IPMU, 2009

Holographic Lifshitz-like Fixed Points in String theory

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Based on

arXiv:0901.0924 with Mitsutoshi Fujita (Kyoto), Wei Li (IPMU)
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arXiv:0905.0688 with Tatsuo Azeyanagi (Kyoto) and Wei Li (IPMU)
Work in progress with Tatsuma Nishioka (Kyoto, IPMU) and Wei Li (IPMU)

Contents

① Introduction

The AdS/CFT offers us many useful examples of holographic duals of conformal field theories.

$$ds^2_{AdS} = -r^2 dt^2 + r^2 \sum_{i=1}^{d} dx_i^2 + \frac{dr^2}{r^2}.$$

⇕

(d+1) dim. CFTs

Question: Can we generalize this duality to other scale invariant theories ?

Relativistic Scale invariance: $(t, x_i, r) \to (\lambda t, \lambda x_i, r/\lambda)$

Non-relativistic Scale invariance:

$$(t, x_i, y_j, r) \to (\lambda^z t, \lambda^z x_i, \lambda y_j, r/\lambda)$$

We expect that the dual geometry looks like

$$ds^2_{Scaling} = r^{2z}\left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + r^2 \sum_{i=1}^{d-p} dy_i^2 + \frac{dr^2}{r^2}.$$

[Kachru-Liu-Mulligan 08']

Comment: Here we do not require the **Galilean** invariance.

[cf. Son 08', Balasubramanian-McGreevy 08']

A typical example of such an anisotropic scale invariance is known as the Lifshitz point.

Multi-critical point

$$V(M) = aM^2 + bM^4 + cM^6 + \cdots \quad (c > 0).$$

$$a = 0 \quad \Rightarrow \quad \text{ordinary critical pt.}$$

$$a = b = 0 \quad \Rightarrow \quad \text{Tricritical pt.}$$

Lifshitz point

[Hornreich-Luban-Shtrikman 1975’]

$$F(M) = aM^2 + bM^4 + c(\nabla_{//} M)^2 + d(\nabla_{//} M)^4 + f(\nabla_{\perp} M)^2$$

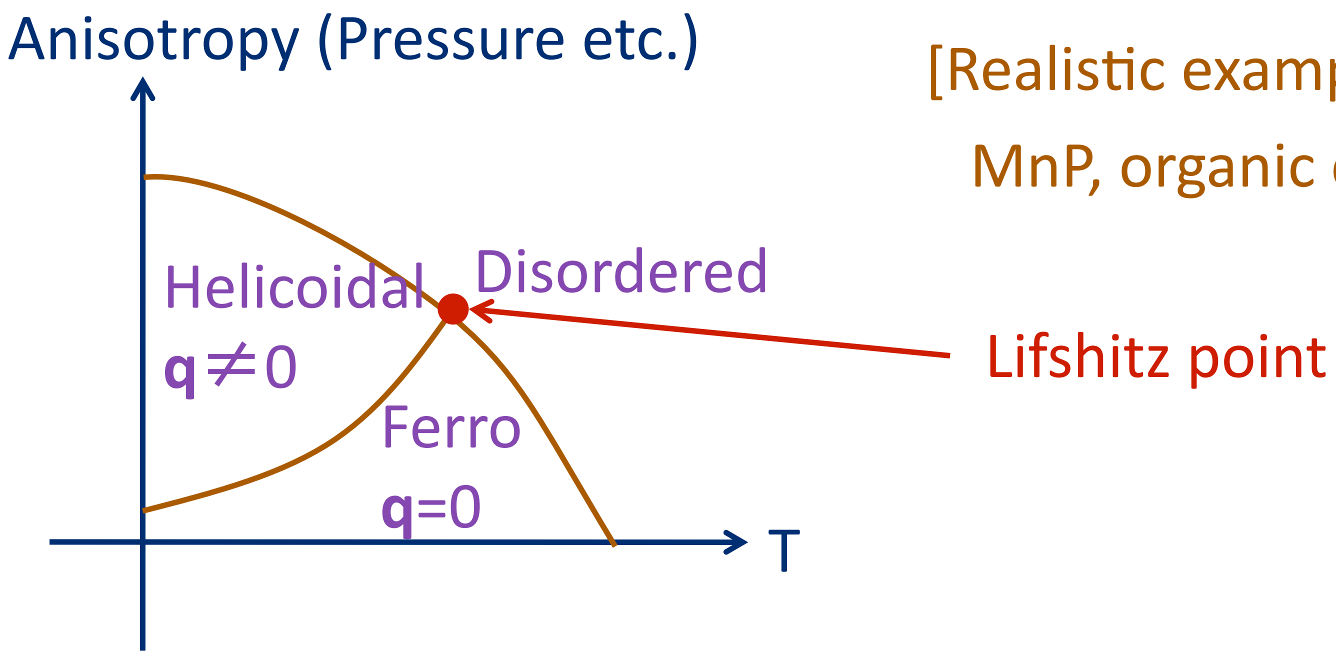
$$a = c = 0 \quad \Rightarrow \quad \text{(classical) Lifshitz point} \quad z = 2$$

Lifshitz points appear magnetic spin systems, typically when the following two interactions compete:

Nearest neighbor **ferro** interaction (**isotropic**)

\+ Next nearest neighbor **anti-ferro** interaction (**anisotropic**).

→ The modulation wave vector **q** begins to be non-vanishing.



[Realistic examples:
MnP, organic crystals, alloy]

Classical Lifshitz model

$$S_E = \int dx^d \left[\frac{1}{2} (\nabla_{//} \phi)^2 + \frac{1}{2} (\nabla_{\perp}^2 \phi)^2 \right].$$

Quantum Lifshitz model

$$S_Q = \int dt dx^d \left[\frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} (\nabla^2 \phi)^2 \right].$$

Free field theory with z=2
In interacting theories, quantum corrections lead to different values of z.

This theory is known to have the remarkable property:

$$\Psi_{Ground\ \ State} = e^{-\int dx^d \frac{1}{2} (\nabla \phi)^2}.$$

Gravity Duals ?

Kachru et.al. showed that this background can be a solution to a five dim. Einstein gravity coupled to 2- and 3-form gauge field.

$$S=\int dx^4\sqrt{-g}(R-2\Lambda)$$
$$-\int F_{(2)}\wedge {}^*F_{(2)} + H_{(3)}\wedge {}^*H_{(3)} - c\int B_{(2)}\wedge F_{(2)}.$$

However, to understand microscopic holographic dual theories, we need *string theory embeddings.*

Actually, no embeddings of such 5D theory have been known….

Ex. AdS4×CP3 (→ dual to 3D N=6 Chern-Simons ABJM theory)

This is the most clear model where all moduli are stabilized within type IIA supergravity and thus is desirable for us.

KLM ansatz: $F_{tr} \neq 0, \quad H_{xyr} \neq 0.$

→ However, we only find unphysical solutions like

z=-4 and imaginary valued H-flux.

$$ds^2_{Scaling} = -r^{2z}dt^2 + r^2\left(dx^2 + dy^2\right) + \frac{dr^2}{r^2}.$$

May be Any No-go theorem ? [Li-Nishioka-TT work in progress]

In this talk, we will take a different ansatz to obtain string theory embedding of anisotropic scale invariant metric.

In particular, we will realize solutions with

z=3/2, p=2 and d=3

in type IIB supergravity, which are dual to D3-D7 systems.

$$ds^2_{Einstein} = \rho^3\left(-dt^2 + dx^2 + dy^2\right) + \rho^2 dw^2 + \frac{d\rho^2}{\rho^2}.$$

② D3-D7 and Pure Chern-Simons Gauge Theory

[Fujita-Li-Ryu-TT 09']

What is the holographic dual of pure Chern-Simons theory ?

$$S_{CS} = \frac{k}{4\pi} \mathrm{Tr} \int AdA + \frac{2}{3}A^3$$

Remember the well-known correspondence:

AdS5 Soliton ×S5 ⟷ Compactified 4D N=4 SYM

→ 3D pure Yang-Mills

[Witten 98']

$$ds^2_{\mathrm{AdS\,BH}} = \frac{R^2 dr^2}{r^2 f(r)} + \frac{r^2}{R^2}(-f(r)dt^2 + dx_1^2 + dx_2^2 + dx_3^2),$$

$$f(r) \equiv 1 - \frac{r_0^4}{r^4},$$

↓

$$ds^2_{\mathrm{AdS\,Soliton}} = \frac{R^2 dr^2}{r^2 f(r)} + \frac{r^2}{R^2}(-dt^2 + f(r)dx_1^2 + dx_2^2 + dx_3^2),$$

If we add the Chern-Simons term to the pure YM, it becomes U(N) Yang-Mills-Chern-Simons theory.

$$S_{3dYM} = -\frac{1}{4g_{YM}^2}\int dx^3 \mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] + \frac{k}{4\pi}\int \mathrm{Tr}[A \wedge dA + \frac{2}{3}A^3].$$

➡ In the low-energy limit (i.e. IR limit), this theory is reduced to pure U(N) Chern-Simons theory !

The Chern-Simons term is dual to the RR 1-form flux because

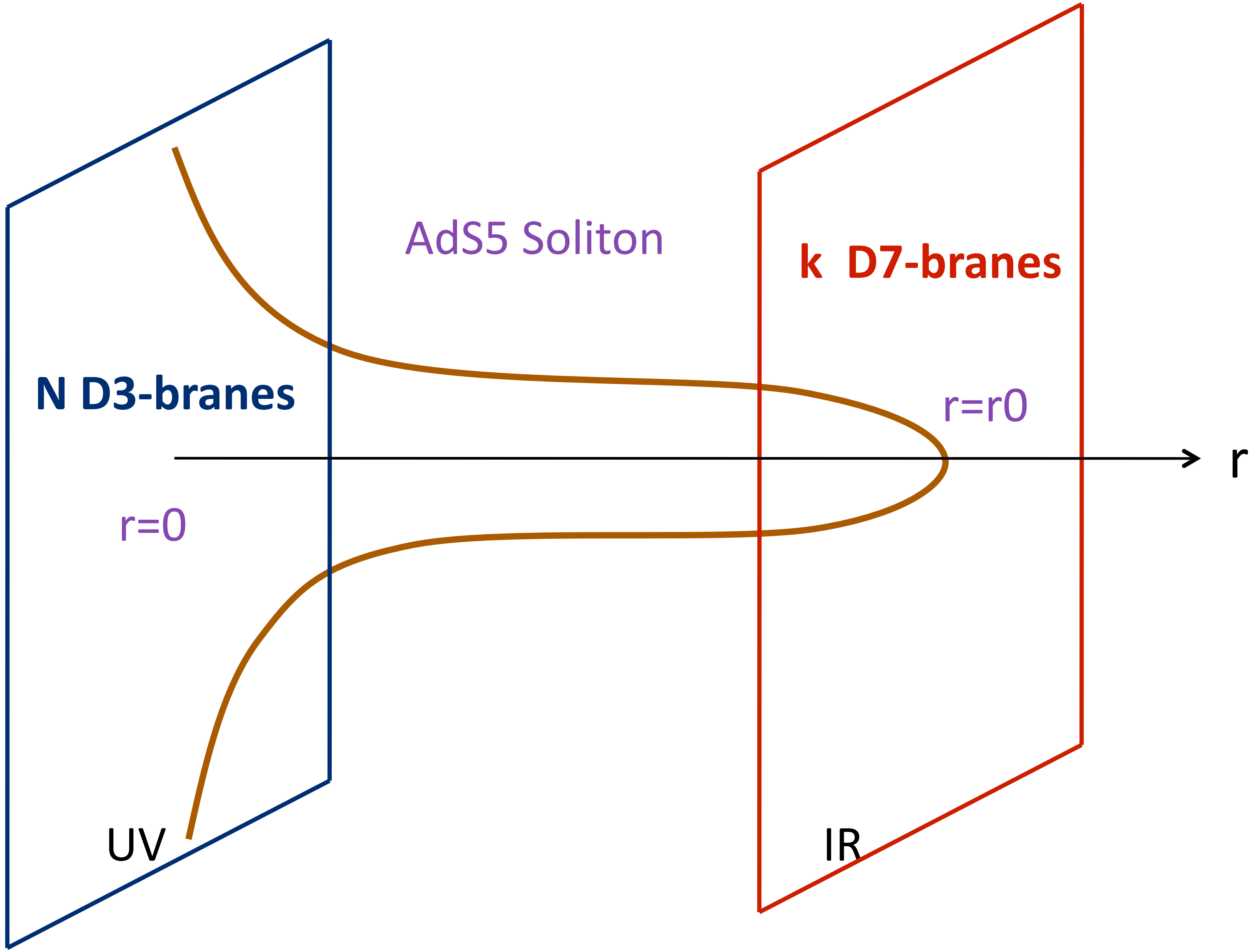
$$S_{WZ} = \frac{1}{4\pi}\int_{D3} \chi\, F \wedge F = -\frac{1}{4\pi}\int_{D3} d\chi\, A \wedge F,$$

$$\mathrm{RR\text{-}flux:} \int_{S^1} d\chi = -k \subset Z$$

$$\rightarrow \frac{k}{4\pi}\int_{R^3} A \wedge F.$$

This k units flux is sourced by k D7-branes.

Therefore we find the dual background:



The AdS/CFT applied to this setup claims

AdS-Side = CFT(QFT)-side

Gravity on AdS5 Soliton + k D7-branes = $U(N)_k$ Yang-Mills-Chern-Simons

Low energy limit

Pure $U(k)_N$ Chern-Simons = Pure $U(N)_k$ Chern-Simons

Level-Rank duality

③ New Supergravity Solutions for D3-D7 Systems

It is interesting to construct back-reacted type II supergravity solutions to such D3-D7 systems.

For this purpose, we assume the following ansatz:

$$ds^2_{string} = e^{2b(r)}(-dt^2 + dx^2 + dy^2) + e^{2h(r)+2a(r)}dw^2$$
$$+ e^{2c(r)-2a(r)}dr^2 + e^{2c(r)}r^2 \underline{ds^2_{X5}} \quad ,$$

Any Einstein Manifold

$$\phi = \phi(r), \qquad B_2 = C_2 = 0,$$
$$F_5 = \alpha\left(\mathrm{Vol}(X_5) + *\mathrm{Vol}(X_5)\right),$$
$$F_1 = d\chi = \beta dw.$$

Equations of motion look like

$$[b'e^{2z}]' = \frac{\beta^2}{4}e^{-2a-h+3b+6c}r^5 + \frac{\alpha^2}{4}e^{-4c+3b+h}r^{-5},$$

$$[(a+h)'e^{2z}]' = -\frac{\beta^2}{4}e^{-2a-h+3b+6c}r^5 + \frac{\alpha^2}{4}e^{-4c+3b+h}r^{-5},$$

$$[(c+\log r)'e^{2z}]' = \frac{4}{r^2}e^{2z-2a} + \frac{\beta^2}{4}e^{-2a-h+3b+6c}r^5 - \frac{\alpha^2}{4}e^{-4c+3b+h}r^{-5},$$

$$[(2z+c-a)'e^{2z}]' = \frac{20}{r^2}e^{2z-2a} - \frac{\beta^2}{4}e^{-2a-h+3b+6c}r^5 - \frac{\alpha^2}{4}e^{-4c+3b+h}r^{-5},$$

$$2z'' + c'' - a'' + 2(z')^2 + \frac{1}{2}(h')^2 + a'h' + 2(c')^2 + (\frac{5}{r} + a')c' + \frac{3}{2}(b')^2 - \frac{10e^{-2a}}{r^2} + \frac{5}{2r^2} = 0,$$

where we have defined

$$z \equiv \frac{3}{2}b + \frac{5}{2}\log r + a + 2c + \frac{1}{2}h - \phi.$$

The derivative of a function f with respect to r is denoted by $f'(r)$. An observation, which will be useful in the next section, is that a linear combination of the first four equations gives

$$[(2b-2a-\phi-2h)'e^{2z}]' = 0.$$

To solve EOMs, we assume that a(r),b(r),h(r),c(r) and ϕ(r) are all proportional to log r. (``Scaling ansatz'')

Then we obtain the scaling solutions:

$$ds^2_{Einstein} = \widetilde{R}^2\left[r^2\left(-dt^2+dx^2+dy^2\right)+r^{\frac{4}{3}}dw^2+\frac{dr^2}{r^2}\right]+R^2 ds^2_{X5} \quad ,$$

$$e^{\phi(r)} = e^{\phi_0} r^{\frac{2}{3}} \quad , \qquad \left(R^2 = \frac{12}{11}\widetilde{R}^2\right)$$

Notice: After the redefinition $\rho = r^{2/3}$, we obtain

$$ds^2_{Einstein} = \frac{9}{4}\widetilde{R}^2\left[\rho^3\left(-dt^2+dx^2+dy^2\right)+\rho^2 dw^2+\frac{d\rho^2}{\rho^2}\right]+R^2 ds^2_{X5} \quad .$$

In terms of N and k (= the number of D3 and D7 - branes),

$$\alpha = \frac{(2\pi)^4 N}{\mathrm{Vol}(X_5)} \quad , \qquad \beta = \frac{k}{L} \quad , \qquad R^2 = 2\sqrt{\frac{\pi^4 N}{\mathrm{Vol}(X_5)}} \quad ,$$

where we assumed the radius of w is L .

Moreover, our solutions allow the black brane generalization:

$$ds^2_{Einstein} = \widetilde{R}^2 \left[r^2 \left(-F(r)dt^2 + dx^2 + dy^2 \right) + r^{\frac{4}{3}} dw^2 + \frac{dr^2}{r^2 F(r)} \right] + R^2 ds^2_{X5} \quad ,$$

$$e^{\phi(r)} = e^{\phi_0} r^{\frac{2}{3}} \quad , \quad F(r) = 1 - \frac{\mu}{r^{11/3}} \quad , \quad \left(R^2 = \frac{12}{11} \widetilde{R}^2 \right) \quad .$$

The temperature and entropy become

$$T = \frac{11}{12\pi} \mu^{3/11}, \qquad S_{BH} = \gamma \cdot N^2 T^{8/3} V_2 L \qquad (\gamma = 3.729...).$$

(1+1+2/3) dim. space → Agree with [w]=2/3 !

④ String Theory Duals of Lifshitz-like Fixed Points

We obtained type IIB solutions whose Einstein metrics possess the anisotropic scale invariance.

Now we would like to interpret our solutions from the viewpoint of the holography in string theory.

Two Problems: (i) The dilaton blows up in the limit $r=\infty$

(ii) What is the holography in this case ?

Any relation to AdS5/CFT4 ?

We can resolve these issues by constructing RG flow from AdS5 !

(4-1) RG Flow from AdS5

Let us require

$$a(r) = 0,$$
$$2b(r) - 2a(r) - \phi(r) - 2h(r) = -2h_0 (= \text{const.})$$
$$\phi(r) = 4c(r) + 4\log r + \widetilde{\phi}_0,$$

in the previous ansatz:

$$ds^2_{string} = e^{2b(r)}(-dt^2 + dx^2 + dy^2) + e^{2h(r)+2a(r)}dw^2$$
$$+ e^{2c(r)-2a(r)}dr^2 + e^{2c(r)}r^2 ds^2_{X5} \quad ,$$
$$\phi = \phi(r), \qquad B_2 = C_2 = 0,$$
$$F_5 = \alpha\left(\mathrm{Vol}(X_5) + {*}\mathrm{Vol}(X_5)\right),$$
$$F_1 = d\chi = \beta dw.$$

Then the equations of motion are reduced to

Defining $B(s) = \frac{db(r)}{d \log r}, \quad C(s) = \frac{dc(r)}{d \log r}$ and $s = \log r$,

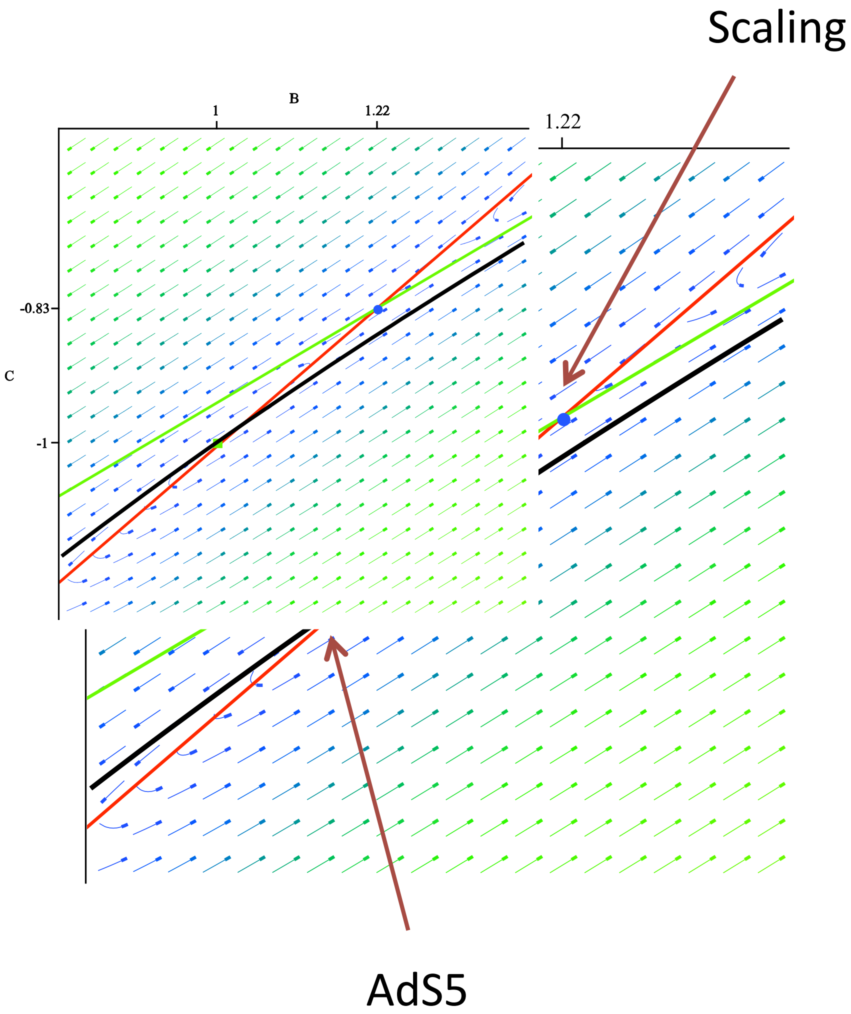
$$\dot{B} = 2 + 24B - 16C - 10B^2 + 24BC - 8C^2,$$
$$\dot{C} = 4 + 14B - 4C - 6B^2 + 14BC - 2C^2,$$

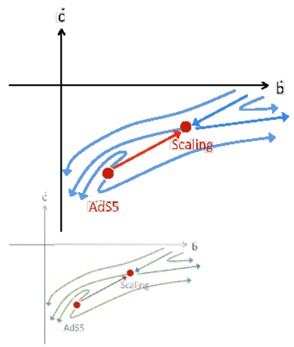
Two fixed points: AdS5 and Scaling

with the constraint :

$$\beta^2 e^{-2b(r)+14c(r)} r^{14} = -2 - 6B^2 + 18BC - 8C^2 + 18B - 16C > 0.$$

Holographic RG flow



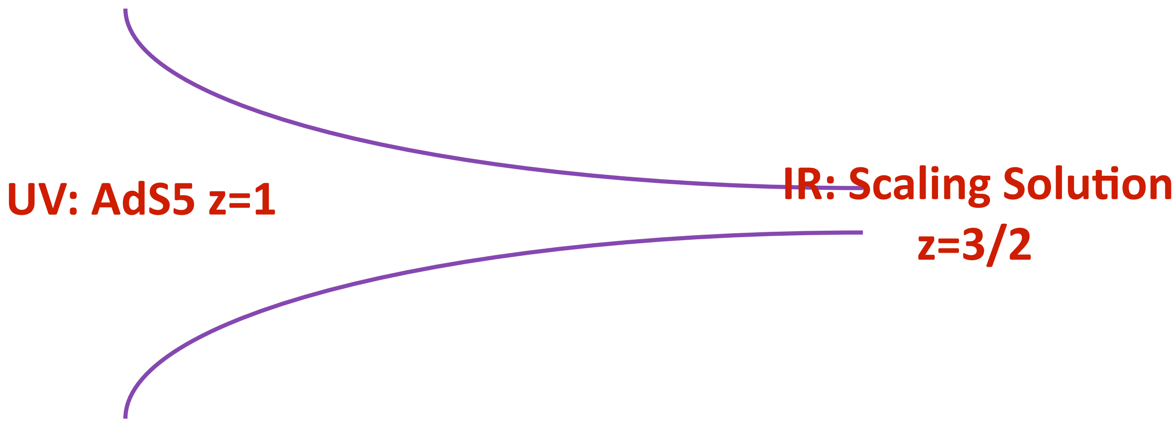


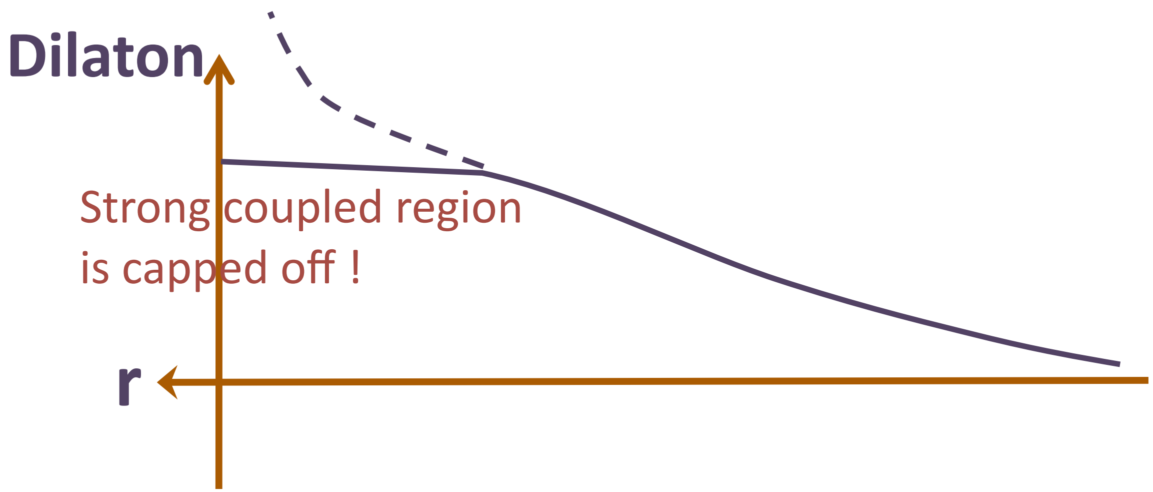
IR

UV

(4-2) Holographic Interpretation

In this way, we find the RG flow in AdS/CFT





When X5=S5,

we start with the UV: N=4 Super Yang-Mills. Then we perform the relevant (`non-local’) perturbation:

$$S_{YM} = -\frac{1}{4g_{YM}^2}\int dx^4 F_{\mu\nu}F^{\mu\nu} + \int dt dx dy dw \theta(w) F \wedge F \ ,$$

$$\theta(w) = \frac{k}{L} w.$$

Relevant perturbation which breaks Lorentz sym.

↓

Equations of motion remain local !

The z=3/2 fixed point looks like an unstable fixed point, which crossovers with z=1 original fixed point. But by fine tuning, we can approach this fixed point as much as we want.

First, eliminating C from equations (3.11) and (3.12) gives

(4-3) Exact Solutions with no D7-branes

When the constraint vanishes β=0, we can somehow find new analytical solutions of IIB SUGRA:

$$
\begin{aligned}
ds_E^2 &= R^2\left[\sqrt{\frac{r^8-1}{r^4}}\left(\frac{r^4+1}{r^4-1}\right)^{\pm\frac{1}{2}\sqrt{\frac{3}{11}}}(-dt^2+dx^2+dy^2)+\sqrt{\frac{r^8-1}{r^4}}\left(\frac{r^4+1}{r^4-1}\right)^{\mp\frac{3}{2}\sqrt{\frac{3}{11}}}dw^2\right.\\
&\qquad\left.+\frac{dr^2}{r^2}+ds_{X_5}^2\right]\\
e^{\phi} &= \left(\frac{r^4+1}{r^4-1}\right)^{\pm 2\sqrt{\frac{3}{11}}}e^{\tilde{\phi}_0}
\end{aligned}
$$

with $R^2 = 2\sqrt{\frac{\pi^4}{\mathrm{Vol}(X_5)}}N$.

(4-4) Perturbative Analysis

We showed that the metric in the Einstein frame has the anisotropic scale invariance. However, this is not true in the string frame metric.

Therefore we have to examine if physical quantities (e.g. correlation functions) can be computed from the Einstein frame metric.

As we will see, indeed this is true for many perturbative modes.

[We will follow the famous analysis
by Kim-Romans-Nieuwenhuizen 85']

In the Einstein frame, (ignoring decoupled three form fluxes) the action simply looks like

$$L_{Ein} = \sqrt{-g}\left(R - \frac{1}{2}e^{2\phi}\partial_I \chi \partial^I \chi - \frac{1}{2}\partial_I \phi \partial^I \phi - \frac{1}{4\cdot 5!}F_{IJKLM}F^{IJKLM}\right).$$

We expand the perturbations of metric and four form potential in terms of spherical harmonic on S5:

$$h_{(\mu\nu)} = h^I_{(\mu\nu)}Y^I, \quad h^\mu_\mu = h^I Y^I, \quad h_{\mu\alpha} = B^I_\mu Y^\mu_\alpha, \quad h_{(\alpha\beta)} = \phi^I Y^I_{(\alpha\beta)}, \quad h^\alpha_\alpha = \pi^I Y^I,$$

$$C_{\alpha\beta\gamma\delta} = b^I \varepsilon^{\tau}_{\alpha\beta\gamma\delta}\nabla_\tau Y^I, \quad C_{\mu\alpha\beta\gamma} = b^I_\mu \varepsilon^{\delta\tau}_{\alpha\beta\gamma}\nabla_\tau Y^I_\delta, \quad C_{\mu\nu\alpha\beta} = b^I_{\mu\nu}Y^I_{[\alpha\beta]}, \quad \ldots\ldots$$

$$\mu,\nu,.. = 0,1,2,3,4, \quad (t,x,y,w,r)$$

$$\alpha,\beta,.. = 5,6,7,8,9 \quad (S^5)$$

We can show that the infinite towers of the perturbative modes

$$\text{Scalar}: \phi^{\mathrm{I}}, \quad (h^I, \pi^I, b^I),$$
$$\text{Vector}: (B_\mu^I, b_\mu^I)$$
$$\text{Tensor}: b_{\mu\nu}^I \quad .$$

satisfy massive free equations of motion in the Einstein frame metric. For example,

$$\left(\Delta_{Sc} + \Delta_{S5} - \frac{2}{R^2} \right) \phi^I Y^I_{(\alpha\beta)} = 0,$$

Laplacian for the scaling metric

or equally

$$\left(\Delta_{\mathrm{Sc}} - \frac{k(k+4)}{R^2} \right) \phi^I = 0. \qquad (k = 2,3,4,..)$$

For a massive scalar

$\left(\Delta - m^2\right)\Phi = 0$ in the background metric

$$ds^2 = R^2\left(\frac{d\rho^2}{\rho^2} + \frac{-dt^2 + dx^2 + dy^2}{\rho^2} + \frac{dw^2}{\rho^{2/z}}\right),$$

$$\Rightarrow \quad -\partial_\rho^2\Phi + \frac{z^{-1}+2}{\rho}\partial_\rho\Phi + \left(\frac{m^2R^2}{\rho^2} + p_i^2 - \omega^2 + p_w^2\rho^{2(1/z-1)}\right)\Phi = 0.$$

$$\Rightarrow \quad \Phi \sim A\rho^{\Delta_+} + B\rho^{\Delta_-},$$

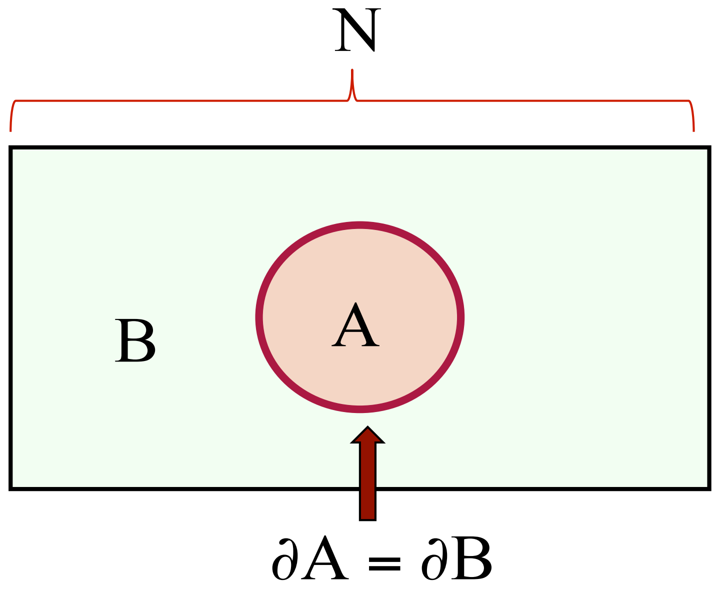
$$\text{Scale dim.} \quad : \quad \Delta_\pm = \frac{z^{-1}+3}{2} \pm \sqrt{\left(\frac{z^{-1}+3}{2}\right)^2 + m^2R^2}.$$

$\Rightarrow$ Information of two point functions

⑤ Entanglement Entropy

When we need to analyze a quantum many-body system whose definition and property are not well understood, the entanglement entropy is very useful.

$$H = H_A \otimes H_B : \ Hilbert \ Space$$



Define the reduced density matrix

$$\rho_A = \mathrm{Tr}_{H_B}[\rho_{tot}].$$

$$S_A = -\mathrm{Tr}[\rho_A \mathrm{Log} \rho_A]$$

= Entropy for an observer who cannot observe the subsystem B

The Lifshitz-like fixed points are also interesting from the viewpoint of **entanglement entropy**, which has a *simple holographic description*.

Usually, in relativistic (d+1) dim. QFTs, we have the area law,

$$S_A = -\mathrm{Tr}[\rho_A \mathrm{Log}\rho_A] = \frac{\mathrm{Area}(\partial A)}{a^{d-1}} + \cdots.$$

This scaling of the leading term will be changed due to the anisotropic scale invariance.

→ It is an alternative of correlation functions, which require more complicated calculations.

The entanglement entropy measures:

(i) Non-local Correlations (like Wilson loops)

(ii) The degrees of freedom (non-vanishing even at zero temp.)

(iii) The missing information hidden inside the subsystem

➡ Sounds like black hole entropy ?

Gravitational origin ?

➡ Indeed, it has a simple holographic description !

Holographic Computation of EE

[Ryu-TT 06', Recent Review: Nishioka-Ryu-TT 0905.0932]

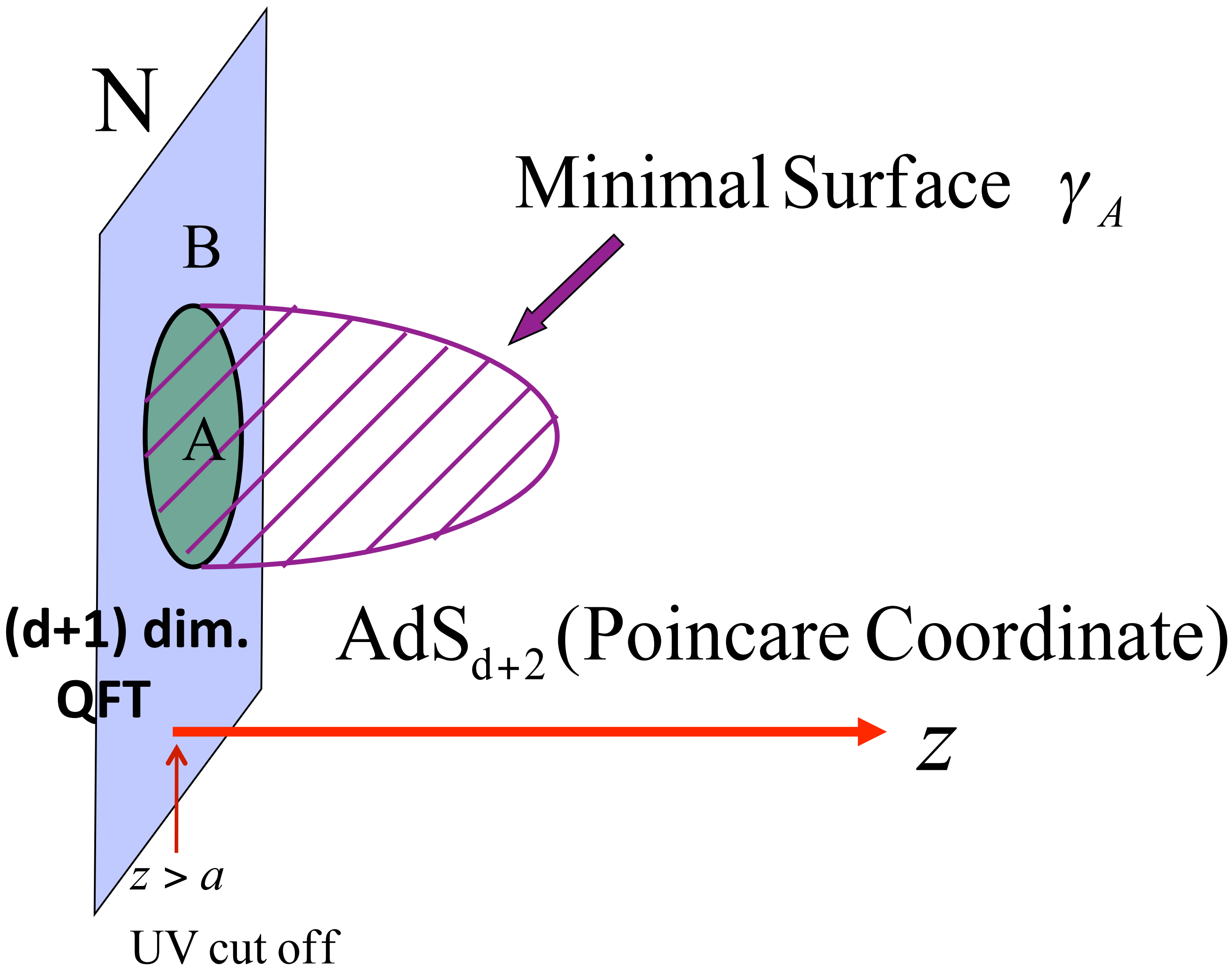
(1) Divide the space N is into A and B.

(2) Extend their boundary ∂A to the entire AdS space. This defines a d dimensional surface.

(3) Pick up a minimal area surface and call this γ_A.

(5) The E.E. is given by naively applying the Bekenstein-Hawking formula as if γ_A were an event horizon.

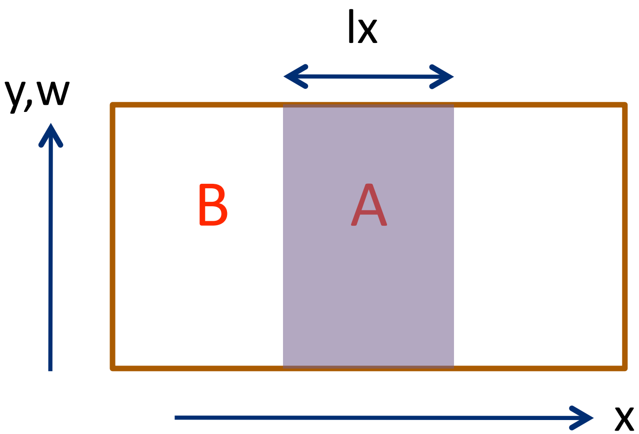
$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N^{(d+2)}}.$$

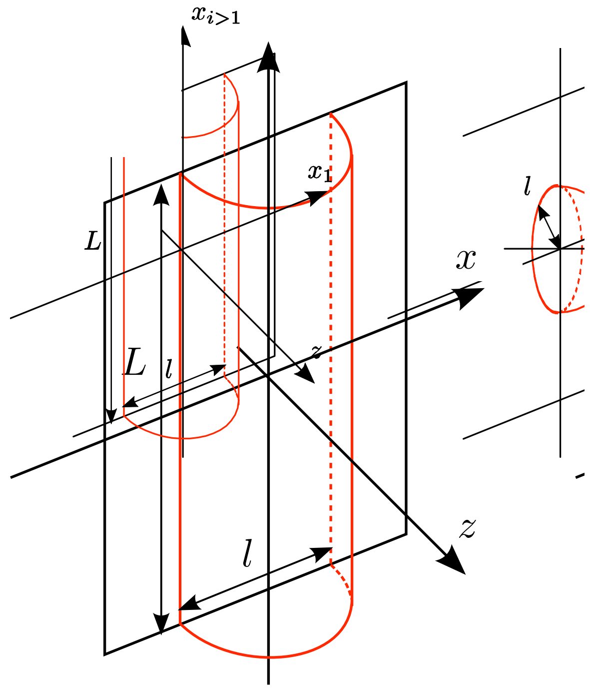


[Derivation from Bulk to Boundary relation: Fursaev 06’]

Holographic EE in D3-D7 scaling solution

(5-1) Case 1: x-Interval



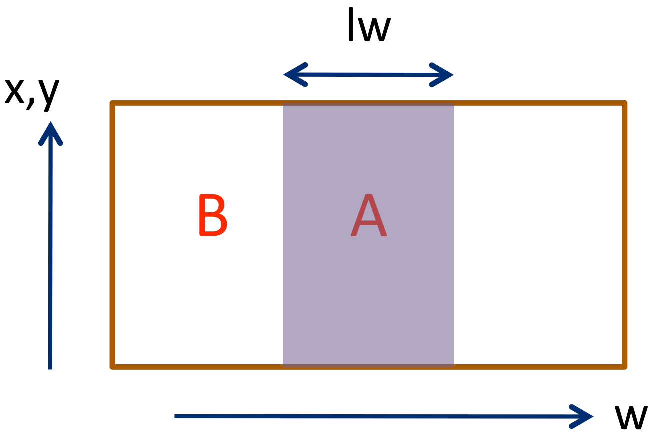


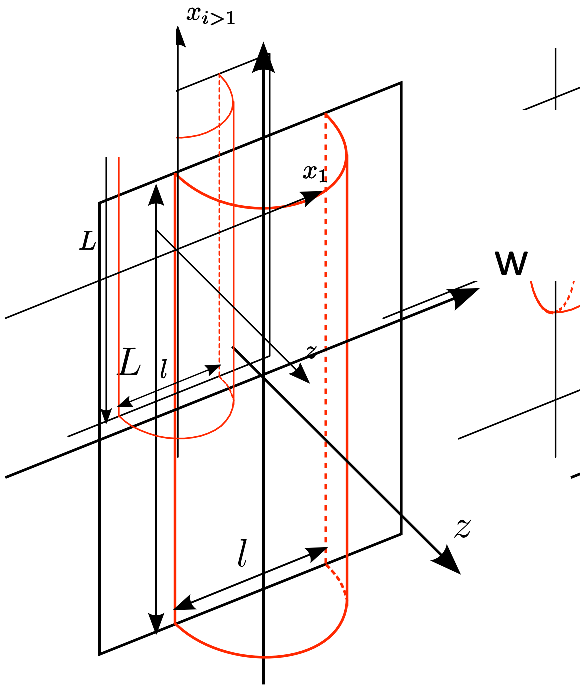
In the end, we obtain

$$S_A = \frac{\text{Area}(\gamma_{\min})}{4G_N} = N^2 L_y L_w \left(\frac{\lambda_1}{a^{5/3}} - \frac{\lambda_2}{l_x^{5/3}} \right) \quad .$$

[w]=2/3, [x]=[y]=1

(5-2) Case 2: w-Interval





In the end, we obtain

$$S_A = \frac{\text{Area}(\gamma_{\min})}{4G_N} = N^2 L_x L_y \left(\frac{\kappa_1}{a^2} - \frac{\kappa_2}{l_w^3} \right) \quad .$$

[w]=2/3, [x]=[y]=1

⑥ Shear and Bulk Viscosity

Finally, we would like to calculate another interesting quantity i.e. viscosity of the `Lifshitz' fluid at finite temperature.

[We follow the approach by Kovtun-Starinets 05', Mas-Tarrio 07']

Starting from the ten dim. IIB supergravity we define the 5D metric perturbation by a Weyl-shift

$$H_{\mu\nu} = h_{\mu\nu} + \frac{1}{3} g_{\mu\nu} h^{\alpha}_{\alpha}.$$

We only consider perturbations which are independent of w and thus we assume the momentum only in y direction

$$H_{\mu\nu} \propto e^{-i(\omega t - qy)}$$

(6-1) Shear Viscosity

By taking into diff. invariance, a shear perturbation of the metric is described by $Z_1(r) = qH_{tx} + \omega H_{yx}$.

By solving EOMs assuming $\varpi = \dfrac{\omega}{2\pi T} << 1$ and $\bar{q} = \dfrac{q}{2\pi T} << 1$, we finally obtain

$$Z_1(r) = F(r)^{-i\varpi/2}\left[1 + \frac{i\bar{q}^2}{2\varpi}F(r) + \ldots\right]$$

By comparing the quasi normal mode with the thermodynamics:

$$\omega = -\frac{i\eta}{Ts}q^2$$

We find the familiar result in our case $\boxed{\dfrac{\eta}{s} = \dfrac{1}{4\pi}.}$

(6-2) Bulk viscosity

To calculate the bulk viscosity, we analyze the sound mode:

$$Z_0(r) = q^2 F(r) H_{tt} + 2q\omega H_{ty} + \omega^2 H_{yy} + g(r)(H_{xx} + H_{yy}).$$

After a bit complicated calculations, we find

$$Z_0(r) = F(r)^{-i\varpi/2}\left[1 - \frac{5(1+2i\varpi)\bar{q}^2}{11\bar{q}^2 - 16\varpi^2} F(r) + \ldots\right].$$

and obtain the dispersion relation of the quasi-normal mode

$$\varpi = \frac{1}{2}\sqrt{\frac{3}{2}}\bar{q} - i\frac{5}{16}\bar{q}^2 + \ldots..$$

By comparing this with the standard formula

$$\varpi = c_s \bar{q} - i \frac{\eta}{Ts}\left(\frac{d-1}{d} + \frac{\zeta}{2\eta}\right)\bar{q}^2 + \ldots\ldots,$$

where d = dim. of space-like coordinates.

Since in our case we have d=2 (i.e. compactifying w direction), we find the sound velocity Cs and the bulk viscosity ζ :

$$c_s = \sqrt{\frac{3}{8}}, \quad \frac{\zeta}{\eta} = \frac{1}{4} \quad .$$

This saturates a conjectured bound (by Buchel 08')

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{d} - c_s^2\right).$$

⑦ Conclusions

- We find solutions in type IIB supergravity whose Einstein frame metrics have a Lifshitz-like anisotropic scale invariance. The backgrounds should be dual to D3-D7 systems.

- We constructed a holographic RG flow between the usual AdS5 and the anisotropic solution. The RG flow is triggered by a non-local theta term of Yang-Mills theory.

- We calculate the thermal and entanglement entropy, which shows the fractional scaling.

- We also computed the bulk and shear viscosity.

Future Problems

- Non-relativistic Lifshitz-like solution ?

D3-D5 systems with F-string soruces lead to the z=7 solution

[Azeyanagi-Nishioka-TT unpublished 07']

$$ds^2_{Einstein} = 10R^2\left[-\rho^{14}dt^2 + \rho^2\left(dx^2 + dy^2 + dw^2\right) + \frac{d\rho^2}{\rho^2}\right] + R^2 ds^2_{X5} \quad ,$$

$$e^{\phi} = \rho^6 \quad .$$

- Non-dilatonic solutions or No-go theorem ?

[Work in progress with Li and Nishioka]

- Stability ? Supersymmetric Solutions ?