Transverse mass kink

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Ref) arXiv:0709.0288, arXiv:0711.4526

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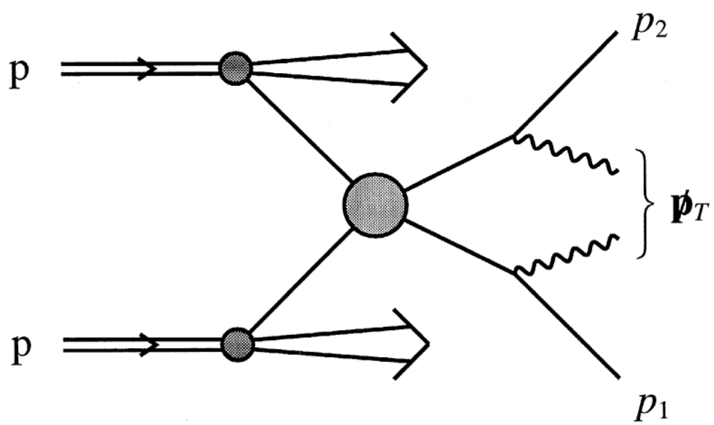
Cambridge m_{T2} variable

(Stransverse Mass)

● Cambridge m_{T2} (Lester and Summers, 1999)



Massive particles pair produced

Each decays to one visible and one invisible particle.

For example,

$$pp \to X + \tilde{l}_R^+ \tilde{l}_R^- \to X + l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$$

For the decay, $\tilde{l} \to l \tilde{\chi}$

$$m_{\tilde{l}}^2 \geq m_T^2(\boldsymbol{p}_{Tl}, \boldsymbol{p}_{T\tilde{\chi}})$$

$$\equiv m_l^2 + m_{\tilde{\chi}}^2 + 2\left(E_{Tl} E_{T\tilde{\chi}} - \boldsymbol{p}_{Tl} \cdot \boldsymbol{p}_{T\tilde{\chi}}\right)$$

(where $E_T = \sqrt{\boldsymbol{p}_T^2 + m^2}$)

If $\boldsymbol{p}_{T\tilde{\chi}_a}$ and $\boldsymbol{p}_{T\tilde{\chi}_b}$ were obtainable,

$$m_{\tilde{l}}^2 \geq \max\left\{ m_T^2(\boldsymbol{p}_{Tl^-}, \boldsymbol{p}_{T\tilde{\chi}_a}), m_T^2(\boldsymbol{p}_{Tl^+}, \boldsymbol{p}_{T\tilde{\chi}_b}) \right\}$$

($\not{\boldsymbol{p}}_T = \boldsymbol{p}_{T\tilde{\chi}_a} + \boldsymbol{p}_{T\tilde{\chi}_b}$: total MET vector in the event)

However, not knowing the form of the MET vector splitting,
The best we can say is that :

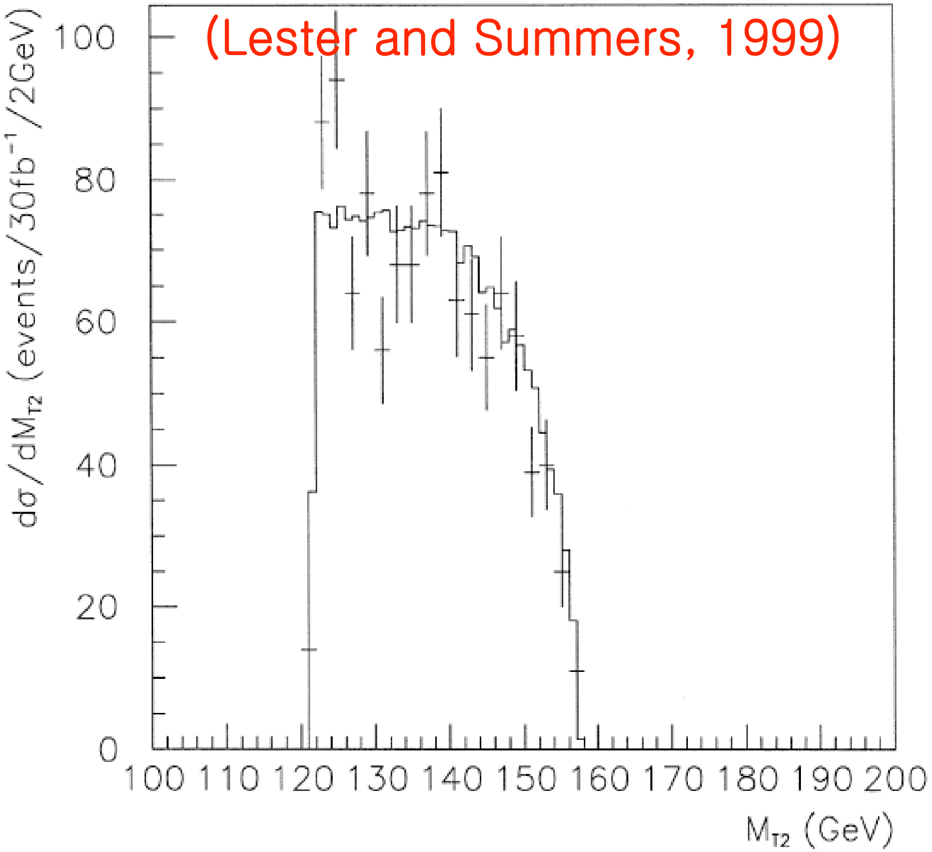
$$m_{\tilde{l}}^2 \geq M_{T2}^2$$
$$\equiv \min_{\not{\boldsymbol{p}}_1 + \not{\boldsymbol{p}}_2 = \not{\boldsymbol{p}}_T} \left[\max\left\{ m_T^2(\boldsymbol{p}_{Tl^-}, \not{\boldsymbol{p}}_1), m_T^2(\boldsymbol{p}_{Tl^+}, \not{\boldsymbol{p}}_2) \right\} \right]$$

with minimization over all possible trial LSP momenta

❖ M_{T2} distribution for $pp \to X + \tilde{l}_R^+ \tilde{l}_R^- \to X + l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$

LHC point 5, with 30 fb^{-1},

$$m_{\tilde{l}_R} = 157.1\,\text{GeV}, \quad m_{\tilde{\chi}_1^0} = 121.5\,\text{GeV}.$$



Endpoint measurement of m_{T2} distribution determines the mother particle mass

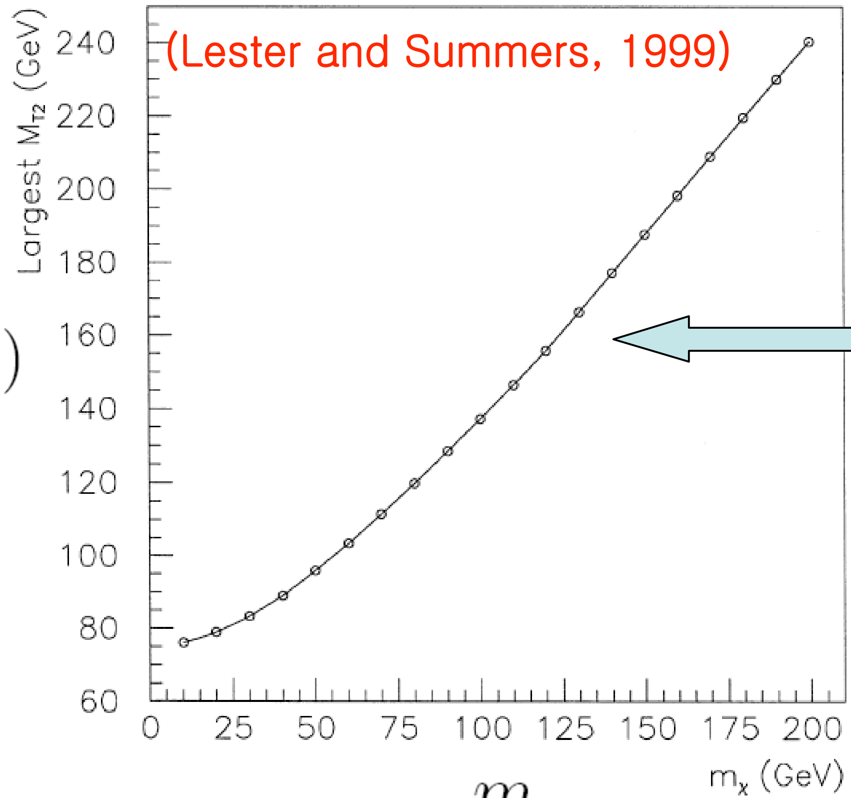
$$m_{T2}^{\max} \simeq 157\ \text{GeV}$$

(with $m_{\tilde{\chi}_1^0} = 121.5\ \text{GeV}$)

The LSP mass is needed as an input for m_{T2} calculation
But it might not be known in advance

m_{T2} depends on a trial LSP mass m_χ
Maximum of m_{T2} as a function of the trial LSP mass

$m_{T2}^{\max}(m_\chi)$



m_χ

Can the correlation be expressed by an analytic formula in terms of true sparticle masses ?

Yes !

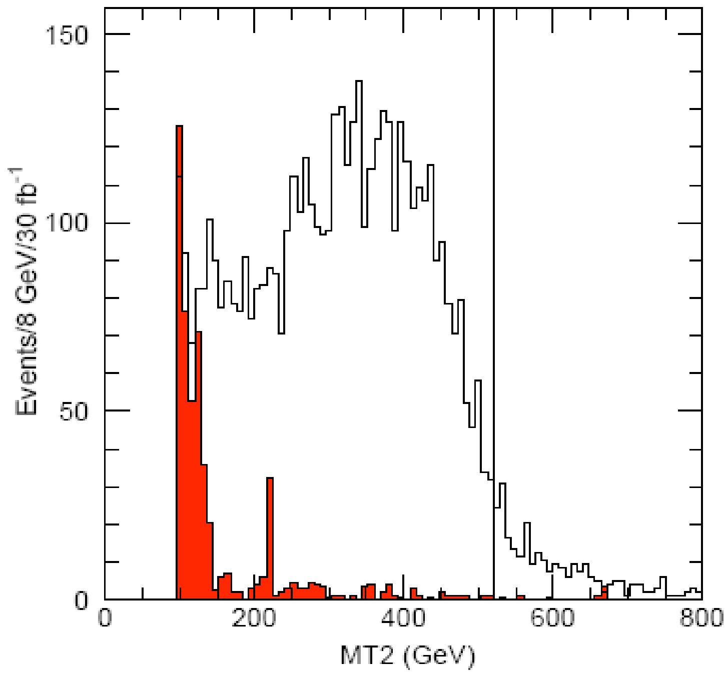
● Right handed squark mass from the m_{T2}

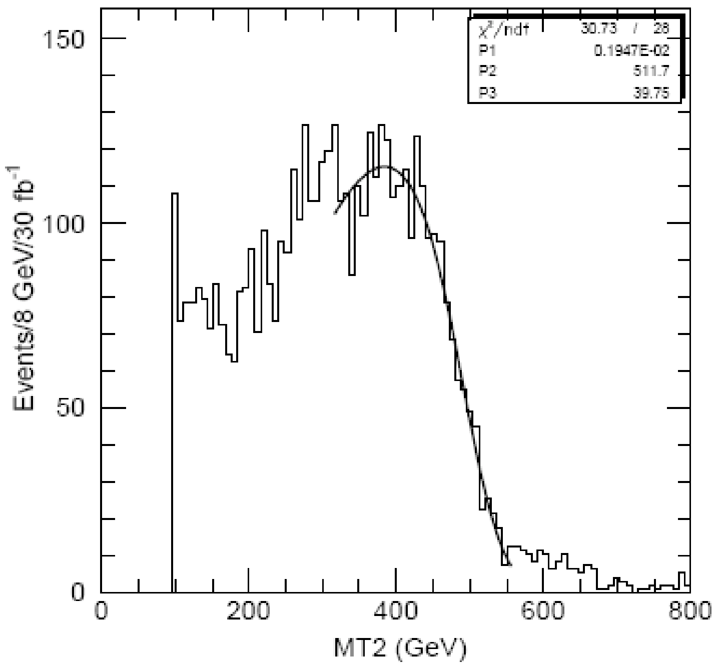
$$\tilde{q}_R \; \tilde{q}_R \longrightarrow q \; \tilde{\chi}_1^0 \; q \; \tilde{\chi}_1^0$$

$$BR(\tilde{q}_R \to q\chi_1^0) \sim 100\%$$

m_qR ~ 520 GeV, mLSP ~96 GeV

SPS1a point, with 30 fb^{-1}





(LHC/ILC Study Group: hep-ph/0410364)

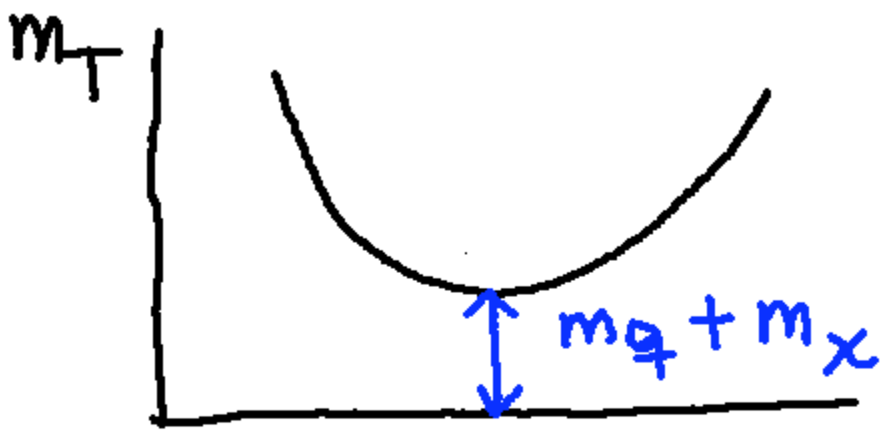
➢ Unconstrained minimum of m_T

$$m_T^2 = m_q^2 + m_\chi^2 + 2(E_T^q E_T^\chi - \mathbf{p}_T^q \cdot \mathbf{p}_T^\chi)$$

$$\frac{\partial m_T^2}{\partial (\mathbf{p}_T^\chi)_k} = 2\left[E_T^q \frac{(\mathbf{p}_T^\chi)_k}{E_T^\chi} - (\mathbf{p}_T^q)_k\right] \qquad (k = 1, 2)$$

At an unconstrained minimum, we have

$$m_T(\text{min}) = m_q + m_\chi \quad \text{with} \quad \frac{\mathbf{p}_T^\chi}{E_T^\chi} = \frac{\mathbf{p}_T^q}{E_T^q}$$

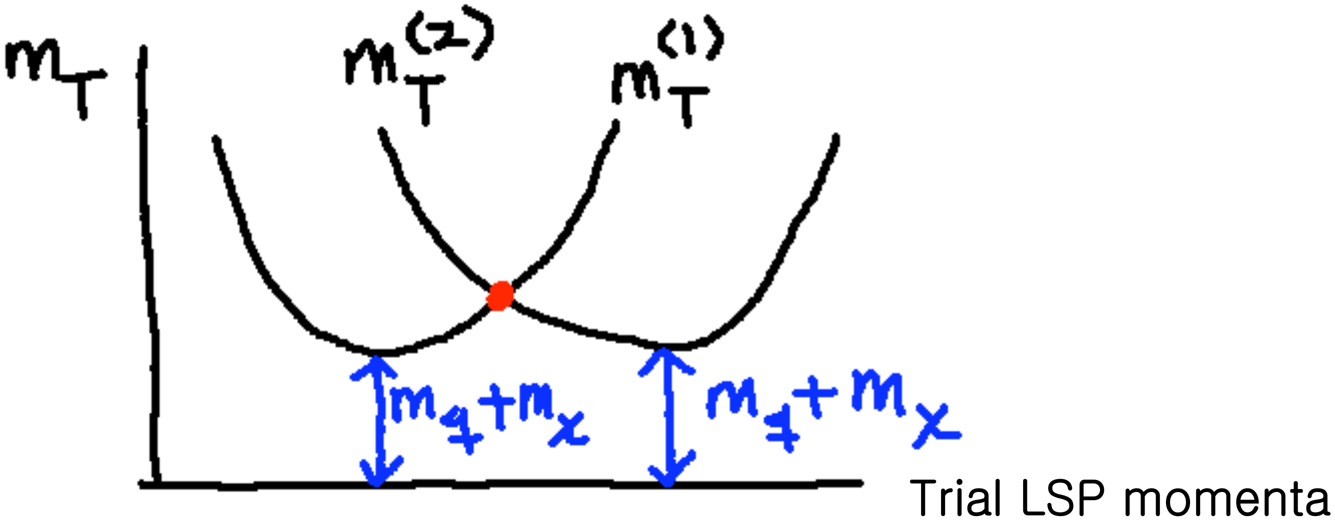


Trial LSP momentum

➢ Solution of m_{T2} (the balanced solution)

$$m_{T2}^2 \equiv \min_{\mathbf{p}_T^{\chi(1)}+\mathbf{p}_T^{\chi(2)}=\mathbf{p}_T^{miss}} \left[\max\{ m_T^2(\mathbf{p}_T^{q(1)}, \mathbf{p}_T^{\chi(1)}), m_T^2(\mathbf{p}_T^{q(2)}, \mathbf{p}_T^{\chi(2)}) \} \right]$$

with $\mathbf{p}_T^{\chi(1)} + \mathbf{p}_T^{\chi(2)} = \mathbf{p}_T^{miss} = -(\mathbf{p}_T^{q(1)} + \mathbf{p}_T^{q(2)})$ (for no ISR)



m_{T2} : the minimum of $m_T^{(1)}$ subject to the two constraints

$m_T^{(1)} = m_T^{(2)}$, and $p_T^{X(1)} + p_T^{X(2)} = p_T^{miss}$

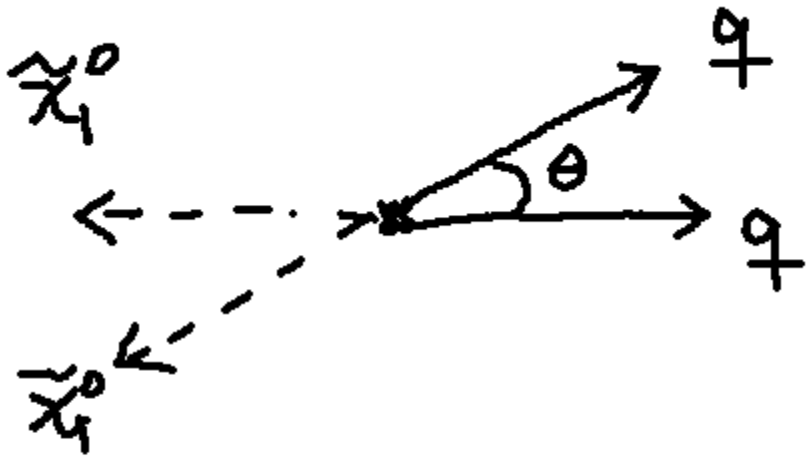
➢ The balanced solution of squark m_{T2} in terms of visible momenta

(Lester, Barr 0708.1028)

$$m_{T2} = P_0 + \sqrt{P_0^2 + m_\chi^2}$$

(m_q = 0)

with $$P_0 = \sqrt{\frac{(1+\cos\theta)}{2}|\mathbf{p}_T^{q(1)}||\mathbf{p}_T^{q(2)}|}$$



- In order to get the expression for m_{T2}^{max} ,

We only have to consider the case where two mother particles are at rest and all decays products are on the transverse plane w.r.t proton beam direction, for no ISR (Cho, Choi, Kim and Park, 2007)

- In the rest frame of squark, the quark momenta

$$|\mathbf{p}_T^{q(i)}| = \frac{m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{q}}}$$

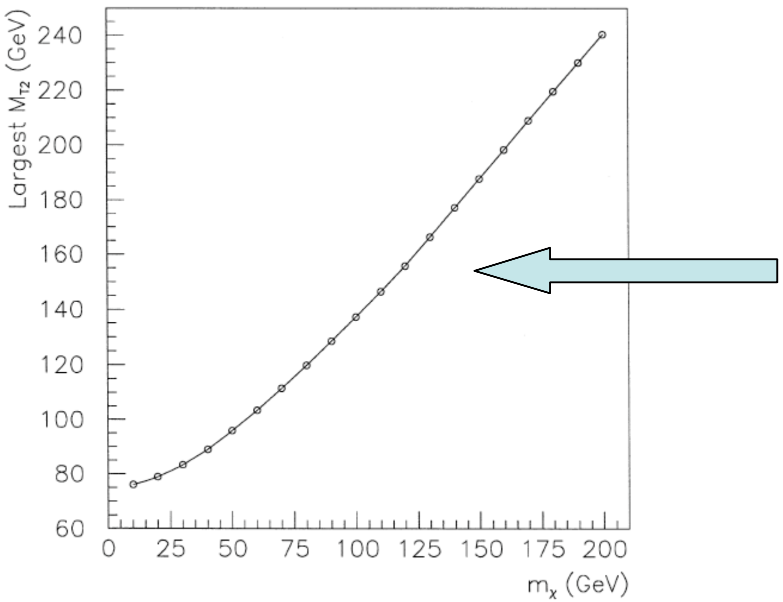
if both quark momenta are along the direction of the transverse plane

The maximum of the squark m_{T2} (occurs at $\theta = 0$)

(Cho, Choi, Kim and Park, 0709.0288)

$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{q}}} + \sqrt{\left(\frac{m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{q}}}\right)^2 + m_\chi^2}$$

- $m_{T2}^{\max}(m_\chi) = m_{\tilde{q}} \quad \text{if } m_\chi = m_{\tilde{\chi}_1^0}$

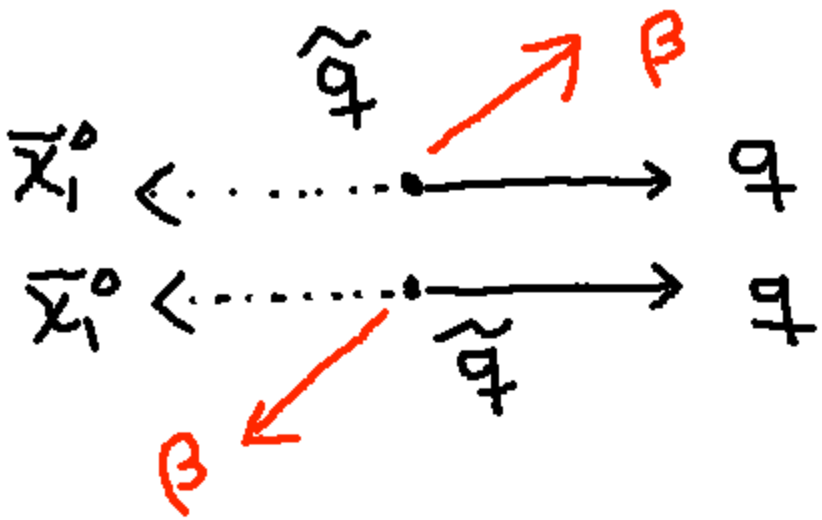


Well described by the above
Analytic expression with true
Squark mass and true LSP mass

Some remarks on the effect of squark boost

In general, squark is produced with non-zero p_T

The m_{T2} solution is invariant under the back-to-back transverse boost of mother squarks
(all visible momenta are on the transverse plane)



For the m_{T2} solution, we can consider
the first decay products as having total mass m_{T2} ,
total transverse momentum $p_T^{(1)} = p_T^{q(1)} + p_T^{\chi(1)}$
and total transverse energy $E_T^{(1)} = E_T^{q(1)} + E_T^{\chi(1)}$

Similarly, for the second products, we have
m_{T2}, $p_T^{(2)} = p_T^{q(2)} + p_T^{\chi(2)}$, $E_T^{(2)} = E_T^{q(2)} + E_T^{\chi(2)}$

$$p_T^{(1)} = -p_T^{(2)} , \quad E_T^{(1)} = E_T^{(2)}$$

Perform arbitrary back-to-back boost the systems

$$p_T^{(1)\prime} = \gamma p_T^{(1)} + \gamma\beta E_T^{(1)}$$
$$p_T^{(2)\prime} = \gamma p_T^{(2)} - \gamma\beta E_T^{(2)}$$

Then,
$$p_T^{(1)\prime} + p_T^{(2)\prime} = \gamma(p_T^{(1)} + p_T^{(2)}) = 0.$$
$$p_T^{\chi(1)\prime} + p_T^{\chi(2)\prime} = -(p_T^{q(1)\prime} + p_T^{q(2)\prime})$$

We have valid splitting of total MET and thus m_{T2} solution.

‘Gluino’ m_{T2} variable

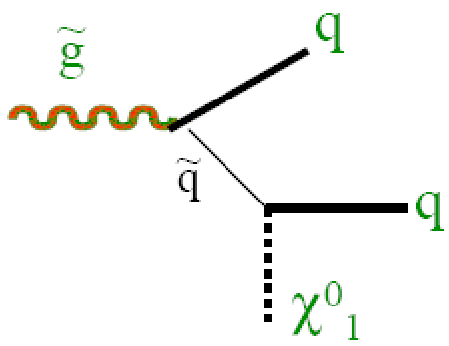
● Gluino m_{T2} (stransverse mass)

A new observable, which is an application of m_{T2} variable to the process

$$pp \rightarrow \tilde{g}\tilde{g} \rightarrow qq\tilde{\chi}_1^0\ qq\tilde{\chi}_1^0$$

Gluinos are pair produced in proton-proton collision

Each gluino decays into two quarks and one LSP



through three body decay (off-shell squark)

or two body cascade decay (on-shell squark)

- For each gluino decay, the following transverse can be constructed

$$m_T^2(m_{qqT}, m_\chi, \mathbf{p}_T^{qq}, \mathbf{p}_T^\chi) = m_{qqT}^2 + m_\chi^2 + 2(E_T^{qq} E_T^\chi - \mathbf{p}_T^{qq} \cdot \mathbf{p}_T^\chi)$$

m_{qqT} and $\mathbf{p}_T^{qq}$: mass and transverse momentum of qq system

m_χ and $\mathbf{p}_T^\chi$: trial mass and transverse momentum of the LSP

$$E_T^{qq} \equiv \sqrt{|\mathbf{p}_T^{qq}|^2 + m_{qqT}^2} \quad \text{and} \quad E_T^\chi \equiv \sqrt{|\mathbf{p}_T^\chi|^2 + m_\chi^2}.$$

- With two such gluino decays in each event, the gluino m_{T2} is defined as

$$m_{T2}^2(\tilde{g}) \equiv \min_{\mathbf{p}_T^{\chi(1)} + \mathbf{p}_T^{\chi(2)} = \mathbf{p}_T^{miss}} \left[\max\{m_T^{2(1)}, m_T^{2(2)}\} \right]$$

(minimization over all possible trial LSP momenta)

- From the definition of the gluino m_{T2}

$$m_{T2}(\tilde{g}) \leq m_{\tilde{g}} \quad \text{for} \quad m_\chi = m_{\tilde{\chi}_1^0}$$

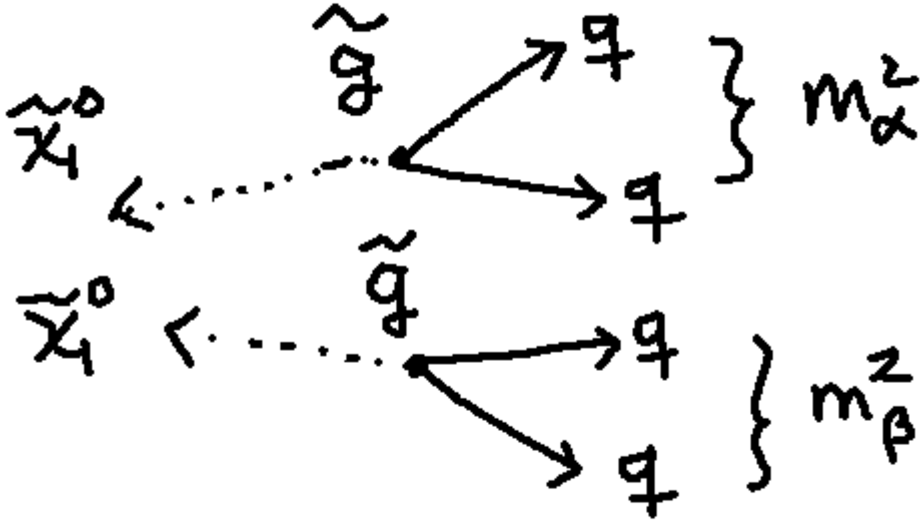
Therefore, if the LSP mass is known, one can determine the gluino mass from the endpoint measurement of the gluino m_{T2} distribution.

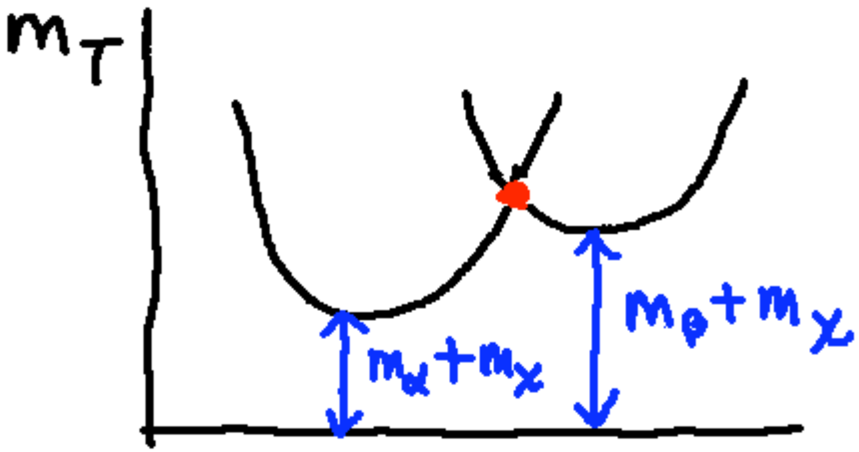
$$m_{T2}^{\max}(m_\chi) \equiv \max_{\text{all events}} [m_{T2}(\tilde{g})]$$

- However, the LSP mass might not be known in advance and then, $m_{T2}^{\max}(m_\chi)$ can be considered as a function of the trial LSP mass m_χ , satisfying

$$m_{T2}^{\max}(m_\chi = m_{\tilde{\chi}_1^0}) = m_{\tilde{g}}$$

Each mother particle produces one invisible LSP and more than one visible particles

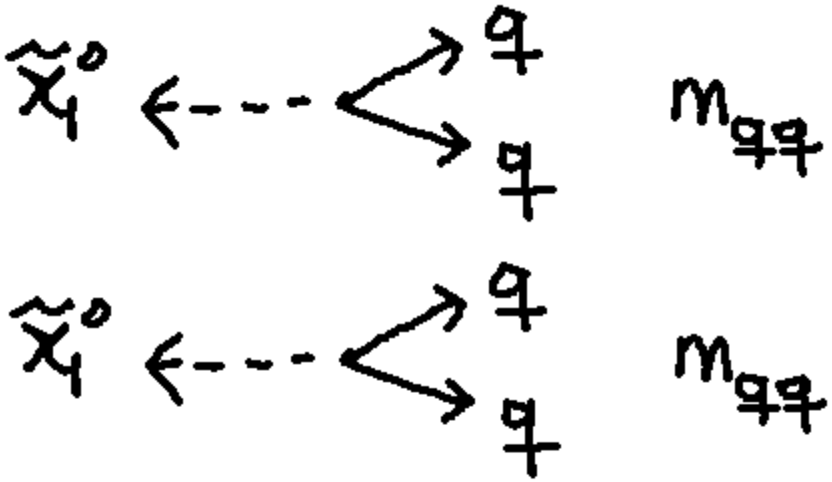




Possible m_{qq} values for three body decays of the gluino :

$$0 \leq m_{qq} \leq m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$$

In the frame of gluino pair at rest



Two sets of decay products have the same m_{qq} and are parallel to each other ($\theta = 0$) on transverse plane

($0 \leq m_{qq} \leq m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$)

Di-quark momenta

$$|\mathbf{p}| = \frac{\sqrt{[m_{\tilde{g}}^2 - (m_{\tilde{\chi}_1^0} + m_{qq})^2][m_{\tilde{g}}^2 - (m_{\tilde{\chi}_1^0} - m_{qq})^2]}}{2m_{\tilde{g}}}$$

Gluino m_{T2}

$$m_{T2} = \sqrt{m_{qq}^2 + |\mathbf{p}|^2} + \sqrt{m_{\chi}^2 + |\mathbf{p}|^2}$$

- The gluino m_{T2} has a very interesting property

$$m_{T2} = \sqrt{m_{qq}^2 + |\mathbf{p}|^2} + \sqrt{m_\chi^2 + |\mathbf{p}|^2}$$

$$(\; 0 \leq m_{qq} \leq m_{\tilde{g}} - m_{\tilde{\chi}_1^0} \;)$$

$$\frac{dm_{T2}}{dm_{qq}} = \frac{m_{qq}}{m_{\tilde{g}}}\left(1 - \frac{(m_{\tilde{g}}^2 + m_{\tilde{\chi}_1^0}^2 - m_{qq}^2)}{\sqrt{(m_{\tilde{g}}^2 + m_{\tilde{\chi}_1^0}^2 - m_{qq}^2)^2 + 4m_{\tilde{g}}^2(m_\chi^2 - m_{\tilde{\chi}_1^0}^2)}}\right)$$

$= 0$ if $m_\chi = m_{\tilde{\chi}_1^0}$ → m_{T2} = m_gluino for all m_{qq}

> 0 if $m_\chi > m_{\tilde{\chi}_1^0}$ → The maximum of m_{T2} occurs when m_{qq}= m_{qq} (max)

< 0 if $m_\chi < m_{\tilde{\chi}_1^0}$ → The maximum of m_{T2} occurs when m_{qq}= 0

This result implies that

$$m_{T2}^{\max}(m_\chi) = \left(m_{\tilde{g}} - m_{\tilde{\chi}_1^0}\right) + m_\chi \qquad \text{for} \quad m_\chi \geq m_{\tilde{\chi}_1^0}$$

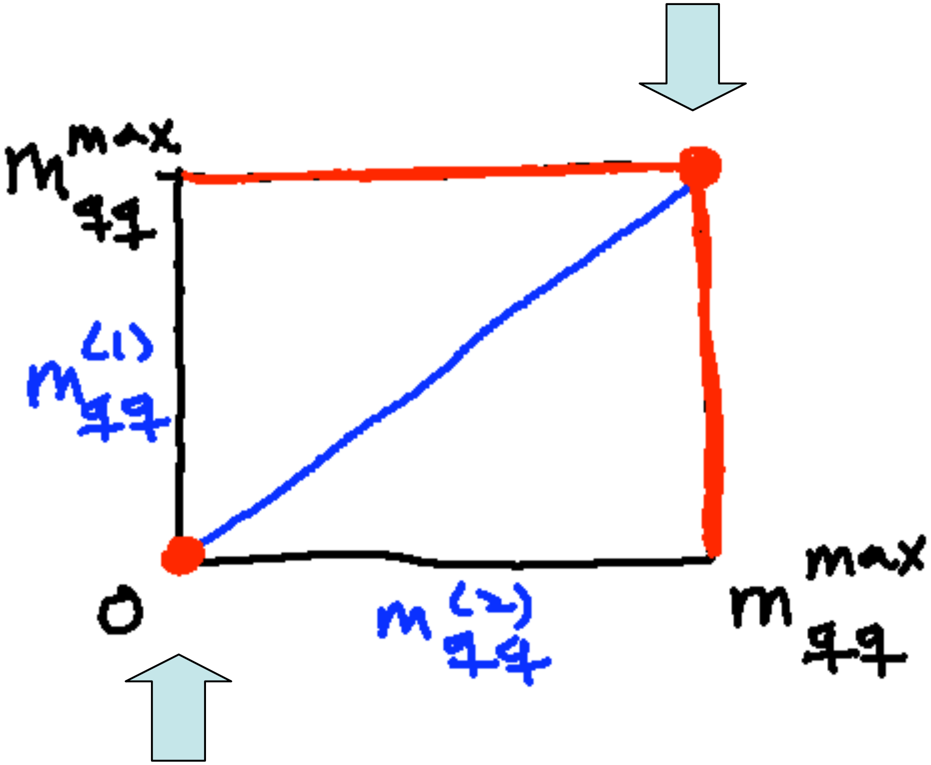
$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2} \qquad \text{for} \quad m_\chi \leq m_{\tilde{\chi}_1^0}.$$

(This conclusion holds also for more general cases where m_{qq1} is different from m_{qq2})

$$\theta = 0$$

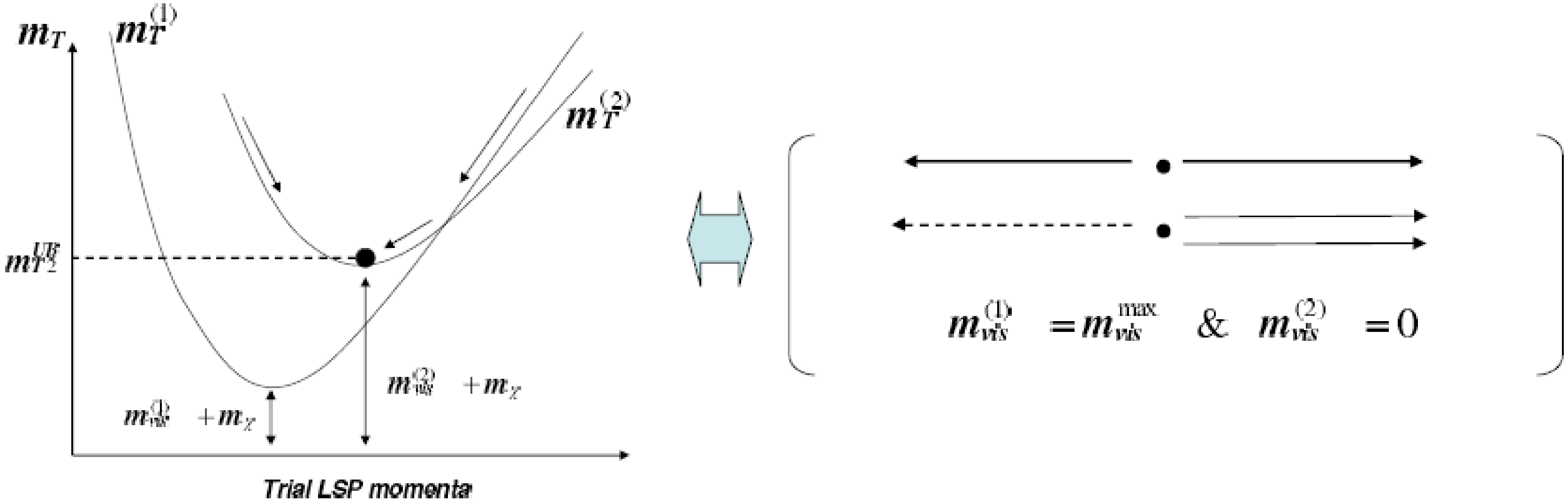
$$m_{T2}^{\max}(m_\chi) = \left(m_{\tilde{g}} - m_{\tilde{\chi}_1^0}\right) + m_\chi$$

for $m_\chi \geq m_{\tilde{\chi}_1^0}$



$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2} \qquad \text{for} \quad m_\chi \leq m_{\tilde{\chi}_1^0}.$$

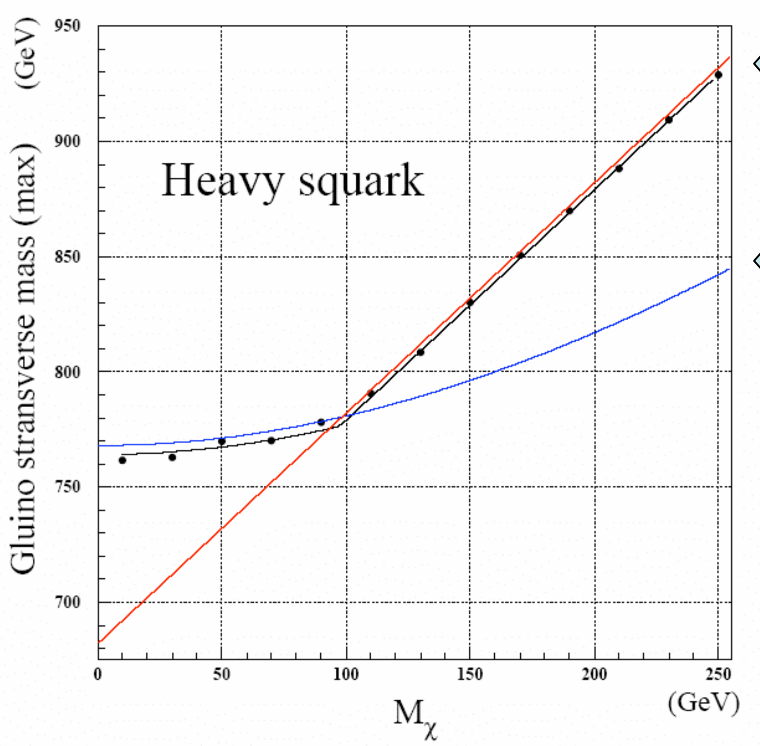
Unbalanced Solution of m_{T2} can appear



In some momentum configuration ,
unconstrained minimum of one $m_T^{(2)}$ is larger than
the corresponding other $m_T^{(1)}$
Then, m_{T2} is given by the unconstrained minimum of $m_T^{(2)}$

$$m_{T2} = m_{qq}^{(i)} + m_x$$

❖ If the function $m_{T2}^{\max}(m_\chi)$ could be constructed from experimental data, which would identify the crossing point, one will be able to determine the gluino mass and the LSP mass simultaneously.



$$m_{T2}^{\max}(m_\chi) = \left(m_{\tilde g} - m_{\tilde\chi_1^0}\right) + m_\chi$$

$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde g}^2 - m_{\tilde\chi_1^0}^2}{2m_{\tilde g}} + \sqrt{\left(\frac{m_{\tilde g}^2 - m_{\tilde\chi_1^0}^2}{2m_{\tilde g}}\right)^2 + m_\chi^2}$$

✓ A numerical example

$$m_{\tilde g} = 780.3 \text{ GeV}, \; m_{\tilde\chi_1^0} = 97.9 \text{ GeV},$$

and a few TeV masses for sfermions

Experimental feasibility

An example (a point in mAMSB)

$$m_{\tilde{g}} = 780.3 \text{ GeV}, \;\; m_{\tilde{\chi}_1^0} = 97.9 \text{ GeV},$$

with a few TeV sfermion masses
(gluino undergoes three body decay)

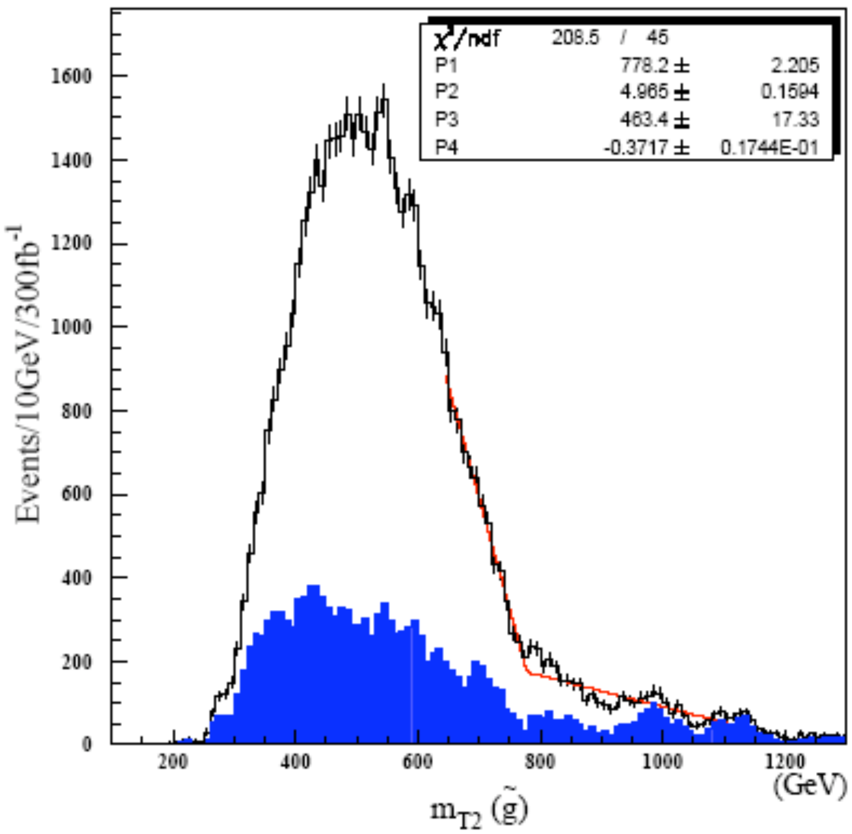
We have generated a MC sample of SUSY events,
which corresponds to 300 fb^{-1} by PYTHIA

The generated events further processed with PGS detector simulation,
which approximates an ATLAS or CMS-like detector

❖ Experimental selection cuts

- At least 4 jets with $P_{T1,2,3,4} > 200, 150, 100, 50$ GeV
- Missing transverse energy $E_T^{miss} > 250$ GeV
- Transverse sphericity $S_T > 0.25$
- No b-jets and no-leptons
- The four hardest jets are divided into two groups of dijets by hemisphere analysis

The gluino m_{T2} distribution
with the trial LSP mass m_x = 90 GeV

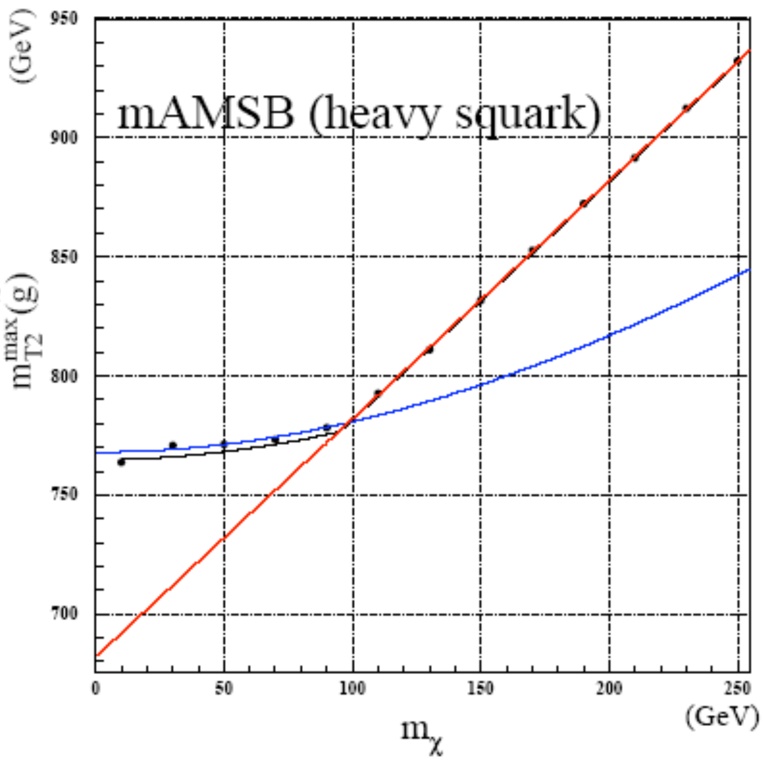


Fitting with a linear function
with a linear background,
We get the endpoints

m_{T2} (max) = 778.2 ± 2.2 GeV

The blue histogram :
SM background

❖ $m_{T2}^{\max}$ as a function of the trial LSP mass for the benchmark point



$$m_{T2}^{\max}(m_\chi) = \left(m_{\tilde{g}} - m_{\tilde{\chi}_1^0}\right) + m_\chi$$

$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2}$$

Fitting the data points with the above two theoretical curves, we obtain

$$m_{\tilde{g}} = 776.5 \pm 1.0 \text{ GeV}$$

$$m_{\tilde{\chi}_1^0} = 94.9 \pm 1.4 \text{ GeV}$$

The true values are

$$m_{\tilde{g}} = 780.3 \text{ GeV}, \; m_{\tilde{\chi}_1^0} = 97.9 \text{ GeV},$$

● Conclusions

We introduced a new observable, ‘gluino’ m_{T2}

We showed that the maximum of the gluino m_{T2} as a function of trial LSP mass has a kink structure at true LSP mass from which the gluino mass and the LSP mass can be determined simultaneously.

BACKUP

➢ Theorem : (Cho, Choi, Kim and Park, arXiv:0711.4526)

m_{T2} of any event induced by mother particle pair having a vanishing total transverse momentum in Lab. frame is bounded from above by another m_{T2} of an event induced by mother particle pair at rest

$$m_{T2}(\mathbf{p}_T^{vis(i)}, m_{vis}^{(i)}, m_\chi) \leq m_{T'2}(\mathbf{q}^{vis(i)}, m_{vis}^{(i)}, m_\chi)$$

for generic $\mathbf{p}^{vis(i)}$ measured in the laboratory frame,

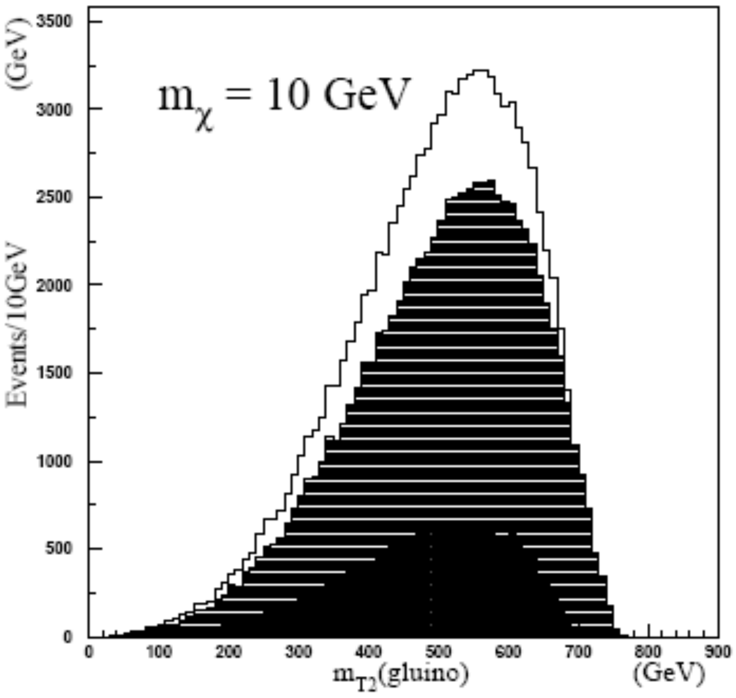
where $\mathbf{q}^{vis(i)}$ is the Lorentz boost of $\mathbf{p}^{vis(i)}$ to the rest frame of the i-th mother particle,

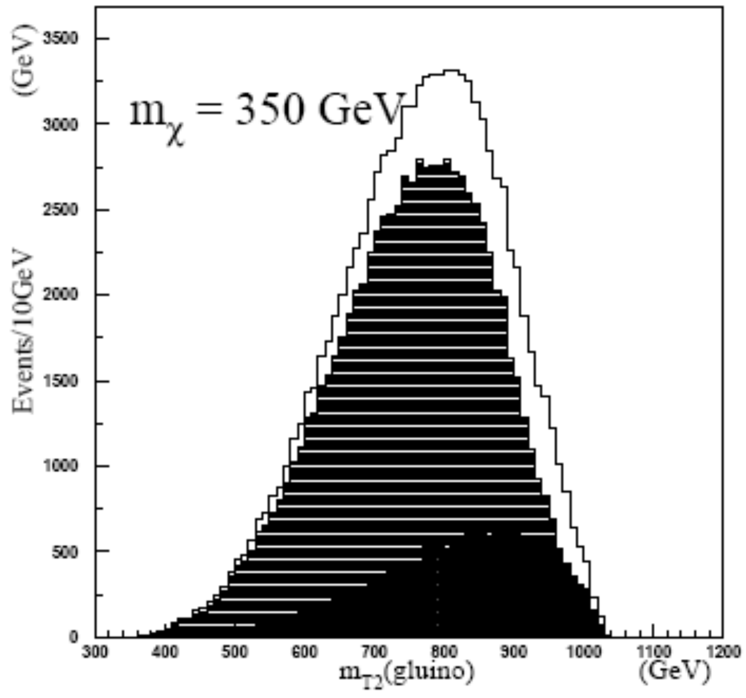
T' is the plane spanned by $\mathbf{q}^{vis(1)}$ and $\mathbf{q}^{vis(2)}$

The equality in the above bound holds when T=T'

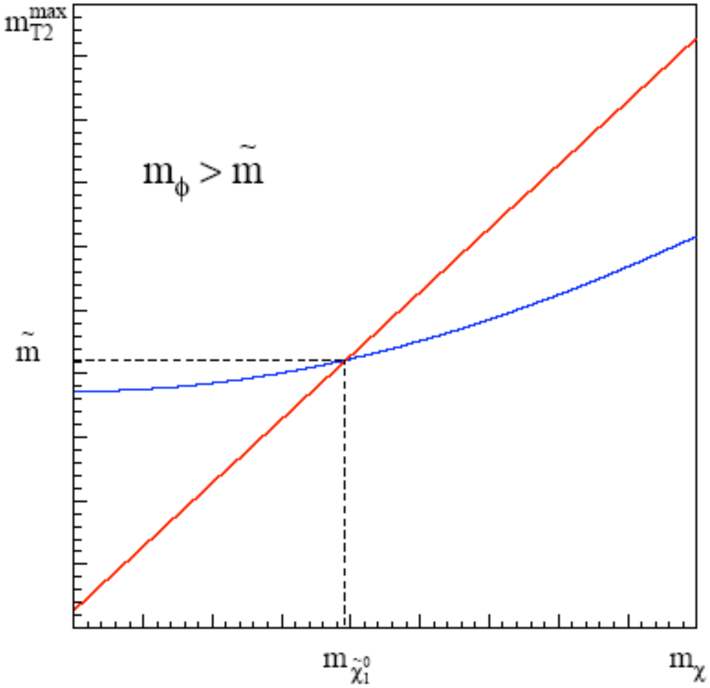
Gluino m_{T2} distributions for AMSB bechmark point

True gluino mass = 780 GeV,   True LSP mass = 98 GeV

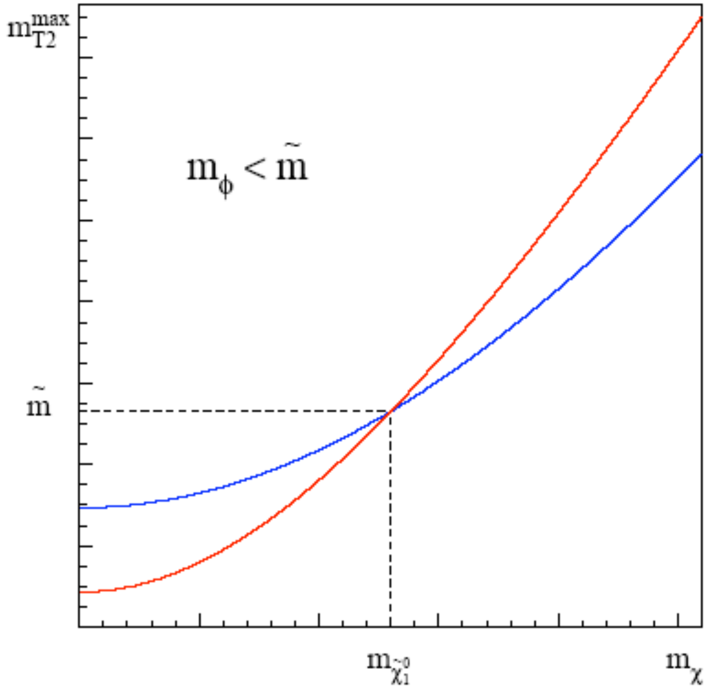




For three body decay



For two body cascade decay



$$\frac{(d\mathcal{F}_{>}^{\max}/dm_\chi)_{m_\chi=m_{\tilde{\chi}_1^0}}}{(d\mathcal{F}_{<}^{\max}/dm_\chi)_{m_\chi=m_{\tilde{\chi}_1^0}}} = 1 + \frac{(m_{vis}^{\max})^2 - (m_{vis}^{\min})^2}{\tilde{m}^2 + m_{\tilde{\chi}_1^0}^2 - (m_{vis}^{\max})^2} > 1.$$

For two body cascade decay

$$m_{qq}^{\max} = \frac{(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)(m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{q}}^2}$$

Therefore, for $m_\chi \geq m_{\tilde{\chi}_1^0}$

$$m_{T2}^{\max} = \left(\frac{m_{\tilde{g}}}{2}(1 - \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}) + \frac{m_{\tilde{g}}}{2}(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{q}}^2})\right)$$

$$+\sqrt{\left(\frac{m_{\tilde{g}}}{2}(1 - \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}) - \frac{m_{\tilde{g}}}{2}(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{q}}^2})\right)^2 + m_\chi^2}.$$

The balanced mT2 solution

$$\left(m_{T2}^{\mathrm{bal}}\right)^2 = m_\chi^2 + A_T + \sqrt{\left(1 + \frac{4m_\chi^2}{2A_T - \left(m_{vis}^{(1)}\right)^2 - \left(m_{vis}^{(2)}\right)^2}\right)\left(A_T^2 - \left(m_{vis}^{(1)} m_{vis}^{(2)}\right)^2\right)},$$

where

$$\begin{aligned} A_T &\equiv \alpha_1^0 \alpha_2^0 + \vec{\alpha_1} \cdot \vec{\alpha_2} \\ &= E_T^{vis(1)} E_T^{vis(2)} + \mathbf{p}_T^{vis(1)} \cdot \mathbf{p}_T^{vis(2)} \end{aligned}$$