Nuclear matrix elements

for $0\nu\beta\beta$ decay

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Focus week on Neutrino Mass
IPMU, Univ. Tokyo, 20/03/2008

Introduction

$$0\nu\beta\beta \Longleftrightarrow \overline{\nu} = \nu,\ m_\nu > 0$$

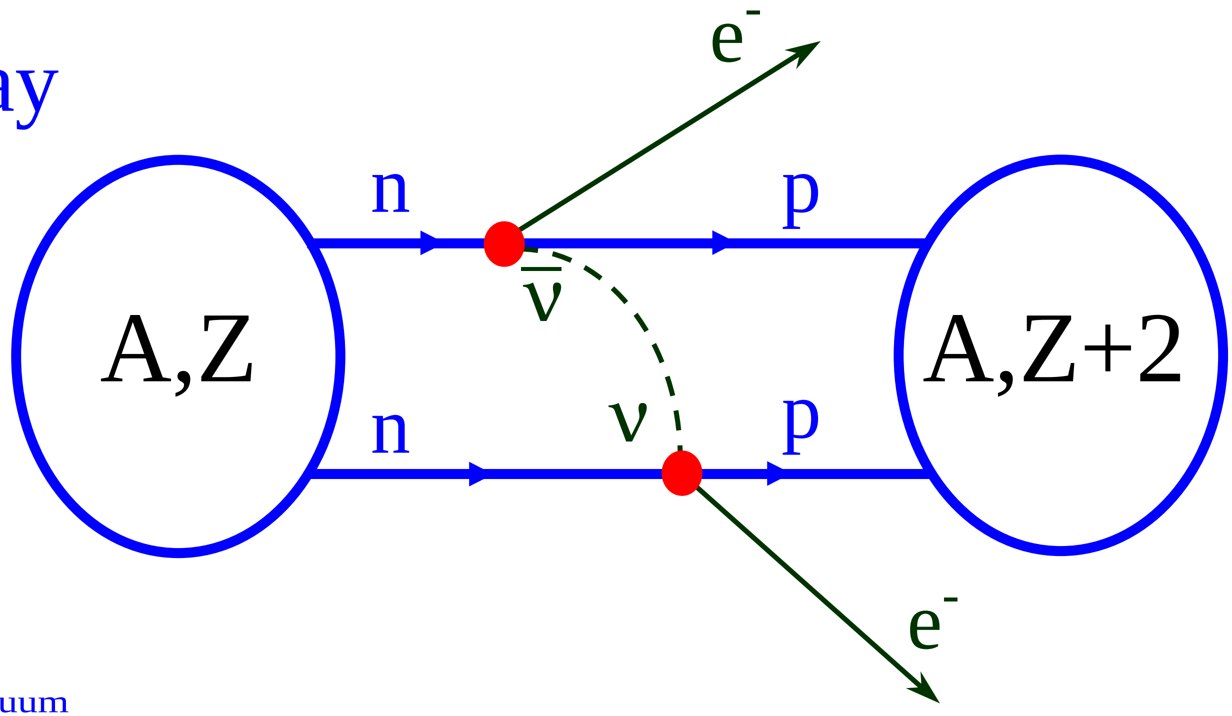
talks of S. Petcov and A. Giuliani

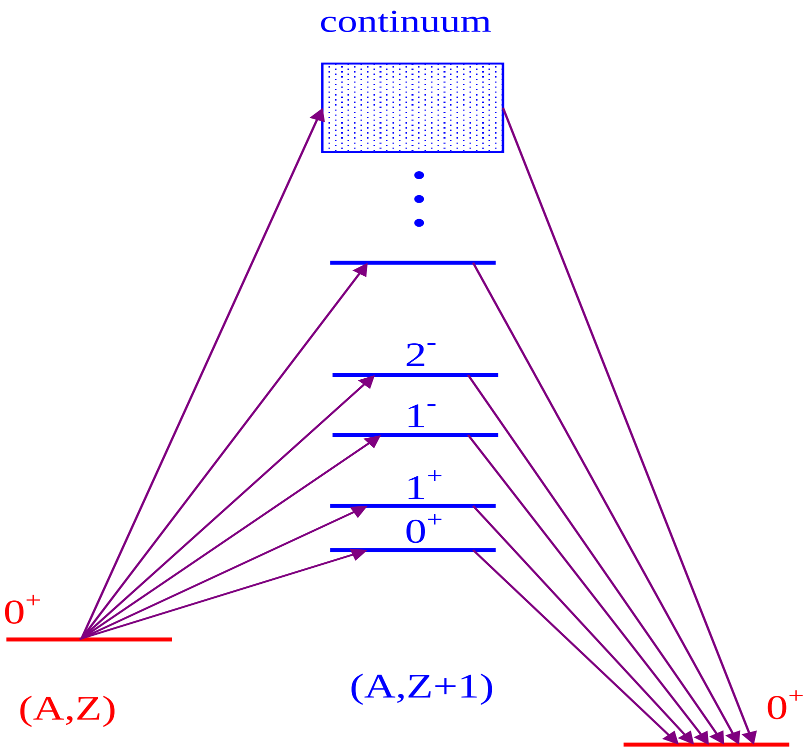
Introduction

More information?

- underlying elementary mechanism of $dd \rightarrow uue^- e^-$
  - Light neutrino exchange
  - Heavy neutrino exchange
  - Right-handed weak currents
  - R-parity violating SUSY
- absolute mass scale
- hierarchy

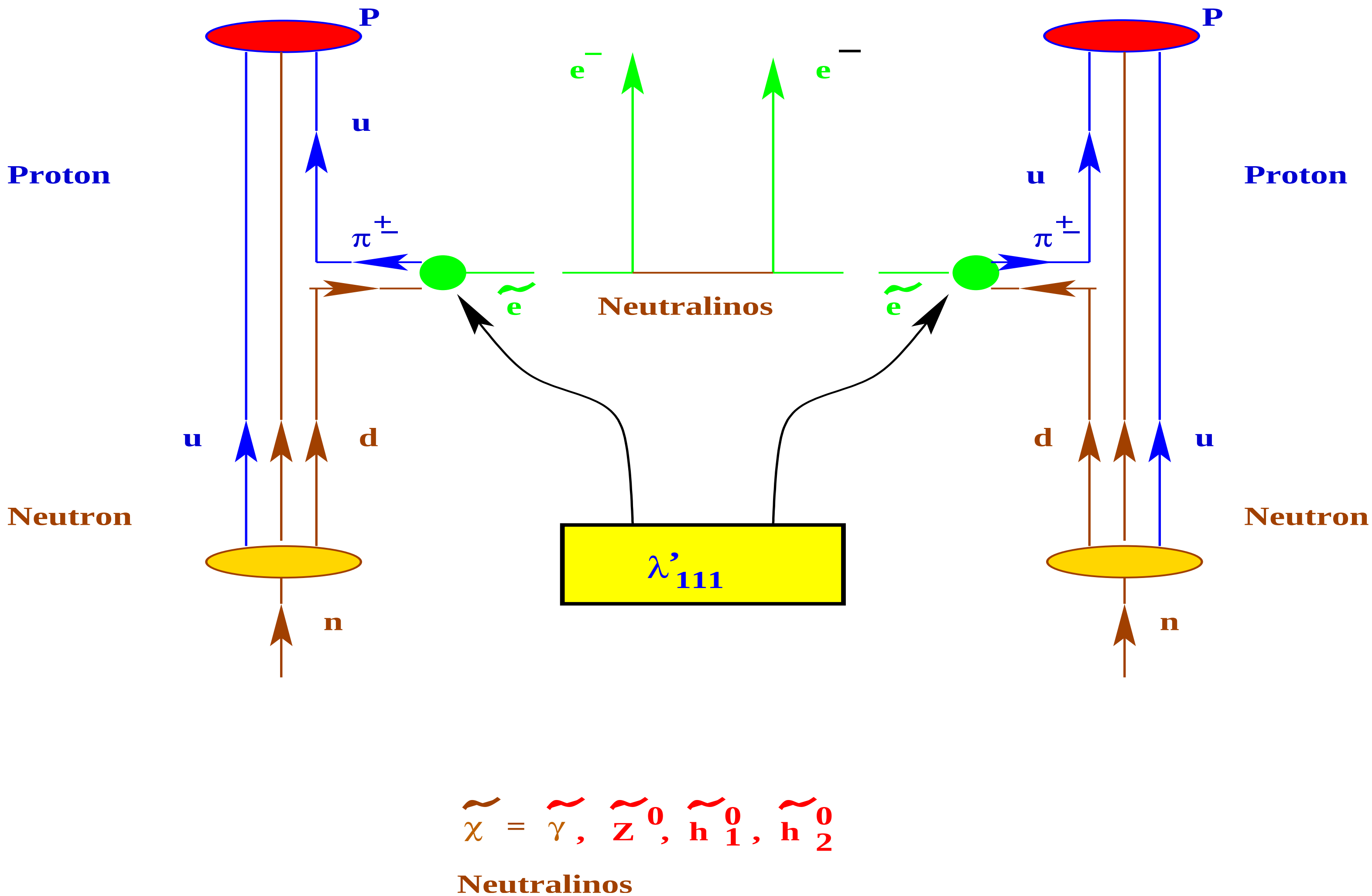
Nuclear $0\nu\beta\beta$-decay





virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

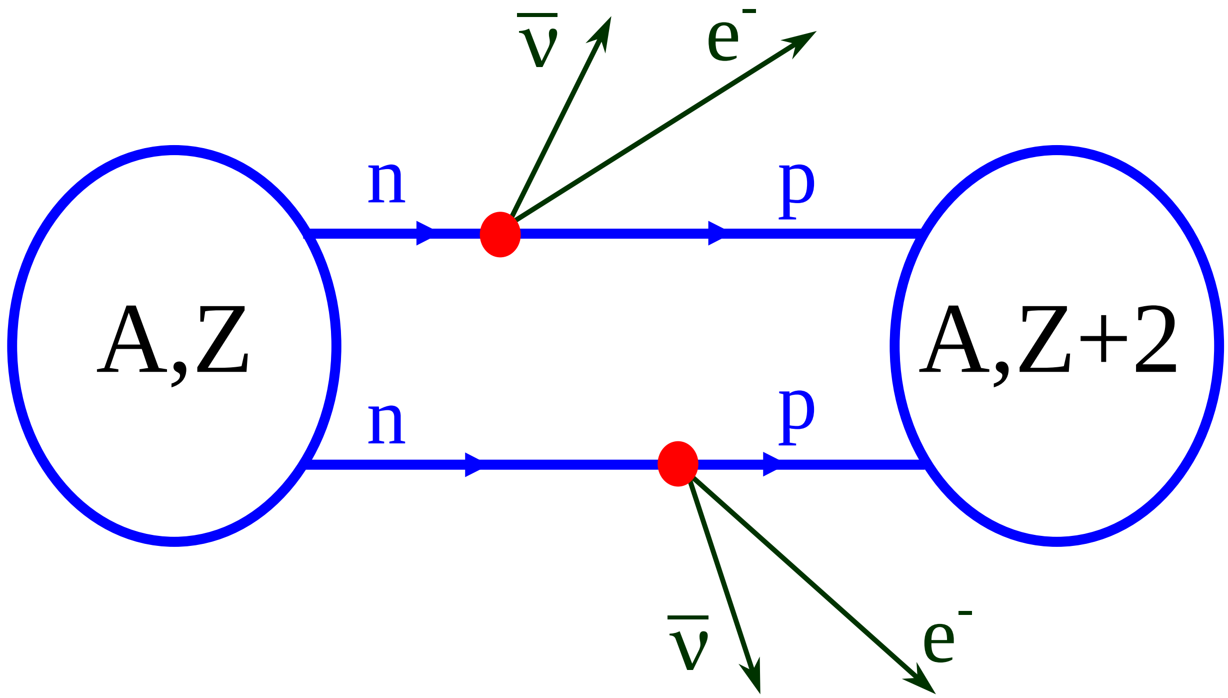
R-parity violating SUSY



Faessler, Kovalenko, Simkovic, PRL (1997) → coupling to pions can increase the SUSY $0\nu\beta\beta$-decay probability by a factor 10000

Nuclear $2\nu\beta\beta$-decay

second order weak process
within SM



measured $T^{2\nu}_{1/2}$ (compilation of A. Barabash, 2005)

Isotope	$T^{2\nu}_{1/2}$, in 10^{19} y
^{48}Ca	$4.2^{+2.1}_{-1.0}$
^{76}Ge	150 ± 10
^{82}Se	9.2 ± 0.7
^{96}Zr	2.0 ± 0.3
^{100}Mo	0.71 ± 0.04
^{116}Cd	3.0 ± 0.2
^{128}Te	$(2.5 \pm 0.3) \times 10^5$
^{130}Te	90 ± 10
^{136}Xe	> 81 (90% CL)
^{150}Nd	0.78 ± 0.07
^{238}U	200 ± 60

$2\nu\beta\beta$ $0\nu\beta\beta$

Inverse Half-Lives $[T_{1/2}(0^+ \to 0^+)]^{-1}$

$$G^{2\nu}(Q,Z)\,|M^{2\nu}_{GT}|^2$$

$$m^2_{\beta\beta}\,G^{0\nu}(Q,Z)\,\left|M^{0\nu}_{GT} - \frac{g_V^2}{g_A^2}M^{0\nu}_F\right|^2$$

Eff. neutrino mass $m_{\beta\beta} = \sum_j m_j U^2_{ej}$

U_{ej} — first raw of the neutrino mixing matrix

$2\nu\beta\beta$ $0\nu\beta\beta$

Nuclear Matrix Elements

$$M_{GT}^{2\nu} = \sum_s \frac{\langle 0_f || \hat{\beta}^- || s \rangle \langle s || \hat{\beta}^- || 0_i \rangle}{E_s - (M_i + M_f)/2}$$

$$\hat{\beta}^- = \sum_k \boldsymbol{\sigma}_k \tau_k^-$$

$$M_{GT}^{0\nu} = \langle 0_f | \sum_{ik} P_\nu(r_{ik}, \overline{\omega}) \tau_i^- \tau_k^- \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k | 0_i \rangle$$

Neutrino potential : $P_\nu(r, \overline{\omega}) =$

$$\frac{2R}{\pi r} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + \overline{\omega})}$$

$$\approx \frac{R}{r} \; \phi(\overline{\omega} r)$$

QRPA vs. SM

Complete theory of nuclear structure does not exist!

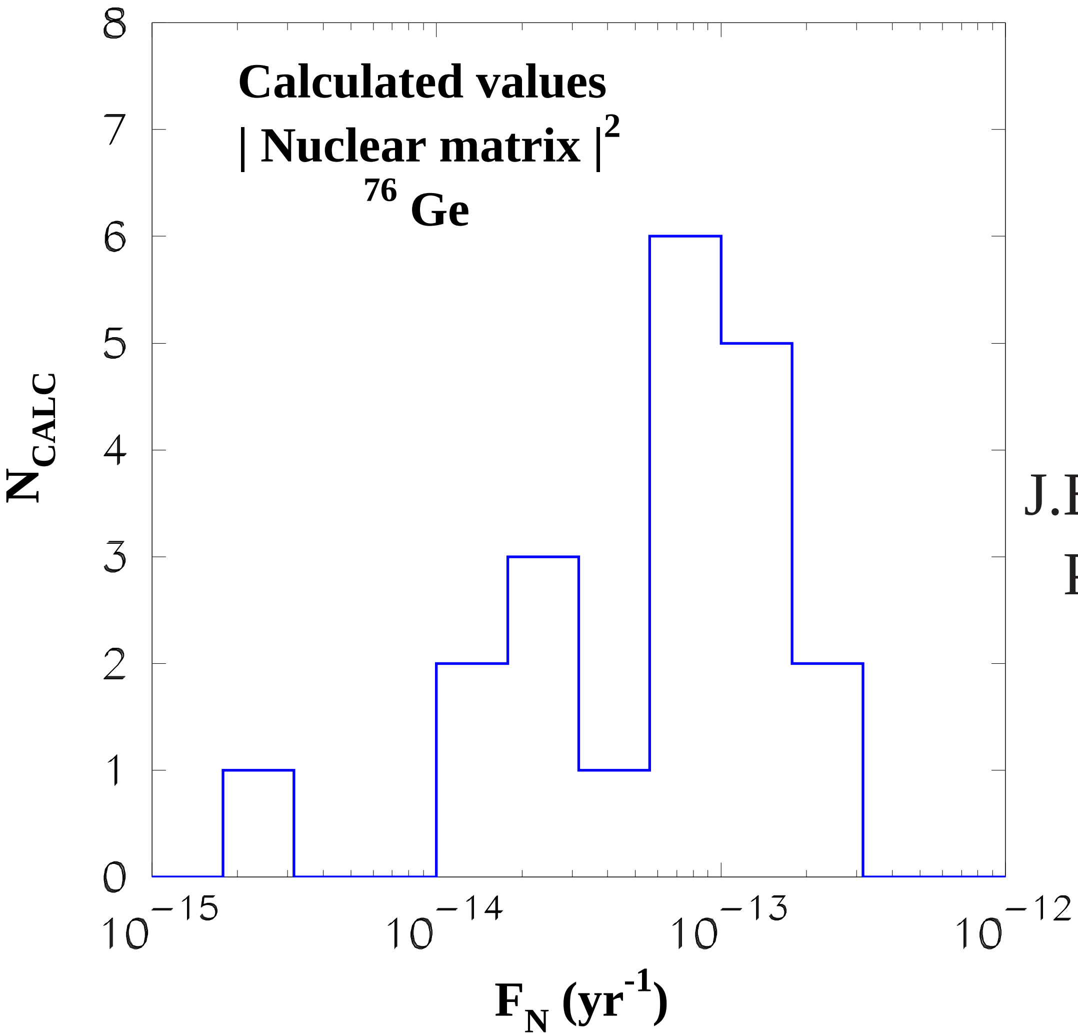
Nuclear models to calculate $\beta\beta$-amplitudes

Mean field $\longrightarrow$ s.p. states $\longrightarrow$ configuration space $\longrightarrow$ diagonalization

Working groups

	Tübingen; Jyväskylä	Strassbourg-Madrid
	QRPA	NSM
s.p. bases	$N\hbar\omega$	$0\hbar\omega$
configurations	limited	all

QRPA vs. SM

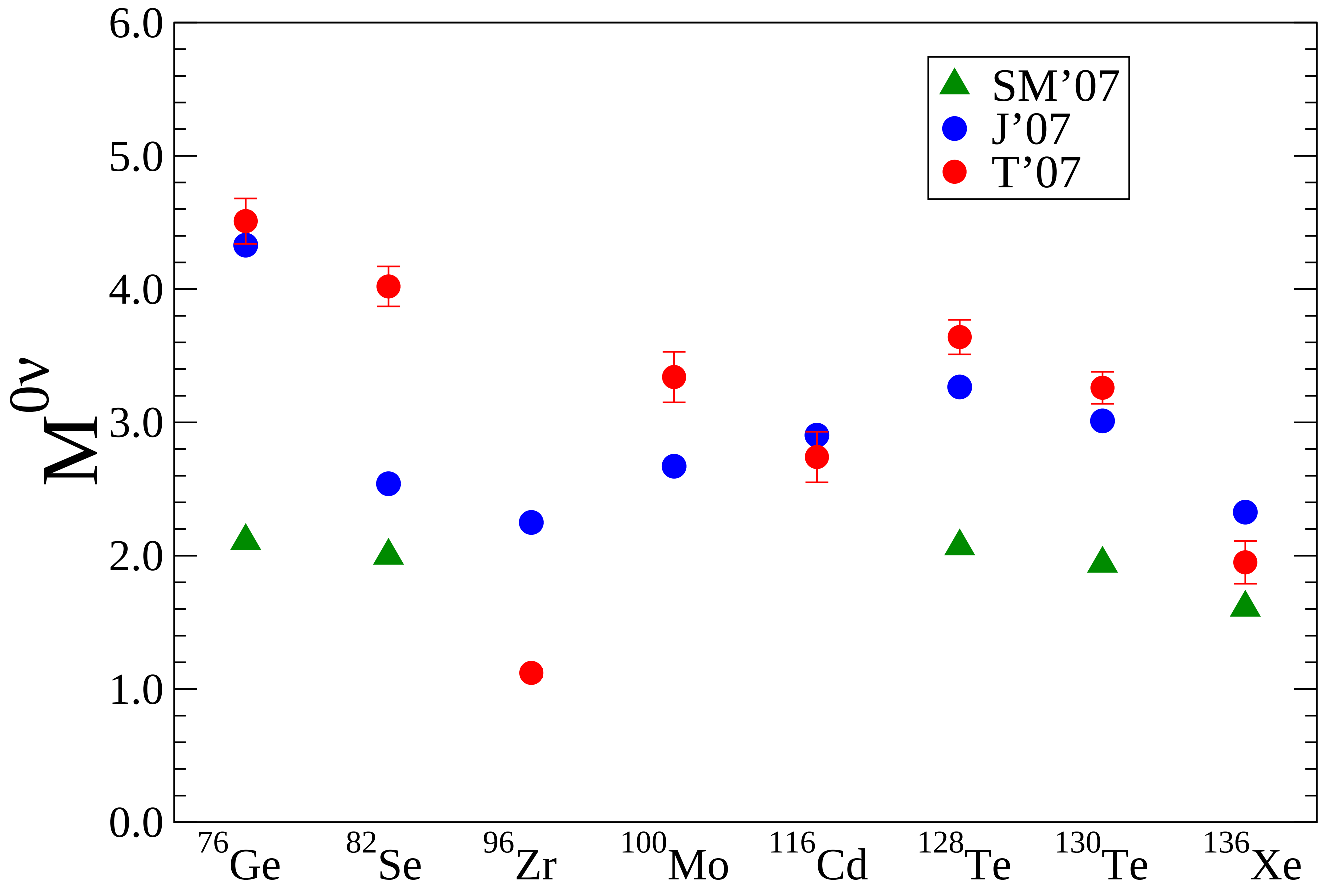


J.Bahcall, H.Murayama, C.Pena-Garay
PRD 70 (2004)

“In the foreseeable future, it does not seem possible to derive in a direct and controlled manner from QCD nuclear matrix elements for large *A*.
Thus, there is no way of quantifying with absolute confidence the range of uncertainties ..."

What is the reason for such a large spread?

QRPA vs. SM



Current status (g_A = 1.25, Jastrow s.r.c.)

QRPA T'07 = V.R., A. Faessler, F. Simkovic, P. Vogel, NPA 793 (2007)

J'07 = M. Korteleinen, J. Suhonen, PRC 76, 051303; 76, 6024315 (2007)

SM SM'07 = E. Caurier, J. Menendez, F. Nowacki, and A. Poves, arXiv:0709.2137 [nucl-th]

QRPA vs. SM

Many input parameters in calculations

- Mean field
- Size of the model space
- Residual nucleon-nucleon interaction

  schematic and realistic interactions (Brueckner G-matrix from Bonn, Argonne, Nijmegen OBEP)

  Renormalizations within QRPA:

  - In pairing channel g_{pair} — BCS model
  - In particle-hole channel g_{ph} — the energy of GTR ($g_{ph} \approx 1$).
  - In particle-particle channel g_{pp}

  The renormalization depends on the basis size (cut-off dependence)

- Renormalization of axial-vector coupl. constant g_A=1.0–1.25
- Nuclear deformation

QRPA vs. SM

- Modifications of the neutrino potential

  - Finite size of the nucleon
  - Two-nucleon short-range correlations (s.r.c.)
    (Jastrow factor, UCOM, etc.)
  - Higher order terms of the nucleon weak current
    (induced pseudoscalar and weak magnetism)

QRPA vs. SM

NSM seems to be appealing *ab initio* approach:

neglects nothing and treats all configurations on the same footing

describes well spectroscopy of low-energy nuclear levels

BUT:

- The Wall: factorial ($N! \sim N^N$) grow of the model space dimension

  $\Rightarrow$ Severe basis truncation in medium and heavy nuclei

- Different phenomenological quenching factors — original *ab initio* is lost

  $\Rightarrow$ effective operators are needed instead of the “bare" ones

One must trust in the "black box" diagonalization

QRPA vs. SM

Shell Model

Model Space: ^{48}Ca — fp; ^{76}Ge, ^{82}Se — $p, f_{5/2}, g_{9/2}$

^{96}Zr, ^{100}Mo — s, d, g; ^{128,130}Te, ^{136}Xe — $s, d, g_{7/2}, h_{11/2}$

- A lot of GT strength is missing (Ikeda Sum Rule is violated up to 40%)

  several spin-orbit partners are missing even in $0\hbar\omega$ model space

- Many $0\nu\beta\beta$-transitions via negative parity intermediate states (dipole, spin-dipole etc.) are missing. They contribute a lot to $M^{0\nu}$ (shown by QRPA)

QRPA vs. SM

Calculated $T^{2\nu}_{1/2}$:

E. Caurier *et al.*, RMP 77 ('05) (F. Nowacki, IDEA meeting, Heidelberg, '04)

Parent	^{48}Ca	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe
$T^{2\nu}_{1/2}$ th.(y)	$3.7 \cdot 10^{19}$	$2.6(1.2) \cdot 10^{21}$	$3.7(3.4) \cdot 10^{19}$	$(4. \cdot 10^{20})$	$(6. \cdot 10^{20})$
$T^{2\nu}_{1/2}$ exp.(y)	$4.2^{+2.1}_{-1.0} \cdot 10^{19}$	$(1.5 \pm 0.1) \cdot 10^{21}$	$(9.2 \pm 0.7) \cdot 10^{19}$	$(9.0 \pm 1.0) \cdot 10^{20}$	$> 8. \cdot 10^{20}$

E. Caurier, F. Nowacki and A. Poves, ArXiV:0709.0277v2 [nucl-th]

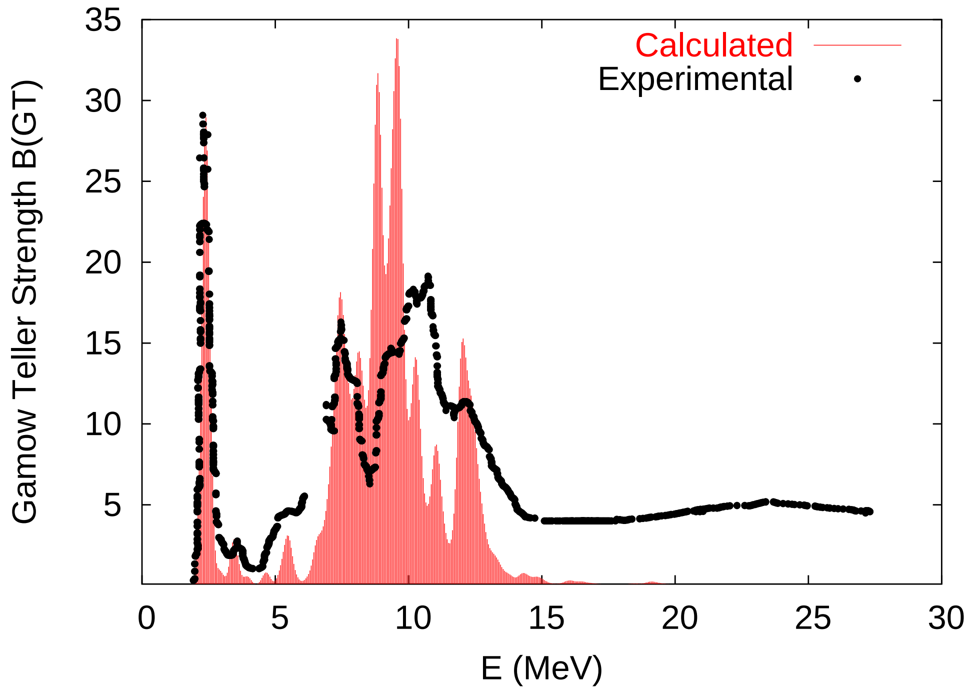
Comparison of the results for ^{48}Ca using the effective interactions KB3, FPD6, and GXPF1

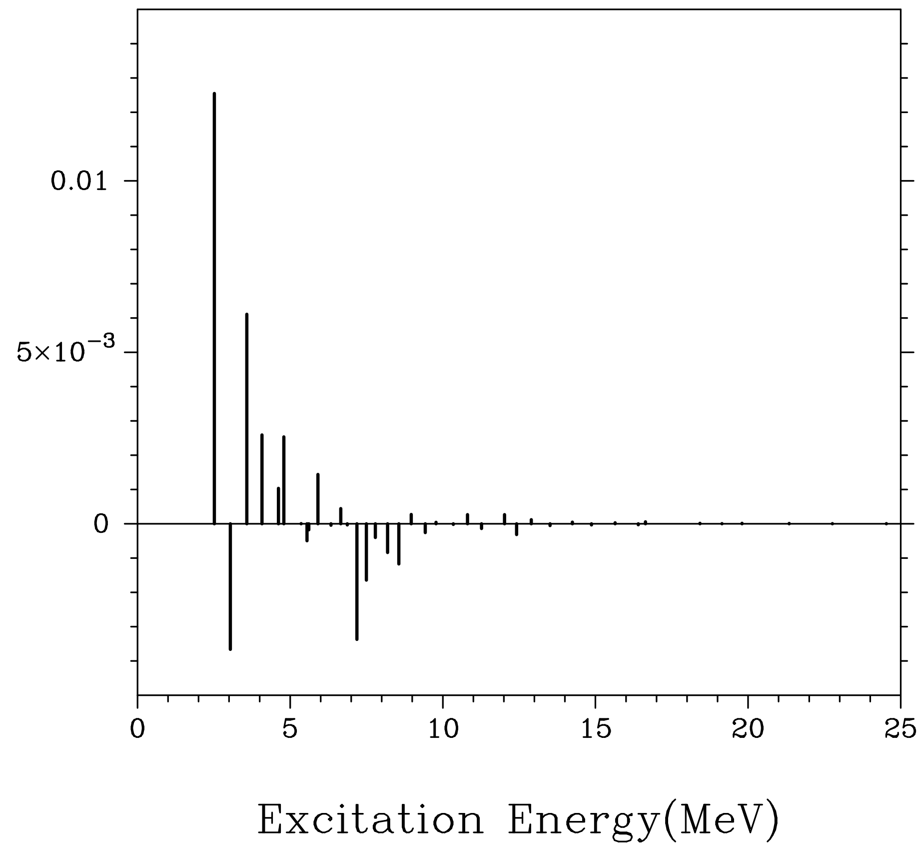
	KB3	FPD6	GXPF1
$M_{GT}(2\nu)$	0.08	0.10	0.11
$M_{GT}(0\nu)$	0.67	0.73	0.62

$M^{2\nu}_{exp} = 0.05$ MeV^{-1}

QRPA vs. SM

Shell Model, ^{48}Ca





QRPA vs. SM

QRPA

- Works quite well when applied to description of collective states

- Fulfills exactly various model-independent sum rules

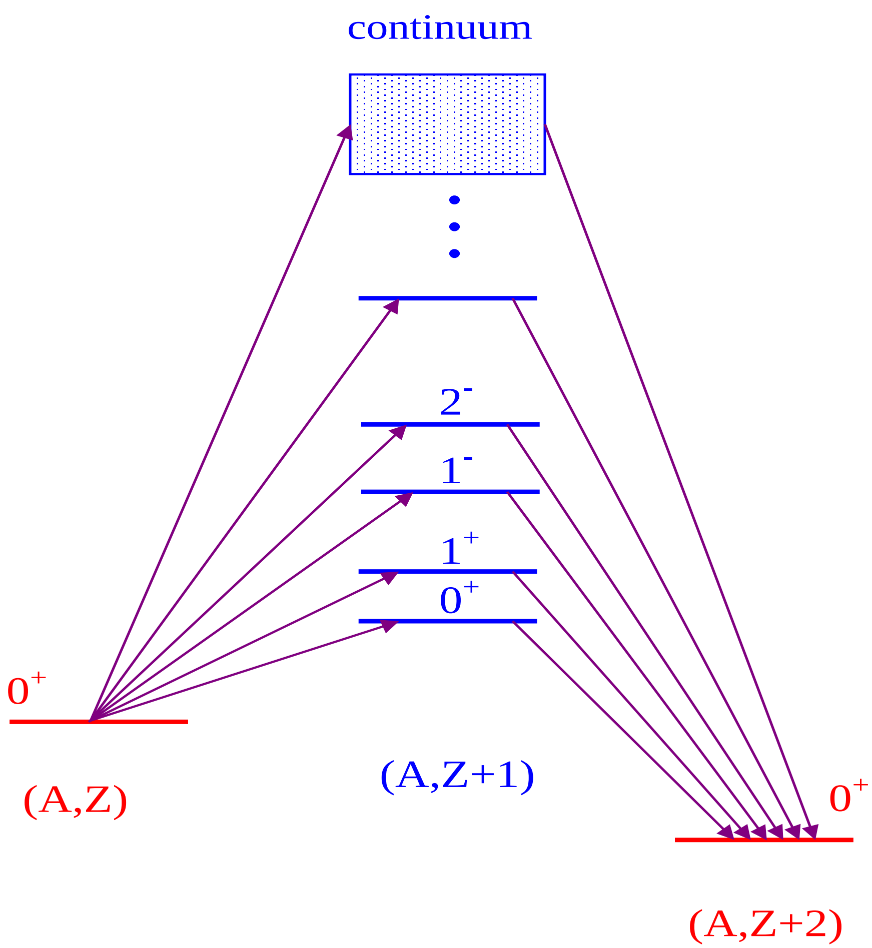
$M^{0\nu}$ and $M^{2\nu}$ are integral quantities (sums over all intermediate states)
challenge for experimental verification, but favors QRPA description

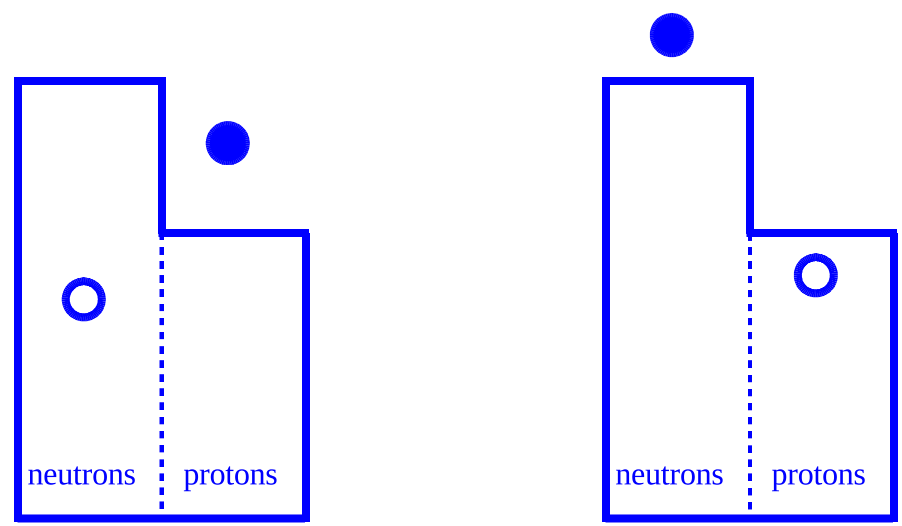
Nuclear dynamics of $\beta\beta$-decay

virtual excitation of states of all multipolarities

in intermediate (A,Z+1) nucleus

($2\nu\beta\beta$ - only 1^+ states, Gamow-Teller transitions)





Sharp Fermi surfaces ⇒ only transitions between Fermi levels

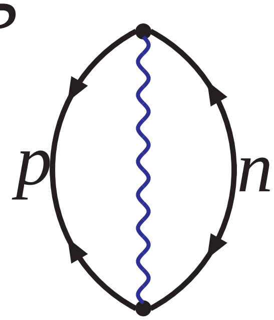
Nucleon pairing interaction ⇒ particle and hole get mixed

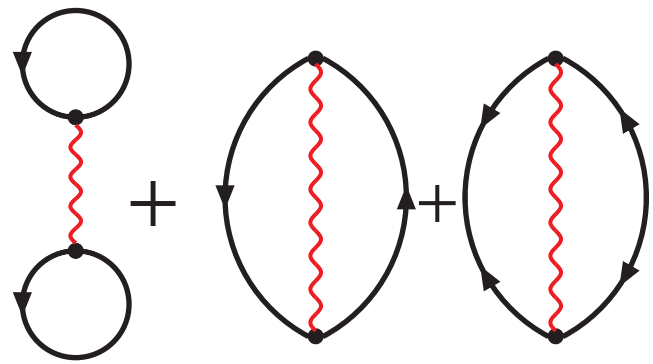
QRPA vs. SM

$0\nu\beta\beta$

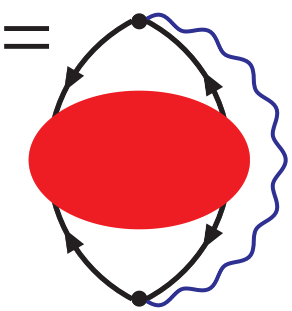
Coulomb

Pairing =



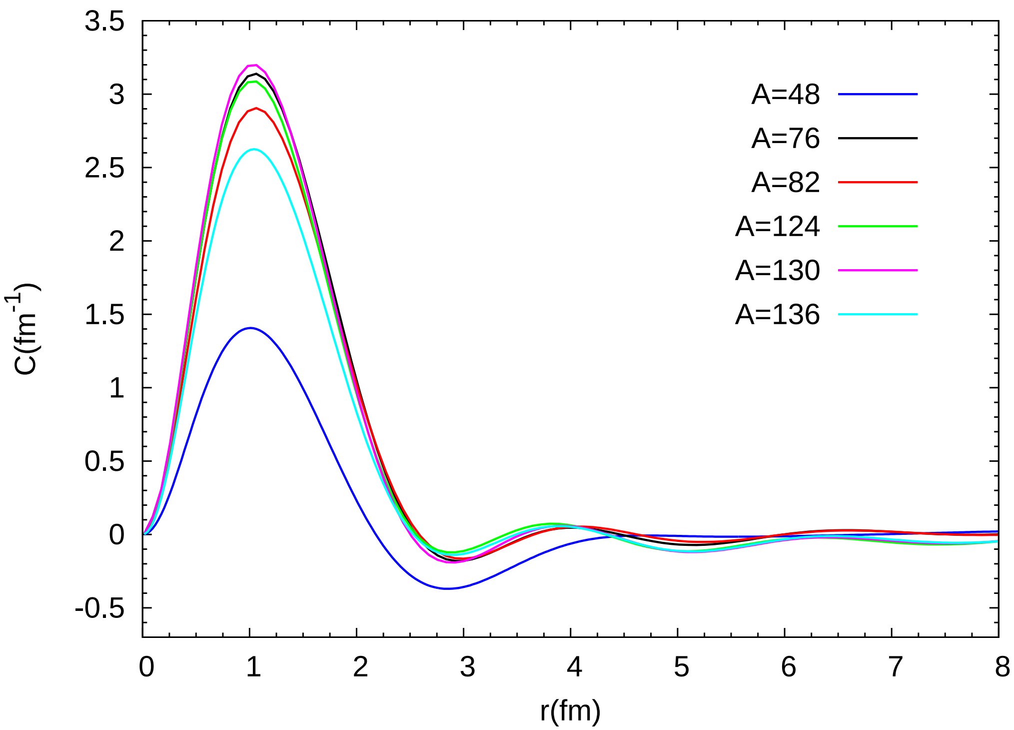


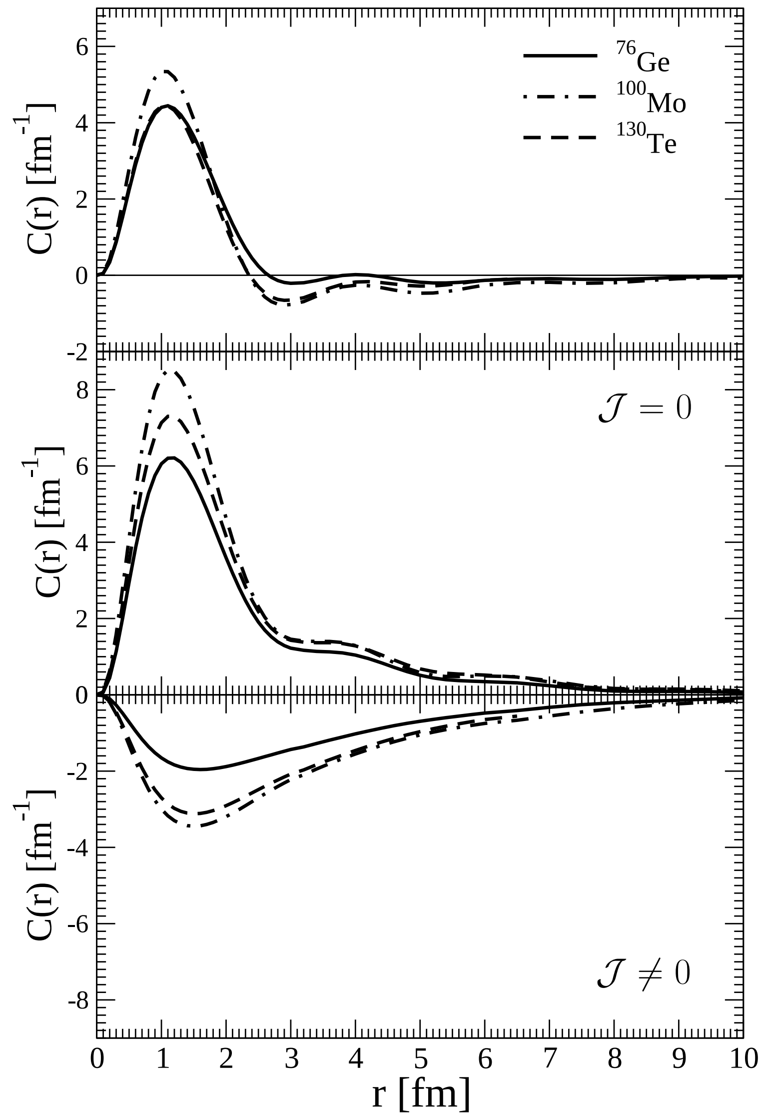
Total =



QRPA vs. SM

$$\int_0^\infty C(r)dr = M^{0\nu}$$

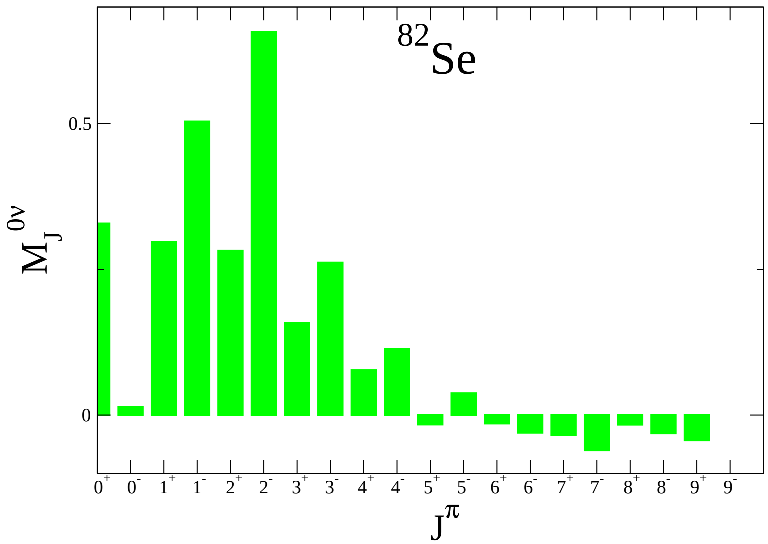




Partial contributions to $M^{0\nu}$

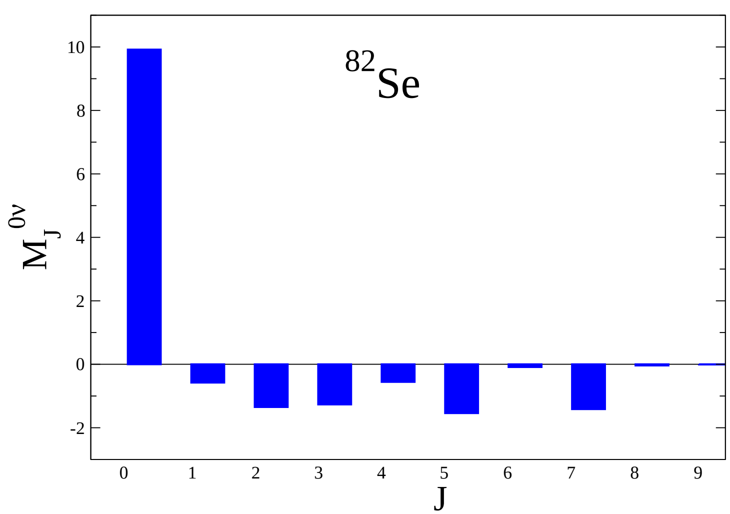
- $M^{0\nu} = \sum_{J^\pi} M^{0\nu}_{J^\pi}$ — particle-hole channel

$$M^{0\nu}_{J^\pi} = \sum_{pnp'n'} a_{pnp'n'} \langle 0_f || \left[c_p^\dagger \tilde{c}_n \right]_J \left[c_{p'}^\dagger \tilde{c}_{n'} \right]_J || 0_i \rangle$$



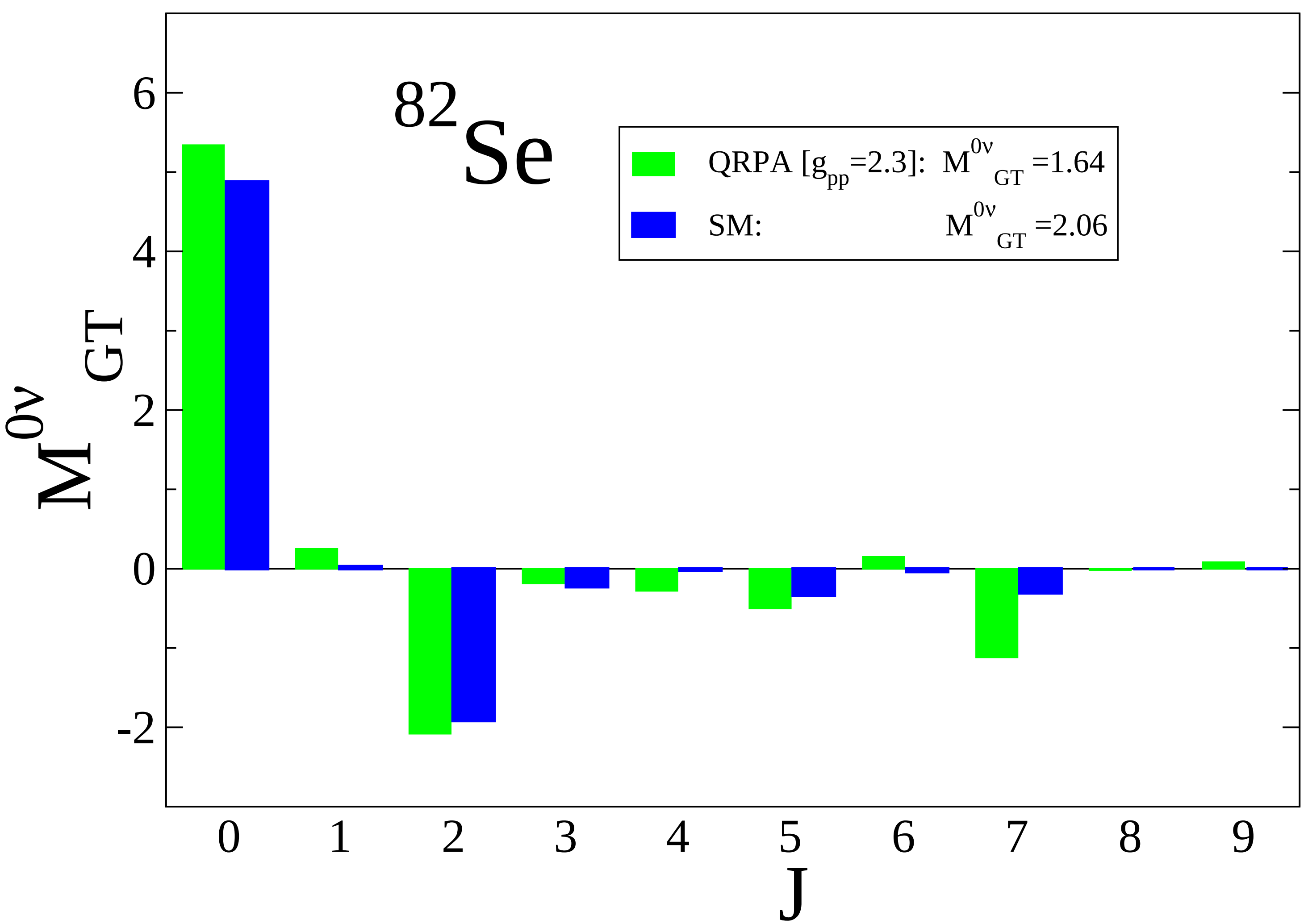
- $M^{0\nu} = \sum_{\mathcal{J}^\pi} M^{0\nu}_{\mathcal{J}^\pi}$ —particle-particle channel

$$M^{0\nu}_{\mathcal{J}^\pi} = \sum_{pnp'n'} b_{pnp'n'} \langle 0_f || \left[c_p^\dagger c_{p'}^\dagger \right]_{\mathcal{J}} \left[c_n c_{n'} \right]_{\mathcal{J}} || 0_i \rangle$$

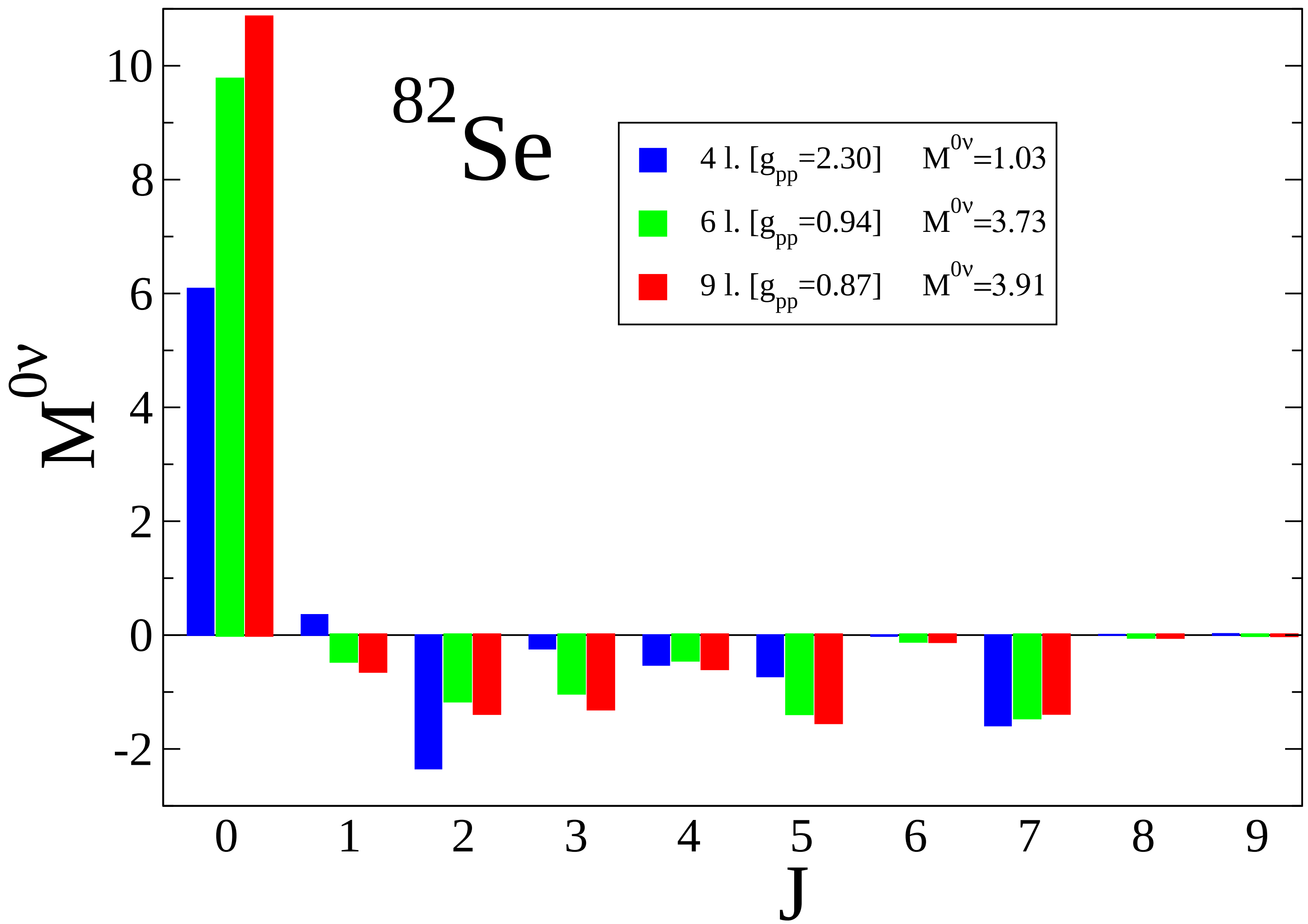


QRPA vs. SM

F. Simkovic, A. Faessler, V.R., P. Vogel, J. Engel, arXiv:0710.2055 [nucl-th]

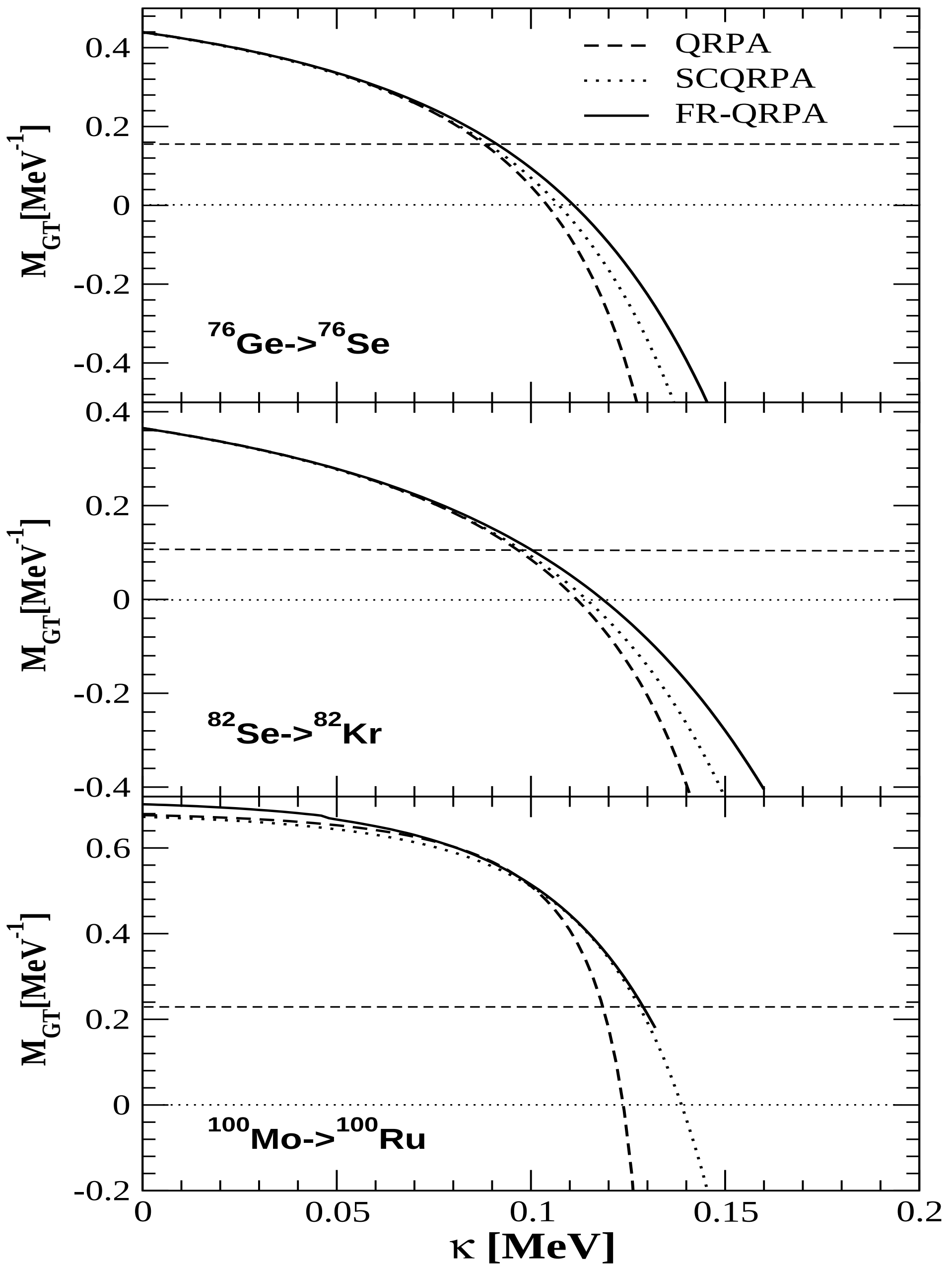


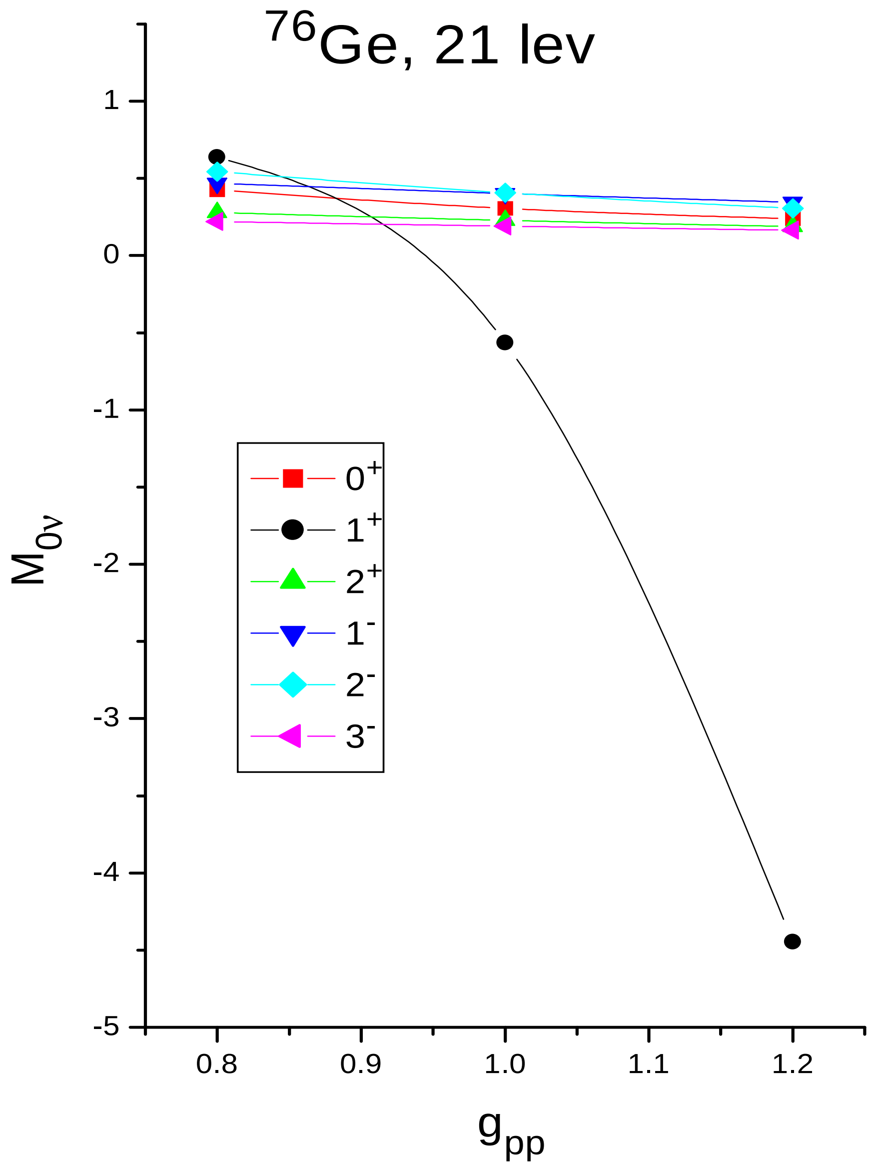
QRPA vs. SM



^{82}Se
4 l. [g_{pp}=2.30] $M^{0\nu}$=1.03
6 l. [g_{pp}=0.94] $M^{0\nu}$=3.73
9 l. [g_{pp}=0.87] $M^{0\nu}$=3.91
$M^{0\nu}$
10
8
6
4
2
0
-2
0 1 2 3 4 5 6 7 8 9
J

Parameter sensitivity of calculated $M^{\beta\beta}$





Broken SU(4) symmetry

Origin of parameter sensitivity of $M^{\beta\beta}$

Artifact of the QRPA, RQRPA, FR-QRPA ... or more general?

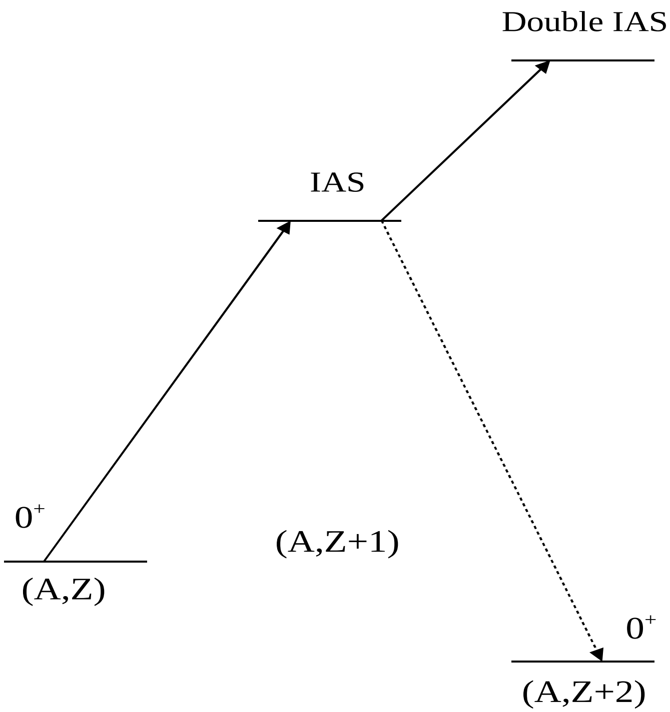
Origin: Violation of the isospin SU(2) and Wigner SU(4) symmetries of the nuclear Hamiltonian
$\Rightarrow$ Sensitivity is unavoidable!

Dynamically broken Wigner spin-isospin SU(4) symmetry
(concept used for many years for qualitative analyses)

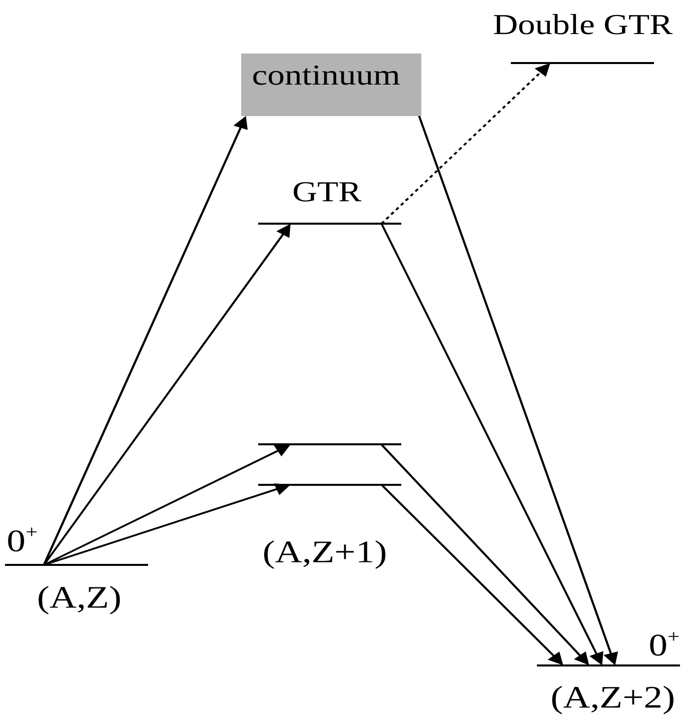
Isolating physical origin of the g_{pp}-sensitivity of $M^{2\nu}$
$\Rightarrow$ a energy-weighted sum rule is at work
V. R., M. Urin, A. Faessler, NPA 747 (2005) and in preparation

Broken SU(4) symmetry

Fermi transitions ($J_s^\pi = 0^+$)



Gamow-Teller transitions ($J_s^\pi = 1^+$)



$M_F^{2\nu} = 0$ if isospin SU(2) symmetry is exact

Violated by Coulomb

$M_{GT}^{2\nu} = 0$ if spin-isospin SU(4) symmetry is exact

Violated by Coulomb, spin-orbit, residual two-body

Model-independent transformation of $2\nu\beta\beta$-amplitude

Partition of $M^{2\nu}$ into a sum of two terms
each sensitive to different components of H_{nucl}

$$M^{2\nu} = \sum_s \frac{g_s f_s}{\omega_s}$$

$$1 = \frac{(\omega_g^2 - \omega_s^2)}{\omega_g^2} + \frac{\omega_s^2}{\omega_g^2} \quad \Rightarrow \quad M^{2\nu} = M'_{2\nu} + \frac{S}{\omega_g^2}$$

Broken SU(4) symmetry

$$M'_{2\nu} = \frac{1}{\omega_g^2} \sum_s \frac{(\omega_g^2 - \omega_s^2) g_s f_s}{\omega_s}$$

$$S^J = \sum_s \omega_s g_s f_s$$

$$= \frac{1}{2} \langle 0_f | \left[\hat{\beta}_J^-, \left[\hat{H}, \hat{\beta}_J^- \right] \right] | 0_i \rangle$$

energy-weighted sum rule

Broken SU(4) symmetry

Which components of the nuclear Hamiltonian contribute to $S^J = \frac{1}{2}\langle 0_f| \left[\hat{\beta}^-, \left[\hat{H}, \hat{\beta}^-\right]\right] |0_i\rangle$?

$$\hat{H} = \hat{H}_{s.p.} + \hat{H}_{p-h} + (\hat{H}_{pair} + \hat{H}_{p-p})$$

$\left[\hat{\beta}^-_J, \left[\hat{H}_{s.p.}, \hat{\beta}^-_J\right]\right] = 0 \leftarrow$ always

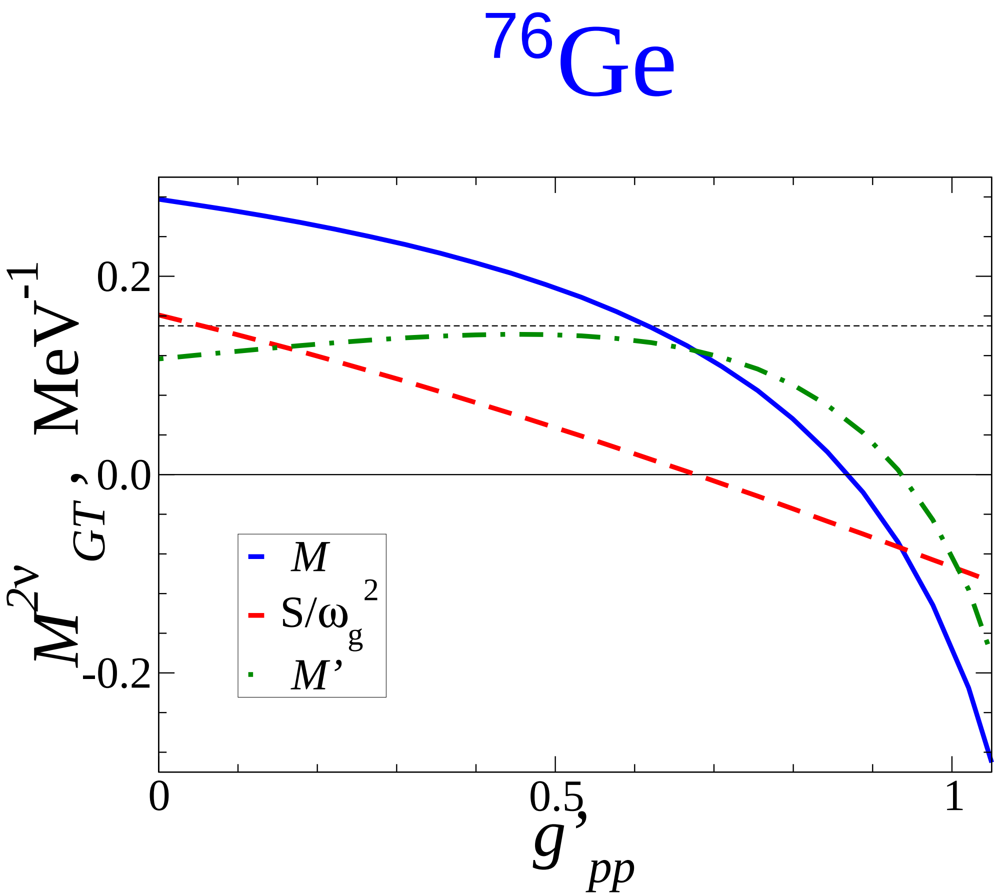
$\left[\hat{\beta}^-_J, \left[\hat{H}_{p-h}, \hat{\beta}^-_J\right]\right] = 0 \leftarrow$ within RPA

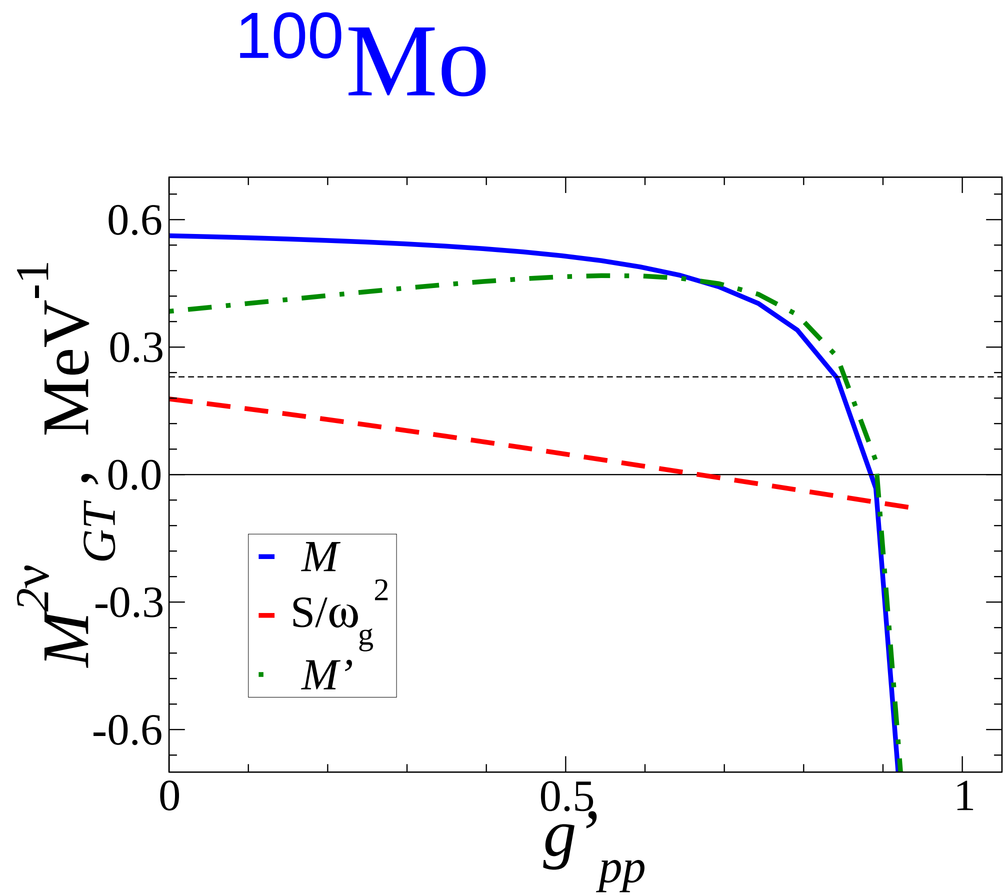
$$S_J = (S_J^{(pair)} + S_J^{(p-p)}) + \delta S$$

Separable p-p interaction

$$S_1 = 3(1 - g'_{pp})\frac{\Delta_n \Delta_p}{G_0}, \quad g'_{pp} = \frac{G_1}{G_0}$$

Broken SU(4) symmetry





Broken SU(4) symmetry

$$M_{GT}(closure) = \sum_s g_s f_s = M''_{GT} + \frac{S_{GT}}{\omega_g}$$

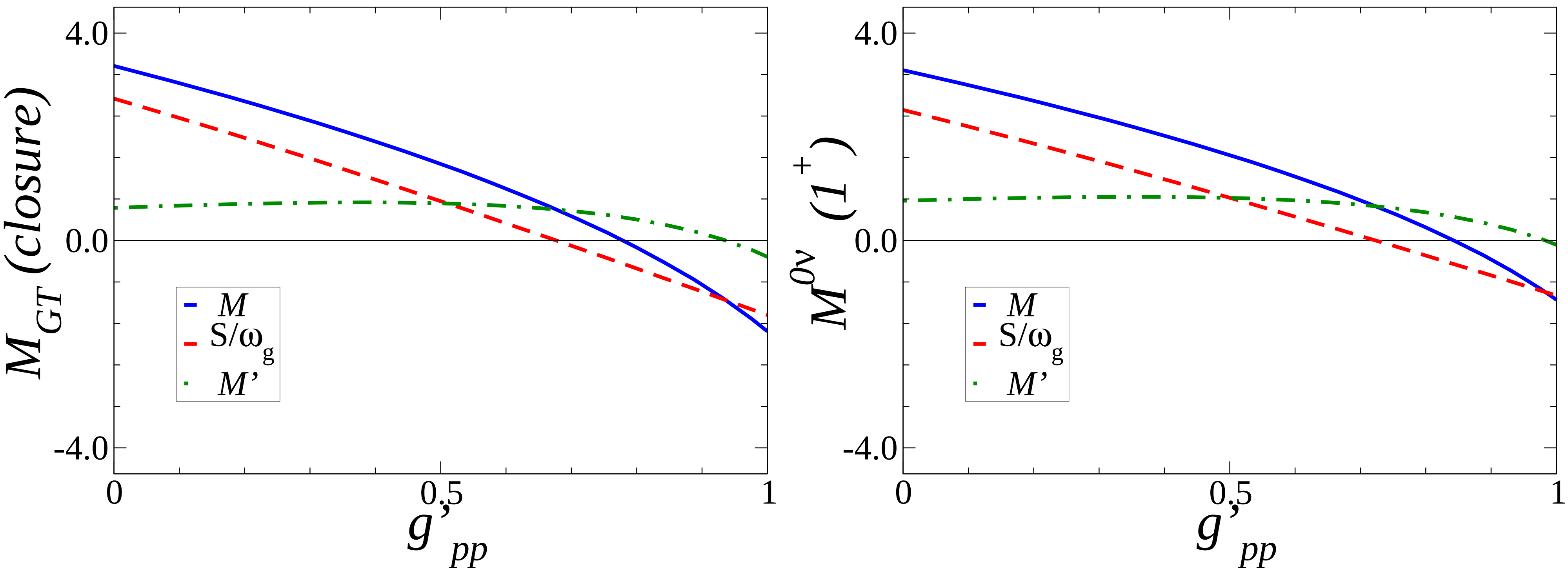
$$\text{Partition of } M^{0\nu} = \sum_J M_J^{0\nu} = \sum_J \sum_s (fg)_s^J$$

$$M_J^{0\nu} = M_J^{0\nu\prime} + \frac{S_J^{0\nu}}{\omega_{gJ}}$$

$$M_J^{0\nu\prime} = \frac{1}{\omega_{Jg}} \sum_s \frac{(\omega_{Jg} - \omega_s)(fg)_s}{\omega_s} \qquad S_J^{0\nu} = \sum_s \omega_s (fg)_s^J$$

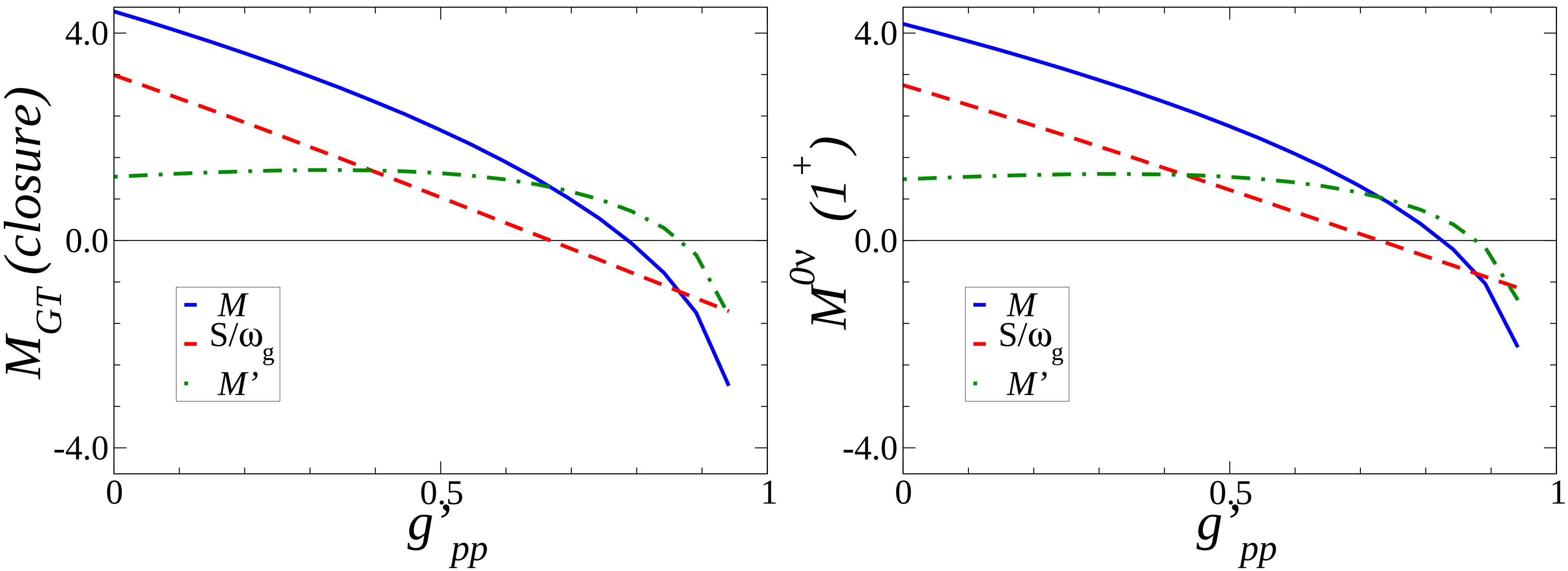
Broken SU(4) symmetry

^{76}Ge



Broken SU(4) symmetry

^{100}Mo

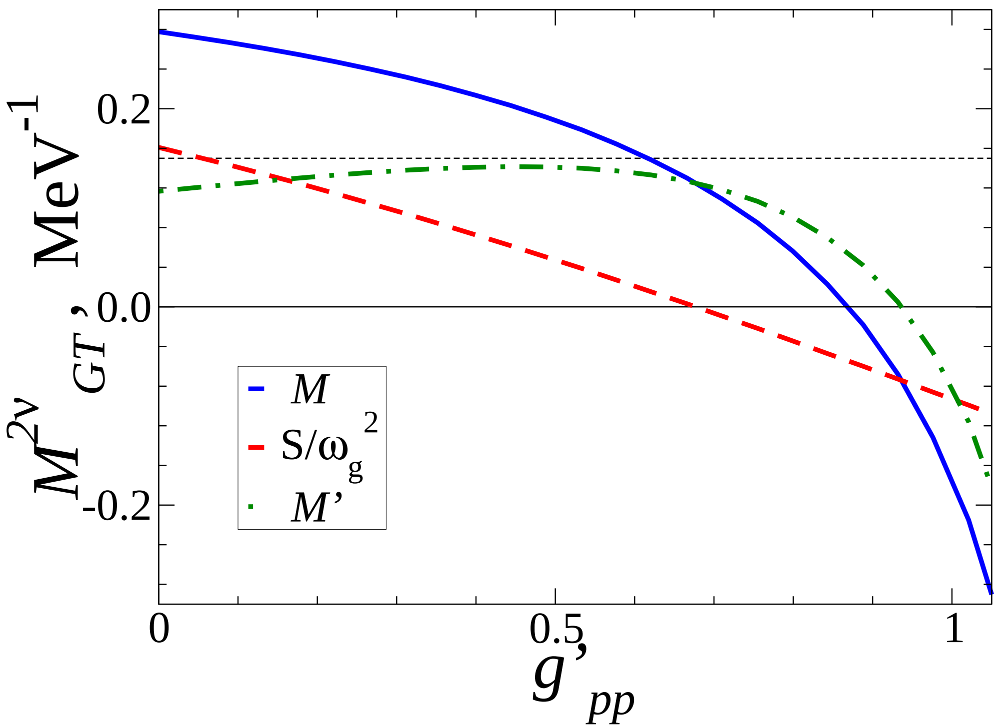


Broken SU(4) symmetry

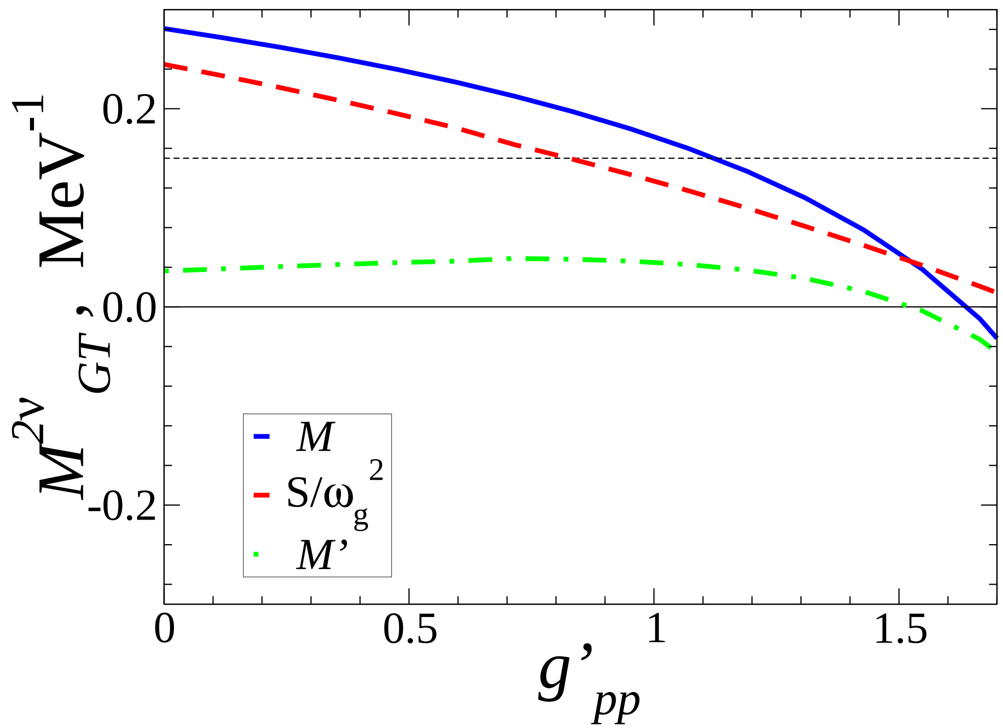
^{76}Ge

Basis

9 lev.



4 lev.



Broken SU(4) symmetry

- The g_{pp}-sensitivity is dictated by the generic structure of the $\beta\beta$ amplitudes. It reflects changing degree of violation of the Wigner SU(4) symmetry in the p-p sector of the Hamiltonian
  Must be present in all nuclear models!

- What is *the realistic* value of g_{pp}?
  S is the best tool to fix g_{pp}! $\Rightarrow$ But how to measure?

- Spin-flip transitions are artificially prohibited within the NSM by the basis choice that most affects Gamow -Teller transitions (the ISR is strongly underestimated).
  Sensitivity to the p-p interaction is unrealistically weak because of too small s.p. basis within the NSM.

$M^{0\nu}$ from $2\nu\beta\beta$

Systematic study of QRPA uncertainties

V.R., A. Faessler, F. Simkovic, P. Vogel, PRC 68 (2003); NPA 766 (2006); NPA 793 (2007)

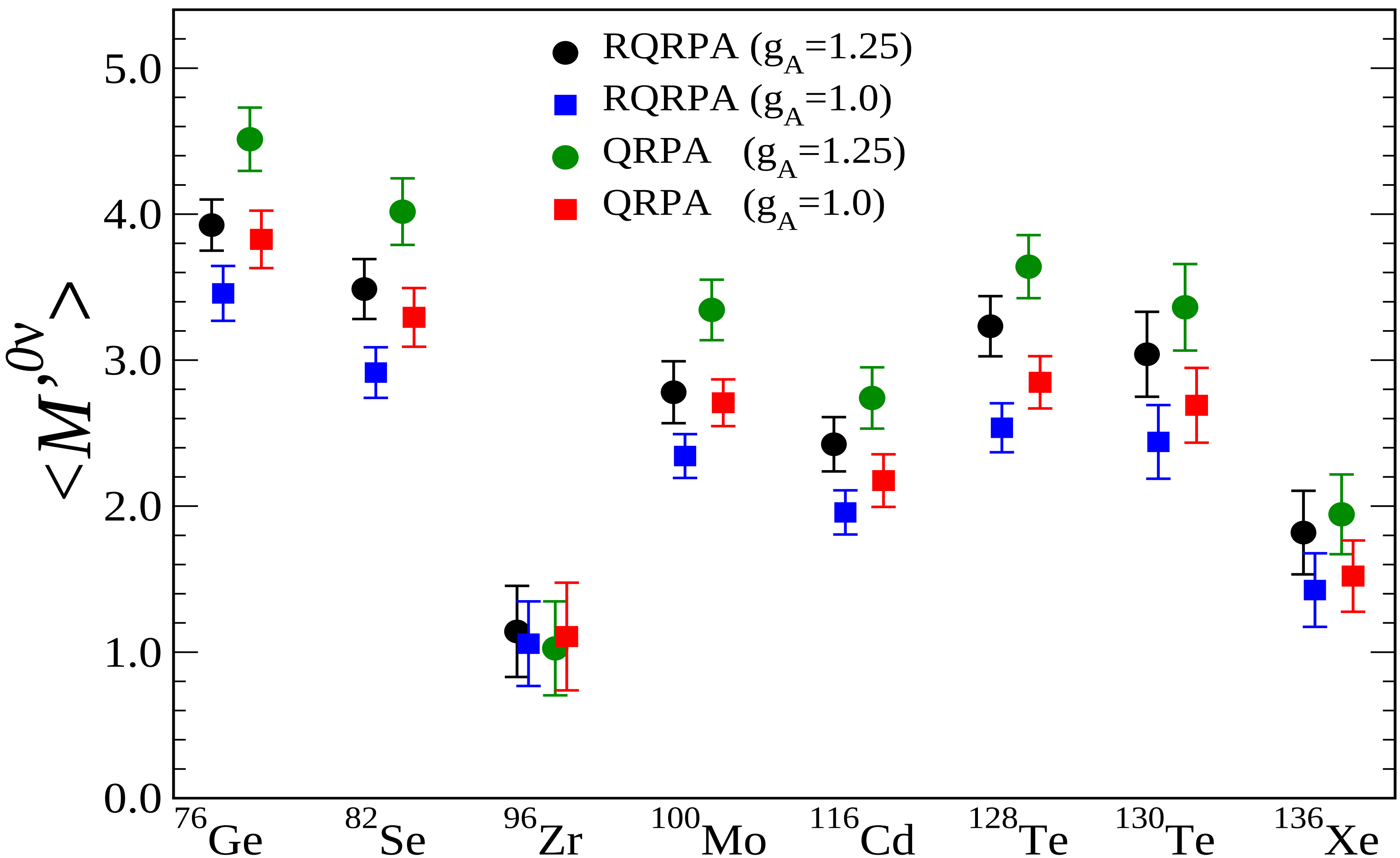
g_{pp} fitted to $2\nu\beta\beta$-decay half-life

Tested sensitivity of $M^{0\nu}$ to:

- size of the single-particle basis ($2, 3, 5\ \hbar\omega$)
- different realistic representations of the nucleon G-matrix (Bonn-CD, Argon, Nijmegen)
- quenching of the axial vector strength g_A

$M^{0\nu}$ from $2\nu\beta\beta$

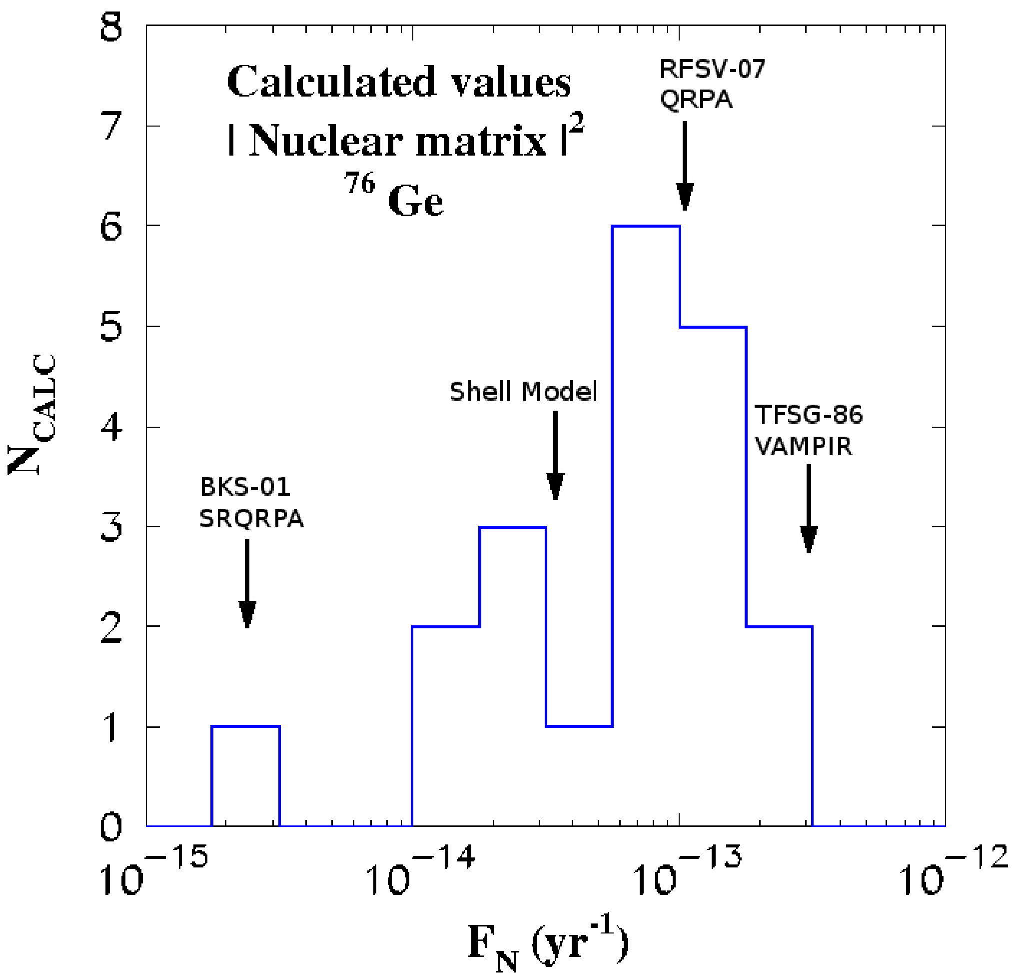
Light Majorana Neutrino Exchange Mechanism



9 calculations for every point

Errors: 1σ "theory" + induced 1σ experiment

$M^{0\nu}$ from $2\nu\beta\beta$



BKS-01 = A. Bobyk, W. Kaminski, F. Simkovic, PRC**63** (2001) ⇐ $2\nu\beta\beta$ 20 times too slow

Shell Model = E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves A. Zuker, RMP**77** (2005)

RFSV-07 = V.R., A. Faessler, F. Simkovic, P. Vogel, NPA**793** (2007) (2003) ⇐ $2\nu\beta\beta$ fitted

TFSG-86 = T. Tomoda, A. Faessler, K. W. Schmid, F. Grummer, NPA**452** (1986)

⇐ $2\nu\beta\beta$ 8 times too fast

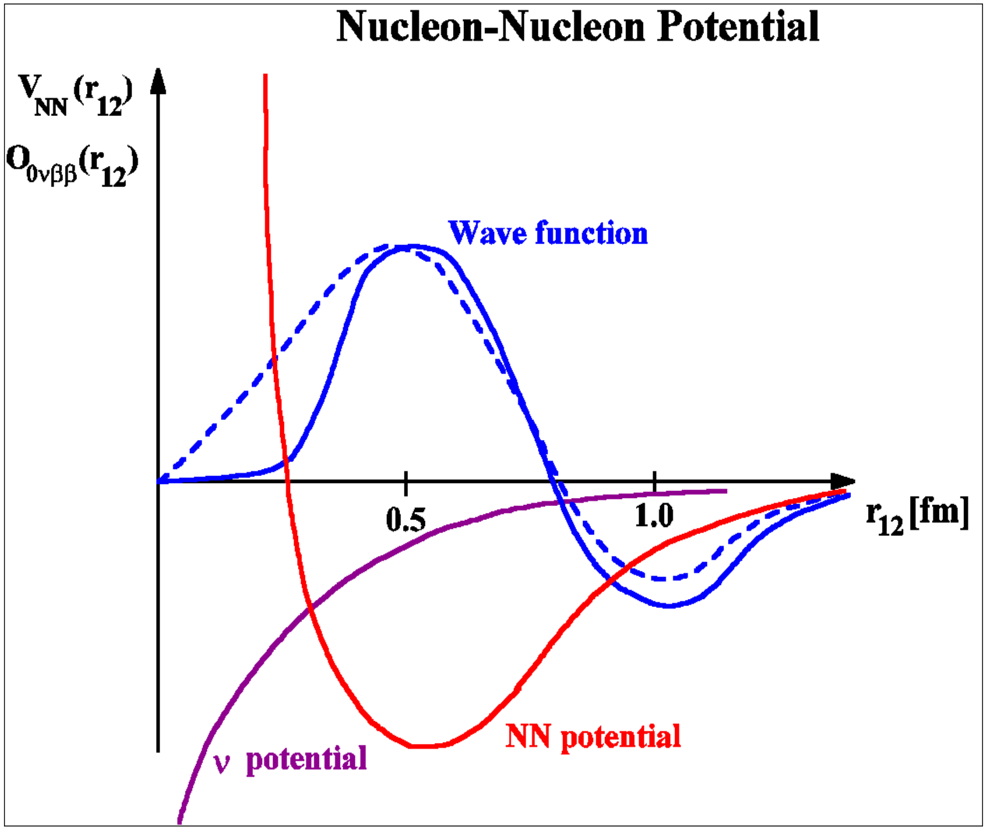
$M^{0\nu}$ from $2\nu\beta\beta$

$0\nu\beta\beta$ half-lives $T^{0\nu}_{1/2}$ (in 10^{27} years) assuming $\langle m_{\beta\beta}\rangle = 50$ meV.

Nuclear transition	QRPA (RFSV-07)	SM (CP-06)
^{76}Ge$\rightarrow$^{76}Se	0.8 – 1.2	2.3
^{82}Se$\rightarrow$^{82}Kr	0.22 – 0.40	0.6
^{130}Te$\rightarrow$^{130}Xe	0.17 – 0.39	0.4
^{136}Xe$\rightarrow$^{136}Ba	0.4 – 1.3	0.5

Effect of short range correlations

Uncorrelated and Correlated Relative N-N-Wavefunction in N-N-Potential



Effect beyond the QRPA and SM

Correlation Functions $f(r)$: $P_\nu(r) \to P_\nu(r) * f(r)^2$

Jastrow — Miller, Spencer, Ann.Phys. (1976): $f_J(r) = 1 - e^{-\alpha r^2}(1 - br^2)$

Unitary Correlator Operator Method — H. Feldmeier, R. Roth

Effect of short range correlations

the QRPA $M^{0\nu}$ with different SRC

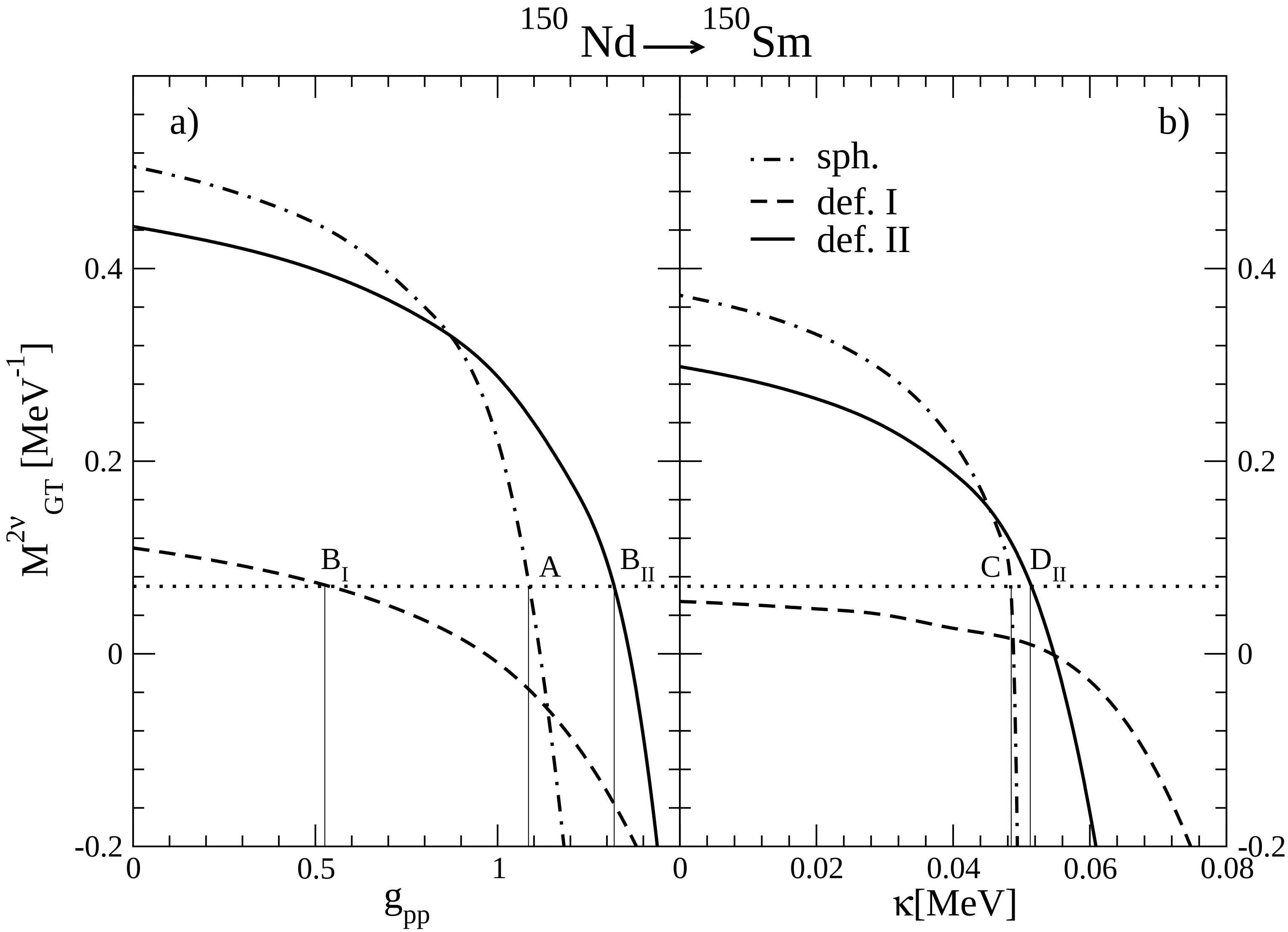
	^{76}Ge	^{100}Mo	^{130}Te
No SRC	5.81	4.70	4.57
Jastrow	4.51	3.34	3.26
UCOM	5.48	4.32	4.15

Further developments

- QRPA with realistic forces in deformed nuclei (^{150}Nd) applied first to $2\nu\beta\beta$ (M. Saleh, V.R., A. Faessler, F. Simkovic — to be submitted soon)

- continuum-QRPA (V.R., A. Faessler, PPNP 57 (2006) & PRC 77 (2008)) to perform “ultimate" QRPA calculations ($N \to \infty$ by including unbound s.p. states)

Further developments



Dot-dashed (sph.) – spherical shape of initial and final nuclei.

Solid (def. I) — exp. defor. (β_2(^{150}Nd)= 0.37 ± 0.09, β_2(^{150}Sm)= 0.23 ± 0.03)

Dashed (def. II) — calc. defor. (β_2(^{150}Nd)= 0.24, β_2(^{150}Sm)= 0.21)

Conclusions

- $0\nu\beta\beta$-decay is an *experimentum crucis* for revealing the Majorana nature of neutrinos and a feasible way to determine the absolute neutrino mass scale down to 10 meV's.

- $2\nu\beta\beta$ is an exotic charge-exchange probe of nuclear structure, sensitive to degree of violation of the Wigner SU(4) symmetry. g_{pp}-sensitivity is unavoidable!

- Uncertainties in the QRPA calculations of $M^{0\nu}$ can be greatly reduced by using the experimental data on $2\nu\beta\beta$.

- The QRPA $M^{0\nu}$ of different groups seem to converge. The $M^{0\nu}$ of the SM are substantially smaller. Reason for such a deviation is under active study now (too small basis of the SM?).

Prospects

F.T. Avignone III, S.R. Elliott, and J. Engel
arXiv:0708.1033 [nucl-ex], to appear in Rev. Mod. Phys.

”In the long term these issues will be solved...

The time it will take is certainly not short, but may be less than the time it will take for expermimentalists to see neutrinoless double-beta decay, even if neutrinos are indeed Majorana particles and the inverted hierarchy is realized.

And the pace of theoretical work will increase dramatically if the decay is seen.

Our opinion is that the uncertainty in the nuclear matrix elements in no way reduces the attractiveness of double-beta decay experiments.
Given enough motivation, theorists are capable of more than current work seems to imply.”

Prospects

- Term by term analysis of sources of parameter sensitivity of $M^{0\nu}$. Trying to separate out parts that can be calculated in less model dependent way.

- Non-accelerator methods playing more and more important role in deciphering new physics.
  $M^{0\nu}$ — probably not the last input needed from nuclear physics.

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