BLACK HOLES AND THE GENERALIZED UNCERTAINTY PRINCIPLE

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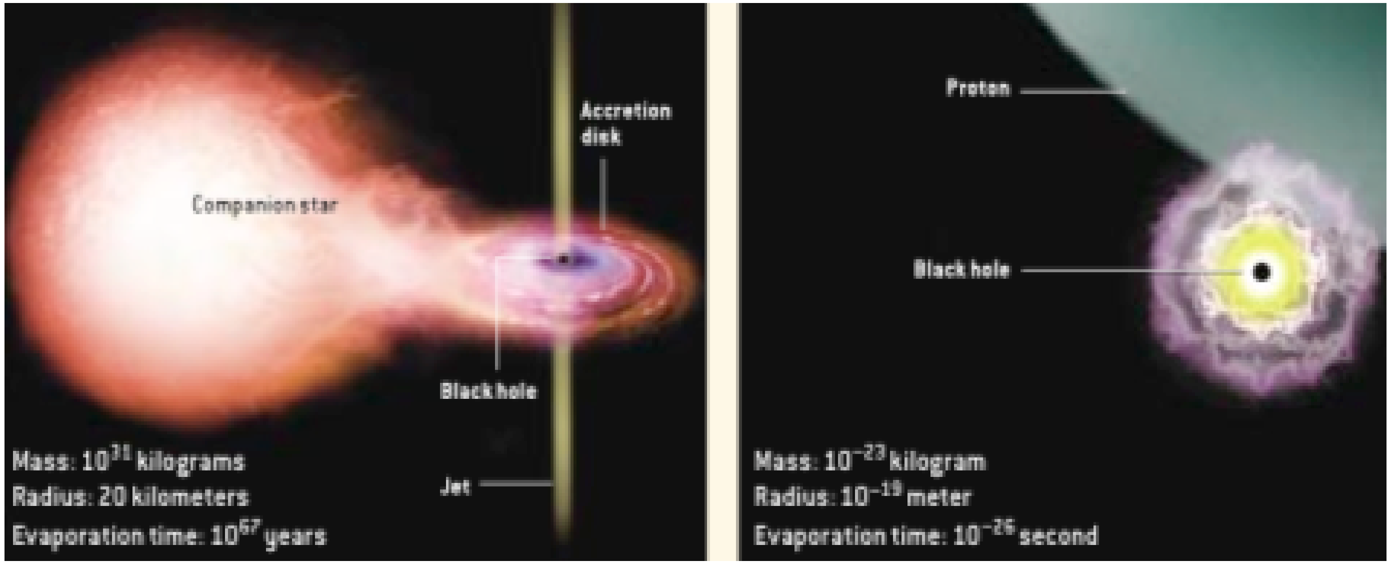
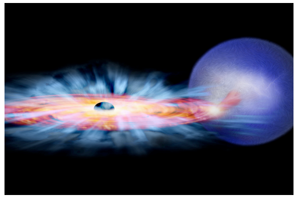


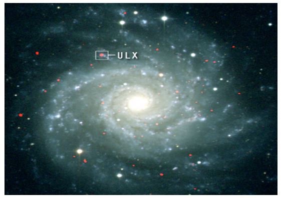
Macroscopic BLACK HOLES Microscopic

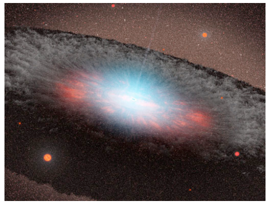
BLACK HOLE FORMATION

$R_S = 2GM/c^2 = 3(M/M_O)$ km => $\rho_S = 10^{18}(M/M_O)^{-2}$ g/cm^3

Good evidence that BHs form at present or recent epochs.
Stellar BH ($M \sim 10^{1\text{-}2} M_O$), IMBH ($M \sim 10^{3\text{-}5} M_O$), SMBH ($M \sim 10^{6\text{-}9} M_O$)







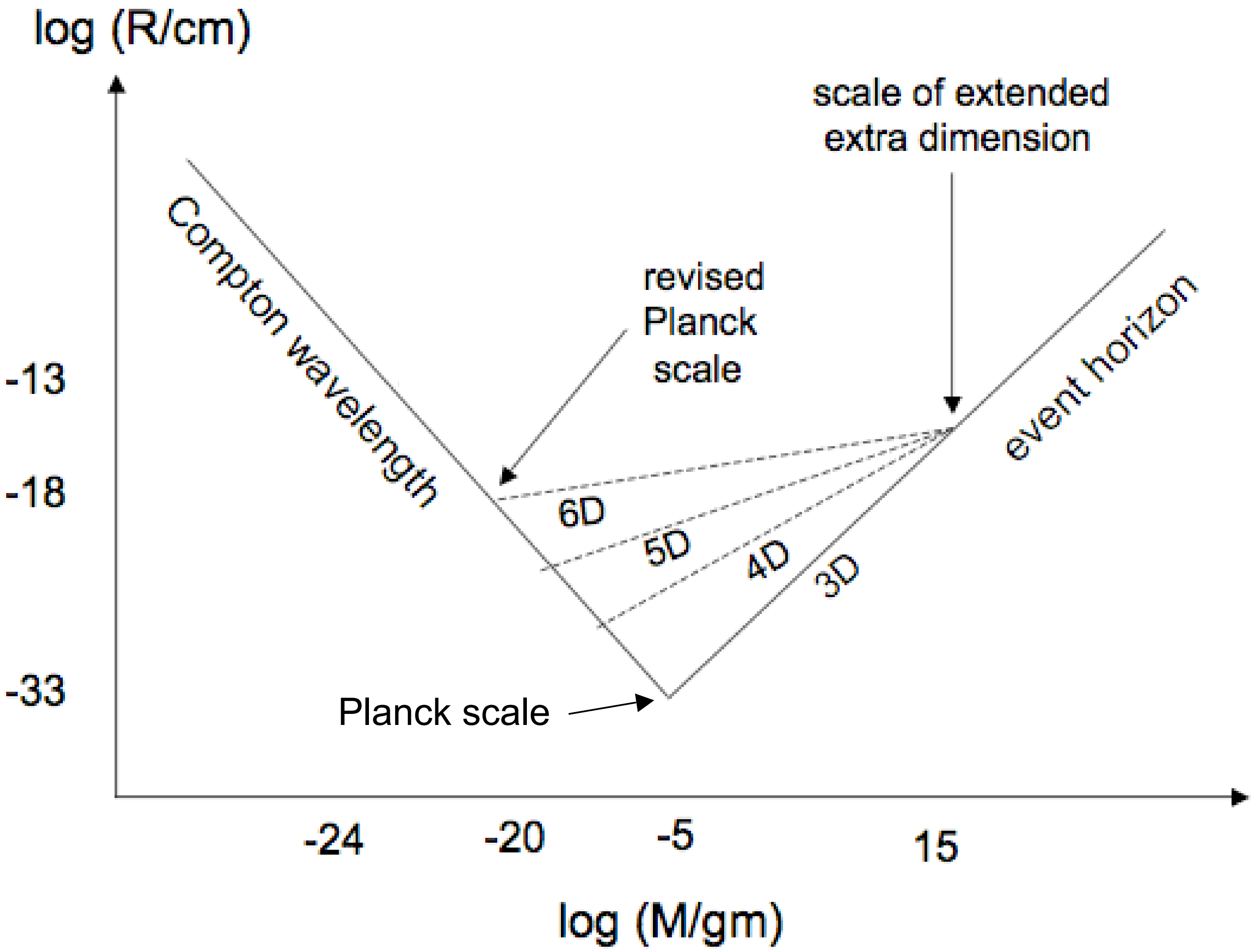
Small "primordial" BHs can only form in early Universe
cf. cosmological density $\rho \sim 1/(Gt^2) \sim 10^6 (t/s)^{-2}$g/cm^3
=> PBHs have horizon mass at formation

10^{-5}g at 10^{-43}s (minimum)

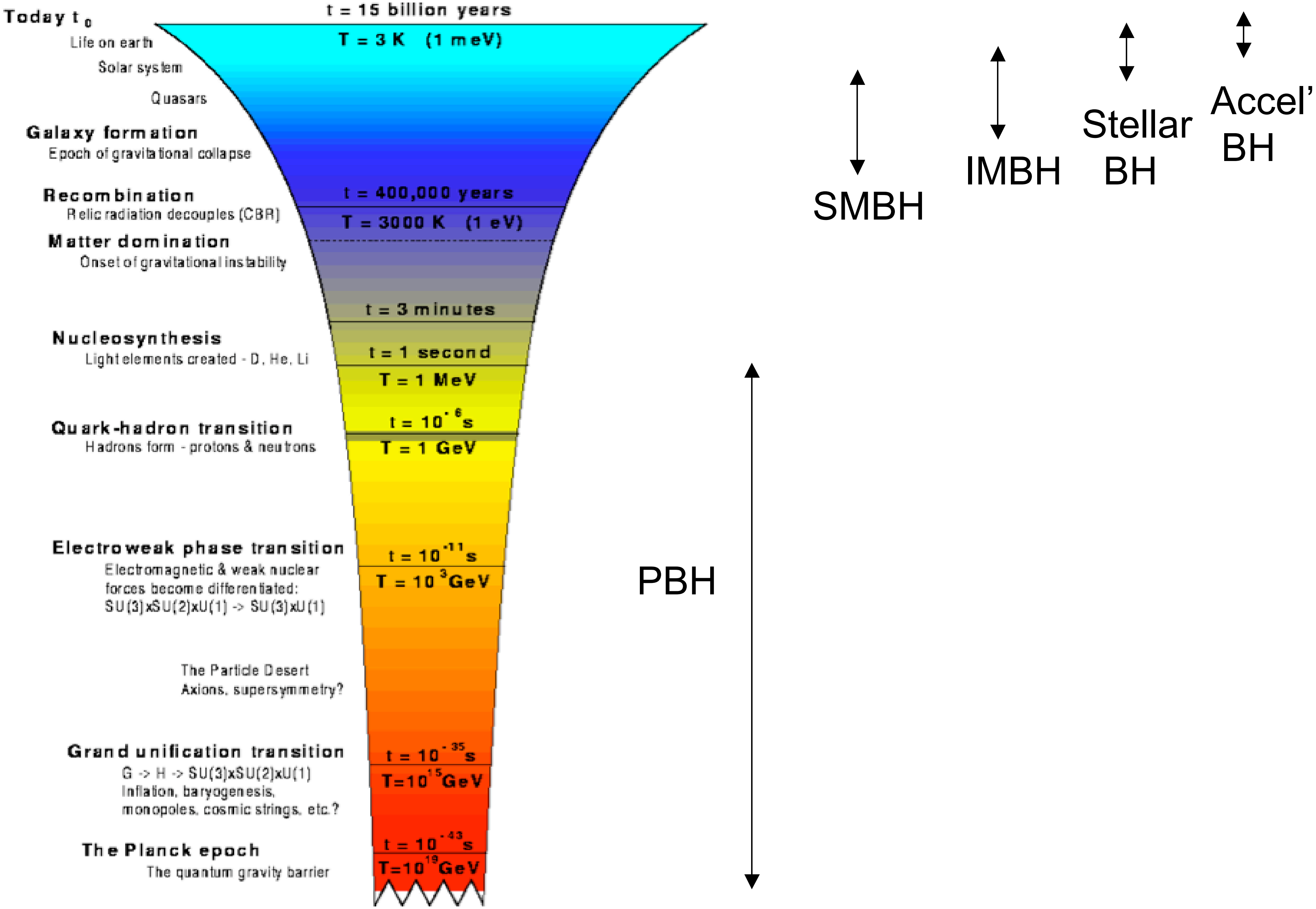
$M_{PBH} \sim c^3t/G =$ 10^{15}g at 10^{-23}s (evaporating now)

$1M_O$ at 10^{-5}s (maximum)

Higher dimensions => TeV quantum gravity => larger minimum?


log (R/cm)
scale of extended extra dimension
revised Planck scale
Compton wavelength
event horizon
-13
-18
-33
6D
5D
4D
3D
Planck scale
-24
-20
-5
15
log (M/gm)

WHEN BLACK HOLES FORM



SMBH

IMBH

Stellar BH

Accel' BH

PBH

PBH EVAPORATION

Black holes radiate thermally with temperature

$$T = \frac{hc^3}{8\pi GkM} \sim 10^{-7} \left[\frac{M}{M_0}\right]^{-1} K$$ (Hawking 1974)

=> evaporate completely in time $t_{evap} \sim 10^{64} \left[\frac{M}{M_0}\right]^{3} y$

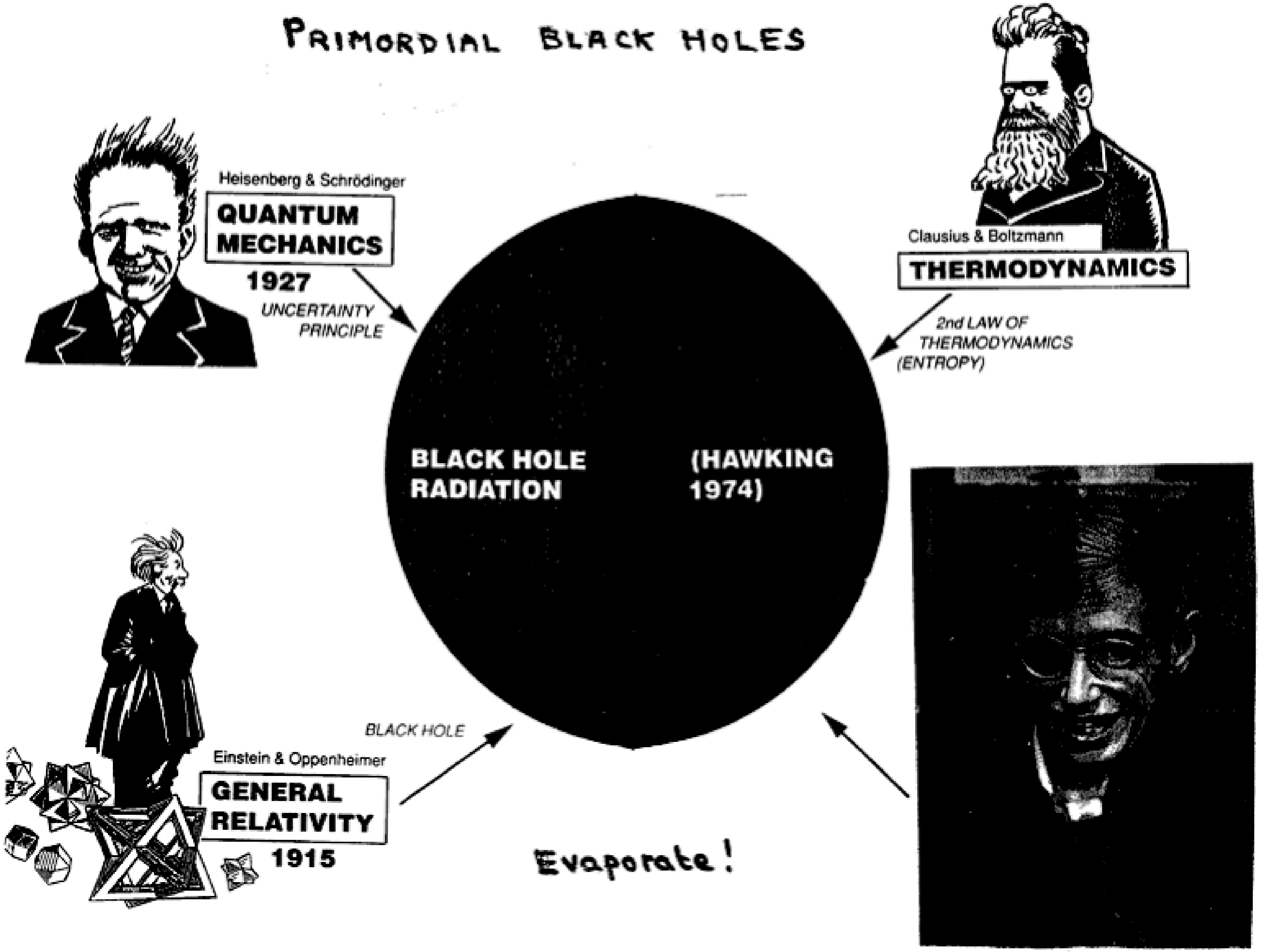
$M \sim 10^{15}g$ => final explosion phase today (10^{30} ergs)

γ-ray bgd at 100 MeV => $\Omega_{PBH}(10^{15}g) < 10^{-8}$

(Page & Hawking 1976)

=> explosions undetectable in standard particle physics model

PRIMORDIAL BLACK HOLES



Heisenberg & Schrödinger

QUANTUM MECHANICS

1927

UNCERTAINTY PRINCIPLE

Clausius & Boltzmann

THERMODYNAMICS

2nd LAW OF THERMODYNAMICS (ENTROPY)

BLACK HOLE RADIATION

(HAWKING 1974)

BLACK HOLE

Einstein & Oppenheimer

GENERAL RELATIVITY

1915

Evaporate!

PBHs important even if never formed!

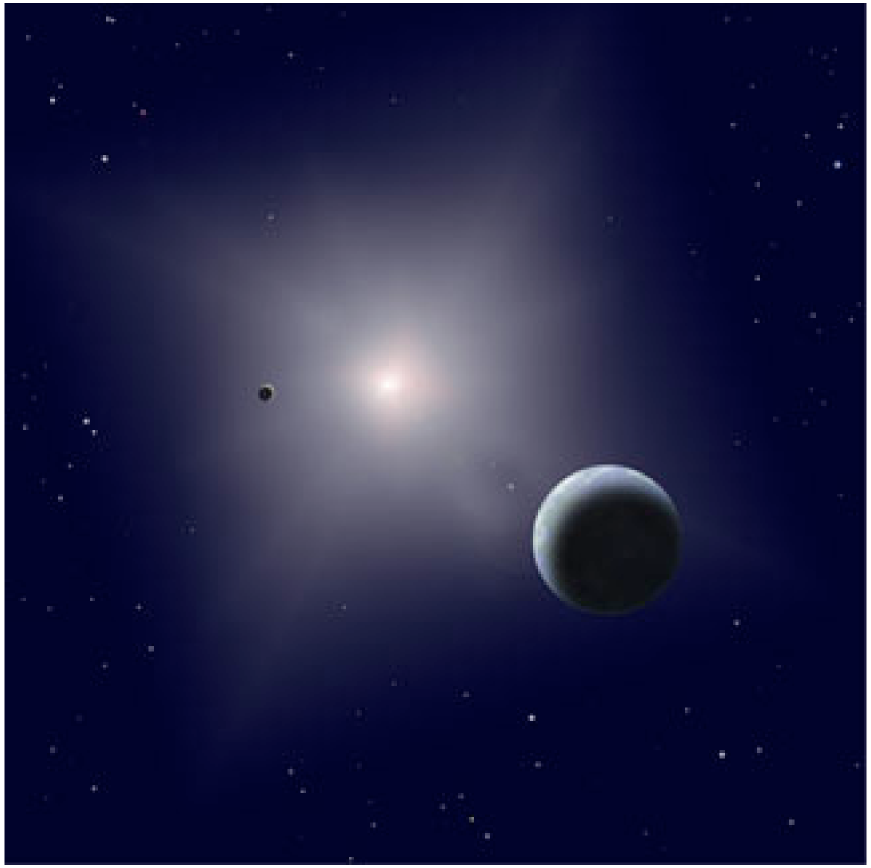
Microlensing searches => MACHOs with 0.5 M_O
PBH formation at QCD transition?
Pressure reduction => PBH mass function peak at 0.5 M_O

But microlensing => < 20% of DM can be in these objects

10^{26}-10^{33}g PBHs excluded by microlensing of LMC
10^{17}-10^{20}g PBHs excluded by femtolensing of GRBs
Above $10^5 M_0$ excluded by dynamical effects
But no constraints for 10^{16}-10^{17}g or 10^{20}-10^{26}g or above 10^{33}g

Stable Planck-mass relics of evaporated BHs?

DETECTION OF 10^{17}G PBHS BY FEMTOLENSING?

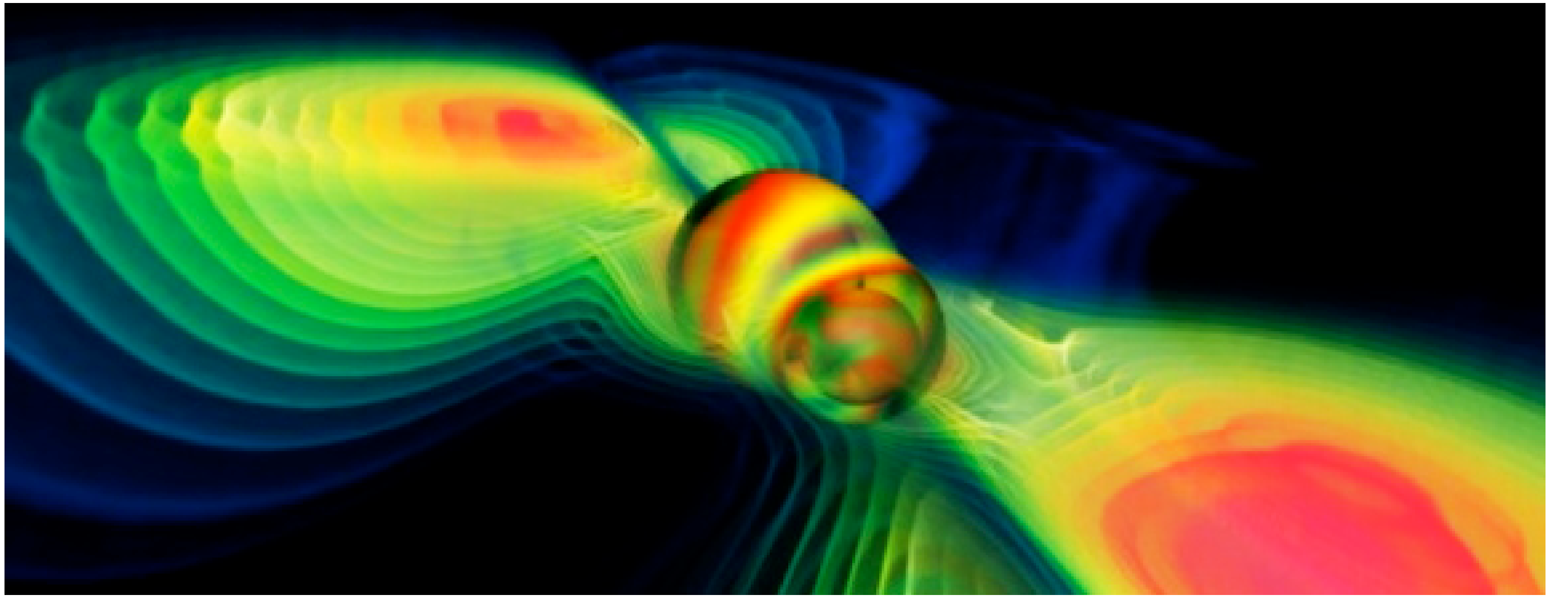


Will measurements of gamma-ray bursts, like the one shown sterilizing a planet in this artist's rendering, reveal the existence of tiny black holes? We may know soon.

Marani et al. (1999)

What Would Happen if a Small Black Hole Hit the Earth?

by IAN O'NEILL ***on*** FEBRUARY 17, 2008

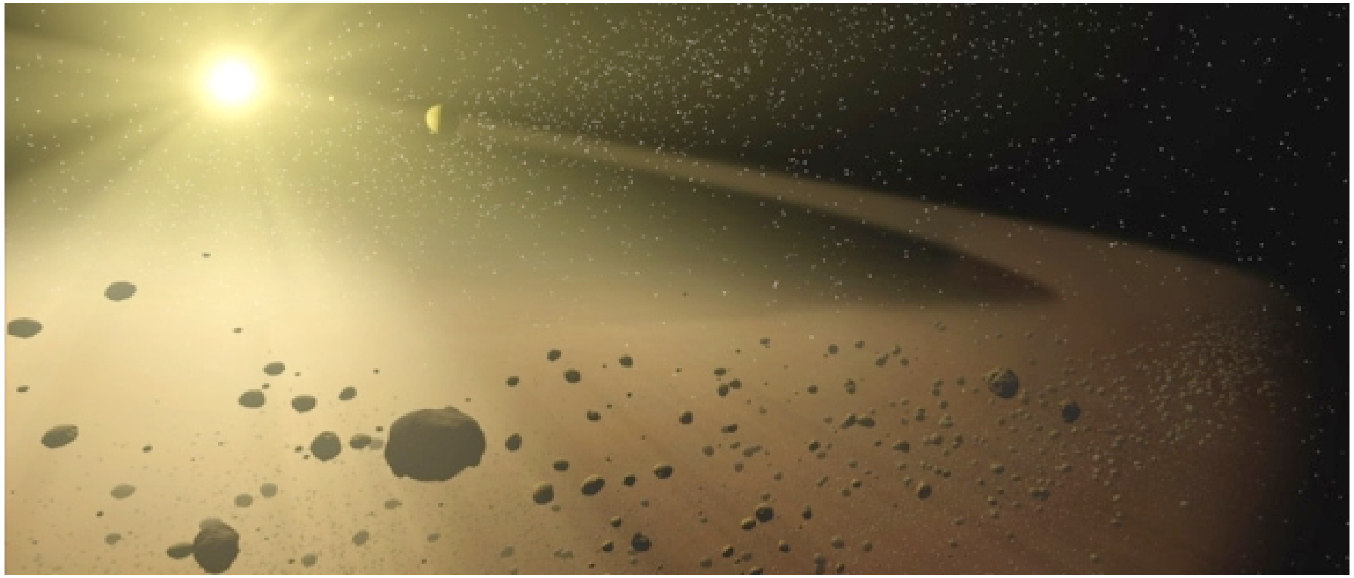


Khriplovich et al. (2008)

Long tube of radiatively damaged material recognisable for geological time

Could Primordial Black Holes Deflect Asteriods on a Collision Course with Earth?

by IAN O'NEILL *on* FEBRUARY 22, 2008

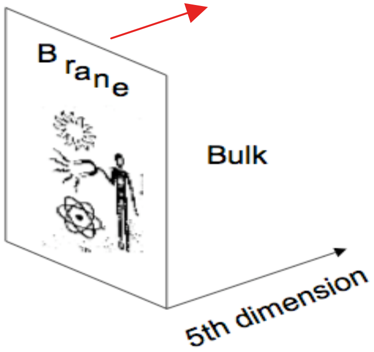


Shatskiy (2008)

Earth-mass PBHs could deflect asteroids onto Earth every 190M years

BRANE COSMOLOGY (Bowcock et al. 2000, Mukohyama et al. 2000)

Brane can be viewed as moving through 5th dimension in static bulk described by 5D Schwarzschild-anti de Sitter :



$$^{(5)}ds^2 = -F(R)dT^2 + F(R)^{-1}dR^2 + R^2\,[(1-Kr^2)^{-1}dr^2 + r^2 d\Omega^2],$$

$$F(R) = K - m/R^2 + (R/L)^2$$

0, +1, -1 mass of BH cosm const scale

5th dimension is identified with cosmic scale factor $R=a(t)$

$m=0$, $a=$const => Randall-Sundrum 1-brane

$K=1$ (closed) and m non-zero => event horizon at “radial” distance

Universe emerges from 5D black hole as $a(t)$ passes through

$$R_h = (1/2)[(L^4+4mL^2)^{1/2}-L^2] = m^{1/2} \text{ for } R_h << L$$

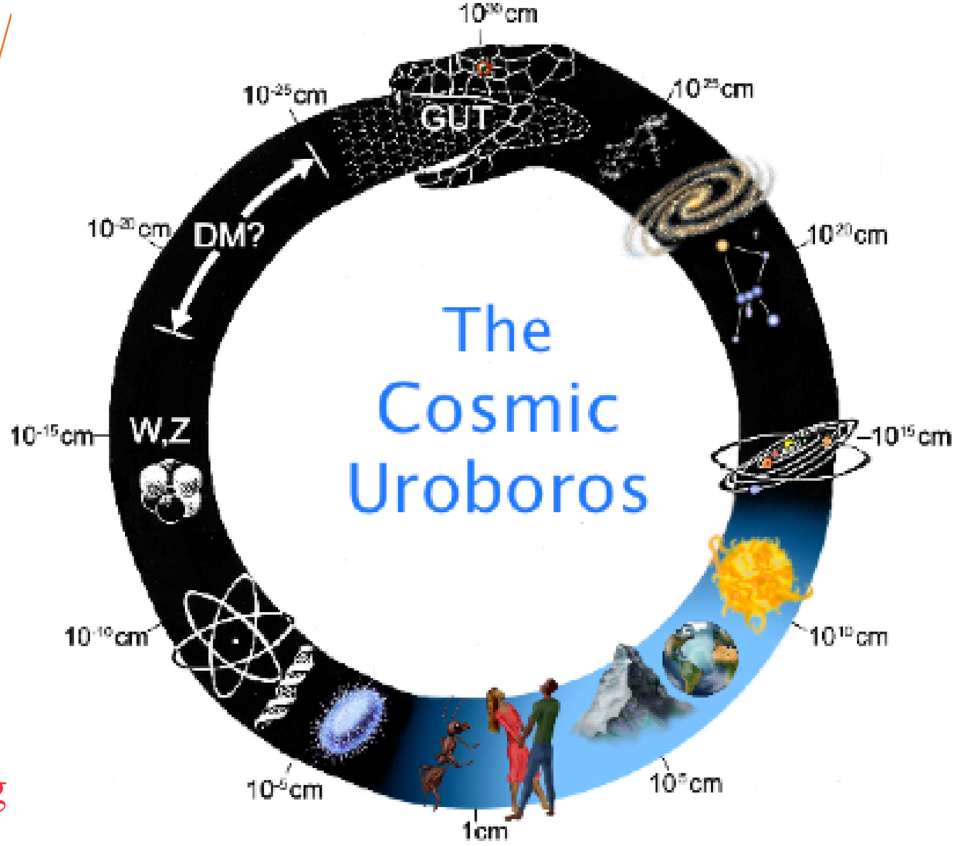
5D BH $F(R)dT^2+F(R)^{-1}dR^2$ 4D FRW $-dT^2+R^2\,(1-Kr^2)^{-1}dr^2$

M-theory **Higher dimensions** Multiverse

Planck 10^{-5}g

10^{22} M_O Universal

extra dimensions



10^{9} M_O QSO

LHC 10^{12}g

10^{6} M_O MW

exploding 10^{15}g

10^{2} M_O IMBH

lunar 10^{21}g

1 M_O Stellar

terrestrial 10^{25}g

BLACK HOLES

GENERALIZED UNCERTAINTY PRINCIPLE

- link with loop quantum gravity

L. Modesto & I. Premont-Schwarz,Self-dual Black holes in LQG: Theory and Phenomenology, Phys. Rev. D. 80, 064041 (2009).

B. Carr, L. Modesto & I. Premont-Schwarz, Generalized Uncertainty Principle and Self-Dual Black Holes, arXiv: 1107.0708 [gr-qc] (2011).

B. Carr, Black Holes, the Generalized Uncertainty Principle and Higher Dimensions, Phys. Lett. A 28, 134001 (2013).

B. Carr, L. Modesto & I. Premont-Schwarz, Loop Black Holes and the Black Hole Uncertainty Principle Correspondence (2013).

See talks by M. Isi, A. Kempf, R. Casadio, A.Bonanno, F.Scardigli

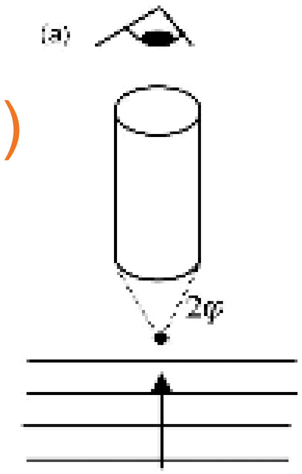
UNCERTAINTY PRINCIPLE

$h \to \hbar$

Photon of momentum p determines position to precision

$\Delta x > \lambda = h / p$ but imparts momentum $\Delta p \sim p$

$$\Rightarrow \Delta x > \frac{h}{(2)\Delta p} \quad \Rightarrow R_C = \frac{h}{Mc}$$ (Compton wavelength)



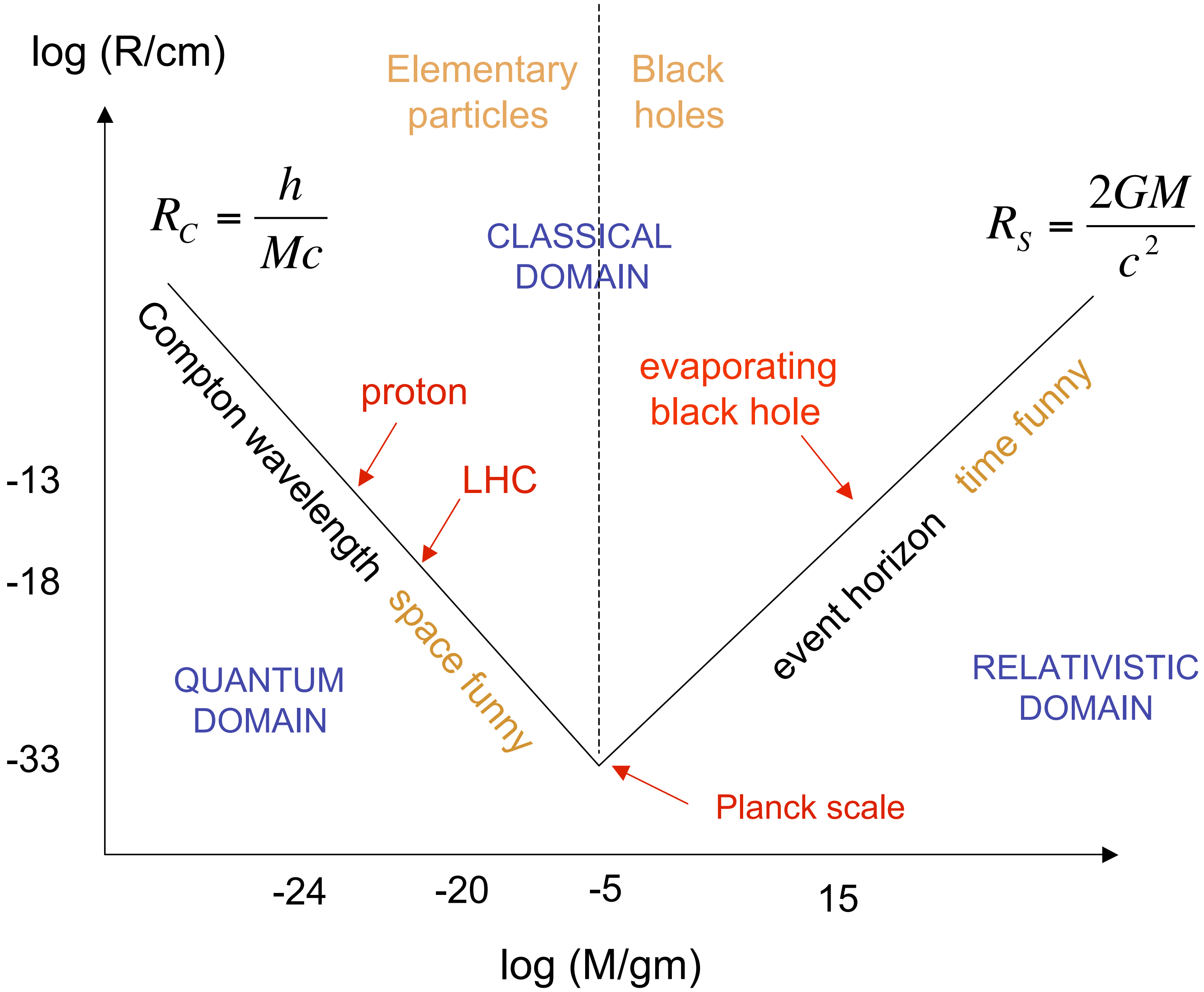
Particle production for $R < R_C \Rightarrow$ QFT

BLACK HOLE EVENT HORIZON

$R < R_S = 2GM/c^2$ (Schwarzschild radius)

Intersect at Planck scales

$$R_P = \sqrt{Gh/c^3} \sim 10^{-33}\,cm, \quad M_P = \sqrt{hc/G} \sim 10^{-5}\,g, \quad \rho_P = \frac{c^5}{h^2 G} \sim 10^{94}\,gcm^{-3}$$


log (R/cm)
Elementary particles
Black holes
$R_C = \frac{h}{Mc}$
$R_S = \frac{2GM}{c^2}$
CLASSICAL DOMAIN
Compton wavelength
proton
LHC
evaporating black hole
time funny
event horizon
space funny
QUANTUM DOMAIN
RELATIVISTIC DOMAIN
Planck scale
-13
-18
-33
-24
-20
-5
15
log (M/gm)

QUANTUM GRAVITY REGIMES

- $R < R_P$ (spacetime foam)

Energy density in gravitational field $\rho_g = \frac{(\nabla\phi)^2}{8\pi G}$

$$\Rightarrow \frac{R^3(\nabla\phi)^2}{8\pi G} > \frac{hc}{R} \Rightarrow \phi \approx R\nabla\phi > \frac{\sqrt{hcG}}{R}$$

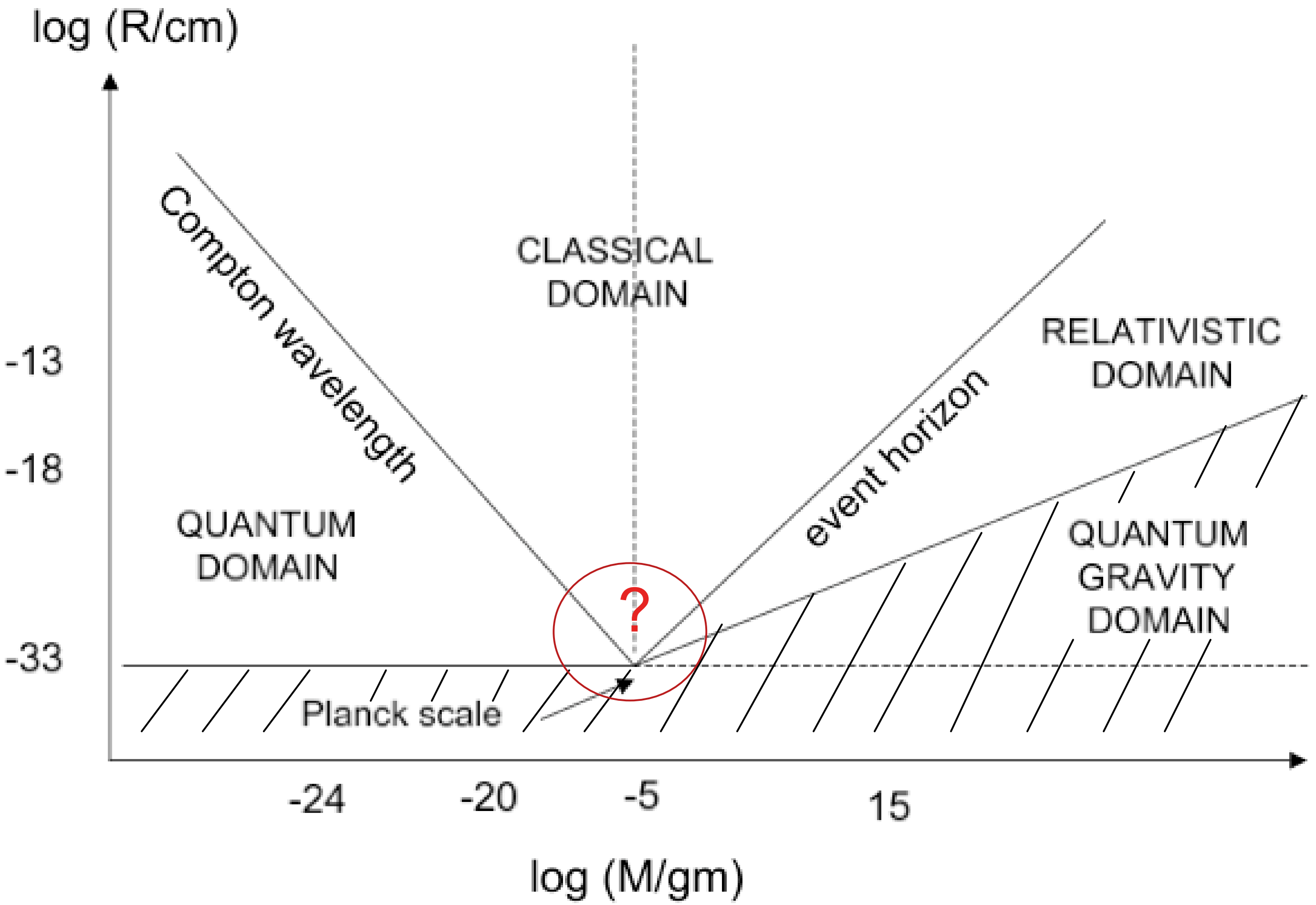
=> spacetime distortion $\frac{\Delta g}{g} \approx \frac{\phi}{c^2} > \frac{R_P}{R}$

- $\rho > \rho_P$ (above Planck density)

$$\Rightarrow R < \left(\frac{3M}{4\pi\rho_P}\right)^{1/3} \sim (M/M_P)^{1/3} R_P$$

Eg. black hole or big bang singularity

What happens to Compton and Schwarzschild lines near M_P…



…is important feature of theory of quantum gravity.

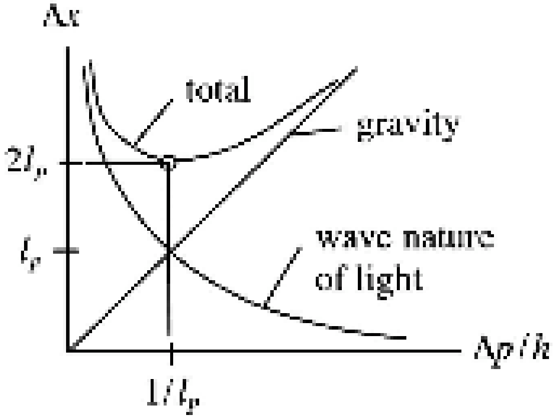
GENERALIZED UNCERTAINTY PRINCIPLE

Newtonian heuristic argument

Adler, Am. J. Phys. 78, 925 (2010)

Photon of frequency ω approaching to distance R induces
=> acceleration $a \sim Gh\omega/(cR)^2$ over time $t \sim R/c$
=> uncertainty in momentum $\Delta p \sim p \sim h\omega/c$ and in position

$$\Delta x_g > at^2 \sim Gh\omega/c^4 \sim G\Delta p/c^3 \sim R_P{}^2 \Delta p/h$$



Einstein heuristic argument

Metric fluctuation $\delta g_{\mu\nu}$ on scale R is

$$\frac{\delta g_{\mu\nu}}{R^2} = \left(\frac{8\pi G}{c^4}\right)\frac{pc}{R^3} \quad \Rightarrow \Delta x_g \sim R\delta g_{\mu\nu} \sim G\Delta p/c^3 \sim R_P{}^2 \Delta p/h$$

Both suggest $$\Delta x > \frac{h}{\Delta p} + R_P{}^2 \frac{\Delta p}{h} > 2R_P$$ (GUP)

GUP in string theory (Veneziano 1986, Witten 1996)

$$\Delta x > \frac{h}{\Delta p} + \alpha \frac{\Delta p}{h}$$

string tension $\alpha \sim (10 R_P)^2$

Minimal length considerations (Maggiore 1993)

Link with back holes (Scardigli 1999, Calmet et al. 2004)

Modifications of commutator [x,p] (Magueio & Smolin 2002)

Principle of relative locality (Doplicher 2010)

Polymer corrections in loop quantum gravity

$$\Delta x > \frac{h}{2\Delta p}\left[1 - \frac{\lambda^2}{2}(\Delta p)^2 + O(\lambda^4)\right]$$

(Hossain et al. 2010)

Experimental probes of GUP (Pikowski et al 2012))

GENERALIZED COMPTON WAVELENGTH

These arguments suggest $\Delta x > \frac{h}{\Delta p} + \alpha R_P{}^2 \frac{\Delta p}{h} = \frac{h}{\Delta p}\left[1 + \alpha\left(\frac{\Delta p}{cM_P}\right)^2\right]$

Putting $\Delta x \to R$ and $\Delta p \to cM$ gives

$$R > R_C{}' = \frac{h}{Mc} + \frac{\alpha GM}{c^2} \approx \begin{cases} \frac{h}{Mc}\left[1 + \alpha\left(\frac{M}{M_P}\right)^2\right] & (M << M_P) \\ \frac{\alpha GM}{c^2}\left[1 + \frac{1}{\alpha}\left(\frac{M_P}{M}\right)^2\right] & (M >> M_P) \end{cases}$$

α=2?

Compton scale becomes Schwarzschild scale for $M>>M_P$?

Compton irrelevant for $M>>M_P$ since $R_C<<R_P$?

Cannot localize on scale below R_S? (Dvali)

BH radiation => quantum boundary becomes BH boundary?

GENERALIZED EVENT HORIZON

Rewrite GUP in the form

$$R > R_C^{/} = \frac{\beta h}{Mc} + \frac{2GM}{c^2}$$

For M>>M_P we obtain generalized event horizon

$$R > R_S^{/} = \frac{2GM}{c^2}\left[1 + \beta\left(\frac{M_P}{M}\right)^2\right]$$

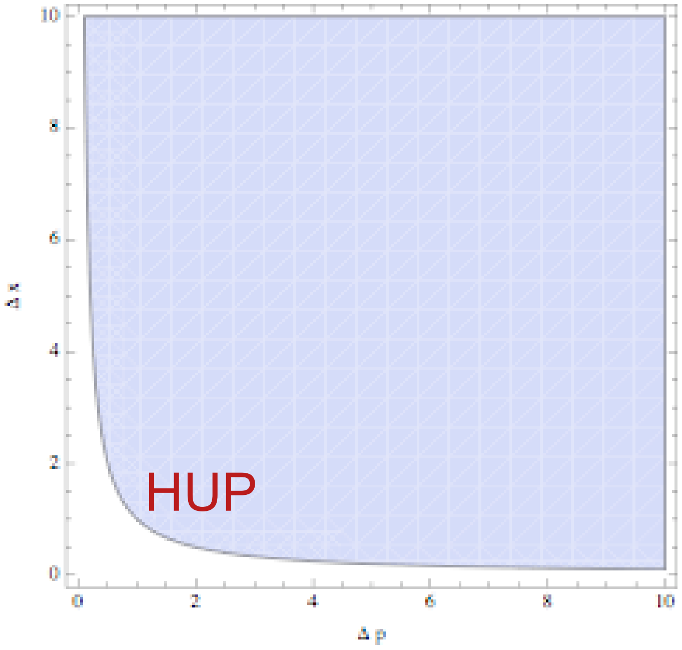
(small correction to Schwarzschild)

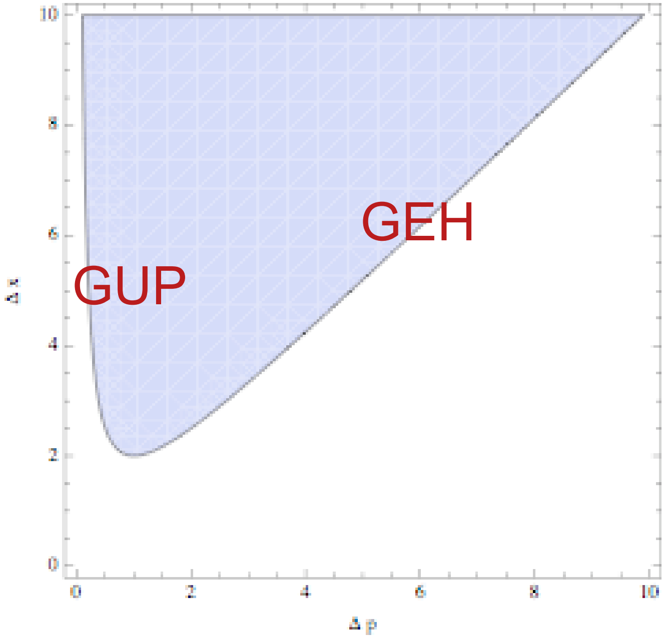
This becomes Compton wavelength for M<<M_P, suggesting

“Black Hole Uncertainty Principle Correspondence”

Generalize/unify Compton/Schwarzschild expressions such that

$$R_C^{/} \equiv R_S^{/} \approx \begin{matrix} h/(Mc) \quad (M << M_P) \\ 2GM/c^2 \quad (M >> M_P) \end{matrix}$$





Interesting alternative

$\alpha < 0$ => cusp rather than smooth minimum

$G \rightarrow 0 \Rightarrow$ asymptotic safety (Bonanno & Reuter 2006)

$\hbar \rightarrow 0 \Rightarrow$ classical theory (Scardigli et al. 2009)

DO GUP UNCERTAINTIES ADD LINEARLY?

Root-mean-square error would give

$$\Delta x > \sqrt{\left(\frac{h}{\Delta p}\right)^2 + \left(\alpha R_P{}^2 \frac{\Delta p}{h}\right)^2} \Rightarrow R_C{}' = \sqrt{\left(\frac{h}{Mc}\right)^2 + \left(\frac{\alpha GM}{c^2}\right)^2} \approx \frac{h}{Mc}\left[1 + \frac{\alpha^2}{2}\left(\frac{M}{M_P}\right)^4\right]$$

$$\Rightarrow R_S{}' = \sqrt{\left(\frac{2GM}{c^2}\right)^2 + \left(\frac{\beta h}{Mc}\right)^2} \approx \frac{2GM}{c^2}\left[1 + \frac{\beta^2}{8}\left(\frac{M_P}{M}\right)^4\right]$$

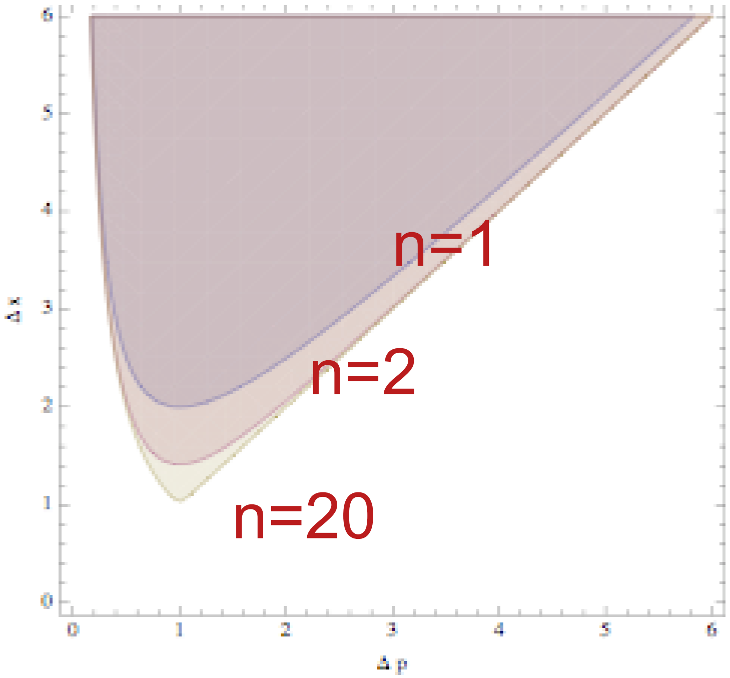
More generally

$$\Delta x > \left[\left(\frac{h}{\Delta p}\right)^n + \left(\alpha R_P{}^2 \frac{\Delta p}{h}\right)^n\right]^{1/n} \Rightarrow R_C{}' = \left[\left(\frac{h}{Mc}\right)^n + \left(\frac{\alpha GM}{c^2}\right)^n\right]^{1/n} \approx \frac{h}{Mc}\left[1 + \frac{\alpha^n}{n}\left(\frac{M}{M_P}\right)^{2n}\right]$$

$$\Rightarrow R_S{}' = \left[\left(\frac{2GM}{c^2}\right)^n + \left(\frac{\beta h}{Mc}\right)^n\right]^{1/n} \approx \frac{2GM}{c^2}\left[1 + \frac{\beta^n}{n}\left(\frac{M_P}{M}\right)^{2n}\right]$$

Minimum at $R_{\min} = 2^{1/n}\sqrt{2\beta}R_P, \quad M_{\min} = \sqrt{\beta/2}M_P$

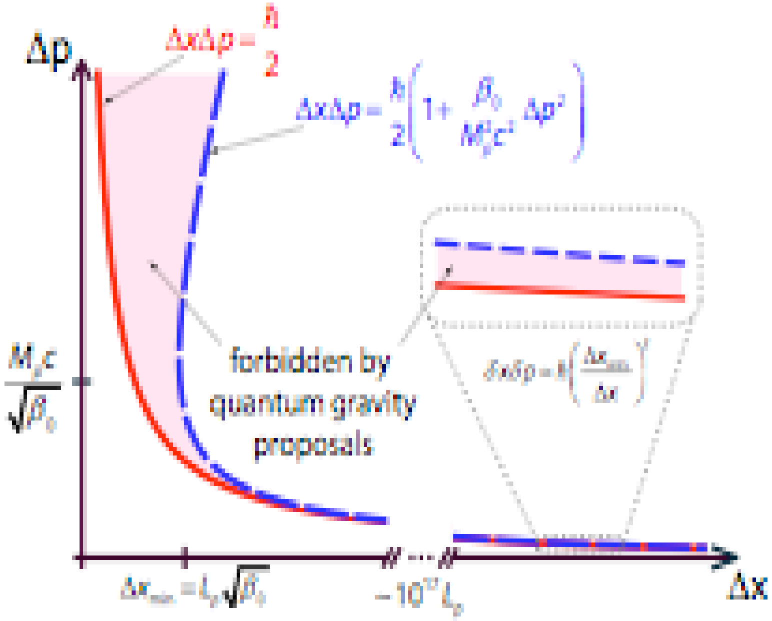
POSSIBLE FORMS OF BHUP CORRESPONDENCE



Could allow <u>any</u> form for $R_C{}'(M) \equiv R_S{}'(M)$ with usual asym' limits

- Does form of $\Delta x(\Delta p)$ determines $R_S(M)$?
- Do we need $M \leftrightarrow 1/M$ symmetry (t-duality)?
- Can there be black holes for $M<M_P$ since $R_S<R_C$?

Pikowski et al. Nature Phys. 8 39 (2012)
Probing Planck-scale physics with quantum optics



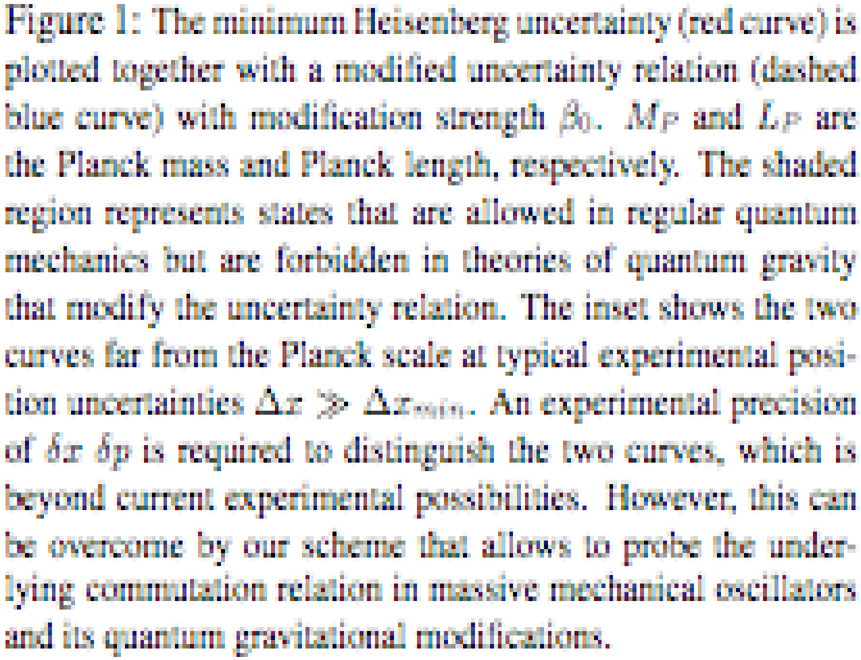
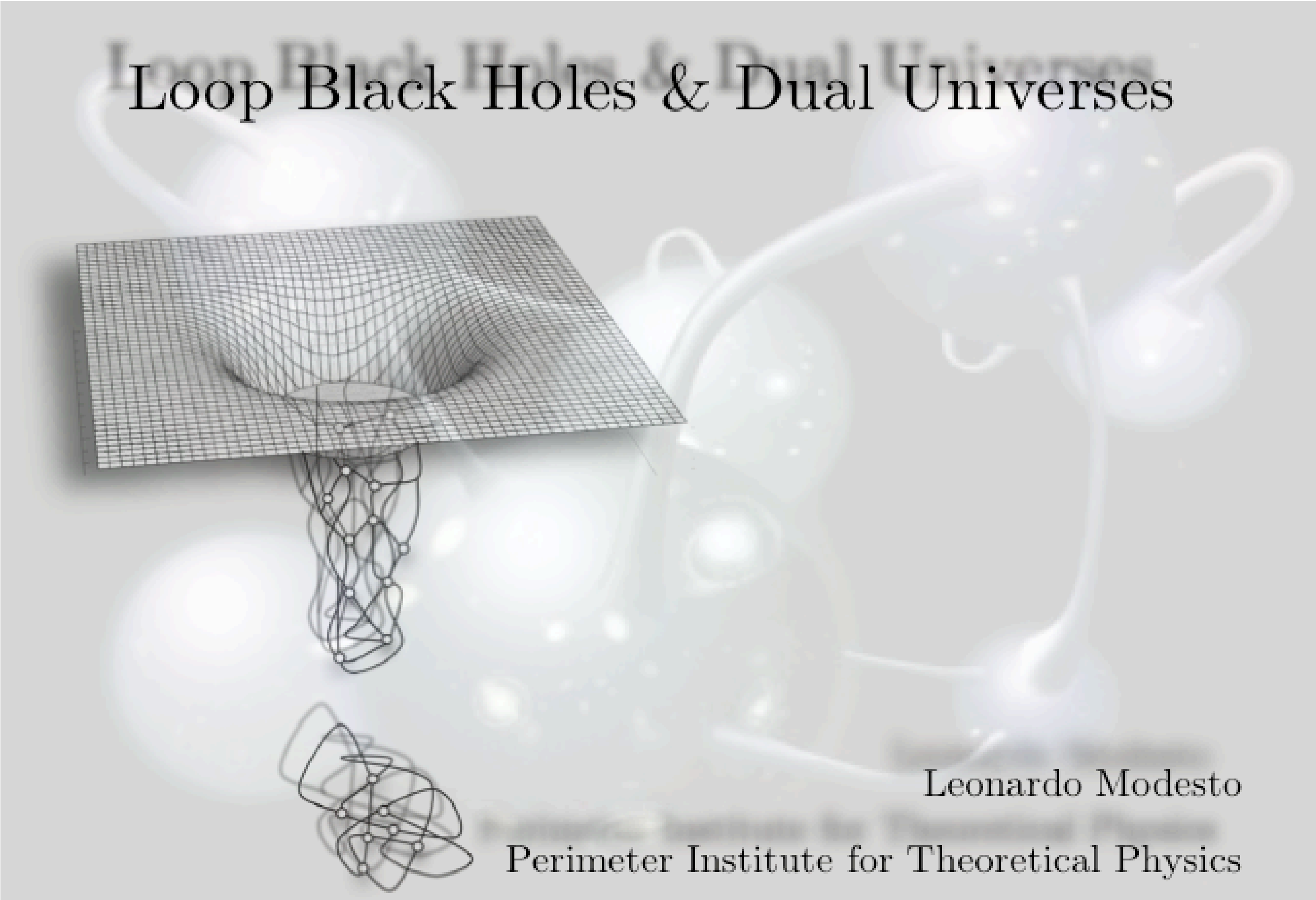


Figure 1: The minimum Heisenberg uncertainty (red curve) is plotted together with a modified uncertainty relation (dashed blue curve) with modification strength β_0. M_P and L_P are the Planck mass and Planck length, respectively. The shaded region represents states that are allowed in regular quantum mechanics but are forbidden in theories of quantum gravity that modify the uncertainty relation. The inset shows the two curves far from the Planck scale at typical experimental position uncertainties $\Delta x \gg \Delta x_{min}$. An experimental precision of $\delta x\ \delta p$ is required to distinguish the two curves, which is beyond current experimental possibilities. However, this can be overcome by our scheme that allows to probe the underlying commutation relation in massive mechanical oscillators and its quantum gravitational modifications.

"Generalized Uncertainty and Self-dual Black Holes"

Carr, Modesto & Premont-Schwarz, arXiv: 1107.0708



Loop Black Holes & Dual Universes

Leonardo Modesto

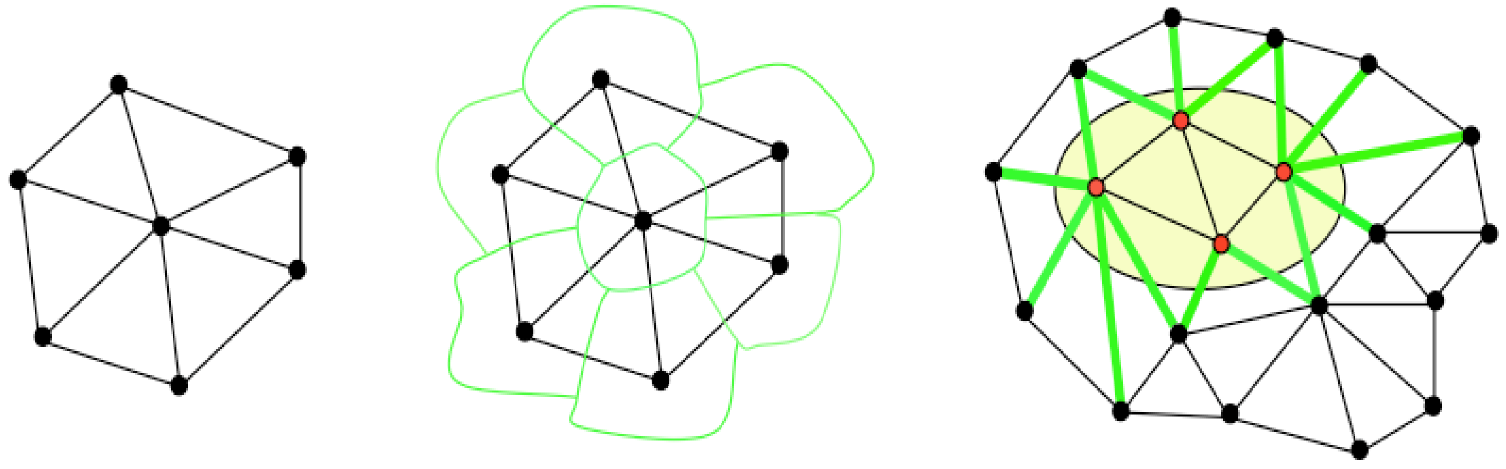
Perimeter Institute for Theoretical Physics

Area Spectrum

$$\widehat{A}_S|\psi >= 8\pi\ell_p^2\gamma\sum_p\sqrt{j_p(j_p+1)}|\psi >$$

Volume Spectrum

$$V(R)|\gamma, j_l, i_1, \ldots, i_N\rangle = \sqrt{(16\pi\hbar G)^3}\ V_{i_n}^{i'_n}|\gamma, j_l, i_1, \ldots, i'_n, \ldots i_N\rangle$$



LOOP BLACK HOLES

Metric $ds^2 = -G(r)dt^2 + \frac{dr^2}{F(r)} + H(r)d\Omega^{(2)},$

$$G(r) = \frac{(r-r_+)(r-r_-)(r+r_*)^2}{r^4+a_0^2} ,$$

$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_0^2)} ,$$

$$H(r) = r^2 + \frac{a_0^2}{r^2} .$$

where

$$r_+ = 2Gm/c^2$$

$$r_- = 2GmP^2/c^2$$

$$r_* \equiv \sqrt{r_+ r_-}$$

and $a_0 = A_{\rm min}/8\pi = \sqrt{3}\,\gamma\zeta R_P^2/2$

Immirzi parameter

1<ζ<4

Polymeric function $P \equiv \frac{\sqrt{1+\epsilon^2}-1}{\sqrt{1+\epsilon^2}+1}$ ~ $\epsilon^2/4 \ll 1$

At large r

$$G(r) \to 1 - \frac{2M}{r}(1-\epsilon^2) ,$$

$$F(r) \to 1 - \frac{2M}{r} ,$$

$$H(r) \to r^2.$$

implies $M = m(1+P)^2$ (ADM mass)

Physical radial coordinate $R = \sqrt{H(r)} = \sqrt{r^2 + \frac{a_o^{\,2}}{r^2}}$

$$\Rightarrow R_S = \sqrt{\left(\frac{2Gm}{c^2}\right)^2 + \left(\frac{c^2 a_o}{2Gm}\right)^2} \approx \begin{array}{ll} \dfrac{2Gm}{c^2} & (m > M_P) \\ \dfrac{\sqrt{3}\gamma\beta}{4}\dfrac{h}{mc} & (m < M_P) \end{array}$$

This removes the singularity, permits existence of black holes with m << M_P, and corresponds to the quadratic GEH.

Introduce dual coordinates

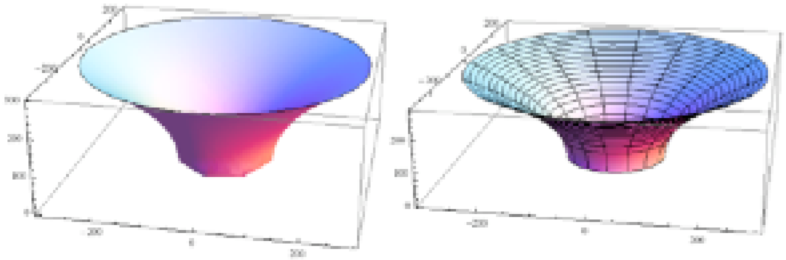
$$\bar{r} = a_o / r \Rightarrow R = \sqrt{r^2 + \bar{r}^2} \qquad \bar{t} = t r_*^{\ 2} / a_o$$

Metric has self-duality with dual radius $\bar{r} = r = \sqrt{a_o}$

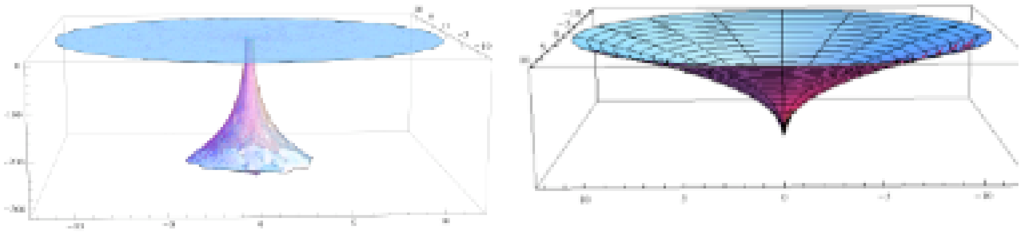
=> another asymptotic infinity (r=0) with BH mass $M_P{}^2/m$

However, sub-Planckian black hole hidden within wormhole.

$m > \sqrt{a_0}/2$

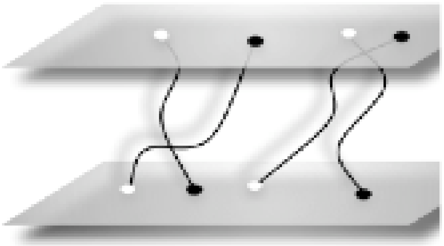


$m < \sqrt{a_0}/2$

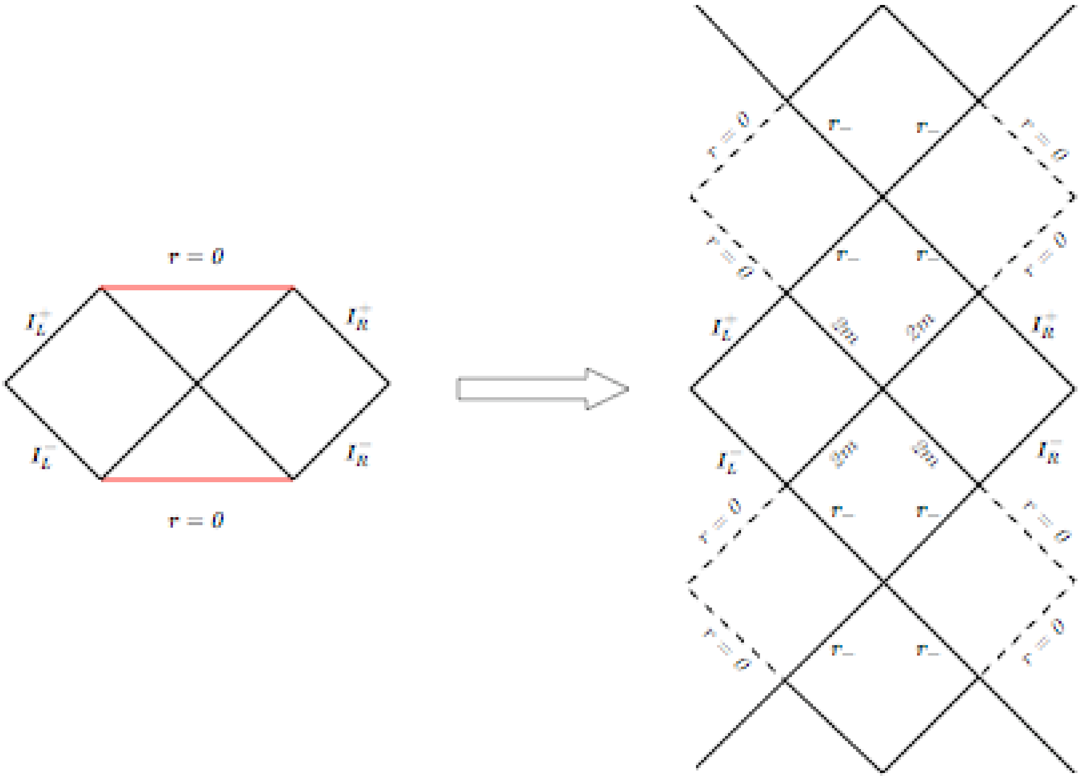


Hidden Horizon *Schwarzschild*

Observer	Dual – Observer
$m_{\text{ADM}} = m(1+P)^2$	$m^d_{\text{ADM}} = \frac{a_0(1+P)^2}{4mG^2P^2}$
$\lambda_c = \frac{\hbar}{2m(1+P)^2}$	$\lambda^d_c = \frac{2\hbar mG^2P^2}{a_0(1+P)^2}$



Penrose diagrams for Schwarzschild and LBH metric



cf. Reissner-Nordstrom

L. Modesto & I. Premont-Schwarz, Phys. Rev. D. 80, 064041 (2009)

“Particles – Black Holes” Duality

If $m \ll m_P \to r_+ \ll \sqrt{a_0}$ (throat radius).

A particle with $\lambda_c \approx \dfrac{\hbar}{2m} \gg l_P$ could have sufficient space in $r < r_+$,

Because :

$$D = 2\left[(2G_N m)^2 + \frac{a_0^2}{(2G_N m)^2}\right]^{1/2} > \lambda_c$$

GUP AND BLACK HOLE THERMODYNAMICS

Heuristic argument

$$kT_{BH} = \eta c\Delta p = \frac{\eta hc}{\Delta x} = \frac{\eta hc^3}{2GM} \sim \frac{M_P{}^2}{M} \quad (M >> M_P) \qquad \eta=1/(4\pi)$$

Putting $\Delta p \sim T$ and $\Delta x \sim GM/c^2$ in linear GUP (Adler & Chen)

$$\Rightarrow \frac{2GM}{c^2} = \frac{\eta hc}{kT} + \frac{\alpha R_P{}^2 kT}{h\eta c}$$

$$\Rightarrow T_{BH} = \frac{\eta Mc^2}{\alpha k}\left(1 - \sqrt{1 - \frac{\alpha M_P{}^2}{M^2}}\right) \approx \frac{hc^3}{8\pi GkM}\left[1 + \frac{\alpha M_P{}^2}{4M^2}\right] \quad (M >> M_P)$$

Complex for $M < \sqrt{\alpha} M_P$ => Planck mass relics.

Quadratic GUP

$$\Rightarrow \frac{2GM}{c^2} = \left[\left(\frac{\eta hc}{kT}\right)^2 \mp \left(\frac{\alpha R_P{}^2 kT}{h\eta c}\right)^2\right]^{1/2}$$

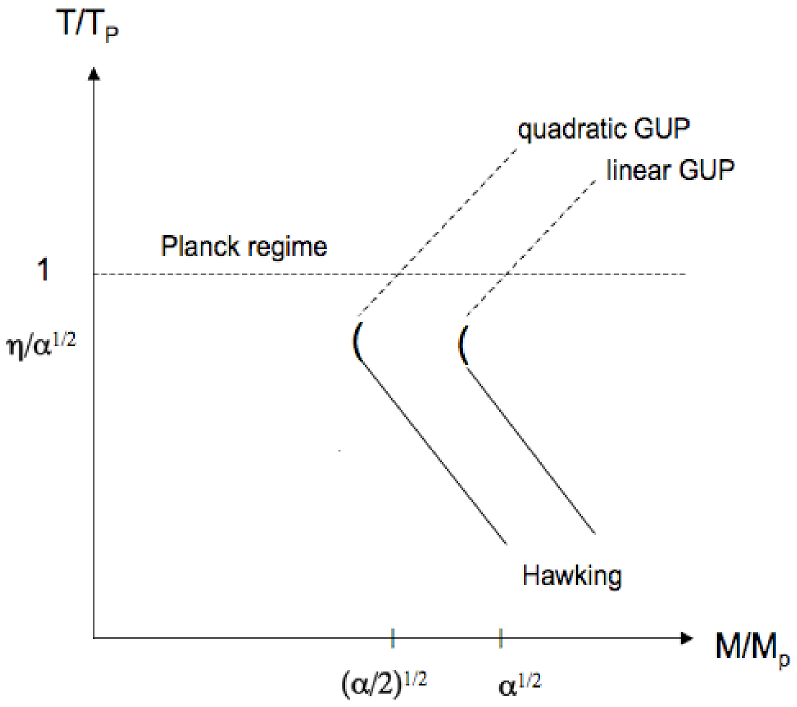
$$\Rightarrow T_{BH} = \frac{\sqrt{2}\eta Mc^2}{\alpha k}\left(1-\sqrt{1-\frac{\alpha^2 M_P{}^4}{4M^4}}\right)^{1/2} \approx \frac{hc^3}{8\pi GkM}\left[1+\frac{\alpha^2 M_P{}^4}{32M^4}\right](M >> M_P)$$

Complex for $M < \sqrt{\alpha/2} M_P$

=> smaller relics

$$T_{\max} = \eta M_P c^2/\sqrt{\alpha}$$

Minus sign gives $T>T_P$
which may be unphysical



Quadratic GUP + GEH

Regard α are β as independent

$$\Rightarrow \left[\left(\frac{2GM}{c^2}\right)^2 + \left(\frac{h\beta}{Mc}\right)^2\right]^{1/2} = \left[\left(\frac{\eta hc}{kT}\right)^2 + \left(\frac{\alpha R_P{}^2 kT}{h\eta c}\right)^2\right]^{1/2}$$

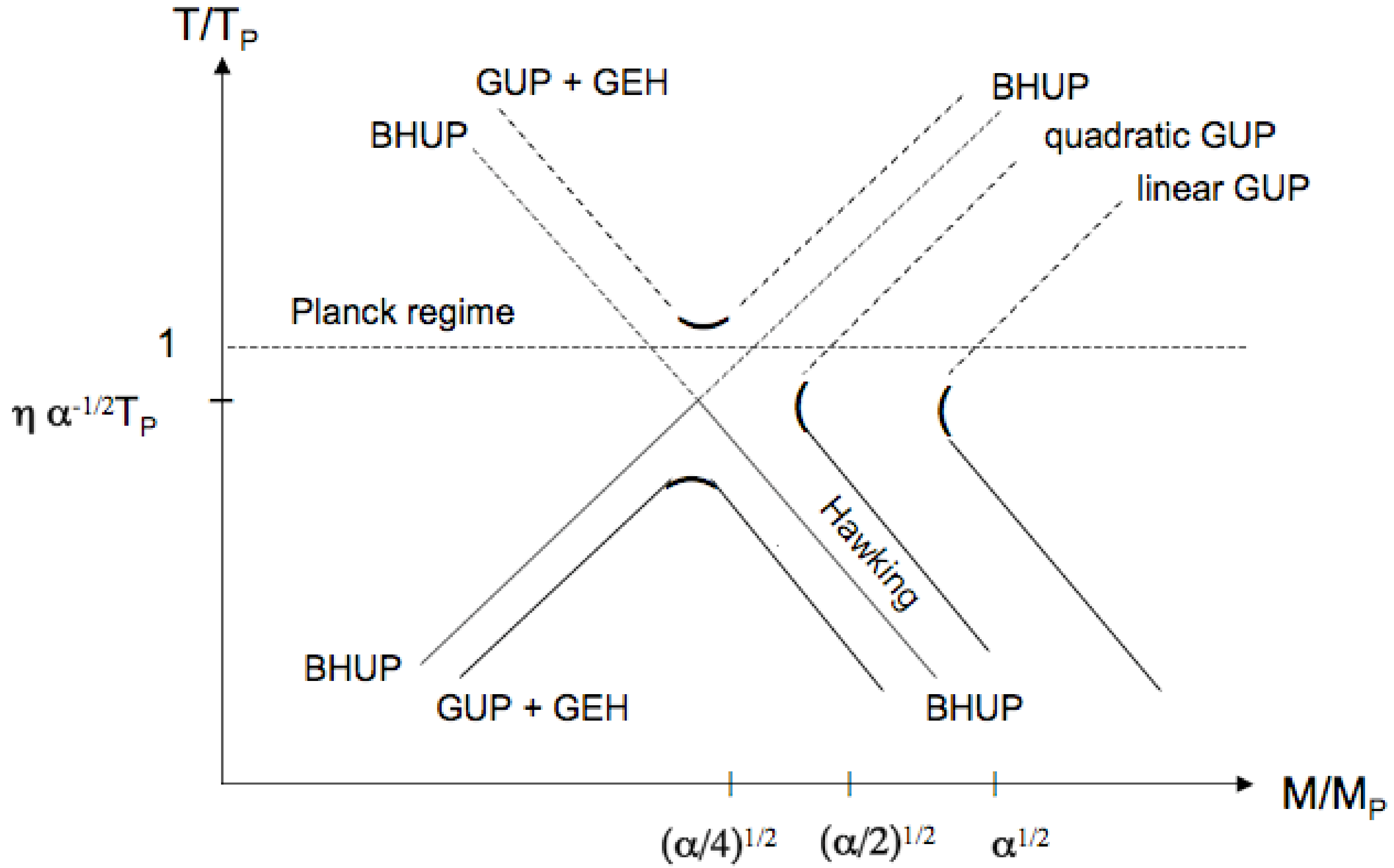
$$\Rightarrow T_{BH} = \frac{\sqrt{2}\eta Mc^2}{\alpha k}\left(1+\frac{\beta^2 M_P{}^4}{4M_P{}^4} - \sqrt{1+\frac{(2\beta^2-\alpha^2)M_P{}^4}{4M^4}+\frac{\beta^4 M_P{}^8}{16M_P{}^8}}\right)^{1/2}$$

Real for all M if $\alpha < 2\beta$

$$\Rightarrow T_{BH} \approx \begin{array}{ll} \dfrac{\eta hc^3}{2GkM}\left[1+\left(\dfrac{\alpha^2-4\beta^2}{32}\right)\dfrac{M_P{}^4}{M^4}\right] & (M \gg M_P) \\ \dfrac{\eta Mc^2}{\beta k}\left[1+\left(\dfrac{\alpha^2-4\beta^2}{2\beta^4}\right)\dfrac{M^4}{M_P{}^4}\right] & (M \ll M_P) \end{array}$$

BHUP => $\alpha = 2\beta$ $\Rightarrow T_{BH} = \dfrac{h\eta c^3}{2kGM}$ or $T_{BH} = \dfrac{\eta}{\beta}Mc^2$ Exact!

T peaks at $M = \sqrt{\beta/2}\,M_P$ with $T_{\max} = \eta\sqrt{\beta/2}\,T_P$


T/T$_P$
GUP + GEH
BHUP
BHUP
quadratic GUP
linear GUP
Planck regime
1
η α$^{-1/2}$T$_P$
Hawking
BHUP
GUP + GEH
BHUP
M/M$_P$
(α/4)$^{1/2}$
(α/2)$^{1/2}$
α$^{1/2}$

SURFACE GRAVITY ARGUMENT

$$T \propto \frac{GM}{R_S{}^2} \propto \left[\frac{M^{3n/2}}{M^{2n} + (\beta/2)^n M_P{}^{2n}}\right]^{2/n} \propto \begin{array}{l} M^{-1} (M >> M_P) \\ M^3 (M << M_P) \end{array}$$

=> different prediction in sub-Planckian range!

Both arguments predict deviations from Hawking formula and imply that T never exceeds T_P but which one is correct?

Black hole entropy (area)

$$S \propto \int \frac{dM}{T} \propto M^2 \overset{+}{-} \frac{\beta^2 M_P{}^4}{4M^2} \qquad \text{(n=2)}$$

plus logarithmic term (n=1)

RESOLUTION: THERE ARE TWO ASYMPTOTIC SPACES

Emission looks different in two spaces!

GUP argument: does Δx mean Δr or ΔR?

$$\frac{\Delta R}{\Delta r} \approx \begin{matrix} 1 & (r >> r_P) \\ (r/r_P)^{-2} & (r << r_P) \end{matrix} \Rightarrow \frac{\Delta R_{BH}}{\Delta r_{BH}} \approx \begin{matrix} 1 & (M >> M_P) \\ (M/M_P)^{-2} & (M << M_P) \end{matrix}$$

So different asymptotic spaces in $M>M_P$ and $M<M_P$ cases.

Need asymptotic space on same side of throat as horizon
=> our space for $M>M_P$, other space for $M<M_P$.

More precise surface gravity argument

$$\kappa^2 = -g^{\mu\nu} g_{\mu\nu} \nabla_\mu \chi^\rho \nabla_\nu \chi^\sigma$$

T = hκ/(2πkc)

$$r \to \infty \Rightarrow g_{00} \to 1 \Rightarrow \chi^\mu = (1,0,0,0) \Rightarrow$$

$$\kappa_- = \frac{4G^3 m^3 c^4 P^4}{16G^4 m^4 P^8 + a_o c^8} \qquad \kappa_+ = \frac{4G^3 m^3 c^4}{16G^4 m^4 + a_o c^8}$$

$$r \to 0 \Rightarrow g_{00} \to r_*^4 / a_0^2 = P^4 (m / M_P)^4 \Rightarrow \chi^\mu = P^{-2} (m / M_P)^{-2} (1,0,0,0) \Rightarrow$$

$$T_\infty = T_0 P^2 (m / M_P)^2$$

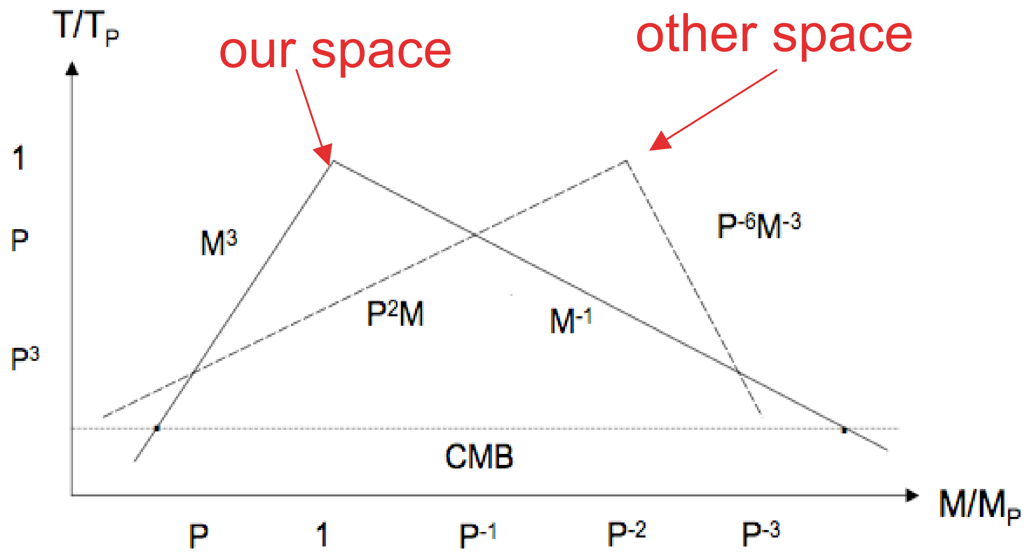
This reconciles to the two arguments!

Three mass regimes

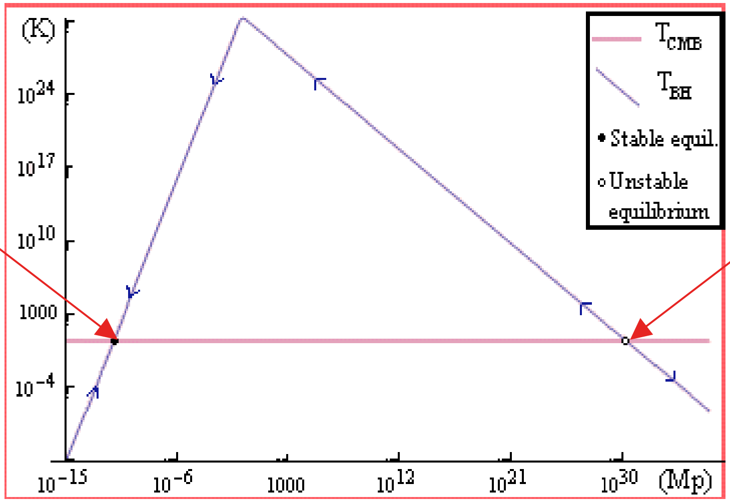
$$M > P^{-2}M_P \Rightarrow r_P < r_- < r_+ \Rightarrow T_\infty \propto M^{-1}, T_0 \propto P^{-6}M^{-3}$$

$$M < M_P \Rightarrow r_- < r_+ < r_P \Rightarrow T_\infty \propto M^3, T_0 \propto P^2 M$$

$$M_P < M < P^{-2}M_P \Rightarrow r_- < r_P < r_+ \Rightarrow T_\infty \propto M^{-1}, T_0 \propto P^2 M$$



CAN SUB-PLANCKIAN RELICS PROVIDE DARK MATTER?



stable

unstable

$T \propto M^3 \Rightarrow$ cooler than CMB for $M < \left(\frac{T_{CMB}}{T_P}\right)^{1/3} M_P \sim 10^{-16} g$

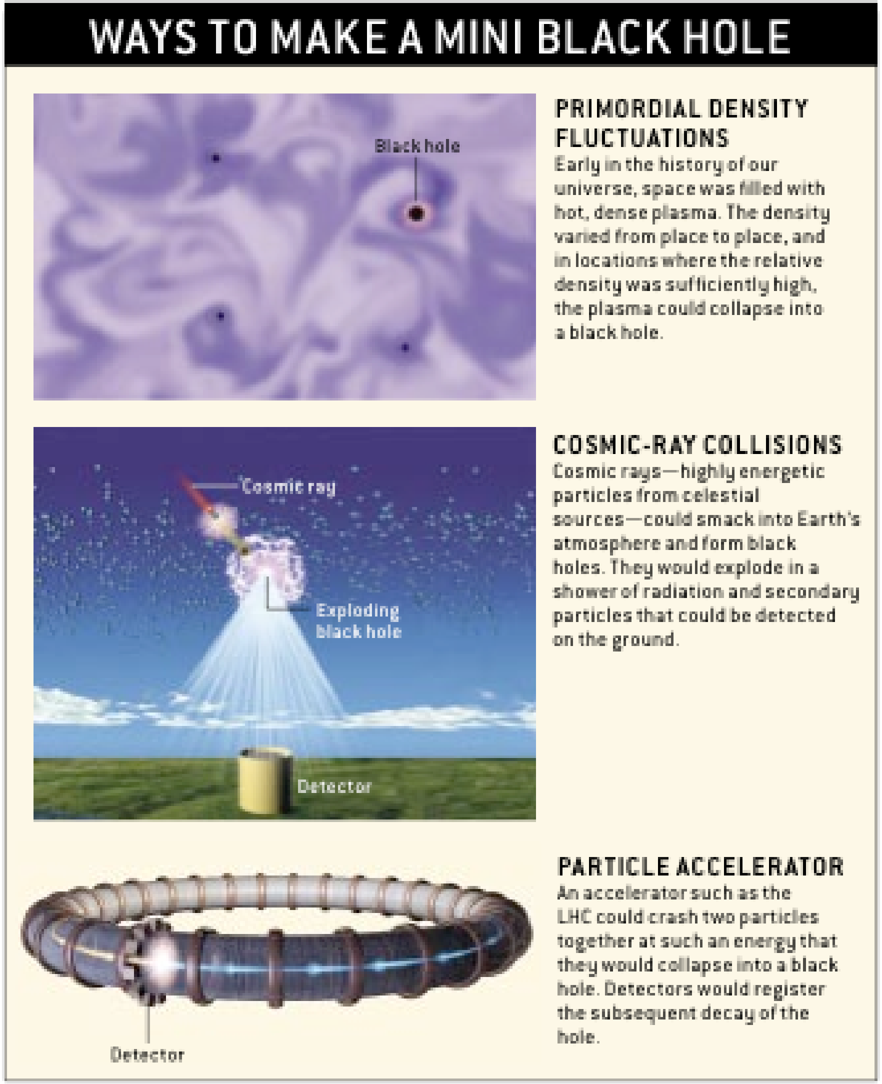
$$\frac{dM}{dt} \propto R^{-2} T^4 \propto M^{10} \Rightarrow M(t) \propto t^{-1/9}$$

=> never evaporate but current mass $M \sim \left(\frac{t_P}{t_0}\right)^{1/9} M_P \sim 10^{-12} g$

=> current temperature $T \sim \left(\frac{t_P}{t_o}\right)^{1/3} T_P \sim 10^{12} K$

=> same observational effects as PBHs with M~10^{15}g!

BLACK HOLES AS A PROBE OF HIGHER DIMENSIONS



WAYS TO MAKE A MINI BLACK HOLE

PRIMORDIAL DENSITY FLUCTUATIONS

Early in the history of our universe, space was filled with hot, dense plasma. The density varied from place to place, and in locations where the relative density was sufficiently high, the plasma could collapse into a black hole.

COSMIC-RAY COLLISIONS

Cosmic rays—highly energetic particles from celestial sources—could smack into Earth's atmosphere and form black holes. They would explode in a shower of radiation and secondary particles that could be detected on the ground.

PARTICLE ACCELERATOR

An accelerator such as the LHC could crash two particles together at such an energy that they would collapse into a black hole. Detectors would register the subsequent decay of the hole.

BLACK HOLES AND EXTRA DIMENSION

Higher dimensions => $M_D^{n+2}V_n \sim M_p^2$

V_n is volume of compactified or warped space

Standard model => $V_n \sim M_P^{-n}$, $M_D \sim M_p$,
Large extra dimensions => $V_n >> M_P^{-n}$, $M_D << M_p$

TeV quantum gravity?

Forming black holes by collisions

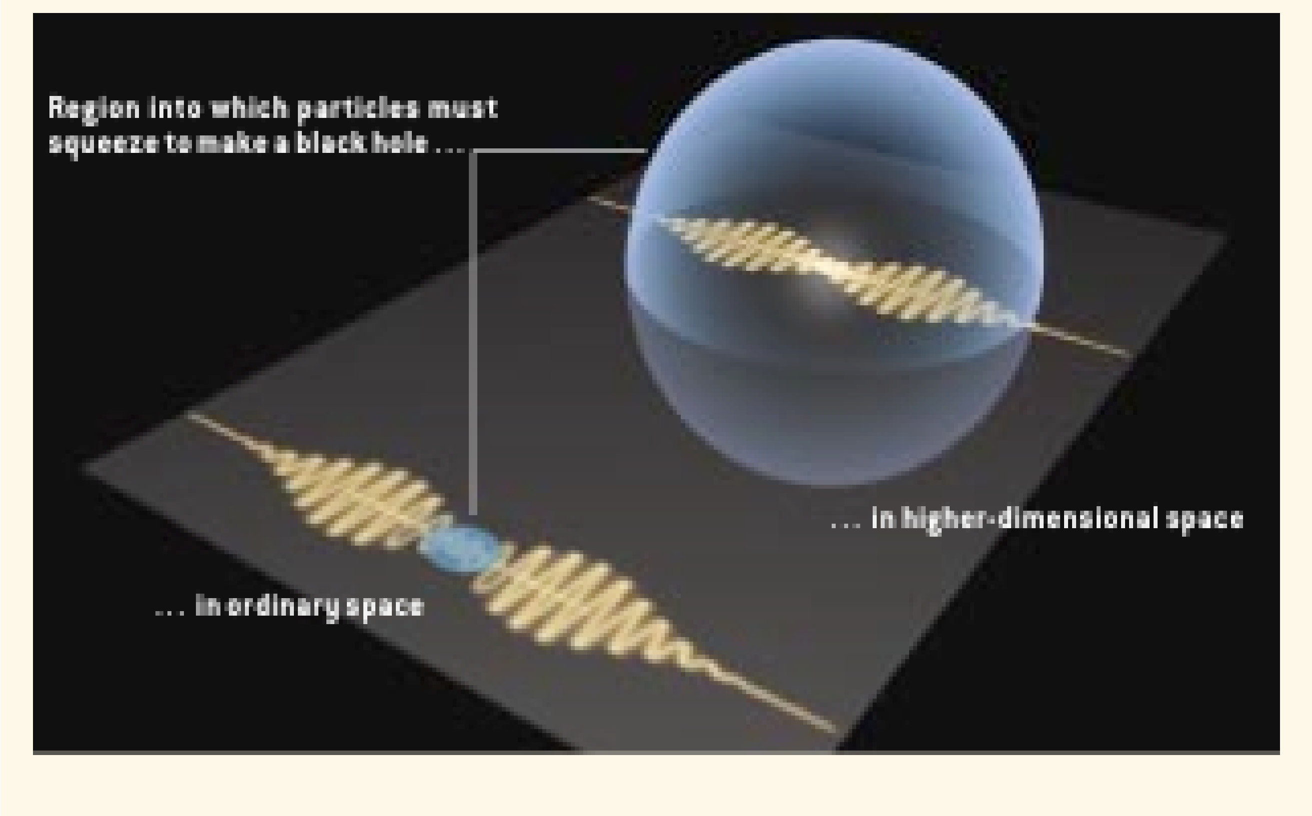
centre of mass energy

Cross-section $\sigma(ij \rightarrow BH) = \pi r_S^2 \Theta(E - M_{BH}^{min})$

Schwarzschild radius $r_S = M_P^{-1}(M_{BH}/M_P)^{1/(1+n)}$

Temperature $T_{BH} = (n+1)/r_S$ < 4D case

Lifetime $\tau_{BH} = M_P^{-1}(M_{BH}/M_P)^{(n+3)/(1+n)}$ > 4D case



BLACK HOLES AND HIGHER DIMENSIONS

Assume D=3+n dimensions for $R<R_C$

Gauss law

$$F_{grav} = \frac{G_D m_1 m_2}{R^{2+n}} \qquad (R<R_C)$$

$$F_{grav} = \frac{G\, m_1 m_2}{R^2} \qquad (R>R_C)$$

$$G = \frac{G_D}{R_C{}^2}$$

3D black hole smaller than R_C for $M < M_C = \frac{c^2 R_C}{2G}$

Black hole condition

$$R < \begin{array}{ll} R_C (M/M_C)^{1/(n+1)} & (M < M_C) \\ R_C (M/M_C) & (M > M_C) \end{array}$$

Intersects Compton boundary at higher dimensional Planck scale

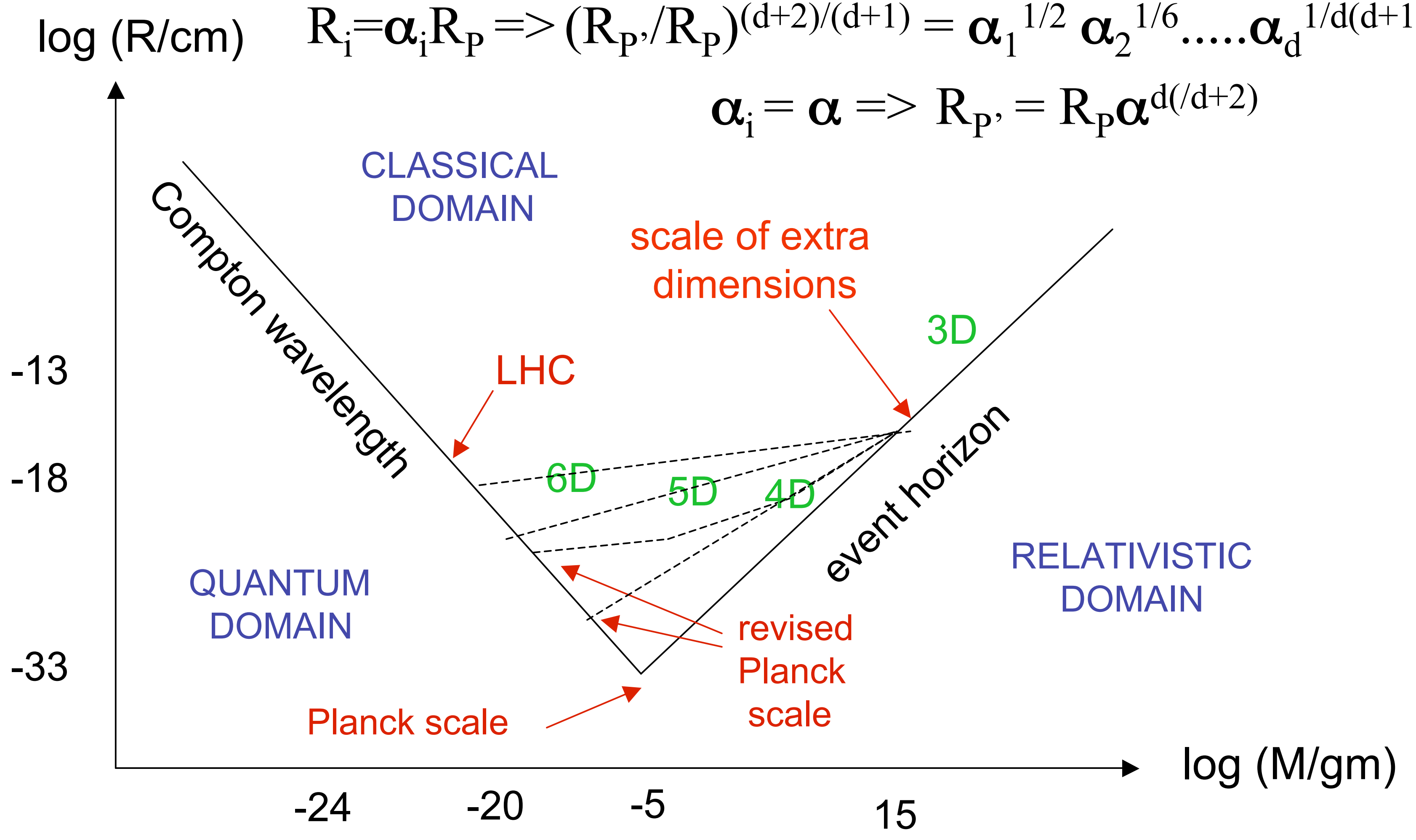
$$M_D = M_P \left(\frac{R_P}{R_C}\right)^{n/(n+2)}, \qquad R_D = R_P \left(\frac{R_C}{R_P}\right)^{n/(n+2)}$$

BH radius $R_S = M_P^{-1}(M_{BH}/M_P)^{1/(1+n)} \Rightarrow R_C{}' \approx \frac{h}{Mc}\left[1+\left(\frac{M}{M_P}\right)^{(n+2)/(n+1)}\right]$

Hierarchy of compactified dimensions

$R_i = \alpha_i R_P \Rightarrow (R_{P'}/R_P)^{(d+2)/(d+1)} = \alpha_1{}^{1/2}\, \alpha_2{}^{1/6} \ldots\ldots \alpha_d{}^{1/d(d+1)}$

$\alpha_i = \alpha \Rightarrow R_{P'} = R_P \alpha^{d(/d+2)}$

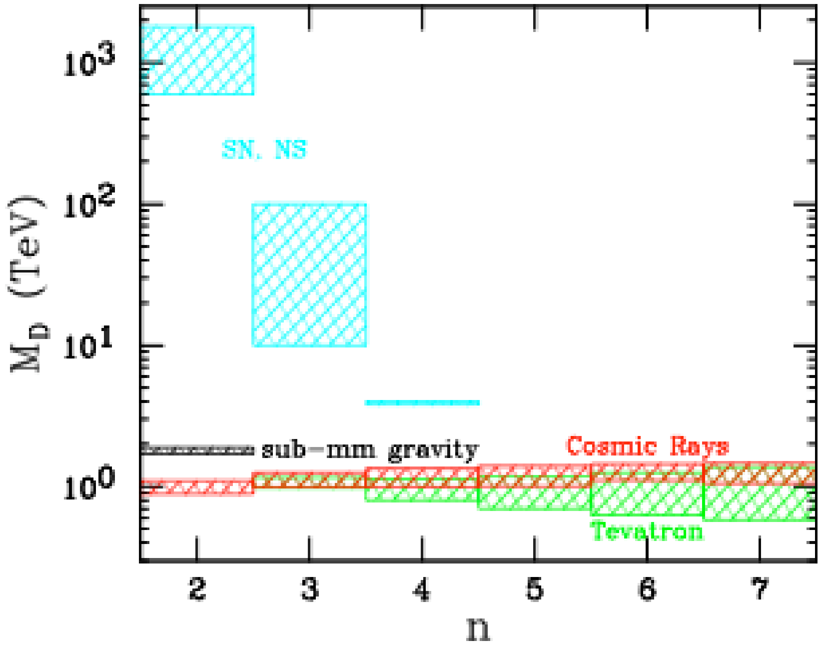


cf. 2D black holes (Mureika & Nicolini 2013)

DETECTABLE AT LHC?

$$M_D \sim TeV \Rightarrow R_C \sim 10^{(32/n)-17}\,cm \sim$$

10^{16} cm	(n=1)	excluded
10^{-1} cm	(n=2)	dark energy?
10^{-6} cm	(n=3)	
10^{-13} cm	(n=7)	

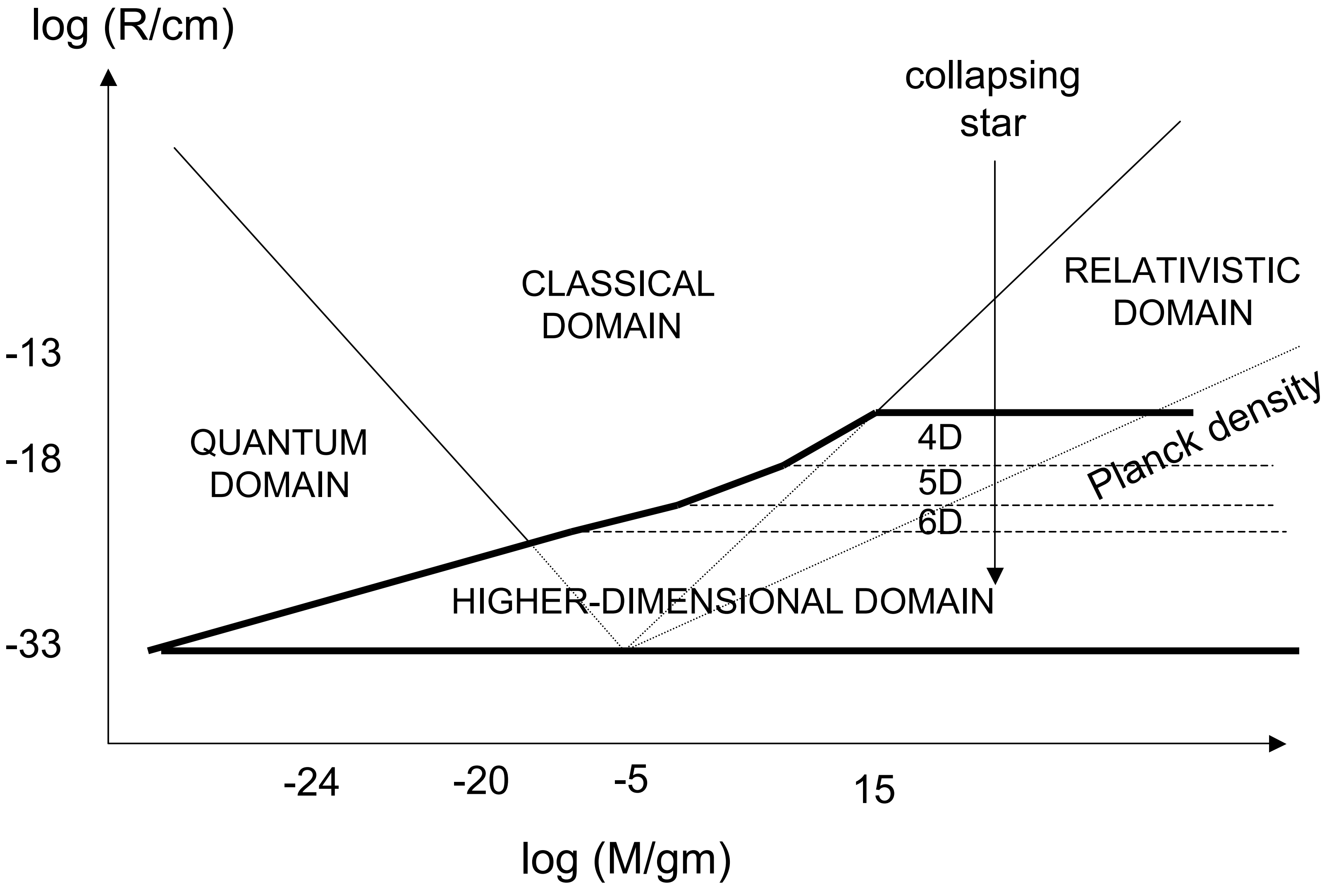


Dark energy

$$\rho_U \sim 10^{-120}\rho_U \sim 10^{-29}\,gcm^{-3}$$

=> intersects Compton line at

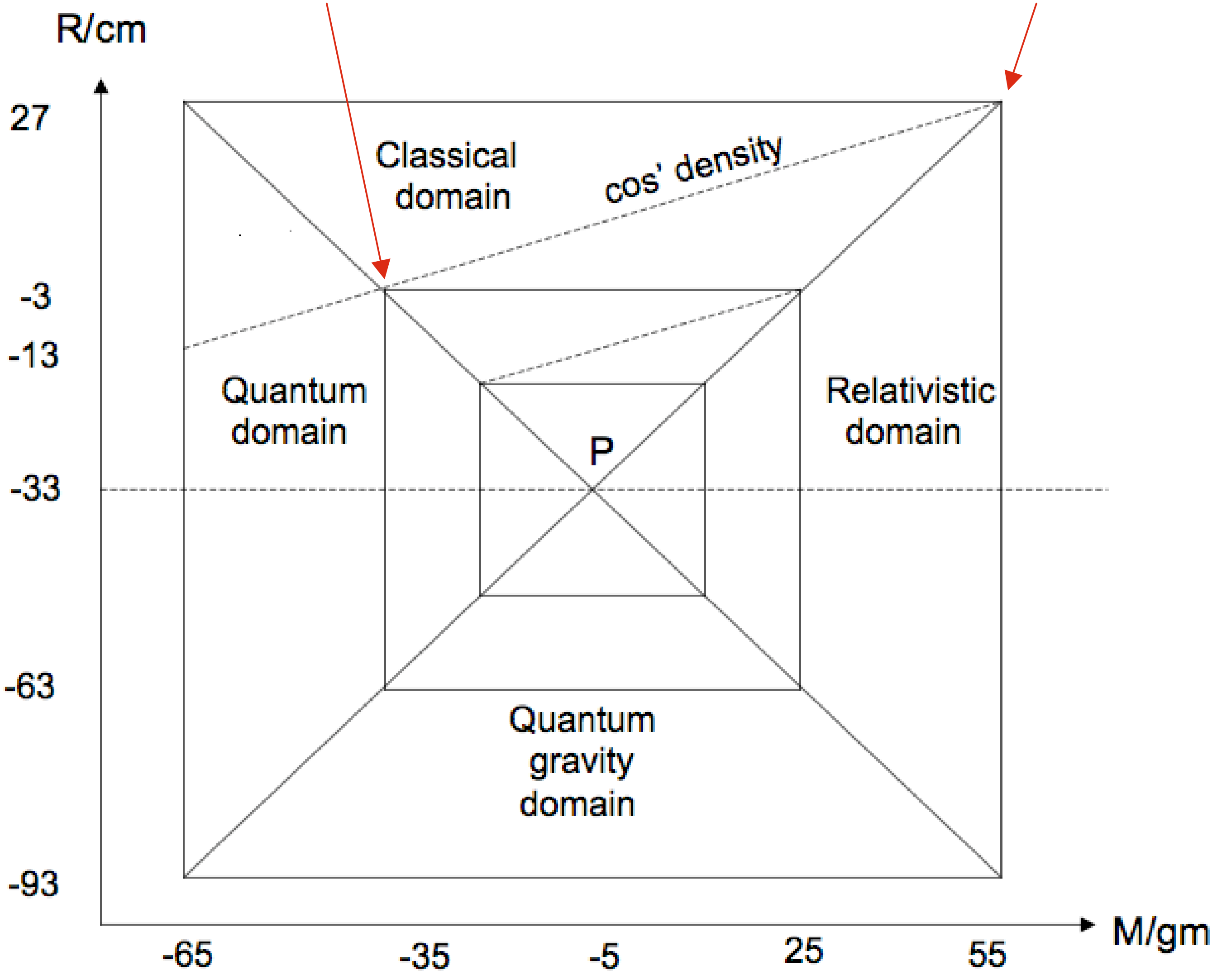
$$M \sim M_P\left(\frac{t_P}{t_U}\right)^{1/2} \sim 10^{-30}M_P \sim 10^{-35}g, \quad R \sim \sqrt{R_U R_U} \sim 100\mu m$$


log (R/cm)
collapsing
star
CLASSICAL
DOMAIN
RELATIVISTIC
DOMAIN
-13
Planck density
QUANTUM
DOMAIN
-18
4D
5D
6D
HIGHER-DIMENSIONAL DOMAIN
-33
-24
-20
-5
15
log (M/gm)

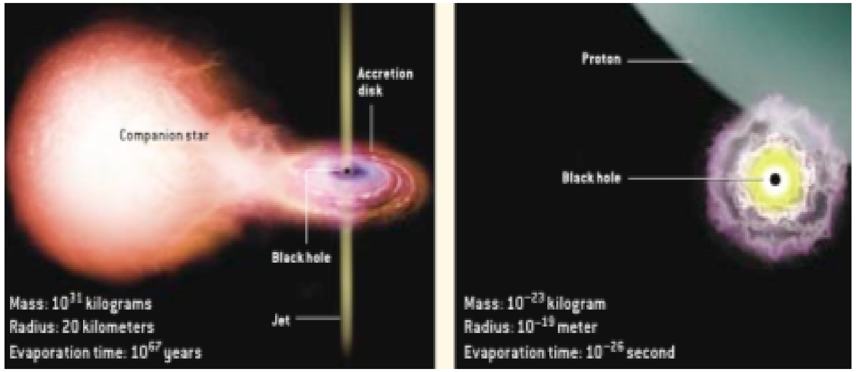
DARK ENERGY

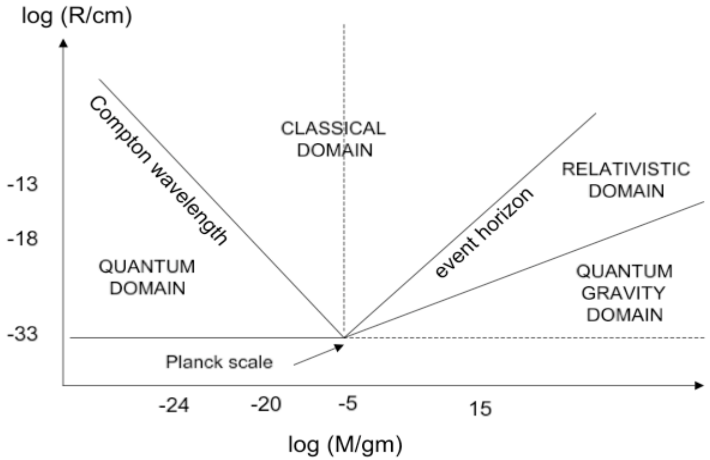
$M \sim 10^{-35}$g, $R \sim 10^{-2}$cm

$M \sim 10^{55}$g, $R \sim 10^{27}$cm



CONCLUSIONS





- Both black holes and Generalized Uncertainty Principle provide important link between micro and macro physics.

- They are themselves linked and black holes with sub-Planckian mass may play a role in quantum gravity.