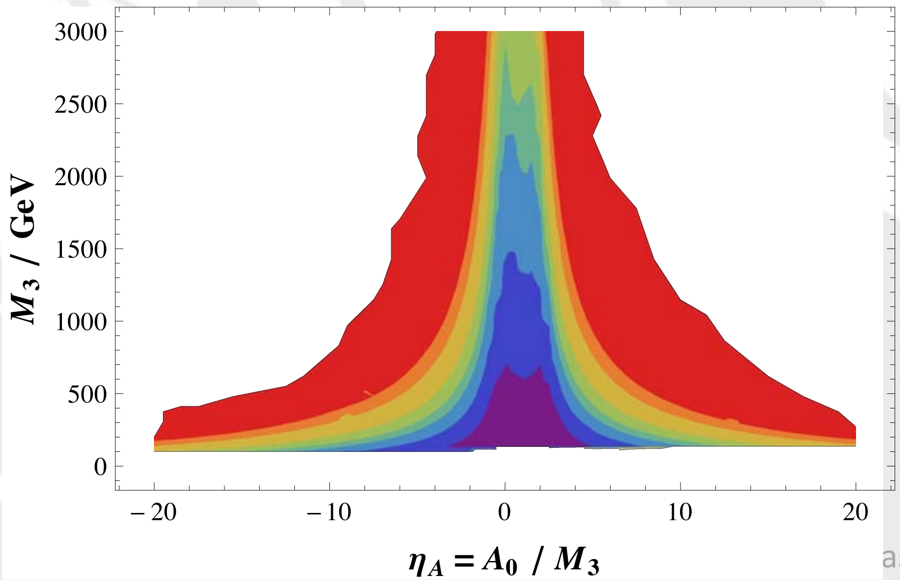
Before the Higgs bound



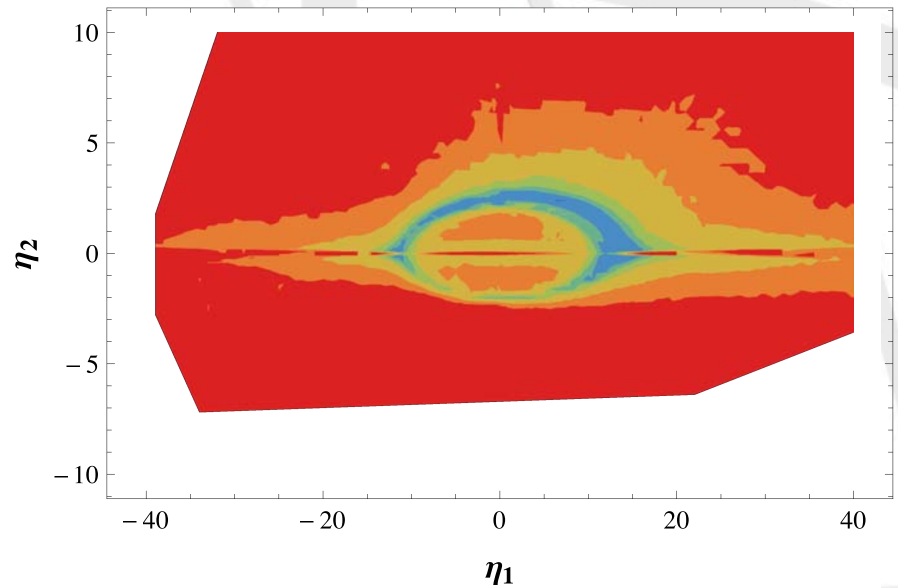
[Antusch, Calibbi, Maurer, Monaco, MS '12]



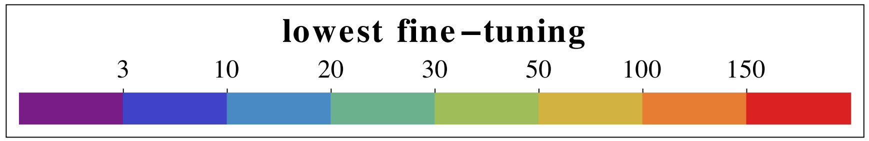


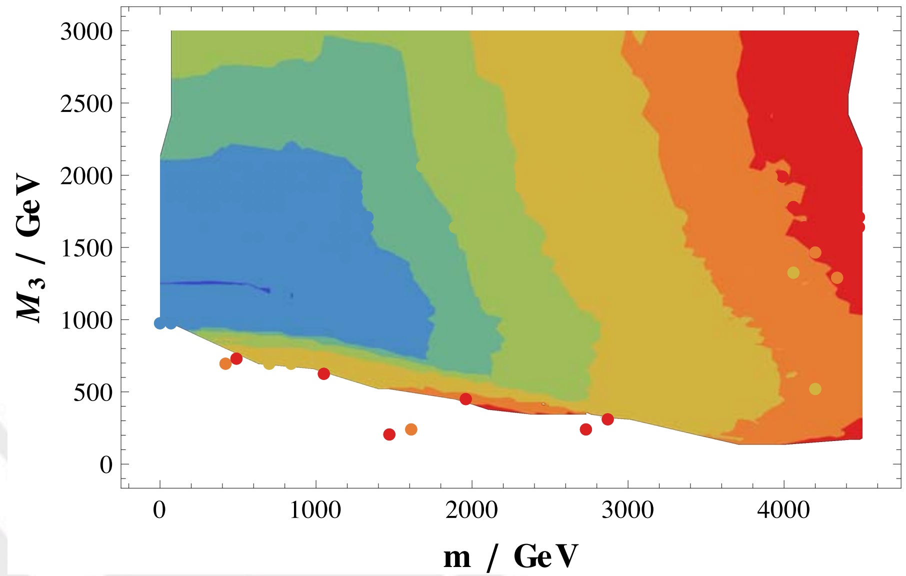


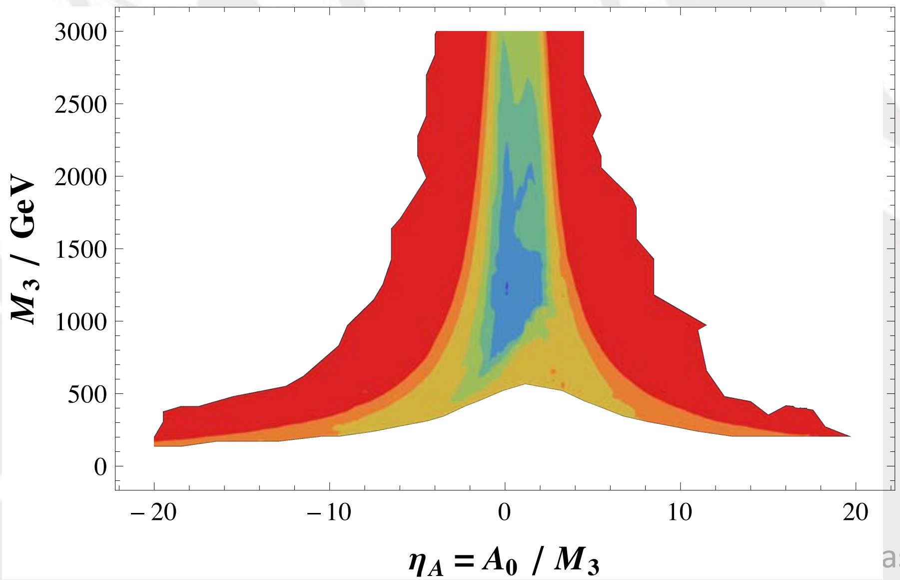
After the Higgs bound



[Antusch, Calibbi, Maurer, Monaco, MS '12]





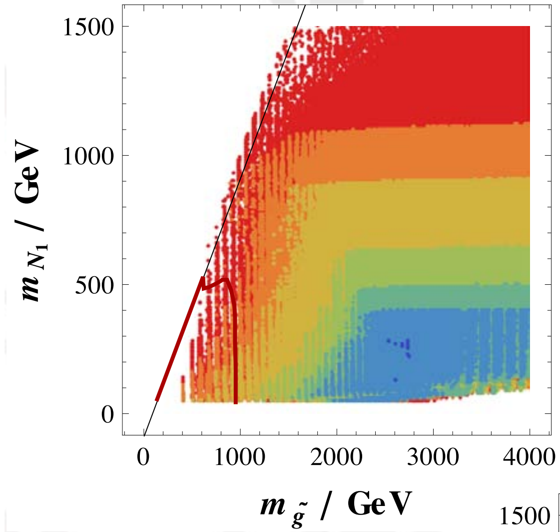


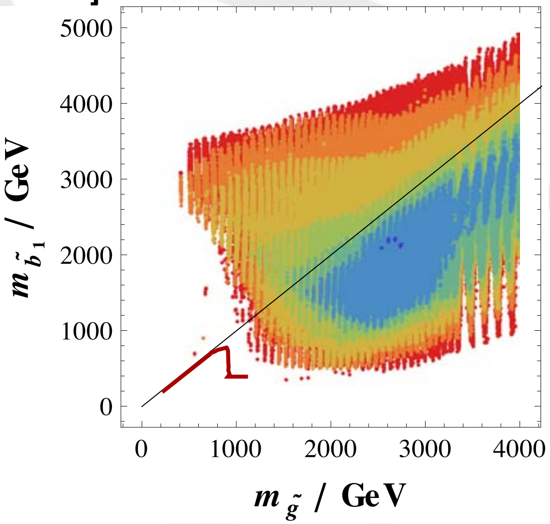
$m_h = 125.3 \pm 0.6_{ex} \pm 3_{th}$ GeV

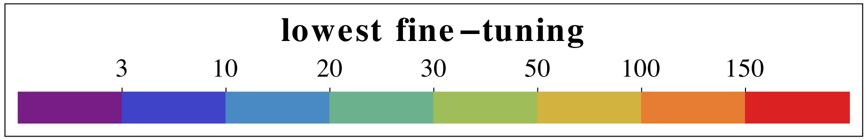
[ATLAS, CMS '12]

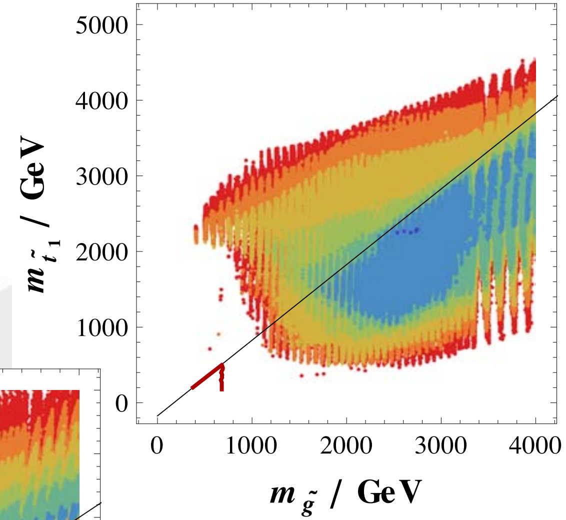
LHC bounds

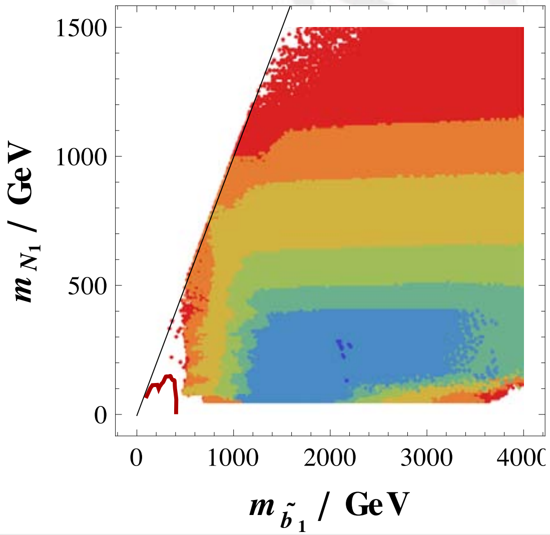
[Antusch, Calibbi, Maurer, Monaco, MS '12]

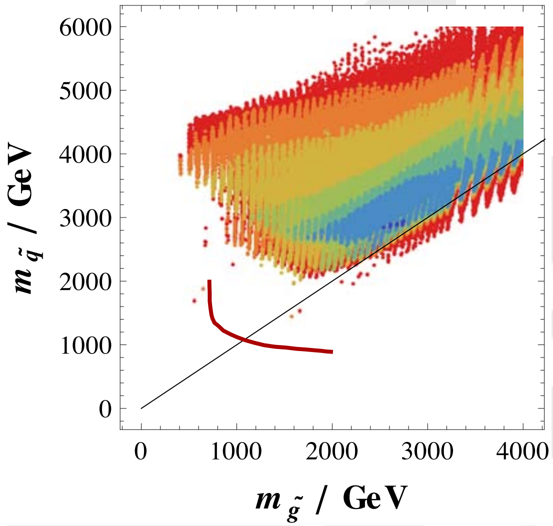






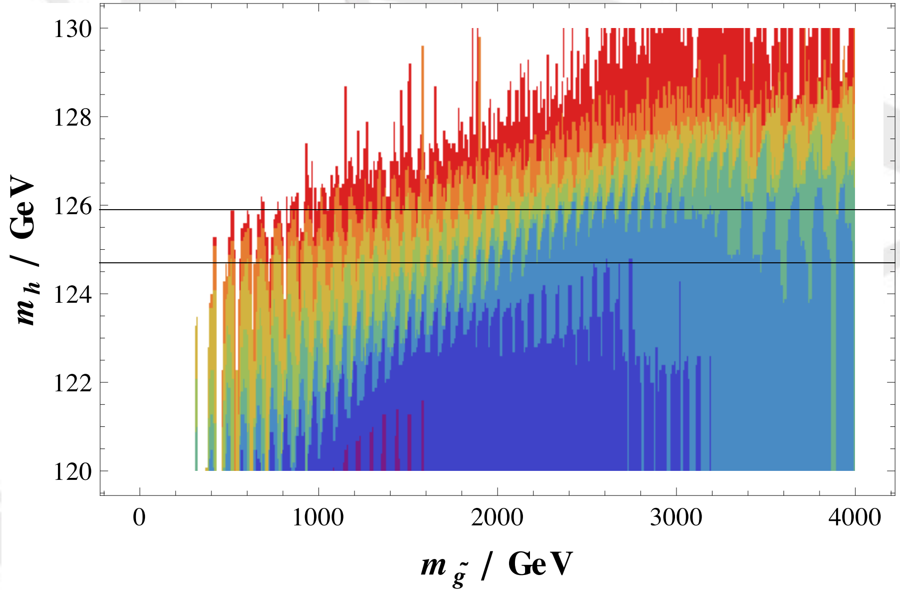


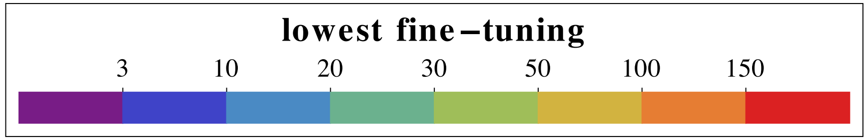


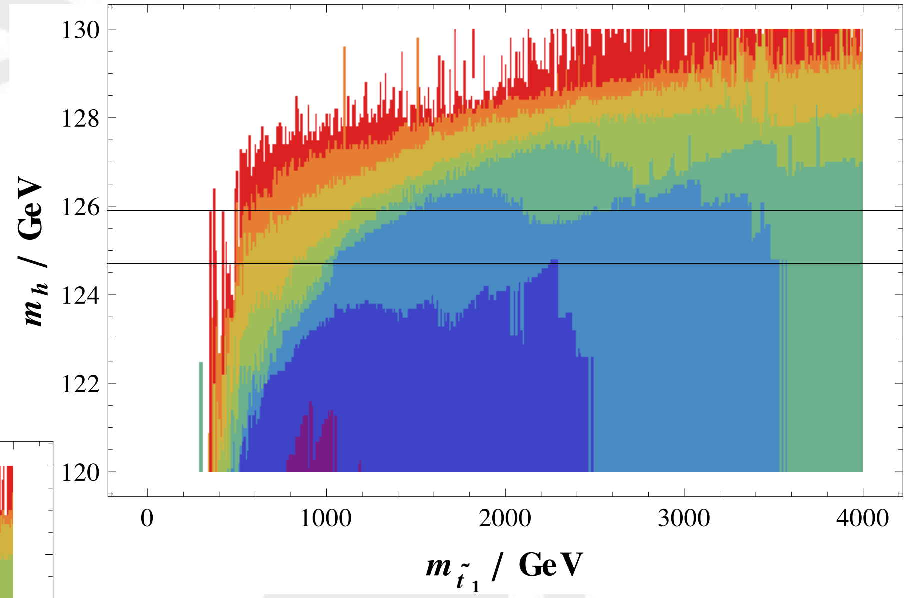


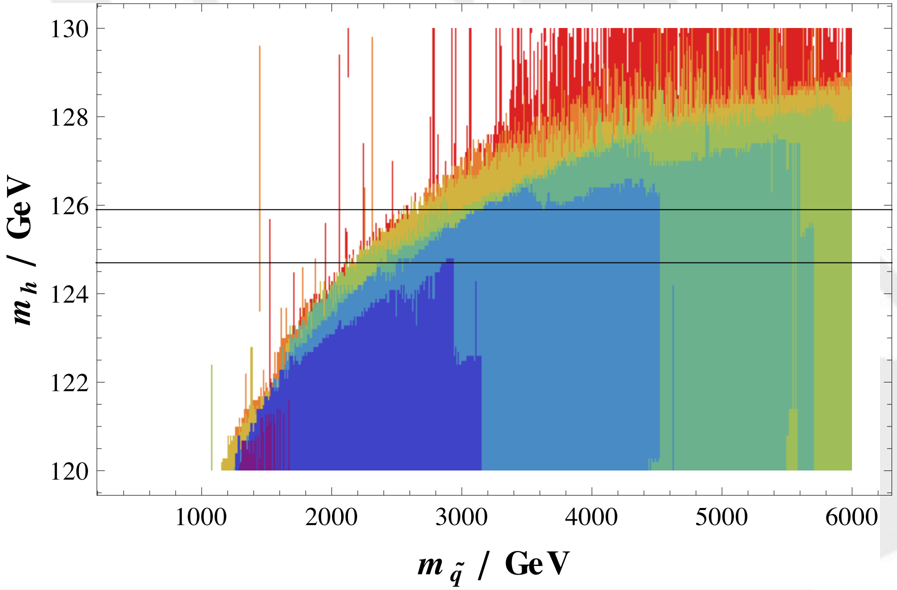
[Based on ATLAS and CMS SUSY searches]

Mass correlations I



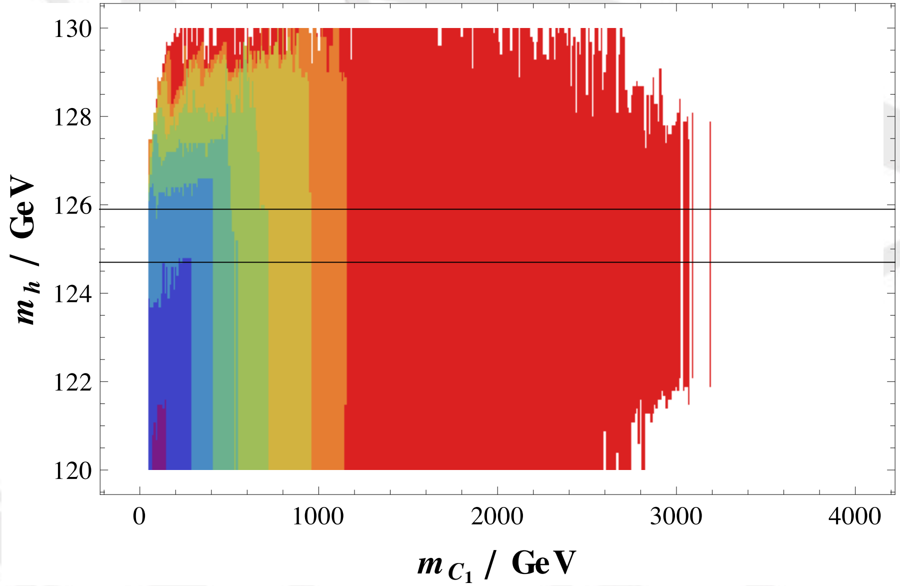


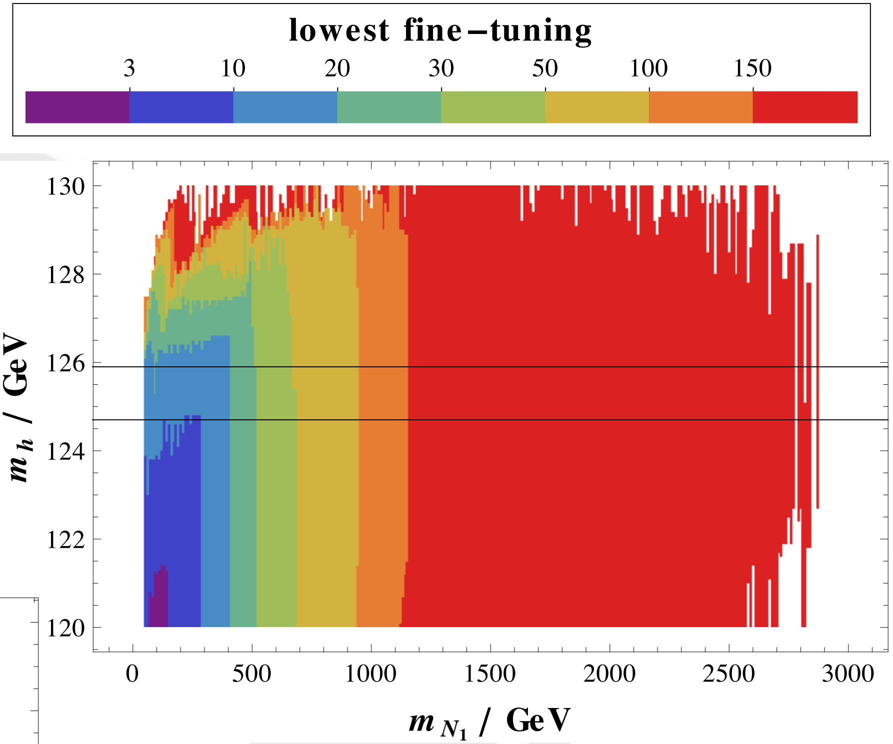


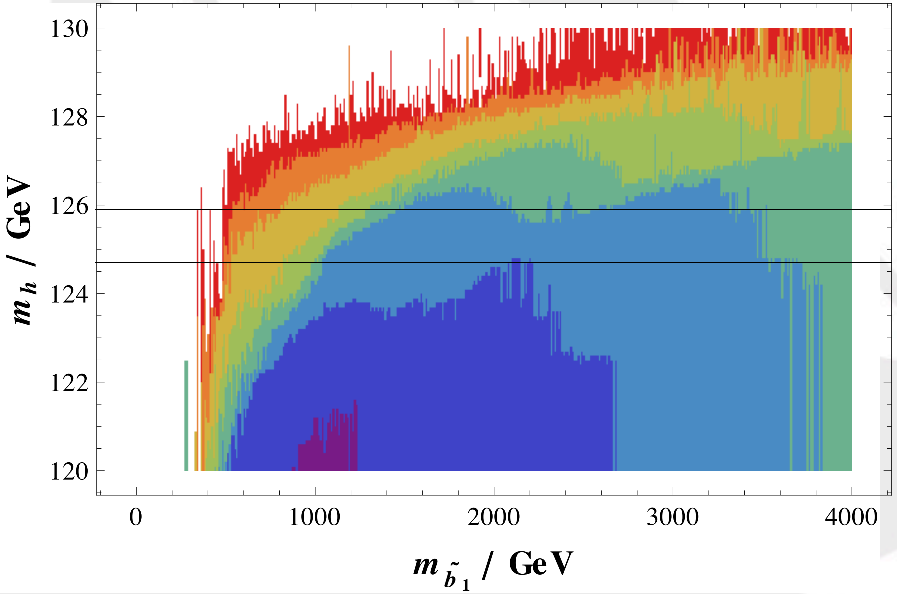


[Antusch, Calibbi, Maurer, Monaco, MS '12]

Mass correlations II

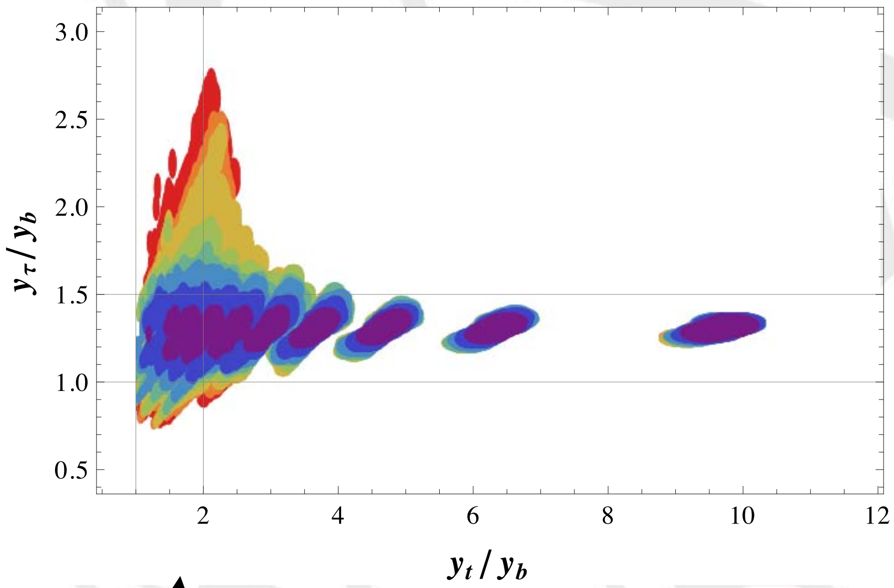


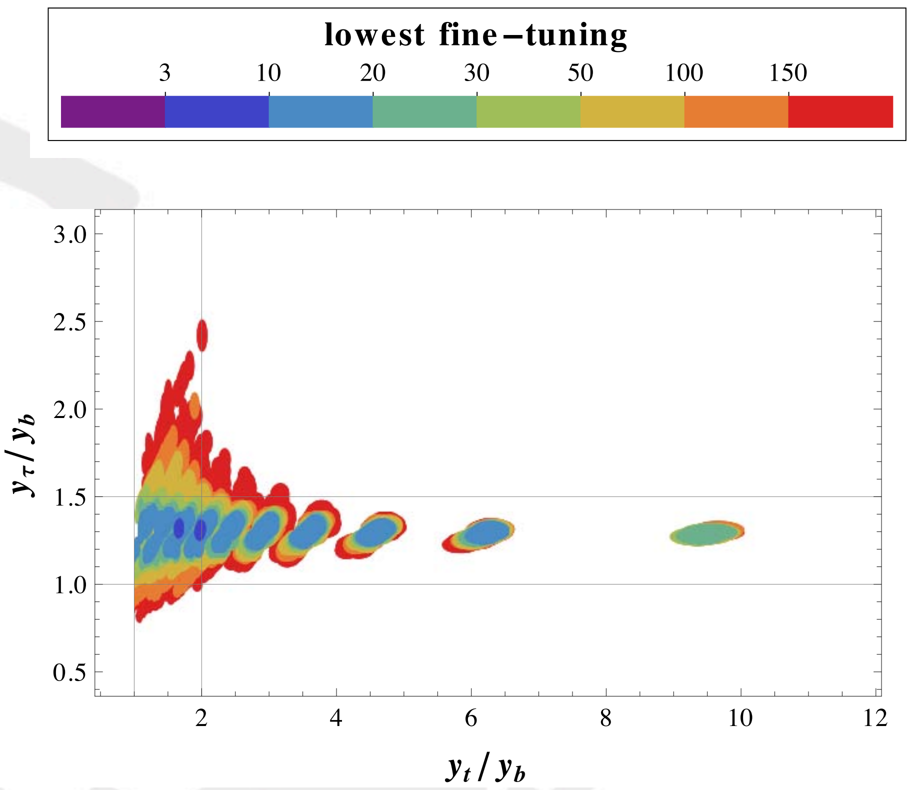




[Antusch, Calibbi, Maurer, Monaco, MS '12]

Fermion Mass Ratios





[Antusch, Calibbi, Maurer, Monaco, MS '12]

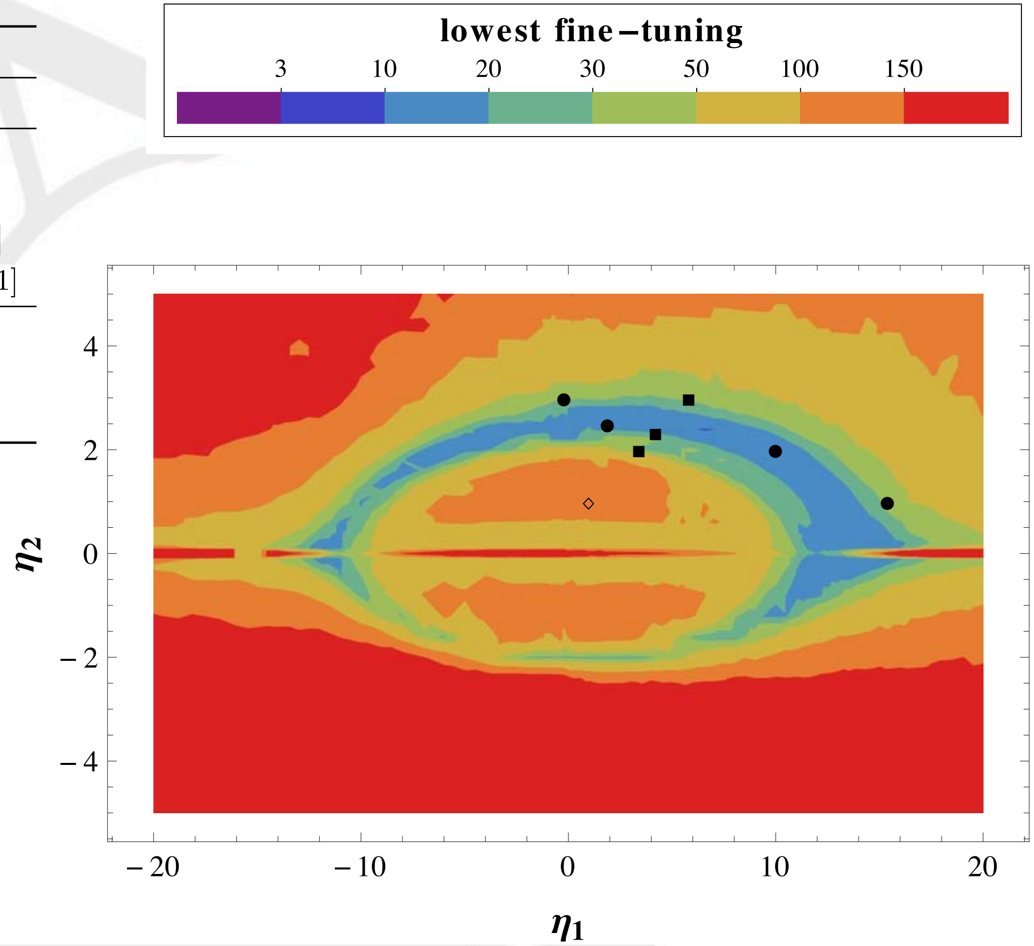
before/after the Higgs:
no clear preference for a ratio

Comparison to Models

η_1, η_2	$\Delta_{\rm min}$	Origin
1, 1	118	CMSSM (Gaugino Unification)
10, 2	12	200 of SU(5) [1]
$\frac{19}{10}, \frac{5}{2}$	18	770 of $SO(10) \to (1,1)$ of $SU(4) \times SU(2)_R$ [1]
$\frac{77}{5}, 1$	36	770 of $SO(10) \to (1,0)$ of $(SU(5)' \times U(1))_{\rm flipped}$ [1]
$-\frac{1}{5}, 3$	46	210 of $SO(10) \to (75,0)$ of $(SU(5)' \times U(1))_{\rm flipped}$ [1]
$\frac{21}{5}, \frac{7}{3}$	13	O-II with $\delta_{\rm GS} = -6$ [2]
$\frac{17}{5}, 2$	28	O-II with $\delta_{\rm GS} = -7$ [2]
$\frac{29}{5}, 3$	44	O-II with $\delta_{\rm GS} = -5$ [2]

[1] Chakrabortty, Raychaudhuri '09; Martin '09
[2] Brignole, Ibanez, Munoz '94, '95; Horton, Ross '09

[Antusch, Calibbi, Maurer, Monaco, MS `12]



Another gauge mediation example:

$(\eta_1, \eta_2) = (13, 17/7) \approx (13, 2.43)$

[Brümmer, Ibe, Yanagida '13]

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Summary and Conclusions I

- There are interesting GUT ratios beyond the standard ones
    - y_μ/y_s = 9/2, 6 and y_τ/y_b = 3/2
- Low energy data constrains GUT models
    - LHC constrains the SUSY spectrum
    - Neutrino data might rule out (tri-)bimaximal mixing with charged lepton corrections

Summary and Conclusions II

- CMSSM is fine-tuned (Δ > 110)
- Little fine-tuning with non-universal gaugino masses (Δ = O(10)) for Δ < 20:
  - gluino: 2-3 TeV
  - squarks: 2.5-4.5 TeV
  - lighter stop: 1.0-3.5 TeV

どうもありがとう

Backup

Fine-tuning in the top mass

For the fine-tuning in measured quantities we have adopted:

$$\Delta_{y_t} = \left| \frac{\sigma_{t_y}}{2M_Z^2} \frac{\partial M_Z^2}{\partial y_t} \right|$$

[For a discussion of this issue, see: Carlos, Casas '93; Giusti, Romanino, Strumia '99; Romanino, Strumia '00; Feng, Matchev '00]