

Simulation of jet physics at hadron colliders

Steffen Schumann



School of Physics
The University of Edinburgh



Science & Technology
Facilities Council

improving QCD jet modelling through multi-leg tree-level calculations

- **The physics:** multi-particle final states at the LHC
- **The tools:** Monte Carlo event generators
 - the parton shower approach
 - (● multi-leg tree-level calculations)
- **The method:** combining matrix elements and parton showers
 - merging prescriptions: anatomy and systematics
 - sample applications: DY+jets, $t\bar{t}$ +jets

Setting the scene: physics at the LHC

main objectives for the LHC era

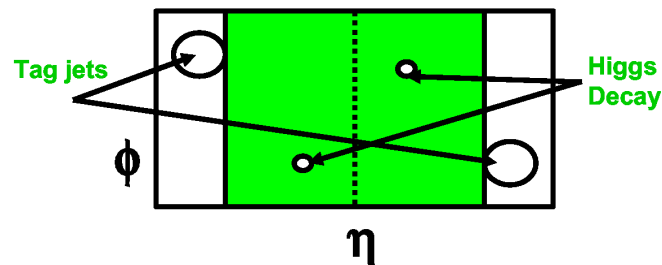
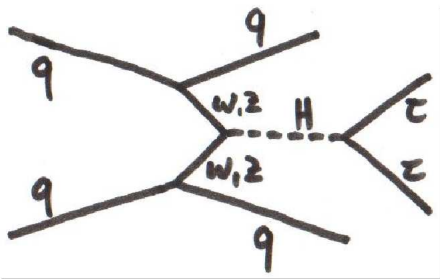
- reveal the mechanism of EWSB [discovery of the Higgs?, alternatives?]
- search for physics beyond the SM: weak scale SUSY, ED, W' & Z' , ...

Setting the scene: physics at the LHC

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- reveal the mechanism of EWSB [discovery of the Higgs?, alternatives?]
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example: Higgs production in weak boson fusion

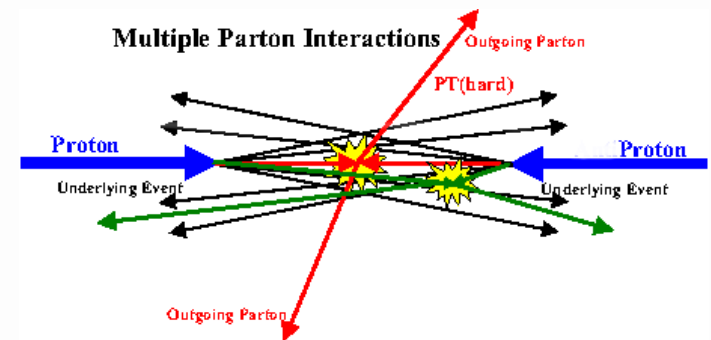


- ➔ rapidity gap between two forward tagged jets
- ➔ signal/background ratio depends on central jet veto

What happens at higher orders?

How well can we model the backgrounds?

Does the underlying event spoil the signal?

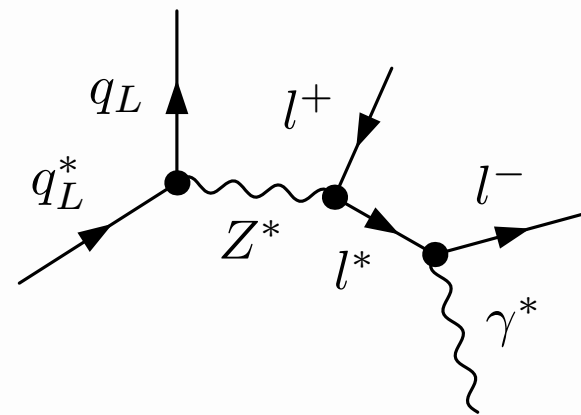
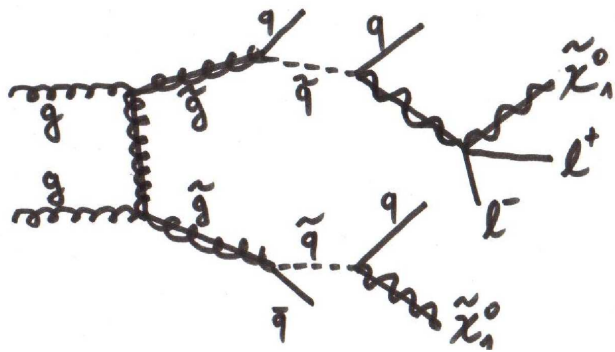


Setting the scene: physics at the LHC

main objectives for the LHC era

- reveal the mechanism of EWSB [discovery of the Higgs?, alternatives?]
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example: cascade decays of new heavy states



- ➔ signals: leptons + n-jets + \cancel{E}_T
- ➔ backgrounds: QCD jets, V +jets, VV +jets, $t\bar{t}$ +jets
- ➔ jet properties depend on nature of new physics [energies, flavours, edges]

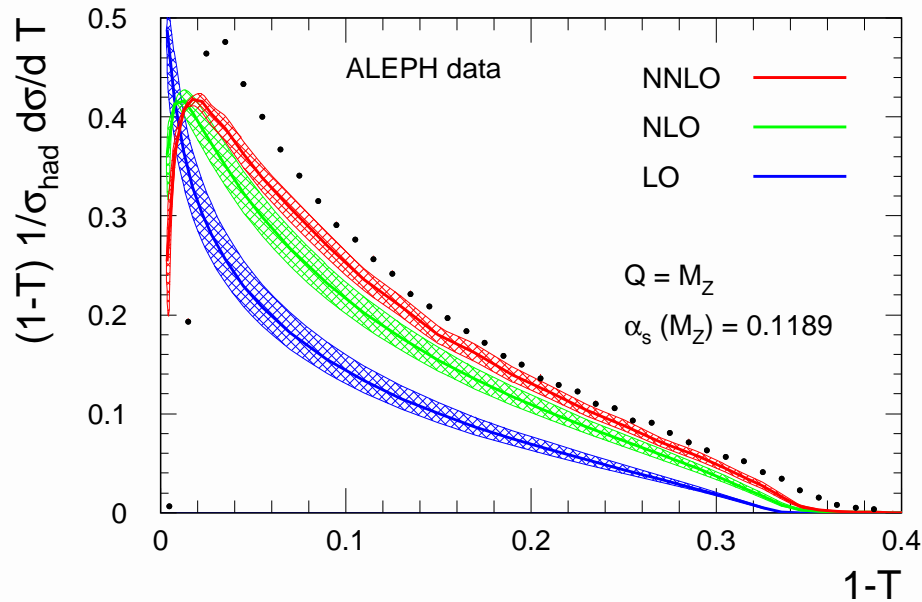
How well can we describe the multi-jet backgrounds?

Setting the scene: perturbative aspects

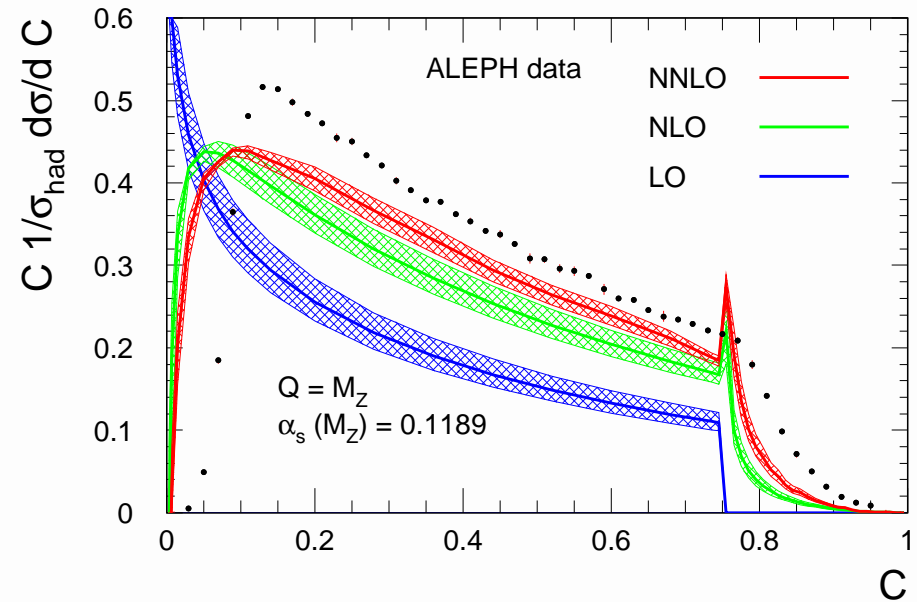
NNLO corrections to event shapes in e^+e^- annihilation

[A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, arXiv:0711.4711]

1-T



C-parameter



➔ strong need for higher order QCD calculations (multiple emissions)

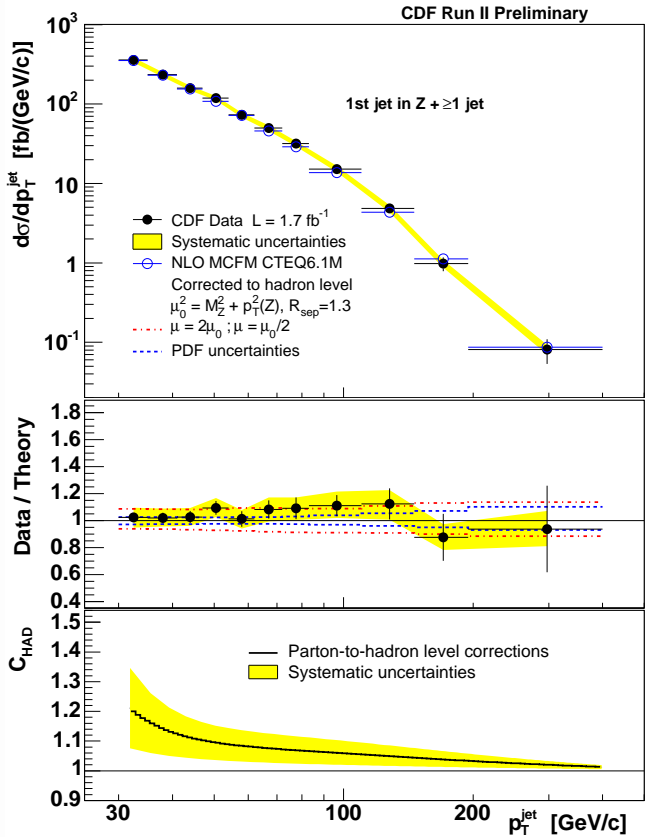
- accurate rate prediction
- reduced scale uncertainties

➔ but, have to consider all-orders resummation of large logarithms as well

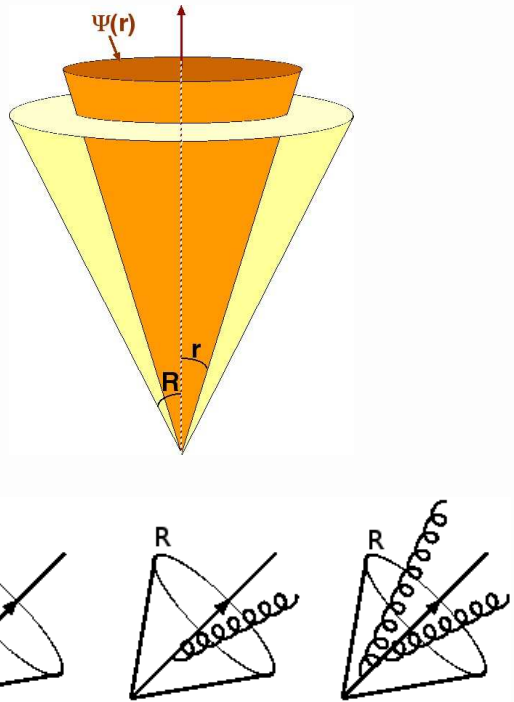
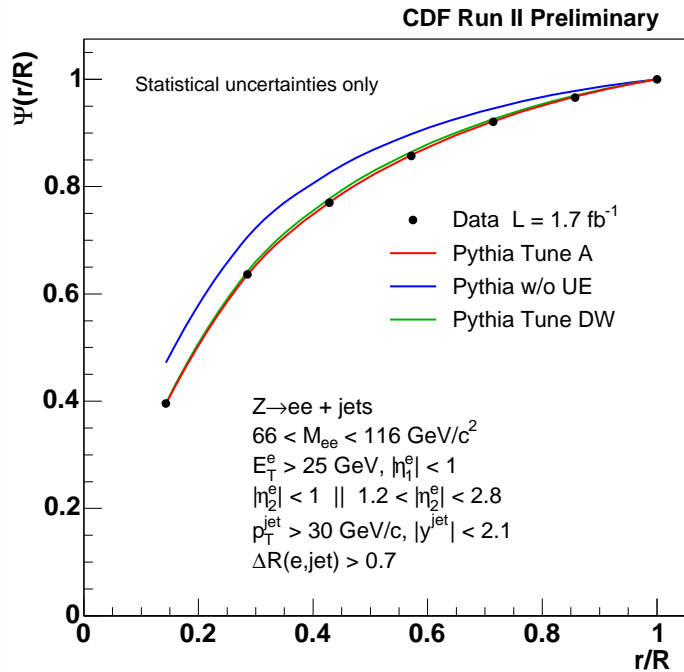
Setting the scene: non-perturbative aspects

jet observables in $p\bar{p} \rightarrow Z(\rightarrow e^+e^-) + X$ @ CDF [CDF note 8827]

p_T -1st jet



jet shape $\Psi(r)$

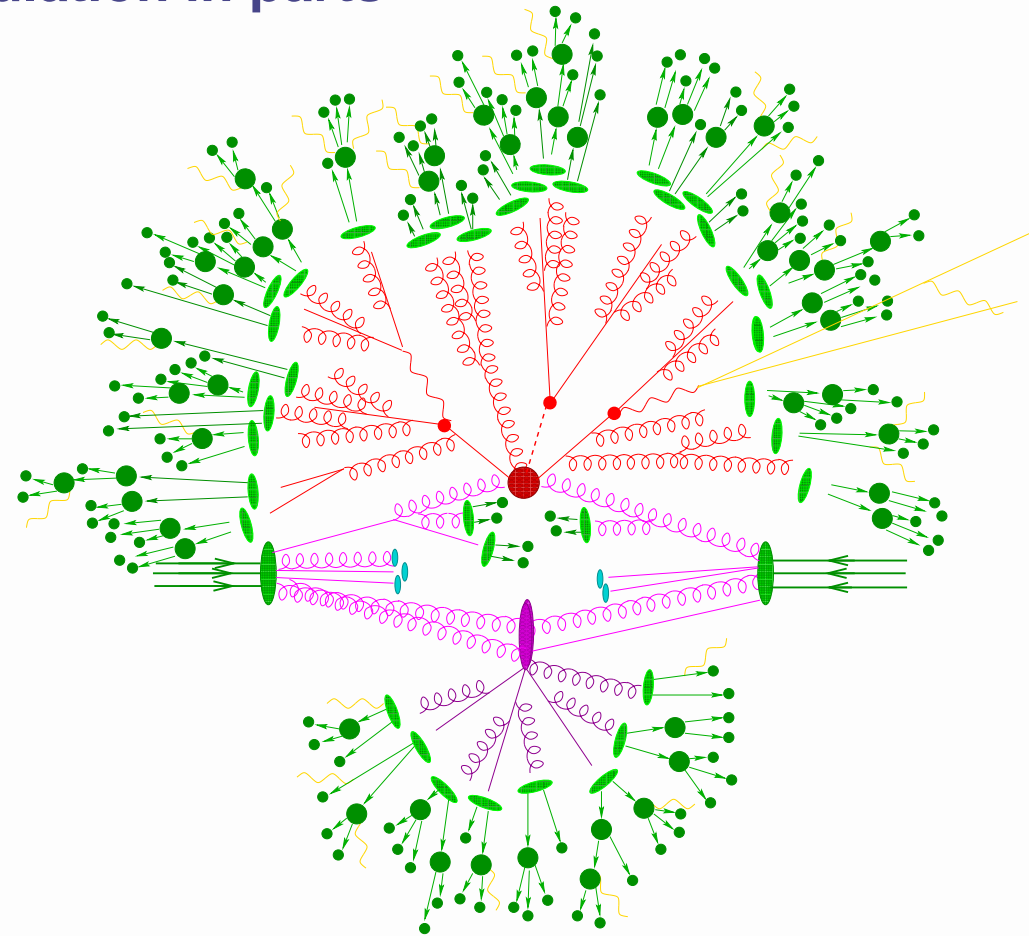


- ➔ corrections needed to compare fixed order with data [Monte Carlo studies]
- ➔ account for non-perturbative aspects like hadronisation & underlying event

Monte Carlo Event Generators

Monte Carlo paradigm: split the simulation in parts

- **Signal/Background process**
exact matrix elements
- **QCD bremsstrahlung**
parton showers in the **initial** and **final** state
- **Multiple Interactions**
beyond factorisation: modelling
- **Hadronisation**
non perturbative QCD: modelling
- **Hadron Decays**
phase space or effective theories



- ➔ fully exclusive hadronic final states
- ➔ can directly be compared with experimental data

Herwig, Pythia, Sherpa

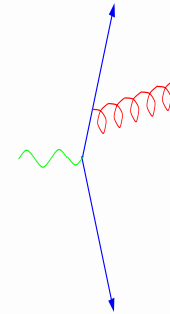
Parton showers in a nutshell

QCD emissions are enhanced near the collinear and/or soft limit

➔ propagator factor for $q \rightarrow qg$ splitting (or $g \rightarrow gg$)

$$\frac{1}{(p_q + p_g)^2} \simeq \frac{1}{2E_q E_g (1 - \cos \theta_{qg})}$$

[soft- and collinear divergences dominate the matrix element]



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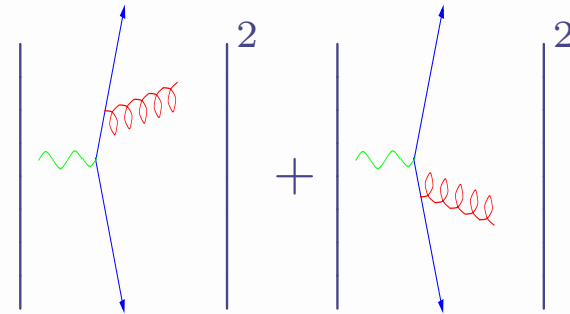
[soft- and collinear divergences dominate the matrix element]

➔ cross section factorises in the collinear limit

$$|\mathcal{M}_{q\bar{q}g}|^2 d\Phi_{q\bar{q}g} \approx |\mathcal{M}_{q\bar{q}}|^2 d\Phi_{q\bar{q}} \left\{ \frac{dt_{qg}}{t_{qg}} \frac{\alpha_S}{2\pi} P_{q,qg}(z_q) dz_q + \frac{dt_{\bar{q}g}}{t_{\bar{q}g}} \frac{\alpha_S}{2\pi} P_{\bar{q},\bar{q}g}(z_{\bar{q}}) dz_{\bar{q}} \right\}$$

with $z_q = p_q^0 / (p_q^0 + p_g^0)$ and $P_{q,qg}(z_q) = C_F \frac{1+z_q^2}{1-z_q}$ (spin averaged Altarelli-Parisi kernels)

$$\int_{t_o}^t \frac{d\tilde{t}}{\tilde{t}} \int_{\sqrt{t_o/\tilde{t}}}^{1-\sqrt{t_o/\tilde{t}}} \frac{dz}{1-z} \sim \ln^2 \frac{t}{t_o} \quad \Rightarrow \quad \text{double leading logarithms}$$

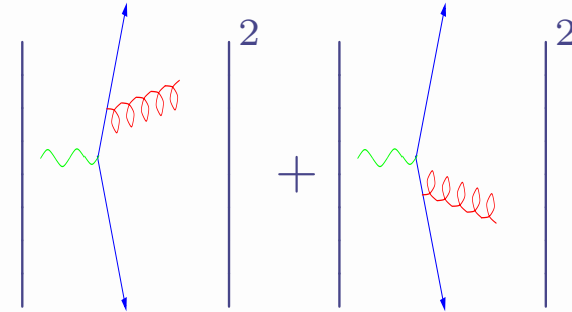


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- ➔ probability for **no emission** off a quark line from t to t_o ($> \Lambda_{\text{QCD}}$)

$$\Delta_q(t_o, t) \equiv \exp \left(- \int_{t_o}^t \frac{d\tilde{t}}{\tilde{t}} \int_{\sqrt{t_o/\tilde{t}}}^{1-\sqrt{t_o/\tilde{t}}} dz \frac{\alpha_S(\tilde{t}, z)}{2\pi} P_{q,qg}(z) \right) \Rightarrow \text{Sudakov form factor}$$

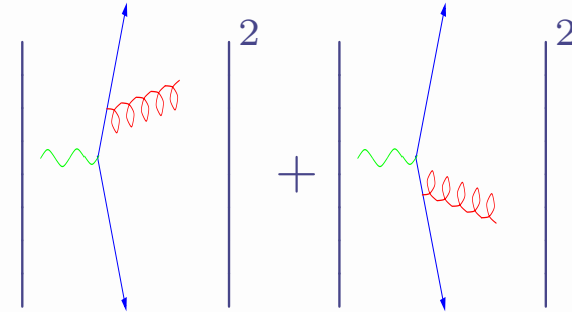
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- ➔ probability for $q \rightarrow qg$ splitting at $t' < t$

$$d\mathcal{P} = \Delta_q(t', t) \frac{\alpha_S(t', z)}{2\pi} \frac{1}{t'} P_{q,qg}(z) dt' dz \Rightarrow \text{emissions ordered in } t$$

Parton showers in a nutshell

main features of parton shower approach

- emissions of soft/collinear partons from initial & final state [resum LL]
- evolve partons from high scales to low scales [ordered in t : virtuality, angle, p_T]
- allow for incorporation of universal hadronisation models [scales $\mathcal{O}(1 \text{ GeV})$]

limitations

- shower seeds are leading-order QCD processes only [Herwig, Pythia]
 - lack of high-energetic large-angle emissions
 - interferences and correlations only approximate
- } needs exact $|\mathcal{M}|^2$

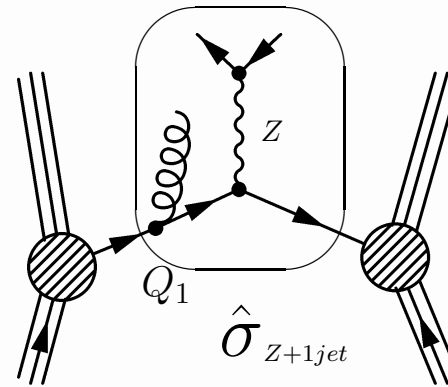
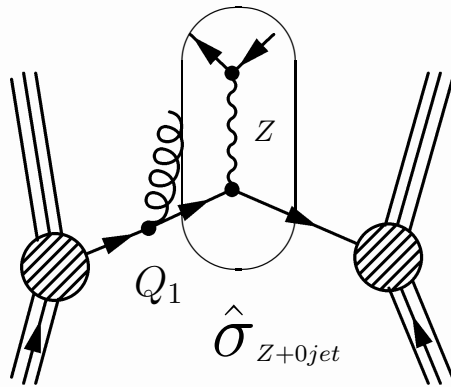
passing lanes

- first few hardest emissions according to exact tree-level matrix elements
[called ME+PS merging – CKKW, MLM]
- take NLO QCD core processes instead [called MC at NLO]
- go beyond the present shower approximations [subleading colour,...]

Combining ME and PS

objectives

- provide good description for soft and hard emissions [PS pros + ME pros]
- incorporate quantum interferences and correlations
- avoid double counting of phase space configurations in PS & ME



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- divide multi-jet phase space in two regimes [measure Q_{cut} or $\Delta R_{\text{min}} \& E_{T,\text{min}}$]
 - jet seeds produced through tree-level ME's [hard partons]
 - jets evolved down to fragmentation scale by PS
- reweight MEs to get exclusive samples at resolution scale
⇒ allows to add samples of different ME jet multiplicities
- reject PS configurations taken into account by higher order ME

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features

- inclusive samples with up to n_{max} -jets described by MEs
- process independent implementation feasible
- rates leading order only, but real corrections included

Combining ME and PS – CKKW

The CKKW method [Catani,Krauss,Kuhn,Webber 2001; Krauss 2002]

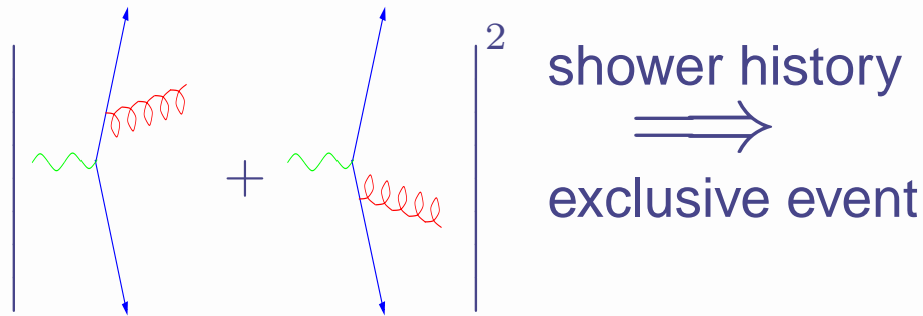
[as implemented in Sherpa, using built-in ME generator Amegic++]

- evaluate MEs $X + 0, 1, \dots, n_{\max}$ -jets at k_T -resolution Q_{cut} [regulator, μ_F, μ_R]
- select a jet multiplicity with probability

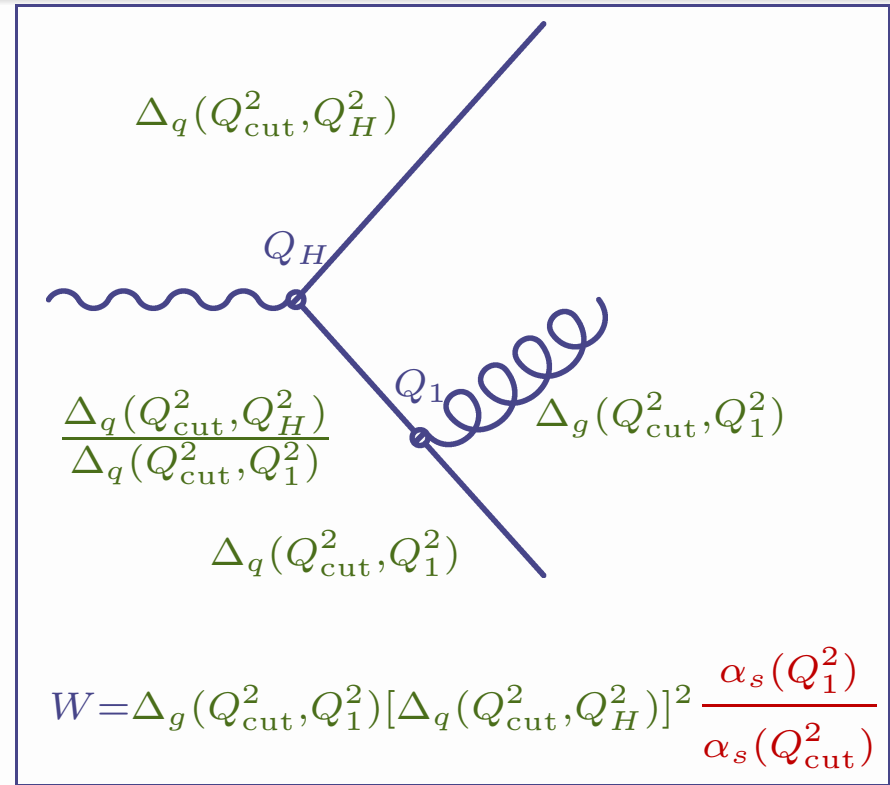
$$P_n = \frac{\sigma_n}{\sum_{i=0}^{n_{\max}} \sigma_i}$$

- generate final-state momenta p_i according to the ME
- reweight ME according to a reconstructed pseudo shower history
 - determine jet emission scales Q_n, \dots, Q_1 with a k_T cluster algorithm
 - calculate corresponding Sudakov weights
 - $\Delta_{q,g}(Q_{\text{cut}}^2, Q_{\text{prod}}^2)$ for outgoing partons
 - $\Delta_{q,g}(Q_{\text{cut}}^2, Q_{\text{prod}}^2) / \Delta_{q,g}(Q_{\text{cut}}^2, Q_{\text{dec}}^2)$ for lines between $Q_{\text{prod}} > Q_{\text{dec}}$
 - recalculate α_S at each vertex in the tree at the corresponding k_T scale
- start initial- or final-state parton shower for all partons of the events
 - at scale where it was produced
 - veto on shower emissions above the scale Q_{cut}

Combining ME and PS – CKKW



\hookrightarrow inclusive events for n_{\max}
 replace Q_{cut} by $Q_{n_{\max}}$



- \rightarrow general implementation for e^+e^- and hadron colliders in Sherpa
- \rightarrow Q_{cut} dependence cancels to (N)LL accuracy
- \rightarrow a variant, the Lönnblad scheme, implemented for Ariadne

Combining ME and PS – MLM

The MLM method [Mangano 2002]

[as implemented for Alpgen, combined with Herwig or Pythia showers]

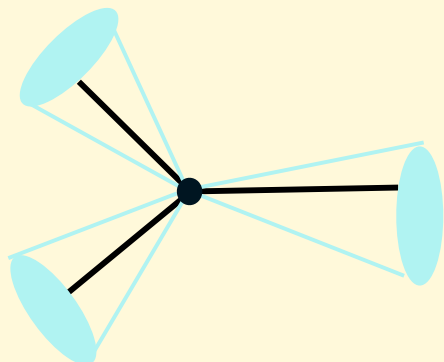
- evaluate the $n = 0, \dots, n_{\max}$ -parton MEs with cone measure R_{\min} & $E_{T,\min}$
- α_S at k_T scales
- generate unweighted events for each multiplicity
- perform showering by Herwig or Pythia [LHA interface]
- after showers (before hadronisation) run cone finder with R_{clus} and $E_{T,\text{clus}}$
- try to geometrically match ME partons with jets after showering
 - for each ME parton select the jet with minimal $\Delta R_{j,\text{parton}}$
 - if $\Delta R_{j,\text{parton}} < R_{\text{match}}$ the parton is “matched” [default $R_{\text{match}} = 1.5R_{\text{clus}}$]
 - a jet can be matched to a single parton only
 - if all partons are matched, keep the event, else reject it
 - ⇒ this defines an inclusive sample, $n_{\text{jet}} \geq n_{\max}$
 - ⇒ for exclusive sample require in addition $n_{\text{jet}} = n$
- after matching combine exclusive and inclusive samples

Combining ME and PS – MLM

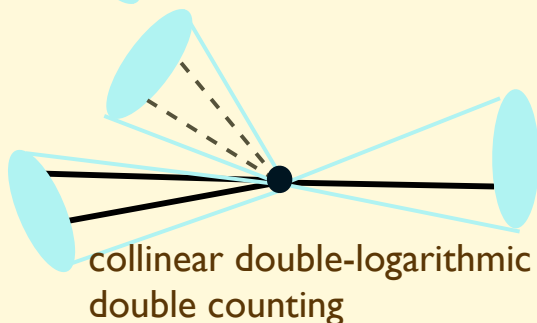
Few examples of matching:

————— hard parton

- - - - - parton emitted by the shower

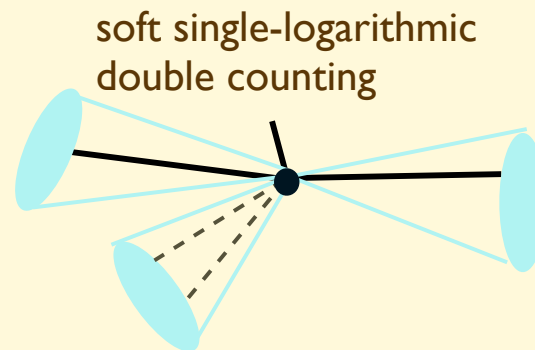


Event matched, $N_{jet} = N_{part} = 3$, keep

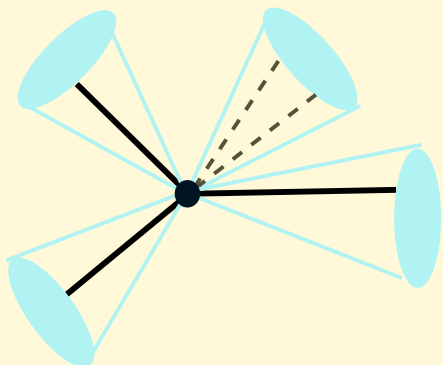


collinear double-logarithmic double counting

NOT matched,
 $N_{jet} = N_{part} = 3$,
but $N_{match} = 2$
Throw away



soft single-logarithmic double counting



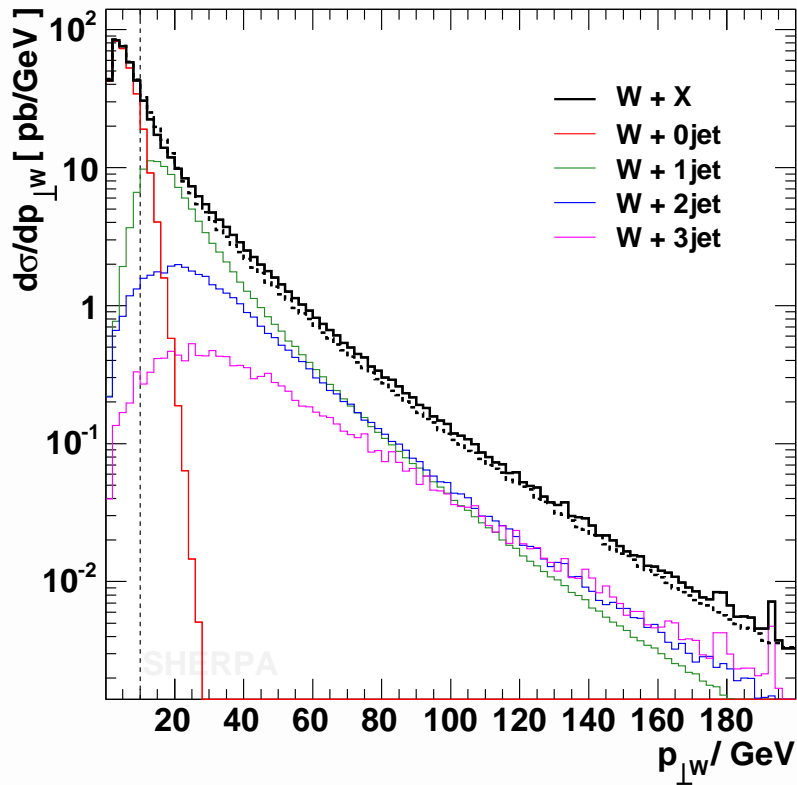
Event matched, $N_{jet} > N_{part}$:

- o Keep for inclusive sample if the unmatched jet is softer than all matched ones.
- o Throw away otherwise, or for exclusive samples.

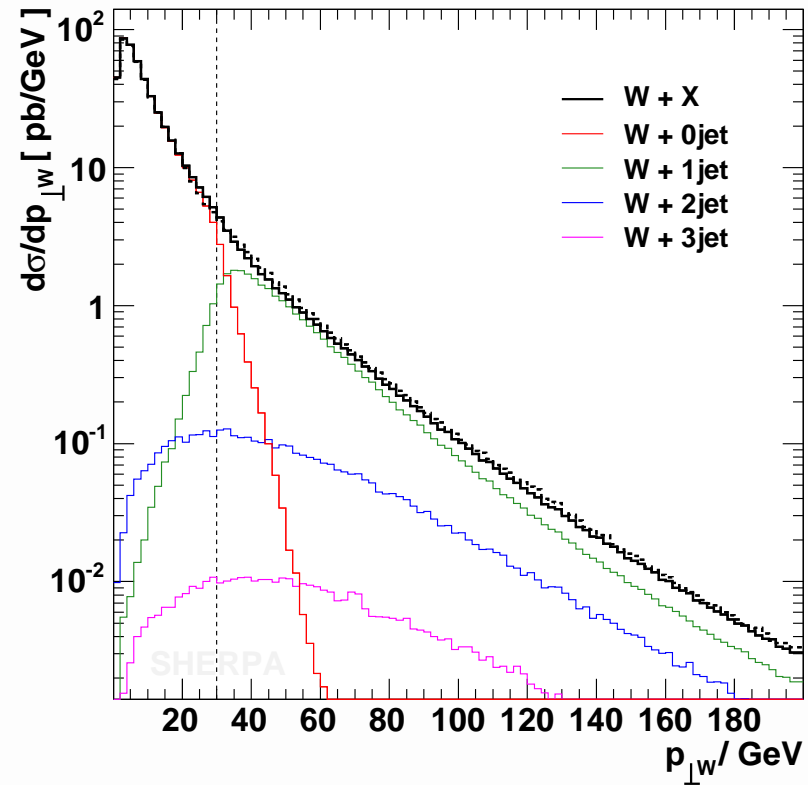
Sherpa consistency checks: variation of Q_{cut}

p_{\perp} distribution of the W^{-} in $p\bar{p} \rightarrow e^{-} \bar{\nu}_e + X$ @ $\sqrt{s} = 1.96$ TeV

[Krauss, Schällicke, S., Soff, 2004]



$Q_{\text{cut}} = 10$ GeV



$Q_{\text{cut}} = 30$ GeV

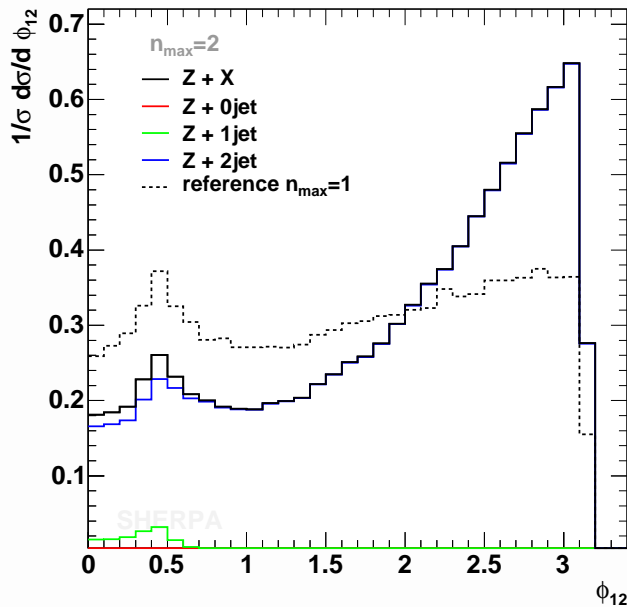
[dashed reference $Q_{\text{cut}} = 20$ GeV]

➔ combines different multiplicity final states into fully inclusive sample

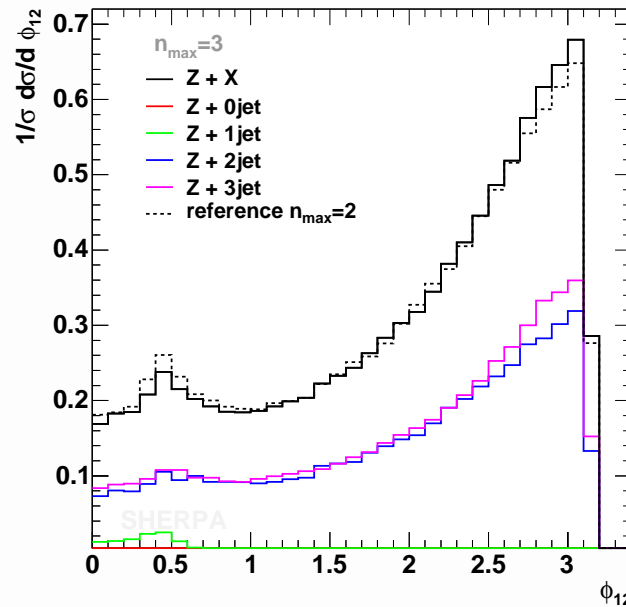
Sherpa consistency checks: variation of n_{max}

The $\Delta\phi$ separation of the two hardest k_T -jets in $pp \rightarrow e^+e^- + X$ @ LHC

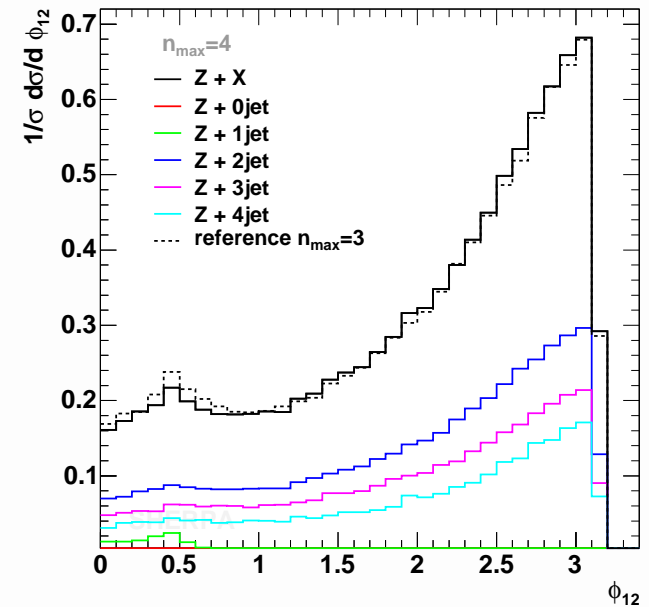
[Krauss, Schällicke, S., Soff, 2005]



$n_{max}=2$



$n_{max}=3$



$n_{max}=4$

two-jet observable in DY

[dashed reference $\tilde{n}_{max} = n_{max} - 1$]

➔ standard shower accuracy $\sum_{n=2}^{\infty} \alpha_S^n \ln^{2n} Q^2/Q_0^2$, uncorrelated emissions

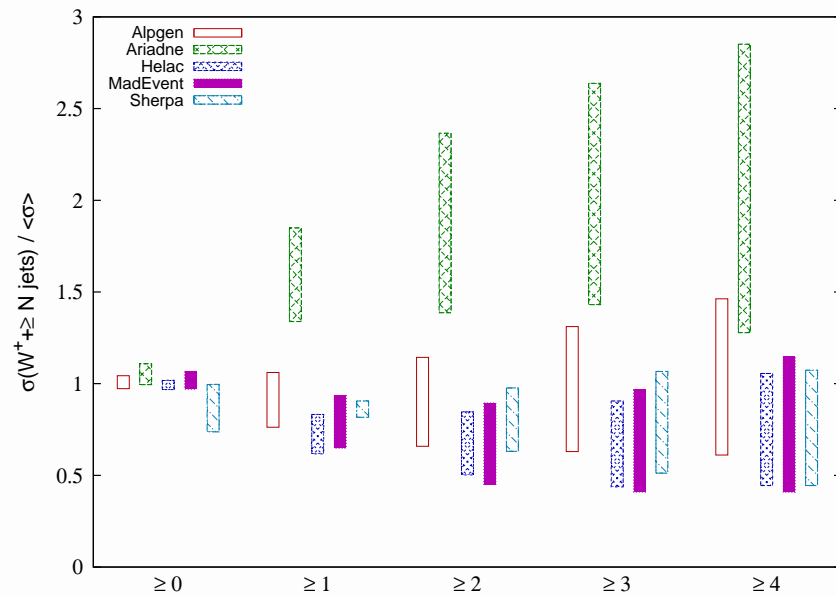
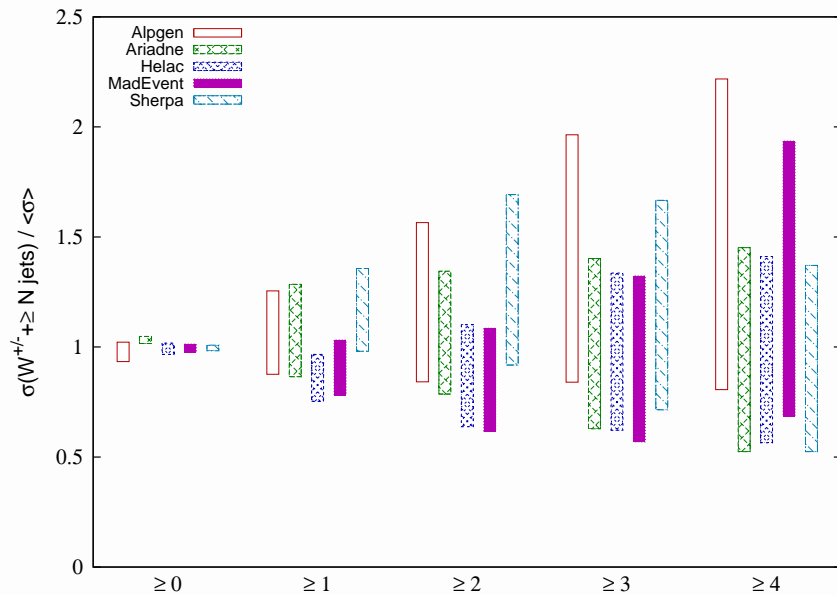
➔ merging approach yields $\alpha_S^2 \sum_{n=0}^{\infty} \alpha_S^n \ln^{2n} Q^2/Q_0^2$, full ME correlations

Comparative study of merging algorithms for W +jets

comparing implementations for ME+PS merging [Alwall et al. 2007]

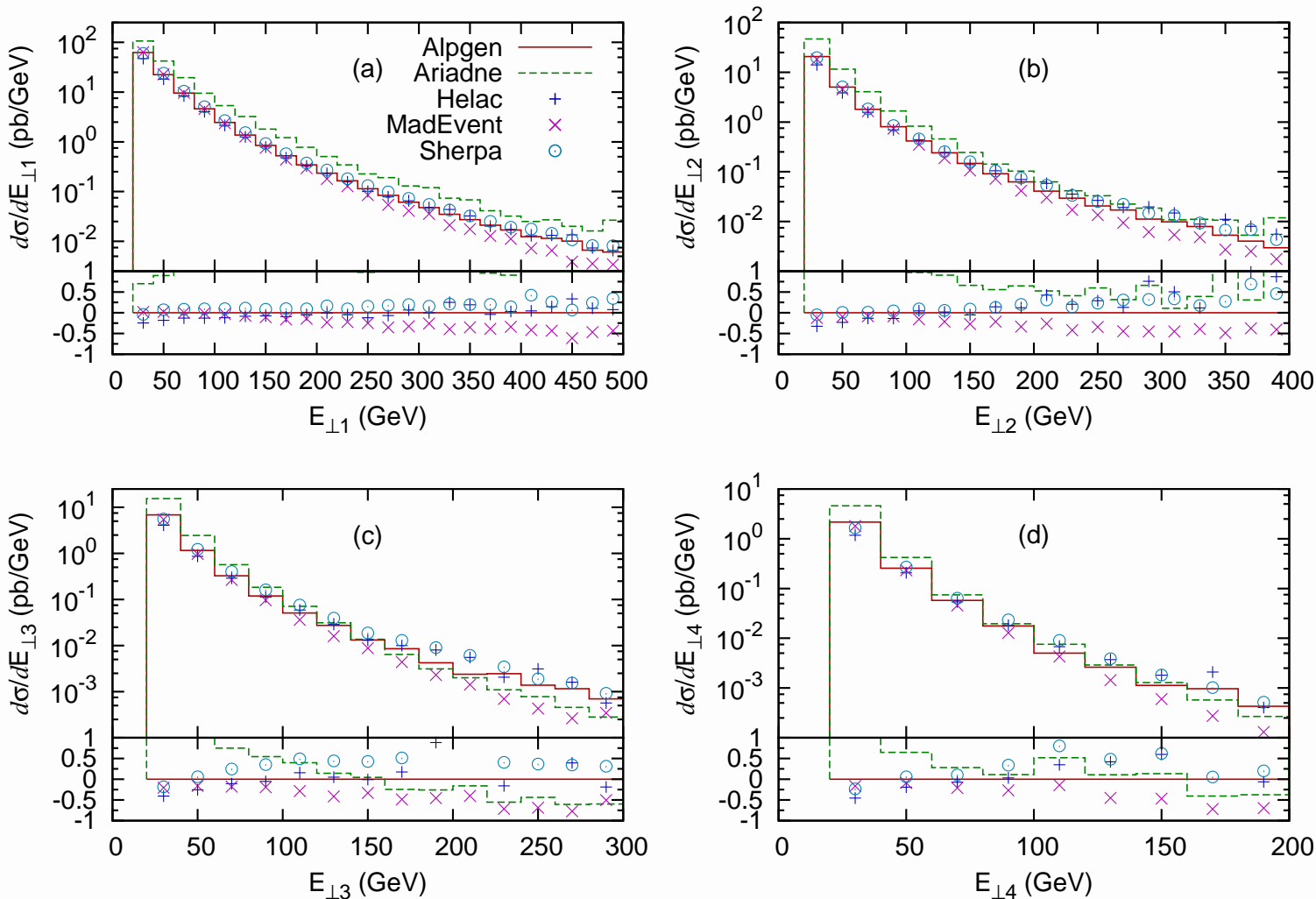
- up-to 4 matrix element jets taken into account [hadron level, UE off]
- analysed for cone jets with $\Delta R = 0.7(0.4)$, $E_T = 10(20)$ GeV at TEV(LHC)
- attempt to estimate the systematics for each generator
 - residual dependence on merging params, Q_{cut} , $E_{T,\text{clus}}$, $(\Delta R_{\text{clus}}, n_{\text{max}})$
 - scale dependencies: $\alpha_S(k_\perp$ vs. $0.5k_\perp$ vs. $2k_\perp)$, Sherpa also μ_F

multi-jet rate variations @ Tevatron & LHC



Comparative study of merging algorithms for W +jets

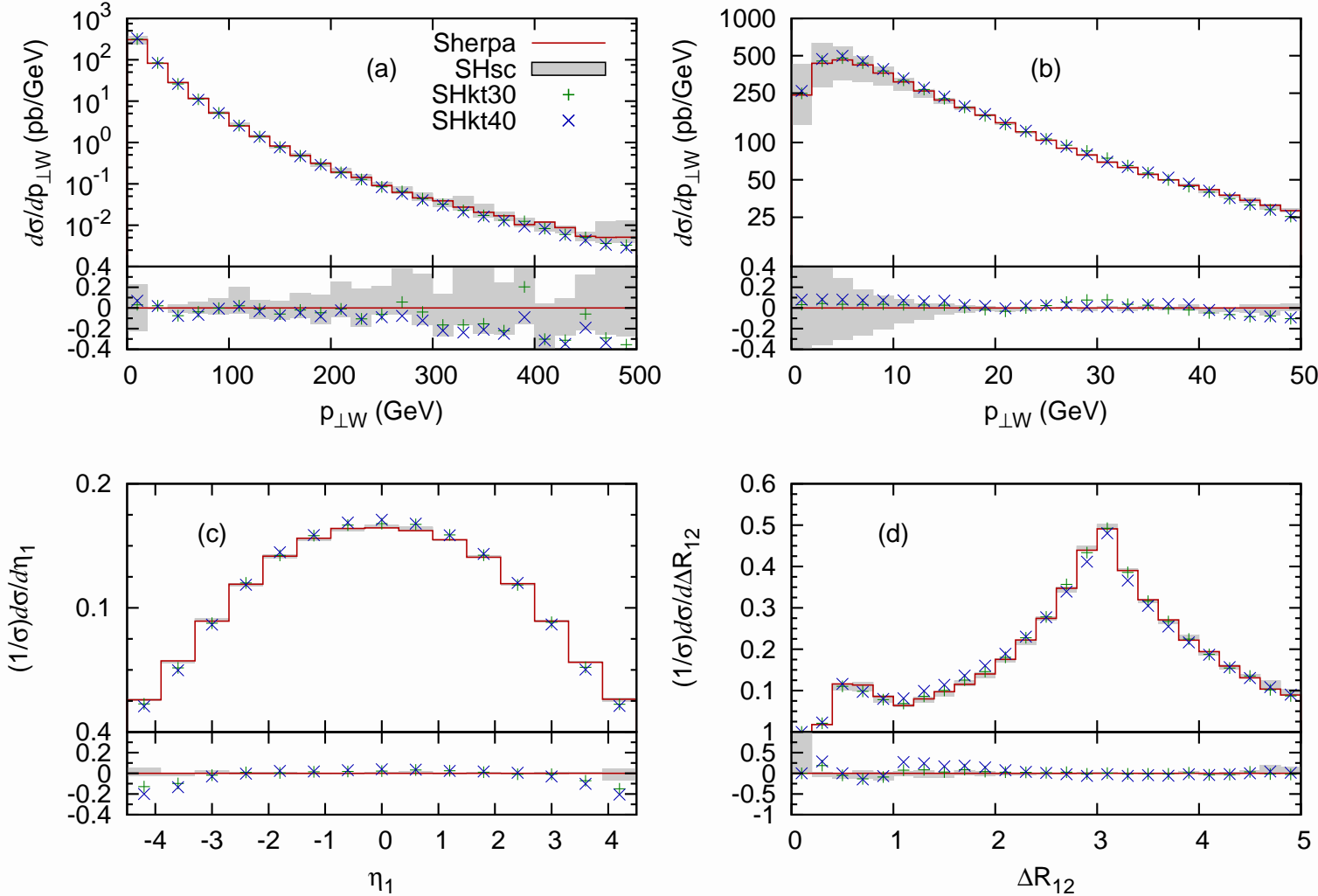
jet transverse energies @ LHC [Alpgen as reference in lower panels]



➔ similar pattern for Tevatron & LHC energies: Tevatron studies desirable

Comparative study of merging algorithms for W +jets

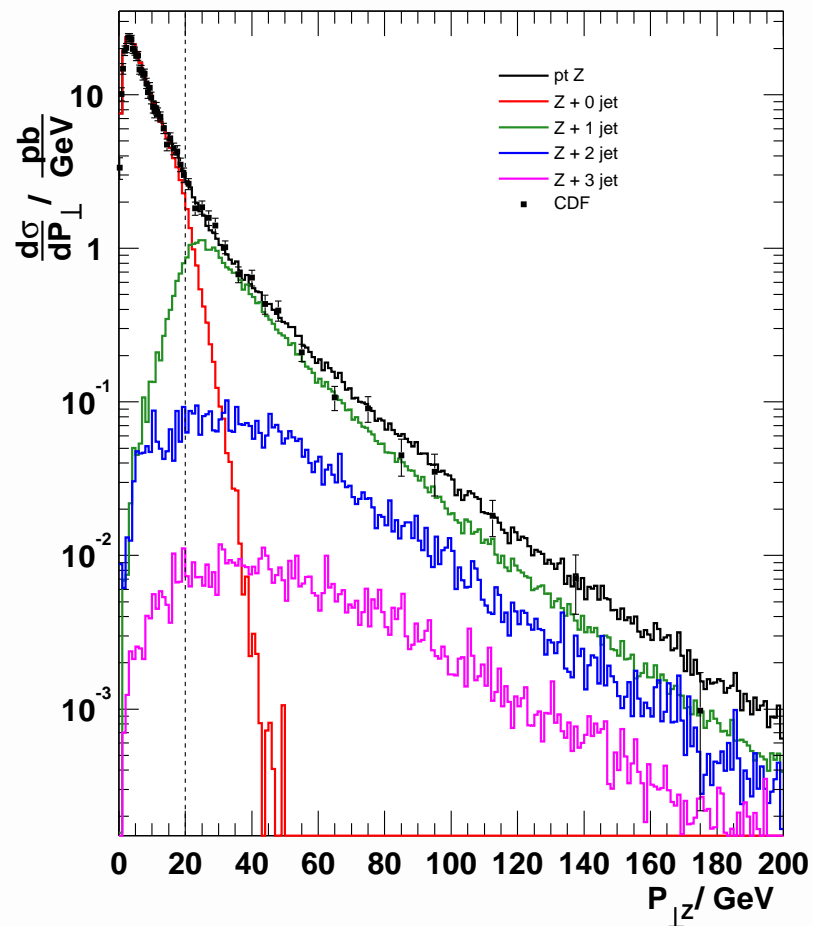
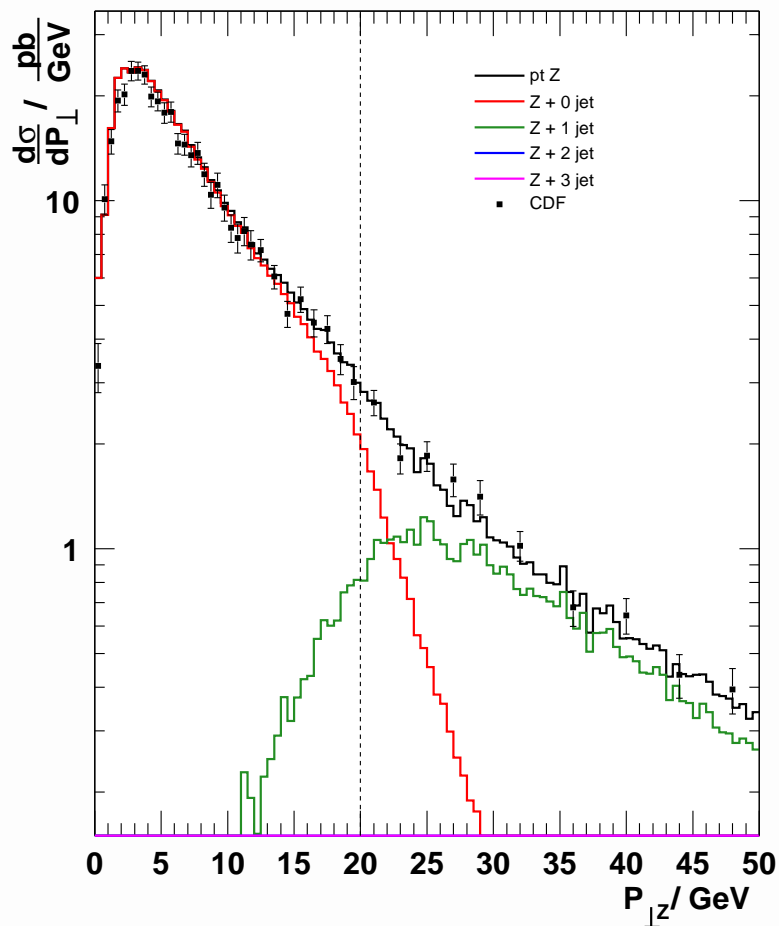
Sherpa's uncertainty band @ LHC



➔ again similar pattern for Tevatron

Sherpa comparison with data: DY

p_{\perp} distribution of the Z measured by CDF



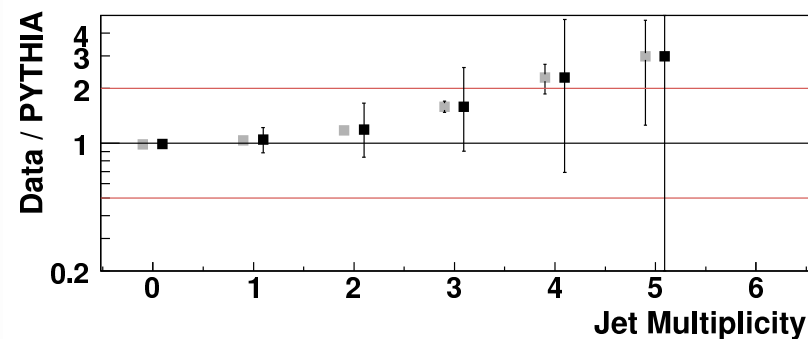
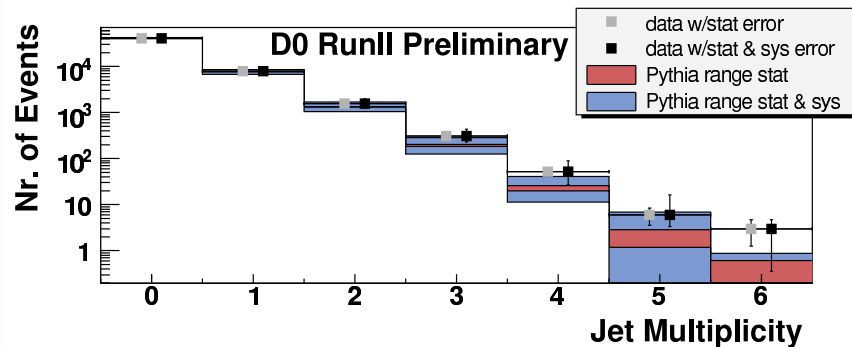
➔ distribution multiplied by appropriate K-factor!

➔ good description of soft and hard region

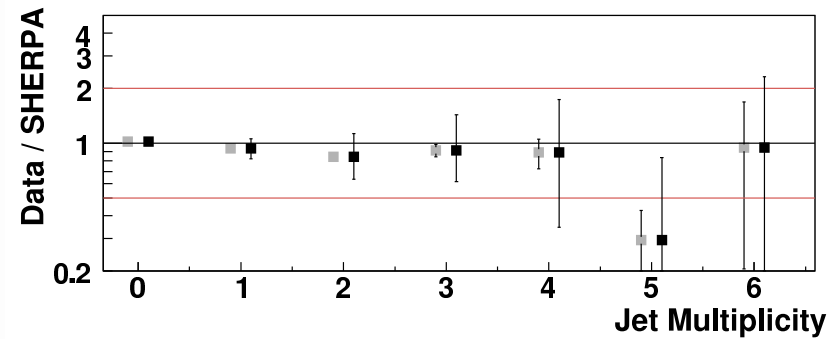
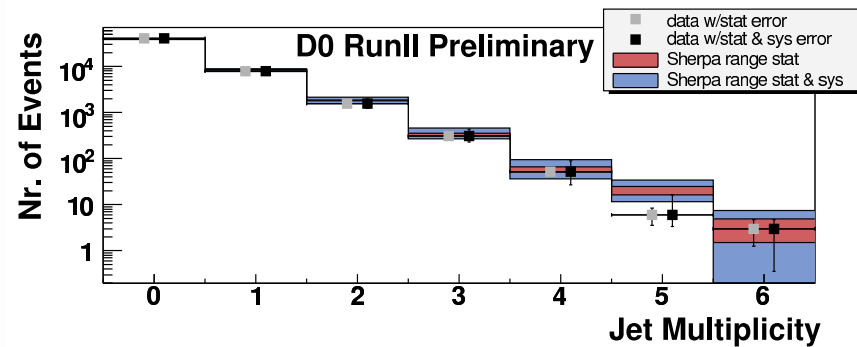
Sherpa comparison with data: DY

e^+e^- +jets @ Tevatron RunII: jet-multiplicities [DØ Note 5066]

Pythia



Sherpa

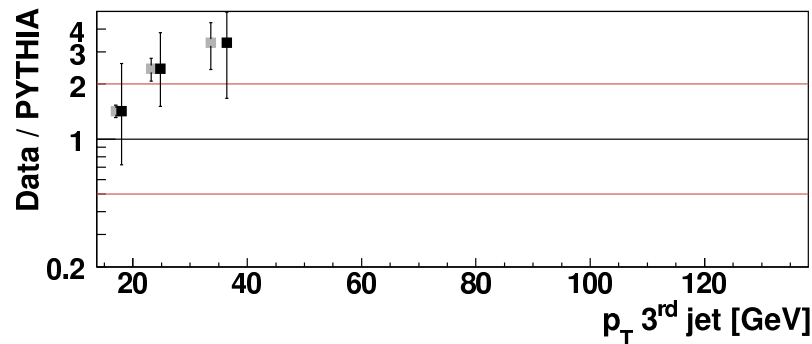
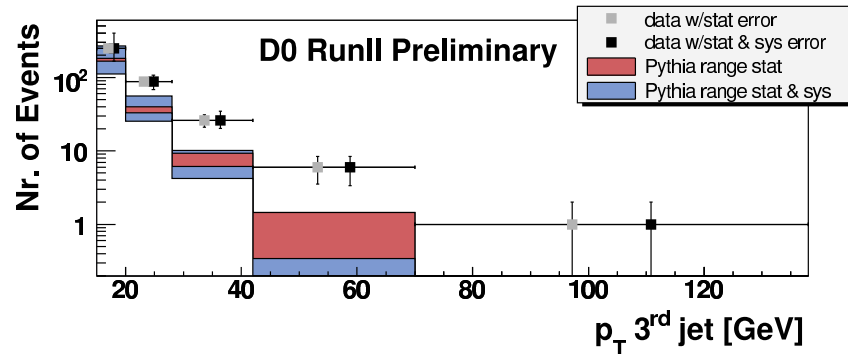


- ➔ inclusive samples normalised to total number of measured events
- ➔ pure shower approach underestimates multi-jet contributions

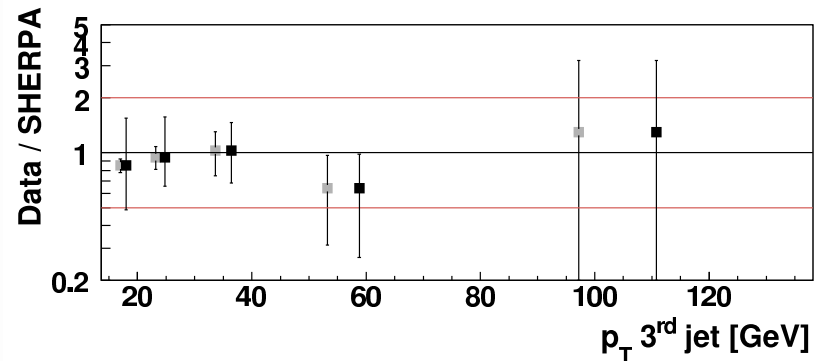
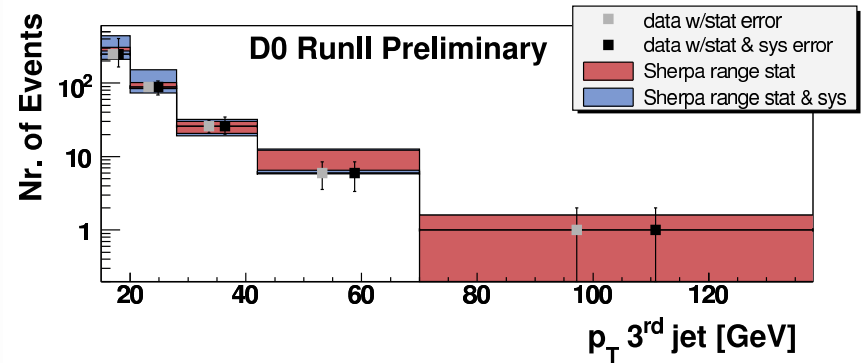
Sherpa comparison with data: DY

$e^+e^- + \text{jets}$ @ Tevatron RunII: p_{\perp} of the third jet [DØ Note 5066]

Pythia



Sherpa



➔ the more hard jets you require the worse gets the leading log approximation

➔ similar analyses done for Alpgen within CDF

Alpgen + Herwig for $t\bar{t}+X$: consistency checks

inclusive $t\bar{t}$ production @ Tevatron & LHC [Mangano et al. 2006]

- ME generation Tevatron(LHC): $E_{T,min} = 20(30)$ GeV, $R_{min} = 0.7(0.7)$
- matching parameters: $E_{T,clus} = 25(36)$ GeV, $R_{match} = 1.5 \times 0.7$
- inclusive xsecs: merged $t\bar{t} + 0, 1$ (S_1) vs. $t\bar{t} + 0, 1, 2, 3$ (S_3) hard partons

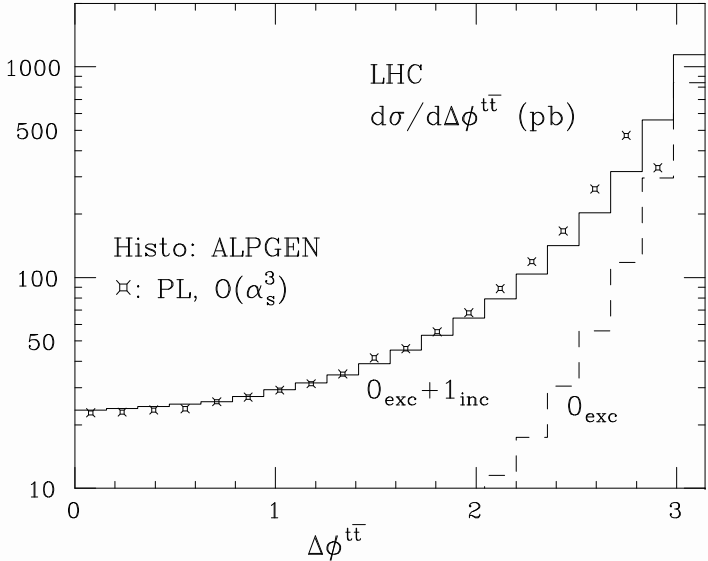
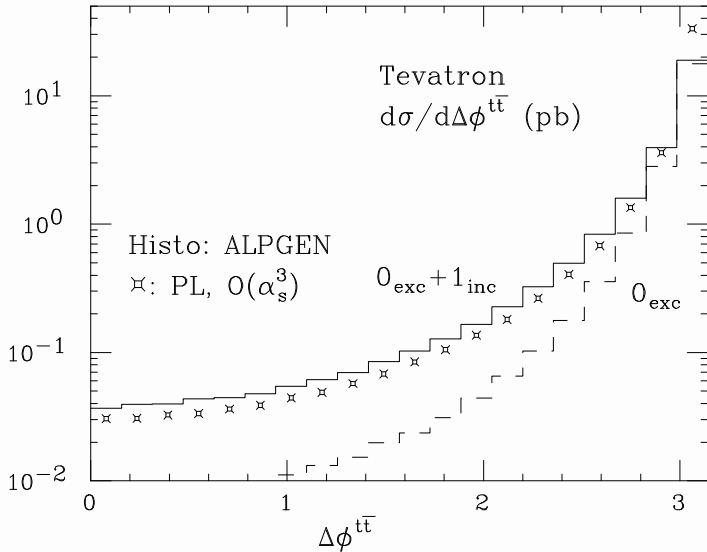
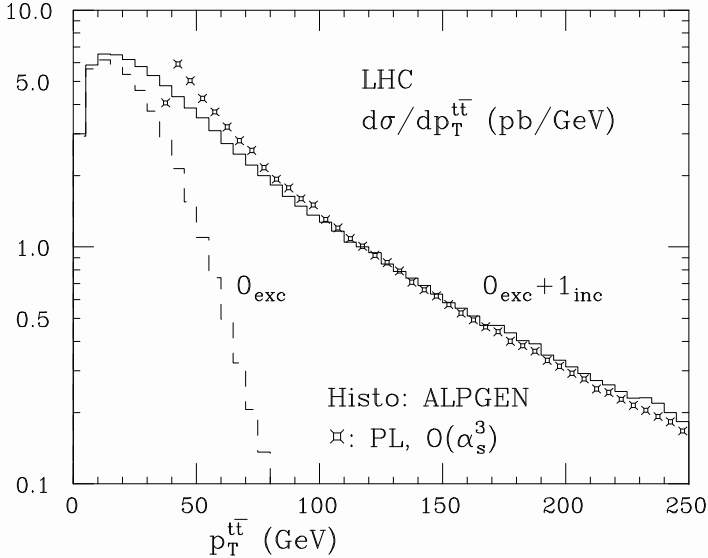
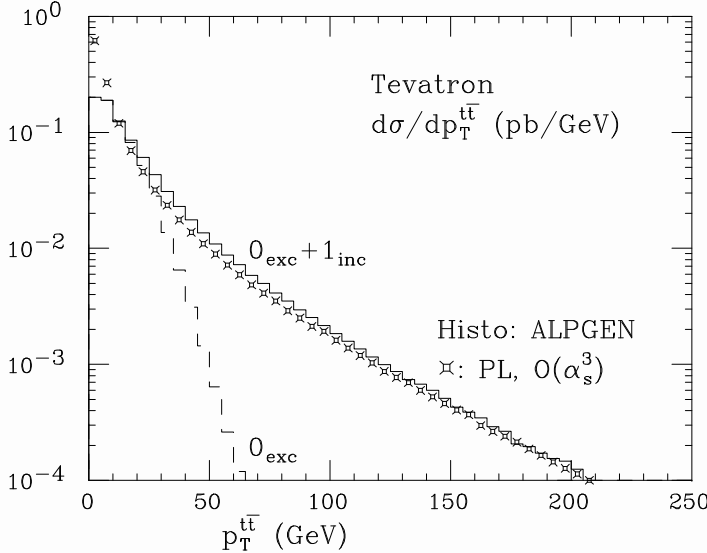
	Tevatron	LHC
0_{exc}	3.42	217
1_{inc}	0.78	252
total	4.20	469

	Tevatron	LHC
0_{exc}	3.42	216.6
1_{exc}	0.66	149.9
2_{exc}	0.09	65.8
3_{inc}	0.010	29.9
total	4.18	462.2

➔ naive leading order results for $t\bar{t}+X$ Tevatron(LHC): **4.37(471) pb**

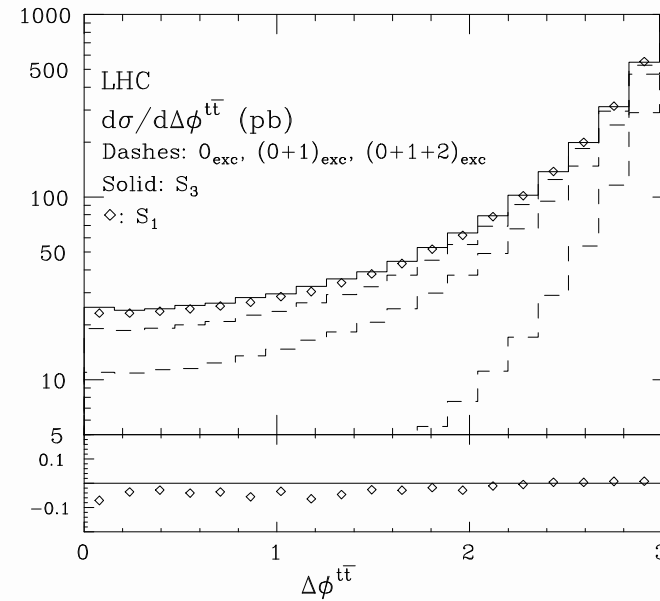
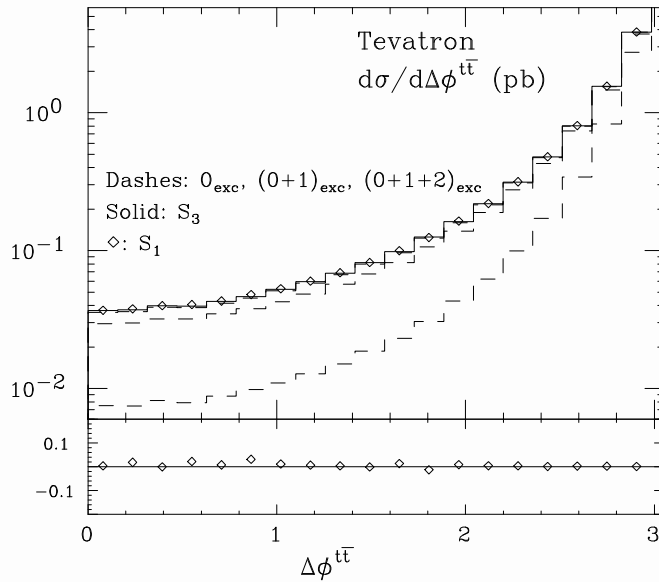
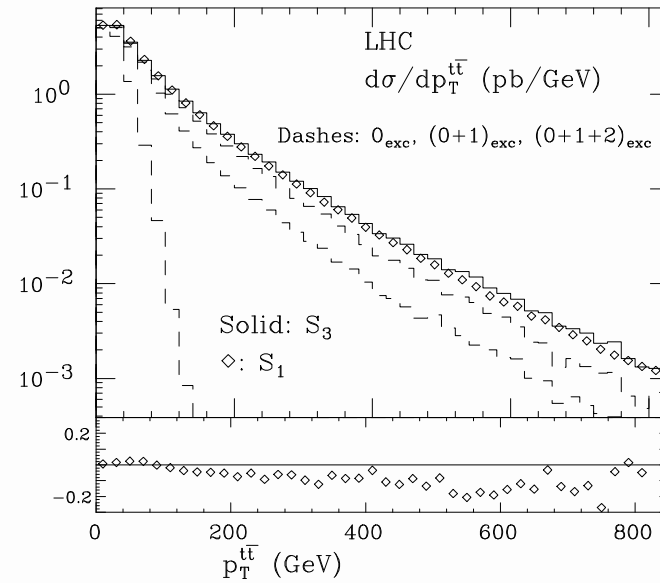
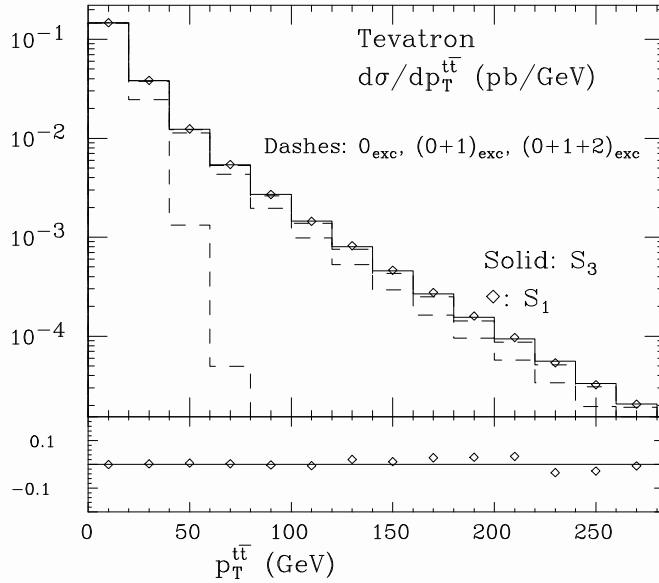
Alpgen + Herwig for $t\bar{t}+X$: consistency checks

$p_T^{t\bar{t}}$ and $\Delta\phi^{t\bar{t}}$ distributions @ Tevatron & LHC (S_1)



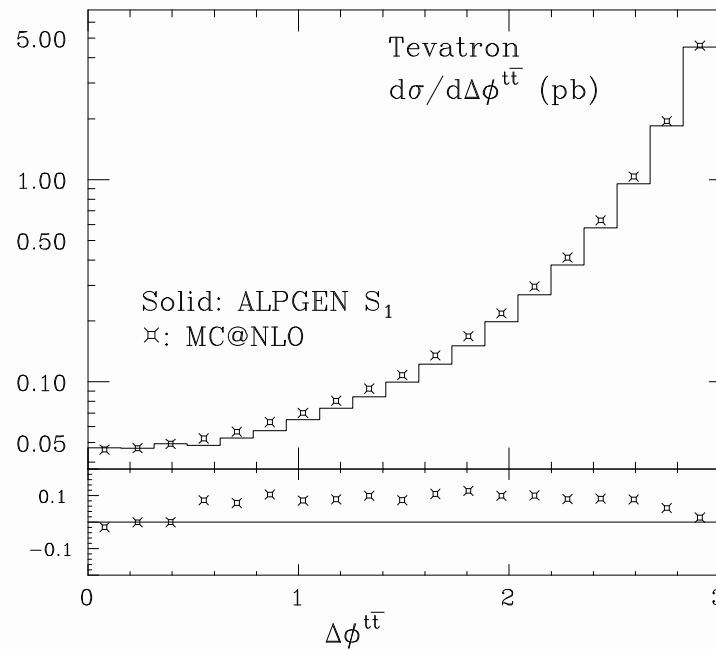
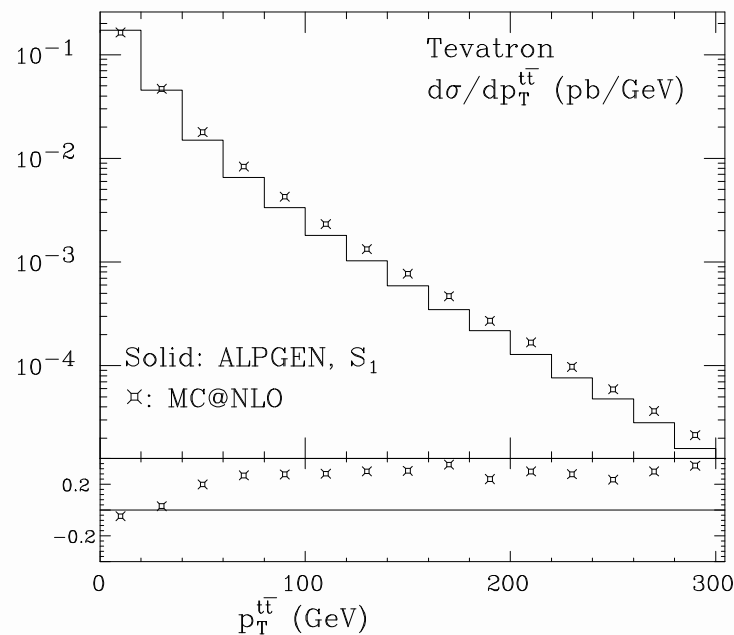
Alpgen + Herwig for $t\bar{t}+X$: consistency checks

$p_T^{t\bar{t}}$ and $\Delta\phi^{t\bar{t}}$ distributions @ Tevatron & LHC (S_1 vs. S_3)



Alpgen + Herwig for $t\bar{t}+X$: comparison with MC@NLO

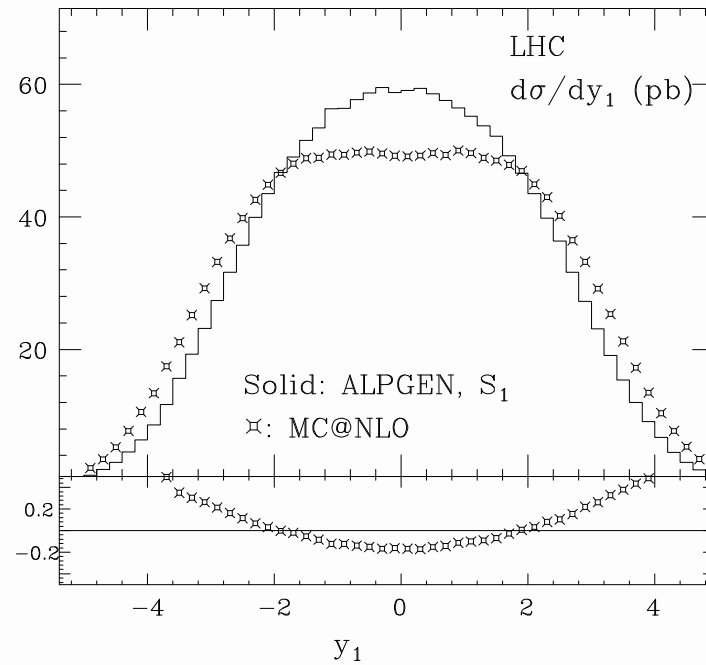
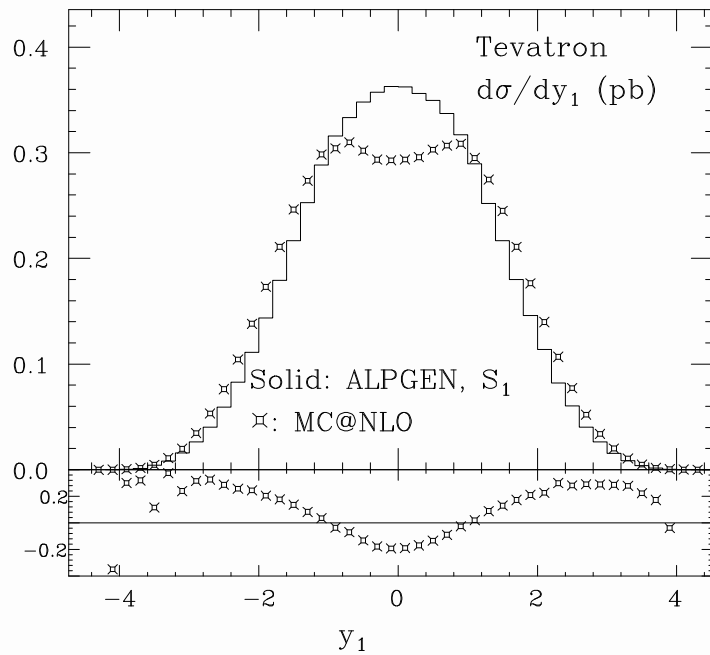
$p_T^{t\bar{t}}$ and $\Delta\phi^{t\bar{t}}$ distributions @ Tevatron (S_1)



- ➔ K -factor of 1.36 applied for Alpgen
- ➔ good agreement for inclusive quantities (both at Tevatron & LHC)
- ➔ but, ...

Alpgen + Herwig for $t\bar{t}+X$: comparison with MC@NLO

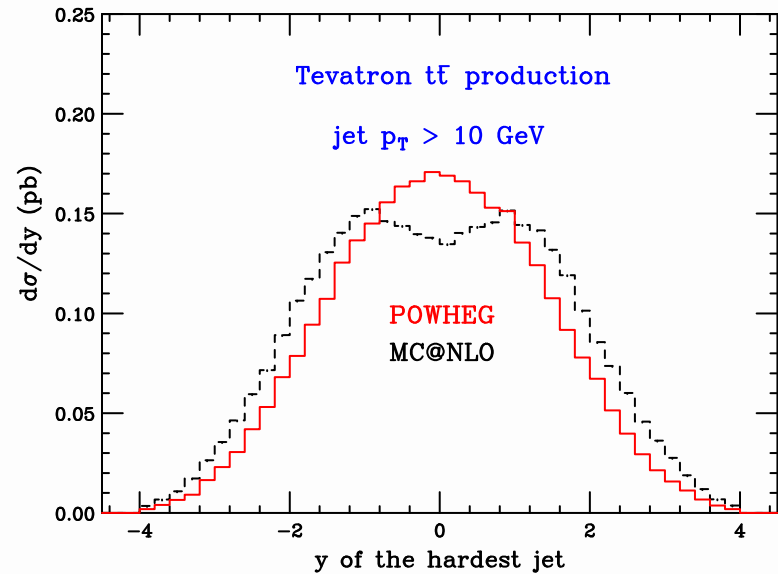
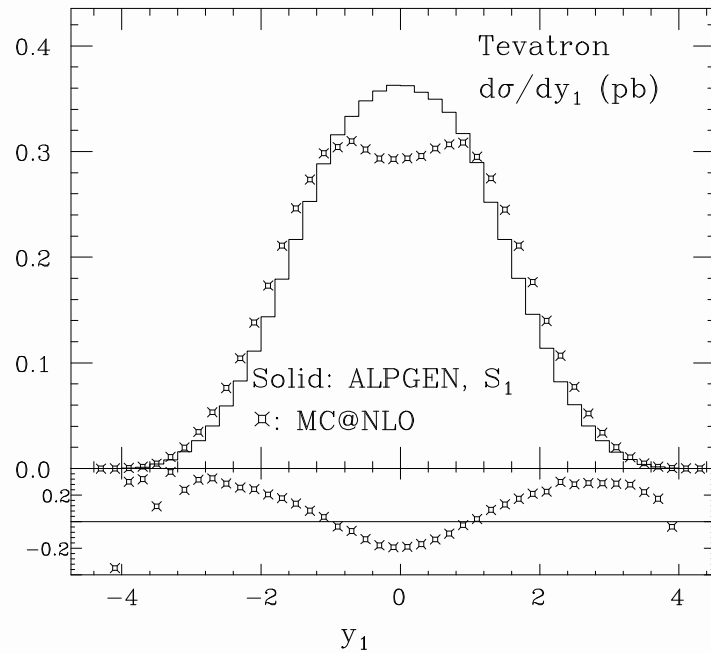
rapidity distribution of hardest jet @ Tevatron & LHC



➔ significant difference in shape around $y_1 \approx 0$

Alpgen + Herwig for $t\bar{t}+X$: comparison with MC@NLO

rapidity distribution of hardest jet @ Tevatron & LHC



- ➔ significant difference in shape around $y_1 \approx 0$
- ➔ fixed order result looks like Alpgen [confirmed by POWHEG, $t\bar{t}+\text{jet}$ @ NLO]
- ➔ effect of shower dead-cone in MC@NLO [new mass treatment in Herwig++]

The LHC physics programme requires a detailed understanding of QCD

improved theoretical modelling of QCD jets in Monte Carlos

- merging higher-order matrix elements and parton showers
 - matrix elements for final states with many hard partons
 - parton showers, hadronisation, UE for realistic events
 - inclusive samples independent on parton level generation
- improved predictions for multi-jet events [SM backgrounds, Higgs signals]



other directions

- MC at NLO: MC@NLO, POWHEG
- new parton shower formalism(s) [Krauss, S. 2007, Weinzierl et al. 2007]

future steps

- apply merging techniques to BSM processes [Alwall, Maltoni, Wacker]
- towards automatisisation of NLO + shower
- MC at NLO + tree-level merging