Minimal SUSY SU(5) GUT in High-scale SUSY

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(accepted for publication in PLB).
J. Hisano, D. Kobayashi, T. Kuwahara, N. Nagata, 1304.3651 .

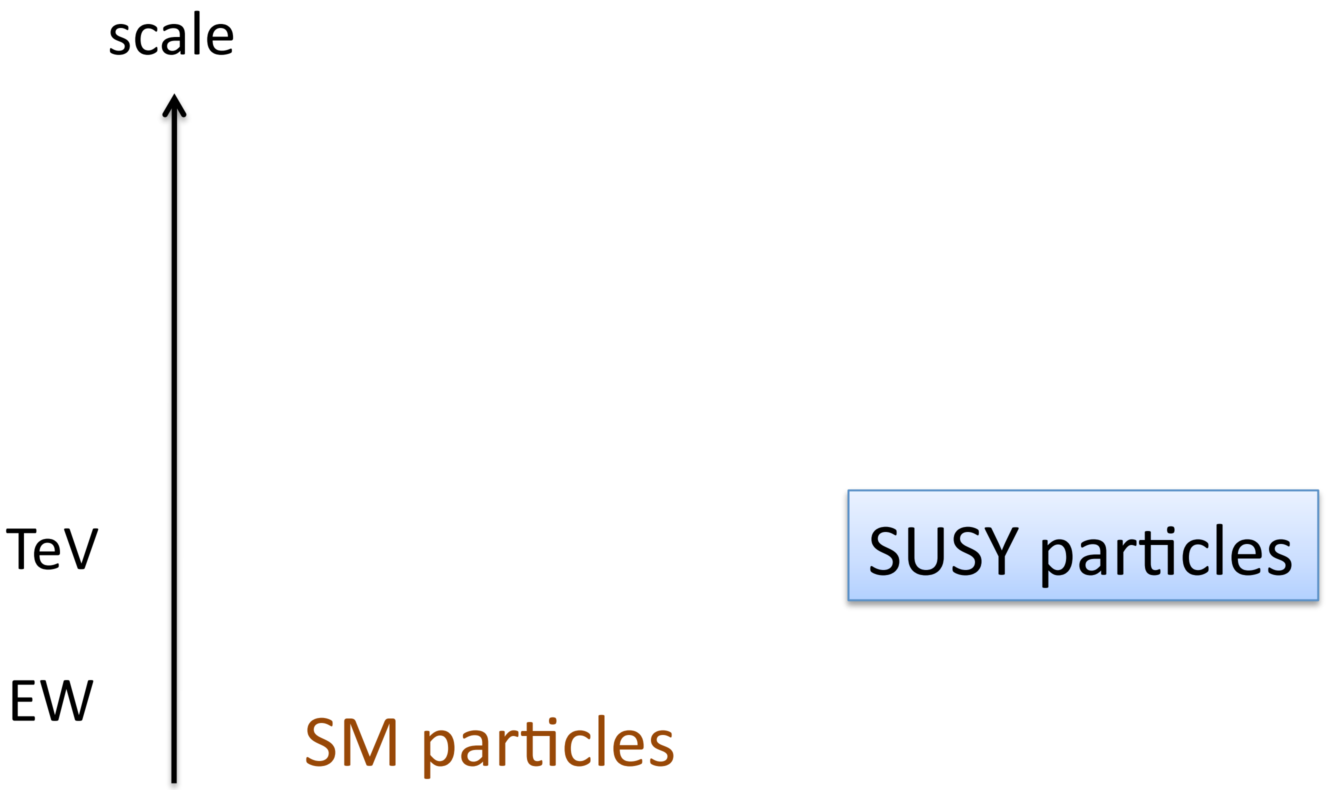
Outline

1. Introduction
2. GUT scale mass spectrum in high-scale SUSY
3. Proton decay in high-scale SUSY
4. Summary

1. Introduction

SUSY SM

Supersymmetric (SUSY) Standard Model (SM)



SUSY SM

Supersymmetric (SUSY) Standard Model (SM)

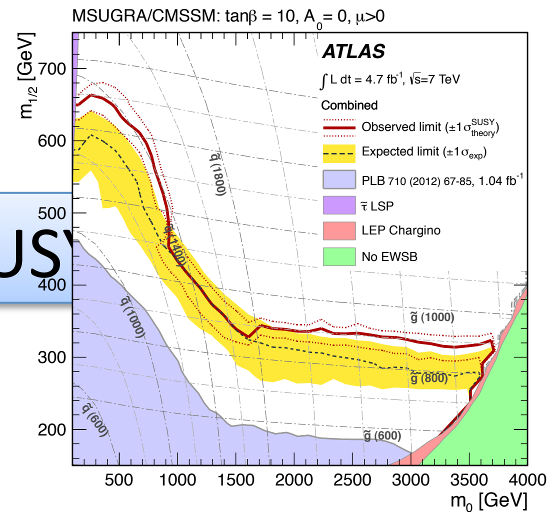
scale

TeV

EW

SM particles

SUS

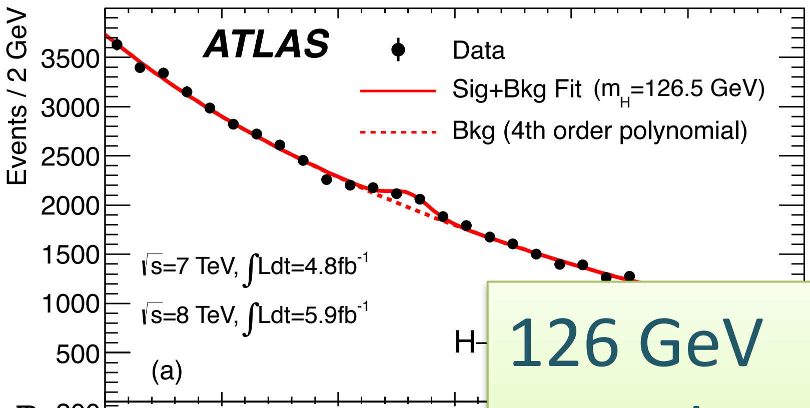


Direct search

SUSY SM

Supersymmetric (SUSY) Standard Model (SM)

scale

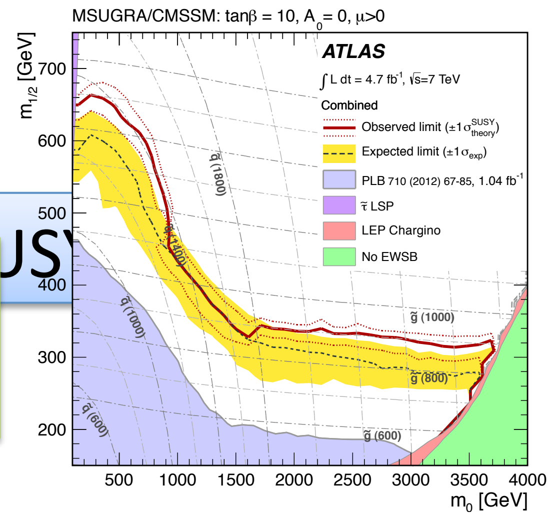


EW

SM particles

126 GeV
Higgs boson

SUSY



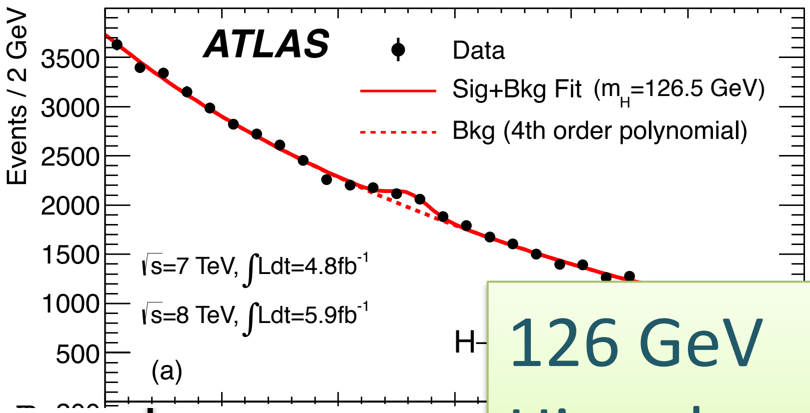
Direct search

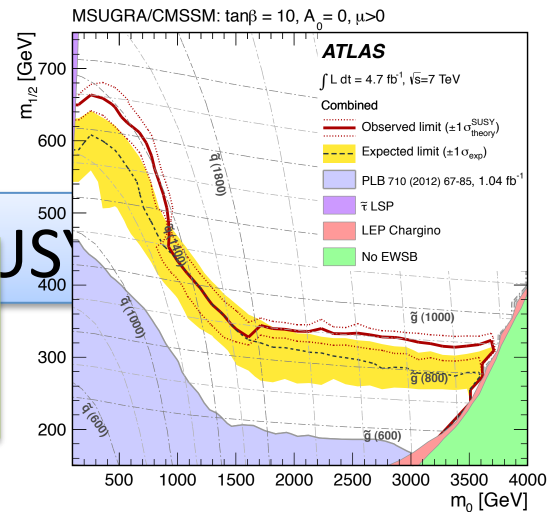
SUSY SM

Supersymmetric (SUSY) Standard Model (SM)

scale

High-scale SUSY ??





EW

SM particles

126 GeV
Higgs boson

Direct search

Attractive features

High-scale SUSY scenario has a lot of fascinating aspects from a phenomenological point of view.

- 126 GeV Higgs boson can be achieved
  (sufficient radiative corrections)
- SUSY flavor & CP problems are relaxed
  (suppressed by sfermion masses)
- Gravitino problem is avoided
  (heavy gravitino)
- DM candidates (wino/higgsino)

w/ light fermions (chiral symmetries)

Today's topic

Additional features of high-scale SUSY in the context of grand unified theories (GUTs):

(i) Gauge coupling unification is improved.

(ii) Proton decay problem in the minimal SUSY SU(5) GUT is avoided.

Defects in the GUTs with low-scale SUSY

Shortcomings of low-scale SUSY GUTs:

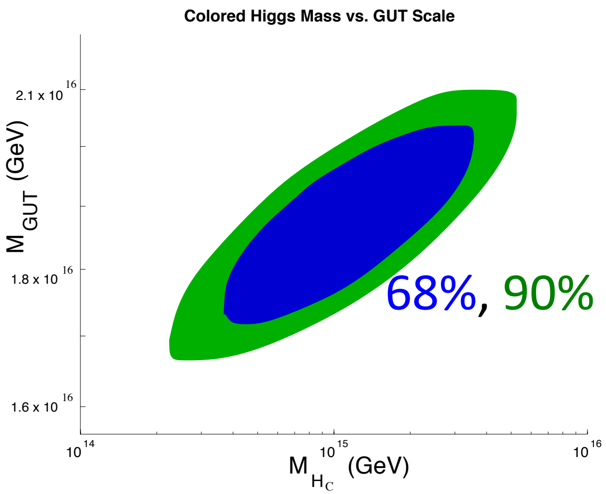
(i) Large threshold corrections at the GUT scale are required to realize the gauge coupling unification.

(ii) Protons decay too fast (Proton decay problem). Some additional mechanism is required to suppress the proton decay.

Gauge coupling unification in low-scale SUSY

In low-energy SUSY, the mass of color-triplet Higgs multiplet is found to be much lower than the GUT scale (RGE analysis).

$$3.5 \times 10^{14}\ \mathrm{GeV} \lesssim M_{H_C} \lesssim 3.6 \times 10^{15}\ \mathrm{GeV}$$



H. Murayama and A. Pierce (2001)

$$\begin{aligned} M_S &= 1\ \mathrm{TeV} \\ M_2 &= 200\ \mathrm{GeV} \\ M_3/M_2 &= 3.5 \\ \tan\beta &= 1.8-4 \end{aligned}$$

Errors are from input parameters.

This implies that the threshold corrections to the gauge coupling constants are still not small, even in the SUSY GUTs.

Defects in the GUTs with low-scale SUSY

Shortcomings of low-scale SUSY GUTs:

(i) Large threshold corrections at the GUT scale are required to realize the gauge coupling unification.

(ii) Protons decay too fast (Proton decay problem). Some additional mechanism is required to suppress the proton decay rate.

Proton decay in low-scale SUSY

Proton decay is induced by the exchanges of

- SU(5) gauge bosons (X-bosons) ⟶ Dim-6 operators
- Color-triplet Higgs multiplets ⟶ Dim-5 operators

yield dominant decay modes, such as $p \to K^+ \bar{\nu}$

Predicted lifetime (in low-scale SUSY): $\tau(p \to K^+ \bar{\nu}) \lesssim 10^{30}$ yrs

T. Goto and T. Nihei (1999), H. Murayama and A. Pierce (2001).

Experimental constraints: $\tau(p \to K^+ \bar{\nu}) > 4.0 \times 10^{33}$ yrs

Super-Kamiokande, arXiv: 1109.3262.

- Minimal SUSY GUT is excluded
- needs some extensions to suppress the dim-5 proton decay

(PQ symmetry, higher-dimensional models, etc.)

Today's topic

Additional features of high-scale SUSY:

(i) Gauge coupling unification is improved

All of the superheavy particles in the minimal SUSY GUT can lie around the GUT scale.

➡ Small threshold corrections are required.

(ii) Proton decay problem in the minimal SUSY SU(5) GUT is avoided

➡ Decoupling can revive the minimal SUSY SU(5) GUT

Simple solution. No need for additional mechanism.

Assumption

A chiral supermultiplet X responsible for ~~SUSY~~ is charged under some symmetry.

➡ ~~SUSY~~ is transferred via operators involving $X^\dagger X$

Soft masses of the scalar particles arise from

$$\frac{1}{M_{\rm Pl}^2}\int d^4\theta X^\dagger X Q^\dagger Q$$

(M_{Pl}: the reduced Planck scale)

Scalar/gravitino mass

➡ $$m_{\rm scalar} \sim \frac{F_X}{M_{\rm Pl}} \sim m_{3/2}$$

VEV of X field

$$X \to \theta\theta F_X$$

Gaugino mass

Since the field X is charged under some symmetry, the gaugino mass terms are not given by the X field linear terms.

$$\int d^2\theta \, X W^\alpha W_\alpha$$

In this case, the gaugino masses are generated by

Anomaly mediation

L. Randall and R. Sundrum (1998)

G.F. Giudice, M.A. Luty, H. Murayama, R. Rattazzi (1998)

$$M_a \simeq -\frac{b_a g_a^2}{16\pi^2} m_{3/2} \qquad b_a = (-33/5, -1, 3)$$

⇒ Wino is the lightest in the gaugino sector

Higgsinos

Origin of Higgsino mass is somewhat model-dependent.

- On the assumption of a generic Kahler potential

$$\mu \sim m_{3/2}$$

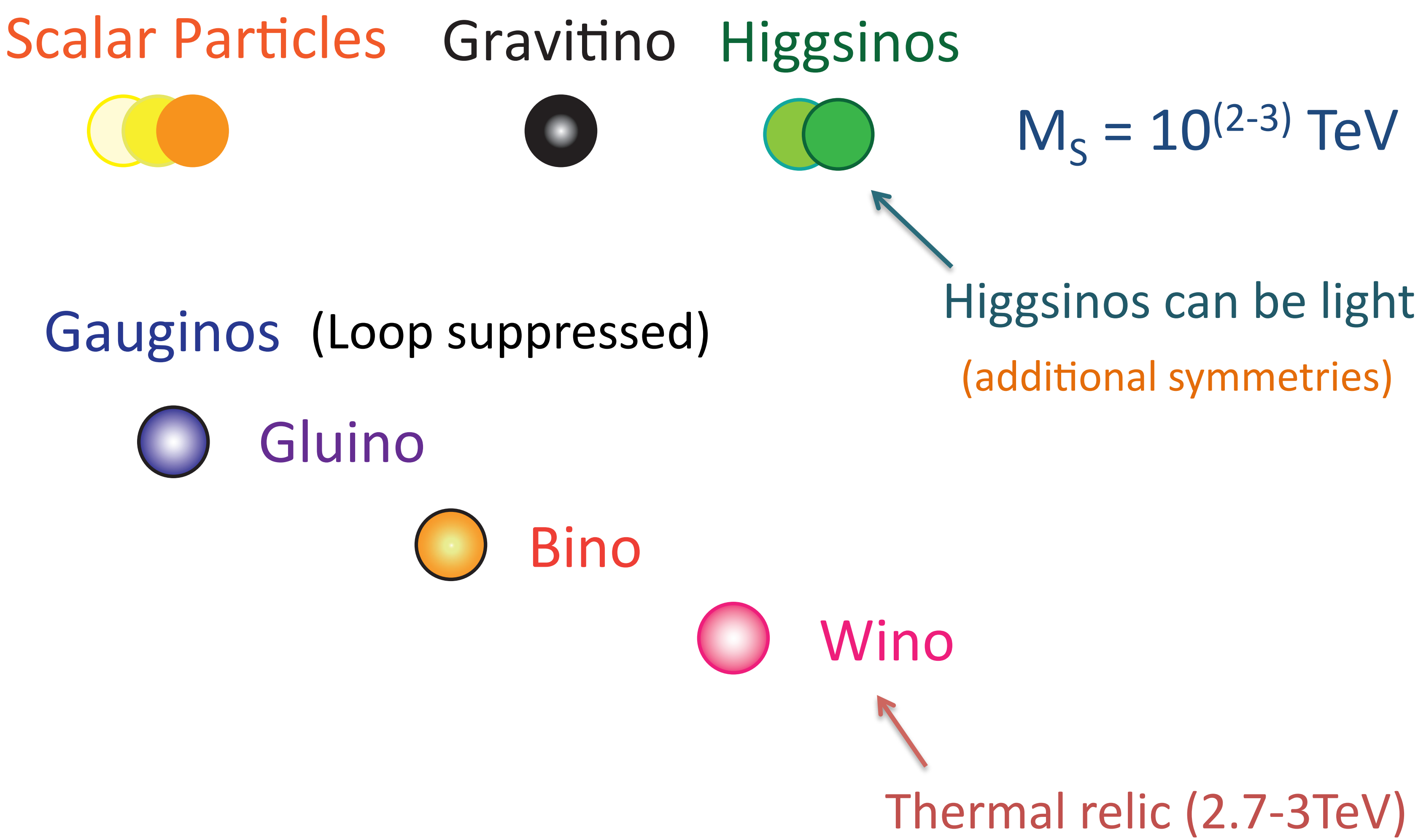
- It can be suppressed by some symmetry.

(*e.g.*, Peccei-Quinn symmetry)

⇒ Higgsinos can be much lighter than the gravitino.

We regard the higgsino mass as a free parameter in the following discussion.

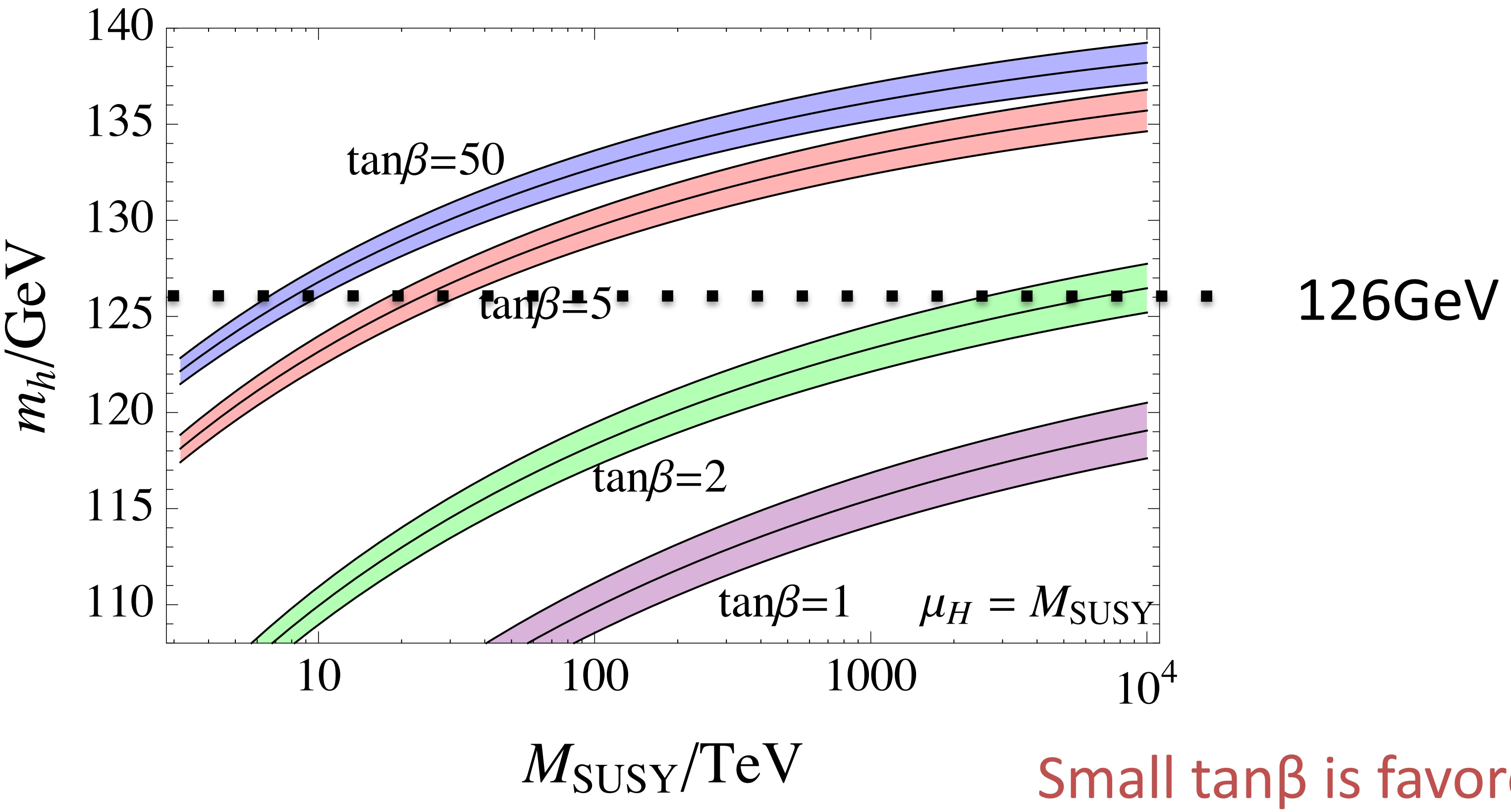
Mass spectrum



J. Hisano, S. Matsumoto, M. Nagai, O. Saito, M. Senami (2006).

Higgs mass

The 126 GeV Higgs boson mass is easily accounted.



Small tanβ is favored

m_t= 173.2 ± 0.9 GeV

M. Ibe, T.T. Yanagida (2012).

2. GUT scale mass spectrum in high-scale SUSY

Minimal SUSY SU(5) GUT

S. Dimopoulos and H. Georgi (1981)
N. Sakai (1981)

MSSM matter fields are embedded in a $\bar{5} \oplus 10$ representation

$$\Phi = \begin{pmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \\ E \\ -N \end{pmatrix}, \qquad \Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{U}_3 & -\bar{U}_2 & U^1 & D^1 \\ -\bar{U}_3 & 0 & \bar{U}_1 & U^2 & D^2 \\ \bar{U}_2 & -\bar{U}_1 & 0 & U^3 & D^3 \\ -U^1 & -U^2 & -U^3 & 0 & \bar{E} \\ -D^1 & -D^2 & -D^3 & -\bar{E} & 0 \end{pmatrix},$$

MSSM Higgs fields are embedded into

$$H = \begin{pmatrix} H_C^1 \\ H_C^2 \\ H_C^3 \\ H_u^+ \\ H_u^0 \end{pmatrix}, \qquad \bar{H} = \begin{pmatrix} \bar{H}_{C1} \\ \bar{H}_{C2} \\ \bar{H}_{C3} \\ H_d^- \\ -H_d^0 \end{pmatrix},$$

Color-triplet Higgs multiplets

induce the baryon #
violating interactions

MSSM Higgs superfields

(M_{HC} : mass of color-triplet Higgs)

Minimal SUSY SU(5) GUT

Gauge superfields

$$\mathcal{V}=\frac{1}{\sqrt{2}}\begin{pmatrix} G-\frac{2}{\sqrt{30}}B & \begin{matrix} X^{\dagger 1} & Y^{\dagger 1} \\ X^{\dagger 2} & Y^{\dagger 2} \\ X^{\dagger 3} & Y^{\dagger 3}\end{matrix} \\ \begin{matrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \end{matrix} & \begin{matrix} \frac{1}{\sqrt{2}}W^3+\frac{3}{\sqrt{30}}B & W^+ \\ W^- & -\frac{1}{\sqrt{2}}W^3+\frac{3}{\sqrt{30}}B \end{matrix} \end{pmatrix} .$$

X bosons (mass: M_X)

MSSM gauge superfields

induce baryon # violating interactions

Adjoint Higgs superfields SU(5) → $SU(3)_C \times SU(2)_L \times U(1)_Y$ $\langle\Sigma\rangle = V\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$

$$\Sigma=\begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(3^*,2)} & \Sigma_3 \end{pmatrix}+\frac{1}{2\sqrt{15}}\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}\Sigma_{24} .$$

SM singlet

Adjoint Higgs (mass: M_Σ)

Longitudinal mode of X multiplets

RGE analysis

The superheavy particles yield threshold corrections to the SM gauge couplings at the GUT scale (μ_{GUT})

$$\alpha_i^{-1}(\mu_{GUT}) = \alpha_G^{-1}(\mu_{GUT}) + \text{threshold corrections}$$

Unified gauge coupling

depending on the masses of superheavy particles

obtained through the renormalization group equations (RGEs)

Indirect method

By using the relation, we can estimate the masses of superheavy particles at the GUT scale.

J. Hisano, H. Murayama, T. Yanagida (1992).

Threshold corrections @ GUT scale

Threshold corrections (1-loop in $\overline{\text{DR}}$ scheme)

$$\frac{1}{g_1^2(\mu)} = \frac{1}{g_G^2(\mu)} + \frac{1}{8\pi^2}\left[\frac{2}{5}\ln\frac{\mu}{M_{H_C}} - 10\ln\frac{\mu}{M_X}\right] ,$$
$$\frac{1}{g_2^2(\mu)} = \frac{1}{g_G^2(\mu)} + \frac{1}{8\pi^2}\left[2\ln\frac{\mu}{M_\Sigma} - 6\ln\frac{\mu}{M_X}\right] ,$$
$$\frac{1}{g_3^2(\mu)} = \frac{1}{g_G^2(\mu)} + \frac{1}{8\pi^2}\left[\ln\frac{\mu}{M_{H_C}} + 3\ln\frac{\mu}{M_\Sigma} - 4\ln\frac{\mu}{M_X}\right] .$$

Then, we have two relations,

$$\frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} - \frac{1}{g_1^2(\mu)} = -\frac{3}{10\pi^2}\ln\frac{\mu}{M_{H_C}} ,$$
$$\frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} = -\frac{3}{2\pi^2}\ln\frac{\mu^3}{M_X^2 M_\Sigma} .$$

The GUT scale

$$M_{\rm GUT} \equiv (M_X^2 M_\Sigma)^{1/3}$$

By using the relations, we evaluate M_{HC} and M_{GUT} in the following analysis.

1-loop results

First, we derive simpler formulae by using

- 1-loop RGEs for gauge couplings
- 1-loop threshold corrections

Results

$$\frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi}\left[\frac{12}{5}\ln\left(\frac{M_{H_C}}{m_Z}\right) - 2\ln\left(\frac{M_S}{m_Z}\right) + 4\ln\left(\frac{M_3}{M_2}\right)\right],$$

$$\frac{5}{\alpha_1(m_Z)} - \frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} = \frac{1}{2\pi}\left[12\ln\left(\frac{M_{\rm GUT}^3}{m_Z^3}\right) + 4\ln\left(\frac{M_2}{m_Z}\right) + 4\ln\left(\frac{M_3}{m_Z}\right)\right].$$

(M_S : scalar mass, $\mu_H = M_S$)

Features

- M_{HC} gets larger as M_S is taken to be higher.

  It originates from the mass difference among the components of the fundamental Higgs multiplets.

- M_{HC} depends only on the ratio of M_2 and M_3.

1-loop results

$$\frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi}\left[\frac{12}{5}\ln\left(\frac{M_{H_C}}{m_Z}\right) - 2\ln\left(\frac{M_S}{m_Z}\right) + 4\ln\left(\frac{M_3}{M_2}\right)\right],$$

$$\frac{5}{\alpha_1(m_Z)} - \frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} = \frac{1}{2\pi}\left[12\ln\left(\frac{M_{\rm GUT}^3}{m_Z^3}\right) + 4\ln\left(\frac{M_2}{m_Z}\right) + 4\ln\left(\frac{M_3}{m_Z}\right)\right].$$

- M_{GUT} is independent of M_S
- M_{GUT} is dependent on the scale of the gauginos

Since it results from the mass difference in the gauge vector multiplets and the adjoint Higgs multiplet.

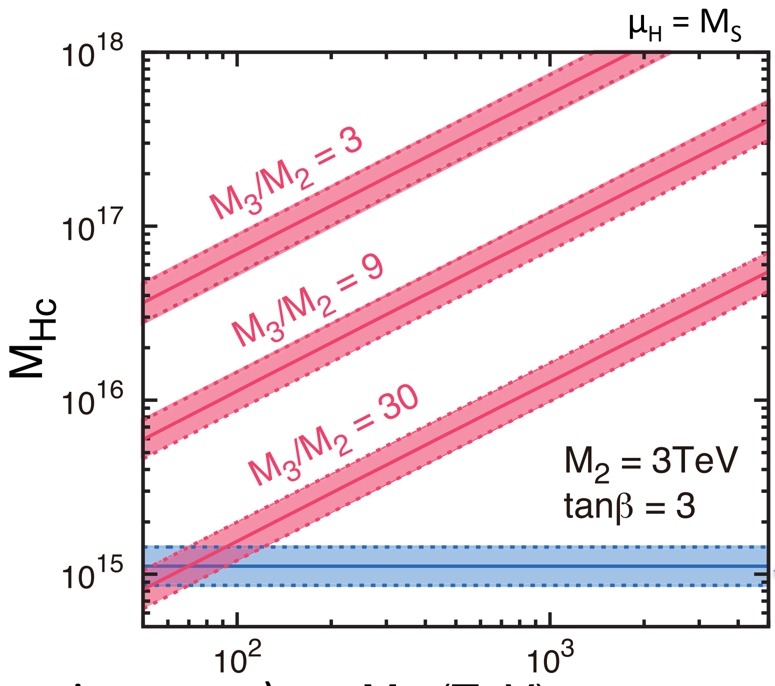
(A part of which is included as the longitudinal mode of the gauge multiplets)

- M_{GUT} decreases when gaugino masses get larger

Owing to the opposite sign of the contributions of the gauge fields to those of matter fields.

c.f.) 1-loop gauge coupling beta function

M_{HC} vs. SUSY scale



(M_S : sfermion mass)

Errors come from the strong coupling constant
$$\alpha_s(m_Z) = 0.1184(7)$$

Low-energy SUSY case
$$M_S = 1\ \mathrm{TeV}$$
$$M_2 = 200\ \mathrm{GeV}$$
$$M_3/M_2 = 3.5$$

- M_{HC} increases as M_S grows while it decreases when M_3/M_2 becomes large
- M_{HC} can be around $\sim 10^{16}$ GeV in high-scale SUSY scenario

J. Hisano, T. Kuwahara, N. Nagata (2013).

Interesting features

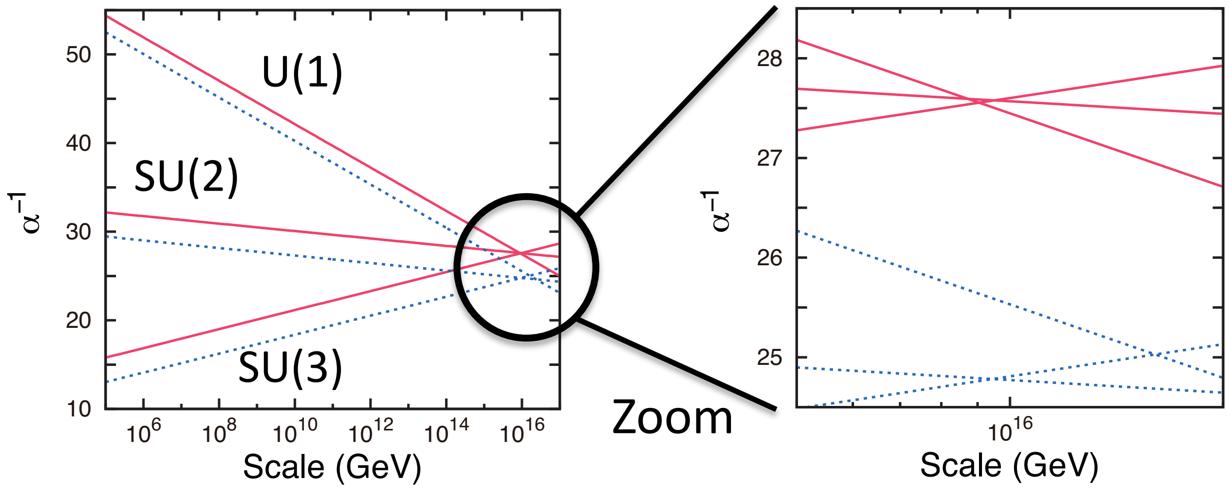
M_{HC} can be around the GUT scale in the high-scale SUSY scenario.

⇓

Small threshold corrections are required
to realize gauge coupling unification.

⇓

Gauge coupling unification is improved !!



High-scale SUSY

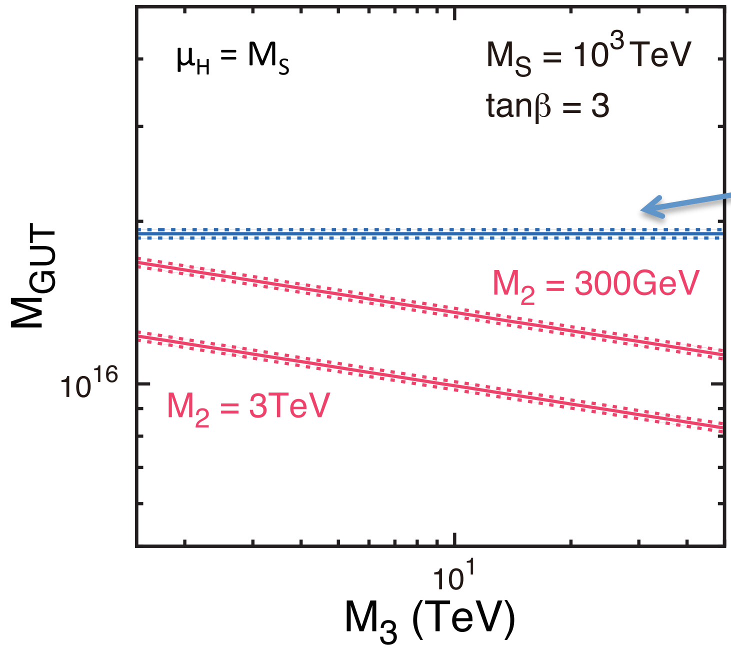
$M_S = 10^2$ TeV
$M_2 = 3$ TeV
$M_3/M_2 = 9$

Low-scale SUSY

$M_S = 1$ TeV
$M_2 = 200$ GeV
$M_3/M_2 = 3.5$

M_{GUT} vs. gluino mass

$$M_{\rm GUT} \equiv (M_X^2 M_\Sigma)^{1/3}$$



Low-energy SUSY case

$$M_S = 1 \text{ TeV}$$
$$M_2 = 200 \text{ GeV}$$
$$M_3/M_2 = 3.5$$

Errors come from

$$\alpha_s(m_Z) = 0.1184(7)$$

- The GUT scale M_{GUT} has little dependence on the gaugino masses.
- M_{GUT} gets lower when the gaugino masses become larger.

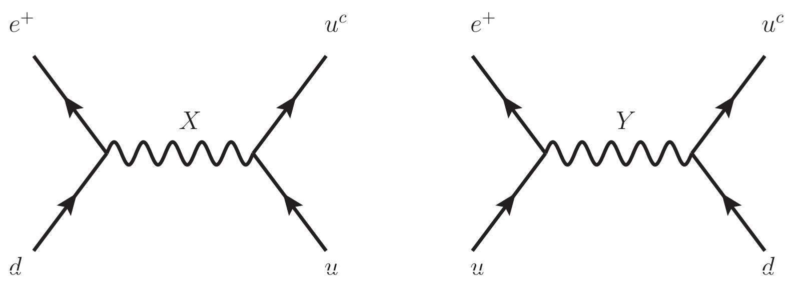
$$M_{\rm GUT} \propto (M_3 M_2)^{-1/9}$$

J. Hisano, T. Kuwahara, N. Nagata (2013).

Interesting features

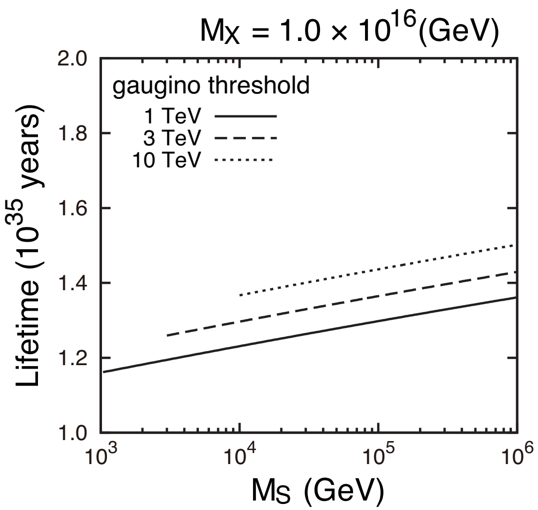
The GUT scale becomes lower in the high-scale SUSY scenario.

Proton decay via the X-boson exchange is enhanced.



$$\Gamma(p \to \pi^0 e^+) \propto \frac{\alpha_G^2}{M_X^4}$$

sensitive to the X-boson mass



J. Hisano, D. Kobayashi, N. Nagata (2013).

Lifetime

$$\tau(p \to \pi^0 e^+) \simeq 1.3 \times 10^{35} \text{ yrs}$$

($M_X = 1.0 \times 10^{16}$ GeV)

c.f.) Hyper-Kamiokande with 10 years exposure

$$\text{expected sensitivity} \simeq 1.3 \times 10^{35} \text{ yrs}$$

[arXiv: 1109.3262]

3. Proton decay in high-scale SUSY

Yukawa interactions

Yukawa coupling terms in superpotential

$$W_{\text{Yukawa}} = \frac{1}{4} h^{ij} \epsilon_{abcde} \Psi_i^{ab} \Psi_j^{cd} H^e - \sqrt{2} f^{ij} \Psi_i^{ab} \Phi_{ja} \bar{H}_b$$

(i,j: generations, a,b,…: SU(5) indices)

D.O.F.

$$h^{ij} \in \mathbb{C}^6 \qquad f^{ij} \in \mathbb{C}^9$$

Field re-definitions

$$\Psi_i, \Phi_j : \ \mathrm{U}(3) \otimes \mathrm{U}(3)$$

$2 \times 6 + 2 \times 9 - 9 \times 2 = 6 + 4 + 2$ (Real D.O.F.)

quark masses CKM extra phases

Parameterization

$$h^{ij} = f_{u_i} e^{i\varphi_i} \delta_{ij} \ , \qquad f^{ij} = V_{ij}^* f_{d_j} \ ,$$

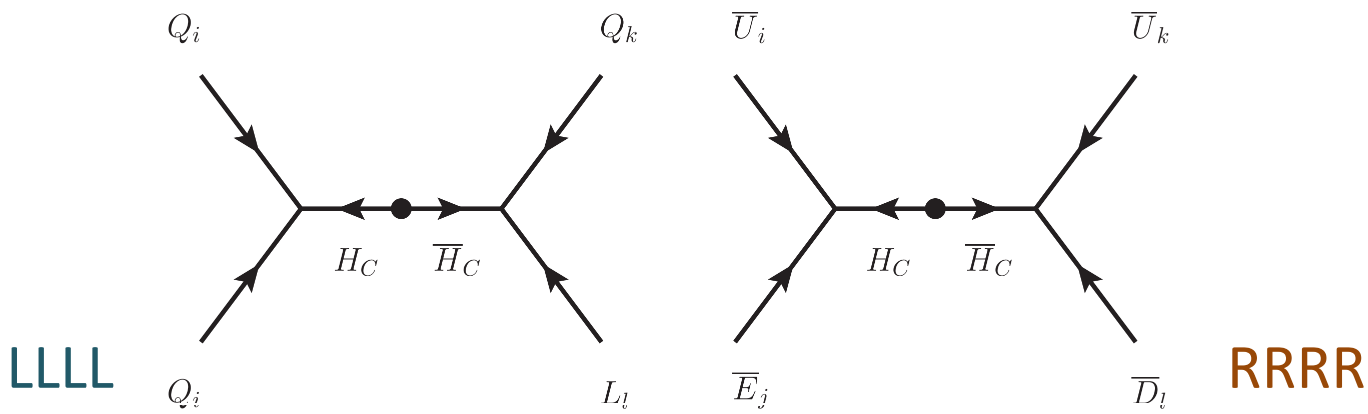
(V_{ij}: CKM matrix)

Constraint

$$\varphi_1 + \varphi_2 + \varphi_3 = 0$$

(Two of them are independent)

Dim-5 effective operators



Exchanges of color-triplet Higgs multiplets induce the dimension-five baryon-number violating operators.

Effective superpotential

$$W_5 = +\frac{1}{2M_{H_C}} f_{u_i} f_{d_l} V_{kl}^* e^{i\varphi_i} \epsilon_{\alpha\beta\gamma}\epsilon_{rs}\epsilon_{tu} Q_i^{\alpha r} Q_i^{\beta s} Q_k^{\gamma t} L_l^u \quad \text{LLLL}$$

$$+ \frac{1}{M_{H_C}} f_{u_i} f_{d_l} V_{kl}^* e^{i\varphi_i} \epsilon^{\alpha\beta\gamma} \overline{U}_{i\alpha} \overline{E}_i \overline{U}_{k\beta} \overline{D}_{l\gamma} \ , \quad \text{RRRR}$$

Dim-5 effective operators

Effective Lagrangian (dim-5)

$$\mathcal{L}_5 = \int d^2\theta W_5 \quad + \quad \text{h.c.}$$

LLLL RRRR

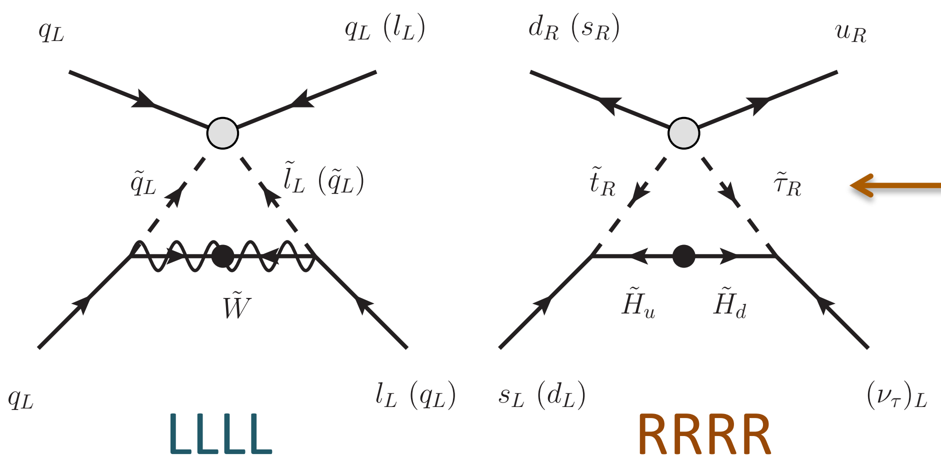
$$\ni \ \epsilon_{\alpha\beta\gamma}\left(\overline{u_i^C}\right)^{\alpha} P_L d_i^{\beta} \underline{(\tilde{u}_L)_k^{\gamma} (\tilde{e}_L)_l}, \quad \epsilon^{\alpha\beta\gamma}\left(\overline{u_k}\right)_{\alpha} P_L d_{l\beta}^{C} \underline{(\tilde{u}_R^*)_{i\gamma} (\tilde{e}_R^*)_i}, \quad \text{etc.}$$

Sfermions are integrated out below the SUSY breaking scale (M_S)

- LLLL operator → Charged wino exchange
- RRRR operator → Charged higgsino exchange

Other contributions (*e.g.*, neutral wino, neutral higgsino, gluino…) are found to be negligible in our scenario.

Dim-5 effective operators



Although it is induced in the flavor changing process, its contribution is sizable because of the large Yukawa couplings of the third generation fermions

T. Goto and T. Nihei (1999)
V. Lucas and S. Raby (1997)

Loop function

$$\propto \frac{M}{M_S^2},$$

Chiral flip [M = wino mass (M_2) or higgsino mass (μ_H)]

Suppressed by sfermion mass (decoupling)

In the case of $\mu_H \gg M_2$, the higgsino exchange contribution dominates the wino exchange one.

Proton decay lifetime

For $\mu_H = M_S$, the proton lifetime is approximately given as

$$\tau(p \to K^+ \bar{\nu}) \simeq 4 \times 10^{35} \times \sin^4 2\beta \left(\frac{0.1}{\overline{A}_R}\right)^2 \left(\frac{M_S}{10^2 \text{ TeV}}\right)^2 \left(\frac{M_{H_C}}{10^{16} \text{ GeV}}\right)^2 \text{ yrs} ,$$

Renormalization factors

The proton lifetime is significantly reduced with large tanβ.

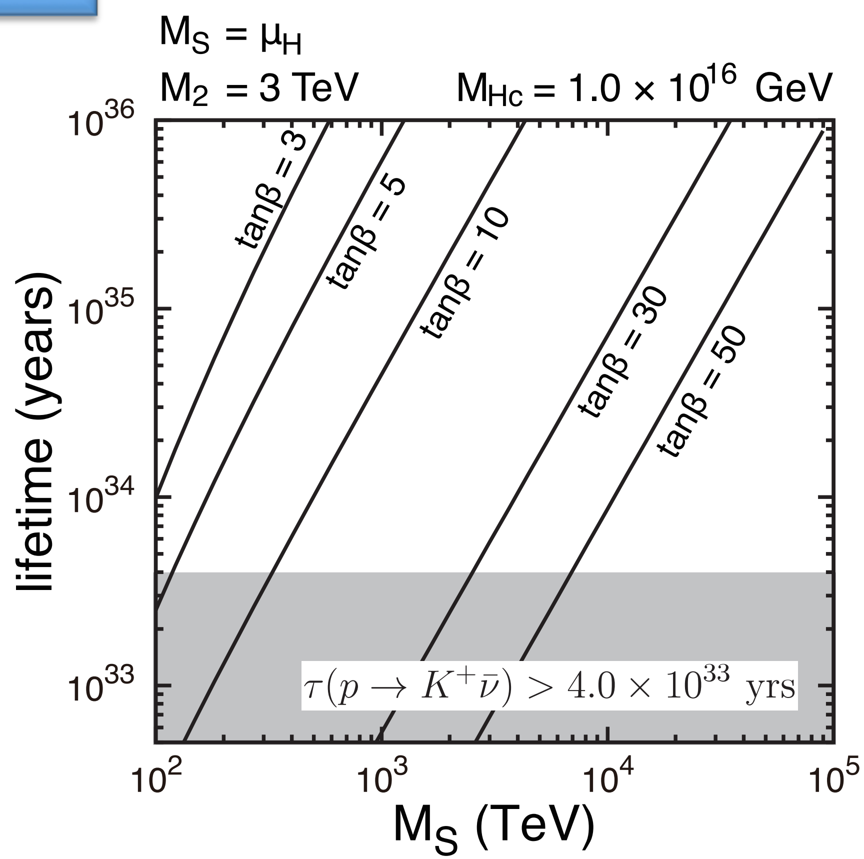
M_{HC} can be around 10^{16} GeV (as discussed above)

The lifetime can be well above the current experimental limit:

$$\tau(p \to K^+ \bar{\nu}) > 4.0 \times 10^{33} \text{ yrs}$$

⇨ Decoupling can revive the minimal SUSY SU(5) GUT !

Results

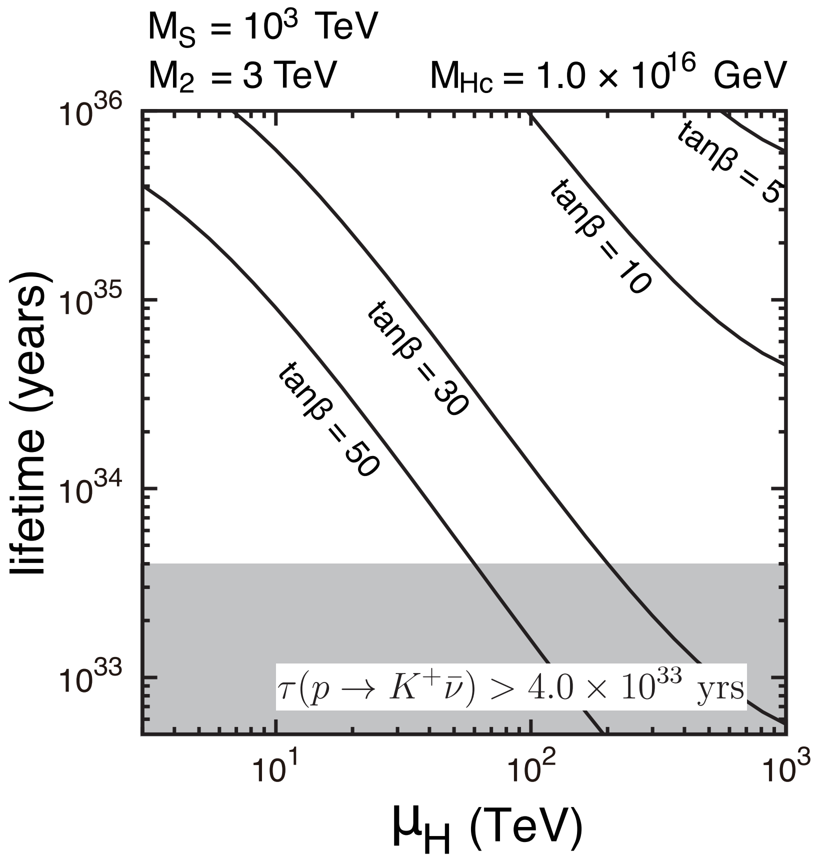


← Experimental limit

Proton decay lifetime in high-scale SUSY scenario can evade the experimental constraints, especially for small tanβ.

J. Hisano, D. Kobayashi, T. Kuwahara, N. Nagata (2013).

Results



We take the new phases so that they yield the maximal amplitude for the proton decay rate.

We are to obtain severe limits on the parameters

← Experimental limit

We have found that the higgsino contribution is dominant in a wide range of parameter region.

J. Hisano, D. Kobayashi, T. Kuwahara, N. Nagata (2013).

5. Summary

Summary

- We discuss the minimal SUSY SU(5) GUT in the high-scale SUSY scenario
- The gauge coupling unification can be improved with high SUSY breaking scale
- The GUT scale is slightly reduced, and thus the dimension-6 proton decay might be promising
- Dimension-5 proton decay can evade the current experimental limits
- We look forward to future proton decay experiments

Backup

Yukawa interactions

Yukawa coupling terms in superpotential

$$W_{\text{Yukawa}} = f_{u_i} e^{i\varphi_i} \epsilon_{rs} \overline{U}_{i\alpha} Q_i^{\alpha r} H_u^s - V_{ij}^* f_{d_j} \epsilon_{rs} Q_i^{\alpha r} \overline{D}_{j\alpha} H_d^s - f_{d_j} \epsilon_{rs} V_{ij}^* \overline{E}_i L_j^r H_d^s$$
$$- \frac{1}{2} f_{u_i} e^{i\varphi_i} \epsilon_{\alpha\beta\gamma} \epsilon_{rs} Q_i^{\alpha r} Q_i^{\beta s} H_C^{\gamma} + V_{ij}^* f_{d_j} \epsilon_{rs} Q_i^{\alpha r} L_j^s \overline{H}_{C\alpha}$$
$$+ f_{u_i} e^{i\varphi_i} \overline{U}_{i\alpha} \overline{E}_i H_C^{\alpha} - V_{ij}^* f_{d_j} \epsilon^{\alpha\beta\gamma} \overline{U}_{i\alpha} \overline{D}_{j\beta} \overline{H}_{C\gamma}$$

Mass and gauge eigenstates

$$Q_i = \begin{pmatrix} U_i' \\ V_{ij} D_j' \end{pmatrix}, \qquad L_i = \begin{pmatrix} N_i' \\ E_i' \end{pmatrix},$$

$$\bar{U}_i = e^{-i\varphi_i} \bar{U}_i', \qquad \bar{D}_i = \bar{D}_i', \qquad \bar{E}_i = V_{ij} \bar{E}_j' \ ,$$

where primes represent the mass eigenstates.

Superheavy masses

Superpotential of Higgs sector

$$W_{\text{Higgs}} = \frac{1}{3}\lambda_\Sigma \text{Tr}\Sigma^3 + \frac{1}{2}m_\Sigma \text{Tr}\Sigma^2 + \lambda_H \bar{H}\Sigma H + m_H \bar{H}H \ .$$

VEV of adjoint Higgs

$$\langle\Sigma\rangle = V \cdot \text{diag}(2,2,2,-3,-3) \qquad (V = m_\Sigma/\lambda_\Sigma)$$

Doublet-triplet mass splitting

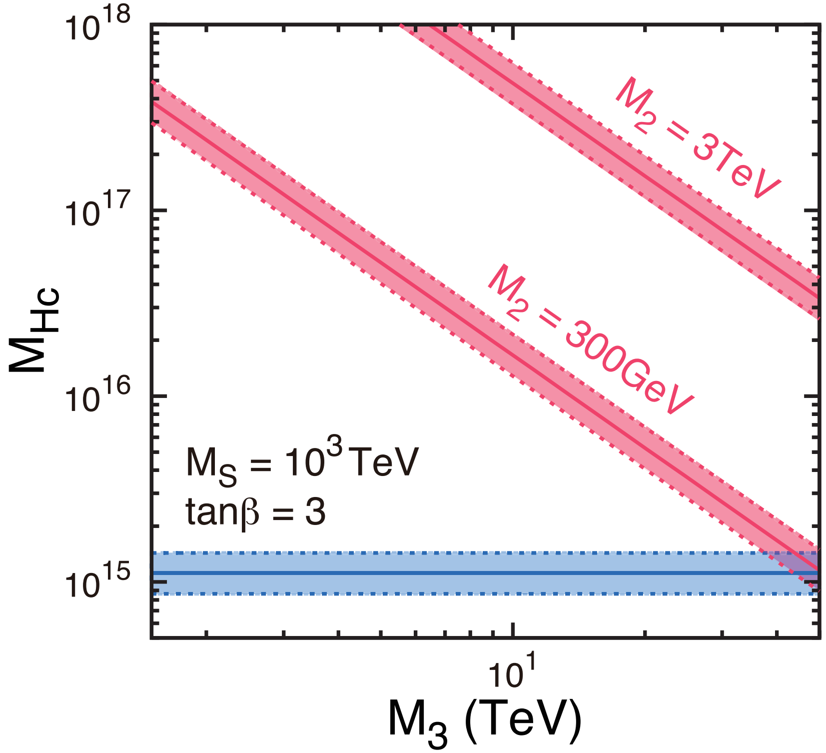
m_H is fine-tuned as $m_H = 3\lambda_H V$

Superheavy masses

$$M_{H_C} = M_{\bar{H}_C} = 5\lambda_H V \qquad M_X = 5\sqrt{2}g_5 V$$

$$M_\Sigma \equiv M_{\Sigma_8} = M_{\Sigma_3} = \frac{5}{2}\lambda_\Sigma V \qquad M_{\Sigma_{24}} = \frac{1}{2}\lambda_\Sigma V$$

M_{HC} vs. gluino mass



Errors come from the strong coupling constant

$$\alpha_s(m_Z) = 0.1184(7)$$

Low-energy SUSY case

$$M_S = 1 \text{ TeV}$$
$$M_2 = 200 \text{ GeV}$$
$$M_3/M_2 = 3.5$$

- M_{HC} decreases when M_3/M_2 becomes large
- M_{HC} can be around ～10^{16} GeV in high-scale SUSY scenario

J. Hisano, T. Kuwahara, N. Nagata (2013).

Effective 4-Fermi operators

$$\mathcal{L}_6 = \frac{\alpha_2^2}{M_{H_C} m_W^2 \sin 2\beta}\Big[2F(M_2, M_S^2) \sum_{i,j=2,3} \overline{m}_{u_i}\overline{m}_{d_j} V_{u_i d} V_{u_i s} V^*_{u d_j} e^{i\varphi_i}$$
$$\times A_R^{(i,j)} \epsilon_{\alpha\beta\gamma}\{(u_L^\alpha d_L^\beta)(\nu_{Lj} s_L^\gamma) + (u_L^\alpha s_L^\beta)(\nu_{Lj} d_L^\gamma)\}$$
$$-\frac{\overline{m}_t^2 \overline{m}_\tau V_{tb}^* e^{i\varphi_1}}{m_W^2 \sin 2\beta} F(\mu_H, M_S^2)\overline{A}_R \epsilon_{\alpha\beta\gamma}\{\overline{m}_d V_{ud} V_{ts}(u_R^\alpha d_R^\beta)(\nu_\tau s_L^\gamma) + \overline{m}_s V_{us} V_{td}(u_R^\alpha s_R^\beta)(\nu_\tau d_L^\gamma)\}\Big]$$
$$+ \text{h.c.},$$

($A_R^{(i,j)}$, $\overline{A}_R$: renormalization factors)

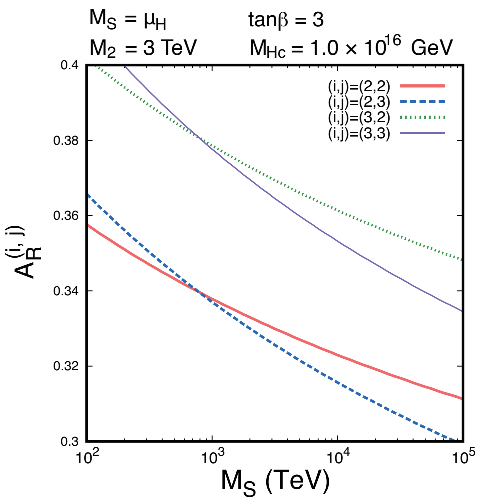
Loop function

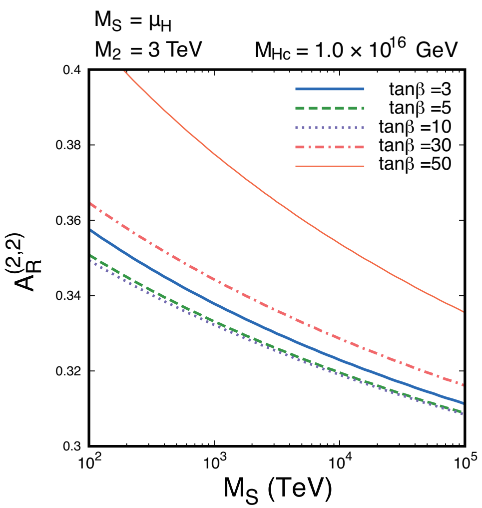
$$F(M, M_S^2) = M\left[\frac{1}{M_S^2 - M^2} - \frac{M^2}{(M_S^2 - M^2)^2}\ln\left(\frac{M_S^2}{M^2}\right)\right],$$

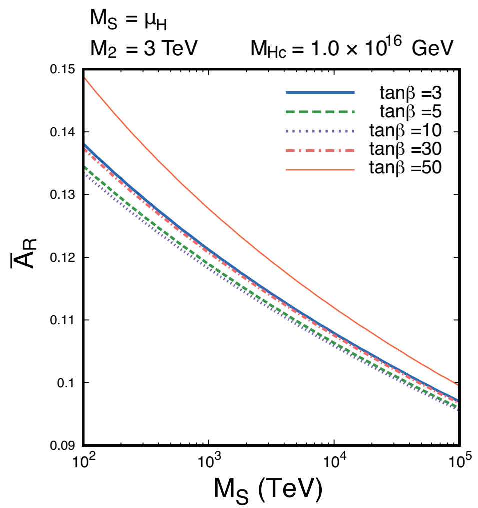
$$F(M, M_S^2) \to \frac{M}{M_S^2}, \qquad (M_S \gg M),$$

$$F(M, M_S^2) \to \frac{1}{2M_S}, \qquad (M \to M_S).$$

Renormalization factors







Renormalization factors are reduced as the SUSY scale gets higher.

Hadron matrix elements

$$\langle K^+|\epsilon_{\alpha\beta\gamma}(u_L^\alpha d_L^\beta)s_L^\gamma|p\rangle = \frac{\beta_p}{\sqrt{2}f_\pi}\left(1+\frac{D+3F}{3}\frac{m_p}{M_B}\right)P_L u_p \ ,$$

$$\langle K^+|\epsilon_{\alpha\beta\gamma}(u_L^\alpha s_L^\beta)d_L^\gamma|p\rangle = \frac{\beta_p}{\sqrt{2}f_\pi}\left(\frac{2D}{3}\frac{m_p}{M_B}\right)P_L u_p \ ,$$

$$\langle K^+|\epsilon_{\alpha\beta\gamma}(u_R^\alpha d_R^\beta)s_L^\gamma|p\rangle = \frac{\alpha_p}{\sqrt{2}f_\pi}\left(1+\frac{D+3F}{3}\frac{m_p}{M_B}\right)P_L u_p \ ,$$

$$\langle K^+|\epsilon_{\alpha\beta\gamma}(u_R^\alpha s_R^\beta)d_L^\gamma|p\rangle = \frac{\alpha_p}{\sqrt{2}f_\pi}\left(\frac{2D}{3}\frac{m_p}{M_B}\right)P_L u_p \ ,$$

$$f_\pi \simeq 92.2\ \mathrm{MeV}$$

$$D \simeq 0.80$$

$$F \simeq 0.47$$

(M_B : baryon mass parameter)

$$\langle 0|\epsilon_{\alpha\beta\gamma}(u_R^\alpha d_R^\beta)u_L^\gamma|p\rangle = \alpha_p P_L u_p \ ,$$

$$\langle 0|\epsilon_{\alpha\beta\gamma}(u_L^\alpha d_L^\beta)u_L^\gamma|p\rangle = \beta_p P_L u_p \ ,$$

Low-energy constants

$$\alpha_p = -0.0112 \pm 0.0012_{(\mathrm{stat})} \pm 0.0022_{(\mathrm{syst})}\ \mathrm{GeV}^3 \ ,$$

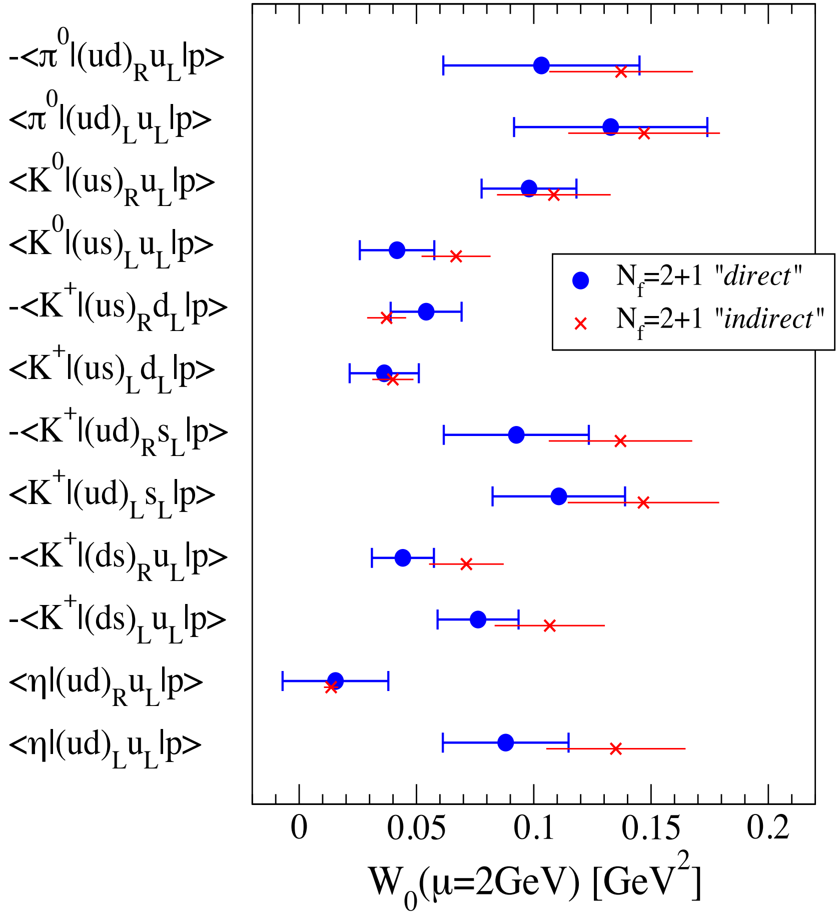
$$\beta_p = 0.0120 \pm 0.0013_{(\mathrm{stat})} \pm 0.0023_{(\mathrm{syst})}\ \mathrm{GeV}^3 \ ,$$

(@ μ = 2 GeV)

Y. Aoki *et al*. [RBC-UKQCD Collaboration] (2008).

Hadron matrix elements

Direct method vs. Indirect method



The results are consistent with each other.

Y. Aoki, E. Shintani, and A. Soni, arXiv:1304.7424

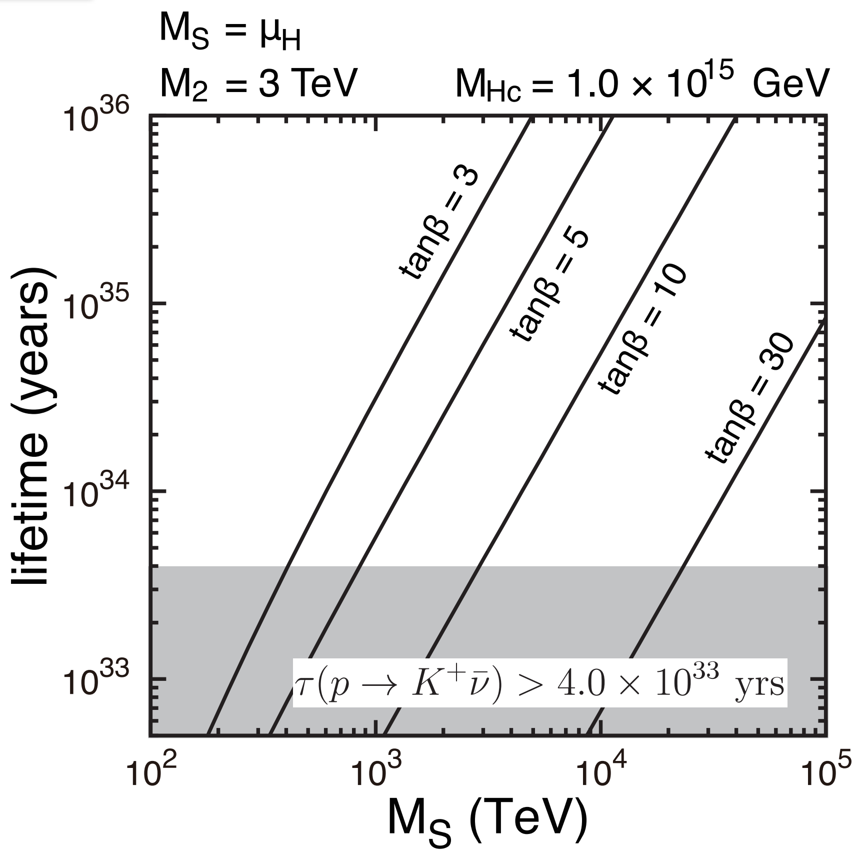
Decay width

$$\Gamma(p \to K^+ \bar{\nu}_i) = \frac{m_p \alpha_2^4 |C_i|^2}{64\pi f_\pi^2 M_{H_C}^2 m_W^4 \sin^2 2\beta} \left(1 - \frac{m_K^2}{m_p^2}\right)^2 ,$$

$$C_\mu = 2\beta_p F(M_2, M_S^2) \{1 + (D+F)\frac{m_p}{M_B}\} \overline{m}_s V_{us}^* \sum_{i=2,3} \overline{m}_{u_i} V_{u_i d} V_{u_i s} e^{i\varphi_i} A_R^{(i,2)} ,$$

$$C_\tau = 2\beta_p F(M_2, M_S^2) \{1 + (D+F)\frac{m_p}{M_B}\} \overline{m}_b V_{ub}^* \sum_{i=2,3} \overline{m}_{u_i} V_{u_i d} V_{u_i s} e^{i\varphi_i} A_R^{(i,3)}$$
$$- \alpha_p \frac{\overline{m}_t^2 \overline{m}_\tau V_{tb}^* e^{i\varphi_1}}{m_W^2 \sin 2\beta} F(\mu_H, M_S^2) \overline{A}_R \left\{ \overline{m}_d V_{ud} V_{ts} \left(1 + \frac{D+3F}{3} \frac{m_p}{M_B}\right) + \overline{m}_s V_{us} V_{td} \frac{2D}{3} \frac{m_p}{M_B} \right\} .$$

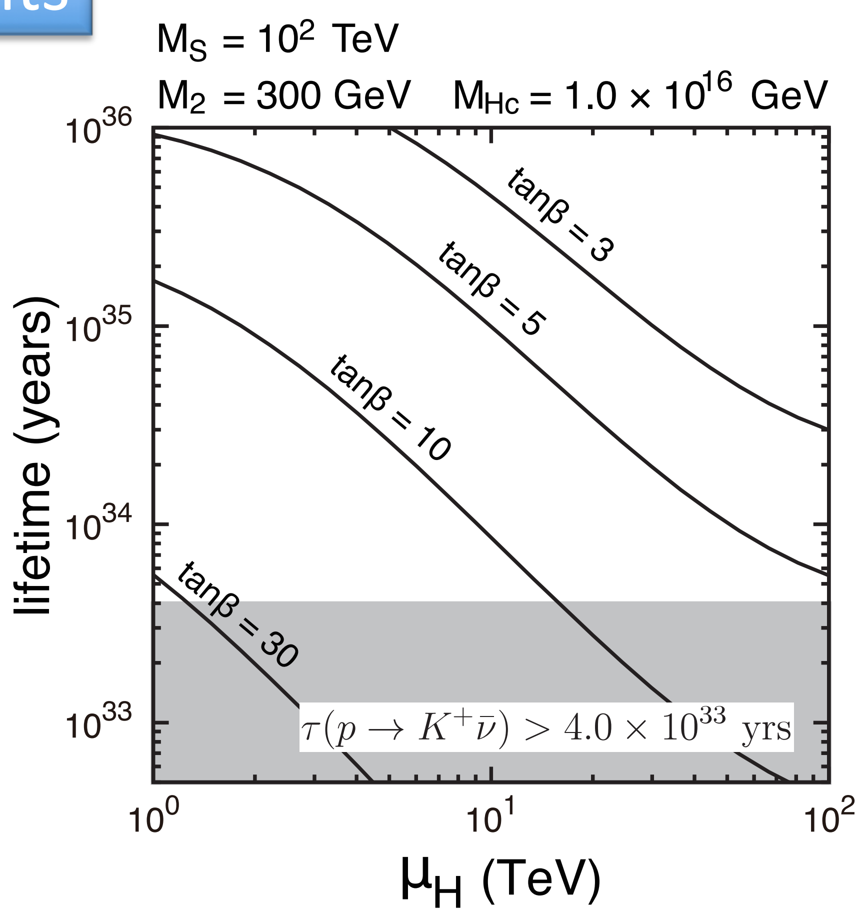
Results



← Experimental limit

J. Hisano, D. Kobayashi, T. Kuwahara, N. Nagata (2013).

Results



← Experimental limit

J. Hisano, D. Kobayashi, T. Kuwahara, N. Nagata (2013).