Striped Order in AdS/CFT

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Based on:

M.R., Darren Smyth, Evgeny Sorkin, Jared Stang:

Holographic Stripes, Phys.Rev.Lett. 110 201603, ArXiv:1211.5600.
Striped Order in AdS/CFT, Phys.Rev.D 87 126007, ArXiv:1304.3130.

Background and Motivation

Stripes are known to form in a variety of strongly coupled systems, but are rare in weakly coupled ones. Examples include:

- Large N QCD, chiral density waves at asymptotically large chemical potential.

  Deryagin, Grigoriev and Rubakov, "Standing wave ground state in high density, zero temperature QCD at large N(c)."

  E. Shuster and D. T. Son, "On finite density QCD at large N(c)," Nucl. Phys. B **573**, 434 (2000) [hep-ph/9905448].

- Systems of strongly correlated electrons

  M. Vojta, "Lattice symmetry breaking in cuprate superconductors: stripes, nematics, and superconductivity," Adv. Phys. 58, Issue 6, 2009.

The formation of stripes is important in some theoretical speculations, e.g:.

- Inhomogeneity induced pairing in high-Tc superconductors:

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- Leading to the concept of *optimal inhomogeneity*

  Kivelson, Steven A.; Fradkin, Eduardo,"How optimal inhomogeneity produces high temperature superconductivity", cond-mat/0507459.

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This talk concentrates on one concrete construction of inhomogeneous black holes, dual to states in which translation invariance is broken *spontaneously*.

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- Lattice induced metal-insulator transitions: A. Donos and S. A. Hartnoll, "Metal-insulator transition in holography," arXiv:1212.2998 [hep-th].
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Spatially inhomogeneous situations are also interesting purely from the bulk gravitational perspective, as they involve a new type of solutions of Einstein's equations. The numerical methods involved are well-understood.

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- Lots of work on linearized instabilities: at some point in phase diagram tachyons develop in the linearized perturbation spectrum, where the dominant one has a non-zero wavenumber.
- Some perturbative solutions, around the phase transition point, or in some other probe limit (e.g. for vortex lattices) exist. They are valid in a corner of parameter space or phase diagram.
- Some fully back-reacted solutions exist with *sourced* inhomogeneity. In most such cases the inhomogeneity is irrelevant, it decreases towards the horizon. New issues arise for *relevant* inhomogeneity, e.g. where it can leads to a metal-insulator transition.
- Some non-linear solutions exist in special cases where helical symmetry allows reduction to ODEs. A. Donos and S. A. Hartnoll, arXiv:1208.4102 [hep-th].

In contrast to all the above, we construct a non-linear co-homogeneity two static solution describing the *spontaneous* breaking of translation invariance. This allows us to explore, hopefully, generic features to do with breaking of translation invariance.

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The Model:

Consider the Einstein-Maxwell system in asymptotically AdS_4, and add a neutral scalar ψ, with $m^2 = -2$ and a coupling

$$\mathcal{L}_{int} = \frac{c_1}{16\sqrt{3}} \psi \, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

A. Donos and J. P. Gauntlett, "Holographic striped phases," JHEP **1108**, 140 (2011) [arXiv:1106.2004 [hep-th]].

- This is axion electrodynamics, which arises as the effective field theory describing the electromagnetic response of topological insulators and their boundary excitations.
- Most other occurrences of inhomogeneous and/or vector-like instabilities involve also bulk topological terms.
- Here we have axion electrodynamics in the bulk. We will also discover a near horizon modulated axion.
- This will result in a bulk magneto-electric effect, more familiar from the study of topological insulator interfaces. This accounts for some unusual properties of the bulk geometry.

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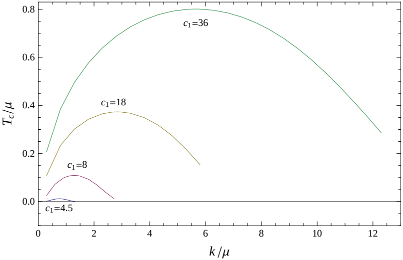
The Instability

In this model the RN black hole develops an unstable mode of a finite wavenumber k_c at some critical temperature T_c. At that point a new branch of solutions develops, which we follow to lower temperatures.

A. Donos and J. P. Gauntlett, "Holographic striped phases," JHEP **1108**, 140 (2011) [arXiv:1106.2004 [hep-th]].

Higher values of the axion coupling c_1 result in higher critical temperatures and a larger range of unstable wave-numbers (the region under the curves).



To leading order the perturbation involves an axion ψ, A_y – current in the y direction, g_{ty} – momentum in the y direction, all modulated in the x direction. The perturbation has two reflection symmetries which simplify the numerics. Higher orders involve the metric elements g_{tt}, g_{xx}, g_{yy} and the gauge potential A_t, all of which acquire significant modulation near the horizon.

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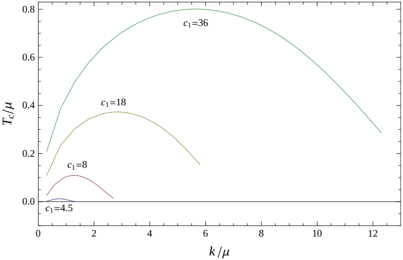

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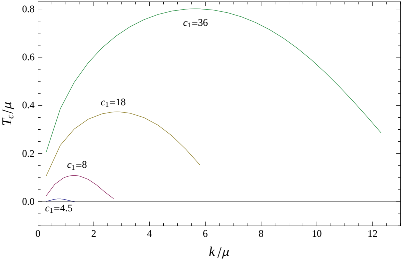

To leading order the perturbation involves an axion ψ, A_y – current in the y direction, g_{ty} – momentum in the y direction, all modulated in the x direction. The perturbation has two reflection symmetries which simplify the numerics. Higher orders involve the metric elements g_{tt}, g_{xx}, g_{yy} and the gauge potential A_t, all of which acquire significant modulation near the horizon.

The Instability

In this model the RN black hole develops an unstable mode of a finite wavenumber k_c at some critical temperature T_c. At that point a new branch of solutions develops, which we follow to lower temperatures.

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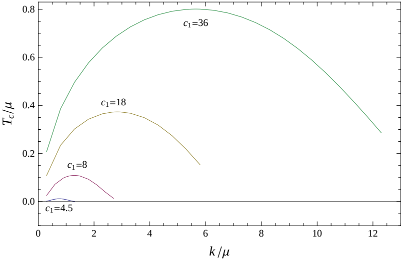

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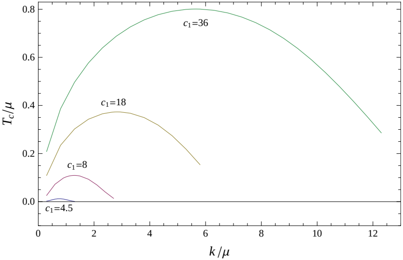

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Technicalities

- We find a fully back-reacted solution of Einstein + Matter equations, which is co-homogeneity two and involves a *spontaneous* breaking of translation invariance.
- We work in a *conformal ansatz*, in which conformal symmetry in the (r,x) directions is instrumental in simplifying the equations, and especially solving the constraints. T. Wiseman, "Static axisymmetric vacuum solutions and nonuniform black strings," Class. Quant. Grav. **20**, 1137 (2003).
- We use finite difference methods to discretize the resulting elliptic equations, then Gauss-Seidel relaxation and multi-grid methods to solve the equations iteratively.
- We performed the standard checks (convergence, constraints, integrated first law, Smarr's formula) to verify we have an approximate solution to the continuum equations. The relative error in the local values of the metric functions is $\sim 10^{-6}$.
- Other methods (Einstein-DeTurck, Pseudo-Spectral discretization) are also possible. A. Donos, "Striped phases from holography," JHEP **1305**, 059 (2013). B. Withers, "Black branes dual to striped phases," arXiv:1304.0129 [hep-th]; "The moduli space of striped black branes," arXiv:1304.2011 [hep-th].

Boundary Observables

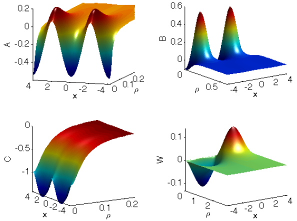
Qualitative features of the boundary observables, all encoded in subleading terms in the asymptotic expansion of the fields near the conformal boundary.

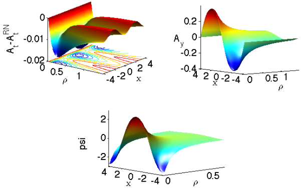
- The boundary theory exhibits a *spontaneous* breaking of translation invariance, i.e. all non-normalizable modes are spatially homogeneous.
- the scalar field ψ represents a modulated order parameter.
- The modulation in A_y and A_t means there are both charge and current density waves. The periodicity of the former is twice that of the latter.
- There is a modulation in the energy-momentum tensor components T_{tt}, T_{yy} and non-zero momentum T_{ty} in the vacuum.
- By virtue of conservation, T_{xx} is spatially homogeneous, which is a good check on the numerics.
- All amplitudes grows steadily with decreasing temperatures, approaching a constant at zero temperatures.

The Geometry

Qualitative features of the geometry:

- We construct solutions for $3 \times 10^{-3} \leq \frac{T}{T_c} \leq 0.9$. The figures are for $T = 0.11 T_c$, $c1 = 4.5$.
- Modulation grows towards the horizon, and is normalizable at infinity, as expected of spontaneous breaking.
- Periodicity is "staggered": the fields g_{tt}, g_{xx}, g_{yy} and A_t have half of the period of ψ, A_y and $W = g_{ty}$.
- The field A_y has to do with bulk magnetic field; $W = g_{ty}$ has to do with rotation. More on their near horizon structure later.
- Inhomogeneity grows as the temperature is lowered, more quantitative discussion later.





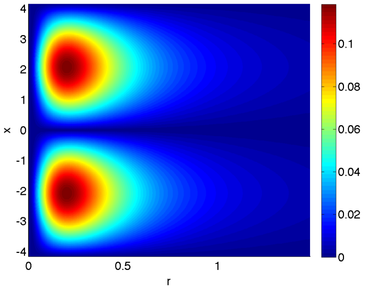
Near Horizon Topological Insulator

Much of the structure of the solution can be understood as a consequence of the magneto-electric effect encoded in axion electrodynamics.

- Axion Electrodynamics describes the electromagnetic response of a topological insulator interface. That type of interface is realized as an axion domain wall.
- For us, the bulk axion $\psi \sim \cos(k_c x)$ drives the striped instability. The presence of axion gradient and an electric field (from the charged black hole) results in electric current $\vec{j} \sim \vec{\nabla}\psi \times \vec{E}$. This is a near horizon current pointing in the y direction.
- The current results in bulk magnetic field and vorticity (represented by the gauge field A_y and metric function g_{ty}).
- The backreaction on the geometry results in some frame dragging near the horizon. But, the horizon itself does not rotate, all rotation is carried by the matter fields.
- Relatedly, the spacetime has no *ergosphere*, despite the rotation.

Near Horizon Topological Insulator

More on bulk rotation:



Contour plot of the metric function g_{ty}. This shows to localized regions of counter-rotating matter near the horizon. The horizon itself does not rotate, this is is an example of *stationary* non-rotating black hole.

Key to decoding the physics?

- The order parameter driving the striped instability is related to topological order, albeit in the bulk.
- The relation to topological order is interesting. Can there be a relation between *local* (=near horizon) topological order, i.e. domains on a length scale $\sim T^{-1}$, and the mechanism for stripe formation?
- Interesting to explore further the consequences of topological terms in the bulk.

H. Liu, H. Ooguri, B. Stoica and N. Yunes, "Spontaneous Generation of Angular Momentum in Holographic Theories," Phys. Rev. Lett. **110**, no. 21, 211601 (2013) [arXiv:1212.3666 [hep-th]].

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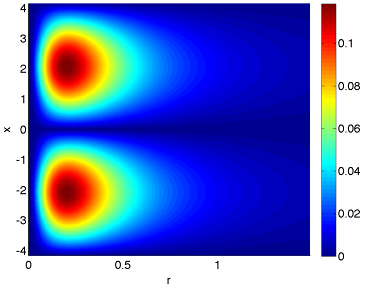

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Low Temperature Limit

We looked at the geometry as function of temperature, in particular as an aid to identify the ground state of the system.

- Some of the variations of the geometry are mild as we lower the temperature. For example, the Ricci scalar on the horizon increases but approaches a constant as $T \to 0$.
- The proper length of the x direction varies radially, and becomes maximal on the horizon (the Archimedes effect). This quantity diverges mildly, as $\sim T^{-0.1}$ at low temperatures.
- The "neck" and "bulge", namely the minimum and maximum size of the transverse direction y, decrease at low temperatures. Their ratio however decreases as $\sim T^{1/2}$ (for $c1 = 4.5$), signalling that the horizon pinches off at low temperatures. This is the case for every c_1, but the precise exponent varies with c_1.
- Interesting to look at the possible ground states of systems with spatial inhomogeneity. Perhaps in this case the natural expectation is an array of lower dimensional structures (as in the Gregory-Laflamme story).

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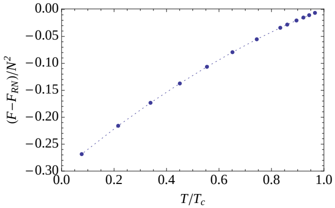
Fixing Temperature and Charge

At first we fix the asymptotic length of the system, $L = \frac{2\pi}{k_c}$, where k_c is the critical wave-number. Later we'll consider the infinite system where all wavenumbers can occur.

In the canonical ensemble we couple the system to a heat bath, fixing the charge. Since the theory is conformal, the free energy is a function of T/Q only.

- This is a second order phase transition at the temperature where tachyons first appear.
- Within the accuracy of the numerics, the scaling of the free energy is consistent with mean field expectations.
- Similar results for other values of the fixed interval length L.
- Some indications that phase transition disappears at smaller values of c_1.

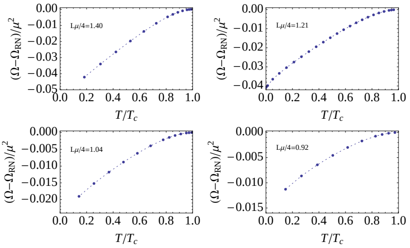


Free energy difference for $c_1 = 8$.

Fixing Temperature and Chemical Potential

Couple system to charged plasma, fixing the temperature and chemical potential. The resulting grand-canonical ensemble is more natural for us, since we fix the chemical potential as part of our boundary conditions. As consequence of conformal invariance, all quantities depend on T/μ only.

- Similarly, second order phase transition at temperature T_c, in which the corresponding tachyon first develops, shown here for various values of L.
- Critical exponents seem to be non mean-field: momentum density ~ 0.41; scalar condensate ~ 0.38; current density $\sim .40$, errors within 10 percent.



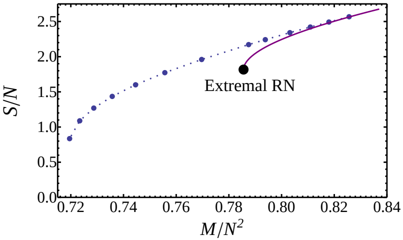
We also get similar results where we fix the tension τ_x conjugate to the length L.

Fixing Mass and Charge

The micro-canonical ensemble describes an isolated system, as appropriate for discussion of dynamical instability. Control parameters are M, Q and the entropy is function of M/Q only.

- Inhomogeneous solutions have higher entropy and lower energy than the critical RN, likely endpoints of dynamical instability.
- As shown in the horizontal axis, mass of stripes goes below that of the extremal RN. In that parameter range there is only inhomogeneous solutions.
- Extrapolating to zero temperature indicates non-zero entropy at $T = 0$, though numerical uncertainties are significant.

Microcanonical ensemble.

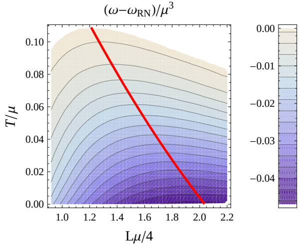


Infinite System

Finally, we can take the infinite L limit, studying relation between thermodynamic *densities*. Now stripes of different widths can be compared.

Working in the grand canonical ensemble, we find again a second order transition at T_c, the temperature where the first instability develops.

As the temperature is lowered, dominant stripe width increases, approaching approximately twice that of the critical width at T_c.



Other results are qualitatively similar to that of the finite system.

Some Work in Progress:

On the stripes front

- Better understanding of mechanism of stripe formation, relation to (semi-local?) topological order.
- Understanding and possible classification of ground states.
- Significance of striped phases in the phase diagram, potential relation to superconductivity.

On the general inhomogeneous holography front:

- Unconventional Fermi liquids at holographic interfaces (via calculation of spectral densities).
- Bound states at holographic interfaces (i.e. Andreev states, mid-gap Majorana fermions).
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