Gravitational Collapse in Hořava-Lifshitz Theory of Gravity

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Based on J. Greenwald, J. Lenells, V. H. Satheeshkumar, AW, "*Gravitational collapse in Hořava-Lifshitz theory,*" arXiv:1304.1167.

Outline

① Motivations

② Lifshitz Scalar Field

③ Minimal Hořava-Lifshitz Gravity (2009)

- Perspectives
- Assumptions
- Challenging Questions
- Different Models

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⑤ Conclusions

A. Why Quantum Gravity?

- Quantum field theory (QFT) provides a general framework for ALL theories, except for gravity.

- The universal coupling of gravity to all forms of energy makes it plausible that gravity should be implemented in such a framework, too.

I. Motivations

- At the singularities of both Big Bang and black holes, all the physics laws become invalid.

- Around these singular points, space-time curvatures become so high, Planck physics is highly involved. So, it is expected that quantum effects of gravity become important, and should be taken into account.

I. Motivations (Cont.)

B. Main Challenges

- GR is described by,

$$S_{EH} = \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g^{(4)}}\left(R^{(4)} - 2\Lambda\right),$$

which is not (perturbatively) renormalizable, so it may not describe the quantum physics in short distances,

$$L \lesssim \frac{1}{M_{pl}}.$$

I. Motivations (Cont.)

- The graviton propagator is given by

$$G(\omega, \mathbf{k}) = \frac{1}{k^2}, \quad (k^2 \equiv \omega^2 - c^2\mathbf{k}^2)$$

- In the Feynmann diagram, each internal line picks up a propagator, while each loop picks up an integral,

$$\int d\omega d^3k \propto k^4$$

I. Motivations (Cont.)

- The total contribution from one loop and one internal propagator is

$$\int d\omega d^3k \cdot G(\omega, \mathbf{k}) = k^4 \cdot k^{-2} \to \infty,$$

as $k \to \infty$. At increasing loop orders, the Feynman diagrams require counterterms of ever-increasing degree in curvature

$\Rightarrow$ Nonrenormalizable.

I. Motivations (Cont.)

- Improved ultraviolet (UV) behavior can be obtained, if higher-derivative curvatures added. For example, when quadratic terms, $\left(R^{(4)}\right)^2$, are added, we have

$$G_k = \frac{1}{k^2 - G_N k^4} \sim k^{-4},$$

as $k \to \infty$.

I. Motivations (Cont.)

- Then, at high energy,

$$\int d\omega d^3k \cdot G(\omega, \mathbf{k}) = k^4 \cdot k^{-4} \rightarrow \text{ finite,}$$

as $k \rightarrow \infty$. But two new problems occur [K. Stelle, Gen. Relativ. Grav. 9 (1978) 353],

$$\mathrm{G}_{\mathbf{k}} = \frac{1}{k^2 - G_N k^4} = \frac{1}{k^2} - \frac{1}{k^2 - 1/G_N},$$

$\frac{1}{k^2}$: **spin-0 graviton;**

$-\frac{1}{k^2 - 1/G_N}$: **ghost.**

I. Motivations (Cont.)

- Solving the ghost problem is not an easy task, and it is still an open question, after ~ 35 years!

- Since 1920's, Quantum Gravity has been one of the main driving forces in gravitational physics, and after about 90 years, still only few candidates exist:
  - Superstring/M-Theory
  - Loop Quantum Gravity
  - ...

II. Lifshitz Scalar Field

- A free scalar field in $3+1$ flat spacetime:

$$S_\phi^{free} = \int dt d^3x \left(\dot{\phi}^2 + \phi \Delta \phi\right),$$

$\Delta \equiv \partial^i \partial_i$, which is of general covariance:

$$t \to \xi^0(t,x), \quad x^i \to \xi^i(t,x).$$

The propagator of the scalar field is,

$$\left(\partial_t^2 - \Delta\right)\phi = 0 \quad \Rightarrow \quad G^\phi(\omega, \mathbf{k}) = \frac{1}{k^2}.$$

II. Lifshitz Scalar Field (Cont.)

- Its nonrenormalizability (when including interaction terms) can be easily understood from the analysis of scaling. For example, the free action S_ϕ^{free} is invariant under the isotropic scaling:

$$t \to b^{-1} t, \quad x^i \to b^{-1} x^i, \quad \phi \to b^{+1} \phi,$$

$$\begin{aligned} S_\phi^{free} &= \int dt d^3x \left(\dot{\phi}^2 + \phi \Delta \phi\right) \\ &= \int b^{-1} dt' b^{-3} d^3x' \left(\left(\frac{b d\phi'}{b^{-1} dt'}\right)^2 + b\phi' b^2 \Delta'(b\phi')\right) \\ &= \int dt' d^3x' \left(\left(\frac{d\phi'}{dt'}\right)^2 + \phi' \Delta' \phi'\right) = S_{\phi'}^{free} \end{aligned}$$

II. Lifshitz Scalar Field (Cont.)

- Units,

$$(\text{Lenght, Time, Mass}) = (L, T, M),$$

so that

$$\begin{aligned} [c] &= \frac{[\Delta x]}{[\Delta t]} = LT^{-1}, \\ [E] &= \left[mc^2\right] = ML^2T^{-2}, \\ [h] &= \frac{[E]}{[\nu]} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1} \end{aligned}$$

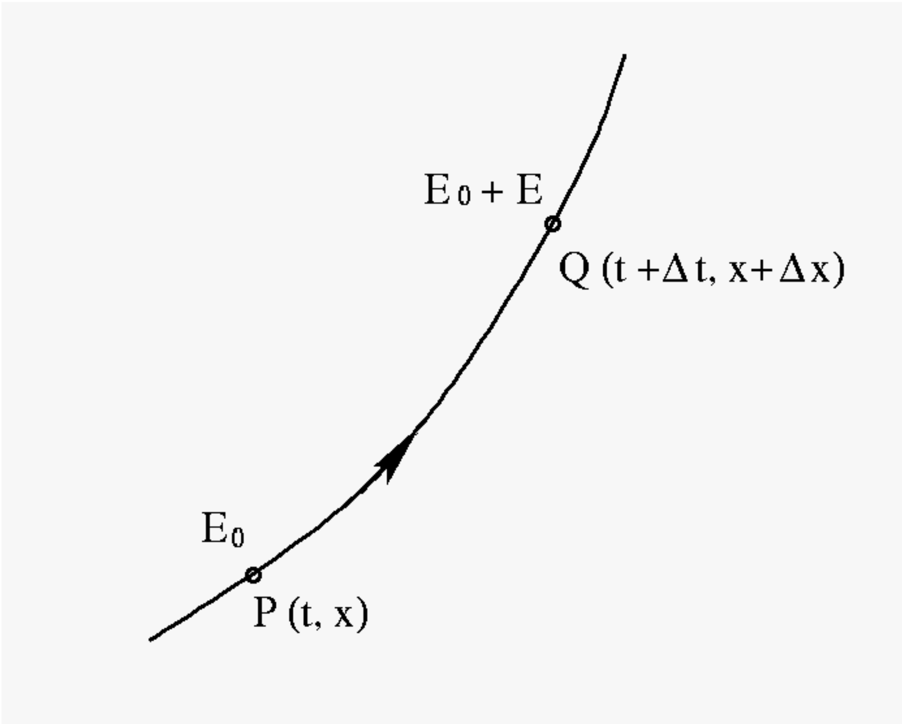
- Natural units, $c = 1 = h \Rightarrow$

$$L = T, \quad E = M, \quad L = M^{-1} = E^{-1}.$$

II. Lifshitz Scalar Field (Cont.)

- For a process with $\Delta\phi \simeq E$, we have

$$\Delta t \simeq E^{-1}$$
$$\Delta \ell \equiv \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} \simeq E^{-1}$$
$$\frac{\Delta\phi}{\Delta t} \simeq \frac{E}{E^{-1}} = E^2,$$
$$\frac{\Delta\phi}{\Delta \ell} \simeq \frac{E}{E^{-1}} = E^2,$$



II. Lifshitz Scalar Field (Cont.)

Then, we find

$$\int_p^{p+\Delta p} dt d^3x \dot{\phi}^2 \simeq \Delta t (\Delta \ell)^3 \left(\frac{\Delta \phi}{\Delta t}\right)^2,$$

$$S_\phi^{free} = \int_p^{p+\Delta p} dt d^3x \left(\dot{\phi}^2 + \frac{1}{c_\phi^2}\phi \nabla^2 \phi\right)$$

$$\simeq \Delta t (\Delta \ell)^3 \left[\left(\frac{\Delta \phi}{\Delta t}\right)^2 + \frac{1}{c_\phi^2}\left(\frac{\Delta \phi}{\Delta \ell}\right)^2\right]$$

$$\simeq E^{-1} \cdot E^{-3} \cdot E^4 \simeq \mathcal{O}(1)$$

$\Rightarrow$ A free scalar field is (power-counting) renormalizable!

II. Lifshitz Scalar Field (Cont.)

Note that the action S_ϕ^{free} defined above is for a free scalar field. Physicists are more interested in its interaction with itself (self-interaction) or with other fields. For example, let us consider the self-interaction term,

$$S_\phi^{inter.} = \int dt d^3x \left(\phi^2 \nabla^2 \phi\right),$$

which is scaling as

$$S_\phi^{inter.} \sim b^{-1-3\,+\,2+2+1} = b.$$

II. Lifshitz Scalar Field (Cont.)

Then, for a process with energy $\Delta\phi \simeq E$, we have

$$S_{\phi}^{inter.} = \int dt d^3x \left(\phi^2 \nabla^2 \phi\right) \simeq \Delta t (\Delta \ell)^3 \left(\frac{(\Delta\phi)^3}{(\Delta\ell)^2}\right) \simeq E \to \infty,$$

as $E \to \infty$. So that

$$S_{\phi}^{total} = S_{\phi}^{free} + S_{\phi}^{inter.} + ... = \mathcal{O}(1) + \mathbf{E}^{+1} + ... \to \infty,$$

$\Rightarrow$ non-renormalizable. Note

$$S_{\phi}^{total} = S_{\phi}^{free} + S_{\phi}^{Int.} + ... \sim b^0 + \mathbf{b}^{+1} + ...$$

II. Lifshitz Scalar Field (Cont.)

- In general, for an operator $\mathcal{O}$, if

$$\int dtd^3x\,(\phi\mathcal{O}\phi) \;\sim\; b^\delta \;\sim\; E^\delta.$$

Therefore, once we know δ, we know whether the operator $\mathcal{O}$ is (power-counting) renormalizable or not [J. Polchinski, arXiv:hep-th/9210046],

$$\delta = \begin{cases} & \quad\quad (E\to 0) \quad\quad\quad\quad (E\to\infty) \\ >0, & \Rightarrow \ \text{Irrelevant,} \quad \text{nonrenormalizable,} \\ =0, & \Rightarrow \ \text{Marginal,} \quad \text{strictly renormalizable,} \\ <0, & \Rightarrow \ \text{Relevant,} \quad \text{superrenormalizable.} \end{cases}$$

II. Lifshitz Scalar Field (Cont.)

- Lifshitz scalar field [E.M. Lifshitz, Zh. Eksp. Teor. Fiz. 11, 255; 269 (1941)]:

$$S_{\phi}^{free} = \int dt d^3x \left(\dot{\phi}^2 + \phi\Delta\phi + \alpha\phi\Delta^z\phi\right),$$

  $\alpha = (-1)^{z+1}/M^{2(z-1)}$,

  $z(\geq 1)$: dynamical critical exponent

- It is a well-established theory, and has achieved great success in explaining various phenomena in condensed matter physics [D. I. Uzunov, Theory of Critical Phenomena (World Scientific, Singapore, 1993)].

II. Lifshitz Scalar Field (Cont.)

- The equation of motion is,

$$\ddot{\phi} - \left(\Delta + \alpha\Delta^{z}\right)\phi = 0,$$

or in the momentum space,

$$\ddot{\phi}_k + \omega_k^2 \phi_k = 0,$$

$$\omega_k^2 \equiv \left(1 + \left(\frac{k}{M}\right)^{2(z-1)}\right) k^2 \simeq \begin{cases} k^2, & k \ll M, \\ \left(\frac{k}{M}\right)^{2(z-1)} k^2, & k \gg M \end{cases}$$

II. Lifshitz Scalar Field (Cont.)

- In low energies $(k \ll M)$,

$$S_{\phi}^{free} \simeq \int dt d^3x \left(\dot{\phi}^2 + \phi \Delta \phi\right), \ (k \ll M),$$

the Lorentz symmetry

$$x^a \to x^a(\bar{t}, \bar{x}),$$

is restored, and S_{ϕ}^{free} is scaling-invariant under,

$$t \to b^{-1}t, \quad x^i \to b^{-1}x^i, \quad \phi \to b^{+1}\phi.$$

II. Lifshitz Scalar Field (Cont.)

- In high energies $(k \gg M)$,

$$S_\phi^{free} \simeq \int dt d^3x \left(\dot{\phi}^2 + \alpha \phi \Delta^z \phi\right),$$

the Lorentz symmetry is *broken*, and S_ϕ^{free} is scaling-invariant only under,

$$t \to b^{-z} t, \quad x^i \to b^{-1} x^i, \quad \phi \to b^{(3-z)/2} \phi.$$

- Remarkable, for $z = 3$, the scalar field is scaling-invariant,

$$\phi \to \phi.$$

In the rest of this talk, we shall take $z = 3$, without loss of generality.

II. Lifshitz Scalar Field (Cont.)

- Then, in high energy the interacting term now scales as,

$$S_\phi^{inter.} = \frac{1}{M}\int dt d^3x \left(\phi^2 \Delta\phi\right)$$

$$\sim b^{-3-3+0+2+0} = b^{-4}.$$

$\Rightarrow$ Superrenormalizable!

- In addition, for a ϕ^n term, we have

$$\int dt d^3x \phi^n \sim b^{-6}.$$

- Thus, for a process with momentum k, we have

$$\int dt d^3x \phi^n \sim \frac{1}{k^6} \sim 0,$$

as $k \gg 1$ $\Rightarrow$ Superrenormalizable!!!

II. Lifshitz Scalar Field (Cont.)

- Adding various sub-leading terms, we have

$$S_\phi = \int dt d^3x \left(\dot{\phi}^2 - \phi \mathcal{O} \phi\right),$$

$$\mathcal{O} = m^2 - c^2 \Delta + \frac{g_1}{M^2} \Delta^2 - \frac{g_2}{M^4} \Delta^3,$$

Δ^3: *marginal;* m^2, Δ, Δ^2 : *relevant*.

- Clearly, the relevant terms do not change the UV behavior of the theory, but do change the IR behavior.

II. Lifshitz Scalar Field (Cont.)

- Similarly, one can add an arbitrary potential term,

$$S_\phi^{Int.} = \int dt d^3x V(\phi) = \int dt d^3x \left(\sum_{n=-\infty}^{\infty} a_n \phi^n \right),$$

where

$$[a_n] = [k]^6, \ \Rightarrow \left[S_\phi^{Int.}\right] = [1].$$

- The theory finally takes the form,

$$S_\phi = \int dt d^3x \left[\dot{\phi}^2 - \phi \mathcal{O} \phi - V(\phi)\right],$$

which is *(power-counting) renormalizable!*

III. Minimal Hořava-Lifshitz (HL) Gravity

Perspectives [P. Hořva, PRD79 (2009) 084008]:

- Construct a theory of quantum gravity in the framework of QFT

- Take the metric as the fundamental variables

- Lorentz symmetry appears only as an emergent symmetry at low energies, but can be fundamentally absent at high energies

III. Minimal HL Gravity (Cont.)

- Breaking of the Lorentz symmetry is accomplished by the anisotropic scaling between time and space,

$$t \to b^{-3} t, \quad x^i \to b^{-1} x^i, \ (i = 1, 2, 3),$$

similar to the Lifshitz scalar field [E.M. Lifshitz, Zh. Eksp. Teor. Fiz. 11 (1941), 255; 269].

III. Minimal HL Gravity (Cont.)

- A natural starting point is the ADM decompositions,

$$N, \quad N^i, \quad g_{ij},$$

with dimensions

$$[N] = 1, \ [N^i] = [k]^2$$

$$[g_{ij}] = 1$$

$$[t] = [k]^{-3}, \ [x] = [k]^{-1}$$

(March 18, 2010)

III. Minimal HL Gravity (Cont.)

Two Basic Assumptions:

- The space-times possess the foliation-preserving diffeomorphisms,

$$t \to f(t), \quad x^i \to \xi^i(t, x),$$

  denoted by Diff$(M, \mathcal{F})$.

- The field equations are *second-order* of time derivatives, and *sixth-order* of spatial derivatives.

III. Minimal HL Gravity (Cont.)

- With the Diff$(M, \mathcal{F})$ symmetry, the building blocks:

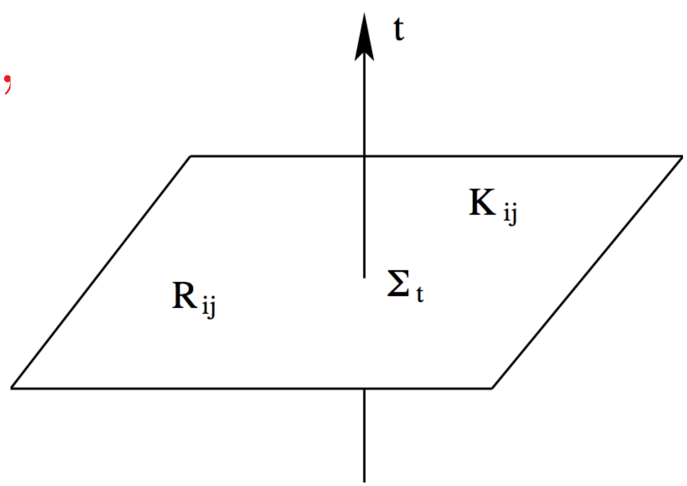
$$K_{ij}, \quad R_{ij}, \quad a_i \equiv \frac{N_{,i}}{N}, \quad \nabla_i,$$

R_{ij}: the 3D Ricci tensor of g_{ij}

K_{ij}: the extrinsic curvature of the leaves $t =$ Constant:

$$K_{ij} = \frac{1}{2N}\left(-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i\right),$$

∇_i: the covariant derivative w.r.t. g_{ij}



- Their dimensions:

$$[K_{ij}] = [k]^3, \quad [R_{ij}] = [k]^2, \quad [a_i] = [k].$$

III. Minimal HL Gravity (Cont.)

- The scalars with the above two assumptions are [Zhu, Shu, Wu & AW, PRD85 (2012) 044053]:

$$
\begin{aligned}
[k]^6 :\ & K_{ij}K^{ij},\ K^2,\ R^3,\ RR_{ij}R^{ij},\ R^i_j R^j_k R^k_i,\ (\nabla R)^2, \\
& \left(\nabla_i R_{jk}\right)\left(\nabla^i R^{jk}\right),\ \left(a_i a^i\right)^2 R,\ \left(a_i a^i\right)\left(a_i a_j R^{ij}\right), \\
& \left(a_i a^i\right)^3, a^i \Delta^2 a_i, \left(a^i{}_i\right) \Delta R, ..., (> 60), \\
[k]^5 :\ & K_{ij}R^{ij},\ \epsilon^{ijk} R_{il} \nabla_j R^l_k,\ \epsilon^{ijk} a_i a_l \nabla_j R^l_k, \\
& a_i a_j K^{ij},\ K^{ij} a_{ij},\ \left(a^i{}_i\right) K, \\
[k]^4 :\ & R^2,\ R_{ij}R^{ij},\ \left(a_i a^i\right)^2,\ \left(a^i{}_i\right)^2,\ \left(a_i a^i\right) a^j{}_j, \\
& a^{ij} a_{ij},\ \left(a_i a^i\right) R,\ a_i a_j R^{ij},\ R a^i{}_i, \\
[k]^3 :\ & \omega_3(\Gamma), \\
[k]^2 :\ & R,\ a_i a^i, \\
[k]^1 :\ & \text{None}, \\
[k]^0 :\ & \gamma_0,
\end{aligned}
$$

$\omega_3(\Gamma)$: the gravitational Chern-Simons term; γ_0: a dimensionless constant, $\Delta = g^{ij}\nabla_i \nabla_j$, and

III. Minimal HL Gravity (Cont.)

$$\epsilon^{ijk} \equiv \frac{e^{ijk}}{\sqrt{g}}, (e^{123} = 1),$$
$$a_{i_1 i_2 \ldots i_n} \equiv \nabla_{i_1}\nabla_{i_2}\ldots\nabla_{i_n} \ln(N),$$
$$\omega_3(\Gamma) \equiv \frac{e^{ijk}}{\sqrt{g}}(\Gamma^m_{jl}\partial_j\Gamma^l_{km} + \frac{2}{3}\Gamma^n_{il}\Gamma^l_{jm}\Gamma^m_{kn}).$$

- The general action is [D. Blas, O. Pujolas & S. Sibiryakov, PRL104 (2010) 181302],

$$S_g = \int dt d^3x N \sqrt{g} \mathcal{L}_g,$$
$$\mathcal{L}_g = \underbrace{a_1 K_{ij}K^{ij} + a_2 K^2 + a_3 R^3 + \ldots}_{(\mathcal{N} \simeq 70)}$$

III. Minimal HL Gravity (Cont.)

Two Additional Assumptions:

- Projectability:

$$N = N(t).$$

This it is preserved by Diff$(M, \mathcal{F})$:

$$\tilde{N}(t') = \frac{df(t')}{dt'} N(t(t')).$$

III. Minimal HL Gravity (Cont.)

- Detailed Balance:

$$\mathcal{L}_g = \zeta^2(K_{ij}K^{ij} - \lambda K^2) - \mathcal{L}_{(V,D)},$$
$$\mathcal{L}_{(V,D)} = E_{ij}\mathcal{G}^{ijkl}E_{kl},$$
$$E^{ij} = \frac{1}{\sqrt{g}}\frac{\delta W_g}{\delta g_{ij}},$$
$$W_g = \frac{1}{w^2}\int_{\Sigma}\omega_3(\Gamma) + \mu\int d^3x\,(R - 2\Lambda),$$

$\mathcal{G}^{ijkl}$: the generalized De Witt metric, λ: a coupling constant. Then, the number of independent coupling constants is,

$$\mathcal{N} = 5$$

III. Minimal HL Gravity (Cont.)

Major Challenges:

- The Minkowski space is not stable [P. Horava, PRD79 (2009) 084008], even without the detailed balance condition [T. Sotiriou, M. Visser & S. Weinfurtner, JHEP 10 (2009) 033; AW, R. Maartens, PRD81 (2010) 024009].

- Note that the de Sitter space-time is stable [Y.Q. Huang, AW, Q. Wu, Mod. Phys. Lett. A25 (2010) 2267; AW, Q. Wu, PRD83 (2011) 044025].

III. Minimal HL Gravity (Cont.)

- It becomes strongly coupled at very low energy even without the detailed balance condition [D. Blas, O. Pujolas & S. Sibiryakov, JHEP 10 (2009) 029; K. Koyama & F. Arroja, JHEP 03 (2010) 061; AW, Q. Wu, PRD83 (2011) 044025].

- Note that strong coupling itself is not a problem, as long as it is consistent with observations. QCD, for example, is strong coupling.

III. Minimal HL Gravity (Cont.)

- Several particular cases show that this is indeed the case. These include:
  - Static spherical case [S. Mukohyama, CQG27 (2010) 223101]
  - Exact cosmological solutions [AW, Q. Wu, PRD83 (2011) 044025]
  - Nonlinear cosmological perturbations with and without matter [K. Izumi, S. Mukohyama, PRD84(2011) 064025; A.E. Gumrukcuoglu, S. Mukohyama, AW, PRD85 (2012) 064042]

III. Minimal HL Gravity (Cont.)

To overcome the above problems, various models have been proposed [S. Mukohyama, CQG**27** (2010) 223101; P. Hořava, CQG**28** (2011) 114012; T. Clifton, P.G. Ferreira, A. Padilla, and C. Skordis, Phys. Rept. **513** (2012) 1]:

- The healthy extension [D. Blas, O. Pujolas & S. Sibiryakov, PRL104 (2010) 181302; PLB688 (2010) 350; JHEP04 (2011) 018]:

$$
\begin{aligned}
S_g &= \int dt d^3x N \sqrt{g} \mathcal{L}_g, \\
\mathcal{L}_g &= \underbrace{a_1 K_{ij} K^{ij} + a_2 K^2 + a_3 R^3 + a_4 (a_i a^i)^3 + ...}_{(\mathcal{N} \simeq 70)}
\end{aligned}
$$

III. Minimal HL Gravity (Cont.)

- Note that in the IR, it reduces to,

$$\mathcal{L}_g^{IR} = \frac{1}{16\pi G}\left(K_{ij}K^{ij} - \lambda K^2 + \beta R + \alpha a_i a^i - 2\Lambda\right).$$

- The ghost and instability problems are absent for the choice,

$$\frac{3\lambda - 1}{\lambda - 1} > 0, \quad 0 < \alpha < 2.$$

III. Minimal HL Gravity (Cont.)

- The strong coupling problem can be avoided by introducing an energy scale M_*,

$$M_* \leq \Lambda_\omega \equiv |\lambda - 1|^{1/2} M_{pl},$$

Λ_ω: the would-be strong coupling energy;

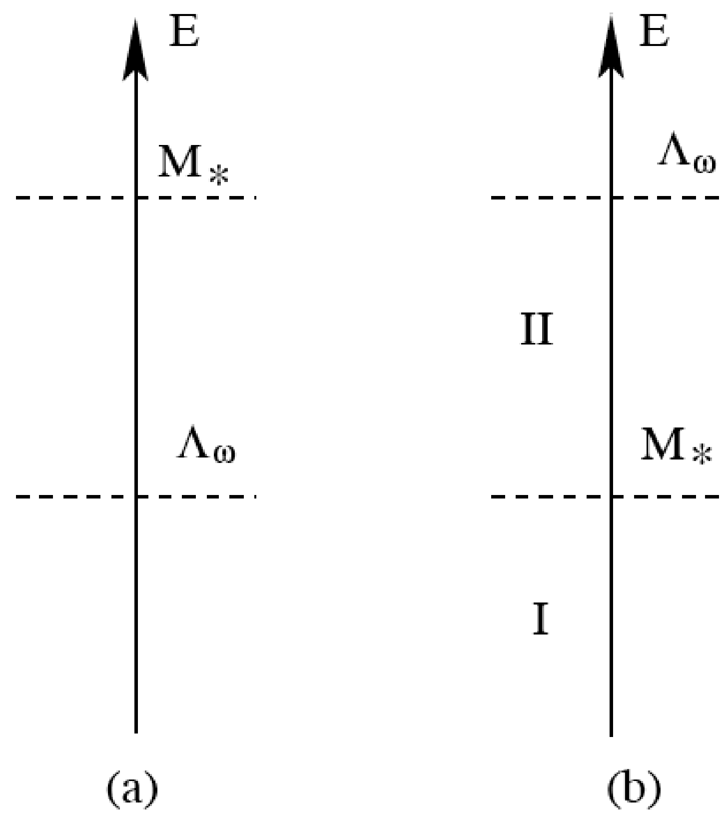
M_*: the suppression energy of high-order derivative terms:

$$\mathcal{O} = \partial^2 + \frac{g_1}{M_*^2}\partial^4 + \frac{g_1}{M_*^4}\partial^6$$

III. Minimal HL Gravity (Cont.)

- Then, before the strong coupling energy Λ_ω is reached, the high-order derivative terms take over, and the scaling becomes anisotropic. So, the interaction terms under the new scaling become either margin or relevant:

$$\mathcal{O} \simeq \frac{g_1}{M_*^2}\partial^4 + \frac{g_1}{M_*^4}\partial^6$$



III. Minimal HL Gravity (Cont.)

- Solar system tests require,

$$|\lambda - 1| \leq 4 \times 10^{-7}$$

$\Rightarrow$

$$M_* \leq 10^{16} \text{ GeV}$$

III. Minimal HL Gravity (Cont.)

- A more dramatic modification is the inclusion of an extra U(1) symmetry [P. Horava & C.M. Melby-Thompson, PRD82 (2010) 064027],

$$U(1) \ltimes \text{Diff}(M,\ \mathcal{F}),$$

realized by introducing a U(1) gauge field A and a Newtonian prepotential $\varphi(\equiv \nu)$.

III. Minimal HL Gravity (Cont.)

- With this extra symmetry, spin-0 gravitons are absent, and the theory has the same degree of freedom as GR [P. Horava & C.M. Melby-Thompson, PRD82 (2010) 064027; AW, Y. Wu, PRD83 (2011) 044031].

- This was initially done in the case $\lambda = 1$, soon generalized to the case with any λ [A.M. da Silva, CQG28 (2011) 055011].

III. Minimal HL Gravity (Cont.)

- Even with any λ, spin-0 gravitons are still eliminated [A.M. da Silva, CQG28 (2011) 055011; Y.-Q. Huang, AW, PRD83 (2011) 104012].
- As a result, problems related to the spin-0 gravitons, such as ghost, instability, different speeds, strong coupling in the gravitational sector, disappear automatically.
- Strong coupling problem in the matter sector can be also resolved by introducing a suppression energy M_*, but now with [K. Lin, AW, Q. Wu, T. Zhu, PRD84 (2011) 044051],

$$M_* \leq |\lambda - 1|^{1/4} M_{pl}$$

III. Minimal HL Gravity (Cont.)

- Consistent with solar system tests, provided that [K. Lin, S. Mukohyama, AW, PRDD 86 (2012) 104024]:

$$ds^2 = -\mathcal{N}^2 dt^2 + g_{ij}\left(dx^i + \mathcal{N}^i dt\right)\left(dx^j + \mathcal{N}^j dt\right),$$

$$\mathcal{N} = N - \frac{\upsilon}{c^2}(A - \mathcal{A}), \quad \mathcal{N}^i = N^i + N\nabla^i\varphi,$$

$$\mathcal{A} \equiv -\dot{\varphi} + N^i\nabla_i\varphi + \frac{1}{2}N(\nabla_i\varphi)^2.$$

υ: a dimensionless coupling constant.

III. Minimal HL Gravity (Cont.)

- It is interesting to note that such defined ds^2 is a scalar under both U(1) and Diff($M,\ \mathcal{F}$) transformations.

- The consistency with solar system tests requires [K. Lin, S. Mukohyama, AW, PRDD 86 (2012) 104024],

$$|v - 1| < 10^{-5}.$$

III. Minimal HL Gravity (Cont.)

- Consistent with cosmology [Y.-Q. Huang, AW, Q. Wu, JCAP10 (2012) 010; Y.-Q. Huang, AW, PRD86 (2012) 103523].

- In particular, the recent Planck results on non-Gaussianity yields [Y.-H. Huang, AW, R. Yousefi, T. Zhu arXiv:1304.1556],

$$\left(\frac{H_{\text{inflation}}}{M_*}\right)^2 \left(\frac{H_{\text{inflation}}}{M_{pl}}\right) \leq 10^{-8}$$

— the strongest constraint found so far.

III. Minimal HL Gravity (Cont.)

- Another model is [T. Zhu, Q. Wu, AW, F.-W. Shu, PRD84 (2011) 101502 (R); T. Zhu, F.-W. Shu, Q. Wu & AW, PRD (2012)]:
  - Non-projectable $N = N(t, x)$
  - Detailed balance condition softly broken reduces the number of independent coupling constant to

$$\mathcal{N} = 15$$

  - Enlarged symmetry, $U(1) \ltimes \text{Diff}(M,\ \mathcal{F})$, can eliminate the spin-0 gravitons.

III. Minimal HL Gravity (Cont.)

- Since spin-0 gravitons can be eliminated, all the problems related to them, such as ghost, instability, different speeds and strong coupling in the gravitational sector, do not exist any longer.

- Strong coupling problem in the matter sector can be also resolved by intruding a suppression energy M_*, so that

$$\Lambda_{SC} = |\lambda - 1|^{1/4} M_{pl},$$

— the same as in the projectable case with $U(1)$ symmetry.

III. Minimal HL Gravity (Cont.):

- Consistent with solar system tests [K. Lin, AW, PRDD87 (2013) 084041].

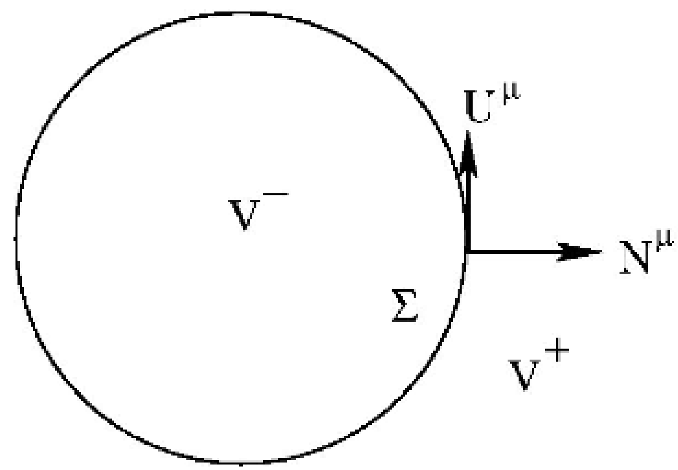
- But, in contrast to the case with the projectability condition [K. Lin, S. Mukohyama, AW, PRDD 86 (2012) 104024], the consistency can be obtained even without the gauge field and Newtonian prepotential being part of the metric.

- Consistent with cosmological observations [T. Zhu, Y.-Q. Huang, AW, JHEP01 (2013) 138; AW, Q. Wu, W. Zhao, T. Zhu, PRD87 (2013) 103512; T. Zhu, W. Zhao, Y.-H. Huang, AW, Q. Wu, arXiv:1305.0600].

IV. Gravitational Collapse

- To consider the gravitational collapse of a star with a finite radius, we first divide the space-time into three different regions: internal V^-, surface Σ, and external V^+, where [J. Greenwald, J. Lenells, V. H. Satheeshkumar, AW, arXiv:1304.1167],

$$
\begin{aligned}
V^- &= \{x^\mu : \Phi(t,r) < 0\}\,,\\
\Sigma &= \{x^\mu : \Phi(t,r) = 0\}\,,\\
V^+ &= \{x^\mu : \Phi(t,r) > 0\}\,,
\end{aligned}
$$



with the normal and tangent vectors,

$$
N_\mu = \frac{\Phi_{,\mu}}{\sqrt{\left|\Phi_{,\lambda}\Phi^{,\lambda}\right|}}, \qquad U_\lambda N^\lambda = 0.
$$

IV. Gravitational Collapse (Cont.):

- The main task is to develop the junction conditions across the hypersurface $\Phi(t, r) = 0$.
- In GR, two different approaches have been developed:
  - Israel's junction conditions [W. Israel, Nuovo Cimento, B**44**, 1 (1966); *ibid.*, B**48**, 463 (1967)]
  - Taub's junction conditions [A. H. Taub, J. Math. Phys. **21**, 1423 (1980)]
  - Of course, they all give the same results.

IV. Gravitational Collapse (Cont.):

In Israel's approach, Israel used the Gauss and Codacci equations [AW, N. Santos, Inter. J. Mod. Phys. A25 (2010) 1661],

$$R^{(D)}_{\mu\nu\lambda\rho} e^{\mu}_{(l)} e^{\nu}_{(i)} e^{\lambda}_{(k)} e^{\rho}_{(j)} = R^{(D-1)}_{likj} - \left(K_{ik}K_{jl} - K_{ij}K_{kl}\right),$$

$$R^{(D)}_{\mu\nu\lambda\rho} N^{\mu} e^{\nu}_{(i)} e^{\lambda}_{(k)} e^{\rho}_{(j)} = K_{ik;j} - K_{ij;k},$$

together with the Lanczos equations [C. Lanczos, Phys. Z. **23**, 539 (1922); Ann. Phys. (Germany), **74**, 518 (1924)],

$$\left[K_{ij}\right]^{-} - g_{ij}\,[K]^{-} = -\kappa^2_D \mathcal{T}_{ij},$$

IV. Gravitational Collapse (Cont.):

with

$$e^{\mu}_{(i)} \equiv \frac{\partial x^{\mu}}{\partial \zeta^{i}},$$

$$\left[K_{ij}\right]^{-} \equiv \text{lim}_{\Phi\to 0^{+}} K^{+}_{ij} - \text{lim}_{\Phi\to 0^{-}} K^{-}_{ij},$$

where on the hypersurface Σ, we have

$$x^{\mu} = x^{\mu}\left(\zeta^{i}\right),$$

ζ^{i}: the coordinates chosen on Σ.

IV. Gravitational Collapse (Cont.):

In Taub's approach, Taub used the generalized function analysis, and wrote the 4D metric as,

$$g_{\mu\nu}(t, x^i) = g^+_{\mu\nu}(t, x^i) H(\Phi) + g^-_{\mu\nu}(t, x^i) \left[1 - H(\Phi)\right],$$

$H(\Phi)$: the Heaviside function, defined as

$$H(\Phi) = \begin{cases} 1, & \Phi > 0, \\ \frac{1}{2}, & \Phi = 0, \\ 0, & \Phi < 0. \end{cases}$$

IV. Gravitational Collapse (Cont.):

- Since we do not know how to generalize the Lanczos equations to the HL theory, in the following we shall follow Taub's treatment.

- The field equations involve second-order derivatives of the metric coefficients with respect to t and sixth-order derivatives with respect to x^i. Thus, one might require that

$$F(t, x^i) = \begin{cases} C^1, & \text{w.r.t. } t, \\ C^5, & \text{w.r.t. } x^i, \end{cases}$$

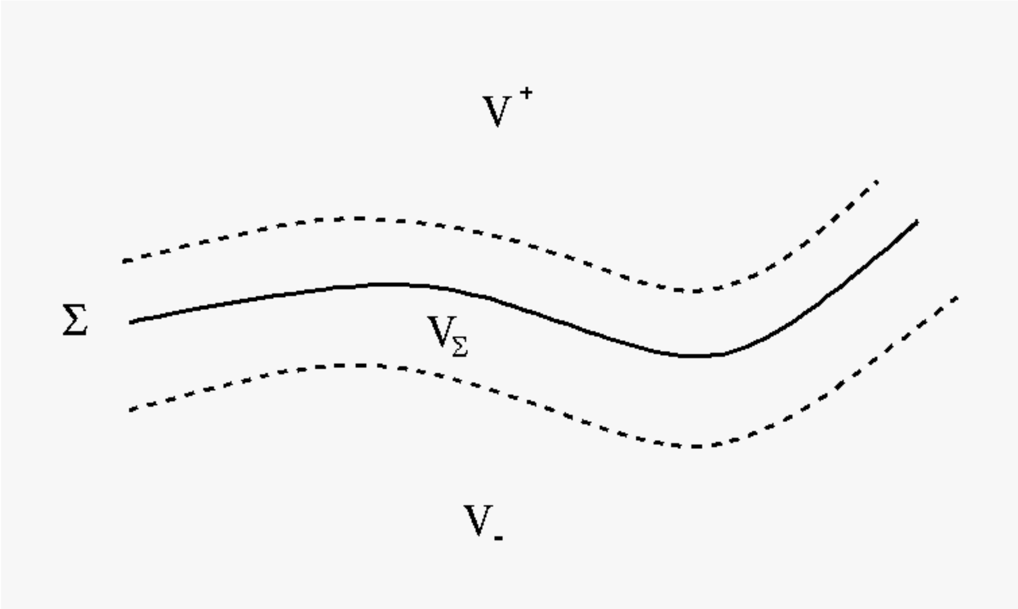
C^n: the first n derivatives exist and are continuous across the hypersurface $\Phi = 0$.

IV. Gravitational Collapse (Cont.):

- However, this assumption eliminates the important case of an infinitely thin shell of matter supported on Σ.
- Instead, we impose the minimal requirement that *the problem be mathematically meaningful:*
  - in $V^{\pm} \cup V_{\Sigma}$,
$$F(t, x^i) = \begin{cases} C^1, & \text{w.r.t. } t, \\ C^5, & \text{w.r.t. } x^i, \end{cases}$$
  - but acorss Σ,
$$F(t, x^i) = \begin{cases} C^0, & \text{w.r.t. } t, \\ C^1, & \text{w.r.t. } x^i, \end{cases}$$

IV. Gravitational Collapse (Cont.):

where V_{Σ} denotes an open neighborhood of Σ.



- It can be shown that these conditions avoid the appearance of the terms, such as

$$\delta^2(\Phi), \quad H(\Phi)\delta(\Phi),$$

which are not mathematically meaningful.

IV. Gravitational Collapse (Cont.):

Then, one can prove the following:

$$\frac{\partial H(\Phi)}{\partial x^\lambda} = \frac{\partial \Phi}{\partial x^\lambda}\delta(\Phi),$$

$$\frac{\partial}{\partial x^\lambda}\delta^{(n)}(\Phi) = \frac{\partial \Phi}{\partial x^\lambda}\delta^{(n+1)}(\Phi), \qquad (n = 0, 1, 2, \ldots),$$

$$\Phi\delta^{(n)}(\Phi) = -n\delta^{(n-1)}(\Phi), \qquad (n = 1, 2, \ldots),$$

where $\delta^{(0)}(\Phi) = \delta(\Phi)$, and

$$\langle \delta(\Phi), \varphi \rangle = \int_\Sigma \varphi d\Sigma,$$

$d\Sigma = \iota_N \ \mathrm{Vol}_g$: the volume three-form induced by g on Σ,

ι_N: the interior multiplication by N_μ.

IV. Gravitational Collapse (Cont.):

Following Taub, one can write any C^0 function $F(t,x)$ in the form,

$$F(t,x) = F^{+}H(\Phi) + F^{-}[1 - H(\Phi)] \equiv F^{D},$$

from which we have

$$\begin{aligned} F^{(1)} &\rightarrow F^{(1)\ D}, \\ F^{(2)} &\rightarrow \delta(\Phi), \\ F^{(3)} &\rightarrow \delta^{(1)}(\Phi), \\ \ldots &\rightarrow \ldots, \\ F^{(6)} &\rightarrow \delta^{(4)}(\Phi), \end{aligned}$$

$F^{(n)} \equiv \partial^n F/\partial y^n,\ (y = t, x)$.

IV. Gravitational Collapse (Cont.):

- Then, the HL equations can be cast in the forms,

$$(...)_{\text{Geometry}} = 8\pi G_N (...)_{\text{Matter}},$$

where

$$(...)_{\text{Geometry}} = G^D + \sum_{k=0}^{n} G^{Im}_{(k)} \delta^{(k)}(\Phi),$$

$G^D = G^+ H(\Phi) + G^-[1 - H(\Phi)]$

$G^{\pm}$: defined, respectively, in $V^{\pm}$

$G^{Im}_{(k)} = G^{Im}_{(k)}(t, x)$: functions defined in a neighborhood of Σ.

IV. Gravitational Collapse (Cont.):

Writing the matter part in the form,

$$(\ldots)_{\text{Matter}} = M^D + \sum_{k=0}^{n} M^{Im}_{(k)} \delta^{(k)}(\Phi),$$

the HL equations can be cast in the form,

$$\Psi = 0,$$

where

$$\begin{aligned} \Psi &\equiv (\ldots)_{\text{Geometry}} - 8\pi G_N (\ldots)_{\text{Matter}} \\ &= \Psi^D + \sum_{k=0}^{n} \Psi^{Im}_{(k)} \delta^{(k)}(\Phi) \end{aligned}$$

IV. Gravitational Collapse (Cont.):

Theorem: The equation $\Psi = 0$ is equivalent to:

$$\Psi^{\pm}(t,x) = 0, \qquad (t,x) \in V^{\pm}, \tag{1}$$

$$\sum_{k=0}^{j} (-1)^k \frac{(n-k)!j!}{(j-k)!} \left. \frac{\partial^{j-k} \Psi^{Im}_{(n-k)}}{\partial \Phi^{j-k}} \right|_{\Sigma} = 0, \tag{2}$$

$0 \leq j \leq n$, where

$$\frac{\partial \Psi}{\partial \Phi} = \frac{1}{\|\nabla \Phi\|_g} d\Psi \cdot N,$$

$$\frac{\partial^n \Psi}{\partial \Phi^n} = \left(\frac{1}{\|\nabla \Phi\|_g} \iota_N d \right)^n \Psi, \quad (n \geq 2).$$

IV. Gravitational Collapse (Cont.):

For $n = 3$, Eq.(2) is equivalent to

$$\Psi^{Im}_{(3)}|_{\Sigma} = 0,$$

$$\left.\left(3\frac{\partial\Psi^{Im}_{(3)}}{\partial\Phi} - \Psi^{Im}_{(2)}\right)\right|_{\Sigma} = 0,$$

$$\left.\left(3\frac{\partial^2\Psi^{Im}_{(3)}}{\partial\Phi^2} - 2\frac{\partial\Psi^{Im}_{(2)}}{\partial\Phi} + \Psi^{Im}_{(1)}\right)\right|_{\Sigma} = 0,$$

$$\left.\left(\frac{\partial^3\Psi^{Im}_{(3)}}{\partial\Phi^3} - \frac{\partial^2\Psi^{Im}_{(2)}}{\partial\Phi^2} + \frac{\partial\Psi^{Im}_{(1)}}{\partial\Phi} - \Psi^{Im}_{(0)}\right)\right|_{\Sigma} = 0.$$

IV. Gravitational Collapse (Cont.):

- In the spherical case, for the projectable case the ADM variables are,

$$N = 1, \quad N^i = \delta^i_r e^{\mu(r,t)-\nu(r,t)},$$
$$g_{ij}dx^i dx^j = e^{2\nu(r,t)}dr^2 + r^2 d\Omega^2.$$

- The continuity conditions required above imply

$$\begin{aligned} \mu_{,t} &= (\mu_{,t})^D, \quad \mu_{,r} = (\mu_{,r})^D, \\ \mu_{,tr} &= (\mu_{,tr})^D + \hat{\mu}_{,t}\delta(\Phi), \\ \mu_{,rt} &= (\mu_{,rt})^D - \dot{\mathcal{R}}\hat{\mu}_{,r}\delta(\Phi), \\ \mu_{,rr} &= (\mu_{,rr})^D + \hat{\mu}_{,r}\delta(\Phi), \end{aligned}$$

IV. Gravitational Collapse (Cont.):

$$
\begin{aligned}
\nu_{,t} &= (\nu_{,t})^D, \quad \nu_{,r} = (\nu_{,r})^D, \\
\nu_{,rr} &= (\nu_{,rr})^D, \quad \nu^{(3)} = (\nu^{(3)})^D, \\
\nu^{(4)} &= (\nu^{(4)})^D + \hat{\nu}^{(3)}\delta(\Phi), \\
\nu^{(5)} &= (\nu^{(5)})^D + 2\hat{\nu}^{(4)}\delta(\Phi) + \hat{\nu}^{(3)}\delta'(\Phi),
\end{aligned}
$$

where on Σ, we have $\Phi(t,r) \equiv r - \mathcal{R}(t) = 0$, and

$$
\hat{\mu} \equiv \tilde{\mu}^+ - \tilde{\mu}^-, \qquad \hat{\nu} \equiv \tilde{\nu}^+ - \tilde{\nu}^-,
$$

are defined on V_Σ.

IV. Gravitational Collapse (Cont.):

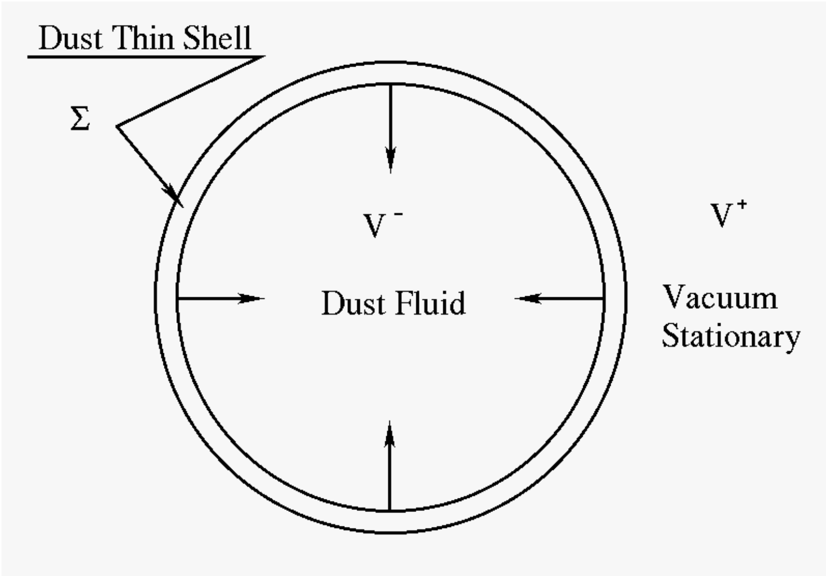
In the projectable case with the extra U(1) symmetry, the fundamental variables are, $\left(N(t),\ N_i,\ A,\ \varphi,\ g_{ij}\right)$, from which we obtain five sets of equations:

$$\begin{aligned}
\int \sqrt{g} d^3x \Psi_{\text{Hamiltonian}} &= 0, \quad (\delta N),\\
\Psi^i_{\text{Momentum}} &= 0, \quad (\delta N_i),\\
\Psi_{\varphi\text{-constraint}} &= 0, \quad (\delta \varphi),\\
\Psi_{A\text{-constraint}} &= 0, \quad (\delta A),\\
\Psi^{ij}_{\text{dynamical}} &= 0, \quad (\delta g_{ij}),
\end{aligned}$$

(i, j = 1, 2, 3), and each of them takes the forms of Eqs.(1) and (2).

IV. Gravitational Collapse (Cont.):

After writing down these equations explicitly in each of the three regions, $V^{\pm}$, Σ, we study a particular case, where



$$\begin{cases} \text{Region } V^{-}: & \text{dust fluid} \\ \text{Region } \Sigma: & \text{dust thin shell} \\ \text{Region } V^{+}: & \text{stationary vacuum spacetimes} \end{cases}$$

IV. Gravitational Collapse (Cont.):

- We find that the matching is possible when the internal is a dust fluid and the external is the Schwarzschild (anti-) de Sitter,

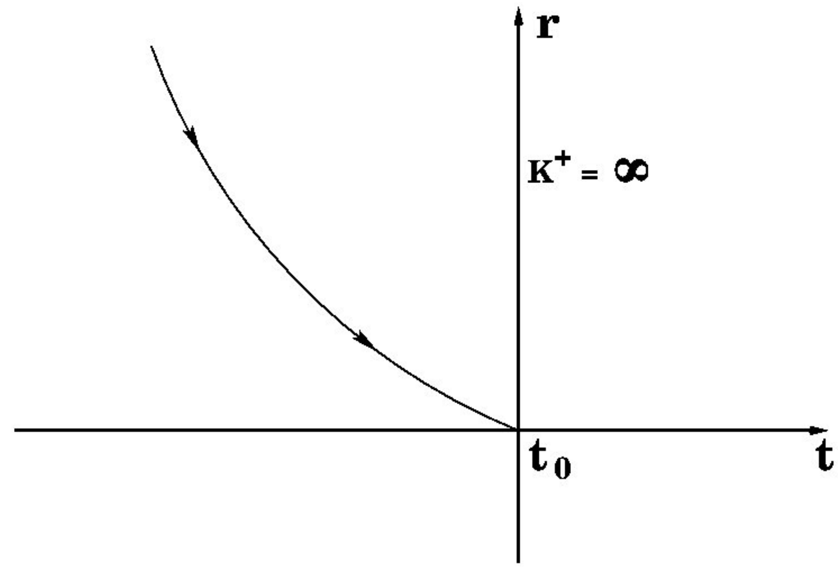
$$
\begin{aligned}
\nu^{+} &= 0, \\
\mu^{+} &= \frac{1}{2}\ln\left(\frac{2M}{r} + \frac{\Lambda}{3}r^{2}\right)
\end{aligned}
$$

- Note that μ^{+} behaves well across the “horizons” in the Painlevé-Gullstrand coordinates.

IV. Gravitational Collapse (Cont.):

- $\Lambda = 0$: In this case, we find that

$$\begin{aligned} \mathcal{R}(t) &= \mathcal{R}_0(t_0 - t)^{2/3}, \\ \rho_H &= \text{Constant} > 0, \\ \rho^{Im}_{(0)} &= \rho^{Im}_{(0)}(t) > 0, \\ M &= \frac{2}{9}\mathcal{R}_0^3 > 0 \end{aligned}$$



- Similar to GR, a space-time singularity is formed finally,

$$K^+(r),\ R^+(r) \to \infty, \quad \text{as } r \to 0.$$

IV. Gravitational Collapse (Cont.):

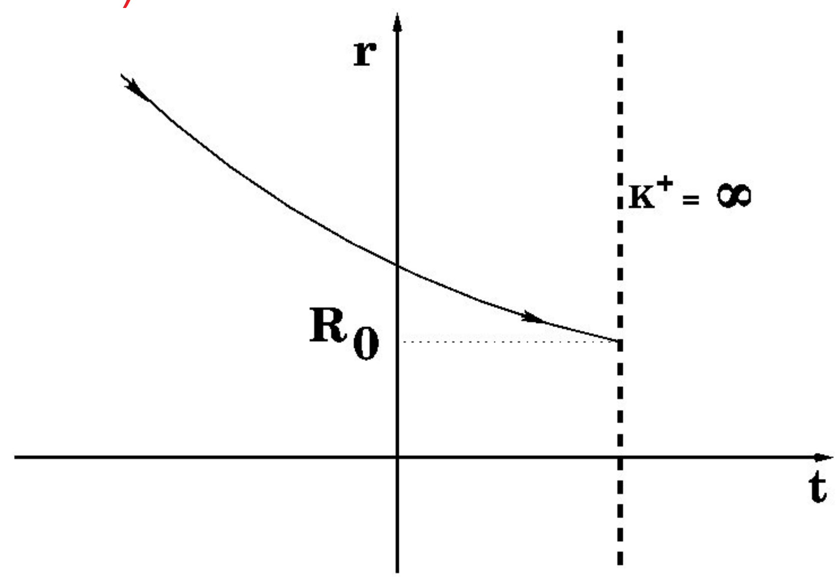
- $\Lambda > 0$: In this case, we find that

$$\mathcal{R}(t) = \mathcal{R}_0 \cosh^{\frac{2}{3}}\left(\frac{\sqrt{3\Lambda}}{2}(t_0 - t)\right),$$

$$\rho_H = \text{Constant} > 0,$$

$$\rho^{Im}_{(0)} = \rho^{Im}_{(0)}(t) < 0,$$

$$M = -\frac{1}{6}\Lambda \mathcal{R}_0^3 < 0,$$



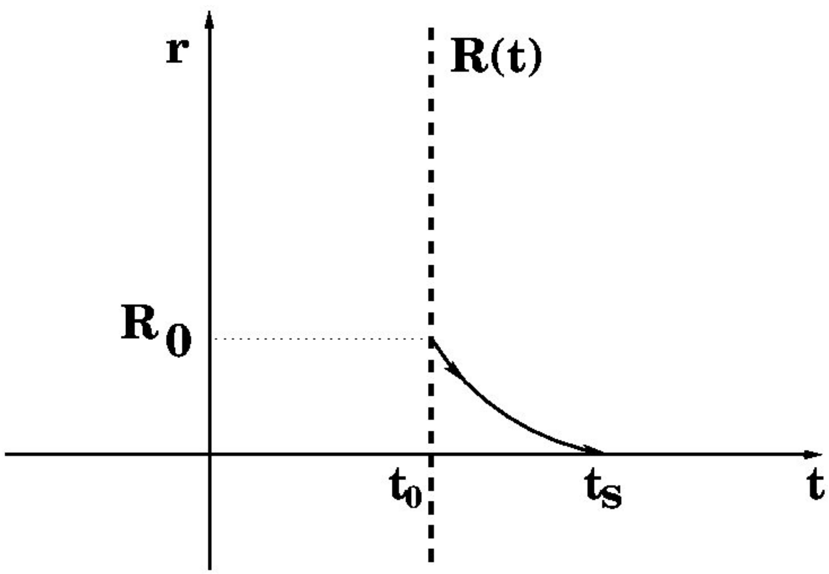
where

$$K^+(r) \to \infty, \quad R^+(r) \to \text{finite}, \quad \text{as } r \to \mathcal{R}_0.$$

IV. Gravitational Collapse (Cont.):

- $\Lambda < 0$: In this case, we find that

$$\mathcal{R}(t) = \mathcal{R}_0 \cos^{\frac{2}{3}}\left(\frac{\sqrt{3|\Lambda|}}{2}(t-t_0)\right), \; ; \; A(t,r) = A_0,$$
$$\rho_H = \text{Constant} > 0,$$
$$\rho^{Im}_{(0)} = \rho^{Im}_{(0)}(t) > 0,$$
$$M = \frac{1}{6}|\Lambda|\mathcal{R}_0^3 > 0,$$



where a space-time singularity is formed finally

$$K^+(r), \quad R^+(r) \to \infty, \quad \text{as } r \to 0.$$

V. Conclusions

- Using distribution theory, we have developed the general junction conditions for the collapse of a spherical star with finite radius $r = \mathcal{R}(t)$, in the framework of the HL theory with the (minimal) requirement: *the junctions be mathematically meaningful in terms of generalized functions.*

V. Conclusions (Cont.)

- Applying them to the projectable case with the extra U(1) symmetry, we have found that: an external Schwarzschild (anti-) de Sitter universe,

$$\nu^+ = 0,$$
$$\mu^+ = \frac{1}{2}\ln\left(\frac{2M}{r} + \frac{\Lambda}{3}r^2\right),$$

can be developed by the gravitational collapse of a dust fluid.

V. Conclusions (Cont.)

- It would be very interesting to consider the gravitational collapse of other sources.
- Our method can be easily applied to other versions of the HL theory.
- It can be also applied to other theories with high-order derivative operators.

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Thank You