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Gauss-Bonnet braneworld redux: A novel scenario for the bouncing universe

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based on

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Contents

Introduction

Relativistic cosmology

- Purpose: Understand the history & fate of our universe
- Zeroth-order approximation of our universe
  - Friedmann-Robertson-Walker cosmological spacetime

$$-d\tau^2 + a(\tau)^2 \left[d\chi^2 + f_k(\chi)^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$f_0(\chi) = \chi,\ f_1(\chi) = \sin\chi, \text{ and } f_{-1}(\chi) = \sinh\chi$$

flat, closed, open

  - Homogeneous & isotropic space, characterized by its curvature k
  - Only 1 dynamical degree of freedom: scale factor a(t)

Dynamics of the FRW universe

- Einstein equation with a perfect fluid:

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \kappa_4^2 T_{\mu\nu}$$ where $$T^\mu{}_\nu = \mathrm{diag}(-\rho, p, p, p)$$

Cosmological constant

Energy density

pressure

- Equation of state: $p = (\gamma - 1)\rho$
- Energy-momentum conservation gives $\rho = \frac{\rho_0}{a^{3\gamma}},$
  - Weak energy condition (WEC): Energy density observed is non-negative
    - γ=[0,2] with non-negative ρ_0
  - Strong energy condition (SEC): Gravity is attractive
    - WEC+SEC: γ =[2/3,2] with non-negative ρ_0
- Einstein equation gives the master equation for a(t)

$$H^2 = \frac{\kappa_4^2}{3}\frac{\rho_0}{a^{3\gamma}} - \frac{k}{a^2} + \frac{1}{3}\Lambda_4,$$ where $$H := \dot{a}/a$$

Friedmann equation

1-dim Potential problem

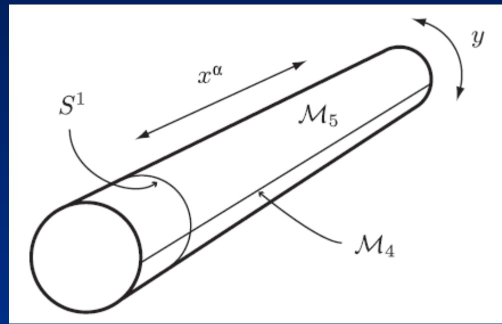
- Friedmann equation: $H^2 = \frac{\kappa_4^2}{3}\frac{\rho_0}{a^{3\gamma}} - \frac{k}{a^2} + \frac{1}{3}\Lambda_4,$ =:V(a)
  - One-dimensional potential problem
  - Allowed domain of a(t) is given by V(a)>0
- a=0 corresponds to the Big Bang (Big Crunch) singularity
  - Zero spatial volume
  - GR breaks down and Quantum effect of gravity dominates
- Qualitative analysis of the evolution is possible
  - Big Bounce (if a=0 not allowed): from contraction to expansion
    - γ =[0,2/3) with k=1 required
  - Bouncing point is given by V(a)=0
  - FACT: No bounce for ordinary matter (WEC + SEC)

Motivation:
Initial singularity problem

- Singularities are generic in GR (Hawking-Penrose, Geroch, 70's)
- Should be cured, but no full quantum gravity available
- (Super)string/M-theory is a strong candidate toward QG
- String-inspired Randall-Sundrum braneworld (`99) is an interesting toy model of the higher-dim universe with a large extra dimension
  - The universe as a domain wall embedded in the 5-dim bulk spacetime
  - Original motivation is for the hierarchy problem (why gravity is so weak)
  - Dynamics of the early universe is modified
- Q. Does the braneworld solve the initial singularity problem?

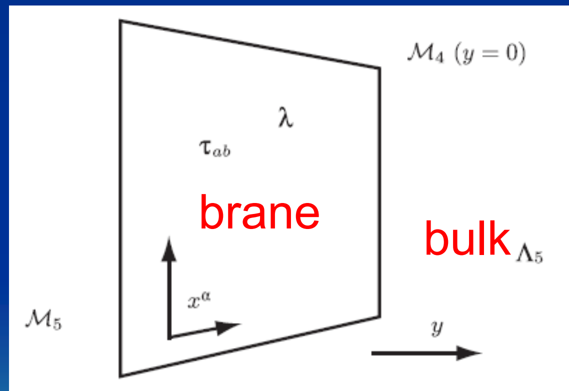
Randall-Sundrum braneworld

- Kaluza-Klein compactification
  - Small & compact extra dimension



Kaluza-Klein

- Randall-Sundrum brane-world (‘99)
  - A (3+1)-dim. timelike hypersurface (brane) in the (4+1)-dim. (asymptotically) AdS bulk spacetime
  - AdS => Confinement of matter on the brane
    - Volcanic potential
  - RS2 model: single-brane model
    - Newtonian gravity on the brane is recovered even with infinitely large extra dimension



Braneworld

Shell dynamics in GR for warming up

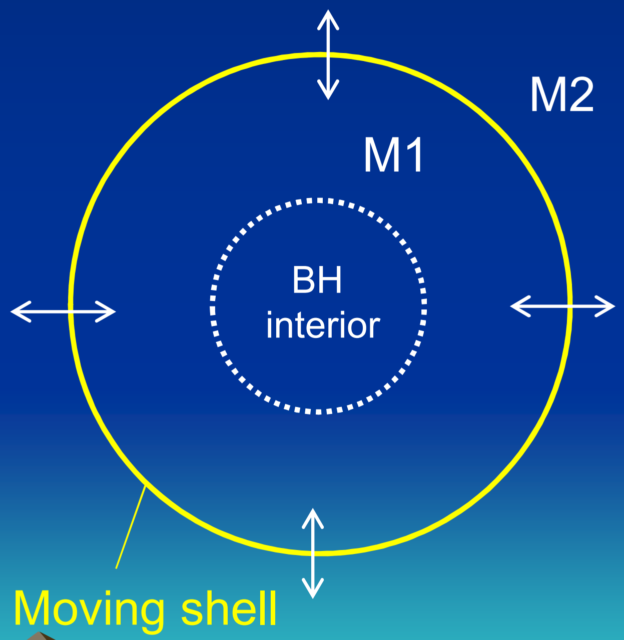
- Consider a spherically symmetric thin-shell in the Schwarzschild vacuum spacetime: $$ds^2 = -\left(1-\frac{M}{r}\right)dt^2 + \left(1-\frac{M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$
  – M=M1 for inside the shell and M=M2(>M1) for outside
- Israel junction condition (=Einstein equation) gives the dynamical equation for the shell

$$[K^a{}_b]_\pm - \delta^a{}_b[K]_\pm = -\kappa_4^2\tau^a{}_b \qquad [X]_\pm := X^+ - X^-,$$

↑ Extrinsic curvature of the shell

↑ Energy-momentum tensor on the shell

- The orbit of the shell is given as r=a(τ), t=T(τ)
  – A timelike hypersurface in the spacetime
  – τ is a proper time of the shell



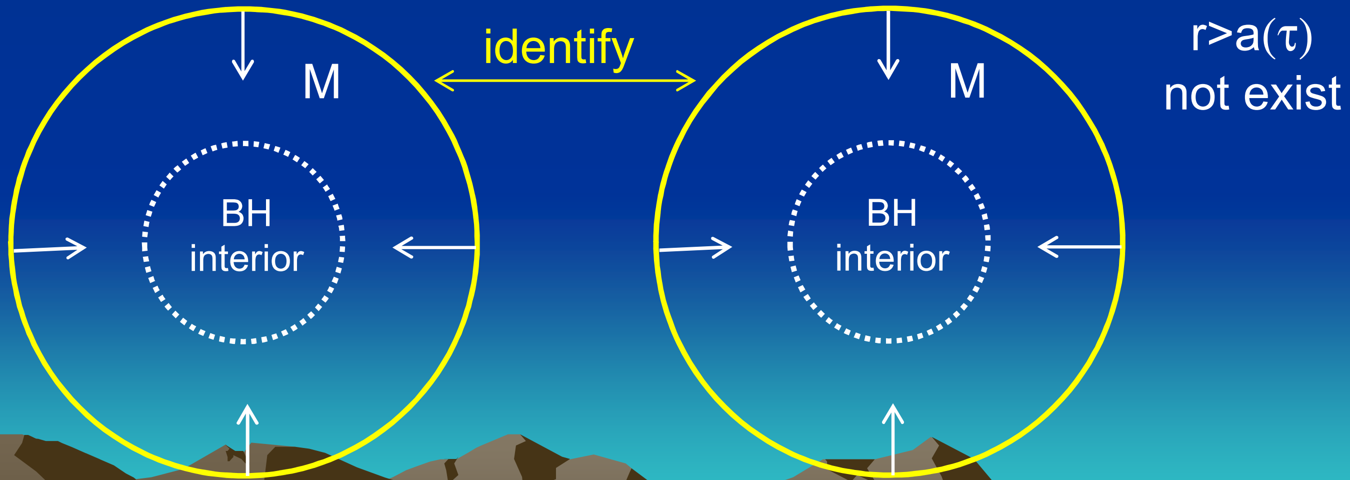
Simplest brane-universe in GR

- Consider a thin-shell (=brane) in the 5-dim Schwarzschild-Tangherlini-AdS vacuum spacetime (Binetruy-Deffayet-Langlois `00 , Kraus `99, Binetruy-Deffayet-Ellwanger-Langlois `00, Ida `00)

$$-h(r)dt^2 + \frac{dr^2}{h(r)} + r^2\left[d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2)\right].$$

$$h(r) = k - \frac{\mu}{8r^2} - \frac{1}{6}\Lambda r^2.$$

  - Position of the brane: r=a(τ), t=T(τ)
  - Take r<a(τ) and paste with the same copy



Friedmann equation on the brane

- Induced metric on the brane => FRW universe

$$-d\tau^2 + a(\tau)^2\left[d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$f_0(\chi) = \chi$, $f_1(\chi) = \sin\chi$, and $f_{-1}(\chi) = \sinh\chi$

- Israel junction condition gives the dynamical equation for a(t)
  - Consistent with a perfect fluid and the EOS $p=(\gamma-1)\rho$
  - Energy-momentum conservation is the same: $\rho = \frac{\rho_0}{a^{3\gamma}}$,
- Modified Friedmann equation:

$$H^2 = \frac{\kappa_5^4}{36}\left(\frac{\rho_0}{a^{3\gamma}} + \sigma\right)^2 - \frac{k}{a^2} + \frac{\mu}{8a^4} + \frac{1}{6}\Lambda =: V_{\rm GR}(a).$$

Brane tension

Modified Friedmann equation

- Modified Friedmann equation

$$H^2 = \frac{\kappa_5^4}{36}\left(\frac{\rho_0}{a^{3\gamma}} + \sigma\right)^2 - \frac{k}{a^2} + \frac{\mu}{8a^4} + \frac{1}{6}\Lambda =: V_{\rm GR}(a).$$

quadratic

Dark radiation

  – Standard 4-dim GR case: $H^2 = \frac{\kappa_4^2}{3}\frac{\rho_0}{a^{3\gamma}} - \frac{k}{a^2} + \frac{1}{3}\Lambda_4,$
  – Existence of the ``dark radiation'' (from the mass term in the bulk)
  – Early-time evolution modified
- Q. Bouncing is possible with perfect fluid satisfying WEC＋SEC?
- A. Still not

Bouncing brane universe?

- Yes, with matter in the bulk
  - U(1) gauge field (Barcelo-Visser `00): but inner horizon in the bulk is unstable (Hovdebo-Myers `03)
  - SU(2) Yang-Mills field (Okuyama-Maeda `04): non-singular bouncing universe is possible with a solitonic bulk spacetime
- In this talk, I consider VACUUM bulk but in modified gravity
  - Einstein-Gauss-Bonnet gravity

$$I = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\Big(R-2\Lambda+\alpha L_{GB}\Big)+I_{\partial M},$$ where

$$L_{GB} := R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}.$$

  - Still 2^{nd}-order theory
  - Low-energy limit of heterotic string theory is 10-dim E-GB with a dilaton
  - We assume $\alpha>0$, $\Lambda<0$, and $1+4\alpha\Lambda/3 \geq 0$

Preliminaries

Bulk solution

- Field equation: $G^{\mu}{}_{\nu} + \alpha H^{\mu}{}_{\nu} + \Lambda \delta^{\mu}{}_{\nu} = 0,$

$$H_{\mu\nu} := 2\Big(RR_{\mu\nu} - 2R_{\mu\alpha}R^{\alpha}{}_{\nu} - 2R^{\alpha\beta}R_{\mu\alpha\nu\beta} + R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}\Big) - \frac{1}{2}g_{\mu\nu}L_{GB}.$$

- Vacuum solution (Boulware-Deser `85, Wheeler `86, Cai `02):

$$-h(r)dt^2 + \frac{dr^2}{h(r)} + r^2\left[d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

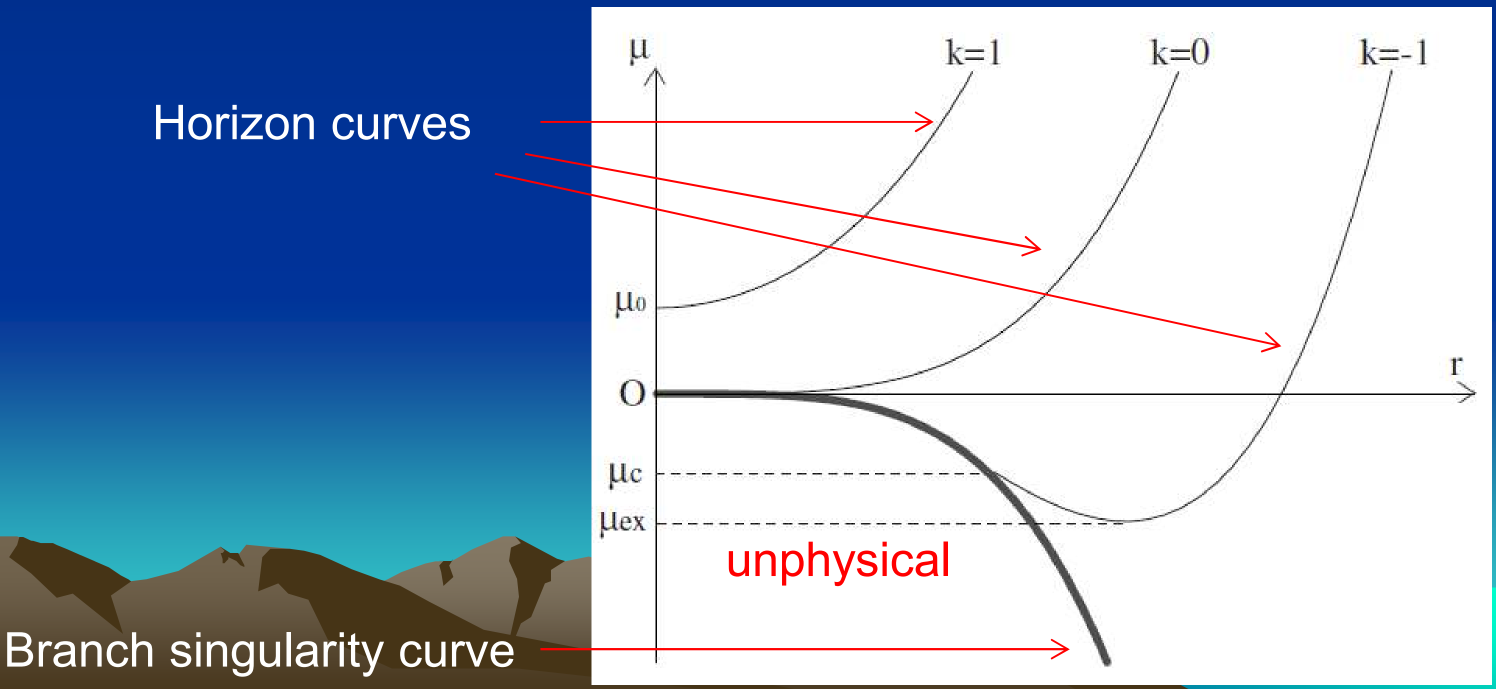
$$h(r) = k + \frac{r^2}{4\alpha}\left(1 \mp \sqrt{1 + \frac{\alpha\mu}{r^4} + \frac{4}{3}\alpha\Lambda}\right),$$

- Two branches of solutions: GR (-) and non-GR (+) branches
  - Only GR branch has the GR limit
  - non-GR branch is dynamically unstable
  - We consider only GR-branch

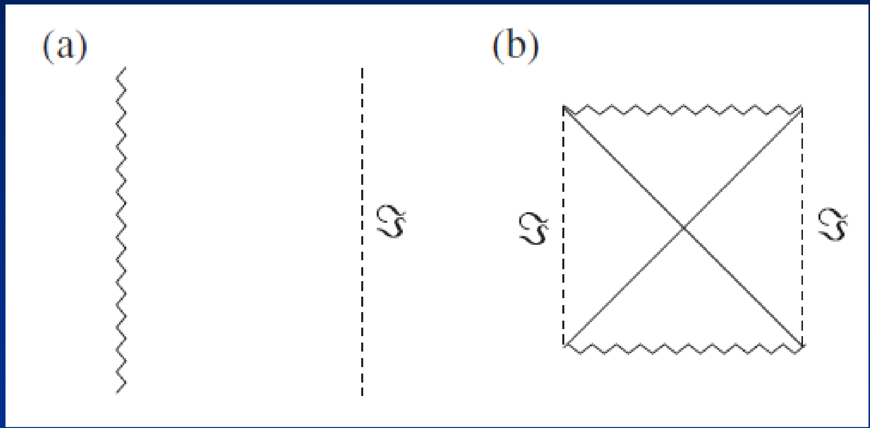
Horizons and singularities in the bulk

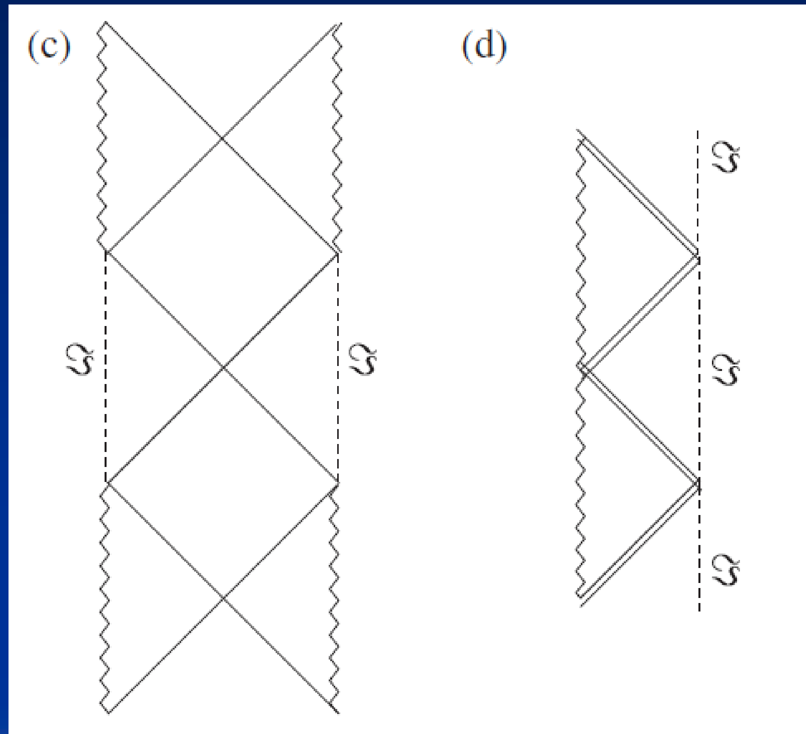
- $\mu>0$: Central singularity (at r=0)
- **$\mu<0$: Non-central ``branch'' singularity (at finite r)**
  - Metric is finite
  - Metric becomes complex and unphysical for $r<r_b$
- Number of horizons depends on k and μ

μ-r diagram



Global structure of the bulk for $\mu<0$





k=1,0

k=-1 depending on μ (<0)

Friedmann equation in the GB braneworld

- Junction condition in EGB (Gravanis-Willison `03, Davis `03)
  - ε=1 (timlike brane), -1(spacelike brane)

$$[K^a{}_b]_\pm - \delta^a{}_b[K]_\pm + 2\alpha\left(3\varepsilon[J^a{}_b]_\pm - \varepsilon\delta^a{}_b[J]_\pm - 2P^a{}_{dbf}[K^{df}]_\pm\right) = -\varepsilon\kappa_5^2\tau^a{}_b, \qquad [X]_\pm := X^+ - X^-,$$

$$\begin{aligned} J_{ab} &:= \frac{1}{3}\left(2KK_{ad}K^d{}_b + K_{df}K^{df}K_{ab} - 2K_{ad}K^{df}K_{fb} - K^2K_{ab}\right) \\ P_{adbf} &:= R_{adbf} + 2h_{a[f}R_{b]d} + 2h_{d[b}R_{f]a} + Rh_{a[b}h_{f]d}. \end{aligned}$$

- Modified Friedmann equation (cubic for H^2) (Charmousis-Dufaux `02)

$$\frac{\kappa_5^4}{36}(\rho+\sigma)^2 = \left(\frac{h(a)}{a^2} + H^2\right)\left[1 + \frac{4\alpha}{3}\left(\frac{3k - h(a)}{a^2} + 2H^2\right)\right]^2$$

  - We assume a perfect fluid on the brane with linear EOS

$$\tau^a{}_b = \mathrm{diag}(-\rho, p, p, p) + \mathrm{diag}(-\sigma, -\sigma, -\sigma, -\sigma,),$$

Modified Friedmann equation in the GB braneworld

$$H^2 = V_{\mathrm{GB}(+)}(a),$$

$$V_{\mathrm{GB}(+)}(a) := \frac{1}{8\alpha}\Bigg[-\frac{8k\alpha}{a^2} - 2 + \left\{A^{3/2} + 256\alpha^3 P^2 + 16\sqrt{2\alpha^3 P^2\left(128\alpha^3 P^2 + A^{3/2}\right)}\right\}^{1/3}$$

$$+ A\left\{A^{3/2} + 256\alpha^3 P^2 + 16\sqrt{2\alpha^3 P^2\left(128\alpha^3 P^2 + A^{3/2}\right)}\right\}^{-1/3}\Bigg],$$

$$A := 1 + \frac{\alpha\mu}{a^4} + \frac{4}{3}\alpha\Lambda,$$

$$P^2 := \frac{\kappa_5^4}{256\alpha^2}\left(\frac{\rho_0}{a^{3\gamma}} + \sigma\right)^2.$$

Still one-dimensional potential problem

Neo bouncing brane universe

New scenario for the Big Bounce

- Dynamics is complicated but still 1-dim potential problem
  - For $\mu>0$, $a=0$ corresponds to the Big-Bang singularity on the brane
  - For $\mu<0$, $a=0$ is NOT allowed since branch singularity is at $a=a_b$ (>0)
- Q. What happens if the brane hits the branch singularity?
  - A singularity on the brane? **NO**.
  - My claim: It is a totally new type of the Big Bounce
- This claim is supported by
  - Fact 1: Collision of the brane with the branch singularity is generic
  - Fact 2: All the curvature invariants on the brane do NOT blow up
  - Fact 3: This bulk singularity is weak

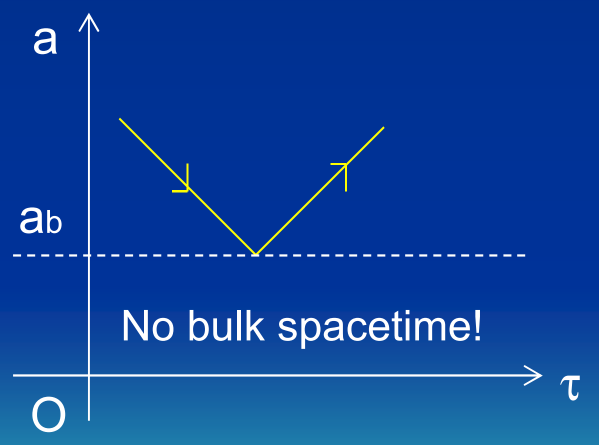
Fact 2: Regularity on the brane

- Asymptotic behavior near $a=a_b$:
  - Curvatures remain finite near $a=a_b$

$$
\begin{aligned}
a &\simeq a_{\rm b} + a_1(\tau-\tau_{\rm b}) + O((\tau-\tau_{\rm b})^2),\\
a_1^2 &:= V_{\rm GB}(a_{\rm b}),
\end{aligned}
$$

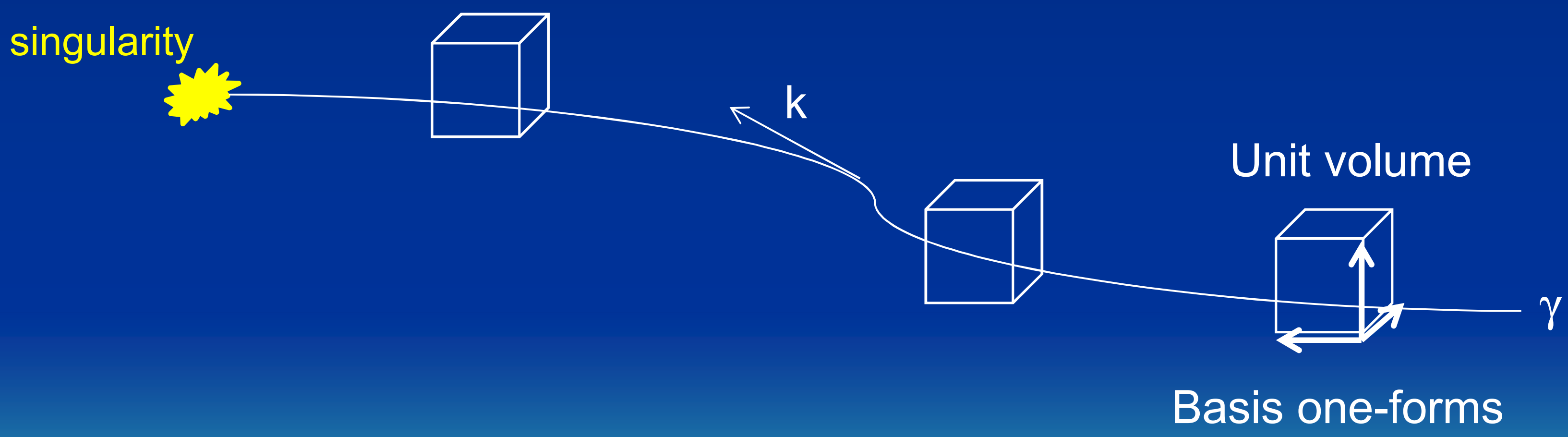
$$
V_{\rm GB}(a_{\rm b}) = \frac{1}{8\alpha a_{\rm b}^2}\left[a_{\rm b}^2\left\{2\kappa_5^4\alpha\left(\frac{\rho_0}{a_{\rm b}^{3\gamma}}+\sigma\right)^2\right\}^{1/3} - 2(a_{\rm b}^2+4k\alpha)\right]
$$

- Derivative of $a(\tau)$ is discontinuous at $a=a_b$:
  - Sudden transition from the collapsing phase to the expanding phase
  - On the brane, there appears an instantaneous matter field
  - With a fine-tuning $a_1=0$, smooth bounce is realized but it is non-generic



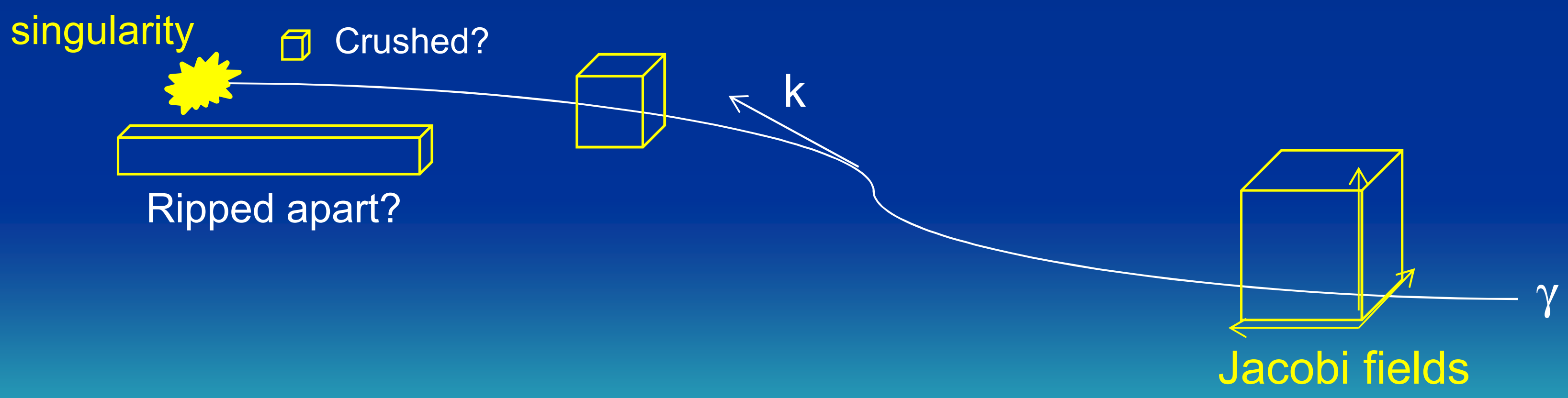
For the Fact 3: Strength of the singularity

- Let γ a causal geodesic to or from the singularity and let k its tangent vector
- Consider a parallel propagated unit orthonormal frame along γ
  - Unit orthonormal basis 1-forms orthogonal to k define a unit volume



For the Fact 3: Strength of the singularity

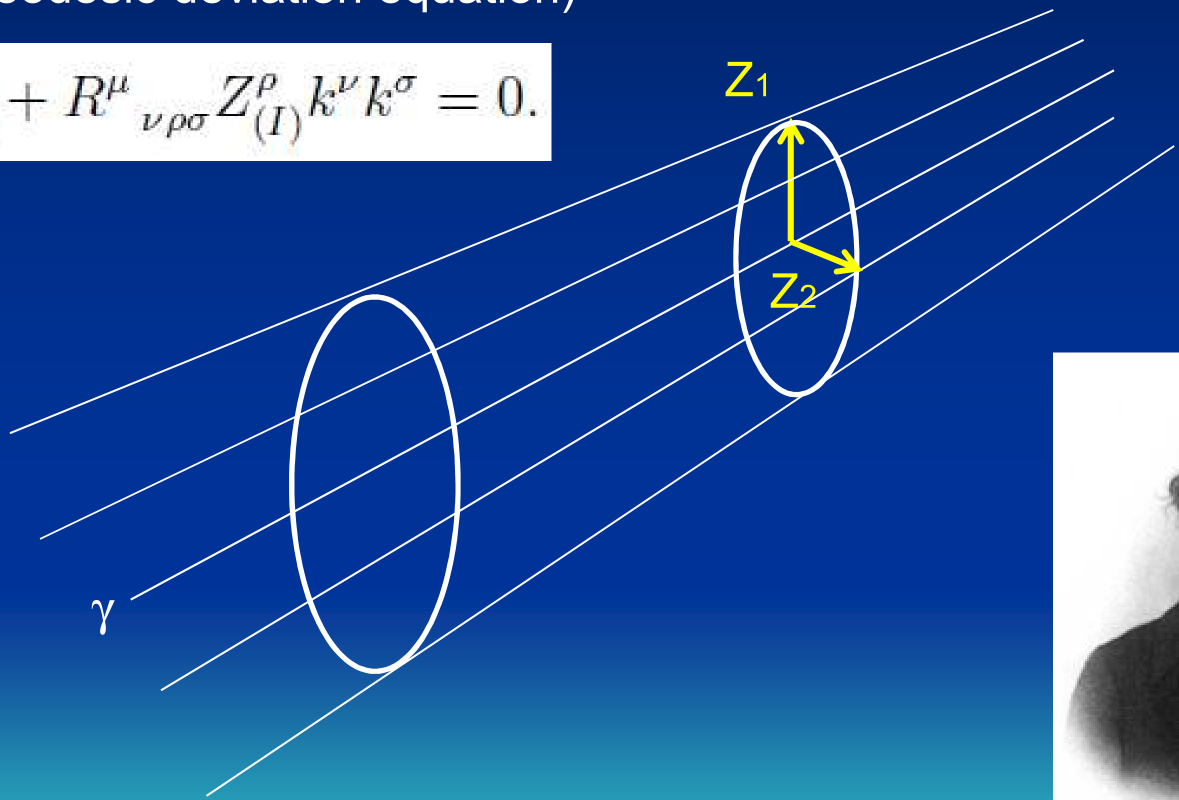
- Consider a set of Jacobi fields, proportional to the basis 1-forms
  - Jacobi fields are governed by the Jacobi equation
- Definition (Tipler ’77)
  A singularity is Tipler weak (strong) if the volume made of the Jacobi fields is finite (zero or infinite) at the singularity
- Tipler weak singularity => Harmless for a finite body



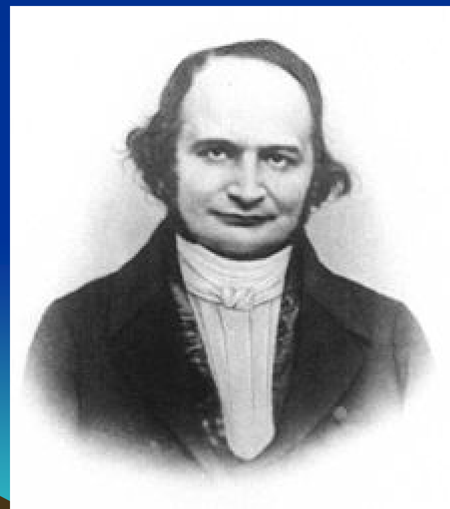
Jacobi field

Jacobi equation
(or geodesic deviation equation)

$$\ddot{Z}^{\mu}_{(I)} + R^{\mu}{}_{\nu\rho\sigma} Z^{\rho}_{(I)} k^{\nu} k^{\sigma} = 0.$$



A bundle of geodesics

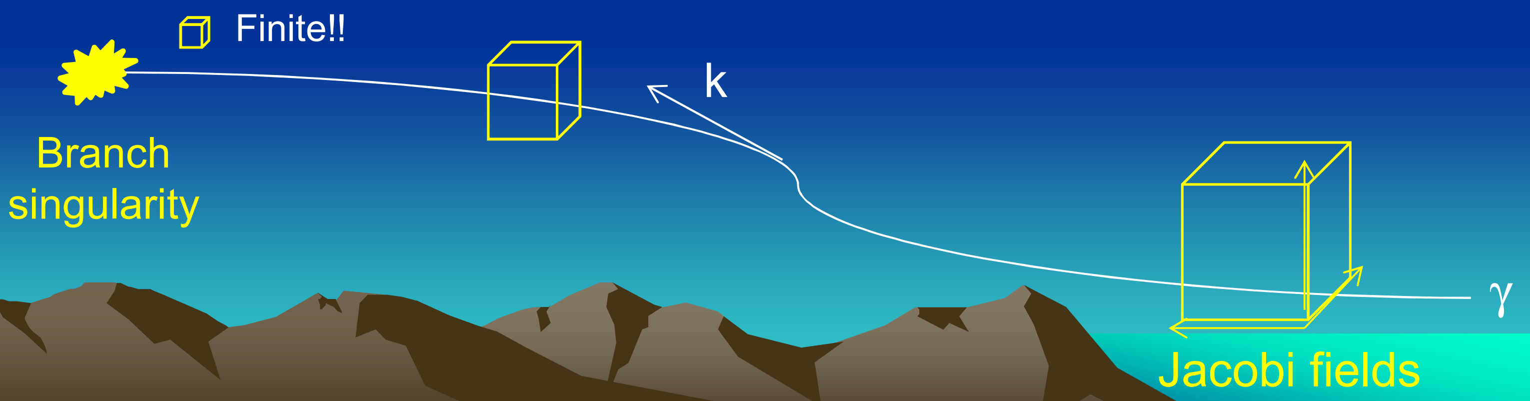


Fact 3: Branch singularity is weak

- Jacobi fields: $Z^{\mu}_{(I)}\frac{\partial}{\partial x^{\mu}} := l_{(I)}(\lambda)\eta^{\mu}_{(I)}\frac{\partial}{\partial x^{\mu}} \quad (I = 1,2,3,4),$

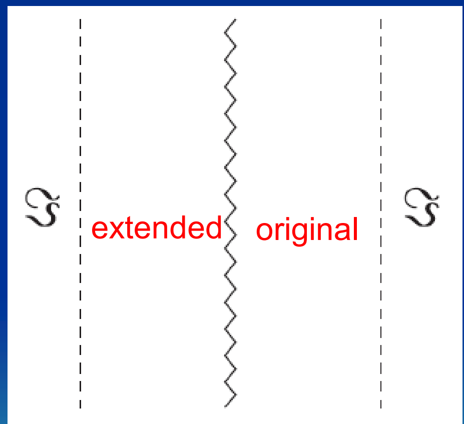
  Basis vector in the orthonormal frame

- Volume made of the Jacobi dual 1-forms: $V := |l_{(1)}||l_{(2)}||l_{(3)}|l_{(4)}|.$
  - Tipler weak: V is finite
  - Deformationally weak (Ori `00): all l's are finite
- Fact 3: the branch singularity is deformationally weak along radial causal geodesics



C^0 extension beyond the branch singularity

- Branch singularity is not the end of the world in the brane universe
- Q. What is the extended region beyond the branch singularity?
  - C^0 extension (because metric is finite)
- A. The same Boulware-Deser-Wheeler spacetime
  - Because of (Birkhoff's theorem) + (dynamical stability)



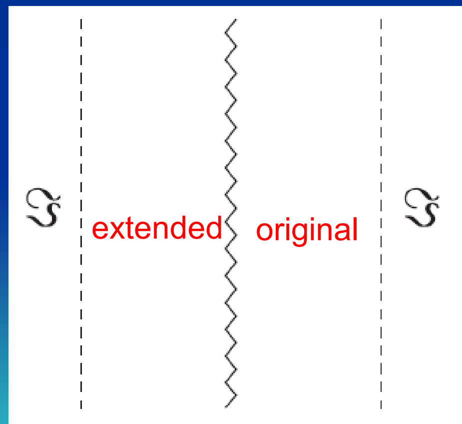
An example of the C^0 extended spacetime

Branch singularity as a massive thin-shell

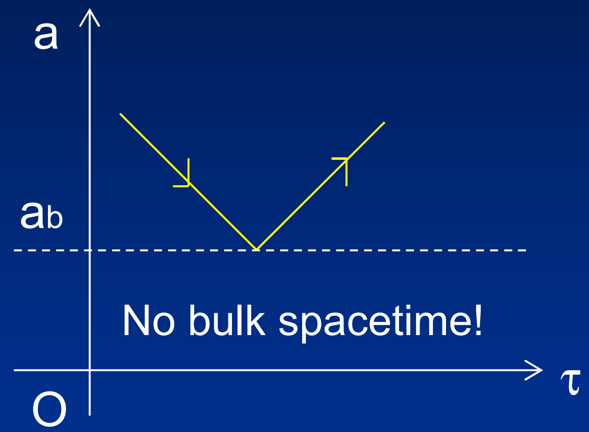
- Derivative of the metric diverges there, but…
- EGB junction condition shows the finite energy-momentum tensor on the branch singularity $\tau^a{}_b = \mathrm{diag}(-\rho_\mathrm{b}, p_\mathrm{b}, p_\mathrm{b}, p_\mathrm{b}),$

  where $$\rho_\mathrm{b} = -16\alpha \frac{h(r_\mathrm{b})^{3/2}}{r_\mathrm{b}^3}, \quad p_\mathrm{b} = \frac{8h(r_\mathrm{b})r_\mathrm{b}^2 - \mu}{2\sqrt{h(r_\mathrm{b})}r_\mathrm{b}^3},$$

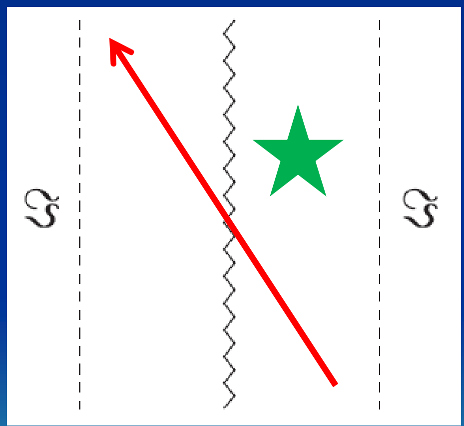
  - WEC is violated
- So, the branch singularity may be considered as a massive thin-shell (=another brane)
- 3-brane arriving branch singularity = collision of two branes



Evolution after the collision

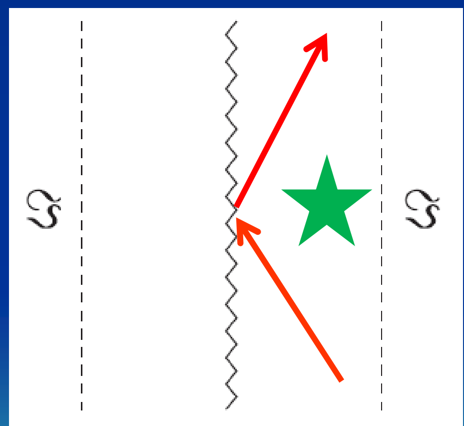


(a)



Red line:
Orbit of the brane

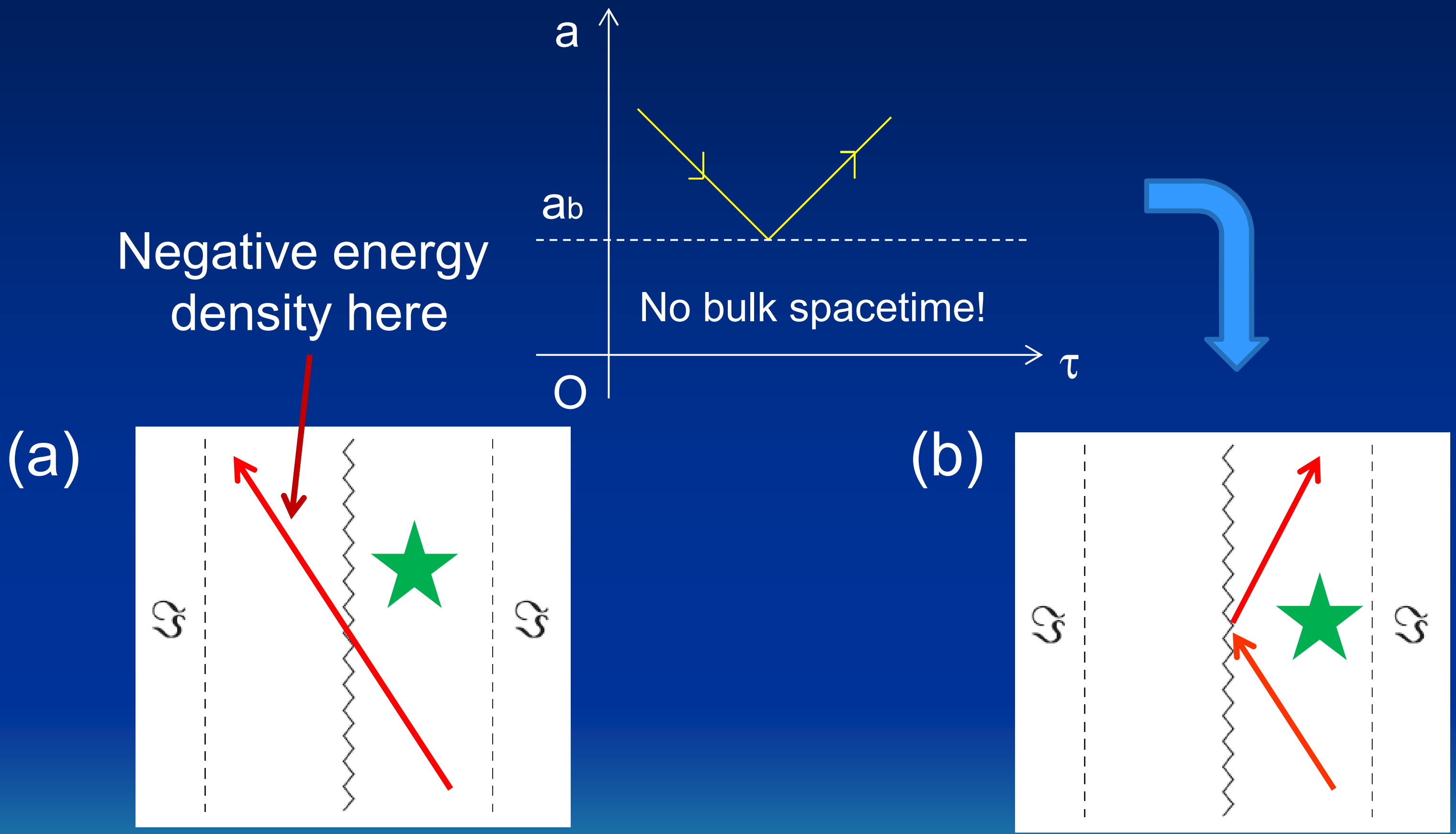
(b)



Right-hand side of the orbit (region with a star) is removed by the Z_2-symmetry

Which is the natural evolution after the collision?

The answer is (b)



However, back reaction to the branch singularity should be considered

Summary

Summary

- In the braneworld, the bulk branch singularity is NOT the end of the world
- It is another massive thin-shell
  - Note: For k=-1 (open FRW universe), the branch singularity can be spacelike (S(pacelike)-brane)
- 3-brane reaches there = collision of two branes
- New scenario for the Big-Bounce

Branch singularity in higher-curvature gravity

- The branch singularity maybe generic in any higher-curvature gravity
  - In EGB with a U(1) gauge field, there is no central singularity and the branch singularity is generic
- **Q. More consequence/applications in cosmology or BH physics?**

FIN