15th Sep 2008
@IPMU workshop on quantum BH

Holographic Nuclei

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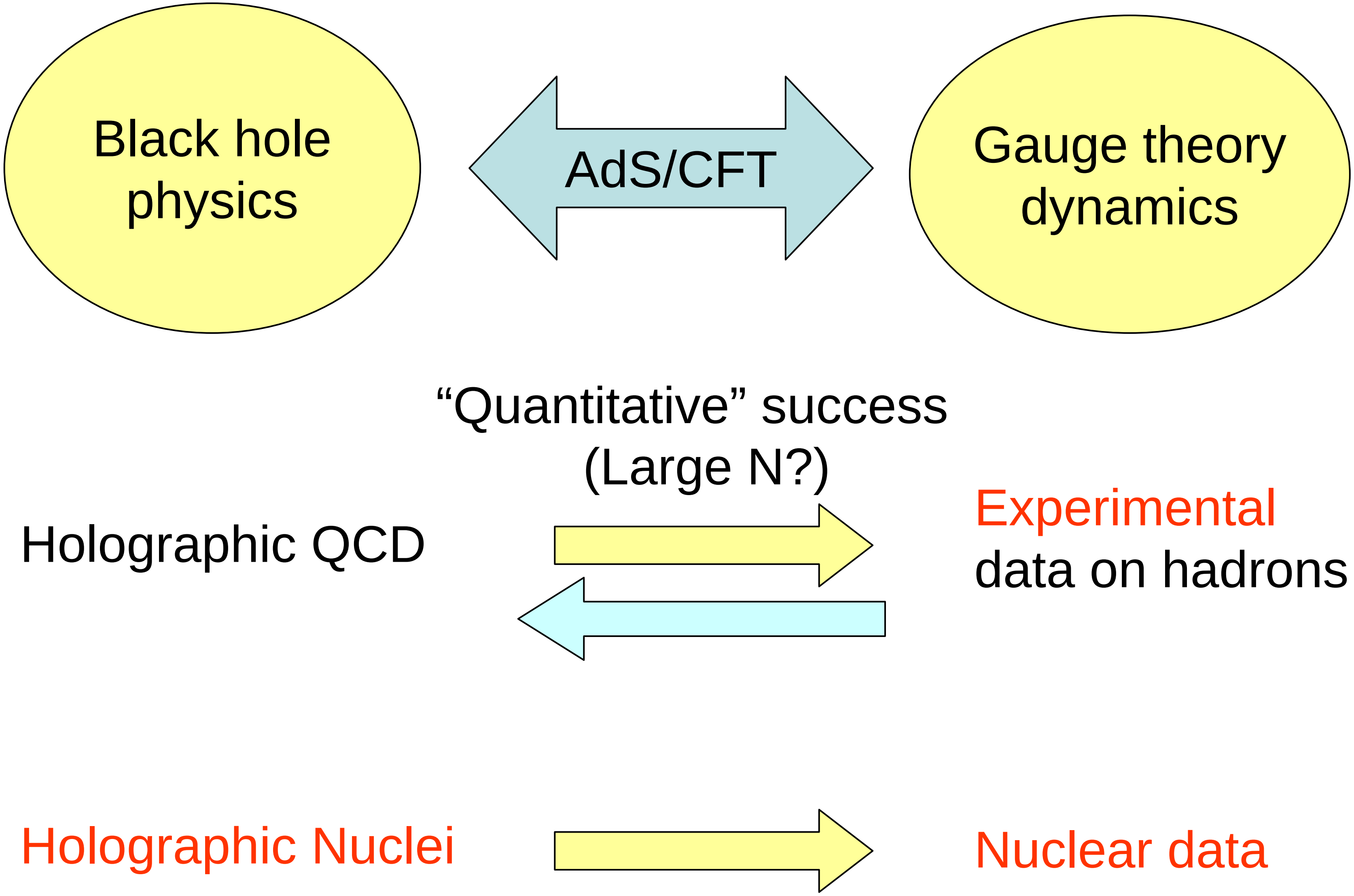
arXiv/0806.3122(hep-th)   T.Sakai, S.Sugimoto, KH

Work in progress   KH

1. A Road to Nuclear Physics


Black hole physics
AdS/CFT
Gauge theory dynamics
“Quantitative” success
(Large N?)
Holographic QCD
Experimental
data on hadrons
Holographic Nuclei
Nuclear data

How to explore the Sakai-Sugimoto model

Achievements of the model

Analytic calculations of :

* Meson spectrum
* Chiral lagrangian of all mesons
* Baryon spectrum
* Finite tenperature and phase transition

....

How to explore further?

Compute more quantities in the model

Resolve problems of the model

Compute more in the model

Glueball Spectrum

[Csaki,Ooguri,Oz,Terning (97)]
[Brower,Mathur,Tan (03)] etc

Meson Spectrum

[Sakai-Sugimoto(04,05)]

Baryon Spectrum

[Hata et al.(07)]

Glueball decay

[Terashima,Tan,KH (07)]

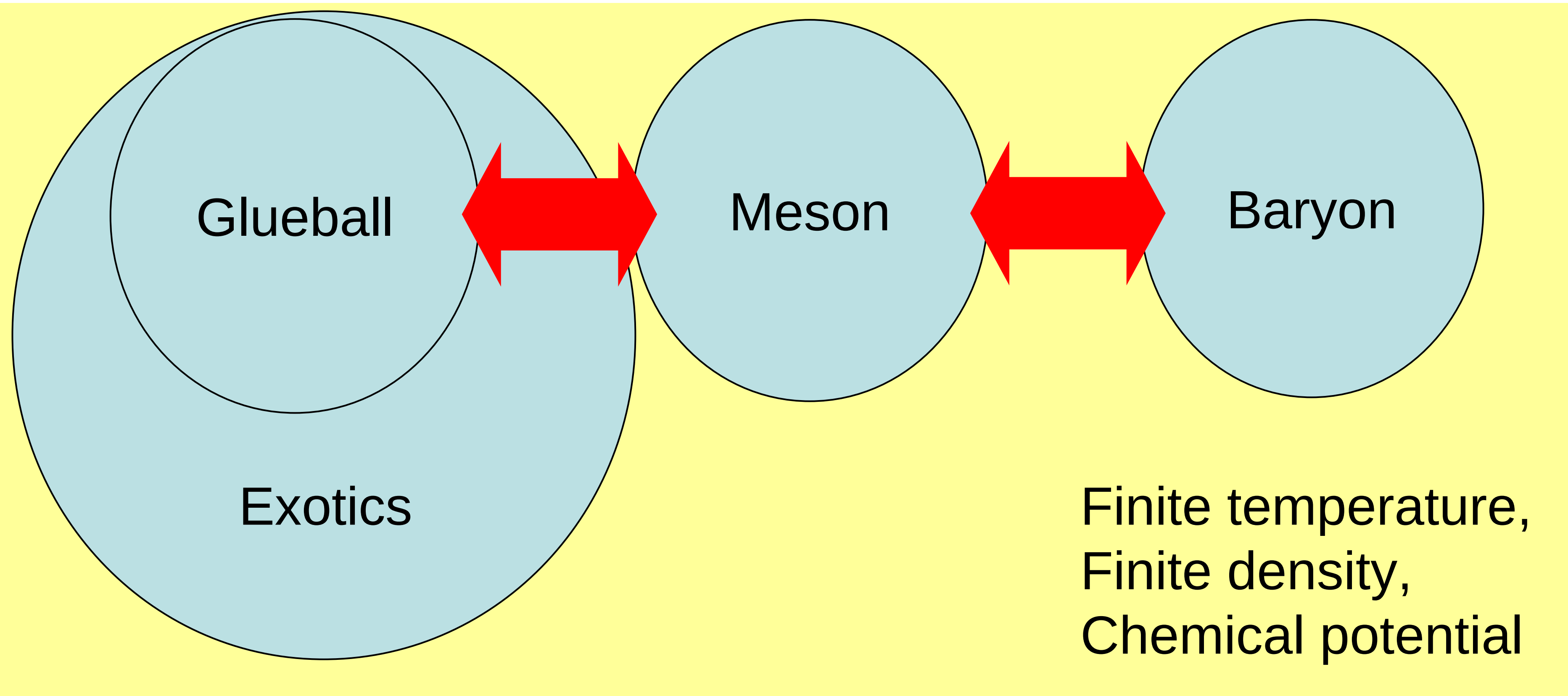
$f_0(1500)$ is identified as a glueball.

Decay $f_0(1500) \to 4\pi^0$ is suppressed.

Charge radius, form factors

[Sakai,Sugimoto,KH, 0806.3122]

Compute more in the model



More dynamical quantities :

Jet quenching parameters, viscosity of plasma, …

A road to nuclear physics?

Most of developments so far : hadron physics,
not really nuclear physics!

Nuclear physics

Light nuclei : Can be studied by nuclear force
Heavy nuclei : Complicated quantum many body problem

Nucleon number || large *A*

Nucleon || D-brane

Possible Gravity dual?

Heavy nuclei have dual geometry description

Plan

☑ 1. A Road to Nuclear Physics — 4 slides

2. Brief Review of Sakai-Sugimoto — 4 slides

3. Holographic Baryons — 7 slides

[T.Sakai, S.Sugimoto and KH, 0806.3122]

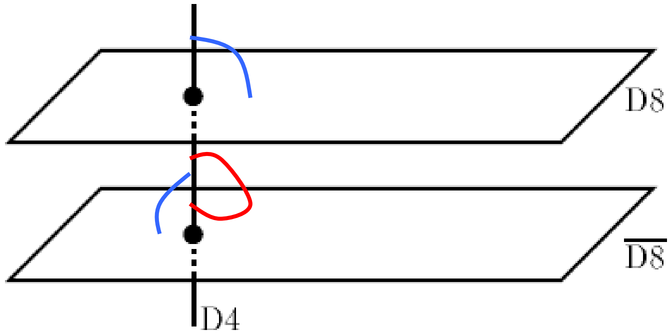
4. Holographic Nuclei — 5 slides

[KH, to appear]

2. Sakai-Sugimoto : Review

Best model of holographic QCD : Sakai-Sugimoto model

- Open string side (D-branes)



	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D4	○	○	○	○					
D8	○	○	○		○	○	○	○	○
$\overline{\text{D8}}$	○	○	○		○	○	○	○	○

- Nc D4 wrap S^1 with radius $2\pi/M_{\mathrm{KK}}$ [Witten(98)]
  Gauginos satisfy anti-periodic boundary condition
  → Massless gluons at low energy, SU(Nc) gauge
- Nf D8 intersecting D4s → Nf massless L-quarks
- Nf $\overline{\text{D8}}$ intersecting D4s → Nf massless R-quarks

Massless QCD is brane-engineered at low energy

D8 x $\overline{\text{D8}}$ gauge group = Chiral symmetry $U(N_f)_{\mathrm{L}} \times U(N_f)_{\mathrm{R}}$

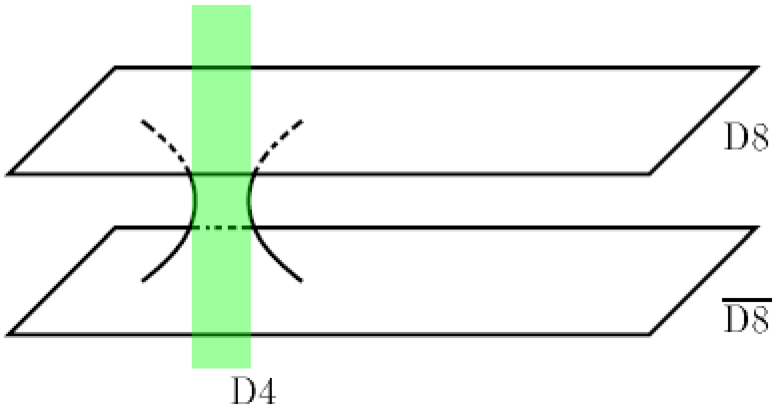
- Closed string side (gravity)

D8s : probe ($N_c \gg N_f$)

Geometry : [Witten(98)]

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$e^\phi = g_s\left(\frac{U}{R}\right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{V_4}\epsilon_4, \quad f(U) = 1 - \frac{U_{KK}^3}{U^3}$$



Geometry truncated at $U = U_{KK}$

→ D8 and $\overline{\text{D8}}$ are connected

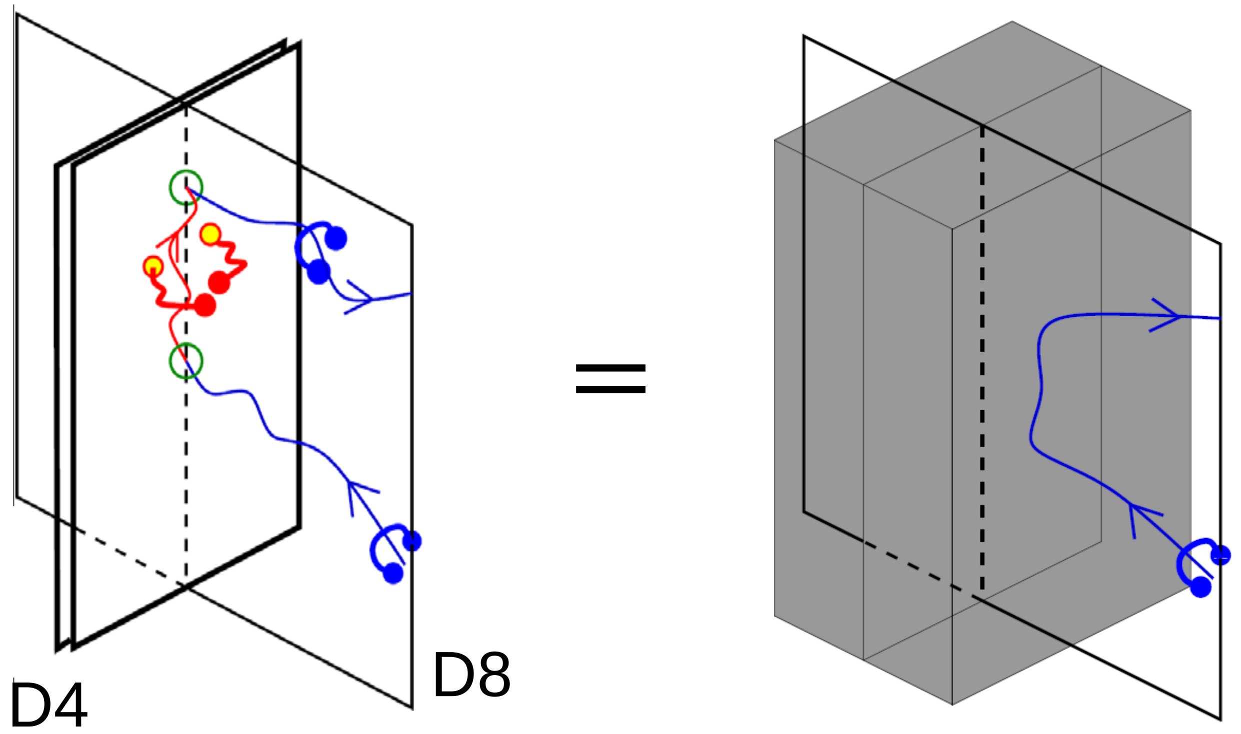
→ Spontaneous Chiral sym. br.

$$U(N_f)_L \times U(N_f)_R \to U(N_f)_V$$

Sugra parameters are

$$R^3 = \frac{g_{YM}^2 N l_s^2}{2M_{KK}}, \quad U_{KK} = \frac{2}{9} g_{YM}^2 N M_{KK} l_s^2, \quad g_s = \frac{g_{YM}^2}{2\pi M_{KK} l_s}$$

Gauge/Gravity correspondence implies….



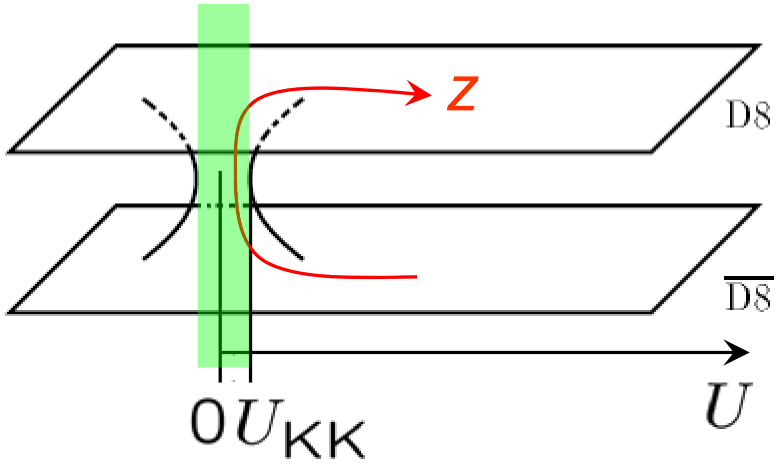
・ D8 → bound states of quarks（Mesons, Baryons）

KK modes of gauge fields on the D8 = Mesons

D8-brane action = Chiral lagrangian

・ gravitons = bound states of gluons（Glueballs）

Meson sector in the model



Redefinition of a coordinate：

$$U^3 \equiv U_{\text{KK}}^3 + U_{\text{KK}} z^2$$

D8-brane action on the curved background $(M_{\text{KK}} = 1)$

$$-\frac{\lambda N_c}{216\pi^3}\int d^4x dz \text{tr}\left[\frac{1}{2}(1+z^2)^{-1/3}F_{\mu\nu}^2 + (1+z^2)F_{\mu z}^2\right]$$

$$+\frac{N_c}{24\pi^2}\text{tr}\int\left[AF^2 - \frac{i}{2}A^3F - \frac{1}{10}A^5\right]$$

KK fluctuation analysis

A_μ → Vector meson tower spectrum
A_z → Massless pion

3. Holographic baryons

arXiv/0806.3122(hep-th)

T.Sakai, S.Sugimoto, KH

Baryons in the model

Baryon = D4 Wrapping S^4 [Witten(98), Gross,Ooguri(98)]

= Instanton in (x^1, x^2, x^3, z) of D8

[Sakai, Sugimoto(04)]

$$-\frac{\lambda N_c}{216\pi^3}\int d^4x dz \mathrm{tr}\left[\frac{1}{2}(1+z^2)^{-1/3}F_{\mu\nu}^2 + (1+z^2)F_{\mu z}^2\right]$$
$$+\frac{N_c}{24\pi^2}\mathrm{tr}\int\left[AF^2 - \frac{i}{2}A^3F - \frac{1}{10}A^5\right]$$

Instanton charge sources U(1)v

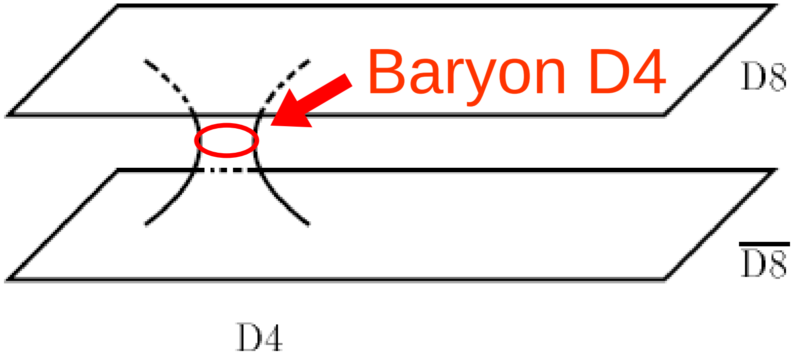
“Electric energy” + “Potential in z”

→ Stabilization of the instanton size $\sim \mathcal{O}(\lambda^{-1/2})$

Quantization of instantons → Baryon spectrum

[Hata, Sakai, Sugimoto, Yamato (HSSY) (07)]

Baryon solution : dyonic instanton [HSSY(07)]



Localized at $z = 0$

Geometry can be expanded around flat space

Solution : BPST instanton + electrostatic potential

$$A_M^{\rm cl} = -if(\xi) g \partial_M g^{-1} , \quad \widehat{A}_0^{\rm cl} = \frac{N_c}{8\pi^2\kappa}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2}\right] , \quad A_0 = \widehat{A}_M = 0$$

$$f(\xi) = \frac{\xi^2}{\xi^2+\rho^2} , \quad g(x) = \frac{(z-Z) - i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi} , \quad \xi = \sqrt{(z-Z)^2 + |\vec{x}-\vec{X}|^2}$$

Inserting this back to the action leads to a potential

$$U(\rho, Z) = 8\pi^2\kappa\left(1 + \frac{\rho^2}{6} + \frac{N_c^2}{5(8\pi^2\kappa)^2}\frac{1}{\rho^2} + \frac{Z^2}{3}\right) \qquad \kappa = \frac{\lambda N_c}{216\pi^3}$$

⇒ Size is stabilized, $\rho_{\rm cl}^2 = \frac{N_c}{8\pi^2\kappa}\sqrt{\frac{6}{5}} , \quad Z_{\rm cl} = 0$

Quantization of the instanton [HSSY(07)]

Moduli space approximation

- Pseudo-moduli : $(\vec{X}, Z)$, y_I $(I = 1, 2, 3, 4)$
- Moduli potential : $L = \frac{M_0}{2}(\dot{\vec{X}}^2 + \dot{Z}^2) + M_0\, \dot{y}_I^2 - U(\rho, Z)$

Quantum mechanics for (Z, y^I)

$M_0 = 8\pi^2\kappa$

$SO(4) \simeq (SU(2)_I \times SU(2)_J)/\mathbb{Z}_2$: Isospin and spin

$I = J = l/2 \quad l = 1, 3, 5, \cdots$

Z, ρ → Harmonic-like potential → n_ρ, n_z

Baryon state : $|\vec{p}, B, s\rangle \qquad B \equiv (l, I_3, n_\rho, n_z)$

Proton $B = (1, 1/2, 0, 0) \qquad |\, p \uparrow\rangle \propto R(\rho)\psi_Z(Z)(a_1 + ia_2)$

Neutron $B = (1, -1/2, 0, 0) \qquad |\, n \uparrow\rangle \propto R(\rho)\psi_Z(Z)(a_4 + ia_3)$

$$R(\rho) = \rho^{-1+2\sqrt{1+N_c^2/5}} e^{-\frac{M_0}{\sqrt{6}}\rho^2} , \quad \psi_Z(Z) = e^{-\frac{M_0}{\sqrt{6}}Z^2} \qquad a_I \equiv y_I/\rho$$

Static properties of baryons [Sakai, Sugimoto, KH]

Chiral currents

$J_L(r), J_R(r)$

→

Static properties

Charge radius
Magnetic moments
Axial radius, coupling

Chiral symmetry = D8 gauge symmetry at $z = \pm\infty$

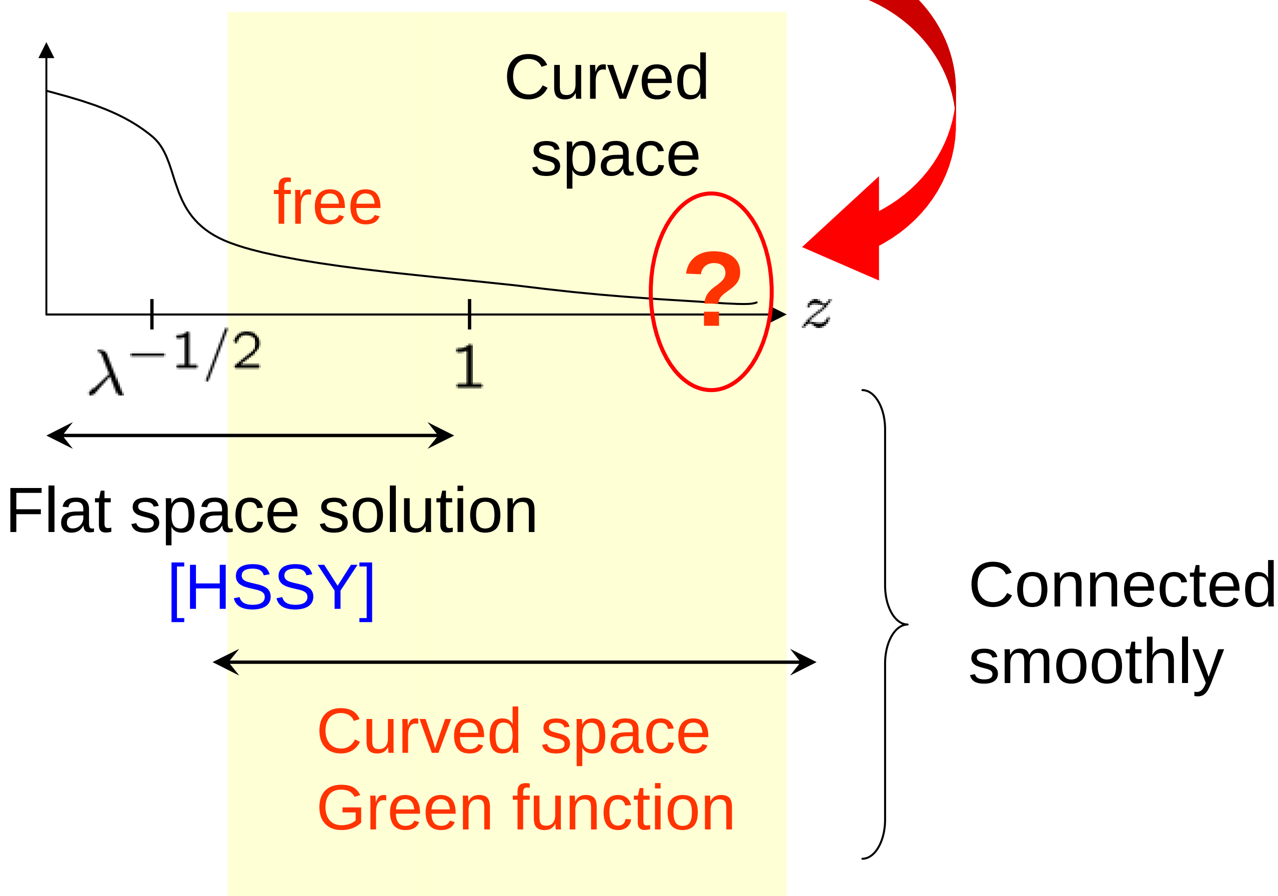
$$S\Big|_{\mathcal{O}(\mathcal{A}_L,\mathcal{A}_R)} = -2\int d^4x \, \mathrm{tr}\left(\mathcal{A}_{L\mu}\mathcal{J}_L^\mu + \mathcal{A}_{R\mu}\mathcal{J}_R^\mu\right)$$

$$\left(\begin{array}{l} \mathcal{A}_\alpha(x^\mu, z) = \mathcal{A}_\alpha^{\mathrm{cl}}(x^\mu, z) + \delta\mathcal{A}_\alpha(x^\mu, z) \\ \delta\mathcal{A}_\mu(x^\mu, z \to +\infty) = \mathcal{A}_{L\mu}(x^\mu) \ , \quad \delta\mathcal{A}_\mu(x^\mu, z \to -\infty) = \mathcal{A}_{R\mu}(x^\mu) \end{array}\right)$$

$$\Rightarrow \quad J_{L,R}(r) = \mp\frac{\lambda N_c}{216\pi^3}\left[(1+z^2)F_{\mu z}^{(\mathrm{sol})}\right]_{z=\pm\infty}$$

Evaluation of chiral currents

We need the solution at $z = \pm\infty$



⇨ Explicit expression of the chiral currents

Static properties of baryons

1) Baryon number current $J_B(r)$

$$1 = N_B = \int_0^\infty dr \rho_B(r), \quad \rho_B(r) \equiv 4\pi r^2 \langle J_B^0(r) \rangle$$

⇒ Isoscalar mean square radius $\langle r^2 \rangle_{I=0} = \int_0^\infty dr\, r^2 \rho_B(r)$

Our result : $\langle r^2 \rangle_{I=0}^{1/2} \simeq 0.742 \text{ fm}$

(Input : $M_{\rm KK} = 949 \text{ MeV}$, $\kappa = 0.00745$)

2) Vector current $J_V^0(r)$

Isovector charge density turns out to be $\rho_{I=1}(r) = \rho_B(r)$

$Q_{\rm em} = I_3 + \frac{N_B}{2}$ ⇒ Charge radius $\langle r^2 \rangle_{\rm E,p} = \langle r^2 \rangle_{I=0}$, $\langle r^2 \rangle_{\rm E,n} = 0$

Our results

	Holographic QCD	Skyrmion	Experiment
$\langle r^2 \rangle^{1/2}_{I=0}$	0.742 fm	0.59 fm	0.806 fm
$\langle r^2 \rangle^{1/2}_{\mathrm{M},\, I=0}$	0.742 fm	0.92 fm	0.814 fm
$\langle r^2 \rangle_{\mathrm{E,p}}$	$(0.742 \text{ fm})^2$	∞	$(0.875 \text{ fm})^2$
$\langle r^2 \rangle_{\mathrm{E,n}}$	0	$-\infty$	-0.116 fm^2
$\langle r^2 \rangle_{\mathrm{M,p}}$	$(0.742 \text{ fm})^2$	∞	$(0.855 \text{ fm})^2$
$\langle r^2 \rangle_{\mathrm{M,n}}$	$(0.742 \text{ fm})^2$	∞	$(0.873 \text{ fm})^2$
$\langle r^2 \rangle^{1/2}_{A}$	0.537 fm	$-$	0.674 fm
μ_p	2.18	1.87	2.79
μ_n	-1.34	-1.31	-1.91
$\left\lvert \frac{\mu_p}{\mu_n} \right\rvert$	1.63	1.43	1.46
g_A	0.73	0.61	1.27
$g_{\pi NN}$	7.46	8.9	13.2
$g_{\rho NN}$	5.80	$-$	$4.2 \sim 6.5$

input : f_π, m_ρ

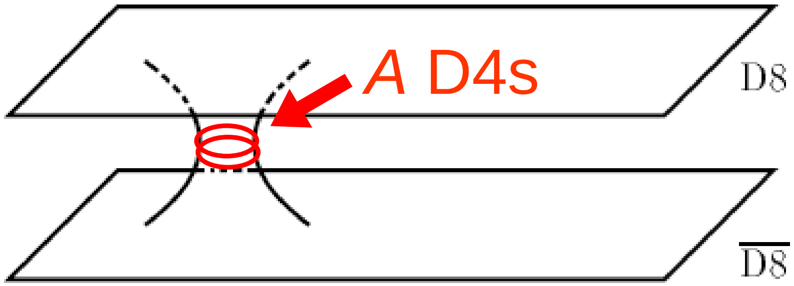
4. Holographic Nuclei

Work in progress

KH

Heavy nuclei have dual geometry?

A : Mass number of nucleus



Nucleon = D4 wrapping S^4

Nucleus = *A* D4s

Large *A*

Heavy nucleus = D4 geometry

Uranium, Plutonium, …

	0	1	2	3	y	z	S^4			
N_f D8s	o	o	o	o		o	o	o	o	o
A D4s	o						o	o	o	o

Fields on the *A* D4s:

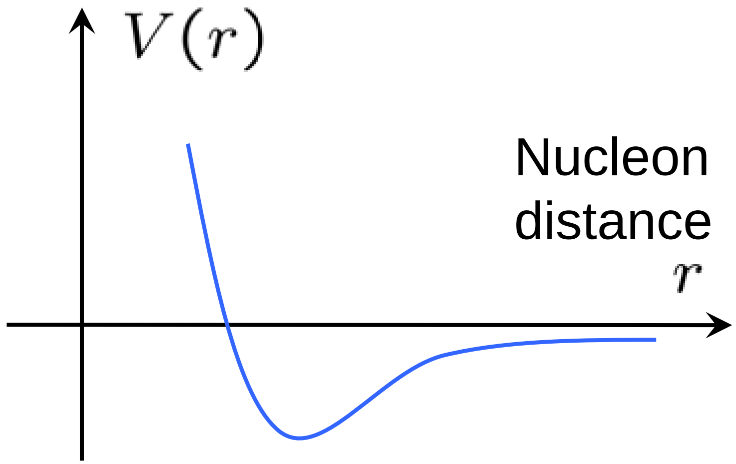
- $\Phi_i \; (i = 1, 2, 3, z, y)$: Adj. scalars (D4-D4)
- $\rho_a \; (a = 1, \cdots, N_f)$: Fund. scalars (D4-D8)

→ *U(A)* "ADHM" gauge theory on S^4

: Complicated. Non-susy, nontrivial potential, ..

Very quick course for Nuclear theory

Nuclear force



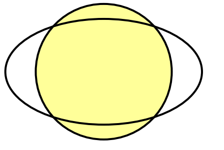
Short range : Strong repulsion
[Sakai,Sugimoto,KH] in progress

Long range : Pion exchange

→ Nucleons are stabilized at a certain distance

→ Heavy nuclei has radius $R_0 \propto A^{1/3}$

Liquid drop model of heavy nuclei [Gamow]



Nuclei are approximated by incompressible fluid ball

Nucleon binding energy is independent of nucleus size

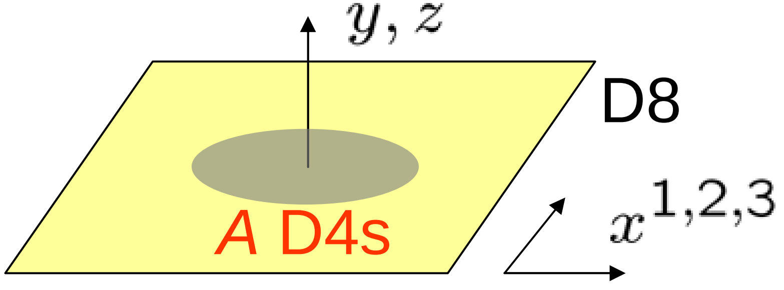
Excitation spectrum of nucleus : $E_n = \omega n \; (n = 1, 2, \cdots)$

Dual geometry?

Backreacted geometry : Gravitating instantons

D4s = Electrically charged *A* instantons on N_f D8

Let's assume isotoropic instanton density on D8 to mimic the nucleus



D4s are localized at the tip of Witten's geometry

$$ds^2 = \left(\frac{U_{\mathrm{KK}}}{R}\right)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{4}{9}\left(\frac{R}{U_{\mathrm{KK}}}\right)^{3/2} (dy^2 + dz^2) + R^{3/2} U_{\mathrm{KK}}^{1/2} d\Omega_4^2$$

→ Dual geometry can be roughly approximated by *A* D4s distributed on a 3-ball with radius $R_{\mathrm{N}} = \dfrac{A^{1/3}}{\sqrt{\lambda} M_{\mathrm{KK}}}$ in this almost flat asymptotics.

Near horizon geometry, Dictionary

Near horizon? ….We don't know explicit solution

But we can argue validity of near-horizon requirement.
Validity condition for "would-be AdS radius": $R_0 > R_{\rm N}$

Indeed, $$R_0 = (A g_s')^{1/3} l_s / \sqrt{g_{ii}} = \frac{A^{1/3}}{N_c^{1/3} M_{\rm KK}} > R_{\rm N} = \frac{A^{1/3}}{\sqrt{\lambda} M_{\rm KK}}$$

⇨ Near horizon argument is suggested to be fine

Dictionary (partial)

Metric fluctuation g_{ij} ⇔ ${\rm tr}[[\Phi_i, \Phi_k][\Phi_j, \Phi_k]]$

Gauge fluctuation A_i ⇔ $\rho^\dagger \gamma_i \rho$

Collective motion of nucleons is described by sugra fields

Spectrum and comparison with nuclear data

The "closest approximation" of the sugra solution is

D3s distributed on a 3-ball of radius R_N

$$H(r) = 1 + \int_{B_3} d^3\vec{a}\,\frac{cR_\mathrm{N}^4}{|\vec{r}-\vec{a}|^4}$$

Fluctuation spectrum

$$\frac{1}{\sqrt{G}}\partial_\mu \sqrt{G} G^{\mu\nu}\partial_\nu \phi = 0$$

$$\Rightarrow \quad E \sim \frac{R_\mathrm{N}}{R_0^2}\, n \quad (n = 1, 2, \cdots)$$

[Freedman,Gubser,Pilch,Warner(99)] [Brandhuber,Sfetsos(99)]

Mass gap due to automatic "truncation" of geometry

$E \propto n$ coincident with the empirical formula for nuclei

$$\frac{R_\mathrm{N}}{R_0^2} \sim \frac{N^{2/3}}{A^{1/3}\sqrt{\lambda}} M_\mathrm{KK} \sim \mathcal{O}(10)[\mathrm{MeV}]$$

Rough agreement

5. Conclusion & Discussions

Conclusion

Holographic baryons :

Static properties of baryons, computed in SS model, nicely match exp. data

Holographic nuclei :

A new AdS/CFT scheme which can be compared with real nuclear data is presented.

Discussions

Using AdS/CFT twice?

"CFT" (QCD) → "AdS" + other D-branes → "AdS"
"CFT"

Cf) Multi-center BH and Coulomb branch of SYM

Holographic nuclei can be black holes?!

Event horizon may happen to form ?
→ Deconfinement phase ("BH embedding")
→ Continuous spectrum…..

But, it leads to fluid dynamics…
→ Possible relation to liquid drop model of nuclei ?!

BH formation and hadron collision? [Nastase] & many others…