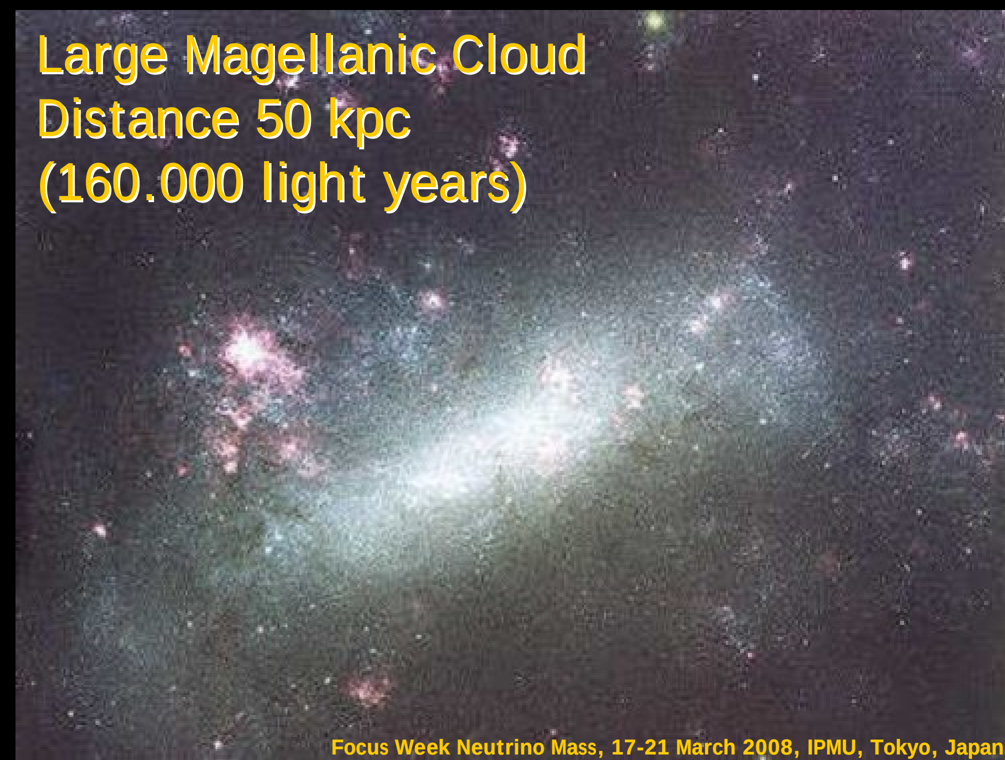
Focus Week: Neutrino Mass, 17-21 March 2008, Tokyo, Japan
Institute for Physics and Mathematics of the Universe (IPMU)

Collective Effects in Supernova Neutrino Oscillations

Georg Raffelt, Max-Planck-Institut für Physik, München

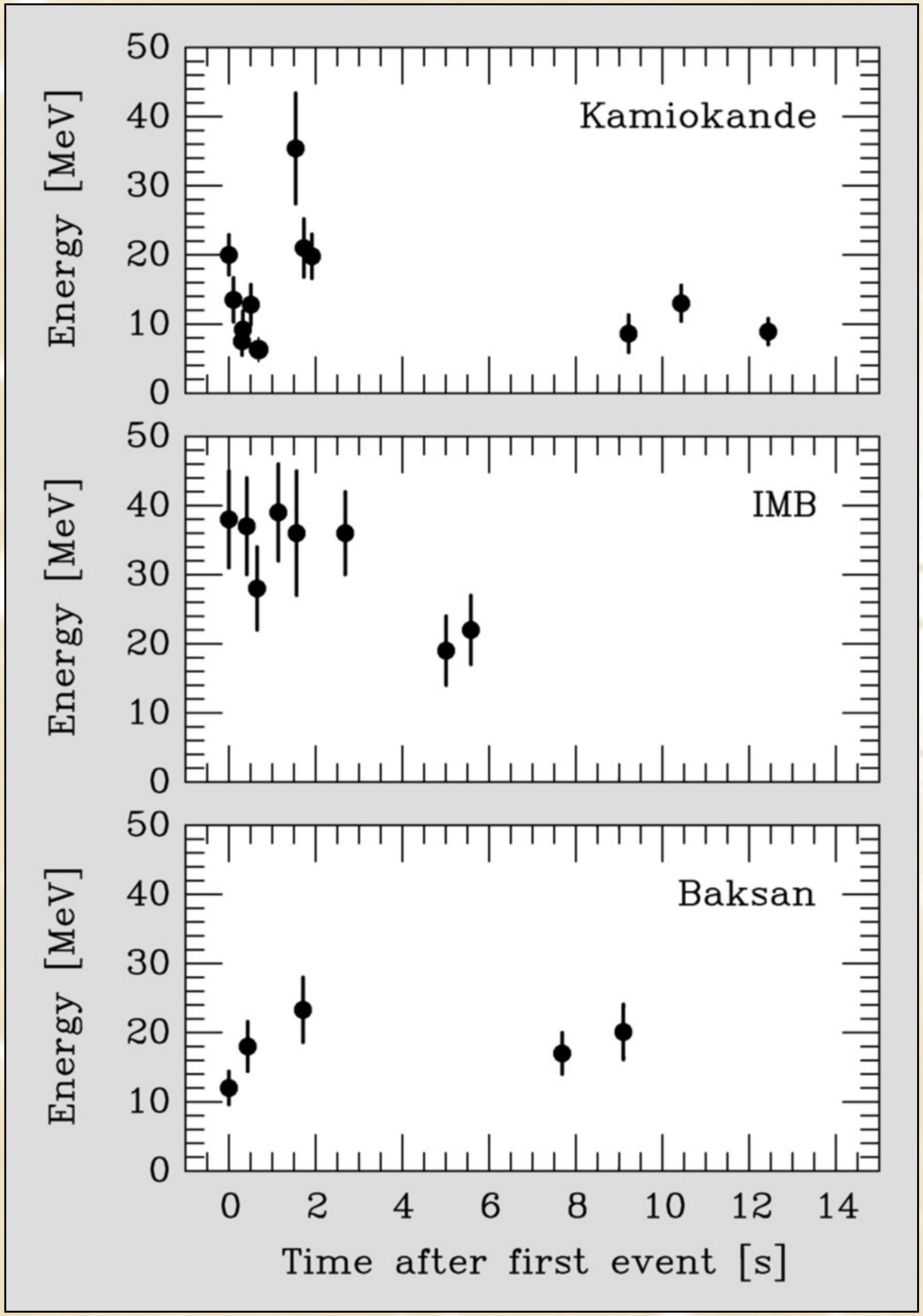
Sanduleak –69 202




Sanduleak –69 202
Georg Raffelt, Max-Planck-Institut für Physik, München


Supernova 1987A
23 February 1987
Focus Week Neutrino Mass, 17-21 March 2008, IPMU, Tokyo, Japan

Neutrino Signal of Supernova 1987A



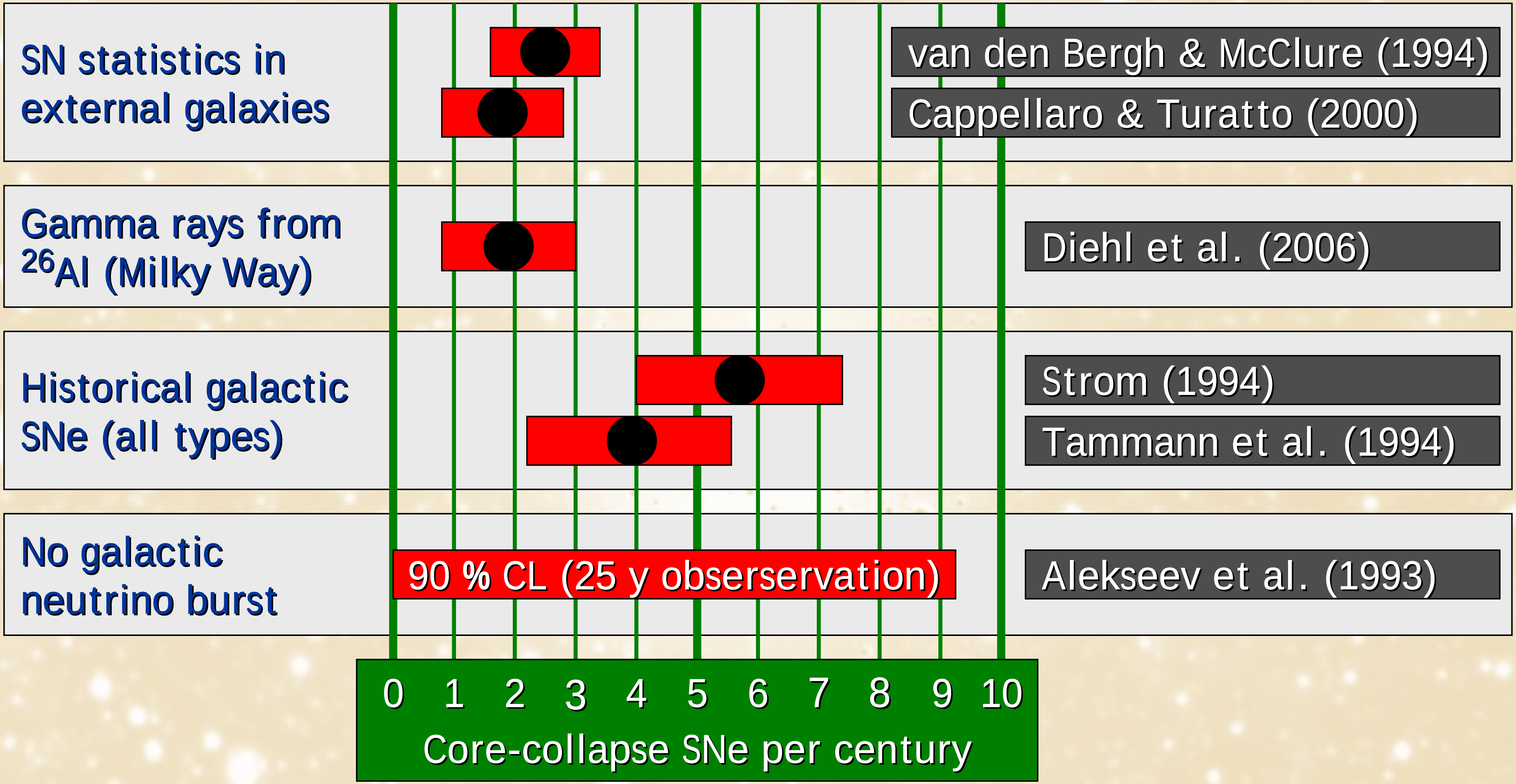
Kamiokande-II (Japan)
Water Cherenkov detector
2140 tons
Clock uncertainty ±1 min

Irvine-Michigan-Brookhaven (US)
Water Cherenkov detector
6800 tons
Clock uncertainty ±50 ms

Baksan Scintillator Telescope
(Soviet Union), 200 tons
Random event cluster ~ 0.7 / day
Clock uncertainty +2 / -54 s

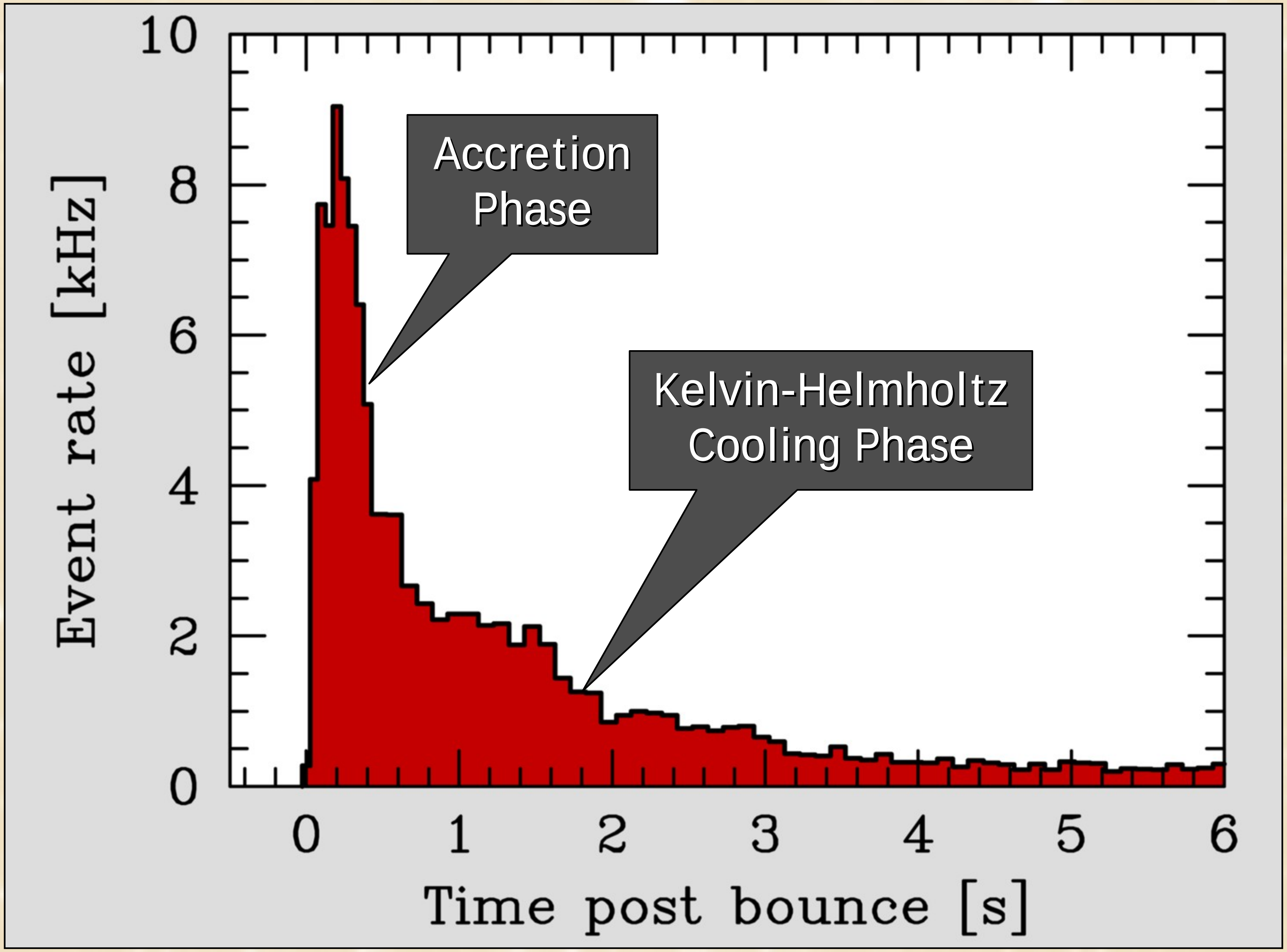
Within clock uncertainties,
signals are contemporaneous

Core-Collapse SN Rate in the Milky Way



References: van den Bergh & McClure, ApJ 425 (1994) 205. Cappellaro & Turatto, astro-ph/0012455. Diehl et al., Nature 439 (2006) 45. Strom, Astron. Astrophys. 288 (1994) L1. Tammann et al., ApJ 92 (1994) 487. Alekeseev et al., JETP 77 (1993) 339 and my update.

Simulated Supernova Signal at Super-Kamiokande

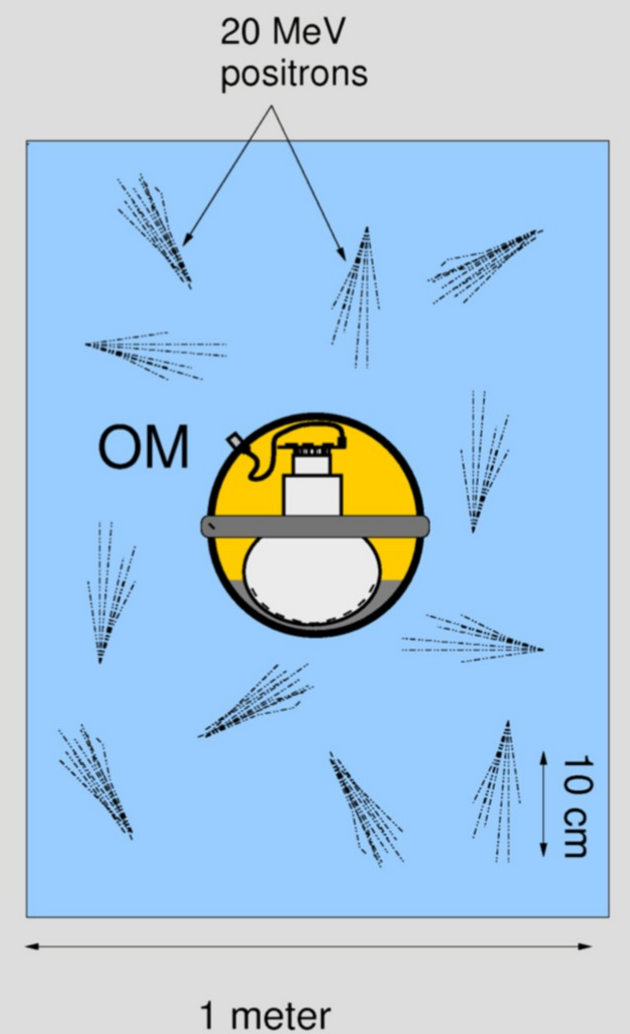


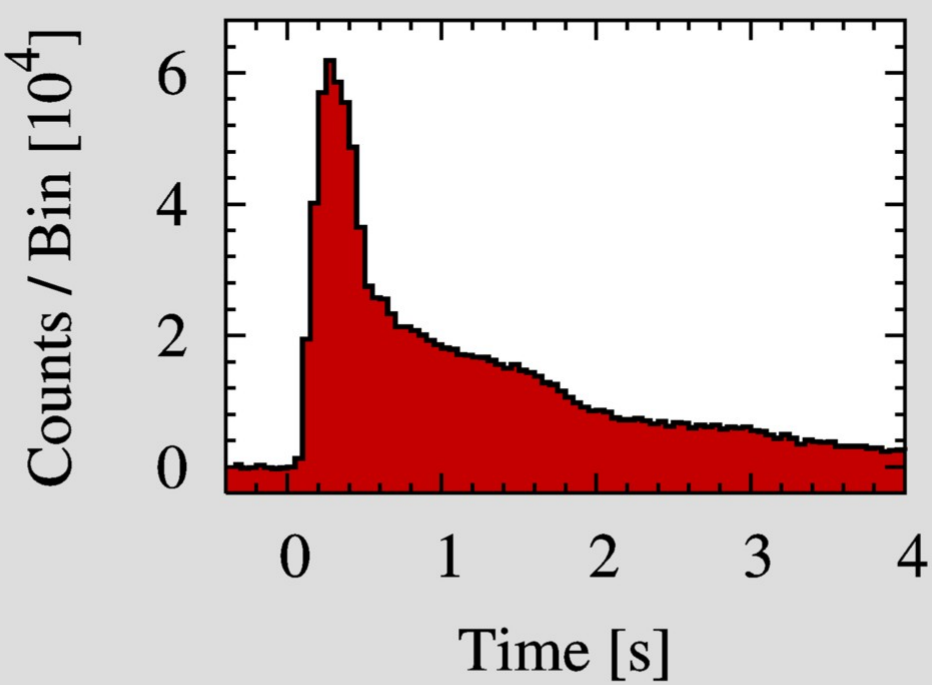
Simulation for Super-Kamiokande SN signal at 10 kpc, based on a numerical Livermore model
[Totani, Sato, Dalhed & Wilson, ApJ 496 (1998) 216]

IceCube as a Supernova Neutrino Detector

Each optical module (OM) picks up Cherenkov light from its neighborhood. SN appears as “correlated noise”.

- About 300 Cherenkov photons per OM from a SN at 10 kpc
- Noise per OM < 260 Hz
- Total of 4800 OMs in IceCube





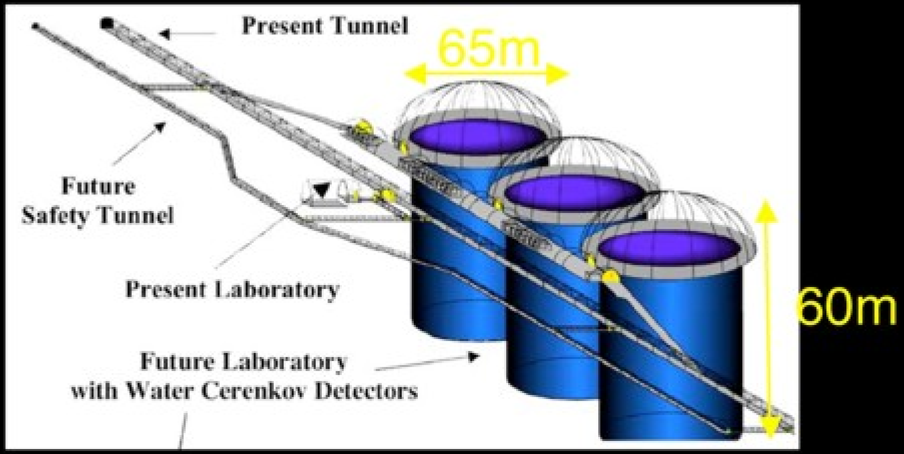
IceCube SN signal at 10 kpc, based on a numerical Livermore model
[Dighe, Keil & Raffelt, hep-ph/0303210]

Method first discussed by
- Pryor, Roos & Webster, ApJ 329:355 (1988)
- Halzen, Jacobsen & Zas astro-ph/9512080

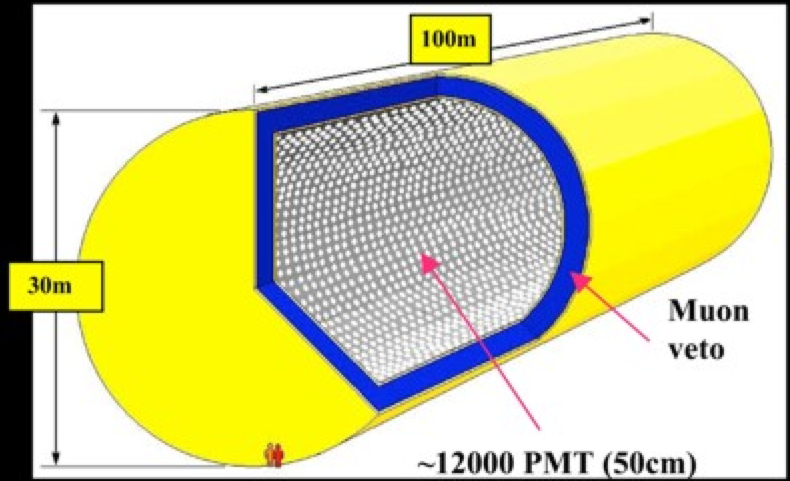
LAGUNA - Approved FP7 Design Study

Large Apparati for Grand Unification and Neutrino Astrophysics
(see also arXiv:0705.0116)

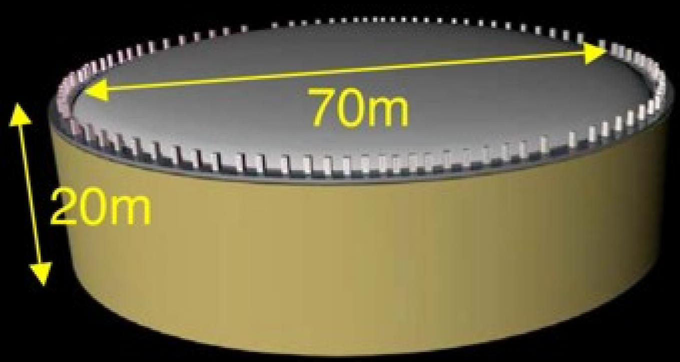
- Three types of large multi-purpose underground detectors with astrophysical program



Water Cherenkov (≈0.5 → 1 Mton)
MEMPHYS



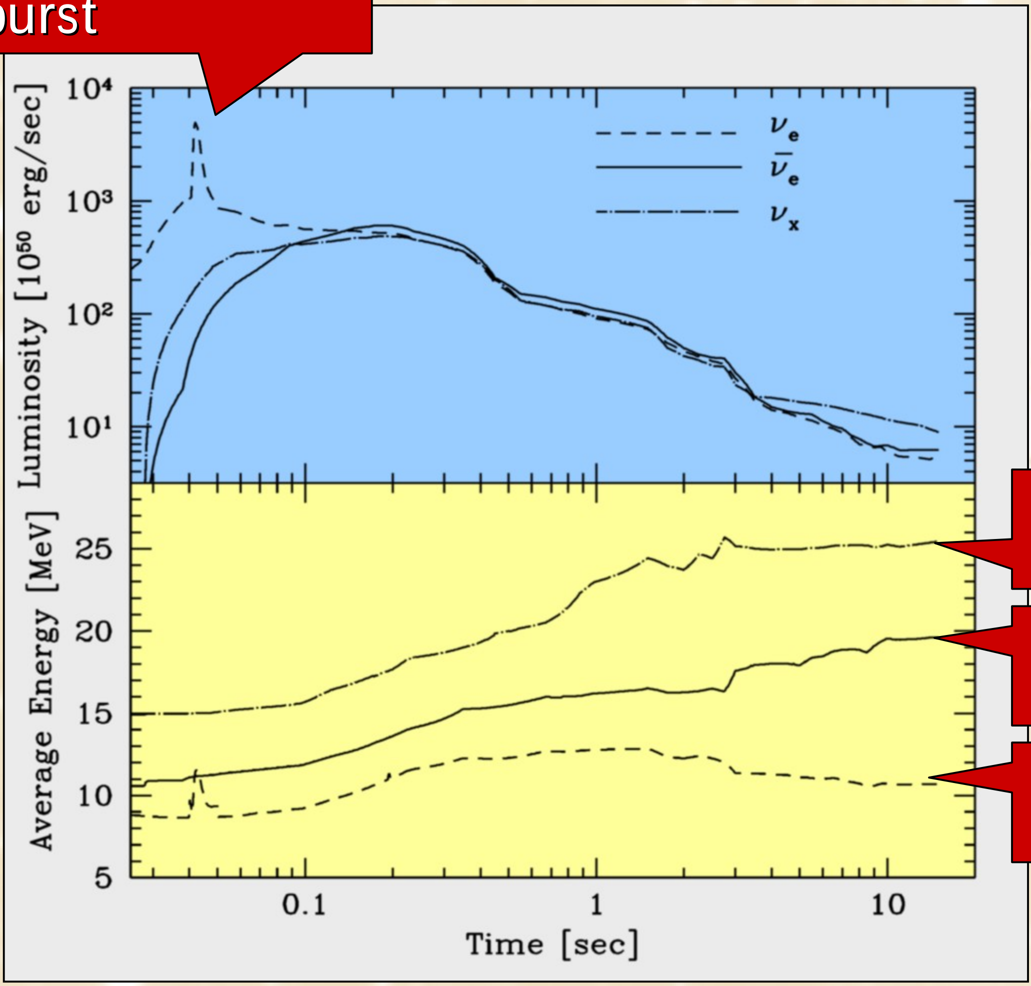
Liquid Scintillator (→ 50 kton)
LENA



Liquid Argon (≈10→100 kton)
GLACIER

Flavor-Dependent Fluxes and Spectra

Prompt ν_e deleptonization burst



ν_x

$\bar{\nu}_e$

ν_e

Livermore numerical model
ApJ 496 (1998) 216

Broad characteristics

- Duration a few seconds
- $\langle E_\nu \rangle \sim$ 10–20 MeV
- $\langle E_\nu \rangle$ increases with time
- Hierarchy of energies

$$\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_x} \rangle$$

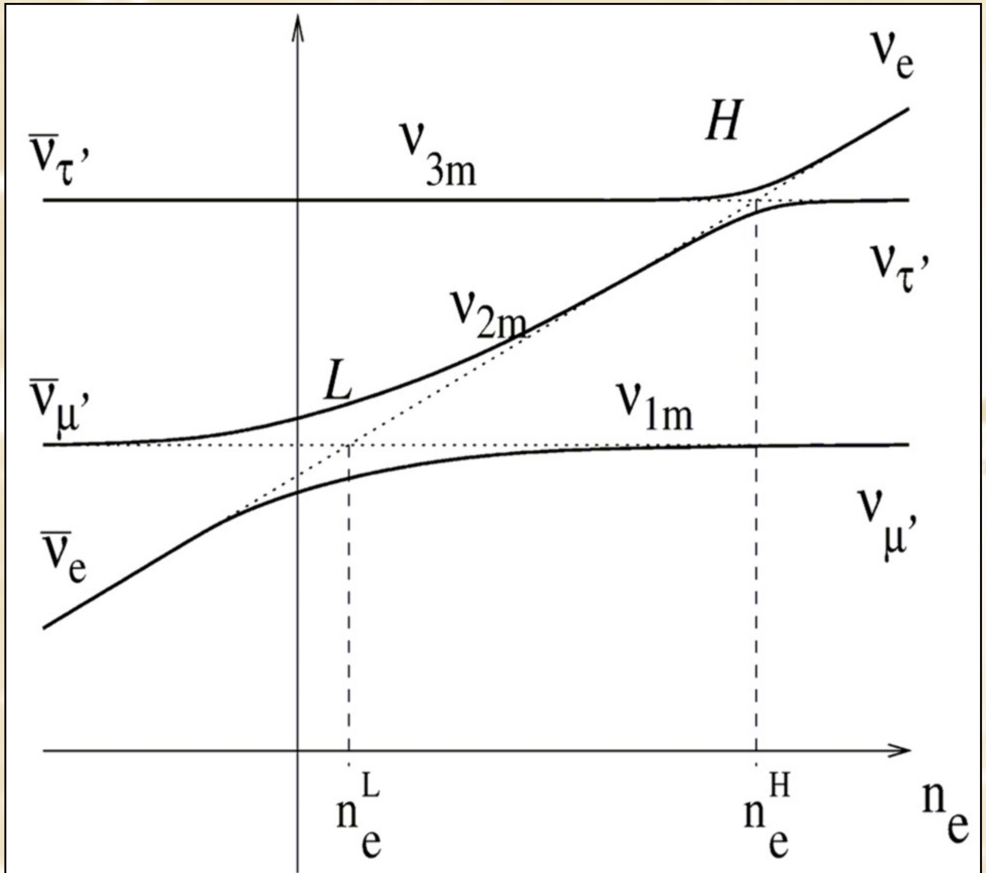
- Approximate equipartition of energy between flavors
- Hierarchy of number fluxes

$$\langle F_{\nu_e} \rangle > \langle F_{\bar{\nu}_e} \rangle > \langle F_{\nu_x} \rangle$$

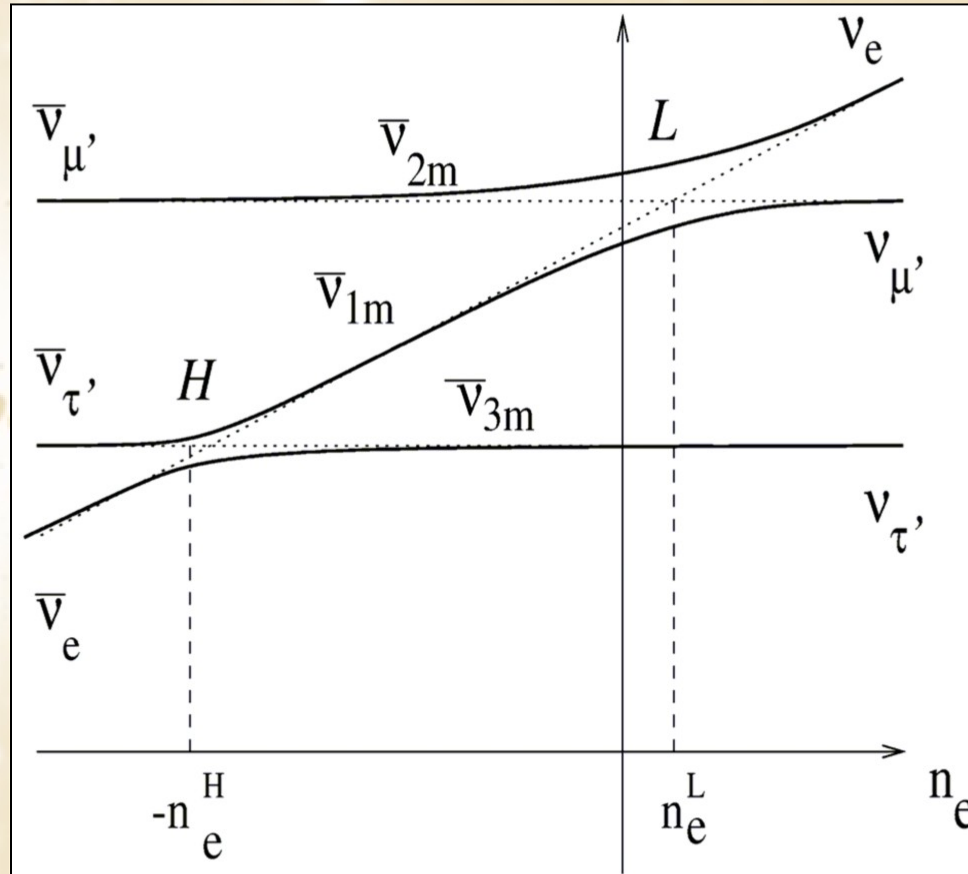
Livermore simulation almost certainly exaggerates the flavor-dependent differences, but no other long-term simulation available

Level-Crossing Diagram in a SN Envelope

Normal mass hierarchy



Inverted mass hierarchy



Dighe & Smirnov, Identifying the neutrino mass spectrum from a supernova neutrino burst, astro-ph/9907423

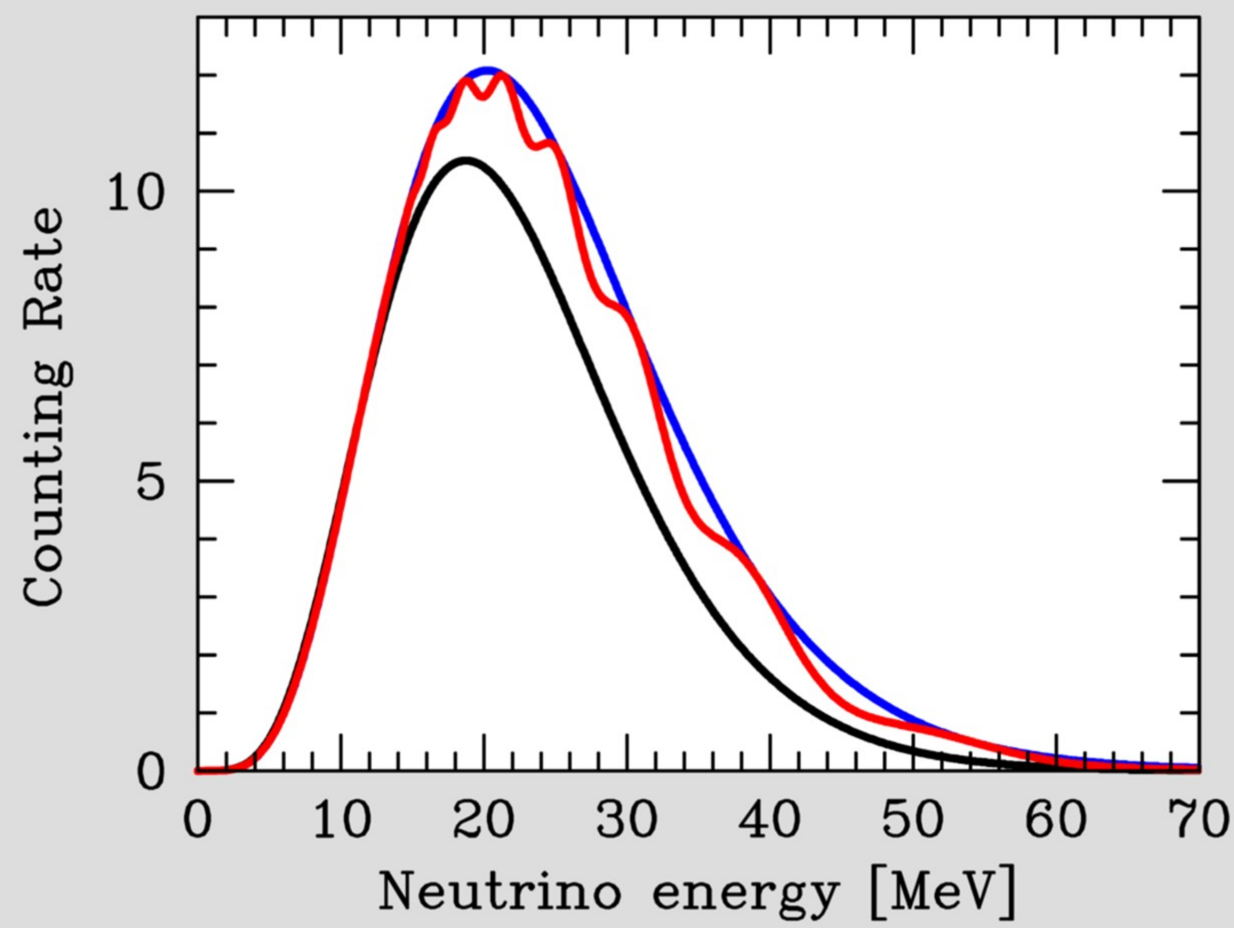
Spectra Emerging from Supernovae

Primary fluxes	F^0_e for ν_e $F^0_{\bar{e}}$ for $\bar{\nu}_e$ F^0_x for $\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$
After leaving the supernova envelope, the fluxes are partially swapped	$F^0_e = p F^0_e + (1-p) F^0_x$ $F^0_{\bar{e}} = \bar{p} F^0_{\bar{e}} + (1-\bar{p}) F^0_x$ $\frac{1}{4}\sum F_x = \frac{2+p+\bar{p}}{4} F^0_x + \frac{1-p}{4} F^0_e + \frac{1-\bar{p}}{4} F^0_{\bar{e}}$

Case	Mass ordering	$\sin^2(2\Theta_{13})$	Survival probability	
			p (for ν_e)	$\bar{p}$ (for $\bar{\nu}_e$)
A	Normal	$\gtrsim 10^{-3}$	0	$\cos^2(\Theta_{12}) \approx 0.7$
B	Inverted	$\gtrsim 10^{-3}$	$\sin^2(\Theta_{12}) \approx 0.3$	0
C	Any	$\lesssim 10^{-5}$	$\sin^2(\Theta_{12}) \approx 0.3$	$\cos^2(\Theta_{12}) \approx 0.7$

Oscillation of Supernova Anti-Neutrinos

Measured $\bar{\nu}_e$ spectrum at a detector like Super-Kamiokande



Assumed flux parameters

Flux ratio $\bar{\nu}_e : \bar{\nu}_\mu = 0.8 : 1$

$\langle E(\bar{\nu}_e) \rangle = 15\,\text{MeV}$

$\langle E(\bar{\nu}_x) \rangle = 18\,\text{MeV}$

Mixing parameters

$\Delta m^2_{\text{sun}} = 60\,\text{meV}^2$

$\sin^2(2\theta) = 0.9$

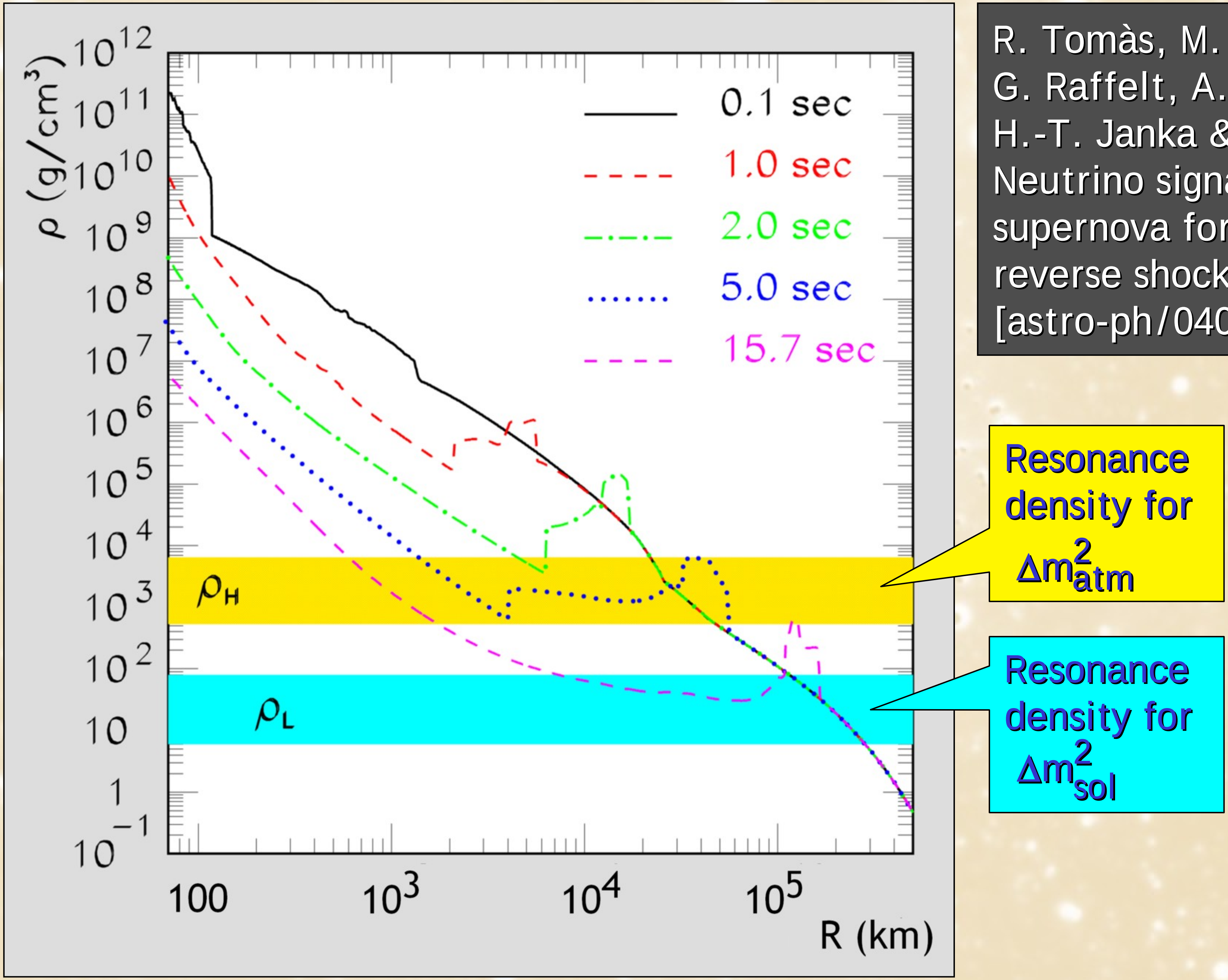
No oscillations

Oscillations in SN envelope

Earth effects included

Π(Dighe, Kachelriess, Keil, Raffelt, Semikoz, Tomàs), hep-ph/0303210, hep-ph/0304150, hep-ph/0307050, hep-ph/0311172

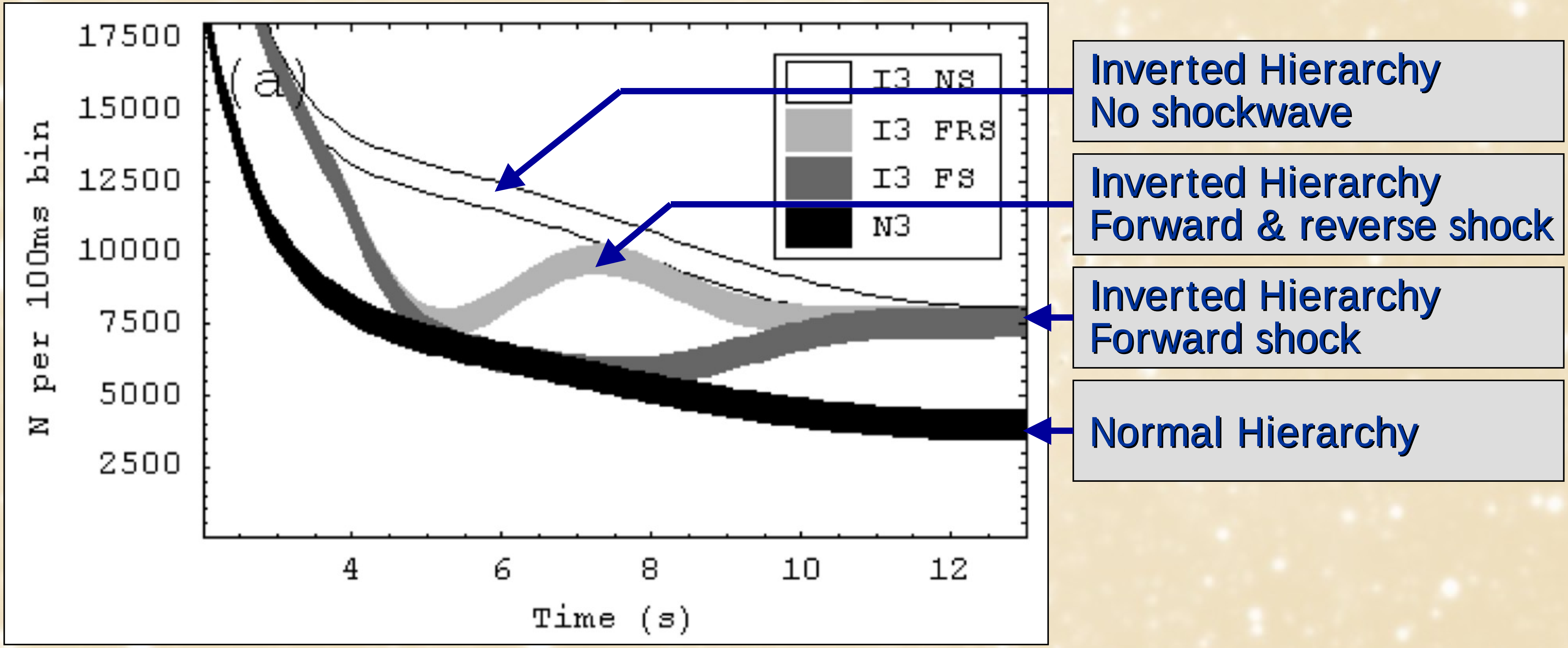
H- and L-Resonance for MSW Oscillations



R. Tomàs, M. Kachelriess, G. Raffelt, A. Dighe, H.-T. Janka & L. Scheck: Neutrino signatures of supernova forward and reverse shock propagation [astro-ph/0407132]

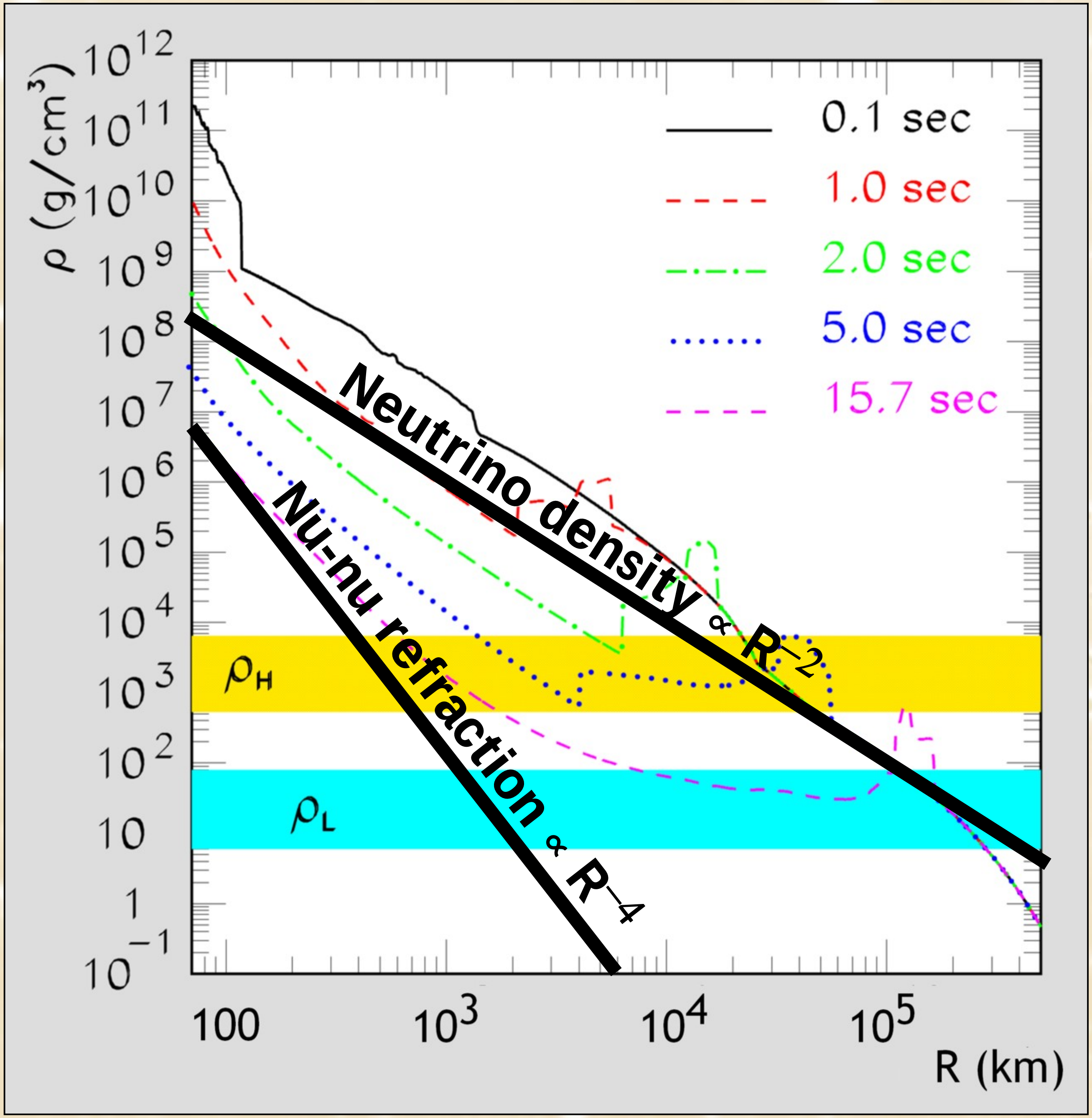
Shock-Wave Propagation in IceCube

$$\frac{\mathrm{Flux}(\overline{\nu}_e)}{\mathrm{Flux}(\overline{\nu}_x)} = 0.8, \qquad \langle E_{\overline{\nu}_e} \rangle = 15\,\mathrm{MeV}, \qquad \langle E_{\overline{\nu}_x} \rangle = 18\,\mathrm{MeV}$$



Choubey, Harries & Ross, "Probing neutrino oscillations from supernovae shock waves via the IceCube detector", astro-ph/0604300

Neutrino Density Streaming off a Supernova Core



Typical luminosity in one neutrino species

$$L_\nu = 3\times10^{52}\,\frac{\text{erg}}{\text{s}}$$

Corresponds to a neutrino number density of

$$n_\nu = 3\times10^{35}\,\text{cm}^{-3}\left(\frac{\text{km}}{R}\right)^2$$

Current-current structure of weak interaction causes suppression of effective potential for collinear-moving particles

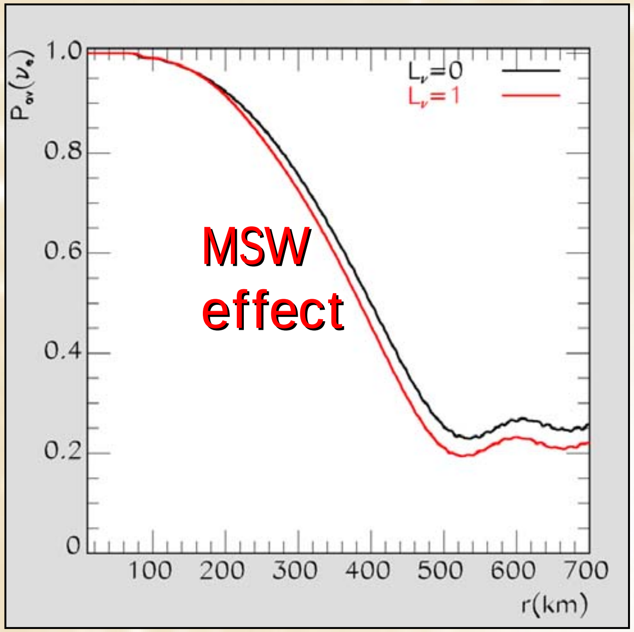
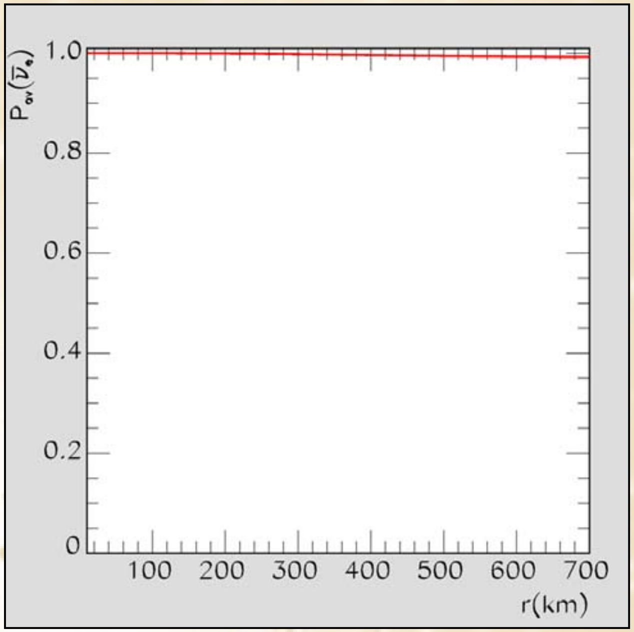
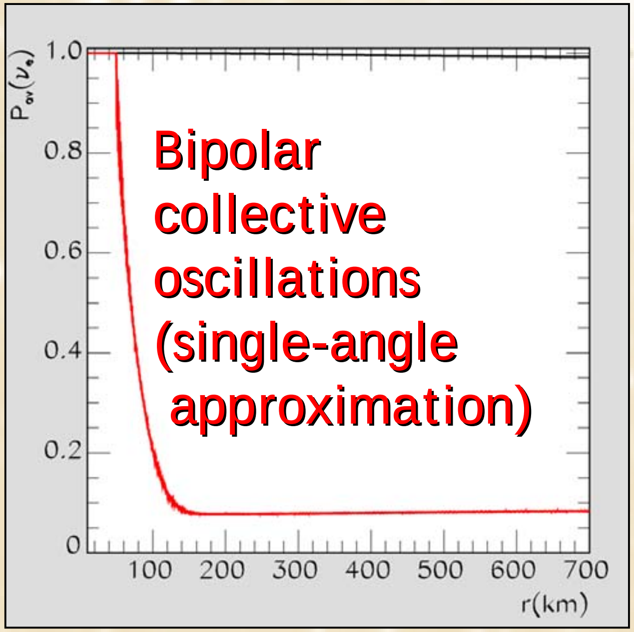
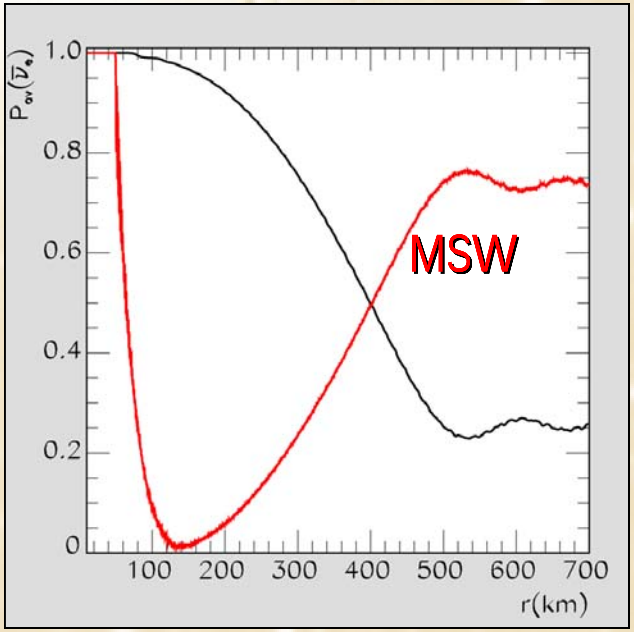
$$V_{\text{weak}} \propto G_F(1-\cos\theta)$$

Nu-nu refractive effect decreases as

$$V_{\nu\nu} \propto R^{-4}$$

Appears to be negligible

Self-Induced Flavor Oscillations of SN Neutrinos

	Survival probability ν_e	Survival probability $\bar{\nu}_e$	
Normal Hierarchy atm Δm^2 Θ_{13} close to Chooz limit			Realistic nu-nu effect No nu-nu effect
Inverted Hierarchy			Realistic nu-nu effect No nu-nu effect

Collective SN neutrino oscillations 2006-2008 (I)

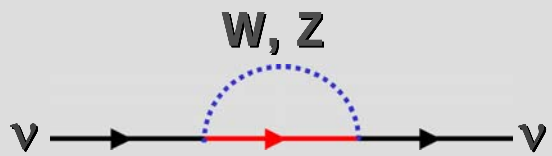
"Bipolar" collective transformations important, even for dense matter	• Duan, Fuller & Qian astro-ph/0511275
Numerical simulations • Including multi-angle effects • Discovery of "spectral splits"	• Duan, Fuller, Carlson & Qian astro-ph/0606616, 0608050
• Pendulum in flavor space • Collective pair annihilation • Pure precession mode	• Hannestad, Raffelt, Sigl & Wong astro-ph/0608695 • Duan, Fuller, Carlson & Qian astro-ph/0703776
Self-maintained coherence vs. self-induced decoherence caused by multi-angle effects	• Sawyer, hep-ph/0408265, 0503013 • Raffelt & Sigl, hep-ph/0701182 • Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl, arXiv:0706.2498
Theory of "spectral splits" in terms of adiabatic evolution in rotating frame	• Raffelt & Smirnov, arXiv:0705.1830, 0709.4641 • Duan, Fuller, Carlson & Qian arXiv:0706.4293, 0707.0290
Independent numerical simulations	• Fogli, Lisi, Marrone & Mirizzi arXiv:0707.1998

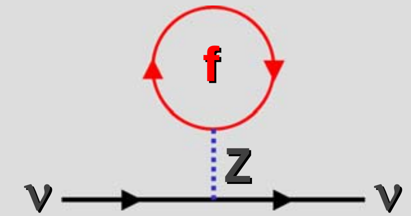
Collective SN neutrino oscillations 2006-2008 (II)

Three-flavor effects in O-Ne-Mg SNe on neutronization burst (MSW-prepared spectral double split)	• Duan, Fuller, Carlson & Qian, arXiv:0710.1271 • Dasgupta, Dighe, Mirrizzi & Raffelt, arXiv:0801.1660
Theory of three-flavor collective oscillations	• Dasgupta & Dighe, arXiv:0712.3798
Second-order mu-tau refractive effect important in three-flavor context	• Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl, arXiv:0712.1137
Identifying the neutrino mass hierarchy at extremely small Theta-13	• Dasgupta, Dighe & Mirizzi, arXiv:0802.1481

Neutrino Oscillations in a Neutrino Background

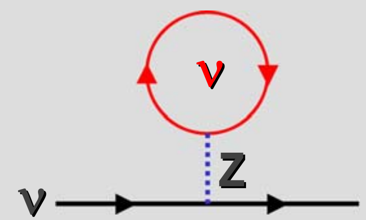
Neutrinos in a medium suffer flavor-dependent refraction (Wolfenstein, PRD 17:2369, 1978)





$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \left[\frac{M^2}{2E} + \sqrt{2}G_F\begin{pmatrix} n_e - \frac{1}{2}n_n & 0 \\ 0 & -\frac{1}{2}n_n\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

If neutrinos form the background, the refractive index has "offdiagonal elements" (Pantaleone, PLB 287:128, 1992)



$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \left[\frac{M^2}{2E} + \sqrt{2}G_F\begin{pmatrix} 2n_{\nu_e} + n_{\nu_\mu} & n_{\langle\nu_e|\nu_\mu\rangle} \\ n_{\langle\nu_\mu|\nu_e\rangle} & n_{\nu_e} + 2n_{\nu_\mu}\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

- One can not operationally distinguish between "beam" and "background"
- Problem is fundamentally nonlinear

Matrices of Density in Flavor Space

Neutrino quantum field

$$\Psi(t,x) = \int \frac{d^3\vec{p}}{(2\pi)^3}\left[a(t,\vec{p})u_{\vec{p}} + b^\dagger(t,-\vec{p})v_{-\vec{p}}\right]e^{i\vec{p}\cdot\vec{x}}$$

Spinors in flavor space

$$\Psi = \begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\end{pmatrix} \qquad a = \begin{pmatrix}a_1\\ a_2\\ a_3\end{pmatrix} \qquad b = \begin{pmatrix}b_1\\ b_2\\ b_3\end{pmatrix}$$

Destruction operators for (anti)neutrinos

Variables for discussing neutrino flavor oscillations

Quantum states (amplitudes)	"Matrices of densities" (analogous to occupation numbers)
$\left\lvert \begin{matrix}\nu_1(t,\vec{p})\\ \nu_2(t,\vec{p})\\ \nu_3(t,\vec{p})\end{matrix}\right\rangle = \begin{pmatrix}a_1(t,\vec{p})\\ a_2(t,\vec{p})\\ a_3(t,\vec{p})\end{pmatrix}^\dagger \lvert 0\rangle$	$\rho_{ij}(t,\vec{p}) = \langle a_j^\dagger(t,\vec{p})\, a_i(t,\vec{p})\rangle$ Neutrinos $\bar{\rho}_{ij}(t,\vec{p}) = \langle b_i^\dagger(t,\vec{p})\, b_j(t,\vec{p})\rangle$ Anti-neutrinos
Sufficient for "beam experiments"	"Quadratic" quantities, required for dealing with decoherence, collisions, Pauli-blocking, nu-nu-refraction, etc.

General Equations of Motion

ν

$$i\partial_t \rho_{\vec p} = +\left[\frac{M^2}{2p}, \rho_{\vec p}\right] + \sqrt{2} G_F [L, \rho_{\vec p}] + \sqrt{2} G_F \int \frac{d^3\vec q}{(2\pi)^3} (1 - \cos\theta_{\vec p \vec q}) [(\rho_{\vec q} - \bar\rho_{\vec q}), \rho_{\vec p}]$$

$\bar\nu$

$$i\partial_t \bar\rho_{\vec p} = -\left[\frac{M^2}{2p}, \bar\rho_{\vec p}\right] + \sqrt{2} G_F [L, \bar\rho_{\vec p}] + \sqrt{2} G_F \int \frac{d^3\vec q}{(2\pi)^3} (1 - \cos\theta_{\vec p \vec q}) [(\rho_{\vec q} - \bar\rho_{\vec q}), \bar\rho_{\vec p}]$$

- Vacuum oscillations
  M is neutrino mass matrix
- Note opposite sign between neutrinos and antineutrinos

Usual matter effect with

$$L = \begin{pmatrix} n_e - n_{\bar e} & 0 & 0 \\ 0 & n_\mu - n_{\bar\mu} & 0 \\ 0 & 0 & n_\tau - n_{\bar\tau} \end{pmatrix}$$

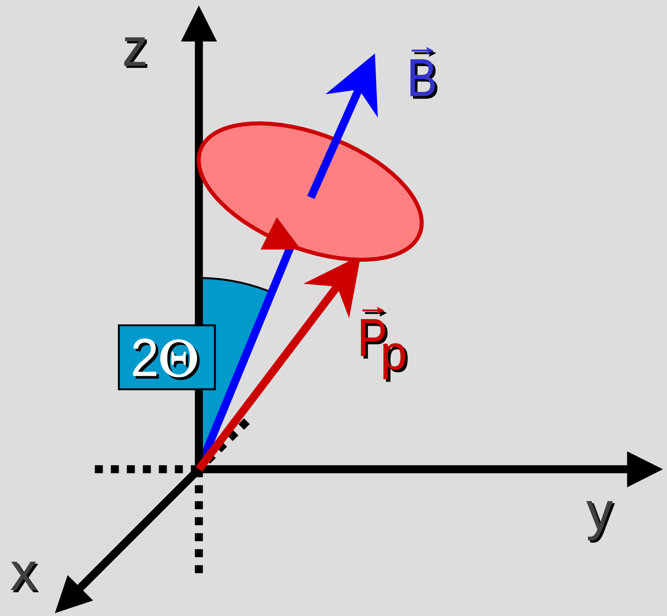
Nonlinear nu-nu effects are important when nu-nu interaction energy exceeds typical vacuum oscillation frequency (Do not compare with matter effect!)

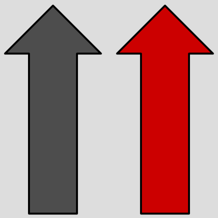
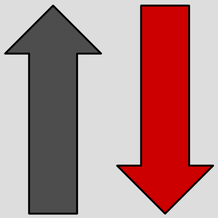
$$\omega_{osc} = \frac{\Delta m^2}{2E} < \mu = \sqrt{2}\, G_F n_\nu \langle 1 - \cos\theta \rangle$$

Two-Flavor Neutrino Oscillations in Vacuum

Polarization vector	$\rho_p = \frac{f_p + \vec\sigma\cdot\vec P_p}{2}$ or different normalization $\rho_p = f_p\,\frac{1+\vec\sigma\cdot\vec P_p}{2}$ σ_i Pauli matrices
"Magnetic field" in flavor space	$\Omega^0_p \approx p + \frac{m_2^2+m_1^2}{4p} + \frac{m_2^2-m_1^2}{2p}\,\frac{\vec\sigma\cdot\vec B}{2}$ $\vec B = \begin{pmatrix}\sin 2\theta\\ 0\\ \cos 2\theta\end{pmatrix}$

Neutrino flavor oscillation as a spin precession



Neutrinos	Anti-neutrinos
$\partial_t \vec P_p = +\frac{\Delta m^2}{2p}\vec B\times\vec P_p$	$\partial_t \vec{\bar P}_p = -\frac{\Delta m^2}{2p}\vec B\times\vec{\bar P}_p$
Spin 1/2  Magnetic moment $+\Delta m^2/2p$	Spin 1/2  Magnetic moment $-\Delta m^2/2p$

Synchronized Oscillations by Self-Interactions

Neutrino ensemble with a broad distribution of momentum modes

$$\partial_t \vec{P}_p = \frac{\Delta m^2}{2p} \vec{B} \times \vec{P}_p + \sqrt{2} G_F \vec{P} \times \vec{P}_p$$

$$\vec{P} = \int \frac{d^3p}{(2\pi)^3} \vec{P}_p$$

Integrated polarization vector

Neutrinos precess with different frequencies in external magnetic field B (in flavor space)

The ensemble of neutrino magnetic moments creates an “internal magnetic field” that is felt by each neutrino

Internal field $\gg$ external B

$\to$ All modes lock to each other and spin-precess together, in analogy to spin-orbit coupling in atoms

Synchronized oscillation frequency

$$\omega_{synch} = \left\langle \frac{\Delta m^2}{2p} \right\rangle$$

Pastor, Raffelt & Semikoz, hep-ph/0109035

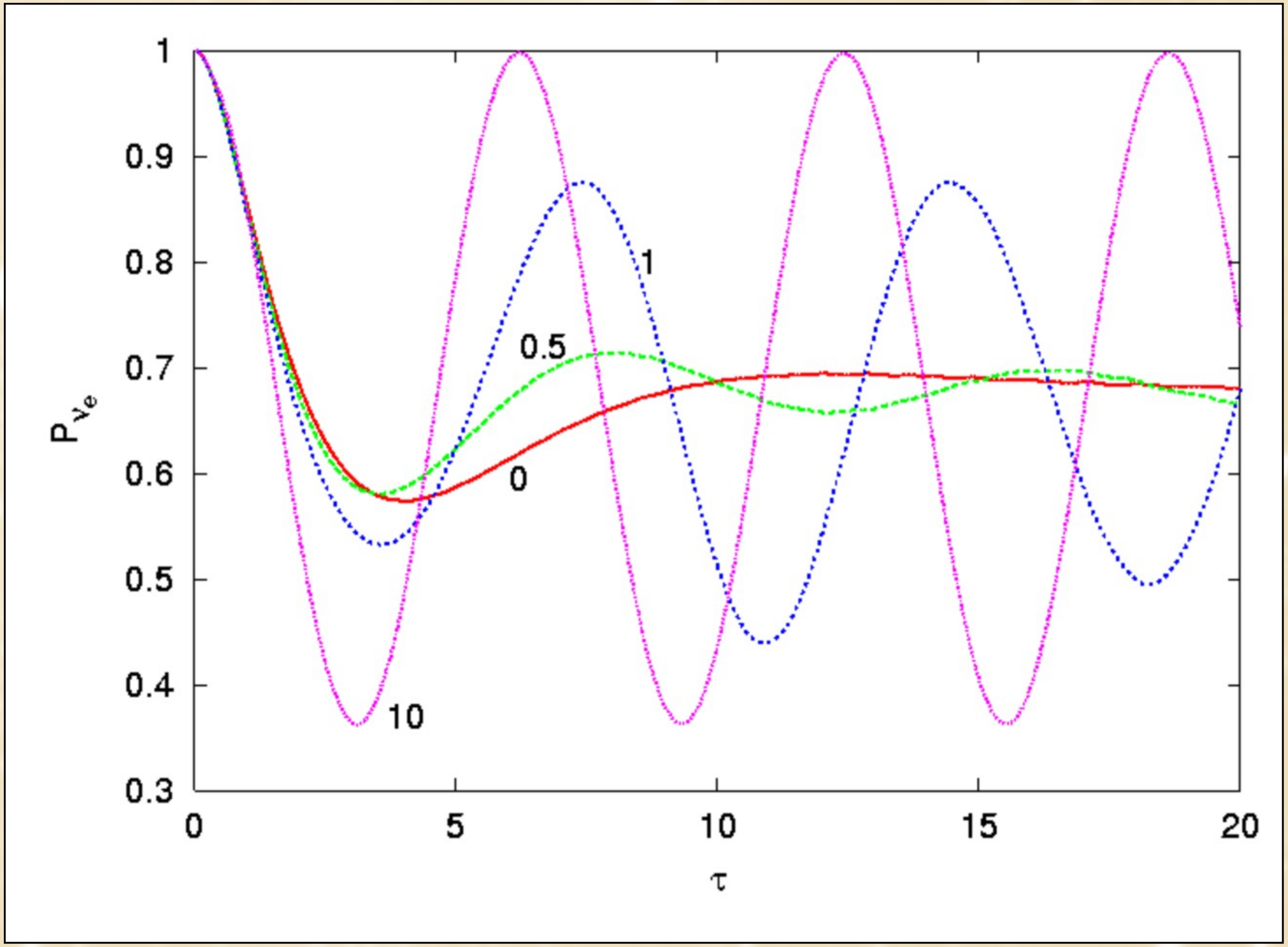
Synchronizing Oscillations by Neutrino Interactions

Vacuum oscillation frequency of mode with momentum p ~ E $\omega_{osc} = \frac{\Delta m^2}{2p}$

Modified in a medium by the usual weak-interaction potential

In an ensemble with a broad momentum distribution, the p-dependent oscillation frequency quickly leads to kinematical flavor decoherence

In a dense neutrino gas, all modes go with the same frequency: "Synchronized flavor oscillations" or "self-maintained coherence"



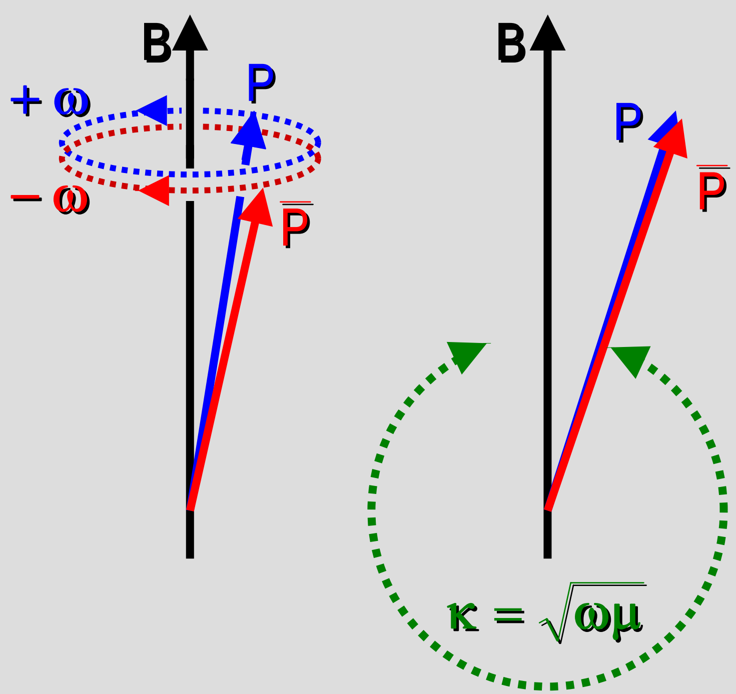
Pastor, Raffelt & Semikoz, hep-ph/0109035

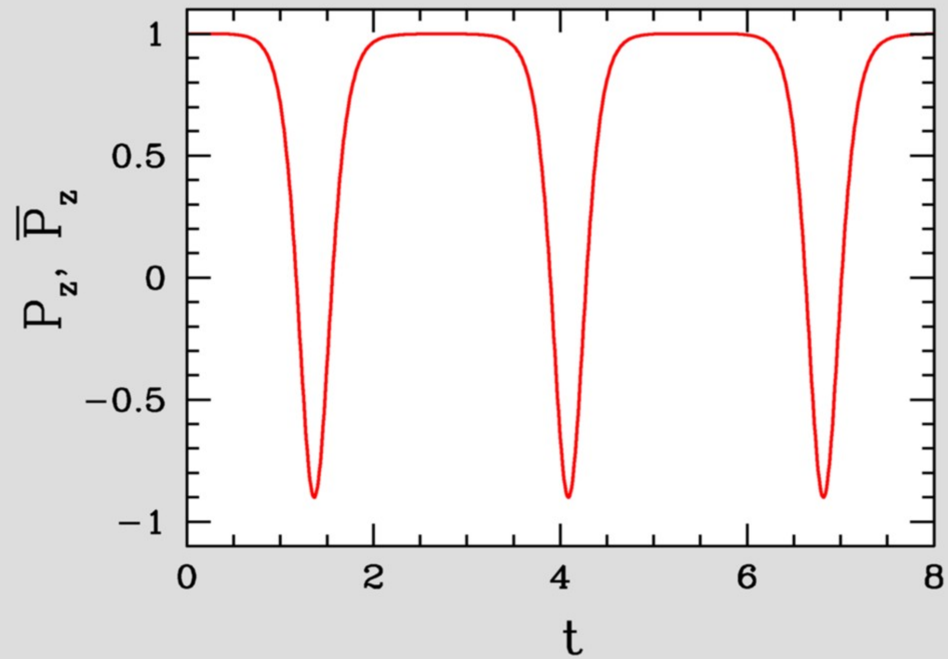
Oscillations of Neutrinos plus Antineutrinos in a Box

Equal ν_e and $\bar{\nu}_e$ densities, single energy E, with $\mu = \sqrt{2}\,G_F n_{\nu_e} \gg \omega = \frac{\Delta m^2}{2E}$

$$(\nu) \quad \partial_t \mathbf{P} = +\omega \mathbf{B} \times \mathbf{P} + \mu(\mathbf{P} - \bar{\mathbf{P}}) \times \mathbf{P}$$

$$(\bar{\nu}) \quad \partial_t \bar{\mathbf{P}} = \underbrace{-\omega \mathbf{B} \times \bar{\mathbf{P}}}_{\text{Opposite vacuum oscillations}} + \underbrace{\mu(\mathbf{P} - \bar{\mathbf{P}}) \times \bar{\mathbf{P}}}_{\text{Equal self terms}}$$



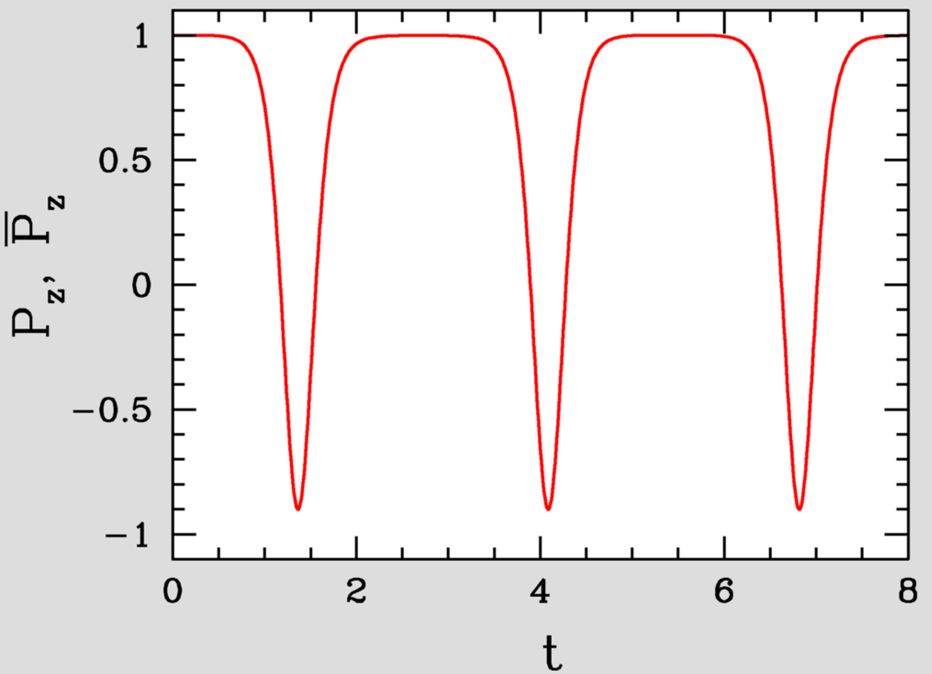


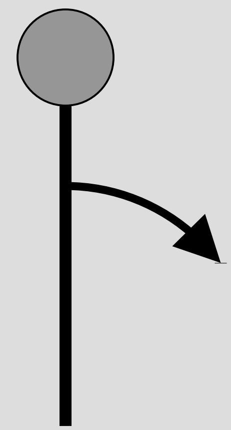
"Pendulum in flavor space"

- Inverted mass hierarchy
  - → Inverted pendulum
  - → Unstable even for small mixing angle
- Normal mass hierarchy
  - → Small-amplitude oscillations

Flavor Conversion Without Flavor Mixing?

Equal ν_e and $\bar{\nu}_e$ densities in a box (inverted hierarchy)





Inverted pendulum:

- Time to fall depends logarithmically on small initial angle Θ
- Stays up forever only for Θ = 0
- Unstable by quantum uncertainty relation ("How long can a pencil stand on its tip?")

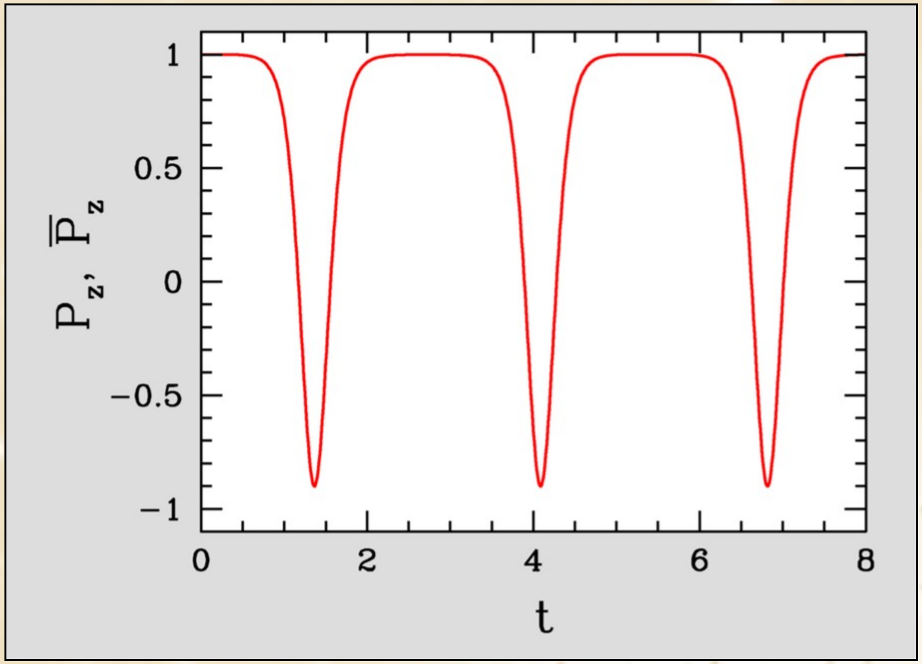
- This is no real "flavor conversion", rather a "coherent pair conversion"

  $\nu_e \bar{\nu}_e \leftrightarrow \nu_\mu \bar{\nu}_\mu$
- Occurs anyway at second order G_F
- Coherent "speed-up effect" (Sawyer)

Not clear (to me) if coherent transformations can be triggered by quantum fluctuations alone (mixing angle Θ = 0)

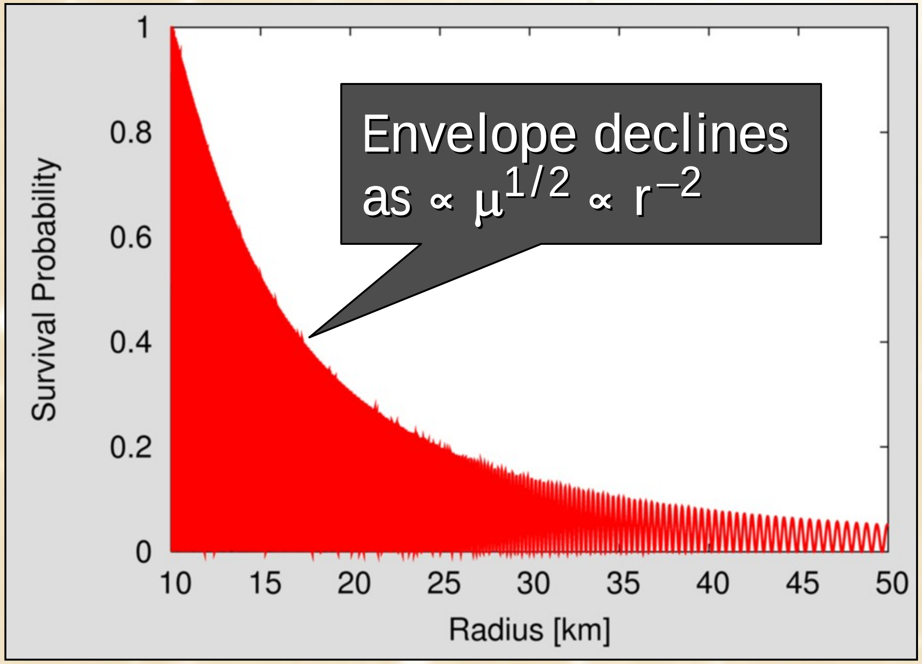
Supernova Neutrino Conversion

Neutrinos in a box



Permanent pendular oscillations

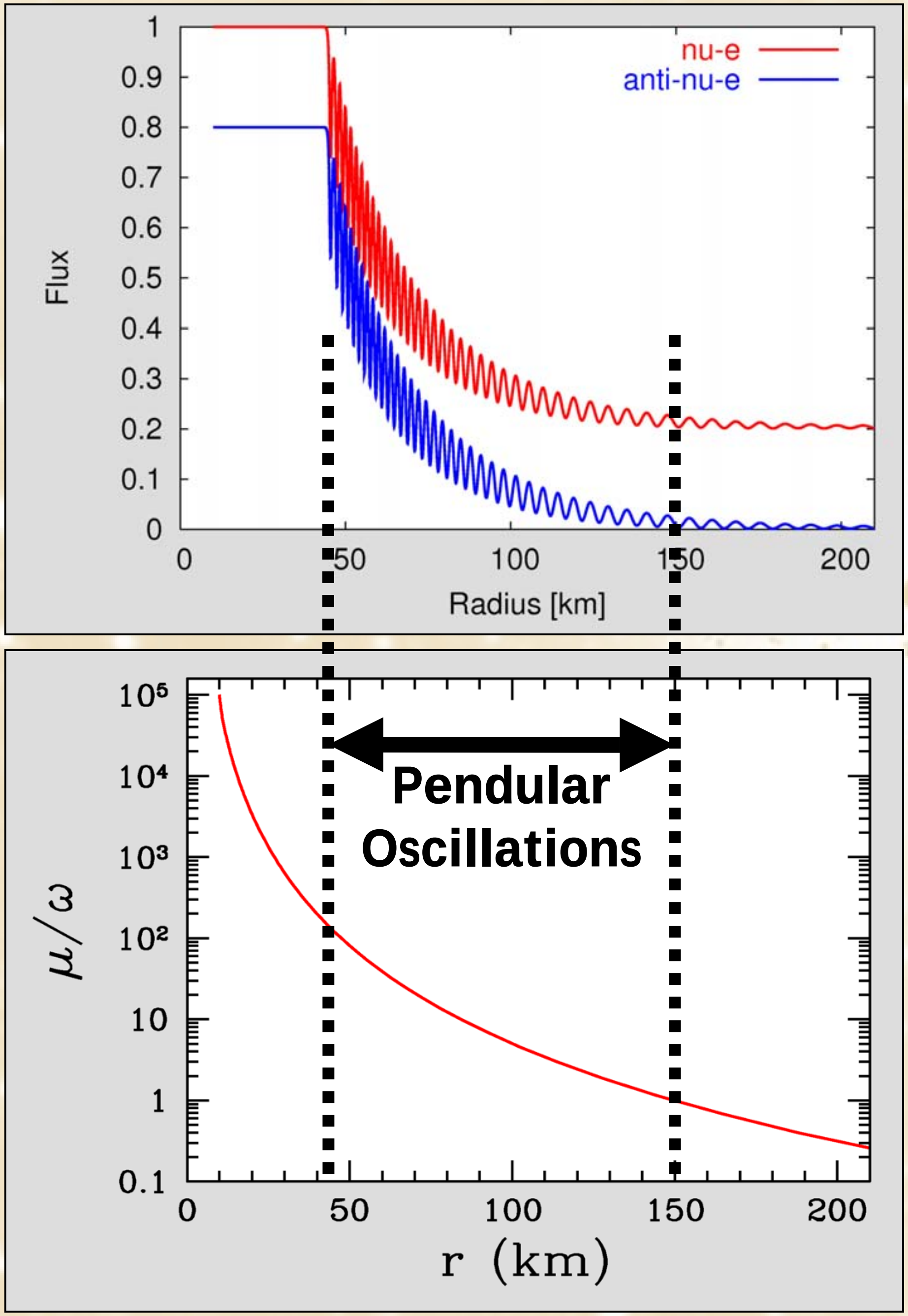
Neutrinos streaming off a supernova core



Complete conversion

- Nu-nu interaction energy $\mu = \sqrt{2} G_F n_\nu$ decreases
- Pendulum's moment of inertia μ^{-1} increases
- Conservation of angular momentum
  → kinetic energy decreases
  → amplitude decreases $\propto \mu^{1/2}$

Flavor Conversion in Toy Supernova



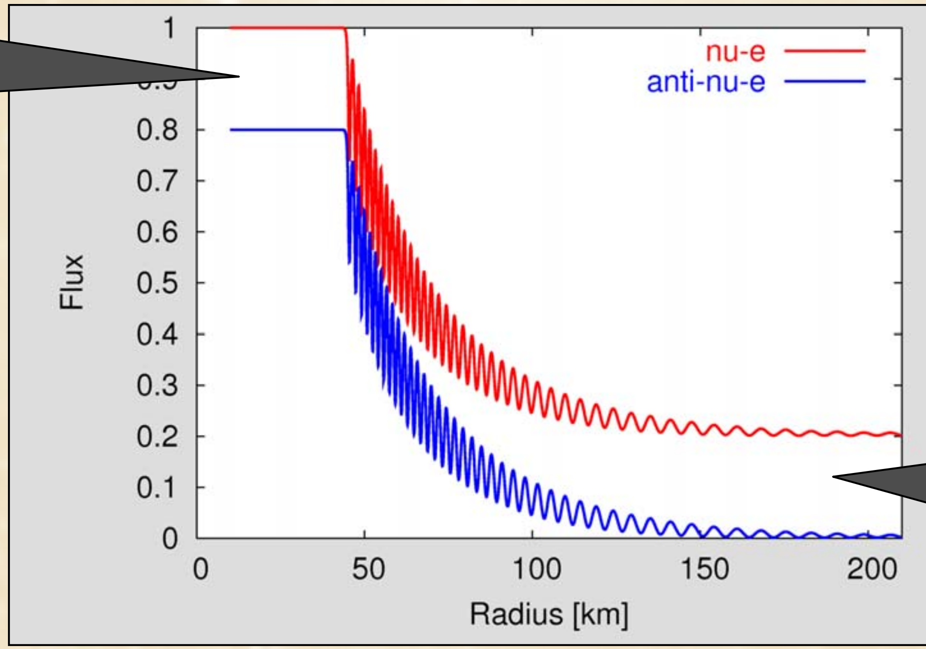
- Assume 80% anti-neutrinos
- Vacuum oscillation frequency $\omega = 0.3\ \mathrm{km}^{-1}$
- Neutrino-neutrino interaction energy at nu sphere (r = 10 km) $\mu = 0.3\times10^5\ \mathrm{km}^{-1}$
- Falls off approximately as r^{-4} (geometric flux dilution and nus become more co-linear)

Decline of oscillation amplitude explained in pendulum analogy by inreasing moment of inertia (Hannestad, Raffelt, Sigl & Wong astro-ph/0608695)

Flavor Pair Conversion vs. Flavor Lepton Conservation

- Flavor-dependent flux hierarchy of neutrinos emerging from a SN core ($\nu_e\bar{\nu}_e$ pair excess) $\langle F_{\nu_e} \rangle > \langle F_{\bar{\nu}_e} \rangle > \langle F_{\nu_x} \rangle$
- Interior of a SN core: Chemical ν_e potential (no pair excess) $\langle n_{\nu_e} \rangle > \langle n_{\nu_x} \rangle > \langle n_{\bar{\nu}_e} \rangle$

20% excess flux of ν_e over $\bar{\nu}_e$ at the source



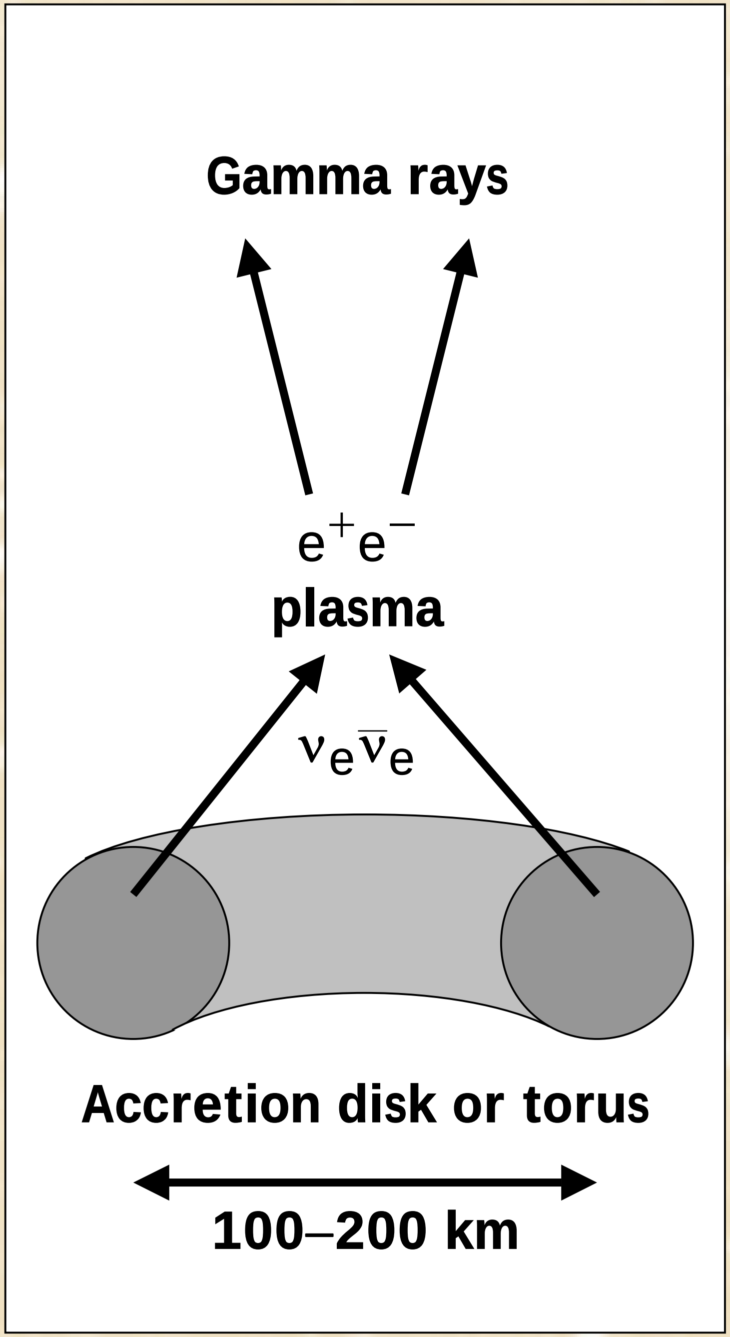
Excess flux of ν_e over $\bar{\nu}_e$ conserverd

- (Collective) oscillations preserve flavor-lepton number in the mass basis
- Essentially identical to weak-interaction basis for small mixing and/or large matter effects
- Of course not true when MSW resonance play a role

Large flavor conversion with small mixing angle

MSW effect	Collective pair conversions
$\nu_e \to \nu_x$	$\nu_e \bar{\nu}_e \to \nu_x \bar{\nu}_x$
Driven by matter density gradient	Driven by neutrino flux dilution with distance from source
Flavor lepton number strongly violated	Flavor lepton number conserved
Effect disappears for small mixing angle (loss of adiabaticity)	• Effect logarithmically delayed with small mixing angle Θ • Effective even for very small Θ
• Solar neutrinos (but mixing angle anyway large) • Neutrinos propagating through mantle and envelope of SN driven by Δm^2_{atm} and Θ_{13}	Dense flux of $\nu_e \bar{\nu}_e$ in excess over other flavors • Core collapse supernova • Coalescing neutron stars (short gamma-ray bursts)

Coalescing Neutron Stars and Short Gamma-Ray Bursts

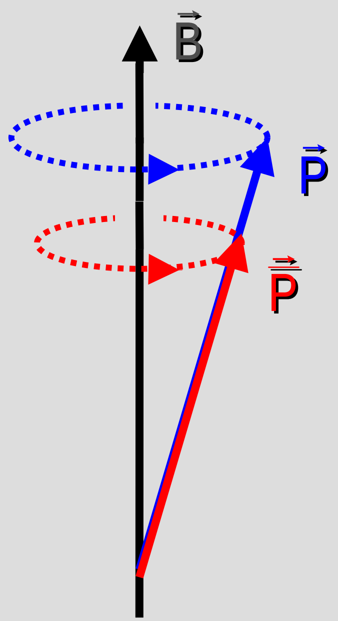


- Annihilation rate strongly suppressed if $\nu_e\bar{\nu}_e$ pairs transform to $\nu_x\bar{\nu}_x$ pairs
- Collective effects important?

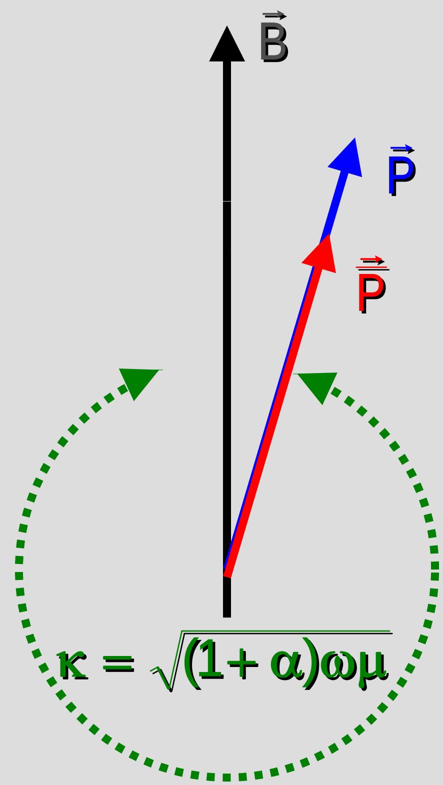
Density of torus relatively small:
- ν_μ and ν_τ not efficiently produced
- Large $\nu_e\bar{\nu}_e$ pair abundance

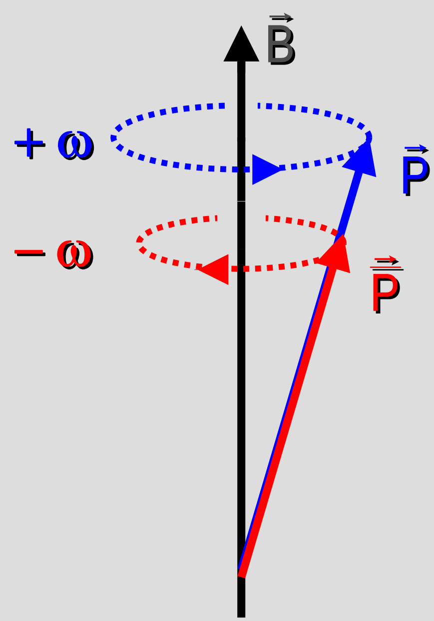
Synchronized vs. Pendular Oscillations

- Ensemble of unequal densities $n_{\bar{\nu}_e} = \alpha\, n_{\nu_e}$ (antineutrino fraction $\alpha < 1$)
- Equal energies (equal oscillation frequency $\omega = \Delta m^2/2E$)
- Interaction energy $\mu = \sqrt{2} G_F n_{\nu_e}$



$$\omega_{\rm synch} = \frac{1+\alpha}{1-\alpha}\,\omega$$





Synchronized oscillations

$$\frac{1+\alpha}{(1-\alpha)^2}\,\omega \ll \mu$$

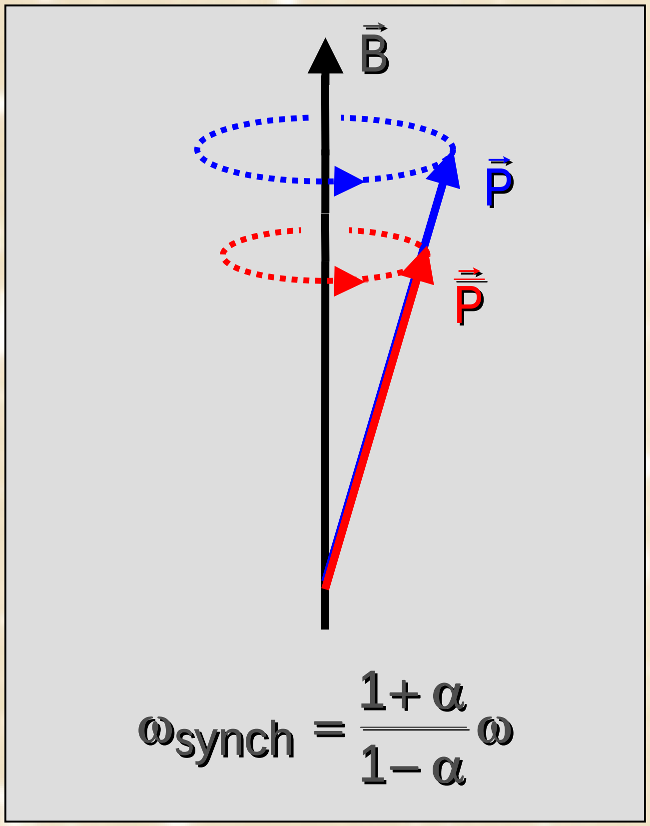
Pendular oscillations

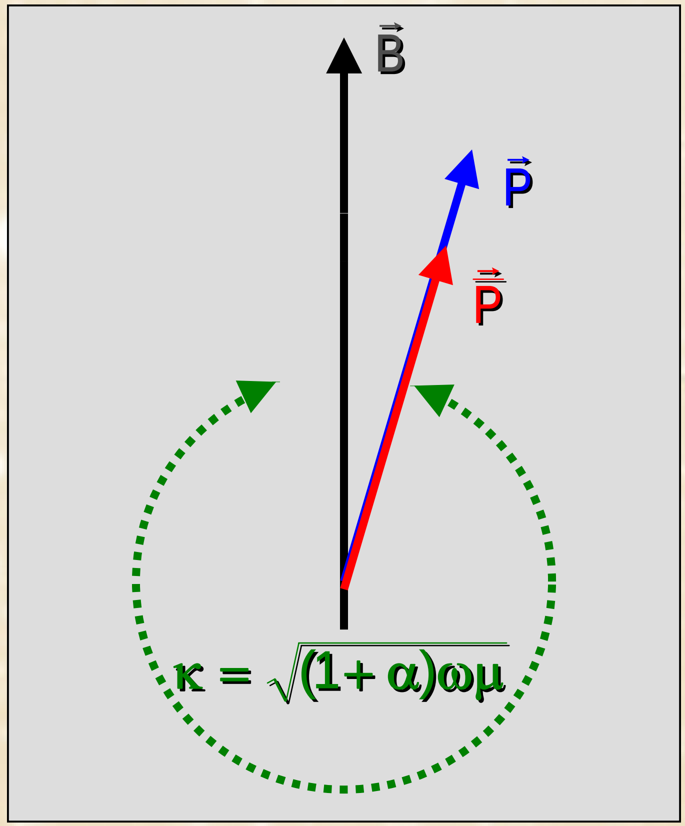
$$\omega \ll \mu \ll \frac{1+\alpha}{(1-\alpha)^2}\,\omega$$

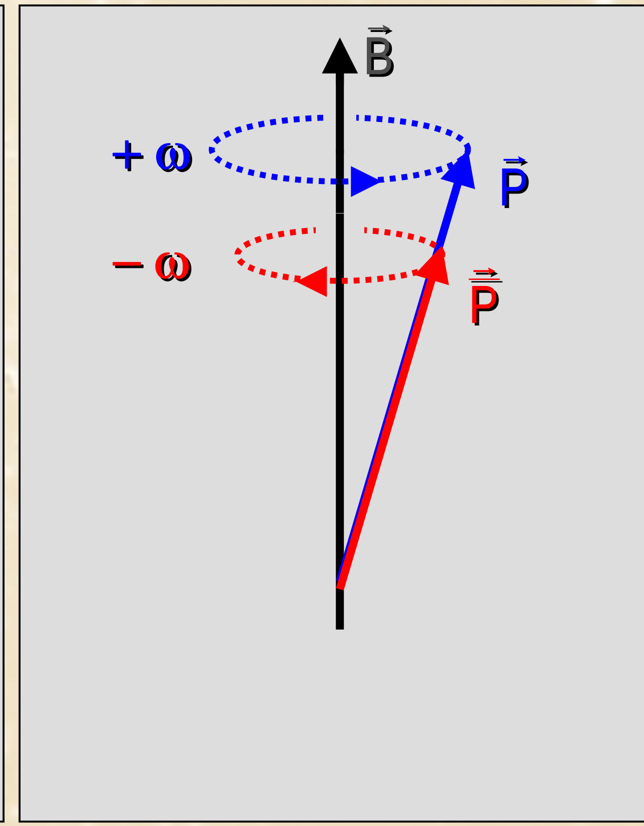
Free oscillations

$$\mu \ll \omega$$

Synchronized vs. Pendular Oscillations







Synchronized oscillations

$$\frac{1+\alpha}{(1-\alpha)^2}\omega \ll \mu$$

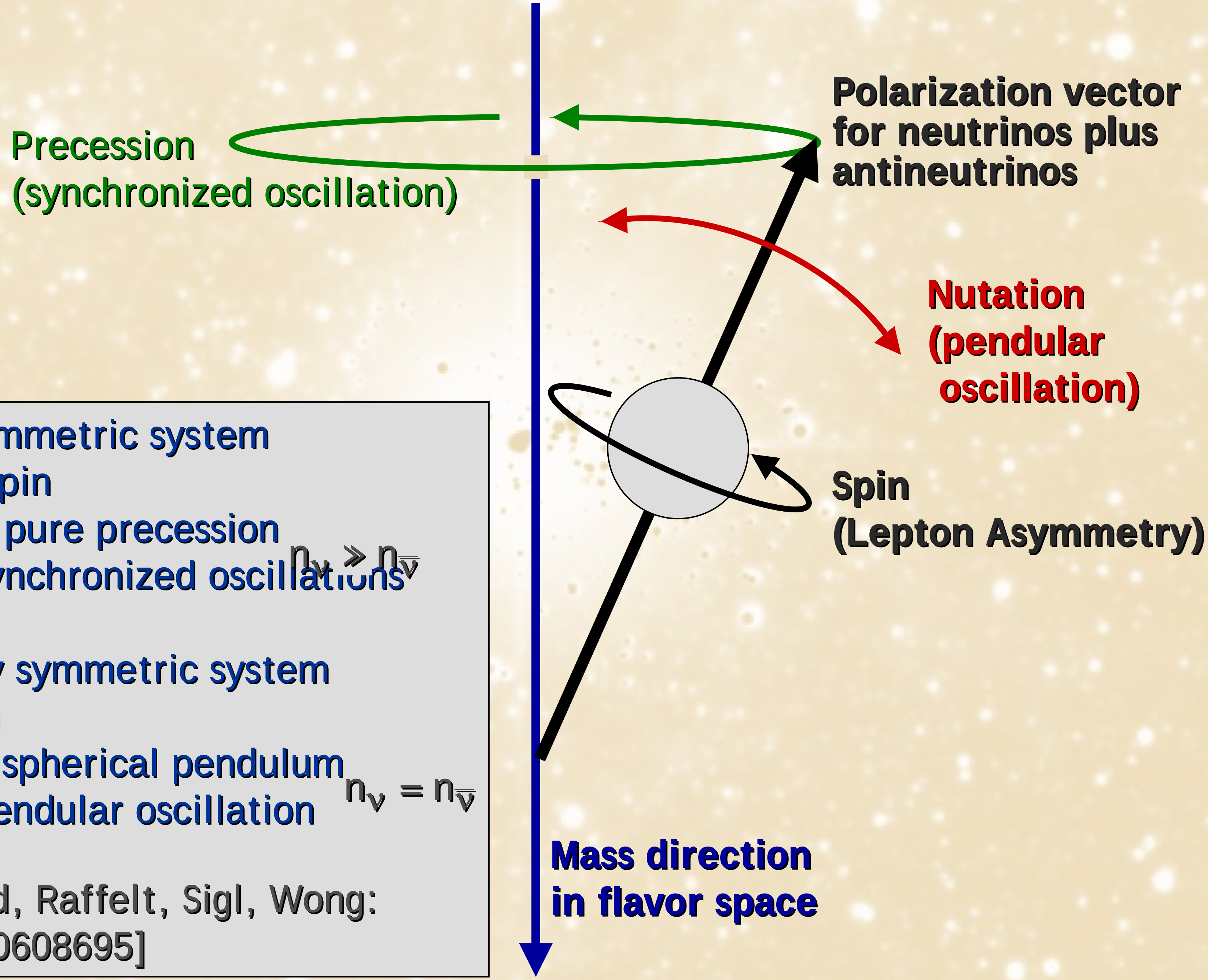
Pendular oscillations

$$\omega \ll \mu \ll \frac{1+\alpha}{(1-\alpha)^2}\omega$$

Free oscillations

$$\mu \ll \omega$$

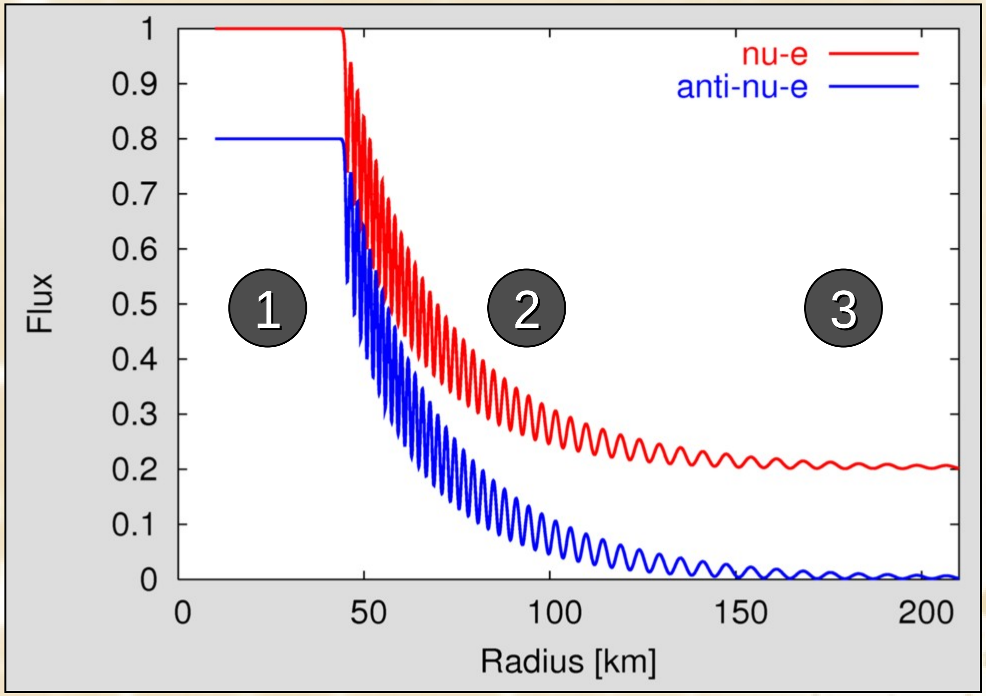
Pendulum in Flavor Space

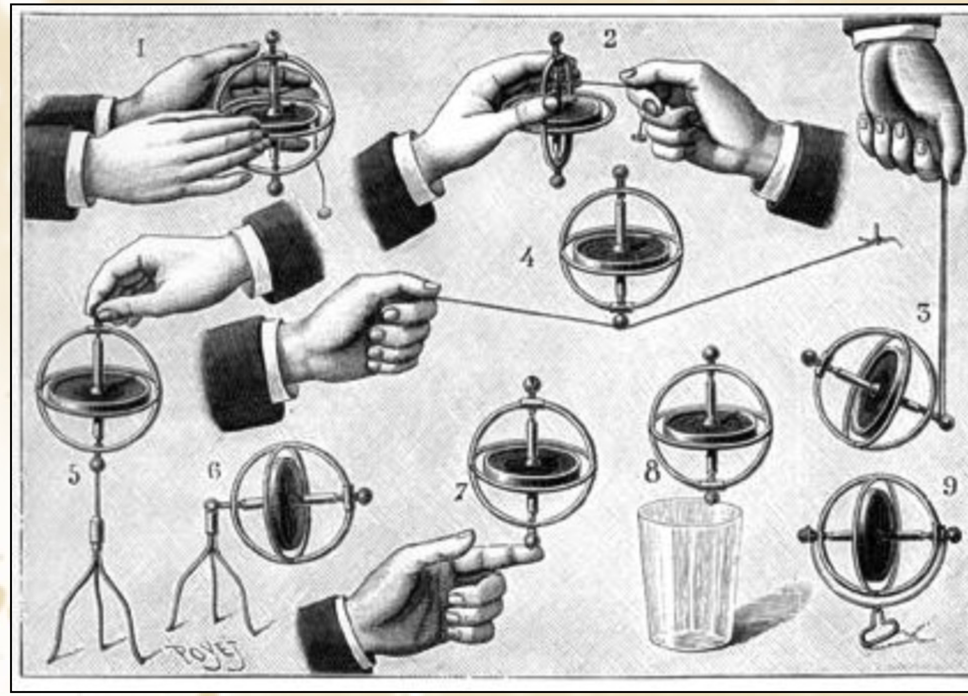


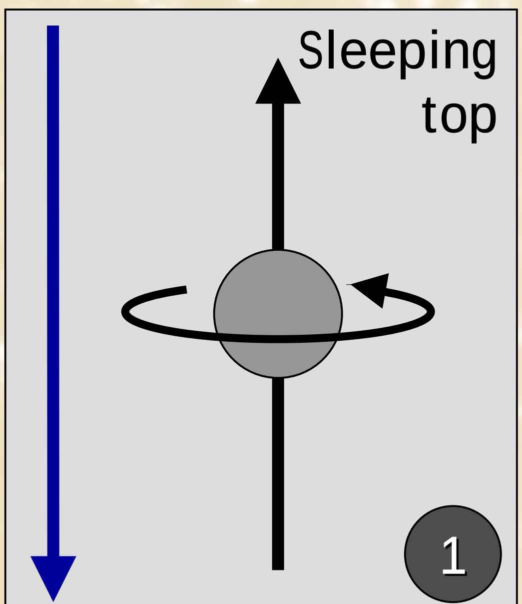
- Very asymmetric system
  - Large spin
  - Almost pure precession
  - Fully synchronized oscillations $n_\nu \gg n_{\bar{\nu}}$
- Perfectly symmetric system
  - No spin
  - Simple spherical pendulum
  - Fully pendular oscillation $n_\nu = n_{\bar{\nu}}$

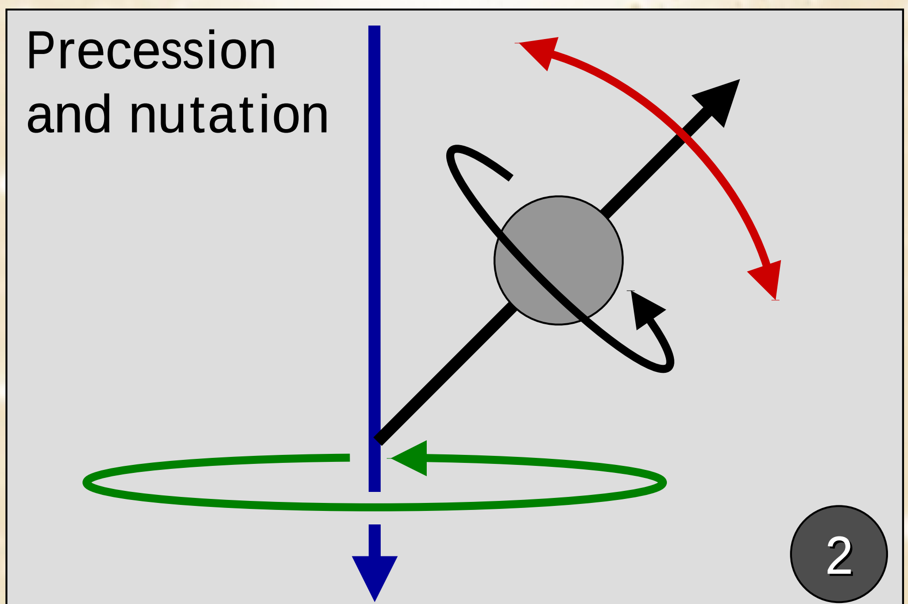
[Hannestad, Raffelt, Sigl, Wong:
astro-ph/0608695]

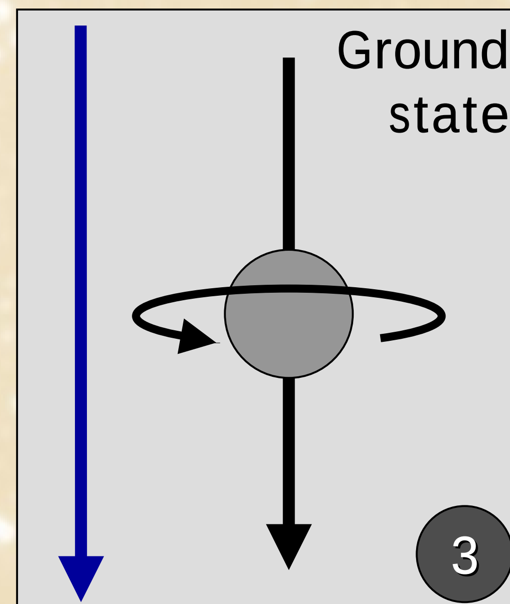
Neutrino Conversion and Gyroscopic Flavor Pendulum











Role of Ordinary Matter

$$(\nu) \quad \partial_t \mathbf{P} = +\omega \mathbf{B}\times\mathbf{P} + \lambda \mathbf{L}\times\mathbf{P} + \mu(\mathbf{P}-\overline{\mathbf{P}})\times\mathbf{P}$$

$$(\overline{\nu}) \quad \partial_t \overline{\mathbf{P}} = -\omega \mathbf{B}\times\overline{\mathbf{P}} + \lambda \mathbf{L}\times\overline{\mathbf{P}} + \mu(\mathbf{P}-\overline{\mathbf{P}})\times\overline{\mathbf{P}}$$

- Matter has identical effect on nus and anti-nus
- In rotating frame (frequency λ) no matter effect (Duan et al. astro-ph/0511275)
- Rotating B-field drives unstable inverted pendulum

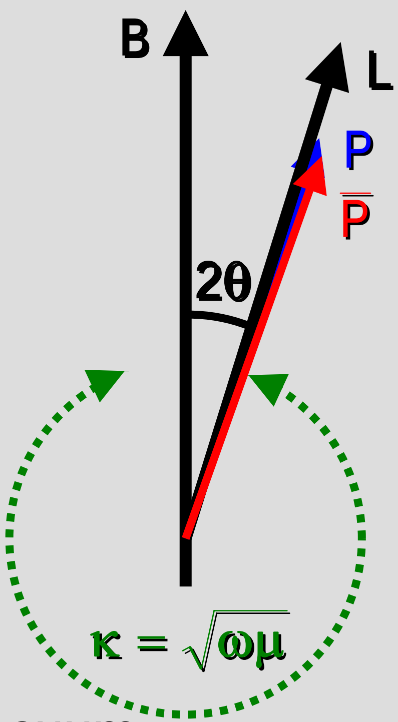
$\omega = \Delta m^2/2E$

$\lambda = \sqrt{2}\, G_F n_e$

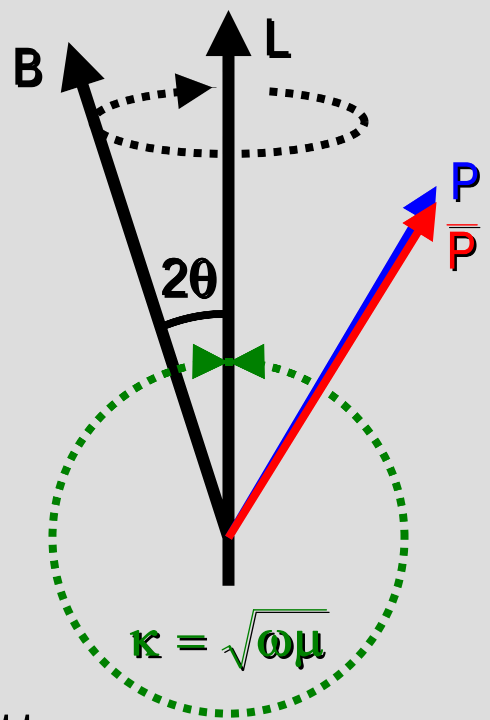
$\mu = \sqrt{2}\, G_F n_\nu$

B = mass direction

L = weak-interaction direction



Vacuum

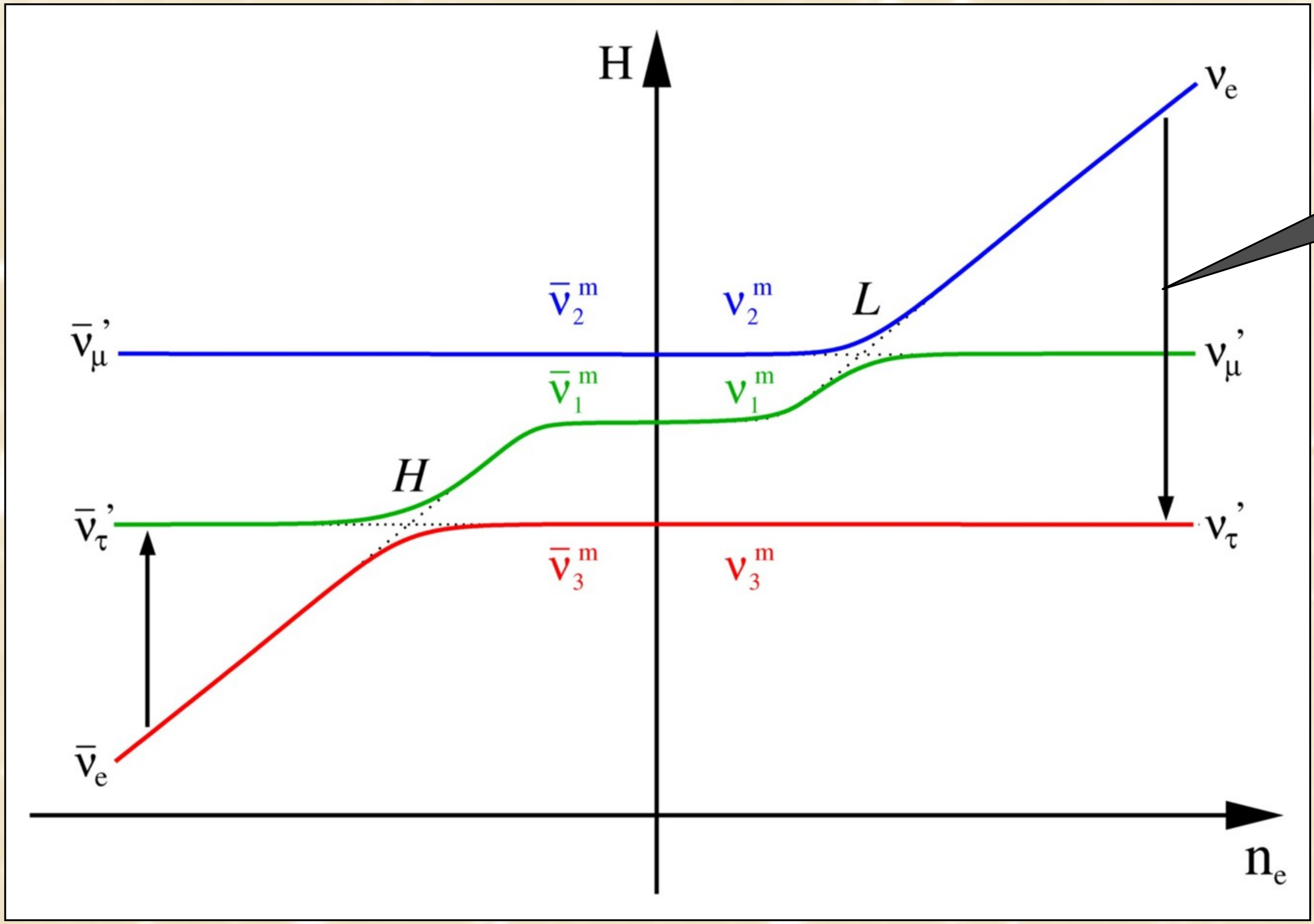


In matter

- B projection on L plays role of B_{eff}
- $\omega_{eff} = \omega \cos(2\theta)$
- No transformation for maximal mixing!
- Oscillation period $\approx \kappa^{-1} \ln\left(\theta\kappa / \sqrt{\kappa^2+\lambda^2}\right)$

Hannestad, Raffelt, Sigl, Wong: astro-ph/0608695

Level-Crossing Diagram for Inverted Hierarchy

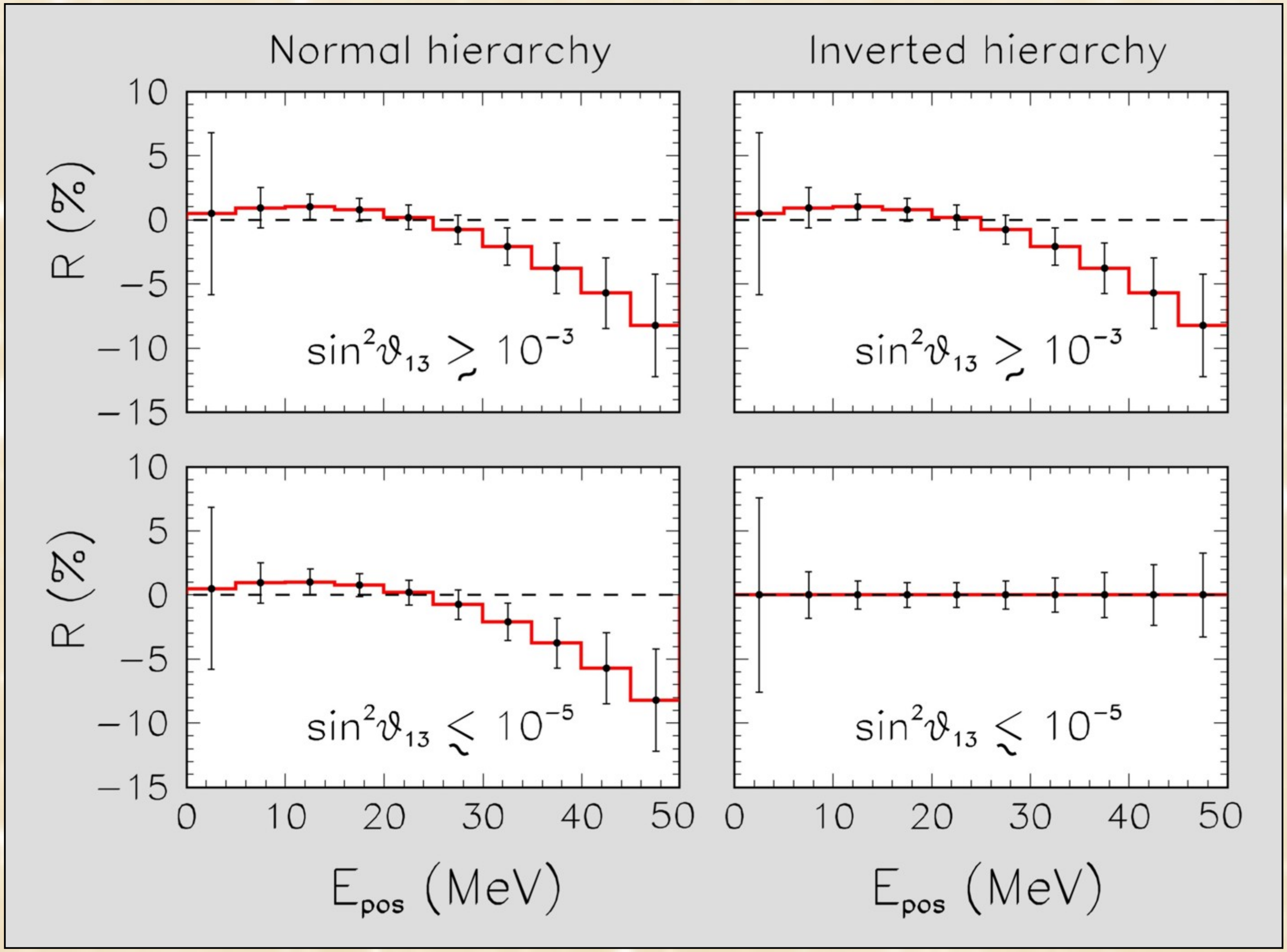


Collective oscillations driven by Θ_{13} and Δm_{atm}

Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl
arXiv:0712.1137

Mass Hierarchy at Extremely Small Theta-13

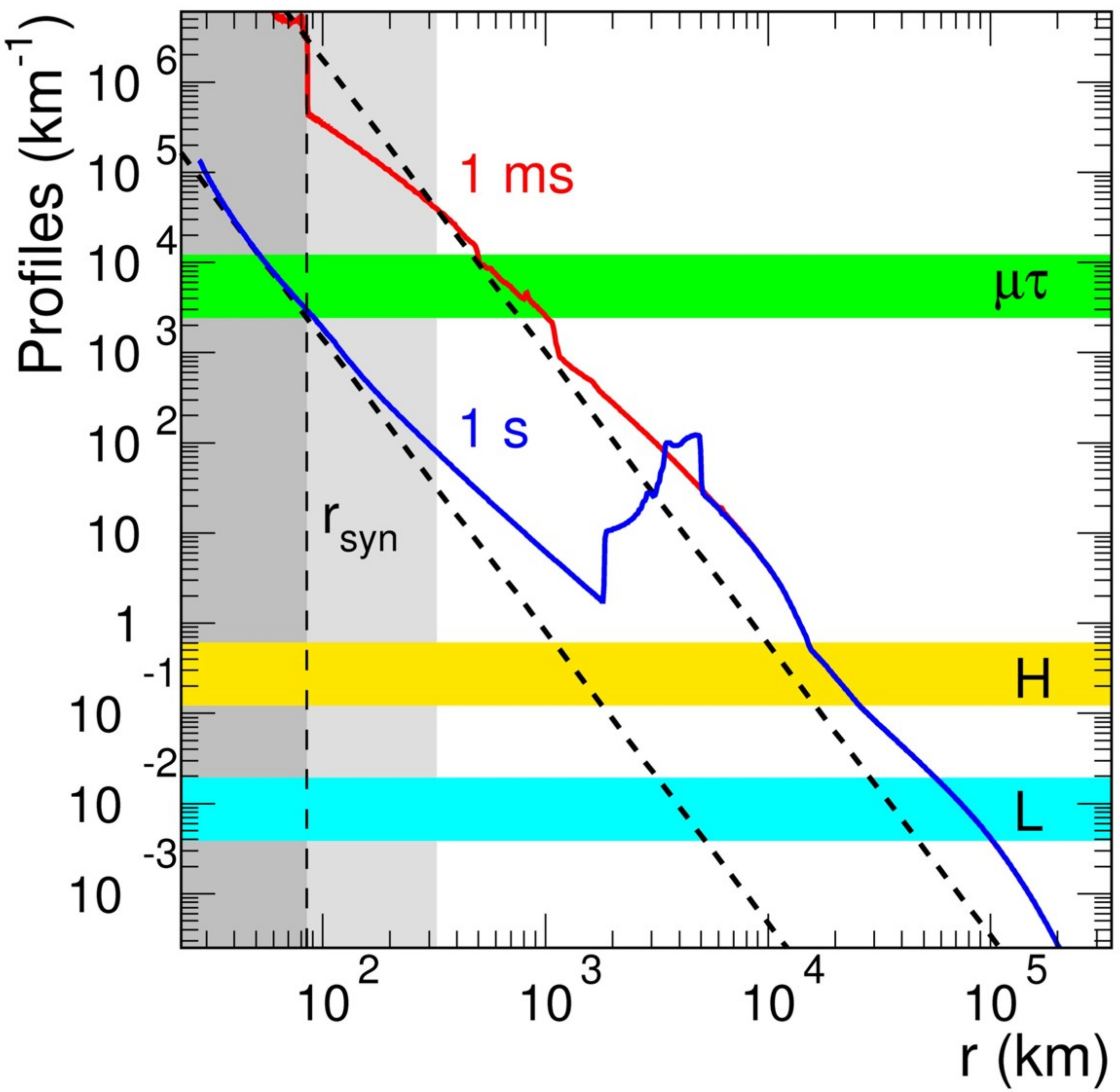
Using Earth matter effects to diagnose transformations



Ratio of spectra in two water Cherenkov detectors (0.4 Mton), one shadowed by the Earth, the other not

Dasgupta, Dighe & Mirizzi, arXiv:0802.1481

Second-Order Mu-Tau Refractive Difference

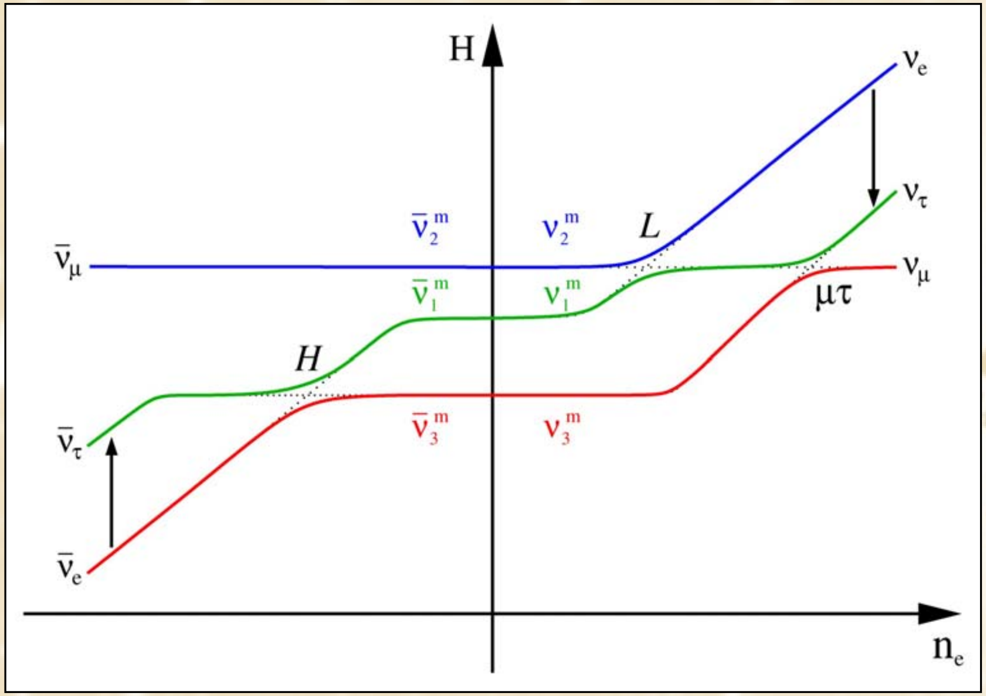


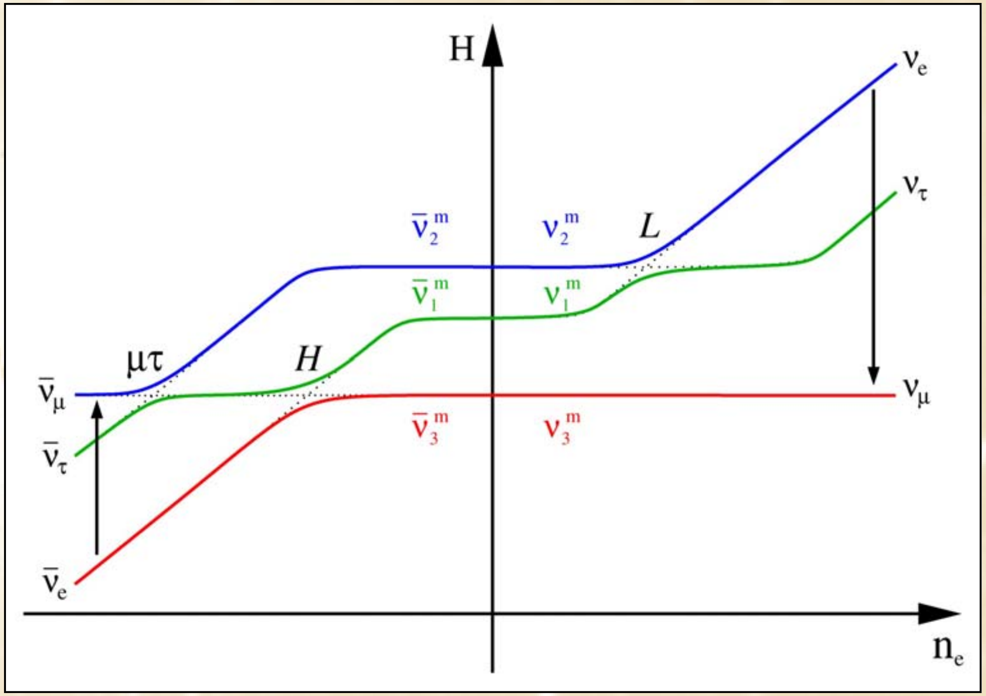
- Second-order difference between ν_μ and ν_τ matter effect causes a level crossing in the 23-flavor subsystem
- Not normally important if ν_μ and ν_τ fluxes are equal
- Even in this case, collective effects cause a large dependence of ν_e and $\bar{\nu}_e$ survival probabilities on matter density and on deviation of 23-mixing from maximal

Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl, arXiv:0712.1137 (Dec. 2007)

Level-Crossing Diagram with Large Mu-Tau Effect

Theta-23 in first octant	Theta-23 in second octant



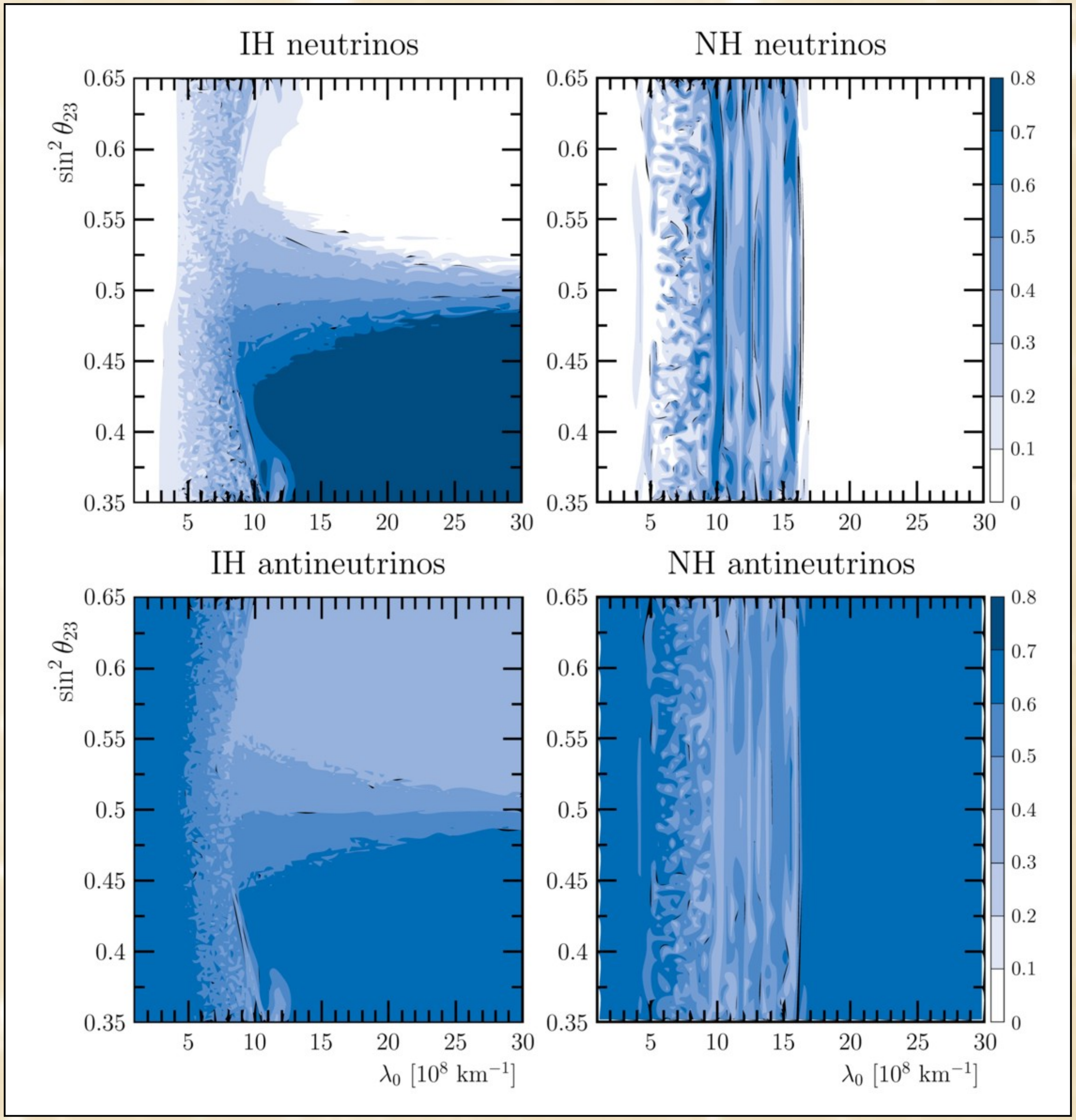


Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl, arXiv:0712.1137

Flavor conversion depending on $\Delta V_{\mu\tau}$ and Θ_{23}

ν_e flux emerging from SN surface (at nu sphere 25% larger than $\bar{\nu}_e$ flux)

$\bar{\nu}_e$ flux emerging from SN surface (normalized to 1 at nu sphere)



Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl, arXiv:0712.1137

Supernova Sensitivity to Neutrino Mixing Parameters

For inverted mass hierarchy, collective flavor conversions cause the flavor neutrino fluxes emerging from a supernova to be sensitive to mixing parameters in counter-intuitive ways

- Theta-13, even if arbitrarily (?) small
- Theta-23, small deviations from maximal mixing (if density is so large that mu-tau matter effect important)
- Dirac phase: has not been investigated

Multi-Energy and Multi-Angle Effects

$$(\nu) \qquad \partial_t \mathbf{P}_p = +\frac{\Delta m^2}{2p}\mathbf{B}\times\mathbf{P}_p + \lambda\mathbf{L}\times\mathbf{P}_p + \sqrt{2}G_F\int\frac{d^3q}{(2\pi)^3}(1-\cos\theta_{pq})(\mathbf{P}_q-\overline{\mathbf{P}}_q)\times\mathbf{P}_p$$

$$(\overline{\nu}) \qquad \partial_t \overline{\mathbf{P}}_p = -\frac{\Delta m^2}{2p}\mathbf{B}\times\overline{\mathbf{P}}_p + \lambda\mathbf{L}\times\overline{\mathbf{P}}_p + \sqrt{2}G_F\int\frac{d^3q}{(2\pi)^3}(1-\cos\theta_{pq})(\mathbf{P}_q-\overline{\mathbf{P}}_q)\times\overline{\mathbf{P}}_p$$

- Different modes oscillate with different frequencies → kinematical decoherence
- Self-maintained coherence by nu-nu interactions
- Can lead to "spectral split"

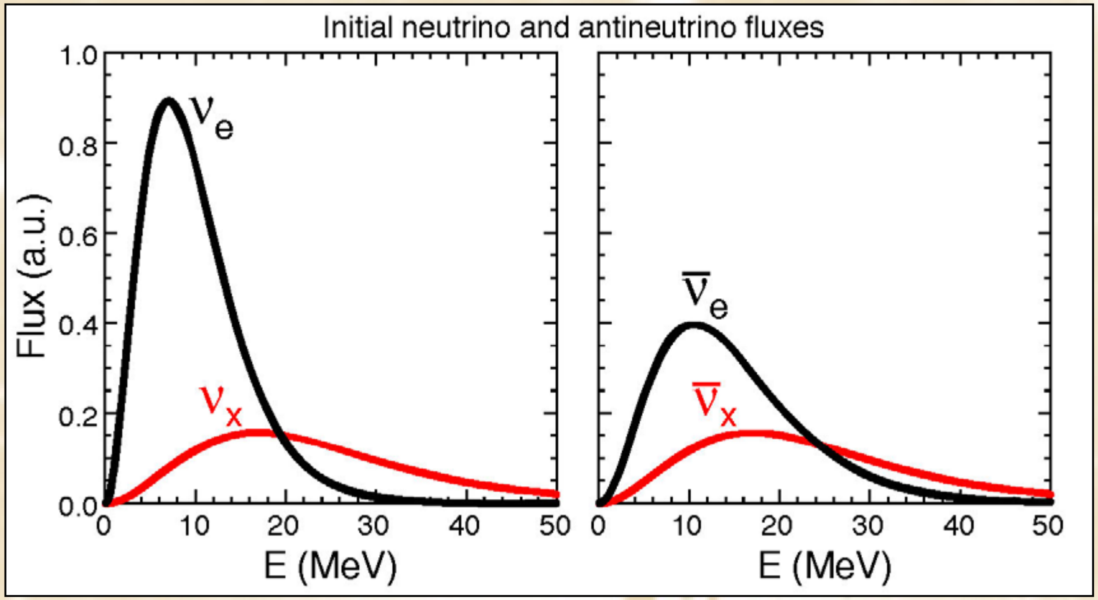
Isotropic matter background affects all modes the same

Multi-angle effects for non-isotropic nu distribution (streaming from SN): Different modes should oscillate differently → kinematical decoherence However, nu-nu interaction can lead to

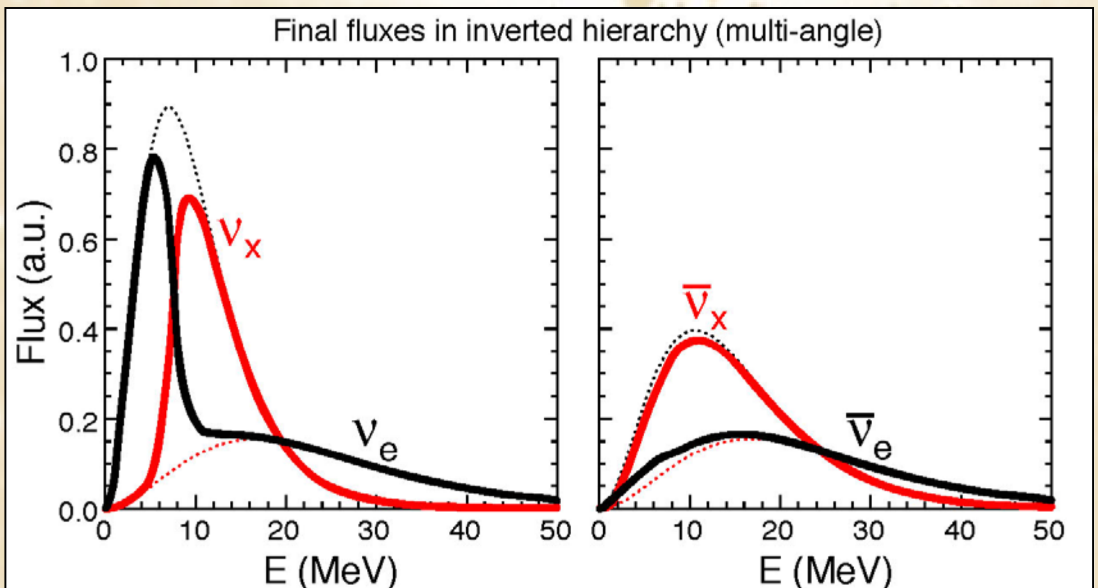
- "Angular synchronization" (quasi-single angle behavior)
- Self-accelerated multi-angle decoherence

Spectral Split (Stepwise Spectral Swapping)

Initial fluxes at nu sphere



After collective trans-formation



For explanation see

Raffelt & Smirnov arXiv:0705.1830
0709.4641

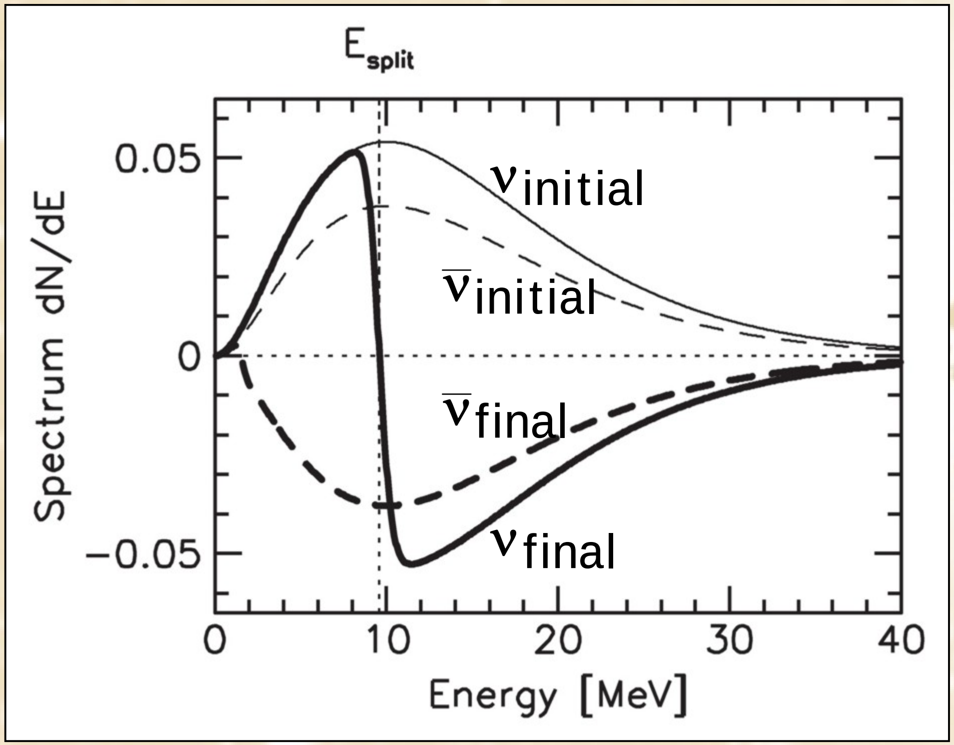
Duan, Fuller, Carlson & Qian arXiv:0706.4293
0707.0290

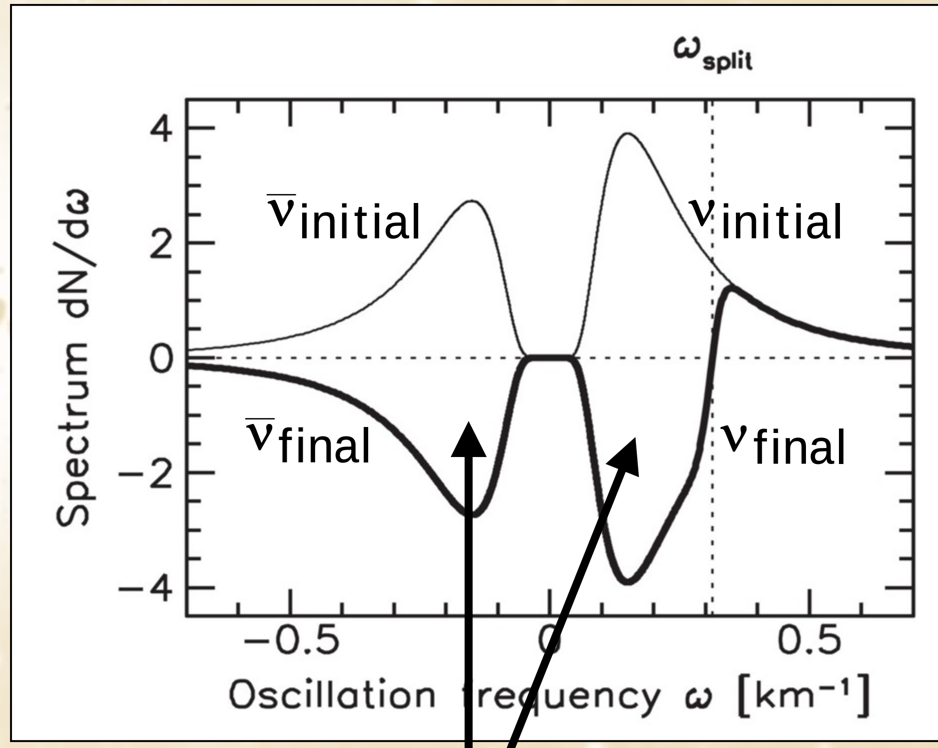
Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Spectral split in terms of the ω variable

Collective conversion of thermal spectra of ν_e and $\bar{\nu}_e$ as in a supernova

Energy spectrum	Spectrum in terms of $\omega = \Delta m^2/2E$





Flavor lepton number conservation:
Equal integrals

Raffelt & Smirnov, arXiv:0709.4641

Adiabatic Evolution in Co-Rotating Frame

$(\nu)\quad \partial_t \mathbf{P}_p = +\frac{\Delta m^2}{2p}\mathbf{B}\times\mathbf{P}_p + \sqrt{2}G_F\int\frac{d^3q}{(2\pi)^3}(\mathbf{P}_q-\bar{\mathbf{P}}_q)\times\mathbf{P}_p$

$(\bar\nu)\quad \partial_t \bar{\mathbf{P}}_p = -\frac{\Delta m^2}{2p}\mathbf{B}\times\bar{\mathbf{P}}_p + \sqrt{2}G_F\int\frac{d^3q}{(2\pi)^3}(\mathbf{P}_q-\bar{\mathbf{P}}_q)\times\bar{\mathbf{P}}_p$

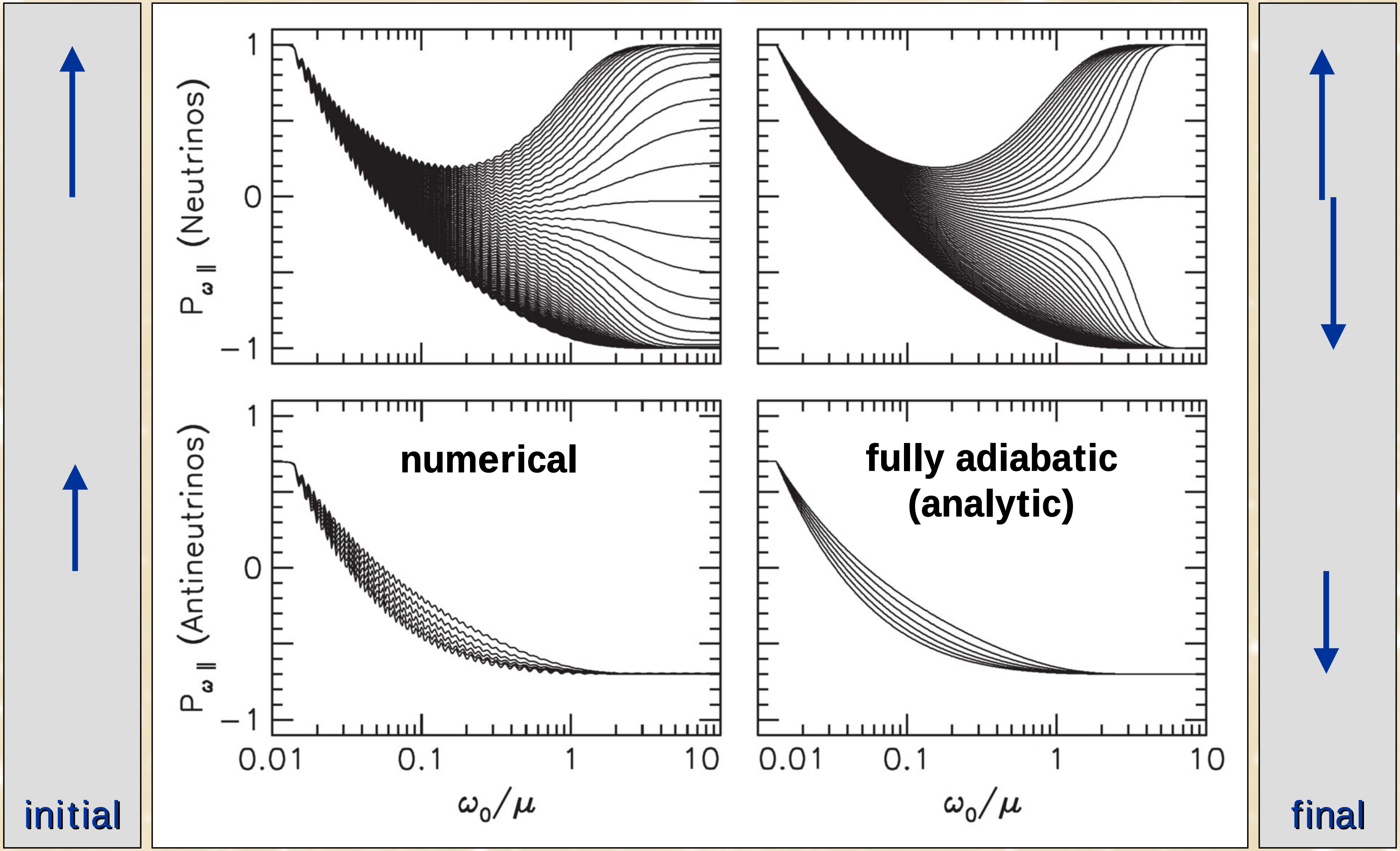
$\partial_t \mathbf{P}_\omega = \omega\mathbf{B}\times\mathbf{P}_\omega + \mu\mathbf{D}\times\mathbf{P}_\omega$

$(\nu)\quad \omega = +\Delta m^2/2E$

$(\bar\nu)\quad \omega = -\Delta m^2/2E$

- Each mode follows its "Hamiltonian" $\quad \partial_t \mathbf{P}_\omega = \mathbf{H}_\omega\times\mathbf{P}_\omega$
- All Hamiltonians are in a single plane $\quad \mathbf{H}_\omega = \omega\mathbf{B} + \mu\mathbf{D}$
- Initially ($\mu = \infty$) all modes are aligned with their Hamiltonians $\quad \mathbf{P}_\omega \parallel \mu\mathbf{D} \approx \mathbf{H}_\omega$
- For adiabatic evolution (μ evolves slowly) always stay aligned with $\mathbf{H}_\omega$
- In the co-rotating plane of B and D, the Hamiltonians $\mathbf{H}_\omega$ are static, evolution is adiabatic $\quad \mathbf{H}_\omega = (\omega-\omega_c)\mathbf{B} + \mu\mathbf{D}$
- In the end ($\mu = 0$) all modes with $\omega > \omega_c$ aligned with B, all modes with $\omega < \omega_c$ anti-aligned
- Final value $\omega_c = \omega_{split}$ determined by flavor-lepton conservation

Evolution of Energy Modes Toward a Spectral Split

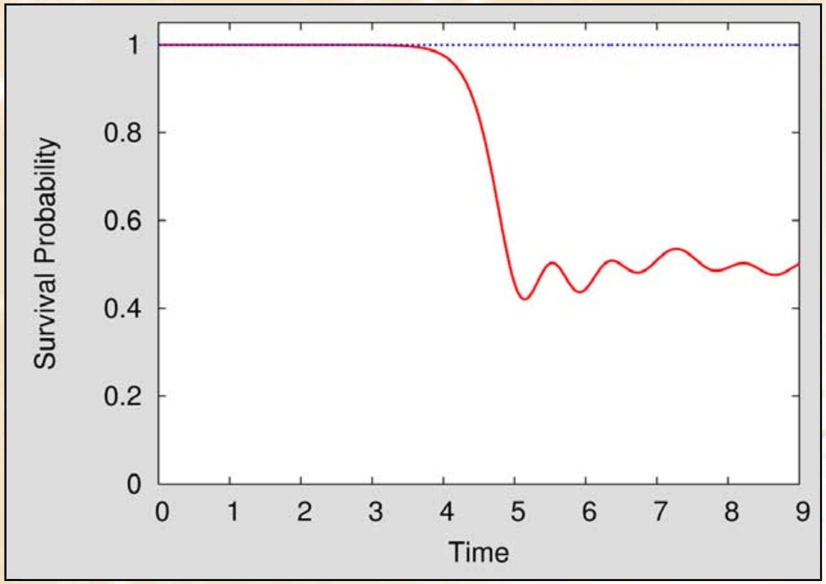


Raffelt & Smirnov, arXiv:0709.4641

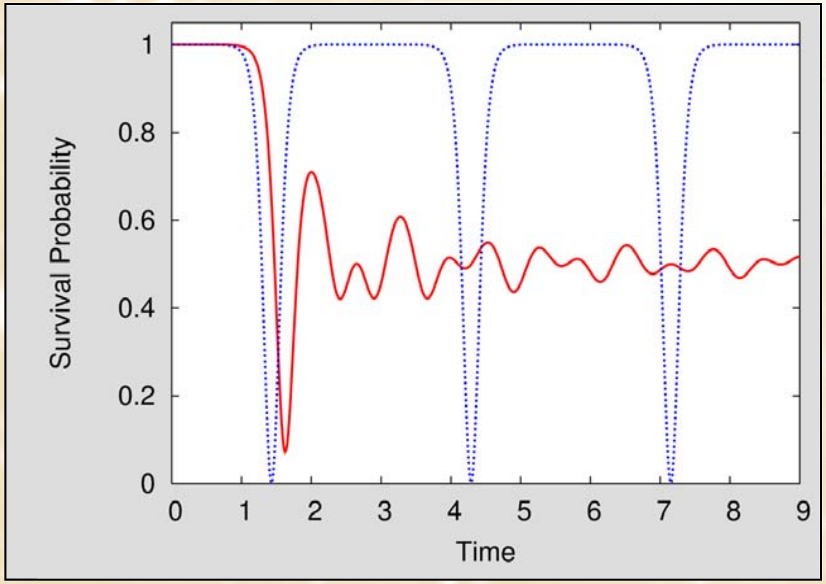
Neutrinos in a Box: Kinematical Multi-Angle Decoherence

Isotropic vs. “half isotropic”

Normal Hierarchy



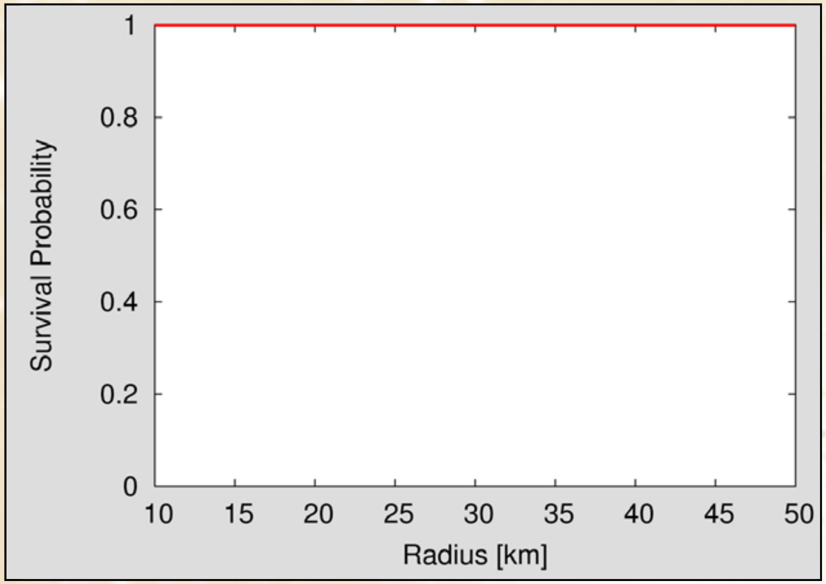
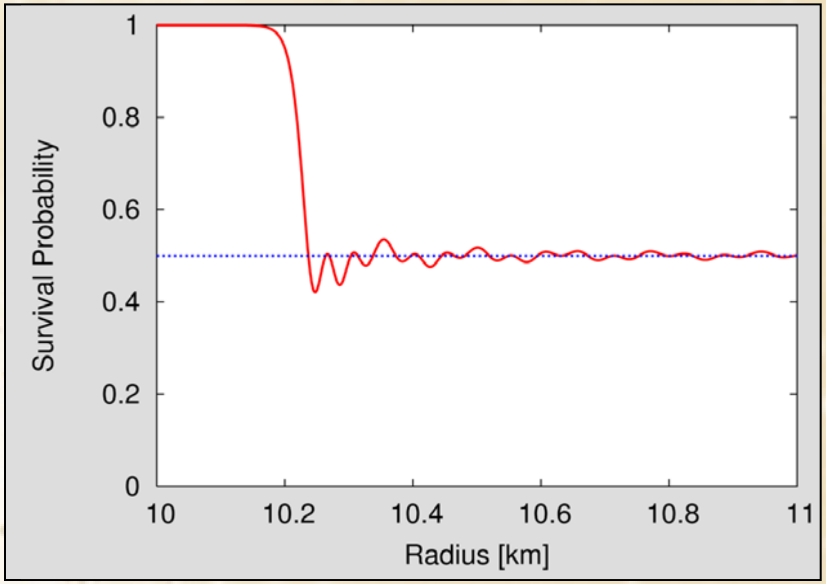
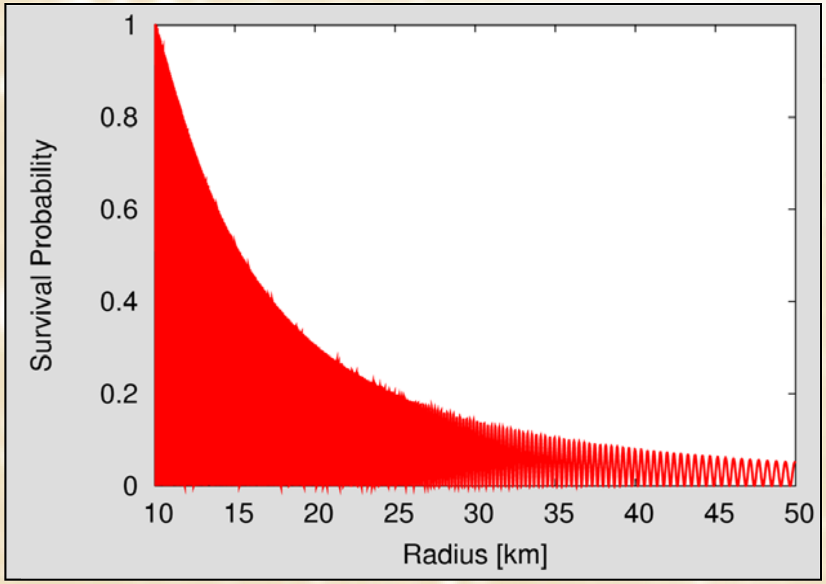
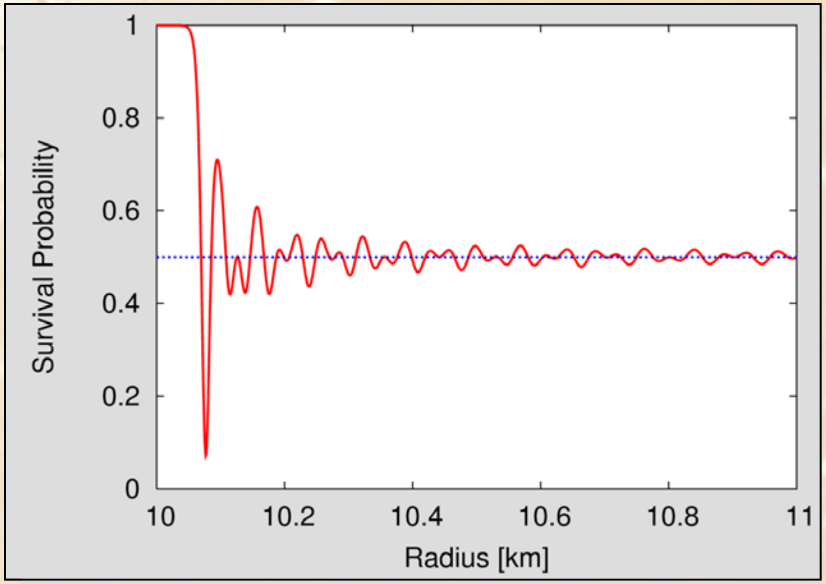
Inverted Hierarchy



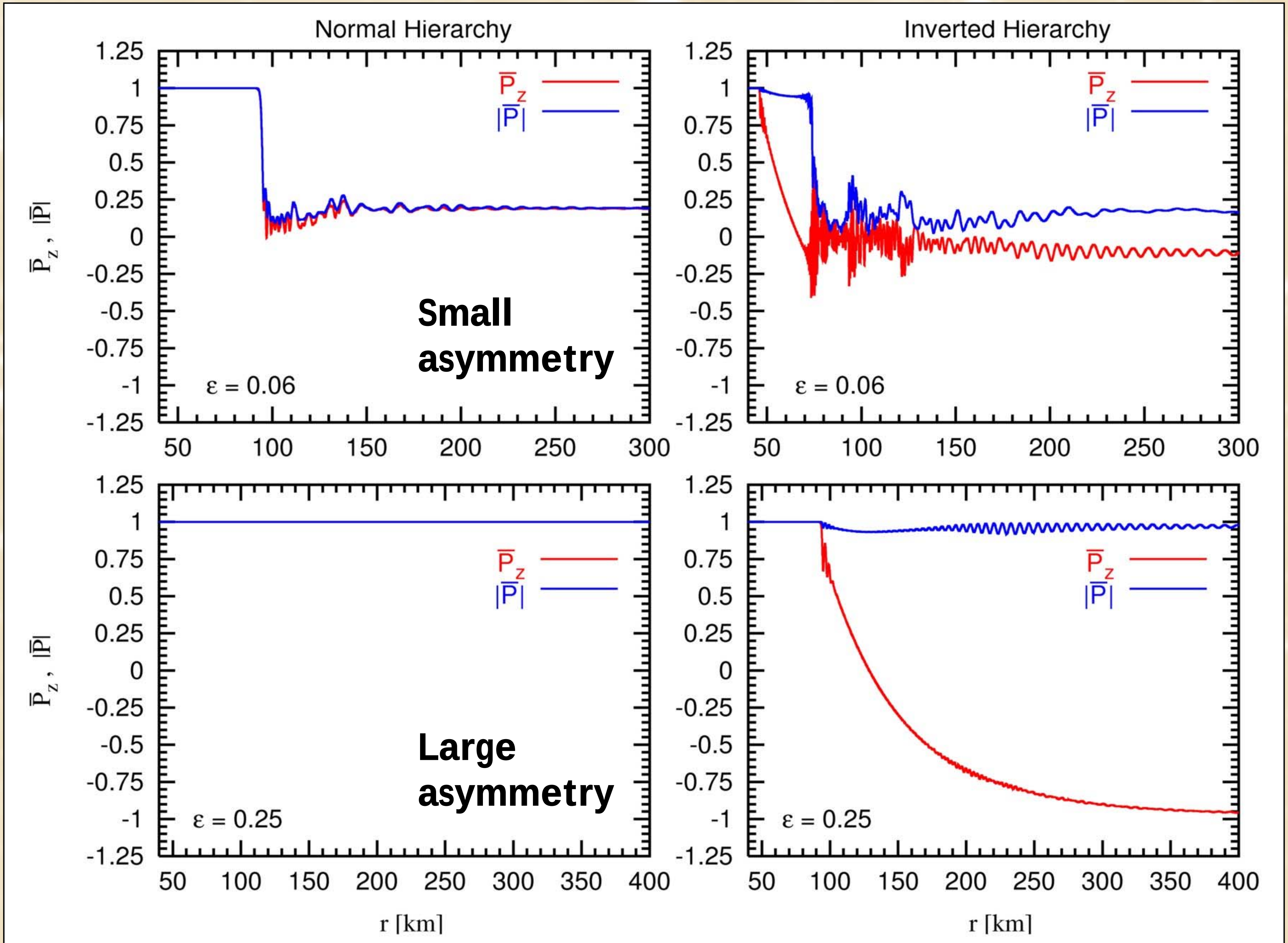
- Complete kinematical decoherence for both hierarchies
- A very small initial deviation from isotropy is enough to trigger a run-away
- Isotropic case an unstable fixed point
- Flavor equipartition generic outcome
- Pure flavor system not stable on the classical level

Raffelt & Sigl:
Self-induced decoherence in dense neutrino gases
[hep-ph/0701182]

Multi-Angle Kinematical Decoherence (Symmetric Case)

	Isotropic (single angle)	Large flux ("half isotropic")
Normal Hierarchy		
Inverted Hierarchy		

Examples for Kinematical Decoherence



End State of Polarization Vectors for Angular Modes

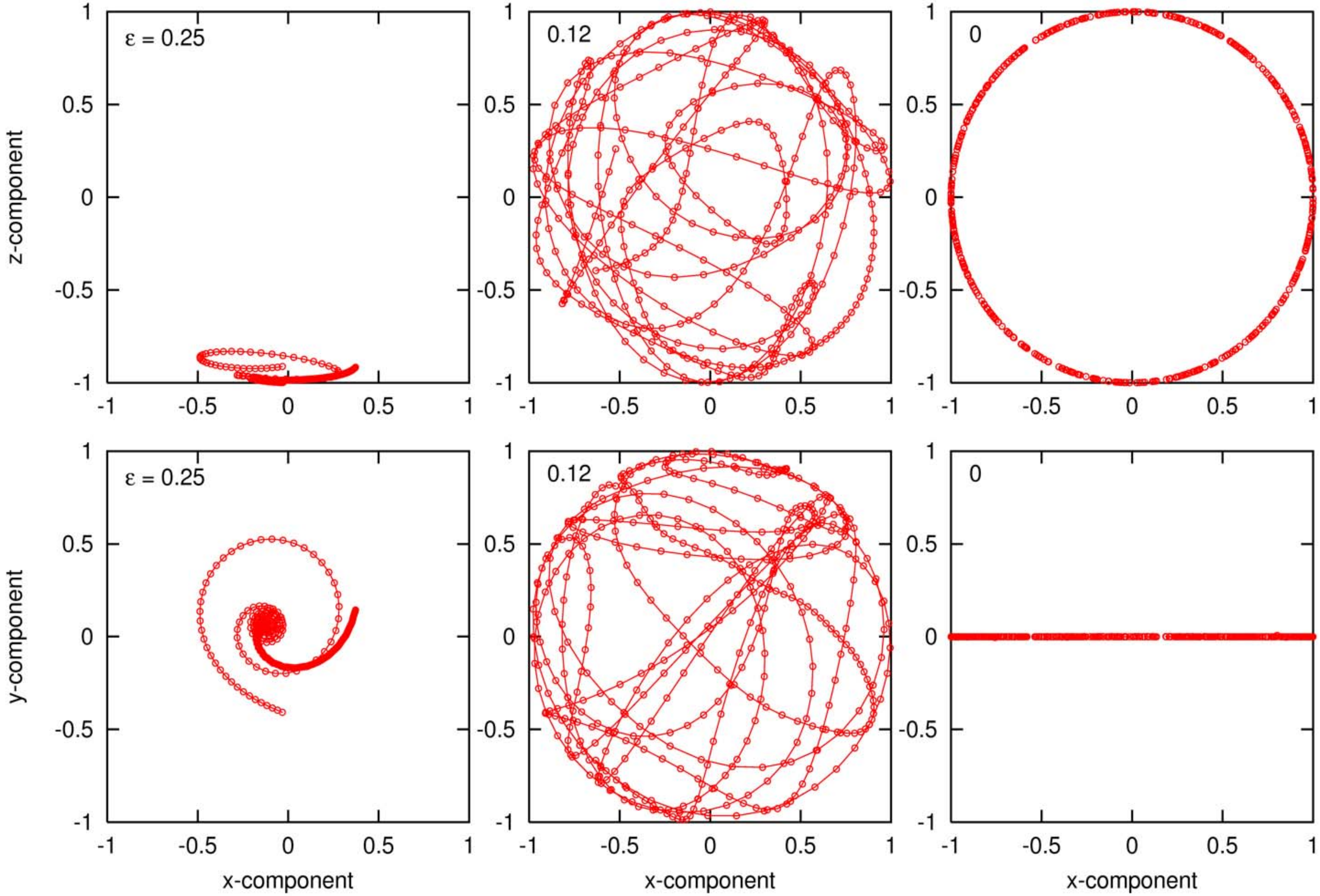
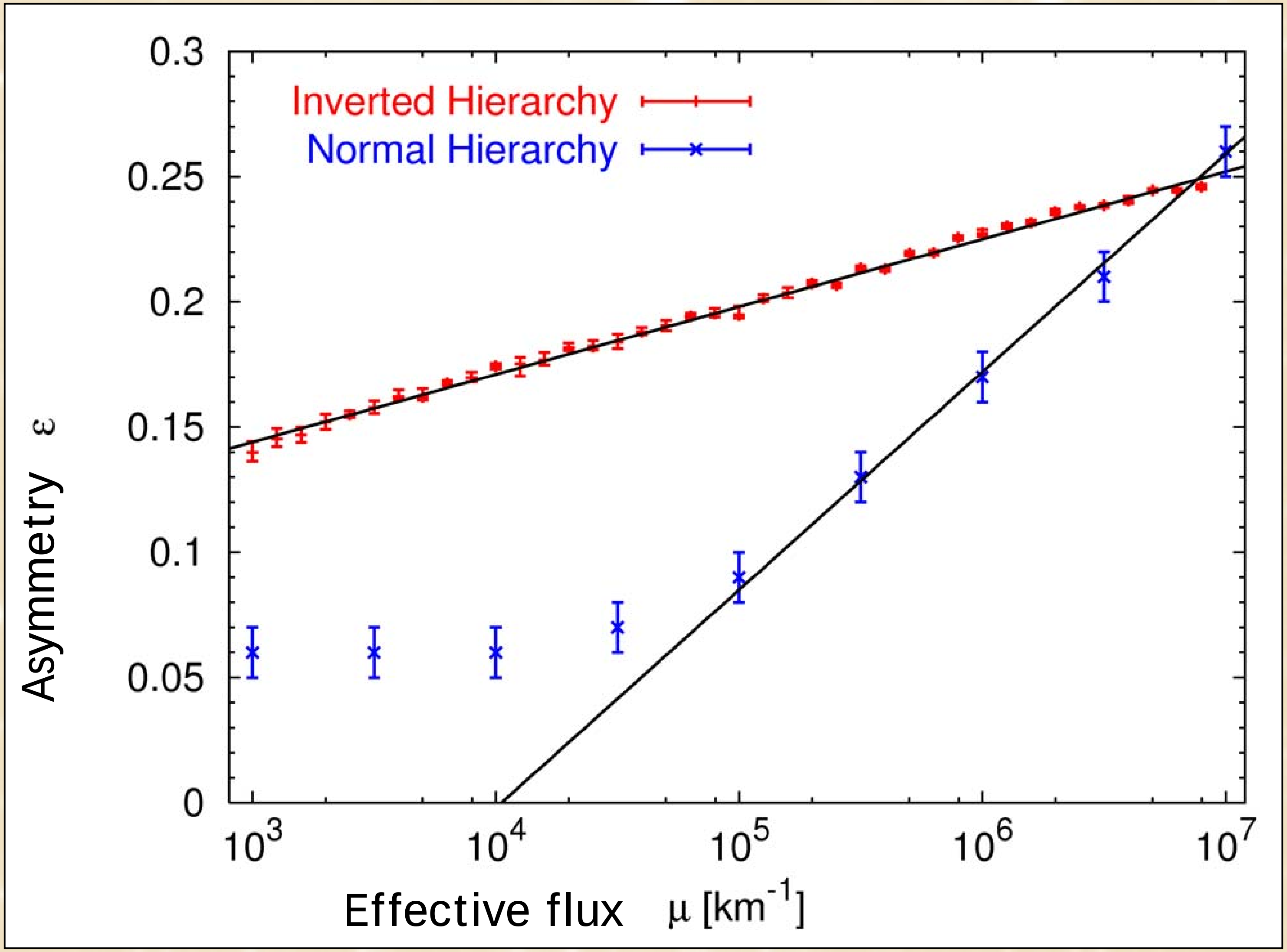


FIG. 3: Final location on the unit sphere of 500 antineutrino polarization vectors for our standard parameters and the inverted hierarchy. The top row is the "side view" (x-z-components), the bottom row the "top view" (x-y-components). Left: quasi single-angle case ($\epsilon = 0.25$). Middle: decoherent case ($\epsilon = 0.12$). Right: symmetric system ($\epsilon = 0$).

Critical Asymmetry for Decoherence

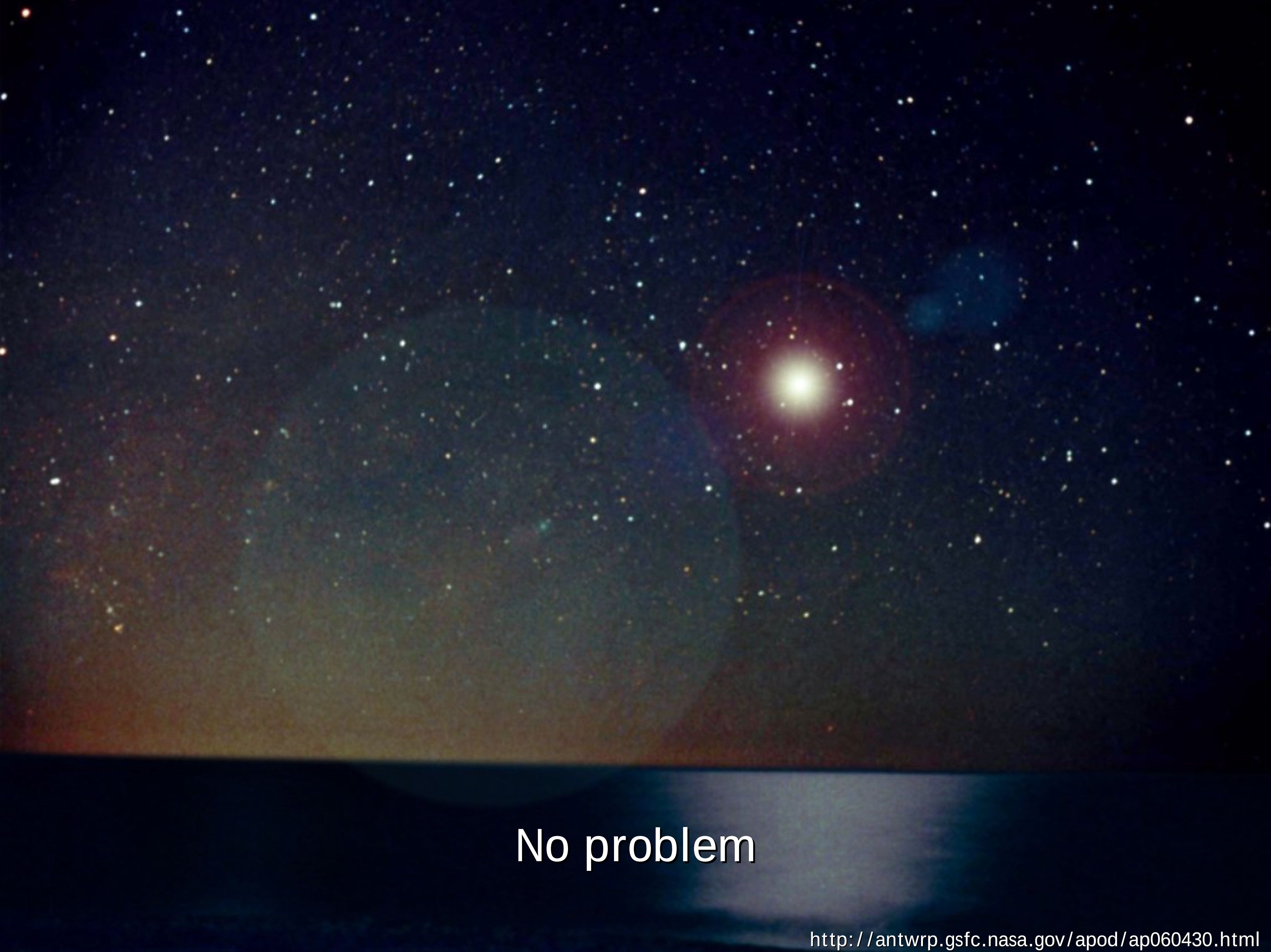


Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl:
Decoherence in supernova neutrino transformations
suppressed by deleptonization astro-ph/0706.2498

Looking forward to the next galactic supernova

http://antwrp.gsfc.nasa.gov/apod/ap060430.html


May take a long time
http://antwrp.gsfc.nasa.gov/apod/ap060430.html


No problem
http://antwrp.gsfc.nasa.gov/apod/ap060430.html


Lots of theoretical work to do!
http://antwrp.gsfc.nasa.gov/apod/ap060430.html