Impact of massive neutrinos on nonlinear matter power spectrum

based on arXiv:0801.0607 [astro-ph]

Focus week: Neutrino Mass @ IPMU

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Neutrino mass from cosmology

Cosmology provides stringent constraints on total neutrino masses

1. Distance test

WMAP5 only $\sum m_\nu \lesssim 1.3\ \mathrm{eV}$

WMAP5 + BAO + SN $\sum m_\nu \lesssim 0.6\ \mathrm{eV}$

Komatsu et al (2008)

This is trustworthy bound, but cannot expect more strict constraint.

CMB is important to determine the other cosmological parameters.

2. Suppression of growth

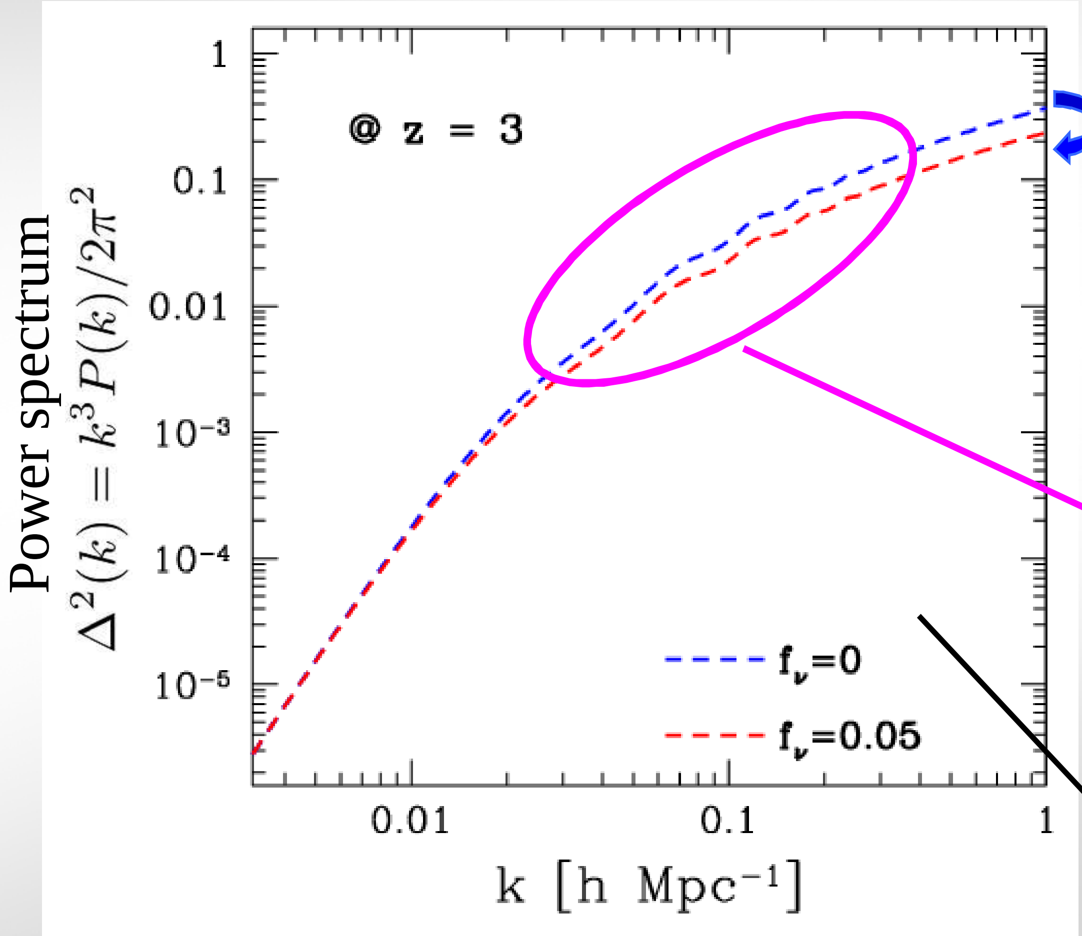
WMAP3 + SDSS LRG $\sum m_\nu \lesssim 0.6\ \mathrm{eV}$ Tegmark et al (2006)

WMAP3 + SDSS Lyα $\sum m_\nu \lesssim 0.2\ \mathrm{eV}$ Seljak et al (2006)

Precision cosmology like CMB will come for next-generation experiment.

Neutrino suppression effect

Neutrino perturbations cannot stay at smaller scale than neutrino free-streaming ➜ weaken gravitational potential



suppression !!

scale-dependent growth

$$\frac{P(k)_{f_\nu \neq 0}}{P(k)_{f_\nu = 0}} - 1 \approx -8 f_\nu \geq 4\%$$

Baryon Acoustic Oscillations (BAOs)

calculated by **Linear** theory ✖

Non-linear regime !!

Future galaxy redshift survey

- Many galaxy redshift surveys, e.g. **WFMOS**, HETDEX, are proposed for measuring BAOs (~100Mpc) to probe the nature of dark energy.

- BAO scale is comparable to sub-eV neutrino free-streaming scale.
Moreover, neutrino suppression effect cannot be neglected.
This is a good chance to constrain or *determine* the neutrino masses!

- BAO scale is in weakly nonlinear regime (k < 0.5 hMpc^-1).
  - Nonlinearity is being understood theoretically.
    - standard perturbation theory — **Makino et al (1992), Nishimichi et al (2007)**, **Crocce & Scoccimarro (2006) Matsubara (2008)**
    - renormalized PT — **Taruya & Hiramatsu (2008)**
    - N-body simulation — **Jeong, Komatsu (2006), Takahashi et al (2008)**
  - All these studies are based on only CDM cosmology without neutrinos.

- Note that other probes of P(k), e.g. weak lensing & Lyα, suffer from more strong nonlinearity (k ~ 1 hMpc^-1).

Our Work

S.S, M. Takada, A. Taruya, arXiv:0801.0607
S.S, M. Takada, A. Taruya, in prep 2008

- For the cosmology with CDM & massive neutrinos, we carefully develop the approach to calculate the nonlinear matter power spectrum based on cosmological perturbation theory.

- Using our refined nonlinear theory, we demonstrate how well neutrino masses are constrained for WFMOS-like survey.

- Our work is definitely first step for nonlinear modeling of most realistic cosmology, ΛCDM with massive neutrinos.
  - Next step is investigating N-body simulations with neutrinos, but very challenging!

Methodology

Perturbation Theory : natural extension of linear theory

multi-fluid component of baryon + mixed dark matter (CDM + Neutrinos)

$$\delta_{\rm m} = f_{\rm cb}\delta_{\rm cb} + f_\nu \delta_\nu \quad \left[f_{\rm cb} \equiv \frac{\Omega_{\rm c}+\Omega_{\rm b}}{\Omega_{\rm m}},\ f_\nu \equiv \frac{\Omega_\nu}{\Omega_{\rm m}} = \frac{\sum m_\nu}{94.1\Omega_{\rm m}h^2} \lesssim 0.05 \right]$$

Power spectrum

$$P(k) = \langle \delta_{\rm m}\delta_{\rm m}\rangle = f_{\rm cb}^2 P_{\rm cb} + 2 f_{\rm cb} f_\nu P_{{\rm cb},\nu} + f_\nu^2 P_\nu$$

⇨ Perturbative expansion of nonlinear Continuity & Euler equations

∗ Contrasted to only CDM case, some difficulties are involved:

① Nonlinear growth functions are also scale-dependent, which complicates the calculation of nonlinear correction.

② Neutrinos cannot be treated as fluid-component.

① One-loop correction for Pcb

calculate next-to-leading order correction for Pcb(k)

From standard perturbation theory **Makino,Sasaki,Suto (1992)**

$$P_{\rm cb}^{\rm Approx}(k) = P_{\rm cb}^{L} + P_{\rm cb}^{(22)} + P_{\rm cb}^{(13)}$$

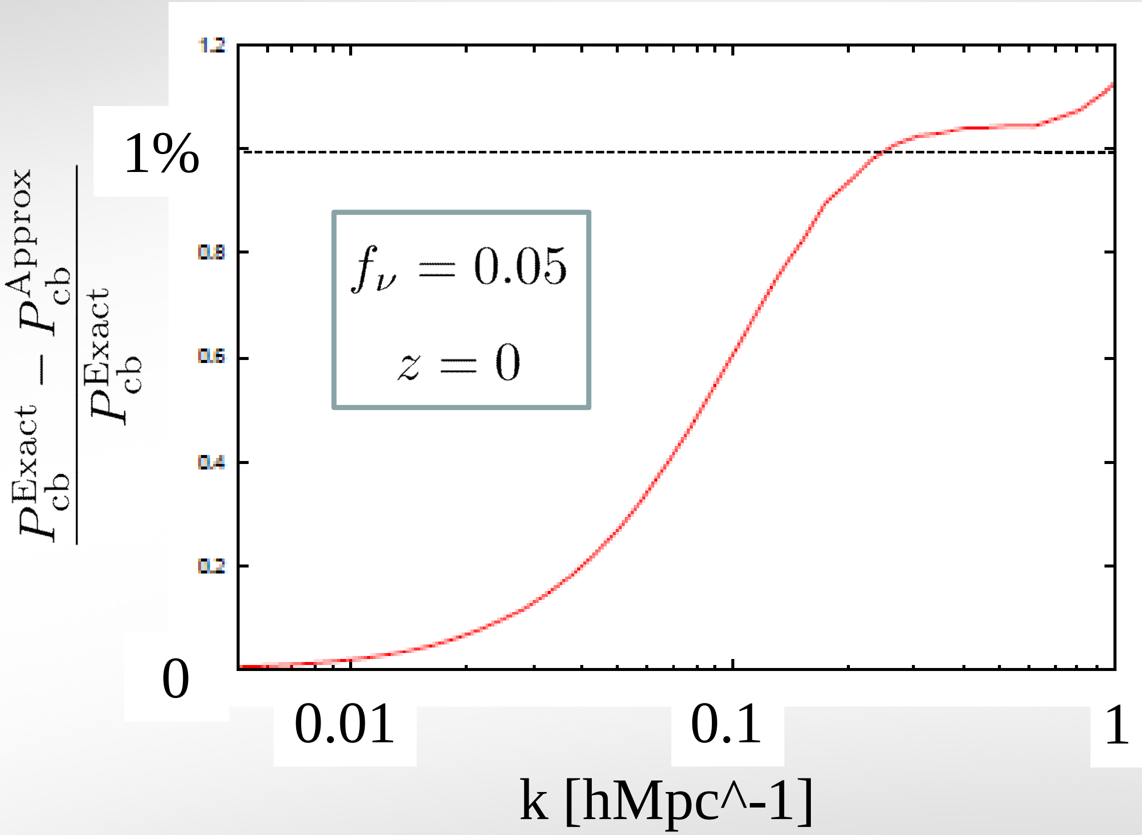
$$P_{\rm cb}^{(22)}(k;z) = \frac{k^3}{98(2\pi)^2}\int_0^\infty dr P_{\rm cb}^L(kr;z)\int_{-1}^{1} d\mu P_{\rm cb}^L(k\sqrt{1+r^2-2\mu r};z)\frac{(3r+7\mu-10r\mu^2)^2}{(1+r^2-2r\mu)^2}$$

$$P_{\rm cb}^{(13)}(k;z) = \frac{k^3}{252(2\pi)^2}P_{\rm cb}^L(kr;z)\int_0^\infty dr P_{\rm cb}^L(kr;z)$$
$$\times\left[\frac{12}{r^2} - 158 + 100r^2 - 42r^4 + \frac{3}{r^2}(r^2-1)^3(7r^2+2)\ln\left|\frac{1+r}{1-r}\right|\right]$$

However, this is an approximation in the sense that scale-dependency of growth functions are neglected.

① One-loop correction for Pcb

difference between exact and approximated $P_{\rm cb}(k) = P_{\rm cb}^{L} + P_{\rm cb}^{(22)} + P_{\rm cb}^{(13)}$



Numerically

solve the 2nd & 3rd order equations

⇩

integrate mode-coupling

⇩

$P_{\rm cb}^{\rm Exact}(k)$

The fractional difference is less than ~ 1%.

Because only nearby mode-coupling contributes to one-loop integration.

② Neutrino fluctuations

菓 Neutrino perturbations cannot be treated as fluid.
We have to solve the Vlasov (collisionless Boltzmann) equation.

菓 The dynamics of neutrino perturbations are controlled by Newton potential, which is supported by CDM + baryon.
(c.f.) dynamics of CDM + baryon are controlled by CDM + baryon itself
➜ causes nonlinearity

菓 { For smaller scales, neutrinos cannot stay due to free-streaming.
Tiny contributions from neutrinos perturbation to total P(k) for $f_\nu \lesssim 0.05$

$$P(k) = f_{\rm cb}^2 P_{\rm cb} + 2 f_{\rm cb} f_\nu P_{{\rm cb},\nu} + f_\nu^2 P_\nu$$

We assume **neutrino perturbations stay at linear level** and add nonlinear corrections **only for Pcb** term.

② Neutrino fluctuations

巣 Is it a good approximation that Pν(k) is calculate from linear theory?

巣 linear Vlasov equations **Ma & Bertschinger (1995)**

$$\dot{\Psi}_0 = -\frac{qk}{a\epsilon}\Psi_1 + H\phi\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_1 = \frac{qk}{3a\epsilon}(\Psi_0 - 2\Psi_2) - \frac{\epsilon k}{3aq}\phi\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_\ell = \frac{qk}{(2\ell+1)a\epsilon}[\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}] \quad (\ell \geq 2)$$

Newton potential

q : 3-momentum
ε : proper energy
Ψ : fluctuated distribution

$$f(x^i, q_j/a, t) = f_0(q)[1 + \Psi(x^i, q, n_j, t)]$$

$$\Rightarrow \quad \delta_\nu^{(1)} = \frac{4\pi}{a^4\rho_m}\int q^2 dq\, \epsilon f_0(q)\Psi_0 \quad \Rightarrow \quad P_\nu(k) = \langle \delta_\nu^{(1)} \delta_\nu^{(1)} \rangle$$

巣 Even if nonlinear CDM + baryon fluctuations are included in Newton potential, less than 0.01% change of Pm(k).

Recipe for nonlinear P(k)

- We can develop the theory to calculate the nonlinear P(k) with massive neutrinos having mass of ~0.1eV.
- Next-to-leading order correction, one-loop correction, is included.

Recipe to calculate the nonlinear P(k;z)

① calculate **linear** power spectra $P_{\rm cb}^L(k;z),\ P_{{\rm cb}\nu}^L(k;z),\ P_\nu^L(k;z)$ for redshift z from CAMB or CMBFAST

② add **one-loop correction** for CDM + baryon term

$$P_{\rm cb}^{\rm Approx}(k) = P_{\rm cb}^L + P_{\rm cb}^{(22)} + P_{\rm cb}^{(13)}$$

③ sum up all components $P(k) = f_{\rm cb}^2 P_{\rm cb} + 2 f_{\rm cb} f_\nu P_{{\rm cb},\nu}^L + f_\nu^2 P_\nu^L$

Notes on nonlinear P(k)

- 粜 For smaller neutrino masses, our PT results become better approximation.
- 粜 From our PT, the limitation of linear theory can be known. Meanwhile, the validity of our PT cannot be provided, which is derived from comparison with N-body simulations with neutrinos.
- 粜 One-loop corrections are roughly proportional to linear P(k).

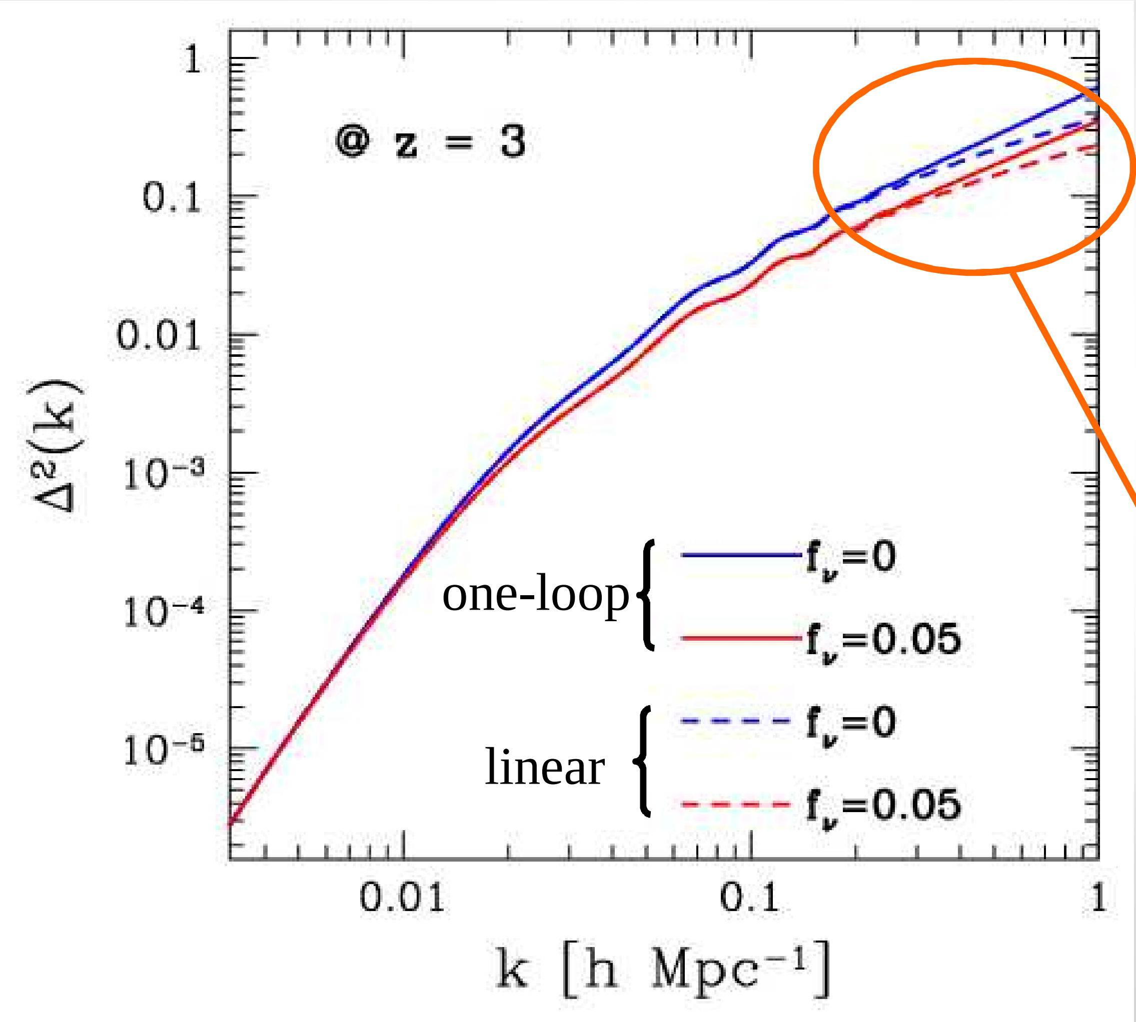
$$P_{\rm cb}^{(22)}(k;z) = \frac{k^3}{98(2\pi)^2}\int_0^\infty dr P_{\rm cb}^L(kr;z)\int_{-1}^{1} d\mu P_{\rm cb}^L(k\sqrt{1+r^2-2\mu r};z)\frac{(3r+7\mu-10r\mu^2)^2}{(1+r^2-2r\mu)^2}$$

$$P_{\rm cb}^{(13)}(k;z) = \frac{k^3}{252(2\pi)^2}P_{\rm cb}^L(kr;z)\int_0^\infty dr P_{\rm cb}^L(kr;z) \times\left[\frac{12}{r^2}-158+100r^2-42r^4+\frac{3}{r^2}(r^2-1)^3(7r^2+2)\ln\left|\frac{1+r}{1-r}\right|\right]$$

➜ neutrino suppression effect is expected to be enhanced in weakly nonlinear regime!

Nonlinear P(k)

dimensionless power $\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$



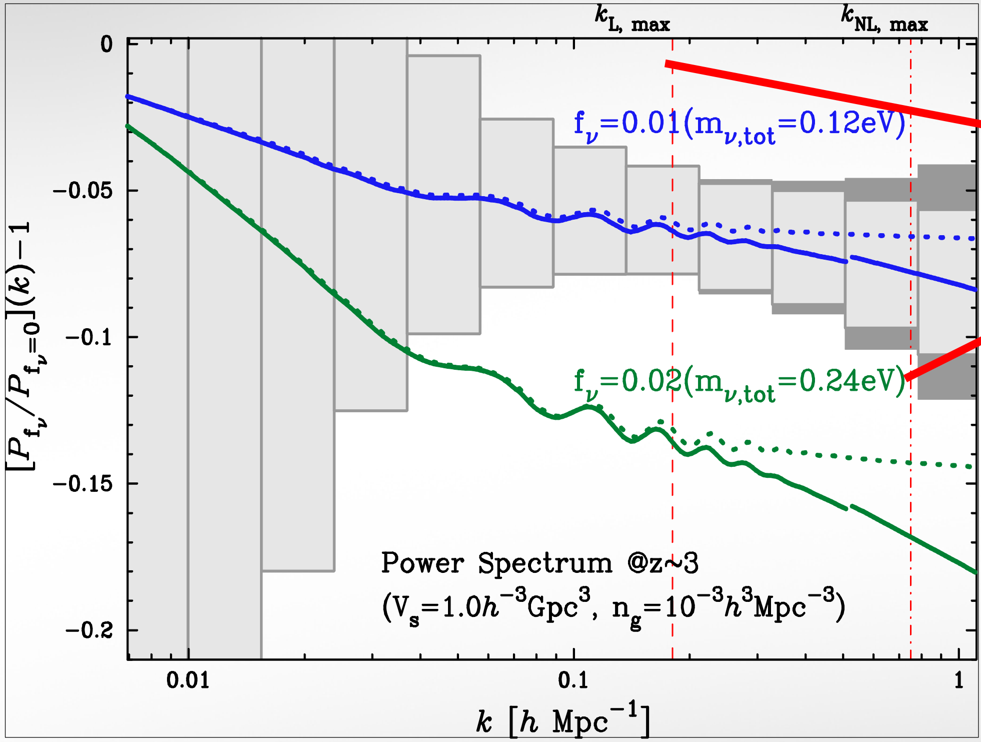
fiducial cosmology

$$\Omega_b h^2 = 0.0223$$
$$\Omega_m h^2 = 0.1277$$
$$h = 0.73, \; w_0 = -1$$
$$\Delta^2_{\mathcal{R}} = 2.35 \times 10^{-9}, \; n_S = 1$$

The amplitude is enhanced by non-linear gravitational evolution.

Neutrino suppression effect

$$P_{f_\nu \neq 0}/P_{f_\nu = 0} - 1$$



Linear

Perturbation Theory

Limitation of linear theory

applicapable range of PT？

$k_{\rm NLmax}$: 0.69 hMpc^-1

$$\Delta^2 = k^3 P^{NL}_{f_\nu=0}(k)/2\pi^2 < 0.4$$

Jeong,Komatsu (2006)

True criterion should be derived from simulations with neutrinos

Neutrino suppression effect is enhanced in weakly nonlinear regime.

The larger amplitude leads to less shot noise error.

Forecast

- How well our PT improve the constraint on neutrino masses for future galaxy redshift survey?

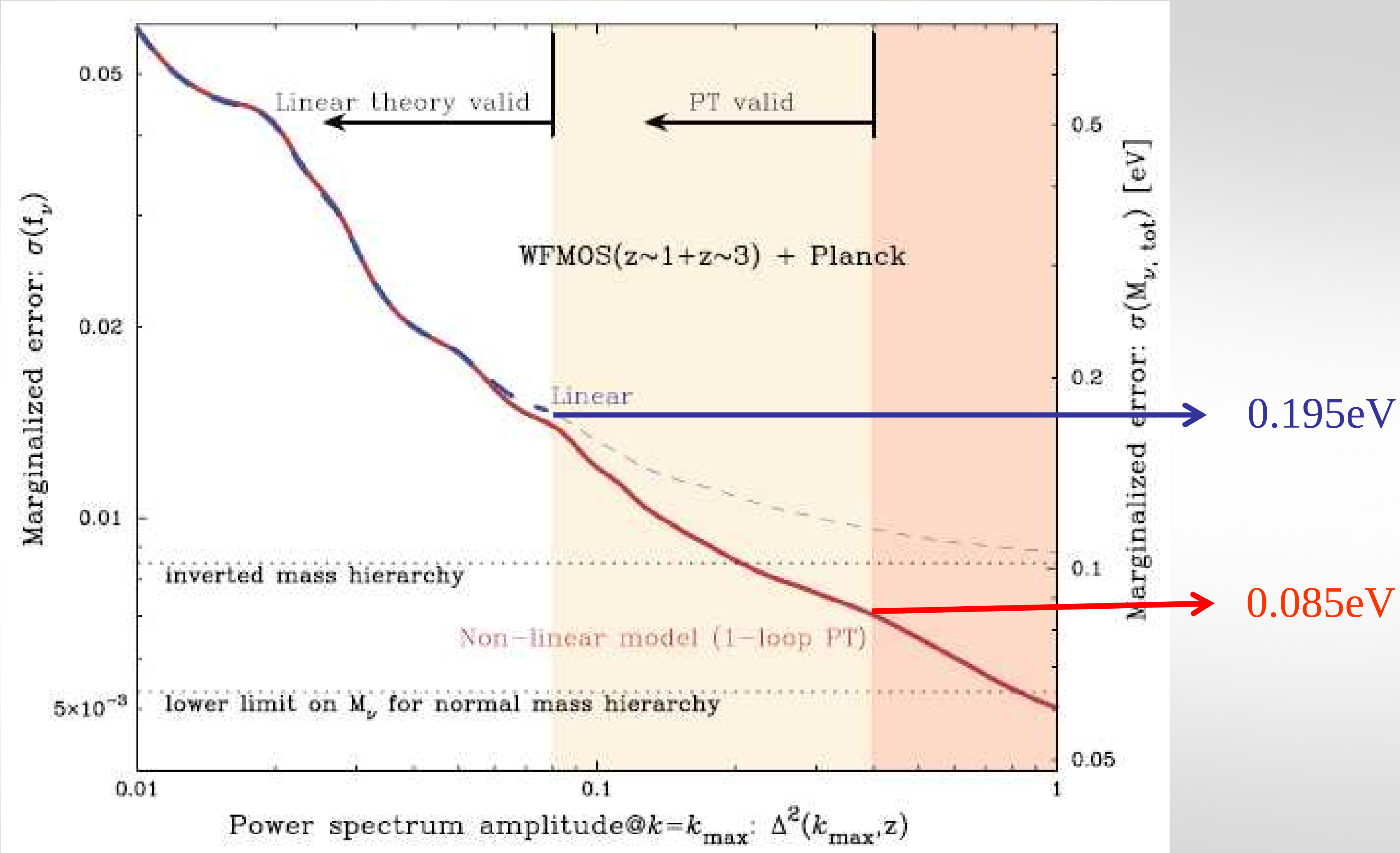
- Fisher information formalism

Seo & Eisenstein (2003)
Takada, Komatsu , Futamase (2006)

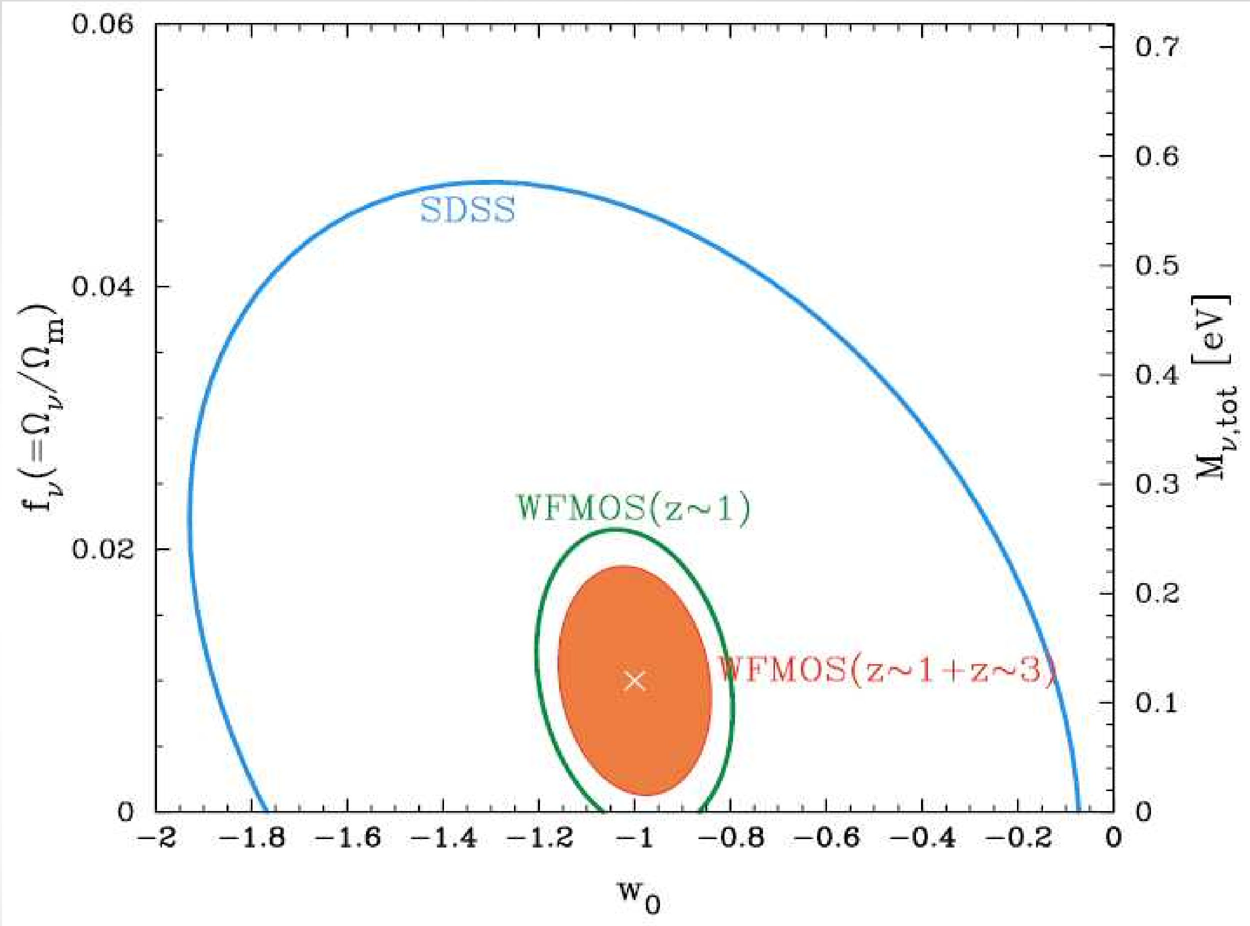
- assumption: linear bias, linear redshift distortion, Nν = 3
- CMB prior : Planck
- 18 free parameters: $\mathbf{p} = (\Omega_{\rm b}h^2, \Omega_c h^2, \Omega_m, \Delta^2_{\mathcal{R}}, n_S, \alpha, w_0, f_\nu, b_1(z_i), \beta(z_i))$
- fiducial parameter: $f_\nu = 0.01, \iff \sum m_\nu = 0.12\text{eV}$
- survey parameters: Wide-field Fiber-fed Multi-Object Spectroscopy

z~1 2000deg^2
z~3 300deg^2

Neutrino mass constraint



Degeneracy with ` w '



Neutrino effect does not shift the BAO peaks.

➜ Break the degeneracy between w & f_nu

Summary

∗ Neutrino effect cannot be neglected for future galaxy redshift survey.

∗ We carefully develop the approach to calculate nonlinear P(k) with massive neutrinos based on perturbation theory.

∗ We show that neutrino effect is enhanced in weakly nonlinear regime and this refined model description leads to improve the constraint on neutrino masses for WFMOS-like survey.

Discussion

S.S, Takada, Taruya in prep

∗ include the other nonlinearities, biasing, redshift distortion.

∗ compare N-body simulation with neutrinos **Brandenbyge et al (2008)**

∗ possibility to improve the theory, e.g. renormalized perturbation theory

The day after tomorrow

"Focused Workshop: The Theoretical Modeling of Cosmic Structure Formation and its Recent Progress``

21 March, 2008 A.M.9:00~

Room 233, Faculty of Science Bld.1 at the Hongo Campus, U. Tokyo

Invited speakers

M. Takada (IPMU)	R. Barkana (Tel Aviv)
A. Taruya (RESCEU)	S. Saito (U Tokyo)
R. Scoccimarro (New York U)	R. Takahashi (U Nagoya)
T. Matsubara (U Nagoya)	Y. Suto (U Tokyo)