Hydrodynamics and beyond in AdS/CFT

Makoto Natsuume (KEK)

arXiv: 0712.2916 [hep-th]
arXiv: 0712.2917 [hep-th]
arXiv: 0801.1797 [hep-th]
arXiv: 0807.1392 [hep-th]
arXiv: 0807.1394 [nucl-th]

in collaboration w/
Takashi Okamura
(Kwansei Gakuin Univ.)

08/9 IPMU

Plan

Aim: what string theory can tell us about "causal hydrodynamics"?

- Review: String theory & quark-gluon plasma
- Basic idea of causal hydrodynamics
- Causal hydrodynamics from string theory

Refs.

- Works overlap w/ ours

  Baier - Romatschke - Son - Starinets - Stephanov, 0712.2451 [hep-th]
  Bhattacharyya - Hubeny - Minwalla - Rangamani, 0712.2456 [hep-th]
  MN - Okamura, 0712.2916 [hep-th]

  ← Our 2nd time to overlap w/ Son & Starinets!

- Early works

  Heller - Janik, hep-th/0703243
  Benincasa - Buchel - Heller - Janik, 0712.2025 [hep-th]
  MN - Okamura, 0712.2917 [hep-th]

- Extensions

  MN - Okamura, 0801.1797 [hep-th]
  Loganayagam, 0801.3701 [hep-th]
  Van Raamsdonk, 0802.3224 [hep-th]
  Bhattacharyya et al., 0803.2526 [hep-th], 0806.0006 [hep-th]
  Buchel - Paulos, 0806.0788 [hep-th]
  Kapusta - Springer, 0802.4175 [hep-th]
  Haack - Yarom, 0806.4602 [hep-th]
  MN, 0807.1392 [hep-th]
  Kinoshita - Mukohyama - Nakamura - Oda, 0807.3797 [hep-th]

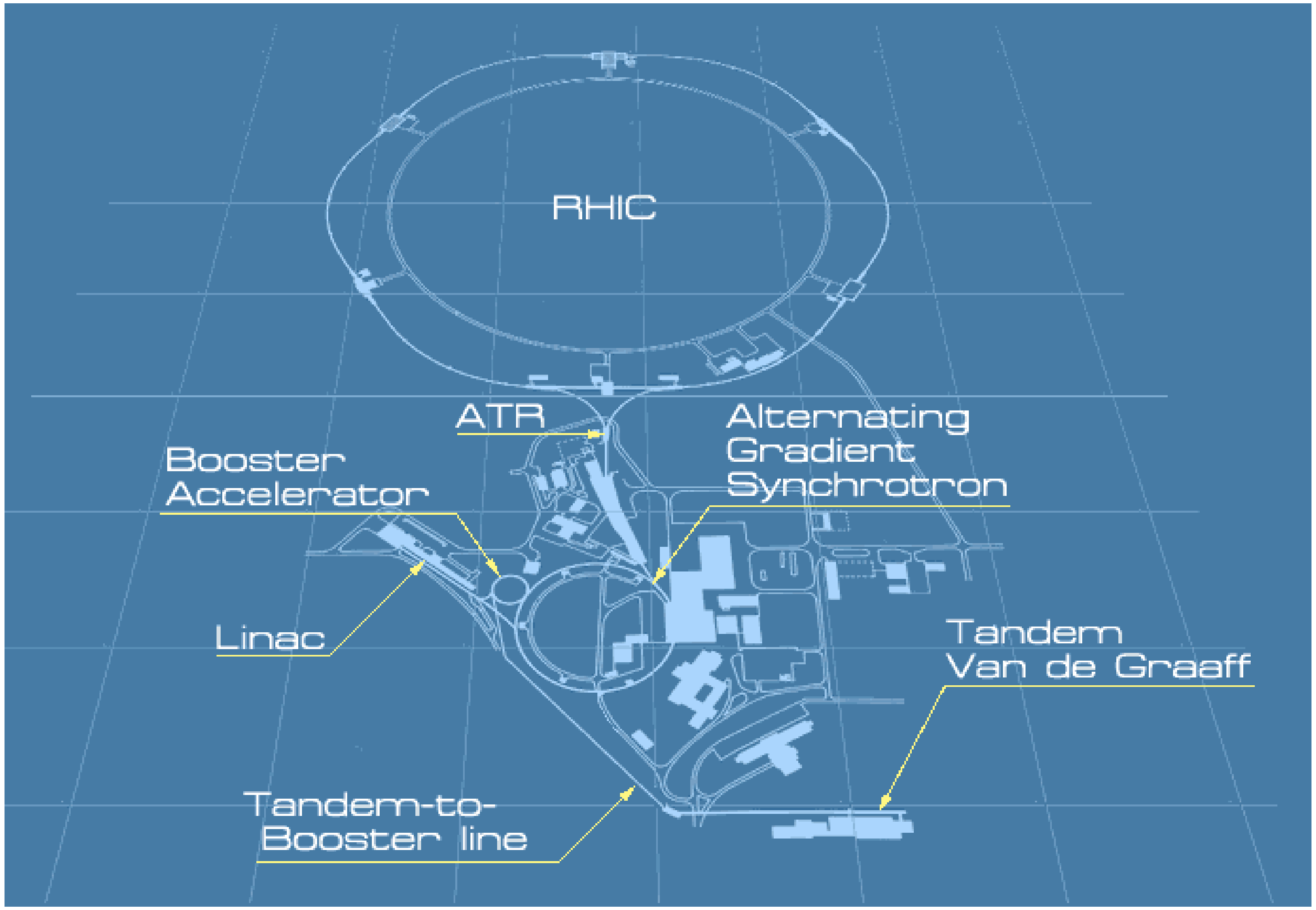
Review

RHIC complex

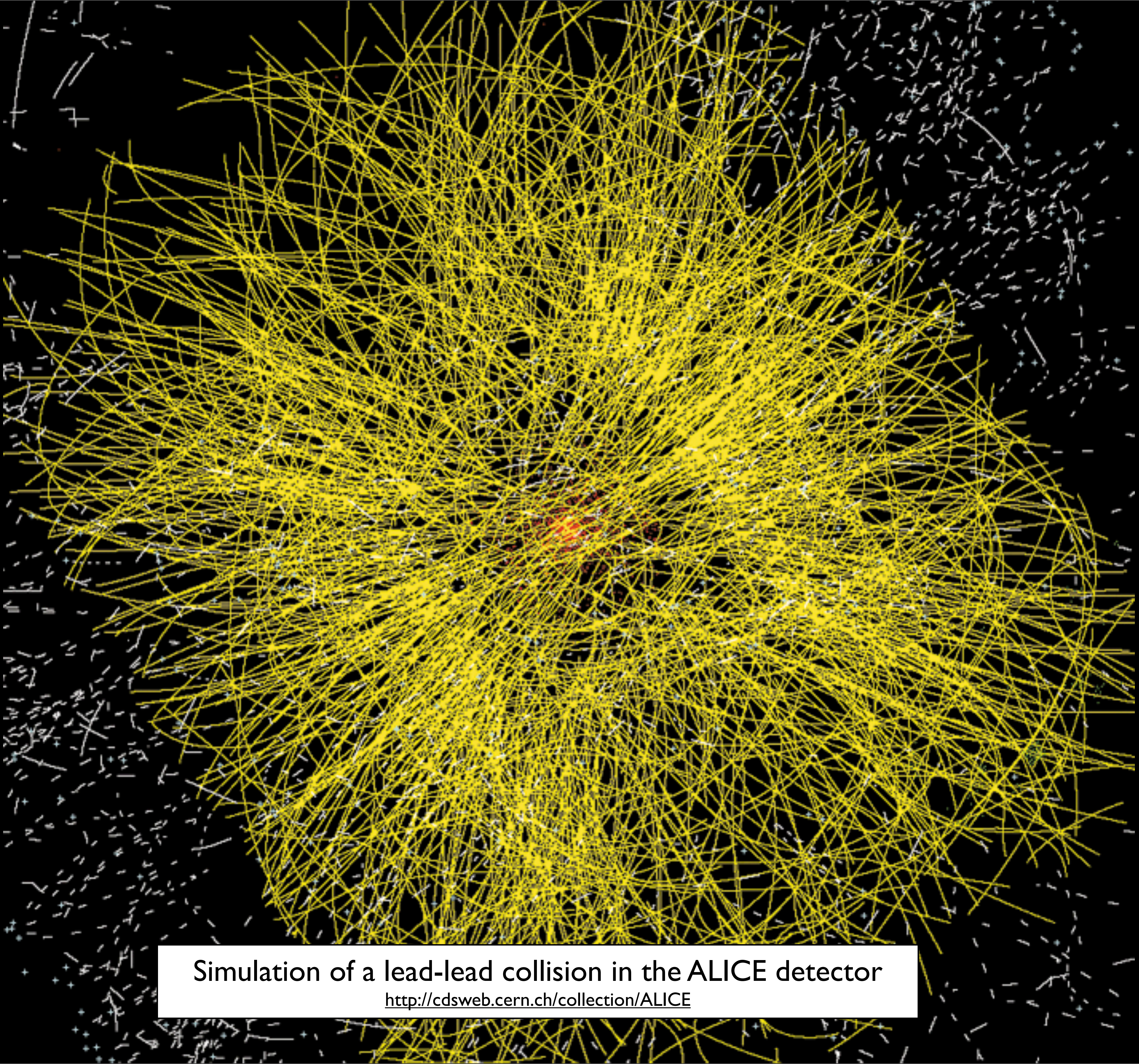
RHIC: Relativitistic Heavy Ion Collider (Bookhaven National Lab.)

heavy ion:
e.g. ^{197}Au

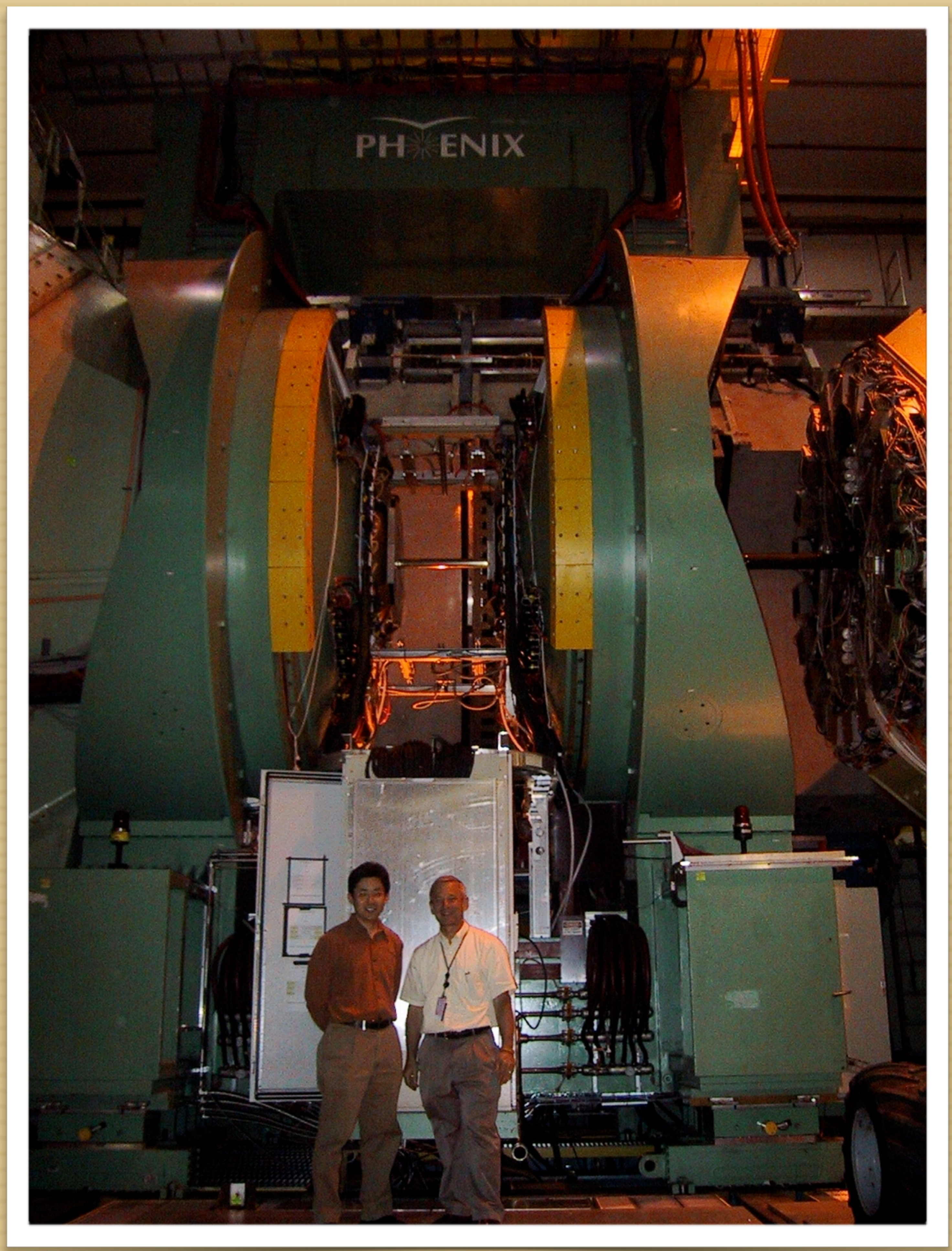
Similar exp. planned at LHC (ALICE/ATLAS/CMS)



http://www.bnl.gov/RHIC/RHIC_complex.htm



Simulation of a lead-lead collision in the ALICE detector
http://cdsweb.cern.ch/collection/ALICE

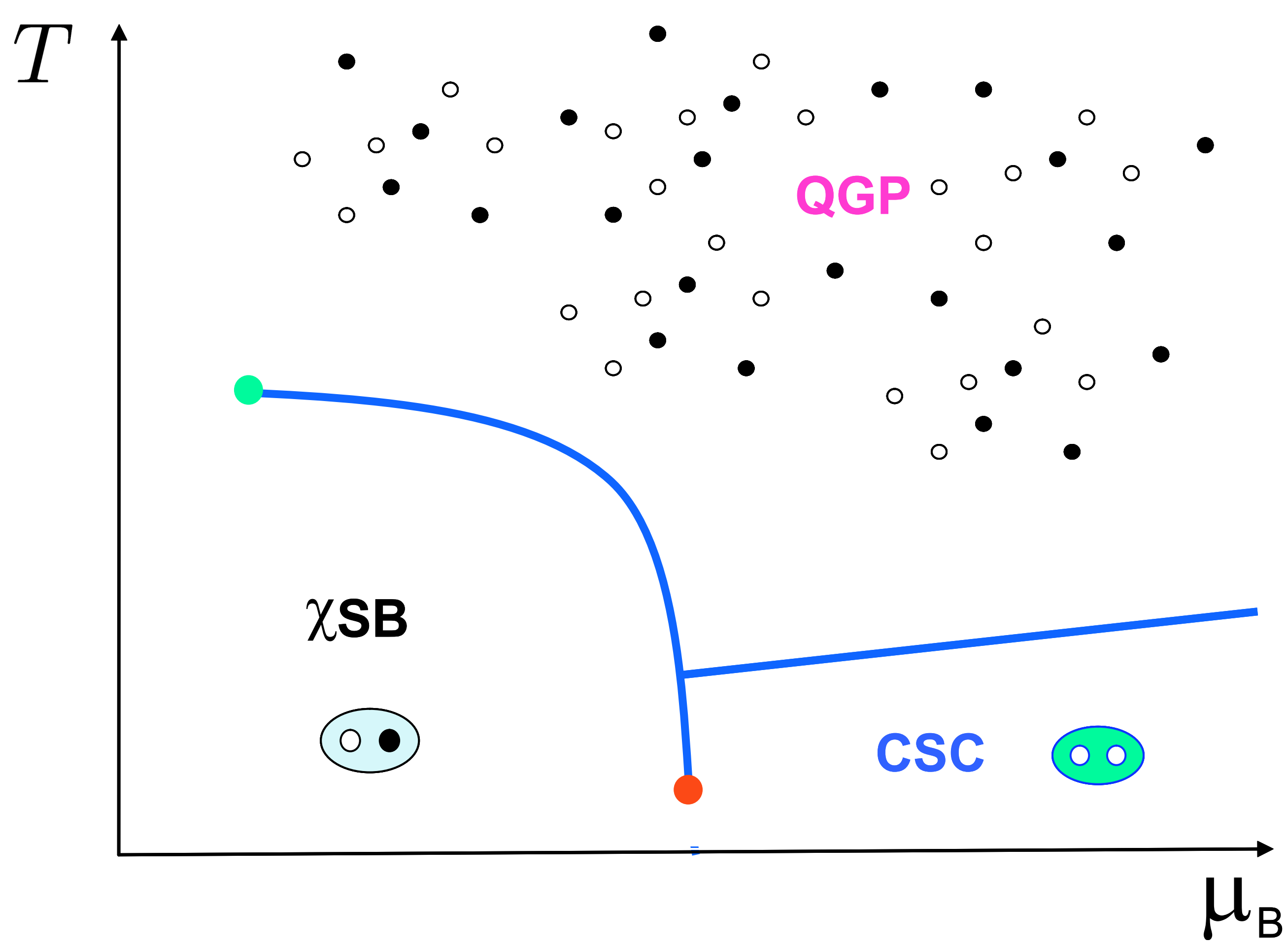


HI & I

Goal of RHIC

- Realize deconfinement transition and form quark-gluon plasma (QGP)
- Measure physical properties of QGP

Adapted from T. Hatsuda, hep-ph/0702293



QCD phase diagram (schematic)

sQGP

QGP: natural phenomenon from QCD

But never formed in exp., let alone physical properties

RHIC: 1st machine dedicated to QGP

QGP: Free gas? (due to asymptotic freedom)

$T \sim O(\Lambda_{QCD}) \rightarrow$: QCD still strongly-coupled

- pQCD: very limited
- Lattice QCD: imaginary-time formulation → not (yet) powerful for dynamical problems

String theory comes to the rescue?

AdS/CFT

QGP

⇕

Finite temperature gauge theory ⇔ Black hole
at strong coupling in AdS space

↓ thermal

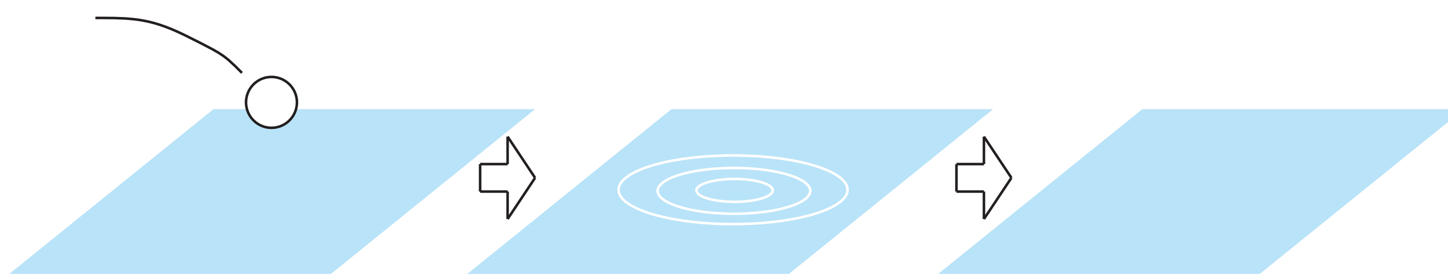
↓ thermal due to the Hawking radiation

BH and hydrodynamics

- RHIC experiments
  → QGP: a fluid w/ a very low viscosity

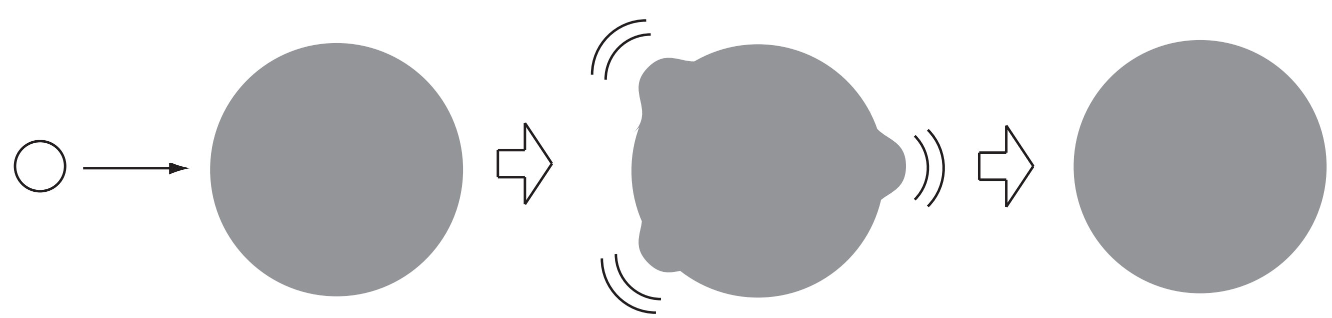
- BHs and hydrodynamic systems in fact behave similarly.

Water pond:



The dissipation: consequence of viscosity

BH:



The dissipation: consequence of BH absorption

Relaxation phenomena

Relaxation phenomena (add perturbs. & see how they decay)

→ Nonequilibrium statistical mechanics or hydrodynamics

→ important quantities: transport coefficients

e.g. (bulk & shear) viscosity
speed of sound
thermal conductivity
. . .

AdS/CFT: especially useful to determine η/s (η: shear viscosity, s: entropy density) due to

universality

Kovtun - Son - Starinets (2004)

Universality of η/s

According to AdS/CFT

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

True for all known examples

- ☑ conformal plasma ($\mathcal{N}$=4 SYM) — Policastro - Son - Starinets, 0104066
- ☑ nonconformal plasmas — Kovtun - Son - Starinets, 0309213; Buchel - J.Liu, 0311175
- ☑ Plasmas in different dimensions — Herzog, 0210126; Kovtun - Son - Starinets, 0309213
- ☑ Plasmas at finite chemical potential — Mas, 0601144; Son - Starinets, 0601157; Saremi, 0601159; **Maeda - Natsuume - Okamura, 0602010**
- ☑ Plasmas w/ flavors — Mateos - Myers - Thomson, 0610184
- ☑ Time-dependent plasma — Janik, 0610144

Comparison

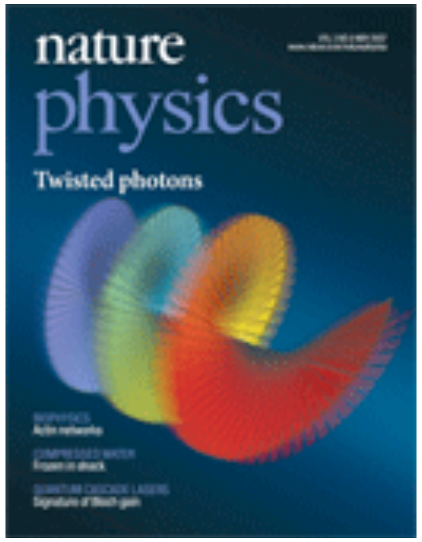
- RHIC: $\frac{\eta}{s} \sim O(0.1) \times \frac{\hbar}{k_B}$?
  Teaney, nucl-th/0301099
  ...
- AdS/CFT: $\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$
- naive extrapolation of perturbative QCD: $\frac{\eta}{s} \sim O(1) \times \frac{\hbar}{k_B}$
- Lattice (pure gauge theory): $1 < 4\pi \frac{\eta}{s} < 2$ for 1.2 Tc < T < 1.7 Tc

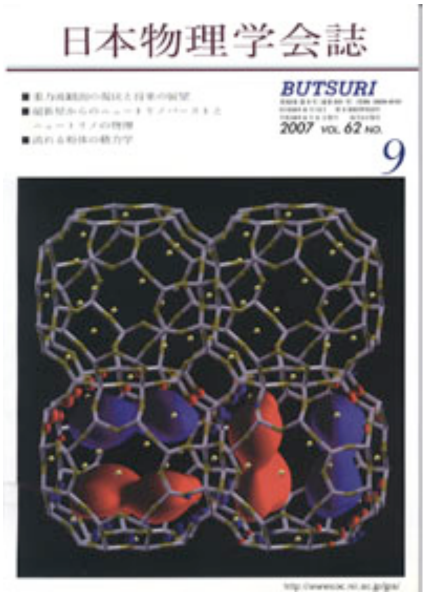
Meyer, 0704.1801 [hep-lat]
(An early work by A. Nakamura & S. Sakai, hep-lat/0406009)

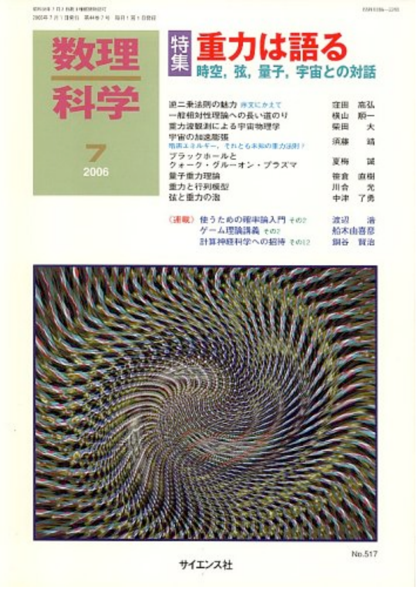
pQCD seems inaccurate and QGP seems strongly-coupled

My reviews

- 日経サイエンス, Feb 2006（翻訳）
- 数理科学, July 2006
- hep-ph/0701201
- Nature Physics, May 2007
- 日本物理学会誌, Sep. 2007
- 数理科学, Feb. 2008
- Various proceedings
  (YKIS, 原子核研究, 素粒子論研究…)







Basic idea of causal hydrodynamics

The hydrodynamic description of QGP using AdS/CFT is very successful,

but this cannot be the END of the story.

1. Standard hydrodynamic eqs. do not satisfy causality
2. In order to restore causality, one is forced to introduce a new set of transport coefficients → causal hydrodynamics/ 2nd order hydrodynamics/ Israel-Stewart theory
3. Such coefficients may become important in the early stages of QGP formation, but little is known about these coeffs.

Müller (1967)
Israel (1976), Israel-Stewart (1979)

Causal hydrodynamics has been widely discussed in heavy-ion literature: e.g. recent (2+1)-dim QGP simulations using causal hydrodynamics.

Romatschke - Romatschke, 0706.1522 [nucl-th]
Chaudhuri, 0708.1252 [nucl-th]
Song - Heinz, 0709.0742 [nucl-th]
0712.3715 [nucl-th]
Dusling - Teaney, 0710.5932 [nucl-th]

There are several potential problems though.
They use the AdS value for η/s, but use the weak coupling result for causal hydrodynamics.

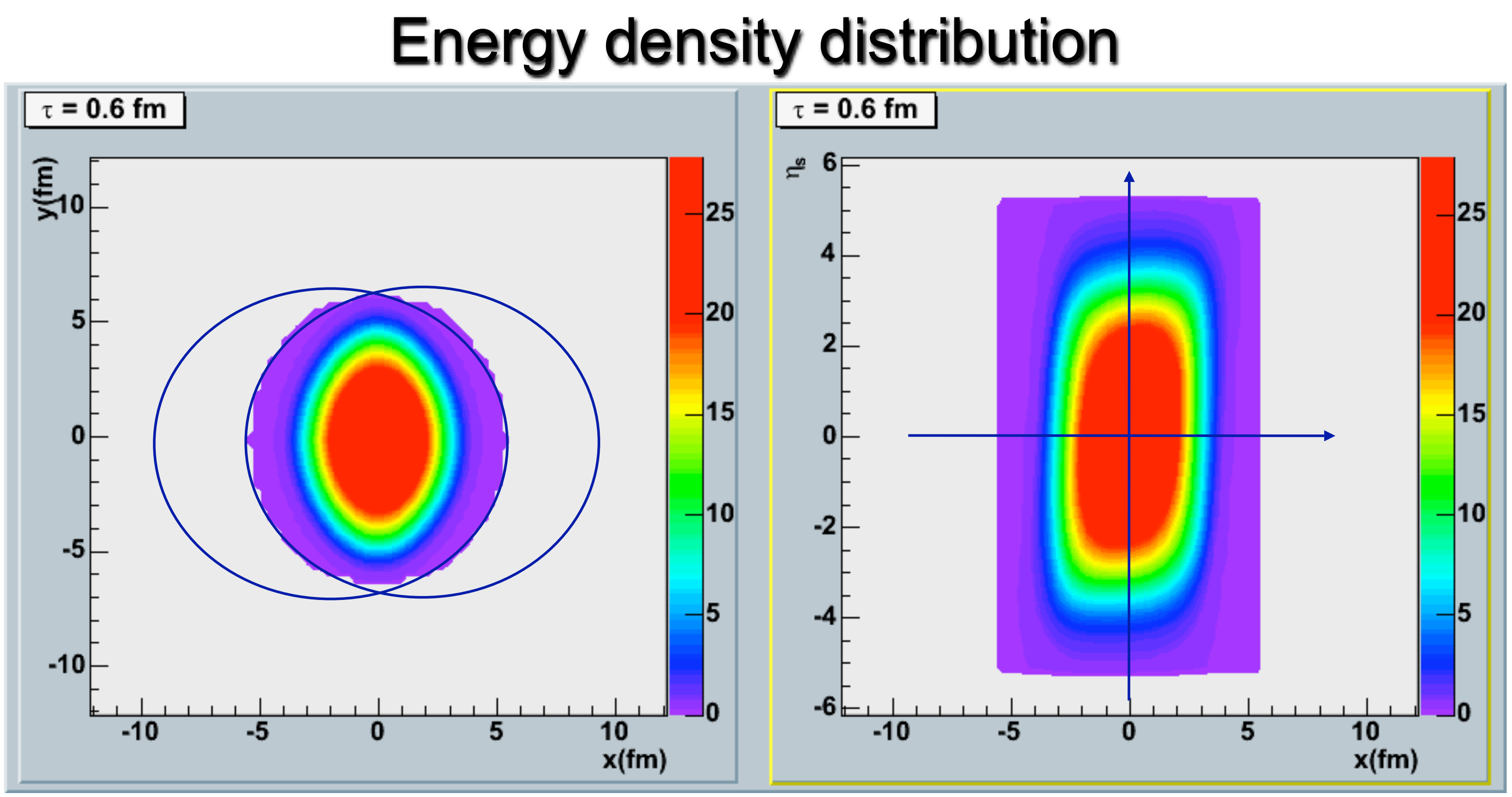
⇨ Is it really OK?

⇨ AdS/CFT

Simulations based on AdS/CFT:
Luzum - Romatschke, 0804.4015 [nucl-th]
Fries - B. Müller - A. Schäfer 0807.4333 [nucl-th]

QGP hydro simulation example

Adapted from slides by T. Hirano (UT, Hongo)



Transverse plane

Reaction plane

initial conditions → evolution (by hydrodynamics) → hadronization

Prototypical example

Hydrodynamic case is complicated due to the tensor nature of $T_{\mu\nu}$
→ R-charge diffusion

R-charge: global U(1) charge in $\mathcal{N}$=4 SYM (analog of baryon # in QCD)

Basic eqs. $J_\mu = (\rho, J_i)$

Conservation eq: $\partial_\mu J^\mu = 0$

Constitutive eq: $J_i = -D\partial_i \rho$ "Fick's law"

↑ Def. of diffusion const. D

(Conserv. eq.) + (Fick's law) → ρ & J_i : decouple

$$\partial_0 \rho - D\partial_i^2 \rho = 0 \quad \rightarrow \quad \boxed{\omega = -iDq^2}$$

Diffusion equation

Hydrodynamic case

Hydrodynamics is similar:

Conservation eq: $\partial_\mu T^{\mu\nu} = 0$

Constitutive eq: $T_{ij} = P\delta_{ij} - \eta(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k) - \varsigma\delta_{ij}\partial_k u_k$

Tensor decomposition (according to little group SO(2)) $(\omega,\underbrace{0,0},q)$

J_μ ⇒ scalar e.g. J_0 ☆
⇒ vector J_i

$T_{\mu\nu}$ ⇒ "sound mode" (scalar) e.g. T_{00} ☆
⇒ "shear mode" (vector) T_{0i} ☆
⇒ tensor T_{ij}

☆ transport coeffs. appear in these channels
(these coeffs. are associated w/ conserved quantities.)

Acausality

Diffusion eq.

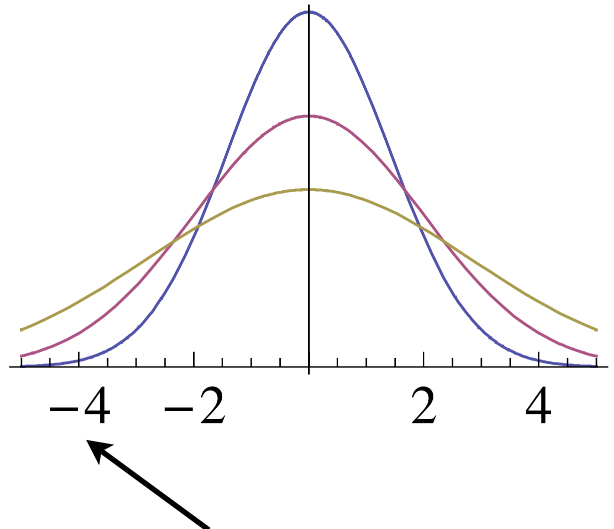
$$\partial_0 \rho - D\partial_i^2 \rho = 0$$

Parabolic (1st derivative in t, 2nd derivative in x)

⇨ acausal

In fact, propagator

$$\rho \sim \frac{1}{\sqrt{4\pi D t}} \exp(-\frac{x^2}{4Dt})$$



nonvanishing even outside the light-cone

To restore causality, hyperbolic eq. such as Klein-Gordon eq.

What's wrong?

conservation eq. → must be true
→ Fick's law should be the source of the prob.

Modify Fick's law (back to Maxwell!):

$$\tau_J \partial_0 J_i + J_i = -D\partial_i \rho$$

↑new parameter (transport coeff.)

- $\tau_J = 0$ → diffusion eq.
- $\partial_i \rho = 0$ at some time → Fick's law: $J_i = 0$ immediately
⇔ Modified law: exponential decay

τ_J: relaxation time for charge current

(Conserv. eq.) + (Modified law) → "telegrapher's eq."

$$\tau_J \partial_0^2 \rho + \partial_0 \rho - D \partial_i^2 \rho = 0$$ → hyperbolic

The new term: important at early time

Just an effective theory expansion in higher orders

Hydrodynamics: just an effective theory, so infinite # of parameters phenomenologically.

From D and τ_J, one gets a speed:

$$v \sim \sqrt{D/\tau_J}$$ → signal propagation

Dispersion relation as an effective theory:

$$\omega = -iDq^2 - iD^2 \tau_J q^4 + \cdots$$

causal hydrodynamic correction

Causal hydrodynamics

Israel (76) carried out a systematic analysis and introduced 5 new coefficients. (3 relaxation times: τ_J, τ_π, τ_Π)

charge diffusion: scalar, vector
EM tensor: sound, shear, tensor

But little is known about these coeffs.

⇨ AdS/CFT

Israel's formalism: highly complicated
→ linearized perturbations

→ charge diffusion & shear mode: just telegrapher's form at the end of the day

Israel's basic procedure

Equilibrium

$s = s(\varepsilon, \rho)$ cf. Ist law $ds = \frac{d\varepsilon}{T} - \frac{\mu}{T} d\rho$

Off-equilibrium Assume $s^\mu = s^\mu(T^{\mu\nu}, J^\mu)$

s^μ: 1st order in currents → standard hydrodynamics

s^μ: 2nd order → Israel-Stewart

Determine the generic form of constitutive eqs. so that ds > 0

$$ds \sim -J^i \partial_i \rho + \cdots$$

$J^i \sim -\partial_i \rho$ ⇨ ds > 0

constitutive eq.

Causal hydrodynamics from AdS/CFT

Outline of computations

Step1: identify appropriate bulk fields
corresponding to the boundary fields we're interested

↓

Step 2: Bulk perturbation eqs.

↓

Step 3: Solutions
→ transport coeffs. from dispersion relations

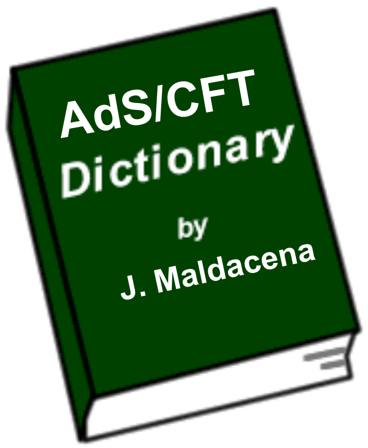
Step 1: identify appropriate modes

Boundary (Gauge) ⇔ Bulk (Gravity)

Deviations from equilibrium — Bulk fluctuations

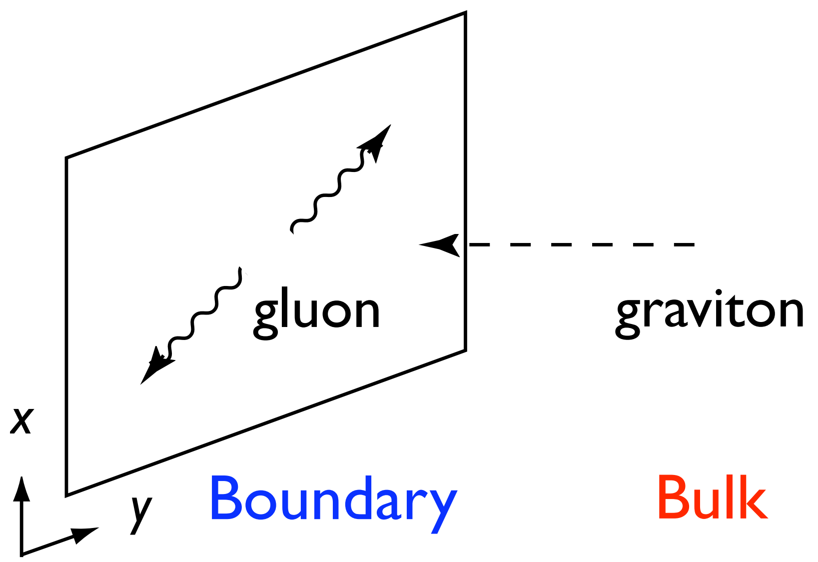
Hydrodynamics: $T_{\mu\nu}$ ⇔ $h_{\mu\nu}$

Charge diffusion: J_μ ⇔ A_μ



© Yarom

Due to the interaction bet. bulk & boundary fields



$$S_{\rm int} \sim \int d^4x\, h^{\mu\nu}T_{\mu\nu} + A^{\mu}J_{\mu} \cdots$$

↑graviton ↑Maxwell (bulk)

Step 2 & 3: R-charge diffusion example

Boundary		Bulk
Global R-charge	$\leftrightarrow$	Gauge field

- Solve Maxwell eq. in BH background
- Look at scalar sector ($\rho \leftrightarrow A_0$)

$$\nabla^\mu F_{\mu\nu} = 0 \quad \Rightarrow \quad A_0''' + \frac{(uf)'}{uf} A_0'' + \frac{\bar{\omega}^2 - \bar{q}^2 f}{uf^2} A_0' = 0$$

$$A_0(x,u) \sim \int d\omega dq\, e^{-i\omega t + iqz} A_0(q,u)$$

$f = 1 - u^2$ u=1: horizon
u=0: asymptotic infinity

$\bar{\omega}$, $\bar{q}$: normalized by temperature $\bar{\omega} = \frac{\omega}{2\pi T}, \bar{q} = \frac{q}{2\pi T}$

More than 3 regular singularities (common for BH problems)

$u = 0, \quad \pm 1, \quad \infty$

→ no analytic solution is known

We are interested only in hydrodynamic limit (ω→0, q→0)

→ Solve the eq. perturbatively in ω, q

$O(\omega, q^2)$:

BC at horizon (ingoing) → determine the solution
BC at asymptotic infinity → possible to impose if $\bar{\omega} = -i\bar{q}^2$

cf. $\omega = -iDq^2$ ⇨ Charge diffusion const. $D = \frac{1}{2\pi T}$

$O(\omega^2, \omega q^2, q^4)$: $\bar{\omega} = -i\bar{q}^2 - i(\ln 2)\bar{q}^4 + \cdots$

cf. $\omega = -iDq^2 - iD^2\tau_J q^4 + \cdots$ ⇨ $\tau_J = \frac{\ln 2}{2\pi T}$

Reminder

Tensor decomposition:

J_μ	scalar	J_0	☆	D	τ_J
	vector	J_i			
$T_{\mu\nu}$	"sound mode" e.g.	T_{00}	☆	vs η ς τ_π τ_Π	
	"shear mode"	T_{0i}	☆	η	τ_π
	tensor	T_{ij}			

↑ transport coeffs.

vs: speed of sound
ς: bulk viscosity

- ς, τ_Π = 0 for conformal theories
- The other 2 coeffs. by Israel-Stewart vanish for BHs w/ no R-charge

⇒ τ_π (relaxation time for shear viscous stress)

Some lessons we learned

- Universality or generic feature ?

  → Analyze various theories: AdS_4, AdS_5 ($\mathcal{N}$=4 SYM), AdS_6 & AdS_7 BHs

- Issue of formalism(s)
- Other issues
  - How does it change w/ coupling?
  - propagation speed vs the speed of sound
  - OK to ignore some terms?

Any info: highly desirable since none is known

Results for τ_π

	relaxation time	
AdS_4 (M2)	$\frac{18-(9\ln 3-\sqrt{3}\pi)}{24\pi T}$	→ **MN - Okamura**
AdS_5 (D3)	$\frac{2-\ln 2}{2\pi T}$	→ **MN - Okamura** Baier et al. Bhattacharyya et al.
AdS_6	$\frac{1}{4\pi T}\left(5-\frac{\pi}{2}\sqrt{1-\frac{2}{\sqrt{5}}}+\frac{\sqrt{5}}{2}\coth^{-1}\sqrt{5}-\frac{5\ln 5}{4}\right)$	→ Haack - Yarom
AdS_7 (M5)	$\frac{36-(9\ln 3+\sqrt{3}\pi)}{24\pi T}$	→ **MN - Okamura**

Different τ_π for different theories

Results for τ_π

	relaxation time	
AdS_4 (M2)	$\dfrac{18-(9\ln 3-\sqrt{3}\pi)}{24\pi T}$	~ 0.18 fm
AdS_5 (D3)	$\dfrac{2-\ln 2}{2\pi T}$	~ 0.21 fm
AdS_6	$\dfrac{1}{4\pi T}\left(5-\dfrac{\pi}{2}\sqrt{1-\dfrac{2}{\sqrt{5}}}+\dfrac{\sqrt{5}}{2}\coth^{-1}\sqrt{5}-\dfrac{5\ln 5}{4}\right)$	~ 0.24 fm
AdS_7 (M5)	$\dfrac{36-(9\ln 3+\sqrt{3}\pi)}{24\pi T}$	~ 0.27 fm

$\hbar c \sim 197\text{MeV fm}$ → Use 1/T = 1 fm

Results for τ_π

These results are simply summarized as

MN, 0807.1392 [hep-th]

$$(4\pi T)\tau_\pi = H_{2/(p+1)} + \frac{p+1}{2}$$

$H_n = \sum_{k=1}^{n} \frac{1}{k}$: harmonic #

p: # of spatial dim (boundary)

If the formula is true generically, for large p

- (signal propagation) $= \sqrt{D_\eta / \tau_\pi} \sim \sqrt{2/p}$ $\quad (4\pi T)\tau_\pi \sim p/2$
- (speed of sound) $= \sqrt{1/p}$ $\quad D_\eta = \eta/(Ts)$

How does it change w/ coupling?

Israel-Stewart made estimate for Boltzmann gas (dilute gas approx)

Boltzmann: $$\frac{\tau_\pi}{\eta} = \frac{6}{Ts}$$

AdS/CFT: $$\frac{\tau_\pi}{\eta} = \frac{2(2-\ln 2)}{Ts} \sim \frac{3}{Ts}$$

Not far from each other

→ τ_π/η does not strongly depend on coupling

The finite-coupling corrections: positive approaching to the Boltzmann value

Buchel - Paulos, 0806.0788 [hep-th]

Different τ_π for different theories,
but a simple formula exists
$\tau_\pi \sim 0.2$ fm (for 1/T =1 fm)

Issue of formalism(s)

Closely related works

Baier - Romatschke - Son - Starinets - Stephanov, 0712.2451 [hep-th]

Baier et al. have done the same analysis (only for $\mathcal{N}$=4 SYM though).

In addition to the coeffs. by IS, they introduced 4 new coefficients from conf. inv.

The Israel-Stewart theory is not complete

See also
Heller - Janik, hep-th/0703243
Benincasa - Buchel - Heller - Janik, 0712.2025 [hep-th]
Bhattacharyya - Hubeny - Minwalla - Rangamani, 0712.2456 [hep-th]
MN - Okamura, 0712.2916 [hep-th]
0712.2917 [hep-th]
0801.1797 [hep-th]

2nd order coefficients (so far)

- Israel-Stewart theory:
  - τ_Π τ_J τ_π ($\beta_{0,1,2}$): relaxation times → τ_Π: irrelevant for conformal
  - α_0 α_1: couplings bet. J_μ and $T_{\mu\nu}$ → irrelevant for 0 chemical potential
- Baier et al. ← conformal only
  - κ: curved (boundary) spacetime effect → irrelevant for flat spacetime
  - $\lambda_{1,2,3}$: nonlinear terms → irrelevant for linear perturbs.

9 coefficients so far

More complications ...

Various formalisms e.g.

1. Israel-Stewart
2. Israel-Stewart modified by Baier et al.
3. "divergence-type theories" — Liu - Müller - Ruggeri (1986), Geroch - Lindblom (1990)
4. Carter's formalism — Carter (1991)

At this moment, unclear how they are related to each other

These formalisms are all equivalent for linear perturbations (in flat space).

Unique formalism in this case (more or less)

Israel-Stewart theory is incomplete.
But all candidates are equivalent for linear perturbations,
so our result must be true for any of these.

Summary

- Different τ_π for different theories, but a simple formula exists
- For practical users,
  - Be careful when you use the Israel-Stewart theory since it's not complete.
  - τ_π ~ 0.2 fm (for 1/T =1 fm), which is similar among the theories we consider.
- String theory may shed more light on this aspect of hydrodynamics