New 3d CFTs with 8 supersymmetries from topological gauging

Bengt E.W. Nilsson

Chalmers University of Technology, Gothenburg

Talk at
Kavli IPMU
Kashiwa, Chiba
June 18, 2013

Talk based on:

- *"Critical solutions in topologically gauged $\mathcal{N} = 8$ CFT's in three dimensions"* , arXiv:1304.2270
- *"Topologically gauged superconformal Chern-Simons matter theories"* with Ulf Gran, Jesper Greitz and Paul Howe, arXiv:1204.2521 in JHEP
- *"Aspects of topologically gauged M2-branes with six supersymmetries: towards a "sequential AdS/CFT"?*, arXiv:1203.5090 [hep-th]

Motivation

Three-dimensional conformal field theories are of interest in

- M-theory: M2-branes, AdS/CFT,....
- condensed matter: phase transitions, quantum critical points,..
- mathematics: 'monopole operators', 3d bosonization,...

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These are conformal field theories in flat space-time!

Motivation: cont.

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Here we will consider "Topologically Gauged "BLG" Theories":

- i.e. matter/Chern-Simons gauge theory with $\mathcal{N} = 8$ ("BLG") superconformal symmetry coupled to conformal supergravity: we find new features like
    - $SO(N)$ gauge groups for any N (instead of just $SO(4)$)
    - higgsing to topologically massive supergravity (super-TMG)
    - a number of possible "critical" backgrounds
    - the possibility of a "sequential AdS/CFT" and connections to higher spin (bosonization)
    - gives a Polyakov-like action (?)

Background for the discussion: classical

We want to understand systems of N conformal M2 branes
with level k and 8 supersymmetries (the 3d IR fix-point theory)!
Examples of such field theories exist but relation to M2-branes tricky!

- BLG: standard classical picture [Bagger, Lambert] [Gustavsson]
  - $\mathcal{N} = 8$ superconformal Chern-Simons (CS)-matter theory
  - only with $SO(4) = SU(2) \times SU(2)$ gauge group (i.e. $N = 2$)
  - parity symmetric
  - no $U(1)$ factor i.e. no center of mass coordinates
  - as a superconformal theory in 3d, any level k is possible

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Reducing to 6 susy's the M2-brane connection is clear [ABJM]

- ABJ(M) is a quiver theory with gauge groups like
  - $U_k(N) \times U_{-k}(N)$, for any k and any N
  - $SU_k(N) \times SU_{-k}(N)$, for any k and any N (for $N = 2$ and $k = 1, 2$ classically equivalent to BLG)
  - $SU_k(M) \times SU_{-k}(N) \times U(1)$, for any k and any M, N

Background for the discussion: quantum

The (non-perturbative) quantum picture for $\mathcal{N} = 8$ is better:

- monopole operators [ABJM, BKKS] can lead to enhanced symmetries for ABJM theories and relations to BLG

This can be checked by comparing moduli spaces [Lambert et al] and by using localization techniques to compute partition functions and superconformal indices [Kapustin et al]:
One finds

- supersymmetry enhancement:
  Ex: ABJM $U_k(N) \times U_{-k}(N)$ has 2 extra susy's for $k = 1, 2$
- $U(1)$ enhancement
- parity enhancement

Motivation: some questions

Questions at the classical level:

- why is classical BLG restricted to only $SO(4)$?
- can CS(supergravity) help? (recall the role of CS(gauge))
- what would such a CS-gravity construction (=topological gauging) mean in M/string theory?
- in AdS/CFT?
- role of HS (higher spin)?

Questions at the quantum level:

- what aspects of CFT_2 can be taken over to CFT_3?
    - 3d bosonization?
      see recent speculations based on HS [Chang et al],[Aharony et al]
      (conjectures by [Sezgin, Sundell] and [Klebanov, Polyakov]
      related by *non-parity symmetric* CS/matter theories)

Outline

1. Brief review:
   - BLG: standard classical picture [Bagger, Lambert] [Gustavsson]
2. Some new results for Chern-Simons-matter theories with 8 susy's
   - topological gauging and $SO(N)$ gauge symmetry
     [Gran,BN][Cederwall, Gran, BN] [Gran, Greitz, Howe, BN]
   - Backgrounds and higgsing to super-TMG
     (topologically massive supergravity)
     [Chu, BN],[Chu, Nastase, BN, Papageorgakis],[BN in prep]
3. Summary and some speculations on
   - "Sequential AdS/CFT", Neumann b.c., higher spin, etc [BN]

3-dim $\mathcal{N} = 8$ superconformal field theory : field content

The 3-dim BLG field content:

- scalars X^i_a
  - i: $SO(8)$ R-symmetry vector index
  - a: three-algebra index related to $[T^a, T^b, T^c] = f^{abc}{}_d T^d$ (structure constants f here antisymmetric in a, b, c)
- spinors ψ_a (2-comp Majorana)
  - with a hidden R-symmetry chiral spinor index (also real 8-dim),
- vector gauge potential $\tilde{A}_\mu{}^a{}_b = A_{\mu cd} f^{cda}{}_b$
  - conformal dimensions (deduced from their kinetic terms):
    - -1/2 for X^i_a
    - -1 for ψ_a
    - -1 for A_μ ("kinetic term" = Chern-Simons term) [Schwarz]

3d $\mathcal{N} = 8$ superconformal field theory: Lagrangian

The BLG Lagrangian

$$\begin{aligned}\mathcal{L} \; = \; & -\tfrac{1}{2}(D_\mu X^i{}_a)(D^\mu X^i{}_a) + \tfrac{i}{2}\bar{\Psi}_a \gamma^\mu D_\mu \Psi_a \\ & + \tfrac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd} A_{\mu ab}\partial_\nu A_{\lambda cd} + \tfrac{2}{3} f^{cda}{}_g f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef}\right) \; , \\ & - \tfrac{i}{4}\bar{\Psi}_b \Gamma_{ij} X^i{}_c X^j{}_d \Psi_a f^{abcd} - V_{BLG}\end{aligned}$$

where $D_\mu = \partial_\mu + \tilde{A}_\mu$ and the potential

$$V^{(st)}_{BLG} = \tfrac{1}{12}(X^i{}_a X^j{}_b X^k{}_c f^{abc}{}_d)(X^i{}_e X^j{}_f X^k{}_g f^{efg}{}_d)$$

- two triple products but a "single trace" (st)
- can introduce a (quantized) level k by rescaling $f^{abc}{}_d$, large k = weak coupling, reduction to string theory [BN, Pope],[ABJM]
- no other free parameters!

BLG transformation rules

The BLG transformation rules for (global) $\mathcal{N} = 8$ supersymmetry are

$$\begin{aligned} \delta X_i^a &= i\epsilon\Gamma_i\Psi^a, \\ \delta\Psi_a &= D_\mu X_a^i \gamma^\mu \Gamma^i \epsilon + \tfrac{1}{6} X_b^i X_c^j X_d^k \Gamma^{ijk} \epsilon f^{bcd}{}_a. \end{aligned}$$

Demanding cancelation on the $(Cov.der.)^2$ terms in $\delta\mathcal{L}$ implies

$$\delta\tilde{A}_\mu{}^a{}_b = i\bar{\epsilon}\gamma_\mu\Gamma^i X_c^i \psi_d f^{cda}{}_b$$

Full susy needs the fundamental identity
[Bagger, Lambert], [Gustavsson]

$$f^{abc}{}_g f^{efg}{}_d = 3 f^{ef[a}{}_g f^{bc]g}{}_d\,,$$

- one finite dim. realization, $\mathcal{A}_4$, with $SO(4)$ gauge symmetry (i.e. with levels $(k, -k)$) [Papadopoulos][Gauntlett,Gutowski]

3-dim $\mathcal{N} = 8$ superconformal gravity

To gauge the global symmetries of the BLG theory we need to introduce 3-dim. $\mathcal{N} = 8$ conformal supergravity:

- Off-shell field content is

$$e_\mu{}^\alpha, \;\; \chi^i_\mu, \;\; B^{ij}_\mu, \;\; b_{ijkl}, \;\; \rho_{ijk}, \;\; c_{ijkl},$$

  but no lagrangian exists[Howe,Izquierdo,Papadopoulos,Townsend]
- On-shell Lagrangian = three CS-like terms [Gran,BN(2008)] (compare $\mathcal{N} = 1$ [Deser,Kay(1983)], [van Nieuwenhuizen(1985)], and for any $\mathcal{N}$ [Lindström,Roček(1989)])

$$\mathcal{L} = \tfrac{1}{2}\epsilon^{\mu\nu\rho}Tr_\alpha(\tilde{\omega}_\mu\partial_\nu\tilde{\omega}_\rho + \tfrac{2}{3}\tilde{\omega}_\mu\tilde{\omega}_\nu\tilde{\omega}_\rho)$$

$$-ie^{-1}\epsilon^{\alpha\mu\nu}(\tilde{D}_\mu\bar{\chi}_\nu\gamma_\beta\gamma_\alpha\tilde{D}_\rho\chi_\sigma)\epsilon^{\beta\rho\sigma} - \epsilon^{\mu\nu\rho}Tr_i(B_\mu\partial_\nu B_\rho + \frac{2}{3}B_\mu B_\nu B_\rho),$$

- supercovariant spin connection: $\tilde{\omega}_{\mu\alpha\beta}(e_\mu{}^\alpha, \chi^i_\mu)$
- CS terms are of 3rd, 2nd and 1st order in derivatives, respectively

Symmetries of 3-dim $\mathcal{N}=8$ superconformal gravity

The local symmetries are here

- 3-dim diff's and local $SO(8)$ R-symmetry
- local $\mathcal{N}=8$ supersymmetry (f^ν is the spin 3/2 field strength)

$$\delta e_\mu{}^\alpha = i\bar{\epsilon}(x)\gamma^\alpha\chi_\mu, \;\; \delta\chi_\mu = \tilde{D}_\mu\epsilon(x),$$
$$\delta B_\mu^{ij} = -\tfrac{i}{2}\bar{\epsilon}(x)\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu,$$

- local scale invariance

$$\delta_\Delta e_\mu{}^\alpha = -\phi(x)e_\mu{}^\alpha, \;\; \delta_\Delta\chi_\mu = -\tfrac{1}{2}\phi(x)\chi_\mu, \;\; \delta_\Delta B_\mu^{ij} = 0,$$

- and local $\mathcal{N}=8$ superconformal symmetry

$$\delta_S e_\mu{}^\alpha = 0, \;\; \delta_S\chi_\mu = \gamma_\mu\eta(x),$$

$$\delta_S B_\mu^{ij} = \tfrac{i}{2}\bar{\eta}(x)\Gamma^{ij}\chi_\mu.$$

Topologically gauged BLG theory

- This supergravity theory has no propagating degrees of freedom!
  - clear in the light-cone gauge: all non-zero field components (plus ∂_+ on them) can be solved for [BN]
    => "topologically gauged BLG"

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- This supergravity theory has no propagating degrees of freedom!
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    => "topologically gauged BLG"
- Conformal supergravity can be coupled to BLG
  by Noether methods
  - to order $(Cov.der.)^3$ and $(Cov.der.)^2$ in δL [Gran,BN(2008)]
  - the full action now derived [Gran, Greitz, Howe, BN(2012)]
- or by other methods
  - demanding on-shell susy (as originally done for BLG)
    [Gran, Greitz, Howe, BN]
  - superspace [Gran, Greitz, Howe, BN],
    following "the Dragon window" in 3d [Cederwall, Gran, BN]
    [Howe, Izquierdo, Papadopoulos, Townsend]

Topologically gauged BLG theory: some details

Supersymmetry to order $(D_\mu)^2$ gives the conformal coupling $-\frac{e}{16}X^2\tilde{R}$:
(f^μ is the dual field strength of the spin 3/2 field χ_μ) [Gran, BN]

$$L^{top}_{BLG} \quad = \quad L^{conf}_{grav} + L^{cov}_{BLG}$$

$$+\frac{1}{\sqrt{2}}ie\bar{\chi}_\mu\Gamma^i\gamma^\nu\gamma^\mu\Psi^a\tilde{D}_\nu X^{ia} \quad (\text{"}the \;\; supercurrent \;\; term\text{"})$$

$$-\frac{i}{4}\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\Gamma^{ij}\chi_\nu(X^i_a\tilde{D}_\rho X^j_a) + \frac{i}{\sqrt{2}}\bar{f}^\mu\Gamma^i\gamma_\mu\Psi_a X^i_a$$

$$-\frac{e}{16}X^2\tilde{R} + \frac{i}{4}X^2\bar{f}^\mu\chi_\mu$$

Topologically gauged BLG theory: more details

The extended transformation rules at order $(D_\mu)^2$ in δL are

$$
\begin{aligned}
\delta e_\mu{}^\alpha &= i\sqrt{2}\bar{\epsilon}\gamma^\alpha\chi_\mu\,, \\
\delta\chi_\mu &= \sqrt{2}\tilde{D}_\mu\epsilon, \\
\delta B_\mu^{ij} &= -\tfrac{i}{\sqrt{2}}\bar{\epsilon}\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu - \tfrac{i}{2\sqrt{2}}\bar{\chi}_\mu\Gamma^{k[i}\epsilon X_a^{j]}X_a^k + \tfrac{i}{16\sqrt{2}}\bar{\epsilon}\Gamma^{ij}\chi_\mu X^2 \\
&\quad -\tfrac{i}{16}\bar{\Psi}_a\Gamma^k\Gamma^{ij}\gamma_\mu\epsilon X_a^k - \tfrac{i}{2}\bar{\Psi}_a\gamma_\mu\Gamma^{[i}\epsilon X_a^{j]}, \\
\delta X_i^a &= i\epsilon\Gamma_i\Psi^a, \\
\delta\Psi_a &= (\tilde{D}_\mu X_a^i - \tfrac{1}{\sqrt{2}}\bar{\chi}_\mu\Gamma^i\Psi_a)\gamma^\mu\Gamma^i\epsilon + \tfrac{1}{6}X_b^iX_c^jX_d^k\Gamma^{ijk}\epsilon f^{bcd}{}_a, \\
\delta\tilde{A}_\mu{}^a{}_b &= i\bar{\epsilon}\gamma_\mu\Gamma^iX_c^i\Psi_d f^{cda}{}_b - \tfrac{i}{\sqrt{2}}\bar{\chi}_\mu\Gamma^{ij}\epsilon X_c^iX_d^j f^{cda}{}_b.
\end{aligned}
$$

Topologically gauged BLG theory: the transformation rules

The complete transformation rules with explicit coupling constants: the level parameter $\lambda = \frac{2\pi}{k}$ and the gravitational coupling g

[Gran, Greitz, Howe, BN(2012)]

$$\begin{aligned}
\delta e_\mu{}^\alpha &= i\bar{\epsilon}_g\gamma^\alpha\chi_\mu, \quad \delta\chi_\mu = \tilde{D}_\mu\epsilon_g, \\
\delta B_\mu^{ij} &= -\tfrac{i}{2e}\bar{\epsilon}_g\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu - \tfrac{ig}{4}\bar{\chi}_\mu\Gamma^{k[i}\epsilon_g X_a^{j]}X_a^k - \tfrac{ig}{32}\bar{\chi}_\mu\Gamma^{ij}\epsilon_g X^2 \\
&\quad -\tfrac{ig}{16}\bar{\Psi}_a\Gamma^{ijk}\gamma_\mu\epsilon_m X_a^k - \tfrac{3ig}{8}\bar{\Psi}_a\gamma_\mu\Gamma^{[i}\epsilon_m X_a^{j]}, \\
\delta X_a^i &= i\epsilon_m\Gamma^i\Psi_a, \\
\delta\Psi_a &= \gamma^\mu\Gamma^i\epsilon_m(\tilde{D}_\mu X_a^i - iA\bar{\chi}_\mu\Gamma^i\Psi_a) + \tfrac{\lambda}{6}\Gamma^{ijk}\epsilon\, X_b^i X_c^j X_d^k\,\epsilon^{bcd}{}_a \\
&\quad +\frac{g}{8}\Gamma^i\epsilon_m X_b^i X_b^j X_a^j - \frac{g}{32}\Gamma^i\epsilon_m X_a^i X^2, \; (NEW) \\
\delta\tilde{A}_\mu{}^a{}_b &= i\lambda\bar{\epsilon}_m\gamma_\mu\Gamma^i X_c^i\Psi_d\,\epsilon^{cda}{}_b - \tfrac{i\lambda}{2}\bar{\chi}_\mu\Gamma^{ij}\epsilon_g X_c^i X_d^j\,\epsilon^{cda}{}_b \\
&\quad +\frac{ig}{4}\epsilon_m\gamma_\mu\Gamma^i\psi_{[a}X_{b]}^i + \frac{ig}{8}\bar{\chi}_\mu\Gamma^{ij}\epsilon_g X_a^i X_b^j. \; (NEW)
\end{aligned}$$

Topologically gauged BLG theory: the Lagrangian

Some of the interesting terms in L are

$$L = \frac{1}{g}L^{SUGRA}_{conf} + L^{BLG}_{cov} - \frac{e}{16}X^2R - V_{new}$$

the scalar potential has a *single-trace* (st) contribution from L^{BLG}_{cov}

$$V^{(st)}_{BLG} = \frac{\lambda^2}{12}(X^i{}_aX^j{}_bX^k{}_c\,\epsilon^{abcd})(X^i{}_eX^j{}_fX^k{}_g\,\epsilon^{efg}{}_d)$$

and a new *triple-trace* (tt) term

$$V^{(tt)}_{new} = \frac{eg^2}{2\cdot 32\cdot 32}\Big((X^2)^3 - 8(X^2)X^j_bX^j_cX^k_cX^k_b + 16X^i_cX^i_aX^j_aX^j_bX^k_bX^k_c\Big)$$

- but *no* new *double-trace* terms as in the ABJ(M) case [Chu, BN]

Topologically gauged BLG theory: new properties

The above Lagrangian has also a new kind of parity non-symmetric Chern-Simons sector ($SO(4) = SU_L(2) \times SU_R(2)$)

$$L_{CS(A)} = \frac{1}{a} L_{CS(A^L)} + \frac{1}{a'} L_{CS(A^R)}$$

where

$$a := \frac{g}{8} - \lambda, \;\; a' := \frac{g}{8} + \lambda$$

which can be seen from $\delta \tilde{A}_\mu^{cd} = \delta A_\mu^{ab} f_{ab}{}^{cd}$ where gauging =>

$$f_{ab}{}^{cd} \rightarrow M_{ab}{}^{cd} = f_{ab}{}^{cd} - \frac{g}{4} \delta_{ab}^{cd} \tag{1}$$

Note: In superspace one starts from a non-parity invariant quiver Chern-Simons theory with independent level parameters a and a'

Topologically gauged BLG theory: new properties

New theories?

$$L_{CS(A)} = \frac{1}{a} L_{CS(A^L)} + \frac{1}{a'} L_{CS(A^R)}$$

where

$$a := \frac{g}{8} - \lambda, \;\; a' := \frac{g}{8} + \lambda$$

New theories arise as follows

- for $\lambda = 0$ the three-algebra indices can be extended arbitrarily: –> gauge group $SO(N)$ for any N
- even with non-zero λ there is an additional new $SO(3)$ theory for certain values of the parameters
- not parity symmetric but this is so already in the gravity sector!

Topologically gauged $\mathcal{N} = 6$ superconformal ABJ(M)

The BLG type of new potential was found first for ABJ(M) [Chu, BN(2009)]

- The complete topologically gauged ABJM lagrangian has about 25 new terms including a new $U_R(1)$ CS gauge field
  - New scalar interaction terms (with explicit λ and g) :
    First: Recall the original ABJ(M) potential (single trace in 3-alg.)

$$V^{(st)}_{ABJ(M)} = \tfrac{2}{3}|\Upsilon^{CD}{}_{Bd}|^2, \;\; \Upsilon^{CD}{}_{Bd} = \lambda f^{ab}{}_{cd} Z^C_a Z^D_b \bar{Z}^c_B + \lambda f^{ab}{}_{cd} \delta^{[C}_B Z^{D]}_a Z^E_b \bar{Z}^c_E \,.$$

The new terms with one structure constant are (double trace)

$$V^{(dt)}_{new} = -\tfrac{1}{8} g\lambda f^{ab}{}_{cd} |Z|^2 Z^C_a Z^D_b \bar{Z}^c_C \bar{Z}^d_D - \tfrac{1}{2} g\lambda f^{ab}{}_{cd} Z^B_a Z^C_b (Z^D_e \bar{Z}^e_B) \bar{Z}^c_C \bar{Z}^d_D \,.$$

and without structure constant (triple trace)

$$V^{(tt)}_{new} = -g^2 \left(\tfrac{5}{12\cdot 64} (|Z|^2)^3 - \tfrac{1}{32} |Z|^2 |Z|^4 + \tfrac{1}{48} |Z|^6\right) .$$

  - also new Yukawa-like terms without structure constants

Higgsing of topologically gauged ABJM

Two observations:

- Higgsing to D2 branes leaves the theory at an AdS chiral point similar to the one of Li, Song, Strominger [Chu, BN]
- Scaling limits can be taken in different ways, rigid susy in AdS [BN],[Chu, Nastase, BN, Papageorgakis]

Several steps needed:

- introduce two parameters $\lambda = \frac{2\pi}{k}$ and g (via the triple product and the trace)
- expand the theory around a real VEV v: $Z^A = v\delta^{A4} + z^A$
- limits are taken in λ, g, v with various combinations kept fixed
- identify the six new ordinary supersymmetries: $Q_{AdS} = Q_{CFT} + S_{CFT}(\eta = ...\epsilon)$

Higgsing of topologically gauged ABJM: the chiral point

The appearance of the chiral point is seen from the scalar/gravitational terms [Chu, BN] (before introducing λ and g_M)

$$L^{ABJM}_{higgsed} = L_{CS(grav)} - \frac{e}{8}v^2 R - \frac{e}{256}v^6$$

Compare to the TMG: (LSS: Li, Song and Strominger)

$$L^{LSS} = \frac{1}{\kappa^2}\left(\frac{1}{\mu}L_{CS(grav)} - (R - 2\Lambda)\right), \quad \Lambda = -\frac{1}{l^2}$$

- thus $\frac{v^2}{8} = \frac{1}{\kappa^2}$ and $\mu = l^{-1} = \kappa^{-2}$, i.e. $\mu l = 1$ (recall $\Lambda = -\frac{1}{l^2}$)
- The sign of the Einstein-Hilbert and cosmological terms => negative energy black holes (non-unitary if present)
    - these features are dictated by the sign of the ABJM scalar kinetic terms (via conformal invariance)!
    - introducing more parameters (levels) does not alter this fact

Higgsing of topologically gauged theories with 8 supersymmetries: the chiral points of the $SO(N)$ model

The appearance of the chiral point for $\mathcal{N} = 8$ is seen from the terms

$$L^{SO(N)} = \frac{1}{g}L_{CS(grav)} - \frac{1}{16}X^2R - \frac{g^2}{2\cdot 32\cdot 32}((X^2)^3 - 8X^2X^4 + 16X^6)$$

where (X^i_a: a=1,2,...,N and i=1,2,..,8)

$$X^{ij} = X^i_a X^j_a,\;\; X^2 = tr(X^{ij}),\;\; X^4 = X^{ij}X^{ij},\;\; X^6 = X^{ij}X^{jk}X^{ki}$$

Compare to the TMG:

$$L^{LSS} = \frac{1}{\kappa^2}(\frac{1}{\mu}L_{CS(grav)} - (R - 2\Lambda)),\;\; \Lambda = -\frac{1}{l^2}$$

- one non-zero $< X >= v$ => $\kappa^2\mu = g$, $\frac{v^2}{16} = \frac{1}{\kappa^2}$, $\frac{2}{\kappa^2 l^2} = \frac{9g^2v^6}{2\cdot 32\cdot 32}$
- =>$\mu l = 1/3$??

Higgsing of topologically gauged theories with 8 supersymmetries: the chiral points of the $SO(N)$ model

$$L^{SO(N)} = \frac{1}{g} L_{CS(grav)} - \frac{1}{16} X^2 R - \frac{g^2}{2 \cdot 32 \cdot 32} ((X^2)^3 - 8X^2 X^4 + 16X^6)$$

where (X^i_a: a=1,2,...,N and i=1,2,..,8)

$$X^{ij} = X^i_a X^j_a, \;\; X^2 = tr(X^{ij}), \;\; X^4 = X^{ij} X^{ij}, \;\; X^6 = X^{ij} X^{jk} X^{ki}$$

There are two well-known critical points on the market:

- chiral AdS with $\mu l = 1$
- null-warped AdS with $\mu l = 3$

New critical points of the $SO(N)$ model

$X^i{}_a$ is an $8 \times N$ rectangular matrix =>
generalize the VEV to a matrix:

$$< X^i{}_a >= v\delta^I{}_A, \quad I, A = 1, 2, .., p \leq 8$$

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- $p = 1, 2, 3, \; 4, \; 5, 6, 7, 8$ give
- $\mu l = \frac{1}{3}, 1, 3, \infty, 5, 3, \frac{7}{3}, 2$

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  corresponding to
  - critical AdS for $p = 2$

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  - critical AdS for $p = 2$
  - null-warped AdS for $p = 3, 6$

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  corresponding to
  - critical AdS for $p = 2$
  - null-warped AdS for $p = 3, 6$
  - Minkowski for $p = 4$

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- $\mu l = \frac{1}{3}, 1, 3, \infty, 5, 3, \frac{7}{3}, 2$

  corresponding to
  - critical AdS for $p = 2$
  - null-warped AdS for $p = 3, 6$
  - Minkowski for $p = 4$
- but in fact $\mu l = 5$ is also known!
  [Ertl, Grumiller, Johansson(2010)]

The Higgsed Chern-Simons sector

Symmetry breaking:

- conformal –> AdS
- $SO(N) \times SO_R(8) \to SO(N-p) \times SO_R(8-p) \times SO_{diag}(p)$

gives (with $m = gv^2$)

$$2\epsilon F(A) + m(A-B) = gXD(A,B)X$$

$$\epsilon G(B) + m(A-B) = -gXD(A,B)X$$

Eliminating B (by first solving the first equation above) gives for zero coupling $g=0$ the exact solution

$$\epsilon F = \frac{4}{m}\epsilon P(\epsilon F) + \frac{8}{m^2}\epsilon(\epsilon F, \epsilon F) \qquad (2)$$

and for non-zero coupling g

$$m(B-A) = \Sigma_{n\geq 0}(\tfrac{X}{v})^n(2\epsilon F - gXP(A)X)(\tfrac{X}{v})^n \qquad (3)$$

The Higgsed Chern-Simons sector: the spectrum

Scalars:

indices split as: $i \to (\hat{i}, I)$, $a \to (\hat{a}, A)$ with indices I and A identified

- $x^i{}_a = (x^{\hat{i}}{}_{\hat{a}}, x^{\hat{i}}{}_A, x^I{}_{\hat{a}}, x^I{}_A)$ where
- $x^I{}_a = (z, w^{(IA)}, y^{[IA]})$

of which the physical ones after higgsing are

- $x^{\hat{i}}{}_{\hat{a}}$, z, w

Vector fields:

the massive ones (YM+CS, see above)

- $A_\mu^{A\hat{a}}$, $B_\mu^{\hat{i}J}$, A_μ^{IJ}

while the rest are massless!

Leads to supermultiplets with an $\hat{a}$ index and without an $\hat{a}$!

Summary

- $SO(N)$ gauge groups for any N possible
  in topologically gauged "free BLG"
- Topologically gauged theories exhibit spontaneous symmetry breaking to topologically massive CS/matter theories coupled to critical super-TMG:
  - topologically gauged "free BLG": special solutions
    - $\mu l = 1$: chiral round AdS
    - $\mu l = 3$: null-warped AdS (or $z = 2$ Schödinger : cold atoms)
    - $\mu l = \infty$: Minkowski
    - $\mu l = 5$: "special" solution of Ertl, Grumiller and Johansson.
  - in the ABJ(M) case: $\mu l = 1$ chiral round AdS and Minkowski

Speculations

- "Sequential AdS/CFT" ??:
  - could the symmetry breaking from CFT_3 to AdS_3 lead to an AdS/CFT sequence: $AdS_4/CFT_3 \rightarrow AdS_3/CFT_2$?

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- "sequential AdS/CFT": speculations

  a) dynamical [BN(2012)]

  b) from AdS foliations ([Compere, Marolf(2008)]

  c) from higher spin algebra/unfolding [Vasiliev(2000, 2012)])

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  a) dynamical [BN(2012)]
  b) from AdS foliations ([Compere, Marolf(2008)]
  c) from higher spin algebra/unfolding [Vasiliev(2000, 2012)])
- the AdS_4 bulk should be an $\mathcal{N} = 8$ higher spin theory (see work by Vasiliev and Sezgin-Sundell)

Possible connection to AdS_4 Vasiliev higher spin theories

Structure of AdS_4 Vasiliev systems very schematically!
(all products $\star$)

Interaction ambiguity and θ parameters:

- $dW + W^2 = J + cc$, $dB + WB - B\pi(W) = 0$
- $J = f(B)dz^2$ with $f(B) = B\, e^{\theta(B)}$
- $\theta(B) = \theta_0 + \theta_2 B^2 + ..$

The parity preserving cases are (with $\theta_{2n} = 0$ for $n \geq 1$)

- $\theta_0 = 0$ dual to free scalar theory on the boundary (operator has $\Delta = 1$) with N bc –> UV
  Klebanov-Polyakov (2002)
- $\theta_0 = \frac{\pi}{2}$ dual to free fermion theory on the boundary (operator has $\Delta = 2$) D bc –> IR
  Sezgin-Sundell (2003)

Possible connection to AdS_4 Vasiliev higher spin theories: non-trivial θ

Other values of θ_0 correspond to finite level CS vector fields added on the boundary:

- parity symmetry broken
- double and triple trace deformations
- supersymmetry only for certain choices of boundary conditions in the bulk: no known case with 8 susy's!
- for boundary quiver theories matrix versions of Vasiliev's theories are needed

Bosonization-like features arise when comparing the different free and interacting boundary theories!

Possible connection to AdS_4 Vasiliev higher spin theories: non-trivial θ

The CFT to BULK dictionary:

Chang, Minwalla, Sharma, Yin (2012)

- finite level k <—> θ_0 non-trivial
- double and triple trace deformations $((\phi^2)^3, \phi^2\psi^2)$ <—> change of scalar field b.c.
- gauging a second gauge group to get a quiver theory <—> change of vector field b.c.

which works only for $\mathcal{N} \leq 6$ CFTs. But what to do here

- $\mathcal{N} = 8$ vector model <—> ????