Transverse Mass for pairs of ‘gluinos’

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In collaboration with♪
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Introduction♪

Measurement of SUSY masses♪

- Precise measurement of SUSY particle masses♪

→ Reconstruction of SUSY theory ♪
(SUSY breaking mechanism)♪

♪

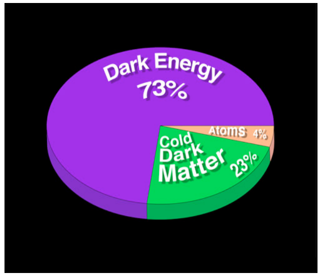
- Hans Peter Nilles' talk in SUSY 08, last week♪

Gaugino masses can serve as a promising tool for an early test for supersymmetry at the LHC

- Rather robust predictions
- 3 basic and simple patterns (Sugra, anomaly, mirage)
- Mirage pattern rather generic

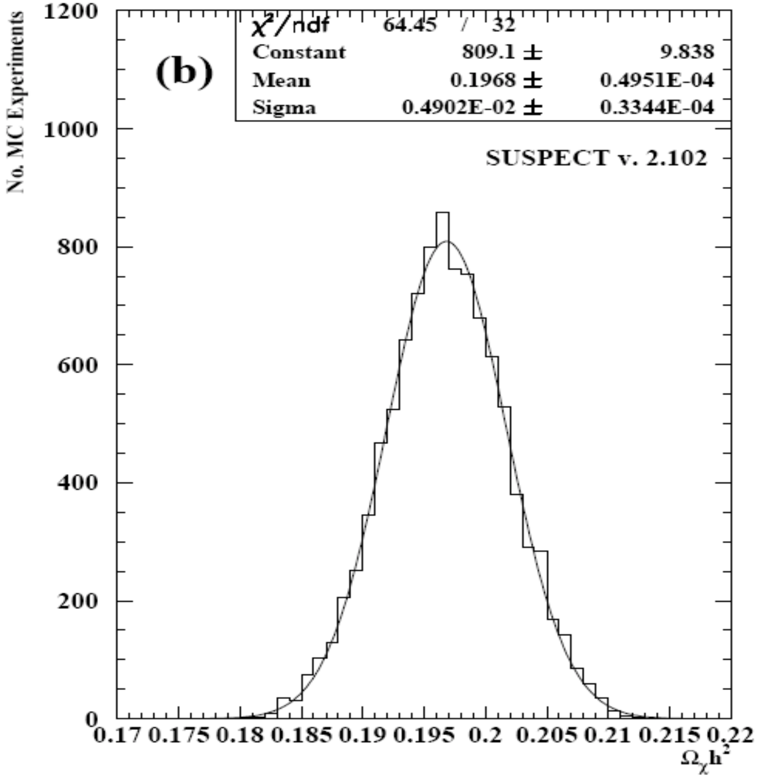
With some luck we might find such a simple scheme at the LHC and measure the ratio $G = M_{\text{gluino}}/m_{\chi_1^0}$!

→ Weighing Dark Matter with collider



Values for thermal relic density from mSUGRA fit to SPS1a invariant mass spectrum end-points

(Polesello and Tovey 2004)



For 300 fb^{-1} of data

~3 % precision

$m_{\ell\ell}^{max}$
$m_{\ell\ell q}^{max}$
$m_{\ell q}^{low}$
$m_{\ell q}^{high}$
$m_{\ell\ell q}^{min}$
$m_{\ell\ell b}^{min}$
$m(\ell_L) - m(\tilde{\chi}_1^0)$
$m_{\ell\ell}^{max}(\tilde{\chi}_4^0)$
$m_{\tau\tau}^{max}$
$m(\tilde{g}) - 0.99 \times m(\tilde{\chi}_1^0)$
$m(\tilde{q}_R) - m(\tilde{\chi}_1^0)$
$m(\tilde{g}) - m(\tilde{b}_1)$
$m(\tilde{g}) - m(\tilde{b}_2)$

The Mass measurement is Not ♪
an easy task at the LHC !♪

- Final state momentum in beam direction ♪
  is unknown a priori, due to our ignorance of♪
  initial partonic center of mass frame♪

- SUSY events always contain two invisible LSPs♪

♪

♪

→ No masses can be reconstructed directly♪

- Several approaches (and variants) ♪ of mass measurements proposed♪

  - Invariant mass Edge method ♪

    Hinchliffe, Paige, Shapiro, Soderqvist, Yao ;♪
    Allanach, Lester, Parker, Webber♪

♪

  - Mass relation method♪

    Kawagoe, Nojiri, Polesello ;♪
    Cheng, Gunion, Han, Marandellea, McElrath♪

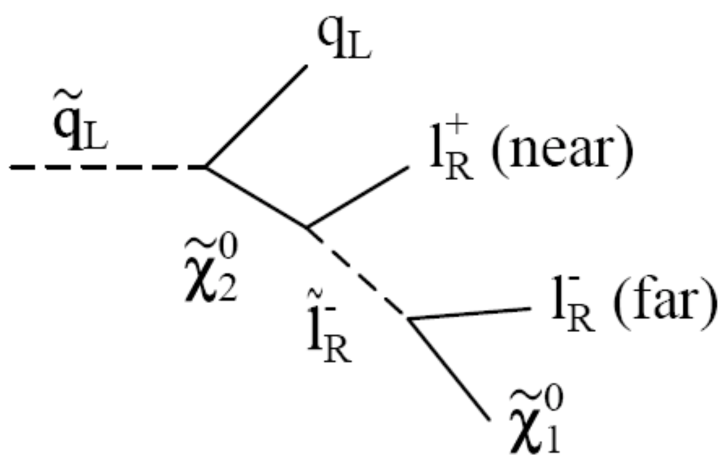
♪

  - Transverse mass (M_{T2}) kink method♪

    Cho, Choi, YGK, Park ; ♪
    Barr, Lester, Gripaios ;♪
    Ross, Serna;♪
    Nojiri, Shimizu, Okada, Kawagoe♪

The Edge method♪

Hinchliffe, Paige, etal.♪
(1997)♪



- Basic idea ♪

♪

→ Identify a particular long decay chain and measure ♪
kinematic endpoints of various invariant mass♪
distributions with visible particles♪

♪

→ The endpoints are given by functions of SUSY♪
particle masses ♪

If a long enough decay chain is identified, ♪
It would be possible to measure sparticle masses ♪
in a model independent way♪

$$(m_{ll}^2)^{\text{edge}} = \frac{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2},$$

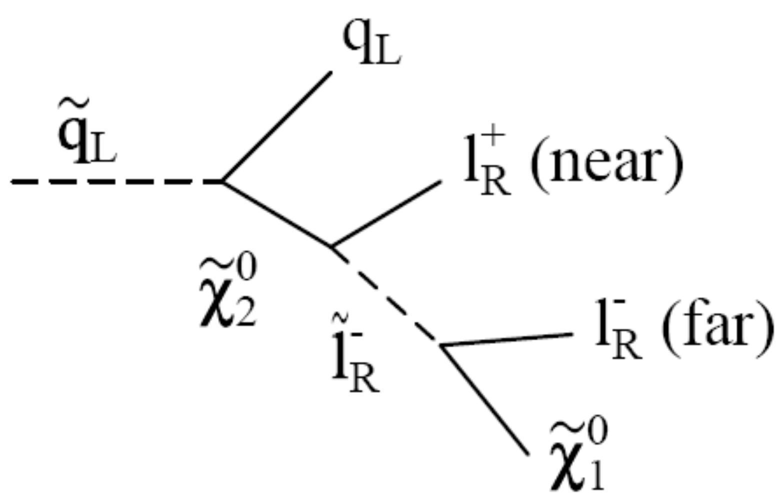
$$(m_{qll}^2)^{\text{edge}} = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{\chi}_2^0}^2},$$

$$(m_{ql}^2)^{\text{edge}}_{\text{min}} = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)}{m_{\tilde{\chi}_2^0}^2},$$

$$(m_{ql}^2)^{\text{edge}}_{\text{max}} = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2},$$

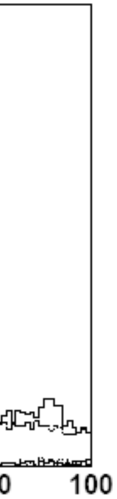
$$(m_{qll}^2)^{\text{thres}} = \Big[(m_{\tilde{q}_L}^2 + m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2) - (m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)\sqrt{(m_{\tilde{\chi}_2^0}^2 + m_{\tilde{l}_R}^2)^2(m_{\tilde{l}_R}^2 + m_{\tilde{\chi}_1^0}^2)^2 - 16 m_{\tilde{\chi}_2^0}^2 m_{\tilde{l}_R}^4 m_{\tilde{\chi}_1^0}^2} + 2m_{\tilde{l}_R}^2(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)\Big] / (4 m_{\tilde{l}_R}^2 m_{\tilde{\chi}_2^0}^2),$$

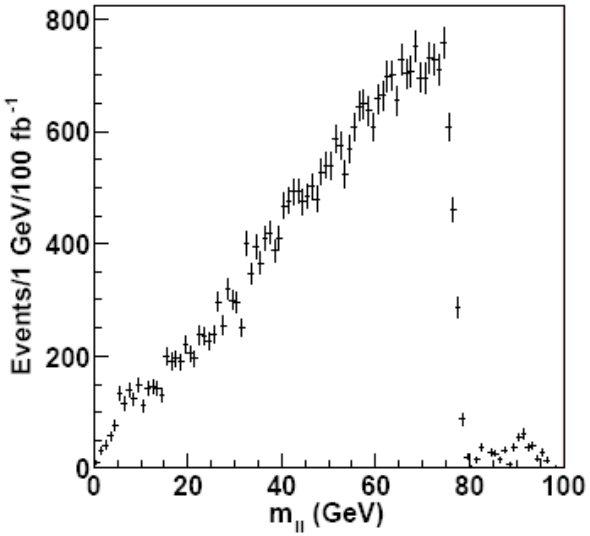
3 step two-body decays♪

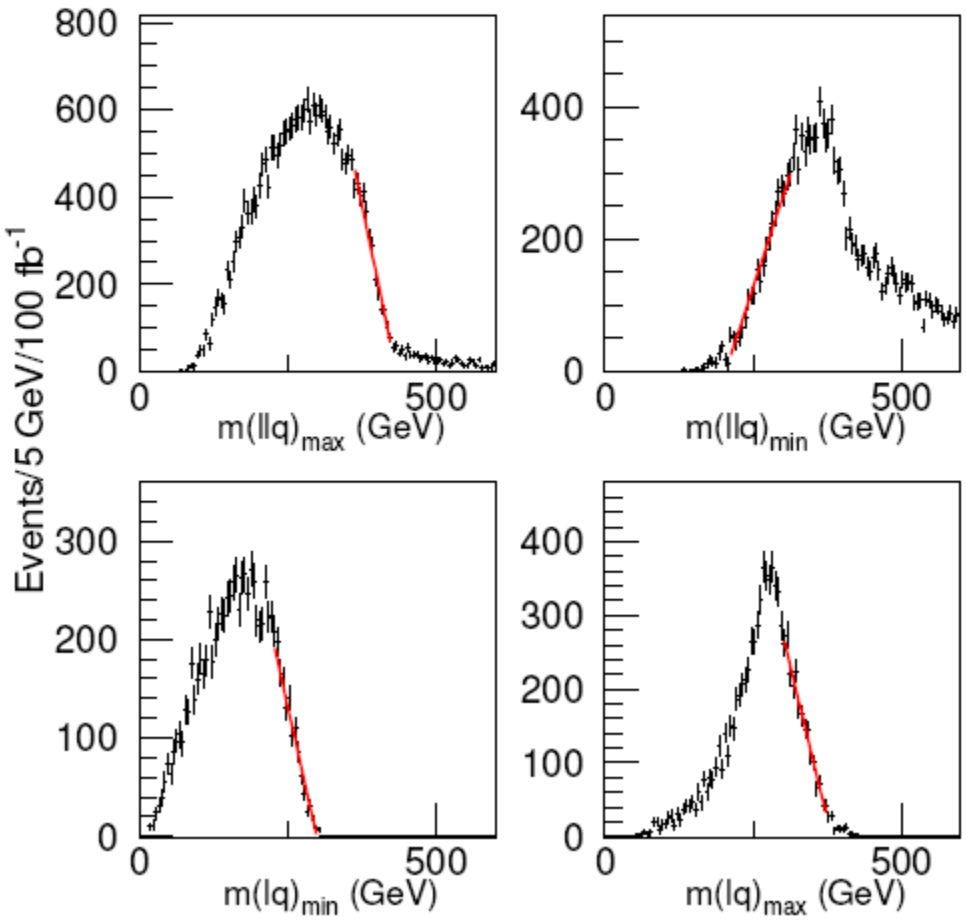


- For SPS1a point♪

[LHC/LC Study Group]♪







From five endpoint ♪
measurements,♪

Four invovled sparticle ♪
masses can be obtained♪

$\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\ell}_R$ masses reconstructed with ~ 5 GeV ,

$\tilde{q}_L$ mass with ~ 9 GeV (300 fb^{-1})

Mass relation method♪

Kawagoe, Nojiri, Polesello (2004)♪

- Consider the following cascade decay chain♪
  (4 step two-body decays)♪

$$\tilde{g} \rightarrow \tilde{b}b_2 \rightarrow \tilde{\chi}_2^0 b_1 b_2 \rightarrow \tilde{\ell} b_1 b_2 \ell_2 \rightarrow \tilde{\chi}_1^0 b_1 b_2 \ell_1 \ell_2$$

- Completely solve the kinematics of the cascade decay♪
  by using mass shell conditions of the sparticles ♪

➢ One can write five mass shell conditions ♪

$$m^2_{\tilde{\chi}^0_1} = p^2_{\tilde{\chi}^0_1}, \qquad m^2_{\tilde{\ell}} = (p_{\tilde{\chi}^0_1} + p_{\ell_1})^2,$$
$$m^2_{\tilde{\chi}^0_2} = (p_{\tilde{\chi}^0_1} + p_{\ell_1} + p_{\ell_2})^2,$$
$$m^2_{\tilde{b}} = (p_{\tilde{\chi}^0_1} + p_{\ell_1} + p_{\ell_2} + p_{b_1})^2,$$
$$m^2_{\tilde{g}} = (p_{\tilde{\chi}^0_1} + p_{\ell_1} + p_{\ell_2} + p_{b_1} + p_{b_2})^2.$$

which contain 4 unknown d.o.f of LSP momentum♪

→ Each event describes a 4-dim. hypersurface ♪
in 5-dim. mass space, and the hypersurfcae♪
differs event by event♪

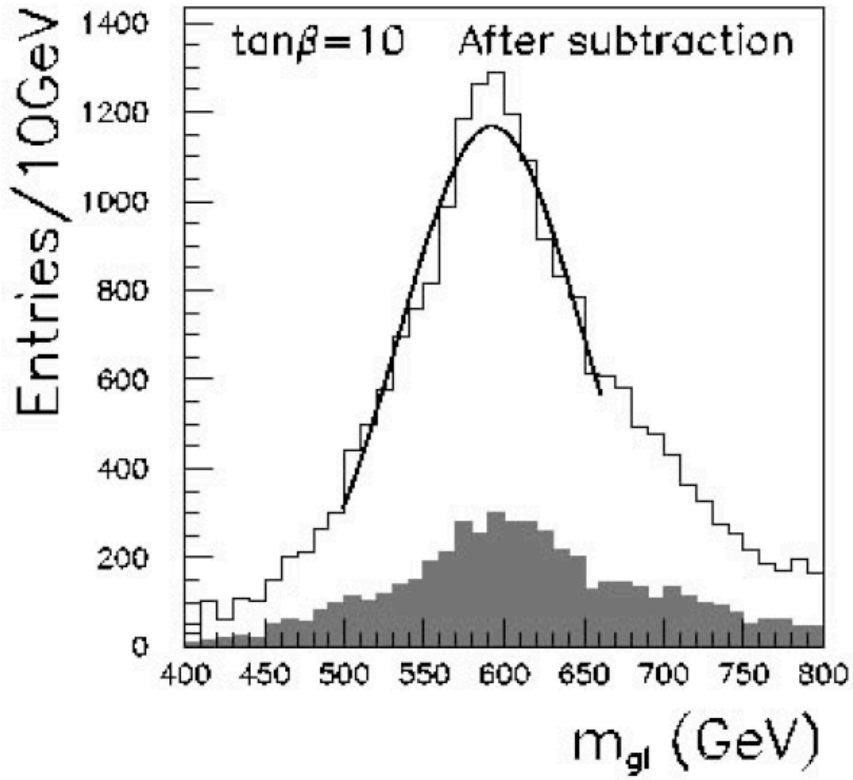
♪

→ Many events determine a solution for masses♪
through intersections of hypersurfaces♪

▪ Measurements of gluino and sbottom masses ♪
(assuming that the masses of two neutralinos and ♪
slepton are already known) in SPS 1a point♪

♪

Kawagoe, Nojiri, Polesello (2004)♪



In this case, each event corresponds to ♪
a different line in $(m_{\tilde{g}}, m_{\tilde{b}})$ plane ♪

Two events are enough to solve the ♪
gluino and sbottom masses altogether♪

Build all possible event pairs ♪
(with some conditions) ♪

m_gluino ~ 592 GeV♪

(300 fb^{-1})♪

Gluino mass distribution with event pair analysis♪

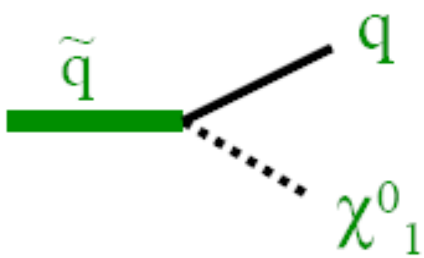
✓ Both of the Edge method and the Mass relation method ♪
rely on a long decay chain to determine sparticle masses ♪

♪

♪

✓ What if we don't have long enough decay chain♪
but only short one ?♪

♪
♪
♪

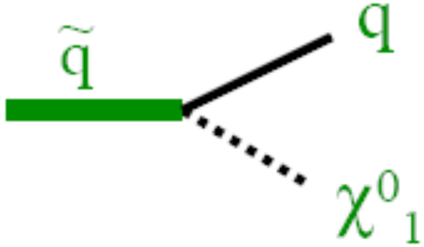


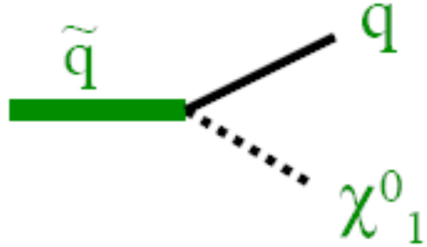
♪
♪
♪

✓ In such case, M_{T2} variable would be useful♪
to get information on sparticle masses♪

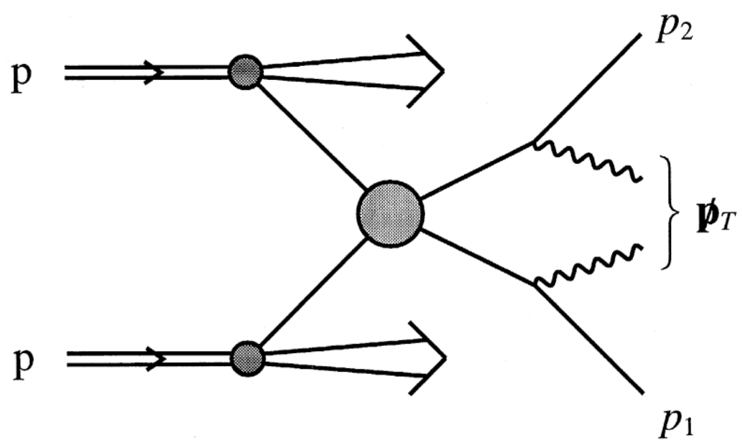
Cambridge m_{T2} variable♪
(Stransverse Mass)♪

Lester, Summers (1999)♪
Barr, Lester, Stephens (2003)♪





● Cambridge m_{T2}♪ (Lester and Summers, 1999)♪



Massive particles pair produced♪
♪
Each decays to one visible ♪
and one invisible particle.♪

For example,♪

$$pp \to X + \tilde{l}_R^+ \tilde{l}_R^- \to X + l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$$

For the decay, $\tilde{l} \to l \tilde{\chi}$

(♪where $E_T = \sqrt{\boldsymbol{p}_T^2 + m^2}$)♪

$$m_{\tilde{l}}^2 \geq m_T^2(\boldsymbol{p}_{Tl}, \boldsymbol{p}_{T\tilde{\chi}})$$

$$\equiv m_l^2 + m_{\tilde{\chi}}^2 + 2\left(E_{Tl} E_{T\tilde{\chi}} - \boldsymbol{p}_{Tl} \cdot \boldsymbol{p}_{T\tilde{\chi}}\right)$$

If $\boldsymbol{p}_{T\tilde{\chi}_a}$ and $\boldsymbol{p}_{T\tilde{\chi}_b}$ were obtainable,

$$m_{\tilde{l}}^2 \geq \max\left\{m_T^2(\boldsymbol{p}_{Tl^-}, \boldsymbol{p}_{T\tilde{\chi}_a}), m_T^2(\boldsymbol{p}_{Tl^+}, \boldsymbol{p}_{T\tilde{\chi}_b})\right\}$$

($\not{\boldsymbol{p}}_T = \boldsymbol{p}_{T\tilde{\chi}_a} + \boldsymbol{p}_{T\tilde{\chi}_b}$: total MET vector in the event)

However, not knowing the form of the MET vector splitting,
the best we can say is that :

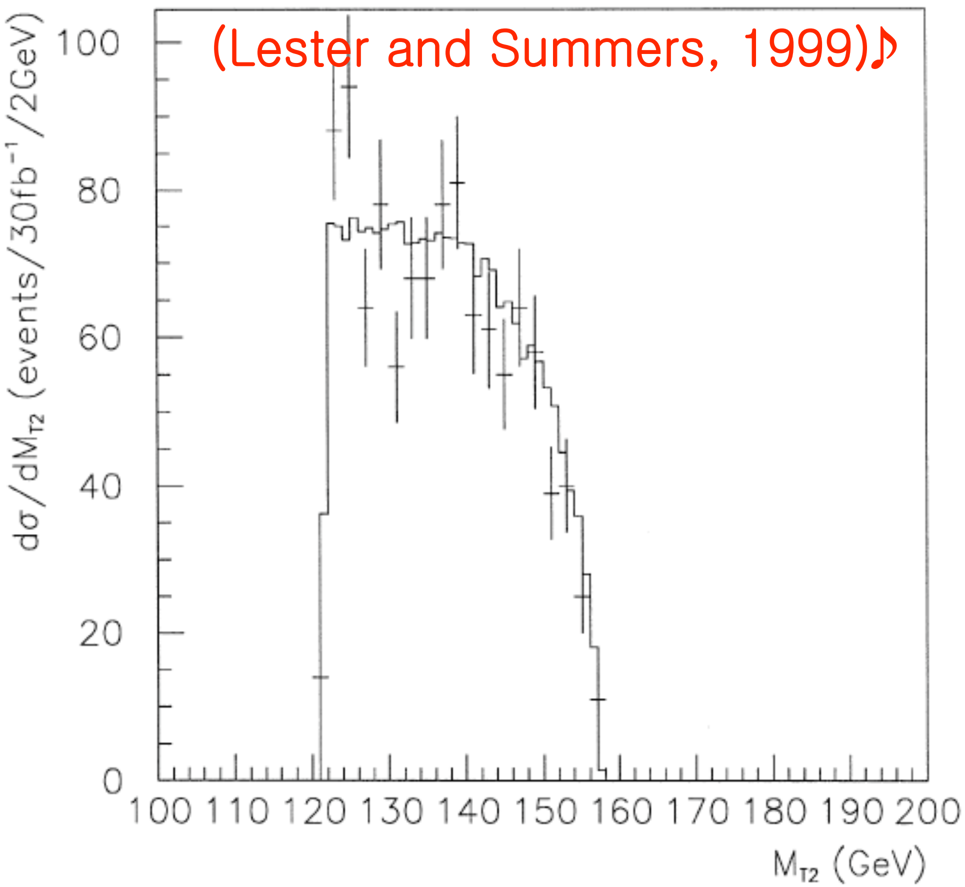
$$m_{\tilde{l}}^2 \geq M_{T2}^2 \equiv \min_{\not{\boldsymbol{p}}_1 + \not{\boldsymbol{p}}_2 = \not{\boldsymbol{p}}_T} \left[\max\left\{m_T^2(\boldsymbol{p}_{Tl^-}, \not{\boldsymbol{p}}_1), m_T^2(\boldsymbol{p}_{Tl^+}, \not{\boldsymbol{p}}_2)\right\}\right]$$

with minimization over all possible trial LSP momenta

❖ M_{T2} distribution for $pp \to X + \tilde{l}_R^+ \tilde{l}_R^- \to X + l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$

LHC point 5, with 30 fb^{-1},

$$m_{\tilde{l}_R} = 157.1\,\text{GeV}\,, \quad m_{\tilde{\chi}_1^0} = 121.5\,\text{GeV}\,.$$



Endpoint measurement of m_{T2} distribution determines the mother particle mass

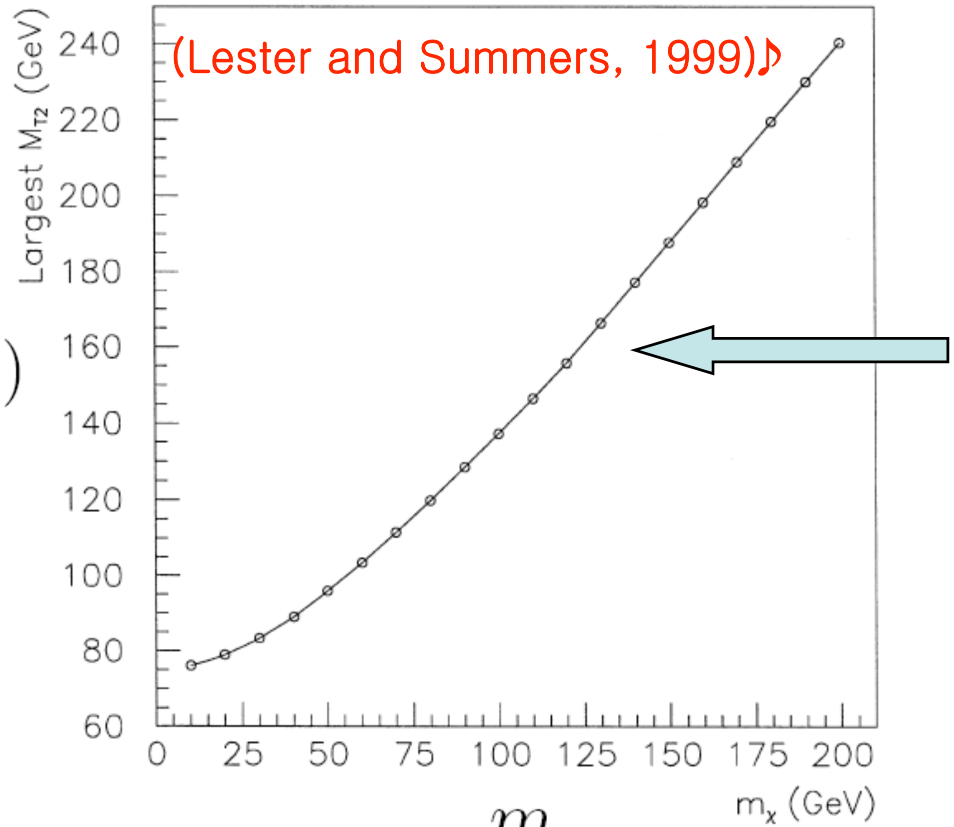
$$m_{T2}^{\max} \simeq 157\ \text{GeV}$$

(with $m_{\tilde{\chi}_1^0} = 121.5\ \text{GeV}$)

The LSP mass is needed as an input for m_{T2} calculation
But it might not be known in advance

m_{T2} depends on a trial LSP mass m_χ
Maximum of m_{T2} as a function of the trial LSP mass

$m_{T2}^{\max}(m_\chi)$



m_χ

The correlation from a numerical calculation can be expressed by an analytic formula in terms of true sparticle masses

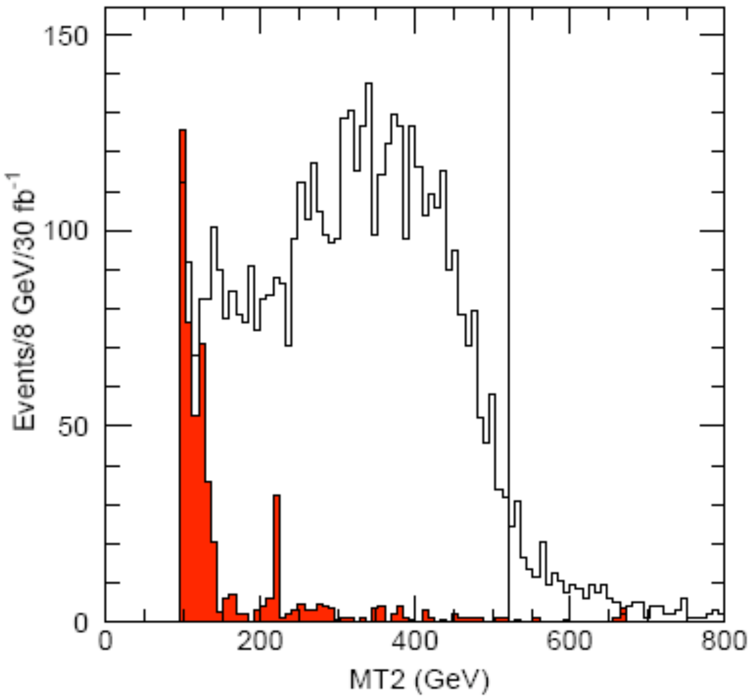
- Right handed squark mass from the m_{T2}

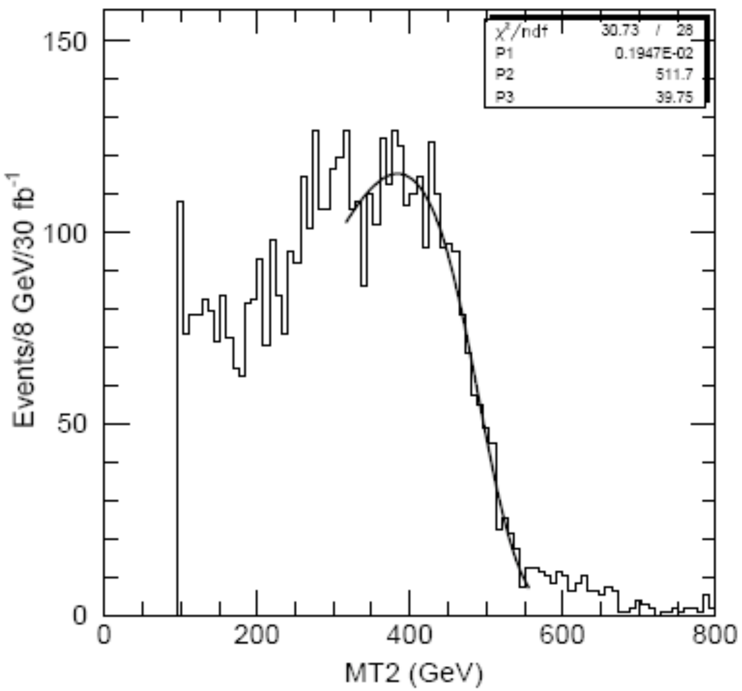
$$\tilde{q}_R\ \tilde{q}_R \rightarrow q\ \tilde{\chi}_1^0\ q\ \tilde{\chi}_1^0$$

$$BR(\tilde{q}_R \rightarrow q\chi_1^0) \sim 100\%$$

m_qR ~ 520 GeV, mLSP ~96 GeV

SPS1a point, with 30 fb^{-1}





(LHC/ILC Study Group: hep-ph/0410364)

➢ Unconstrained minimum of m_T

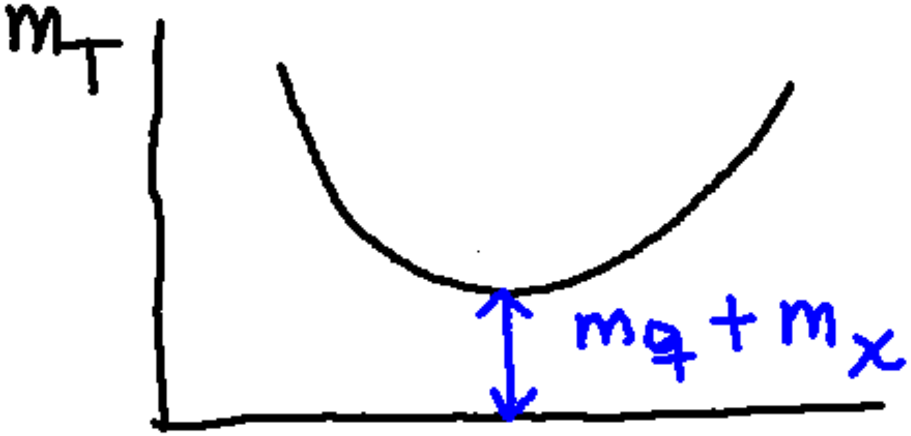
Barr, Lester, Stephens (2003)

$$m_T^2 = m_q^2 + m_\chi^2 + 2(E_T^q E_T^\chi - \mathbf{p}_T^q \cdot \mathbf{p}_T^\chi)$$

$$\frac{\partial m_T^2}{\partial(\mathbf{p}_T^\chi)_k} = 2\left[E_T^q \frac{(\mathbf{p}_T^\chi)_k}{E_T^\chi} - (\mathbf{p}_T^q)_k\right] \qquad (k = 1, 2)$$

We have a global minimum of the transverse mass when $\frac{\mathbf{p}_T^\chi}{E_T^\chi} = \frac{\mathbf{p}_T^q}{E_T^q}$

$$m_T(\min) = m_q + m_\chi$$

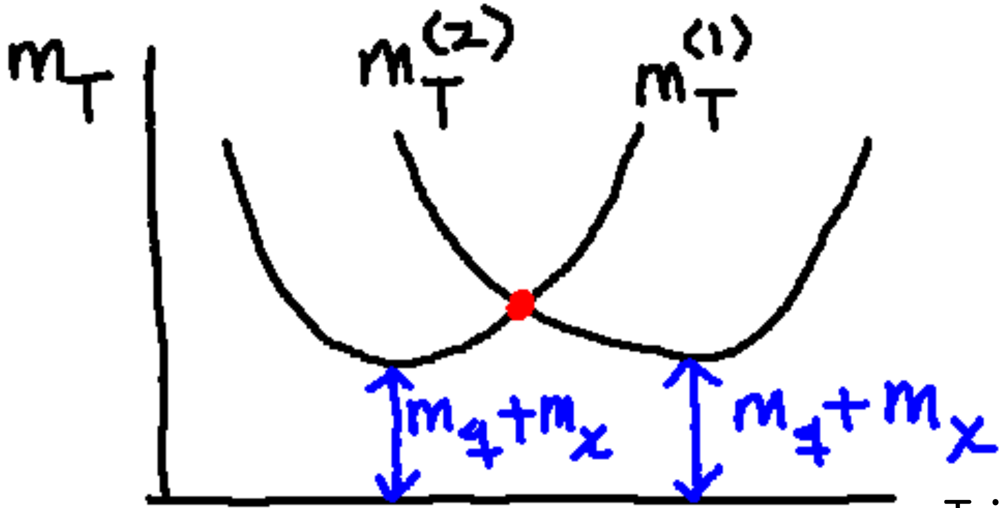


Trial LSP momentum

➢ Solution of m_{T2} (the balanced solution)

$$m_{T2}^2 \equiv \min_{\mathbf{p}_T^{\chi(1)}+\mathbf{p}_T^{\chi(2)}=\mathbf{p}_T^{miss}} \left[\max\{ m_T^2(\mathbf{p}_T^{q(1)}, \mathbf{p}_T^{\chi(1)}), m_T^2(\mathbf{p}_T^{q(2)}, \mathbf{p}_T^{\chi(2)}) \} \right]$$

with $\mathbf{p}_T^{\chi(1)} + \mathbf{p}_T^{\chi(2)} = \mathbf{p}_T^{miss} = -(\mathbf{p}_T^{q(1)} + \mathbf{p}_T^{q(2)})$ (for no ISR)



Trial LSP momenta

m_{T2} : the minimum of $m_T^{(1)}$ subject to the two constraints

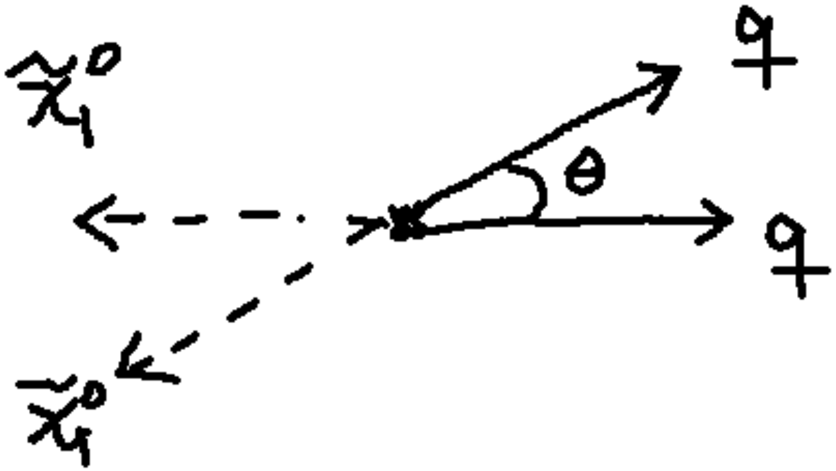
$m_T^{(1)} = m_T^{(2)}$, and $p_T^{\chi(1)} + p_T^{\chi(2)} = p_T^{miss}$

➢ The balanced solution of squark m_{T2} in terms of visible momenta

(Lester and Barr 0708.1028)

$$m_{T2} = P_0 + \sqrt{P_0^2 + m_\chi^2} \qquad (m_q = 0)$$

with $$P_0 = \sqrt{\frac{(1+\cos\theta)}{2}|\mathbf{p}_T^{q(1)}||\mathbf{p}_T^{q(2)}|}$$



- In order to get the expression for m_{T2}^{max} ,

We can only consider the case where
two mother particles are at rest and all decay products
are on the transverse plane w.r.t proton beam direction,
for no ISR

(Cho, Choi, YGK and Park, 2007)

- In the rest frame of squark, the quark momenta

$$\left|\mathbf{p}_T^{q(i)}\right| = \frac{m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{q}}}$$

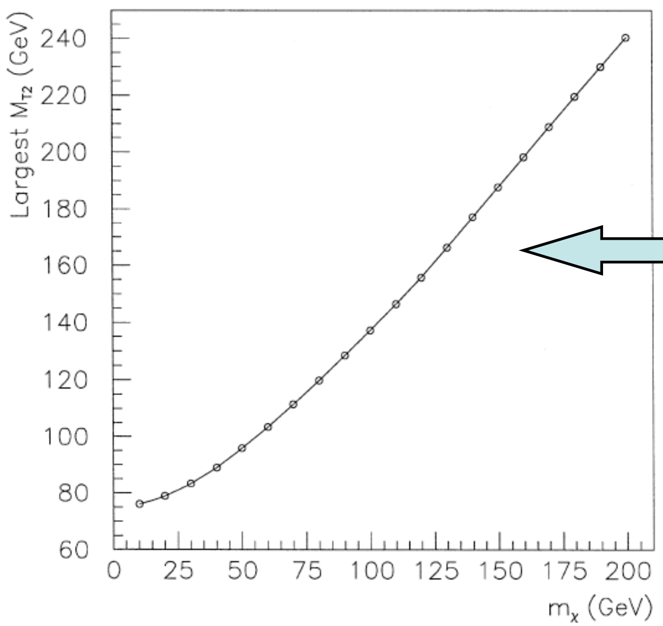
if both quark momenta are along the direction of the transverse plane

The maximum of the squark m_{T2} (occurs at $\theta = 0$)♪

(Cho, Choi, YGK and Park, 0709.0288)♪

$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{q}}} + \sqrt{\left(\frac{m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{q}}}\right)^2 + m_\chi^2}$$

- ♪ $m_{T2}^{\max}(m_\chi) = m_{\tilde{q}} \quad \text{if } m_\chi = m_{\tilde{\chi}_1^0}$

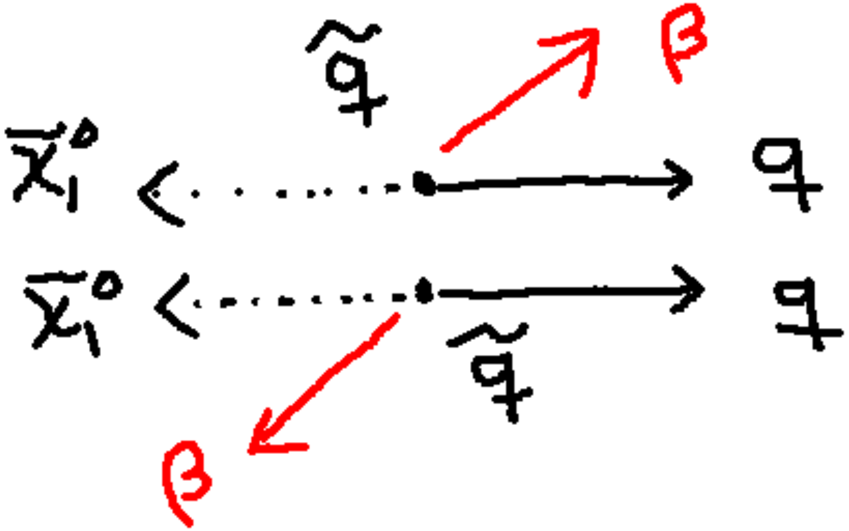


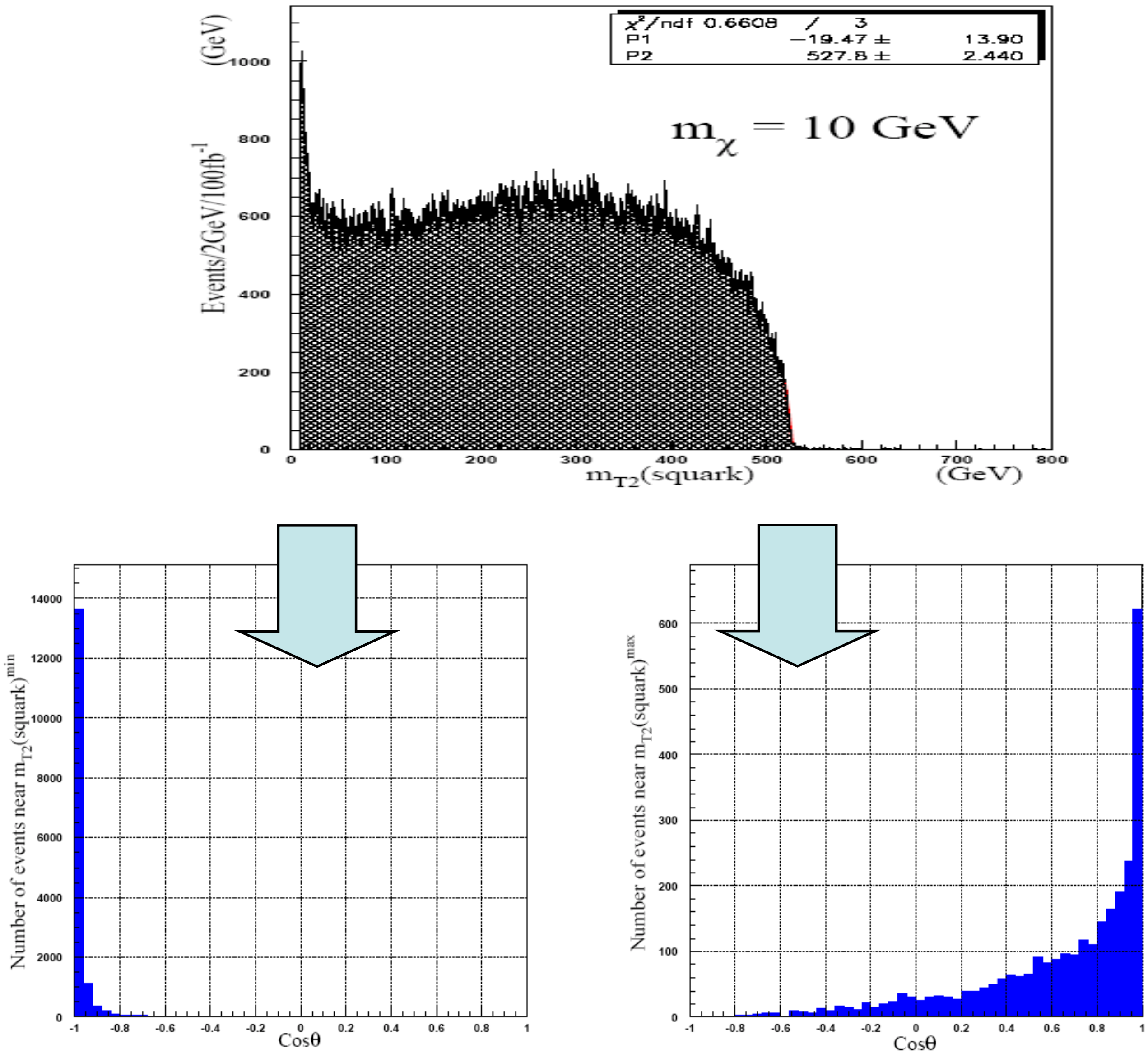
Well described by the above ♪
Analytic expression with true♪
Squark mass and true LSP mass♪

✓ Squark and LSP masses are♪
Not determined separately♪

Some remarks on the effect of squark boost♪

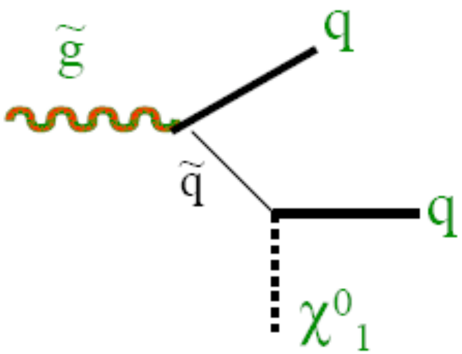
In general, squarks are produced with non-zero p_T♪
♪
The m_{T2} solution is invariant under ♪
back-to-back transverse boost of mother squarks♪
(all visible momenta are on the transverse plane)♪

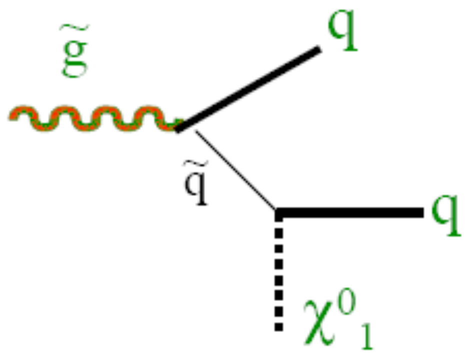




Cos(theta) distribution ♪

‘Gluino’ m_{T2} variable♪





In collaboration with♪

W.S.Cho, K.Choi, C.B.Park ♪

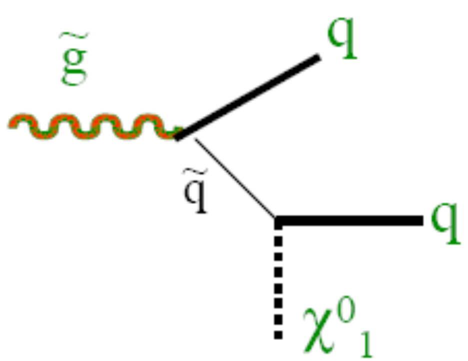
Ref) arXiv:0709.0288, arXiv:0711.4526 ♪

• Gluino m_{T2} (stransverse mass)♪

A new observable, which is an application of m_{T2} variable to♪
the process ♪

$$pp \rightarrow \tilde{g}\tilde{g} \rightarrow qq\tilde{\chi}_1^0 \; qq\tilde{\chi}_1^0$$

Gluinos are pair produced in proton-proton collision♪
♪
Each gluino decays into two quarks and one LSP ♪



through three body decay (off-shell squark)♪

or two body cascade decay (on-shell squark) ♪

- For each gluino decay, ♪
  the following transverse mass can be constructed♪

$$m_T^2(m_{qqT}, m_\chi, \mathbf{p}_T^{qq}, \mathbf{p}_T^\chi) = m_{qqT}^2 + m_\chi^2 + 2(E_T^{qq} E_T^\chi - \mathbf{p}_T^{qq} \cdot \mathbf{p}_T^\chi)$$

m_{qqT} and $\mathbf{p}_T^{qq}$: mass and transverse momentum of qq system♪

m_χ and $\mathbf{p}_T^\chi$: trial mass and transverse momentum of the LSP♪

$$E_T^{qq} \equiv \sqrt{|\mathbf{p}_T^{qq}|^2 + m_{qqT}^2} \quad \text{and} \quad E_T^\chi \equiv \sqrt{|\mathbf{p}_T^\chi|^2 + m_\chi^2}$$

- With two such gluino decays in each event,♪
  the gluino m_{T2} is defined as ♪

$$m_{T2}^2(\tilde{g}) \equiv \min_{\mathbf{p}_T^{\chi(1)} + \mathbf{p}_T^{\chi(2)} = \mathbf{p}_T^{miss}} \left[\max\{m_T^{2(1)}, m_T^{2(2)}\} \right]$$

(minimization over all possible trial LSP momenta)♪

- From the definition of the gluino m_{T2}♪

$$m_{T2}(\tilde{g}) \leq m_{\tilde{g}} \quad \text{for} \quad m_{\chi} = m_{\tilde{\chi}_1^0}$$

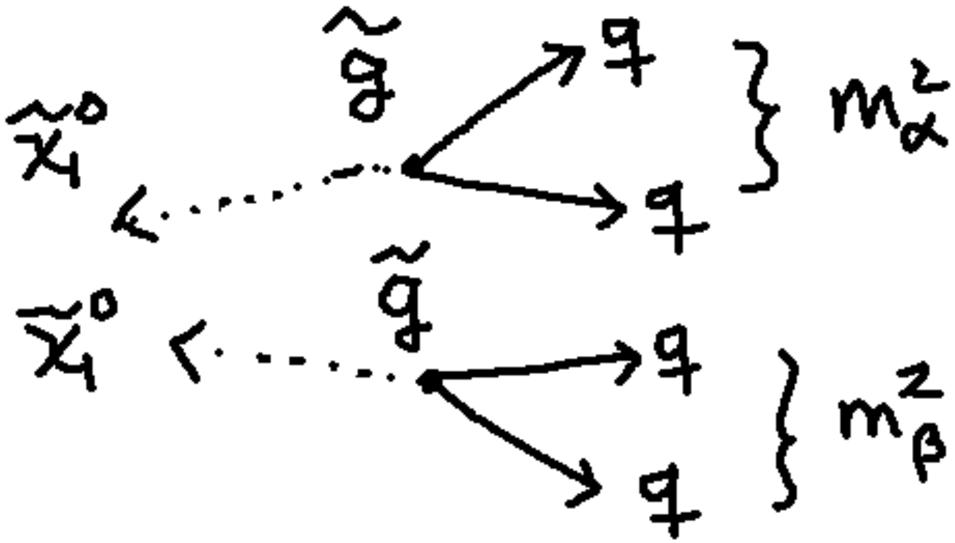
Therefore, if the LSP mass is known, one can determine♪
the gluino mass from the endpoint measurement of the gluino♪
m_{T2} distribution. ♪

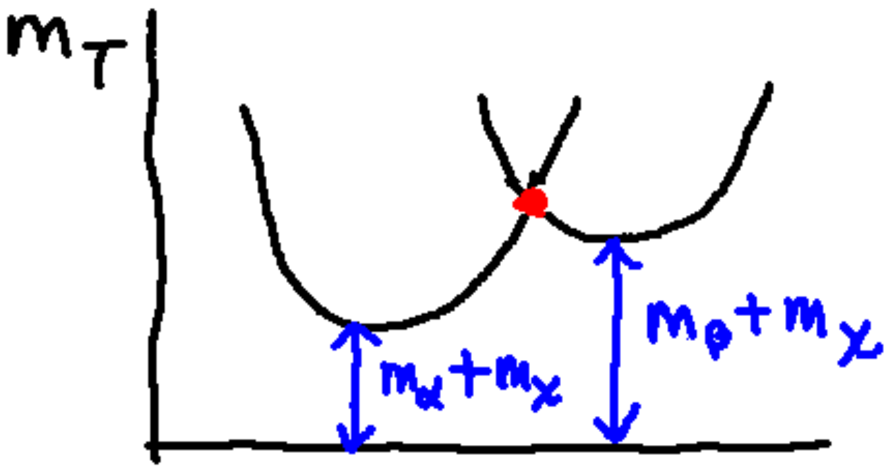
$$m_{T2}^{\max}(m_{\chi}) \equiv \max_{\text{all events}} \left[m_{T2}(\tilde{g}) \right]$$

- However, the LSP mass might not be known in advance♪
and then, $m_{T2}^{\max}(m_{\chi})$ can be considered as a function of♪
the trial LSP mass m_{χ} , satisfying ♪

$$m_{T2}^{\max}(m_{\chi} = m_{\tilde{\chi}_1^0}) = m_{\tilde{g}}$$

Each mother particle♪
produces♪
one invisible LSP♪
and more than one ♪
visible particles♪

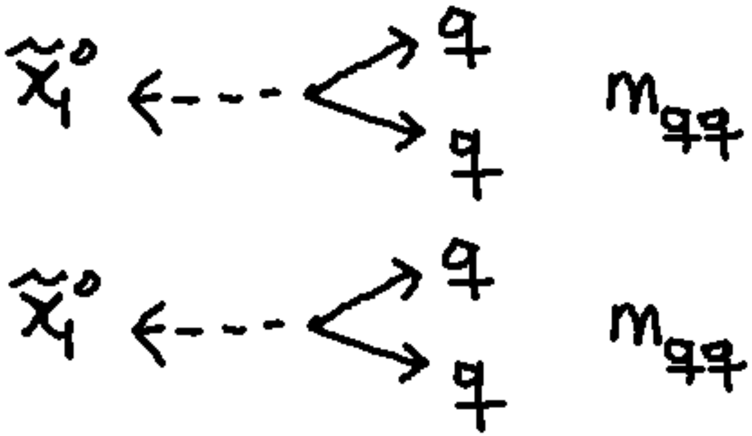




Possible m_{qq} values♪
for three body decays ♪
of gluino :♪

$$0 \leq m_{qq} \leq m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$$

Case : two di-quark invariant masses are equal to each other♪



(♪$0 \le m_{qq} \le m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$)♪

In the frame of gluinos at rest, the di-quark momentum is ♪

$$|\mathbf{p}| = \frac{\sqrt{[m_{\tilde{g}}^2 - (m_{\tilde{\chi}_1^0} + m_{qq})^2][m_{\tilde{g}}^2 - (m_{\tilde{\chi}_1^0} - m_{qq})^2]}}{2m_{\tilde{g}}}$$

Gluino m_{T2}♪ (Two sets of decay products are parallel to each other)♪

$$m_{T2} = \sqrt{m_{qq}^2 + |\mathbf{p}|^2} + \sqrt{m_{\chi}^2 + |\mathbf{p}|^2}$$

- The gluino m_{T2} has a very interesting property♪

$$m_{T2} = \sqrt{m_{qq}^2 + |\mathbf{p}|^2} + \sqrt{m_\chi^2 + |\mathbf{p}|^2}$$

(♪$0 \leq m_{qq} \leq m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$)♪

$$\frac{\mathrm{d}m_{T2}}{\mathrm{d}m_{qq}} = \frac{m_{qq}}{m_{\tilde{g}}}\left(1 - \frac{(m_{\tilde{g}}^2 + m_{\tilde{\chi}_1^0}^2 - m_{qq}^2)}{\sqrt{(m_{\tilde{g}}^2 + m_{\tilde{\chi}_1^0}^2 - m_{qq}^2)^2 + 4m_{\tilde{g}}^2(m_\chi^2 - m_{\tilde{\chi}_1^0}^2)}}\right)$$

$= 0$ if $m_\chi = m_{\tilde{\chi}_1^0}$ → m_{T2} = m_gluino for all m_{qq}♪

> 0 if $m_\chi > m_{\tilde{\chi}_1^0}$ → The maximum of m_{T2} occurs when $m_{qq} = m_{qq}$ (max)♪

< 0 if $m_\chi < m_{\tilde{\chi}_1^0}$ → The maximum of m_{T2} occurs when $m_{qq} = 0$♪

This result implies that♪

$$m_{T2}^{\max}(m_\chi) = \left(m_{\tilde{g}} - m_{\tilde{\chi}_1^0}\right) + m_\chi \qquad \text{for} \quad m_\chi \geq m_{\tilde{\chi}_1^0}$$

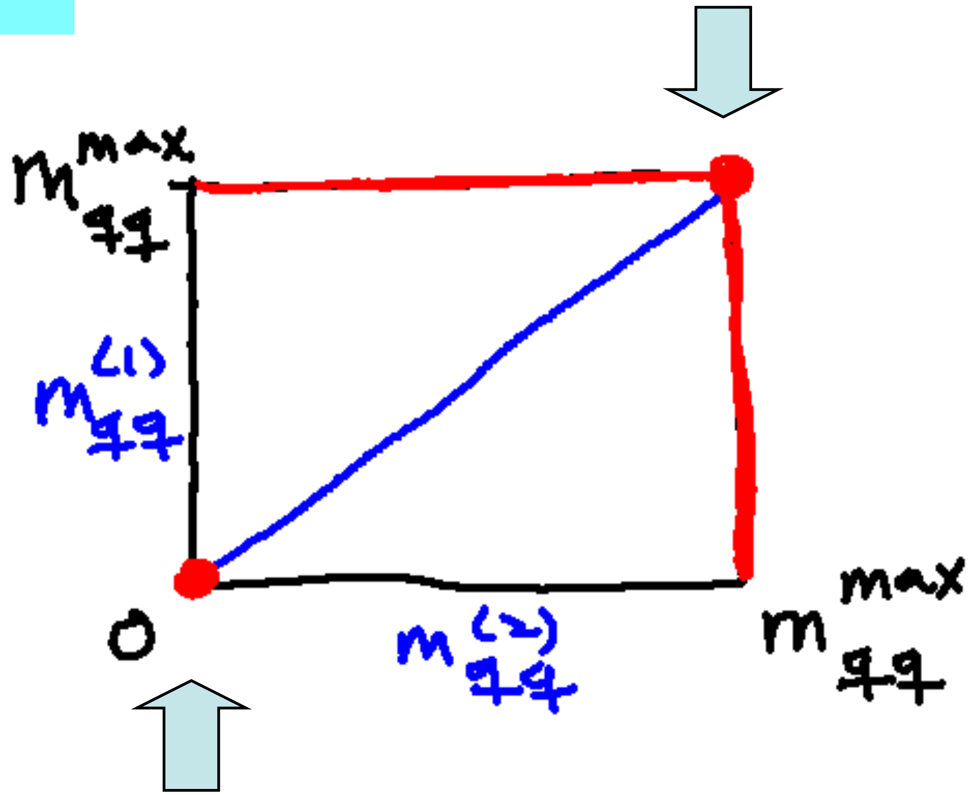
$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2} \qquad \text{for} \quad m_\chi \leq m_{\tilde{\chi}_1^0}.$$

(This conclusion holds also for more general cases where m_{qq1} is different from m_{qq2})♪

$\theta = 0$

$$m_{T2}^{\max}(m_\chi) \;=\; \left(m_{\tilde{g}} - m_{\tilde{\chi}_1^0}\right) + m_\chi$$

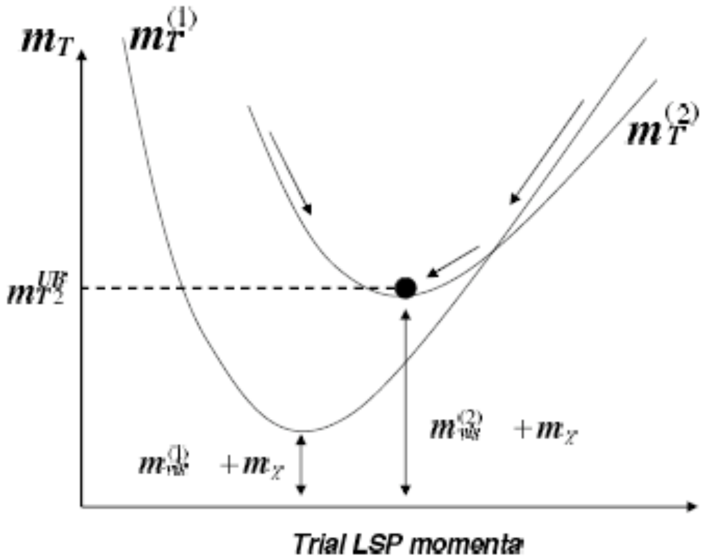
for $\quad m_\chi \geq m_{\tilde{\chi}_1^0}$



$$m_{T2}^{\max}(m_\chi) \;=\; \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2}$$

for $\quad m_\chi \leq m_{\tilde{\chi}_1^0}.$

For the red-line momentum configuration, ♪
Unbalanced Solution of m_{T2} appears♪



for $m_\chi \geq m_{\tilde{\chi}_1^0}$

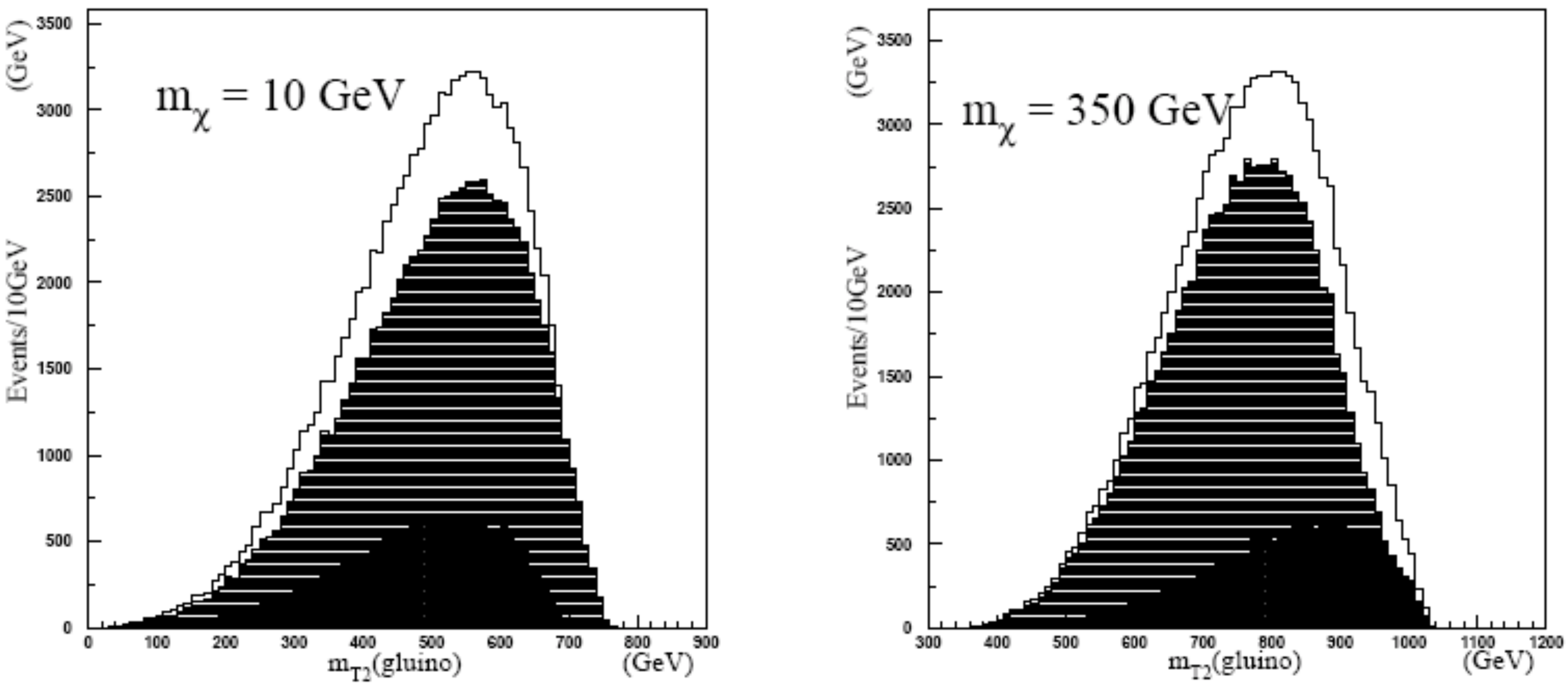
$$m_{vis}^{(1)} = m_{vis}^{max} \quad \& \quad m_{vis}^{(2)} = 0$$

In some momentum configuration , ♪
unconstrained minimum of one $m_T^{(2)}$ is larger than ♪
the corresponding other $m_T^{(1)}$ ♪
Then, m_{T2} is given by the unconstrained minimum of $m_T^{(2)}$♪

$$m_{T2}^{(max)} = m_{qq}^{(max)} + m_\chi$$♪

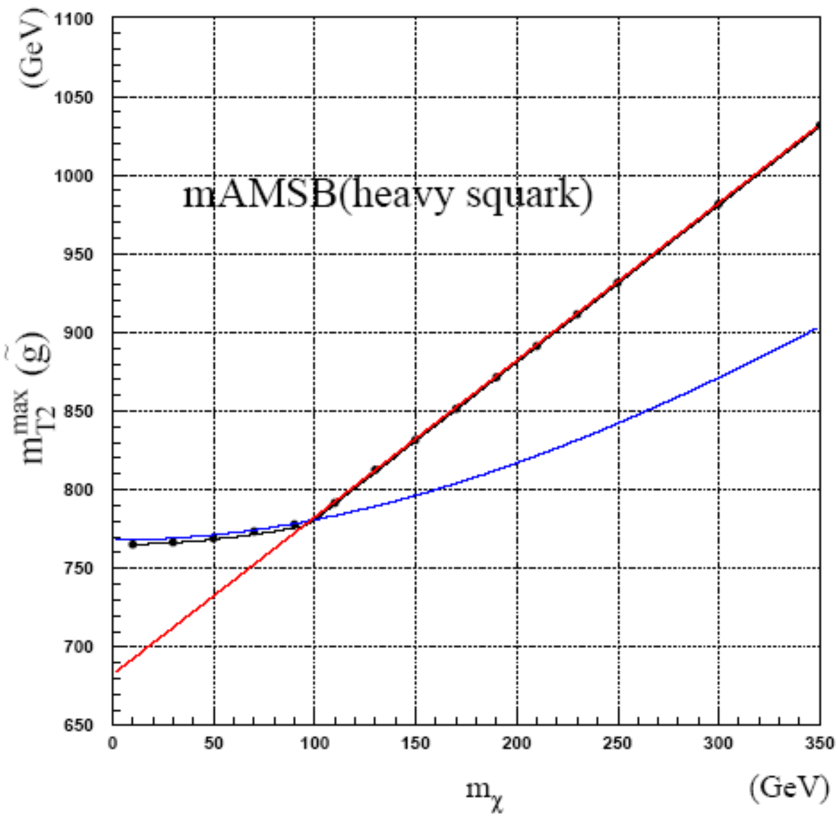
Gluino m_{T2} distributions for a bechmark point♪

True gluino mass = 780 GeV, ♪
True LSP mass = 98 GeV♪



Hatched : balanced mT2, Black : unbalanced mT2♪

❖ If the function $m_{T2}^{\max}(m_\chi)$ can be constructed from experimental data, which identify the crossing point, one will be able to determine the gluino mass and the LSP mass simultaneously.



$$m_{T2}^{\max}(m_\chi) = \left(m_{\tilde{g}} - m_{\tilde{\chi}_1^0}\right) + m_\chi$$

$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2}$$

✓ A numerical example

$$m_{\tilde{g}} = 780.3 \text{ GeV}, \; m_{\tilde{\chi}_1^0} = 97.9 \text{ GeV},$$

and a few TeV masses for sfermions

● Experimental feasibility♪

An example (a point in mAMSB)♪
♪
♪ $$m_{\tilde{g}} = 780.3 \text{ GeV}, \; m_{\tilde{\chi}_1^0} = 97.9 \text{ GeV},$$

with a few TeV sfermion masses ♪
(gluino undergoes three body decay)♪
♪
♪ $\sigma(\tilde{g}\tilde{g}) \sim 1.1\text{pb}$ $\quad B(\tilde{g} \to \tilde{\chi}_1^0 q q) \sim 32\%,$

Wino LSP♪ $\quad B(\tilde{g} \to \tilde{\chi}_1^{\pm} q q') \sim 64\%.$

♪
We have generated a MC sample of SUSY events, ♪
which corresponds to 300 fb^{-1} by PYTHIA♪
♪
The generated events further processed with PGS detector simulation,♪
which approximates an ATLAS or CMS-like detector ♪

❖ Experimental selection cuts ♪

- At least 4 jets with♪ $P_{T1,2,3,4} > 200, 150, 100, 50$ GeV♪

♪

- Missing transverse energy♪ $E_T^{miss} > 250$ GeV

♪

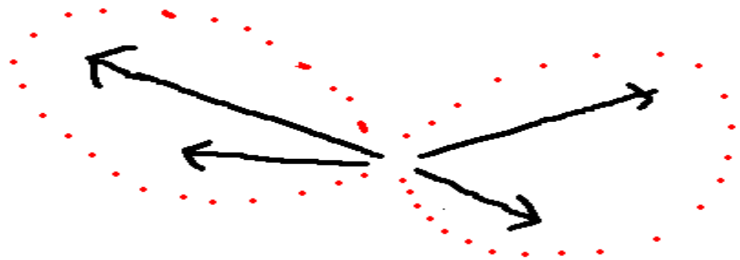
- Transverse sphericity♪ $S_T > 0.25$

♪

- No b-jets and no-leptons♪

♪

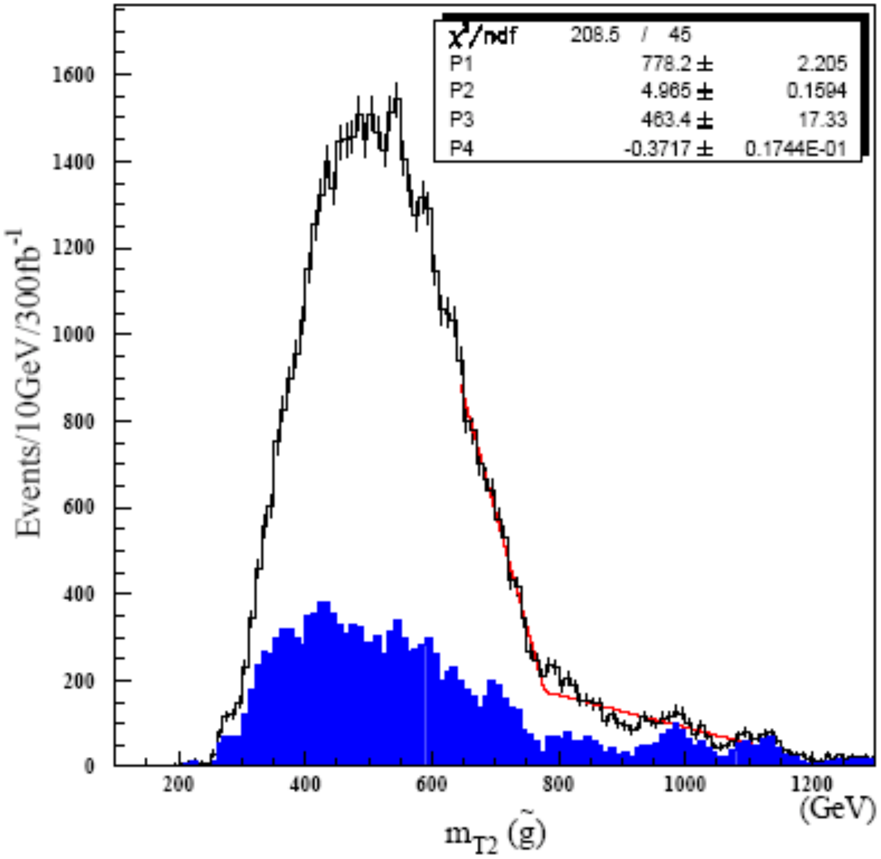
- The four leading jets are divided into two groups of dijets by hemisphere analysis♪



Seeding : The leading jet and the other jet which has ♪
the largest $|p_{jet}|\Delta R$ with respect to the leading jet♪
are chosen as two 'seed' jets for the division♪
♪
Association : Each of the remaining jets is associated to♪
the seed jet making a smaller opening angle♪

If this procedure fail to choose two groups of jet pairs, ♪
We discarded the event♪

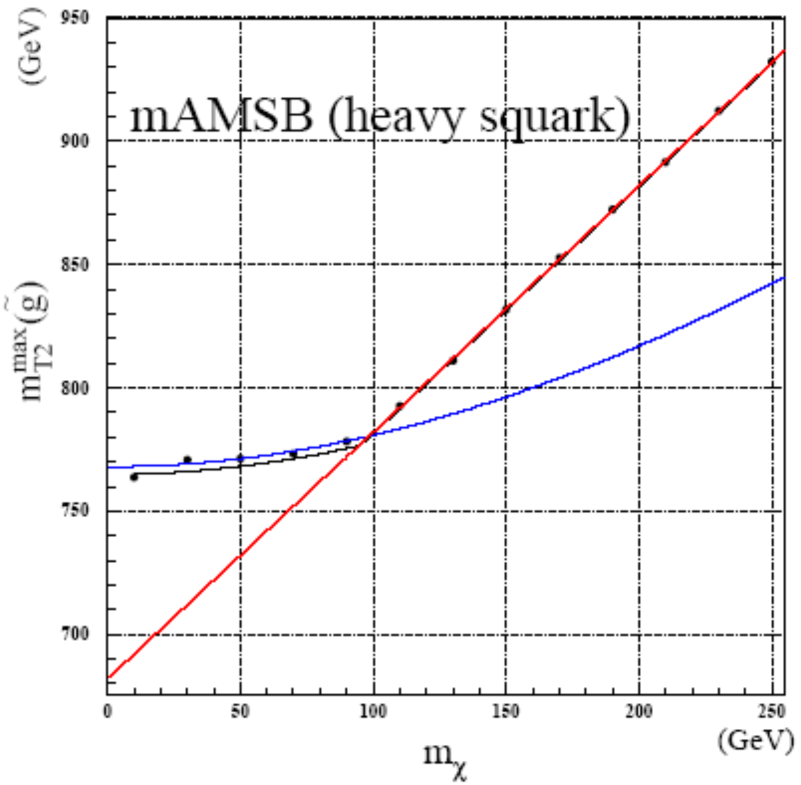
The gluino m_{T2} distribution♪
with the trial LSP mass m_x = 90 GeV ♪



Fitting with a linear function♪
with a linear background,♪
We get the endpoints♪
♪
m_{T2} (max) =♩ 778.2 ± 2.2 GeV

The blue histogram :♪
SM background♪

❖ $m_{T2}^{\max}$ as a function of the trial LSP mass ♪
for the benchmark point♪



$$m_{T2}^{\max}(m_\chi) = \left(m_{\tilde{g}} - m_{\tilde{\chi}_1^0}\right) + m_\chi$$

$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2}$$

Fitting the data points with the above♪
two theoretical curves, we obtain ♪

$$m_{\tilde{g}} = 776.5 \pm 1.0 \text{ GeV}$$

$$m_{\tilde{\chi}_1^0} = 94.9 \pm 1.4 \text{ GeV}$$

The true values are♪

$$m_{\tilde{g}} = 780.3 \text{ GeV}, \ m_{\tilde{\chi}_1^0} = 97.9 \text{ GeV},$$

● Some Remarks ♪

- The above results DO NOT include systematic uncertainties♪
  associated with, for example, fit function, fit range and ♪
  bin size of the histogram etc. to determine the endpoint of♪
  mT2 distribution.♪

♪

♪

- SM backgrounds are generated by PYTHIA. It may ♪
  underestimate the SM backgrounds.♪

- For case of two body cascade decay♪

$$m_{\tilde{q}} < m_{\tilde{g}}, \quad \tilde{g} \to q\tilde{q} \to qq\tilde{\chi}_1^0$$

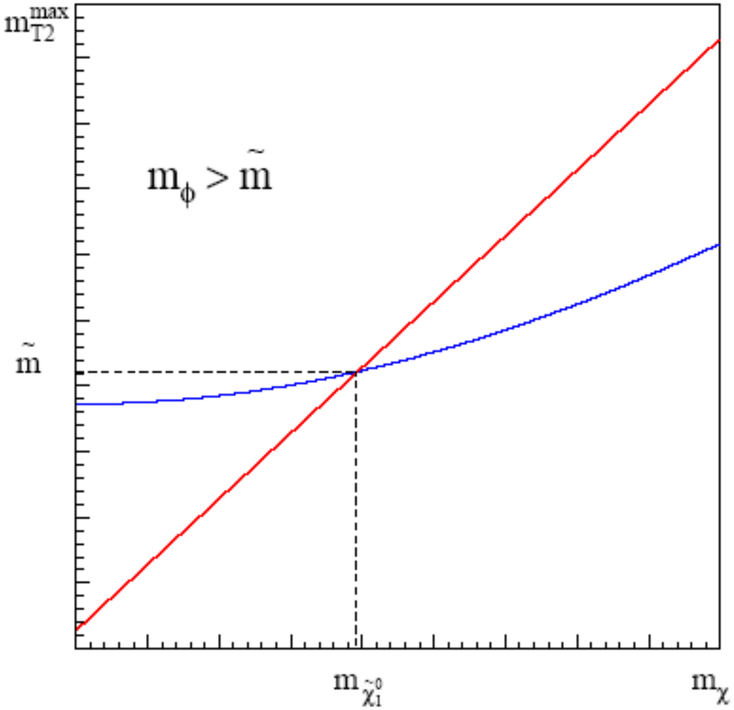
$$0 \leq m_{vis}^{(1)}, m_{vis}^{(2)} \leq \sqrt{\frac{(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)(m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{q}}^2}}.$$

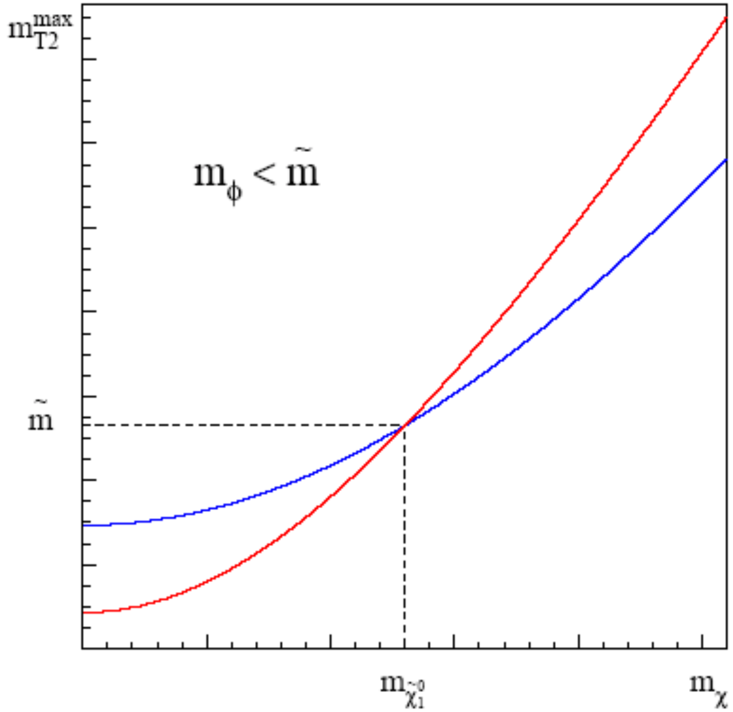
Therefore, . for $m_\chi \geq m_{\tilde{\chi}_1^0}$

$$m_{T2}^{\max} = \left(\frac{m_{\tilde{g}}}{2}(1 - \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}) + \frac{m_{\tilde{g}}}{2}(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{q}}^2})\right)$$

$$+\sqrt{\left(\frac{m_{\tilde{g}}}{2}(1 - \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}) - \frac{m_{\tilde{g}}}{2}(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{q}}^2})\right)^2 + m_\chi^2}.$$

For three body decay♪ For two body cascade decay♪





$$\frac{(d\mathcal{F}^{\max}_{>}/dm_\chi)_{m_\chi = m_{\tilde{\chi}_1^0}}}{(d\mathcal{F}^{\max}_{<}/dm_\chi)_{m_\chi = m_{\tilde{\chi}_1^0}}} = 1 + \frac{(m_{vis}^{\max})^2 - (m_{vis}^{\min})^2}{\tilde{m}^2 + m_{\tilde{\chi}_1^0}^2 - (m_{vis}^{\max})^2} > 1.$$

● 6-quark m_{T2} (I)♪

If m_squark > m_gluino and squark is not decoupled,♪
♪

Squark → quark + gluino (→ q q LSP)♪

→ 3-quarks + LSP ♪

Maximum of the Invariant mass of 3-quarks♪

M_qqq (max) = m_squark – m_LSP ,♪

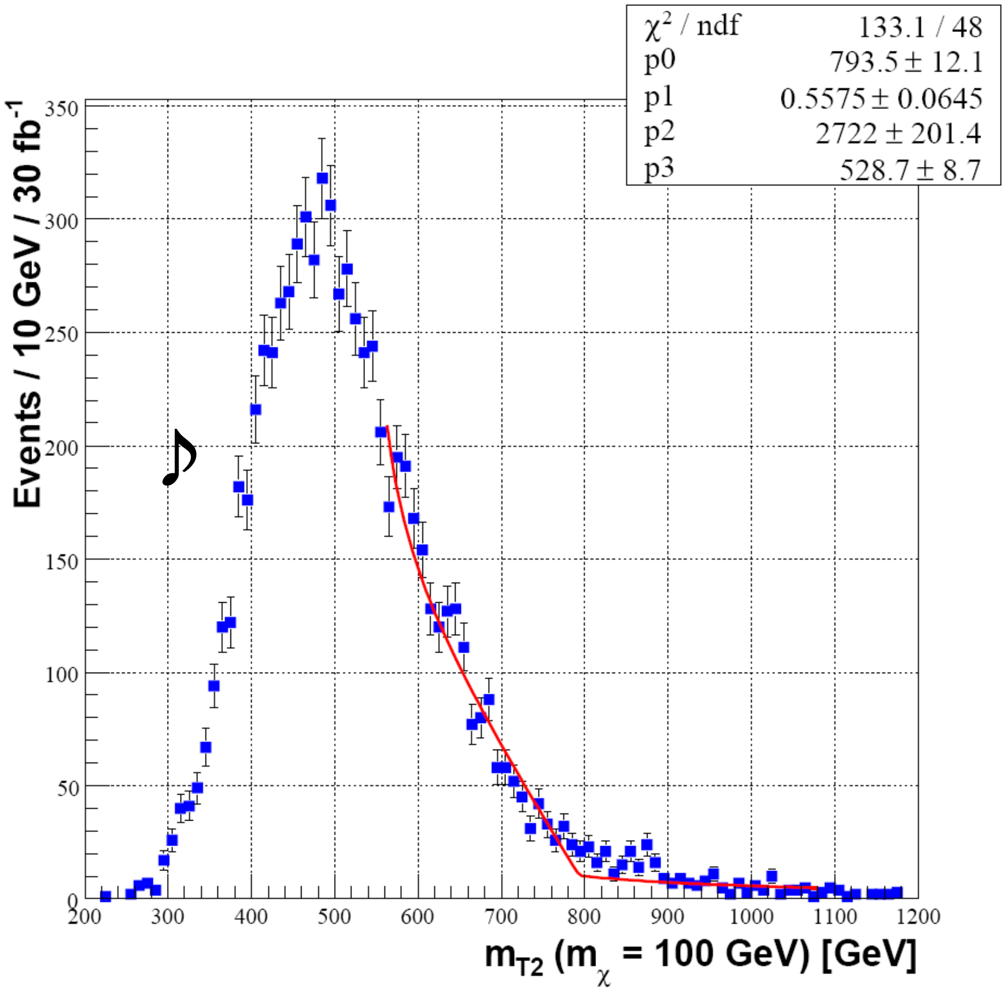
if (m_gluino)^2 > (m_squark * m_LSP)♪

● 6-quark m_{T2} (II)♪

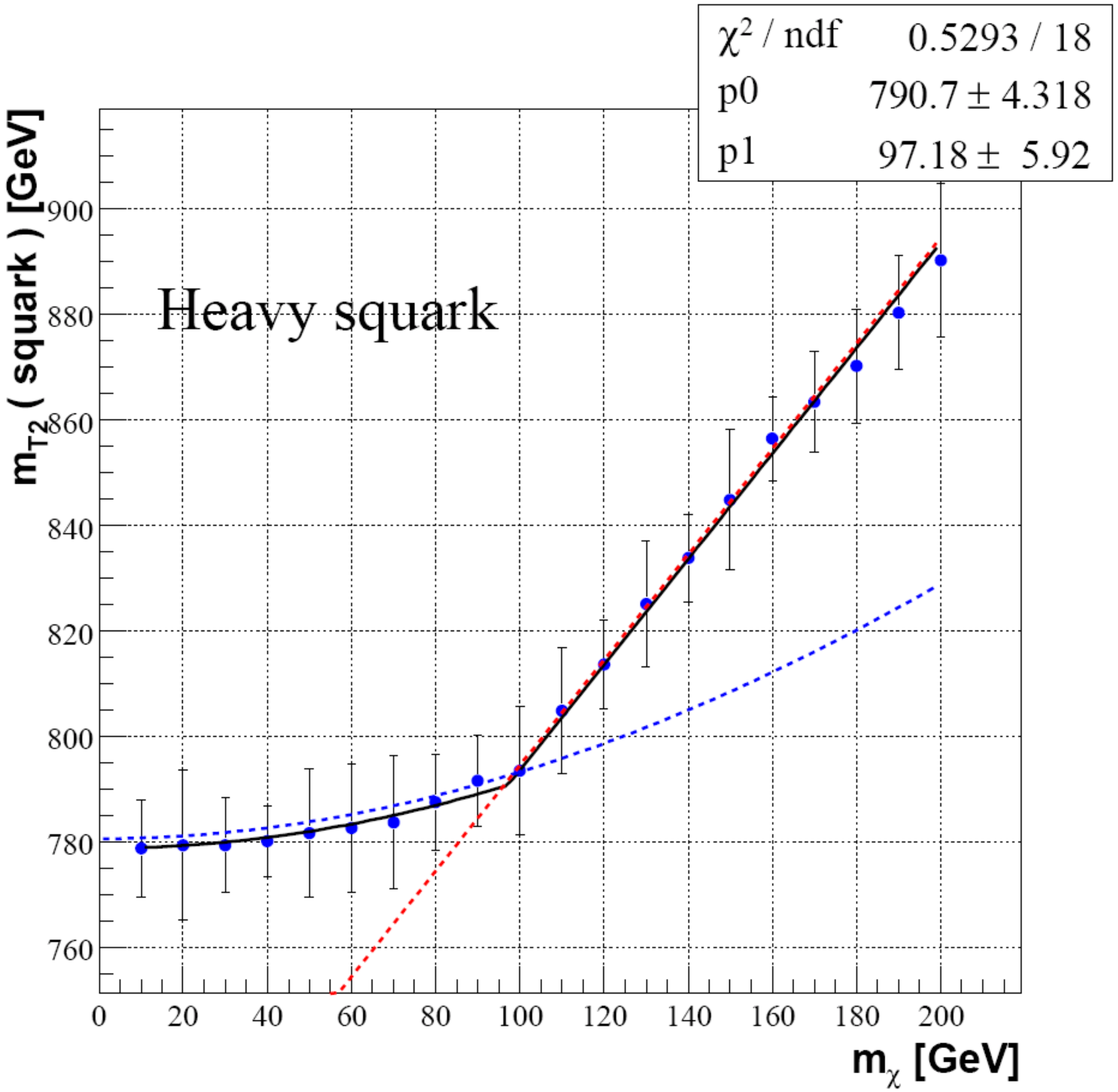
A mSUGRA point, ♪

m_squark ~ 791 GeV, ♪
m_gluino ~ 636 GeV,♪
m_LSP ~ 98 GeV

P_T (7^{th}-jet) <50 GeV♪
Hemishpere analysis♪



● 6-quark m_{T2} (III)♪



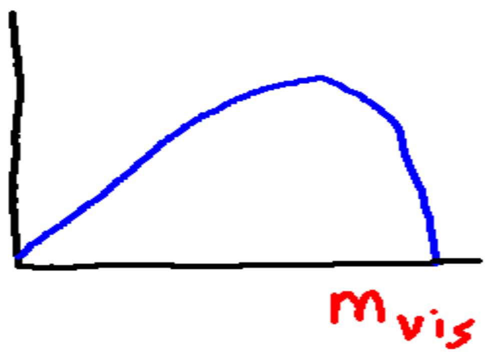
- In principle, we don't have to identify particular chain, if ♪
we can measure visible invariant mass range experimentally.♪

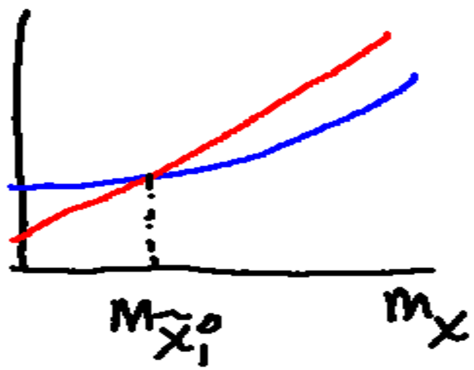
$$m_{T2}^{\max}(m_\chi) = \begin{cases} \mathcal{F}_<^{\max}(m_\chi) = \tilde{\mathcal{F}}(m_{vis} = m_{vis}^{\min}, m_\chi) & \text{if } \; m_\chi < m_{\tilde{\chi}_1^0}, \\ \mathcal{F}_>^{\max}(m_\chi) = \tilde{\mathcal{F}}(m_{vis} = m_{vis}^{\max}, m_\chi) & \text{if } \; m_\chi > m_{\tilde{\chi}_1^0}. \end{cases}$$

where♪

$$\tilde{\mathcal{F}}(m_{vis}, m_\chi) = \frac{\tilde{m}^2 + m_{vis}^2 - m_{\tilde{\chi}_1^0}^2}{2\tilde{m}} + \frac{\left[\left(\tilde{m}^2 - m_{vis}^2 + m_{\tilde{\chi}_1^0}^2\right)^2 + 4\tilde{m}^2\left(m_\chi^2 - m_{\tilde{\chi}_1^0}^2\right)\right]^{1/2}}{2\tilde{m}}$$

If we know minimum and maximum of the visible invariant mass for ♪
mother particle decay, we can use two theoretical curves to identify kin
k position.♪





- M_{TGen} vs. Hemisphere analysis

➢ Barr, Gripaios and Lester (arXiv:0711.4008 [hep-ph])

Instead of jet-paring with hemisphere analysis, we may calculate m_{T2} for all possible divisions of a given event into two sets, and then minimize m_{T2}

- M_{2C} (A Variant of 'gluino' mT2)

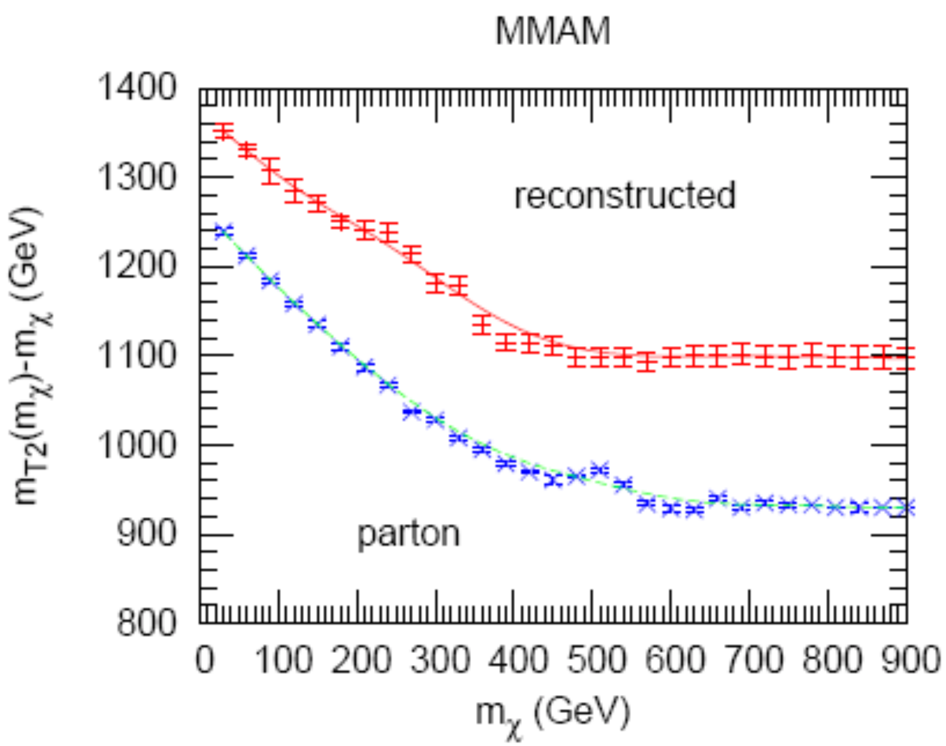
➢ Ross and Serna (arXiv:0712.0943 [hep-ph])

A Variant of 'gluino' m_{T2} with explicit constraint from the endpoint of 'diquark' invariant mass (M_{2C})

Inclusive m_{T2} ♪

➢ Nojiri, Shimizu, Okada and Kawagoe (arXiv:0802.2412)♪
♪

Even without specifying the decay channel, m_{T2} variable♪
still shows a kink structure in some cases.♪



This might help to determine ♪
the sparticle masses at the early♪
stage of the LHC experiment♪

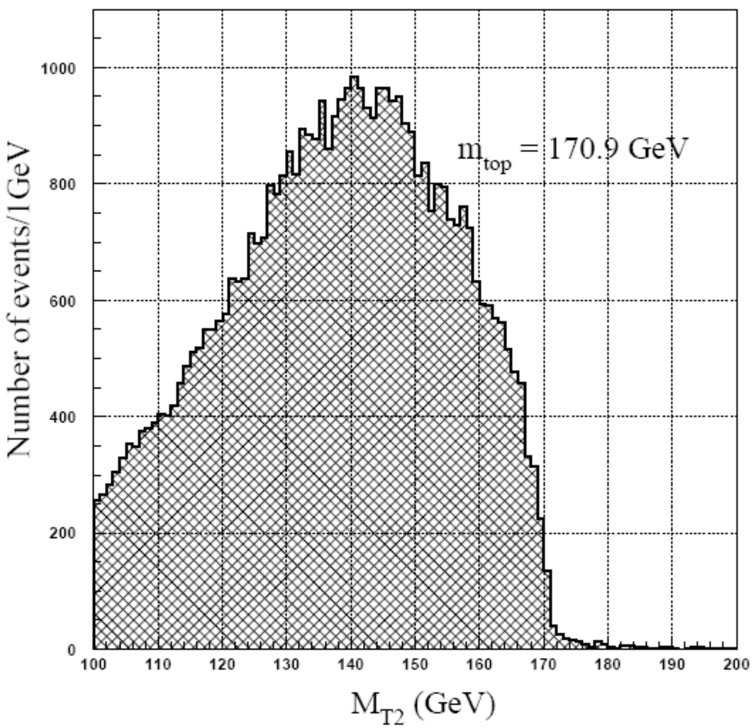
	A: MMAM
	$n_i = 0,\ R = 20,$ $M_3(\text{GUT}) = 650$
$\tilde{g}$	1491
$\tilde{u}_L$	1473
$\tilde{u}_R$	1431
$\tilde{d}_R$	1415
$\tilde{\chi}_1^0$	487

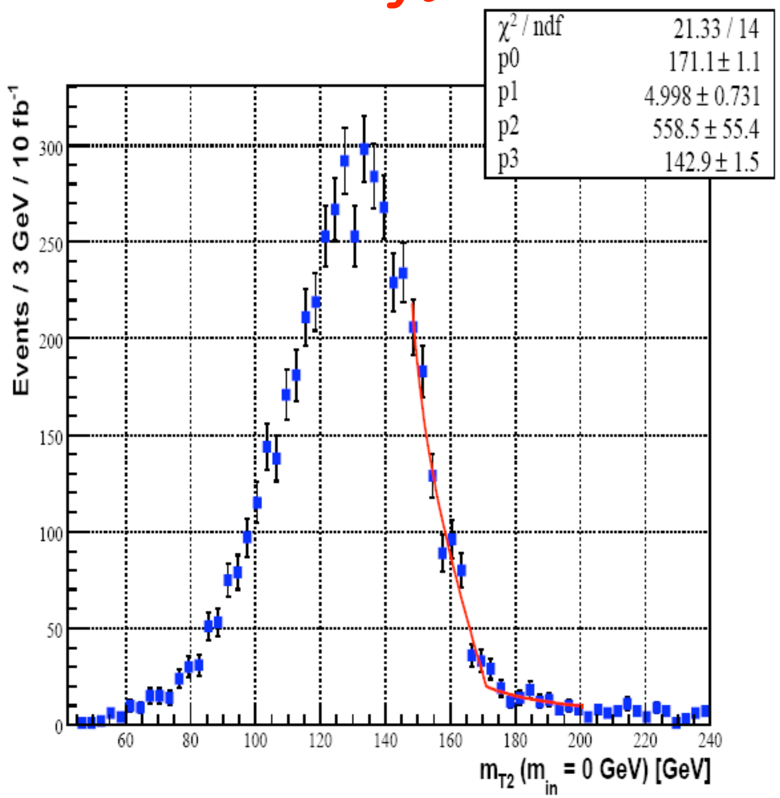
Measuring the top quark mass with m_{T2} at the LHC

(Cho,Choi, YGK, Park, arXiv:0804.2185)

$$t\bar{t} \rightarrow b l^+ \nu \bar{b} l^- \nu$$

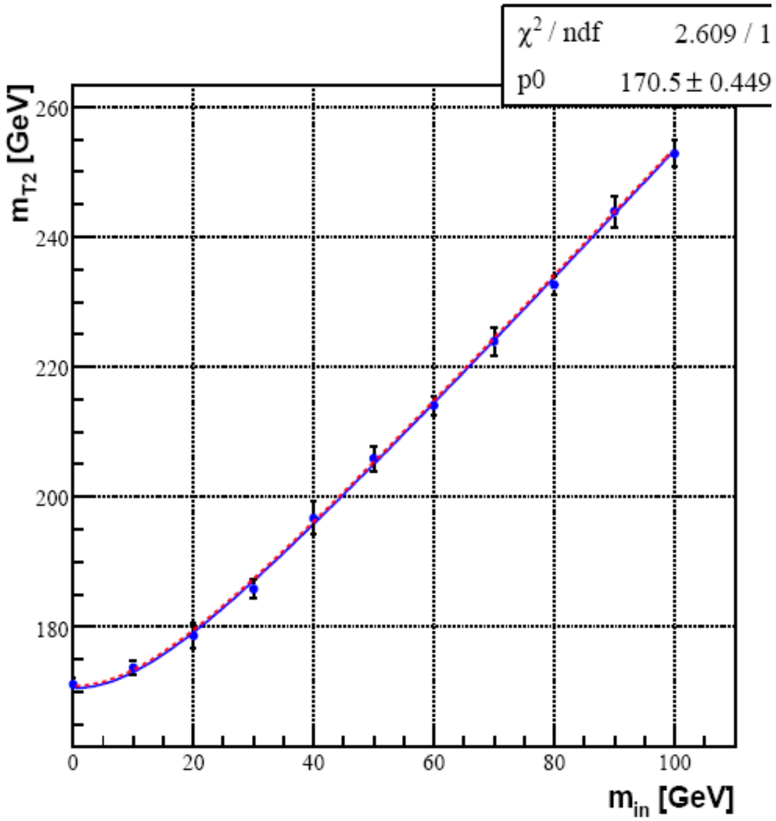
Standard Candle for MT2 study



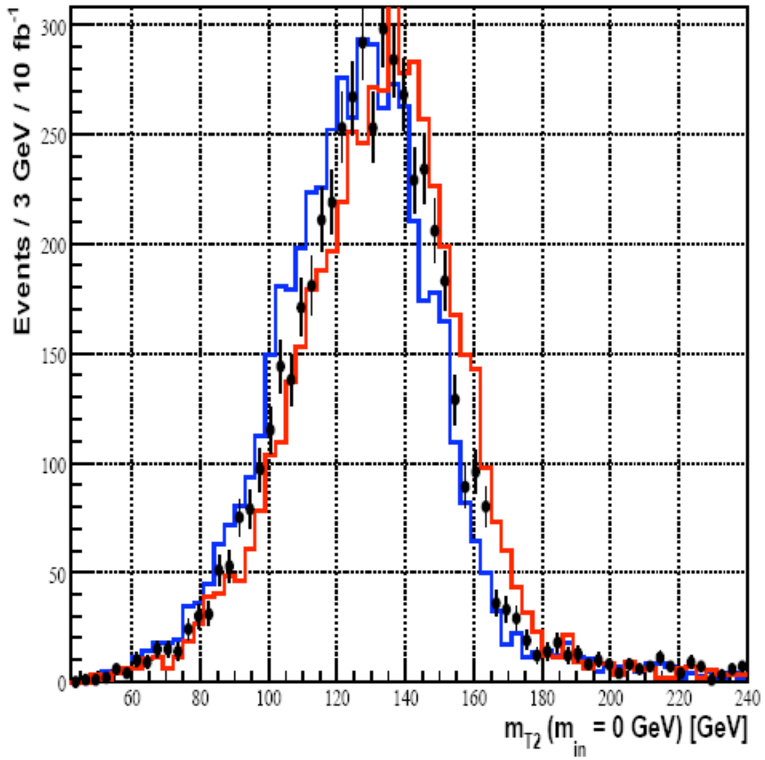


Standard Candle for MT2 study

mT2 max vs. trial neutrino mass



Shape of mT2 distribution



The dileptonic channel will provide a good playground for mT2 excercise

Z polarization in SUSY decays♪

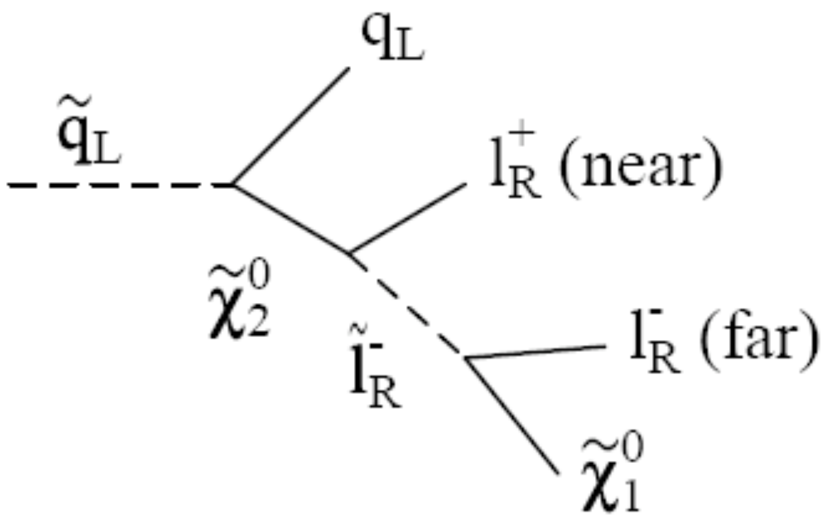
Determining the spin of supersymmetric particles at the LHC using lepton charge asymmetry.

A.J. Barr

Ref. PLB 596 (2004) 205, (hep-ph/0405052)♪

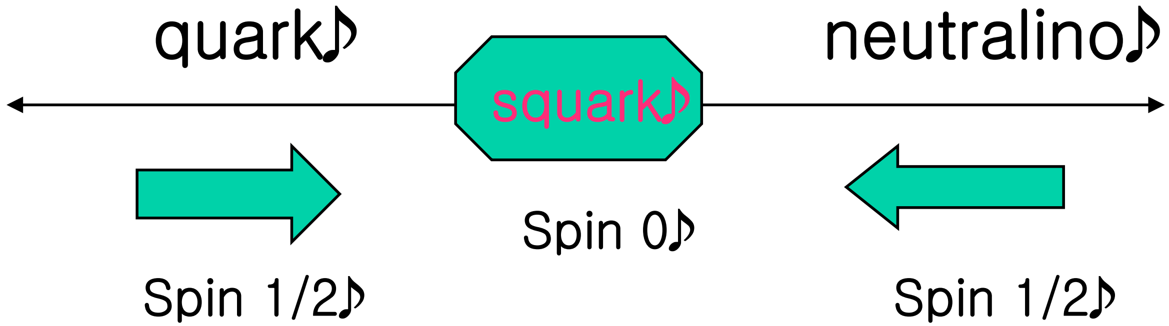
Decay chain under investigation♪

A.J. Barr (2004)♪



Spin correlations can play a significant role ♪
in the kinematics of the emitted particles♪
♪
Consider invariant mass of quark (from the squark) an
d near lepton (from chi_2^0)♪

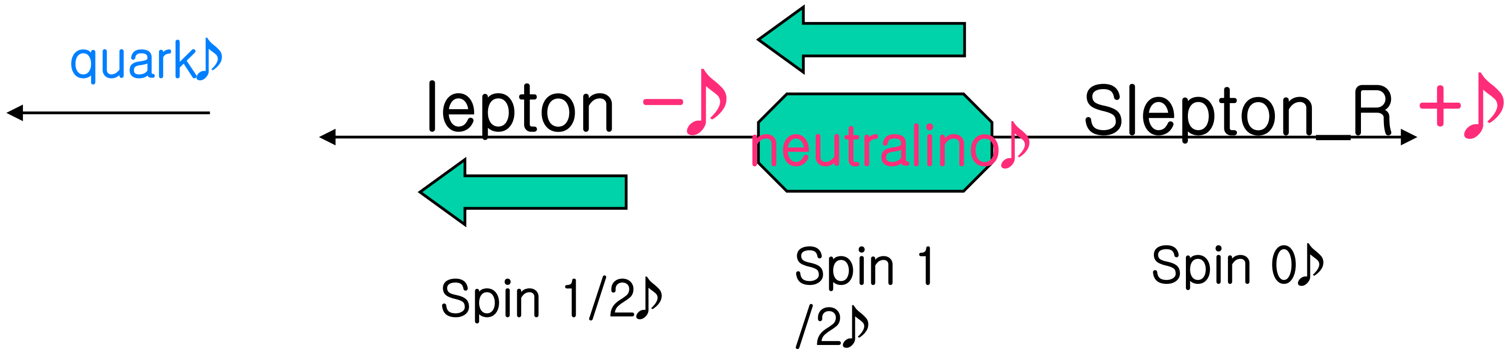
$\tilde{q}_L \rightarrow q + \tilde{\chi}_2^0$ decay♪



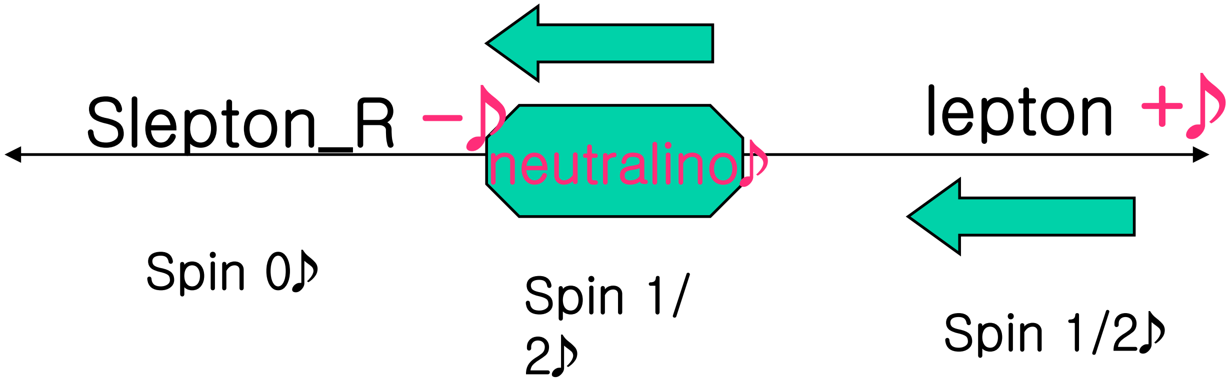
It is assumed that neutralino $\tilde{\chi}_2^0$ is largely Wino, ♪
so the branching ratios $\tilde{q}_R \rightarrow \tilde{\chi}_2^0 q$ are ♪
highly suppressed compared to the above decays♪

Polarized♪ $\tilde{\chi}_2^0 \to \tilde{l}_R^{\pm} + l^{\mp}$ decay♪

Right-handed lepton goes the same direction to the quark direction ♪



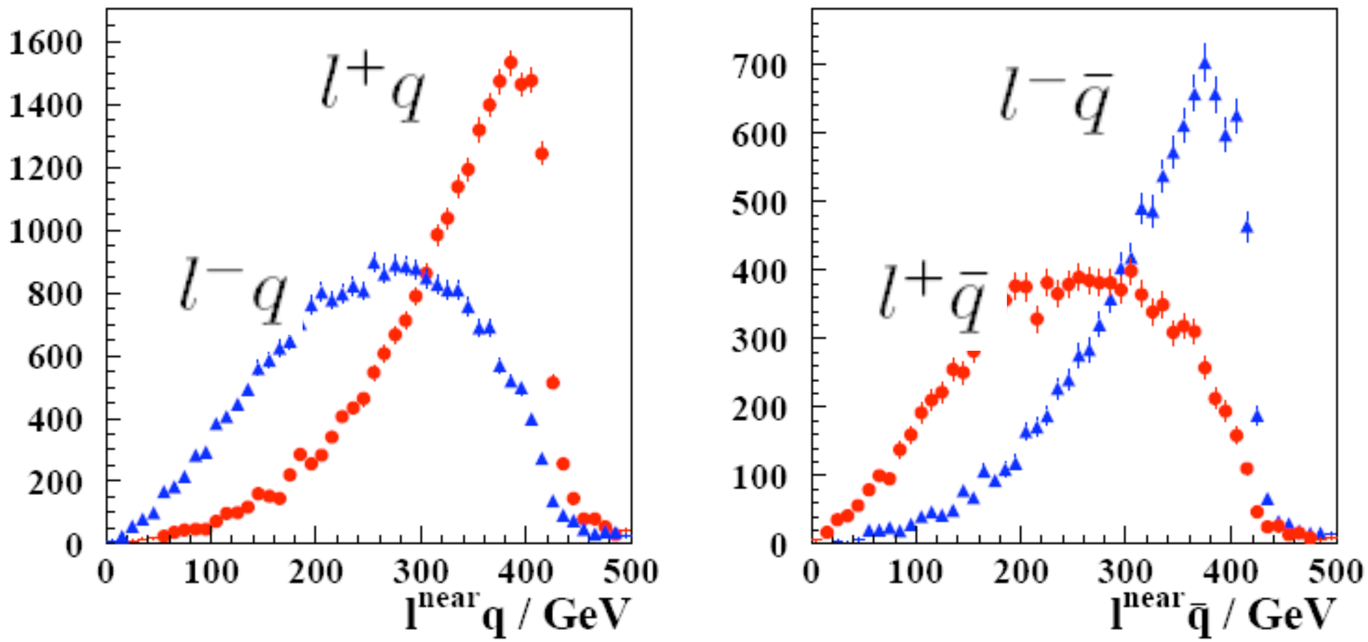
Right-handed anti-lepton goes the opposite to the quark direction♪



- Invariant mass distribution of quark + (near) lepton♪
  at the parton level for a test point♪

$\tilde{g}$	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{u}_L$	$\tilde{d}_L$	$\tilde{e}_R$	$\tilde{e}_L$
717	116	213	631	634	153	229

(mSUGRA point with m_0=100 GeV, $m_{1/2}$=300 GeV, A_0=300 GeV)♪



shows nice charge asymmetry ! ♪

(caused by spin correlations carried by the spin ½ neutralino)♪

● Experimental difficulties ♪
in making such a measurement♪

- In the decay of anti-squark ♪
  the asymmetry in the lepton charge distribution is ♪
  in the opposite sense to that from squark decays♪

♪

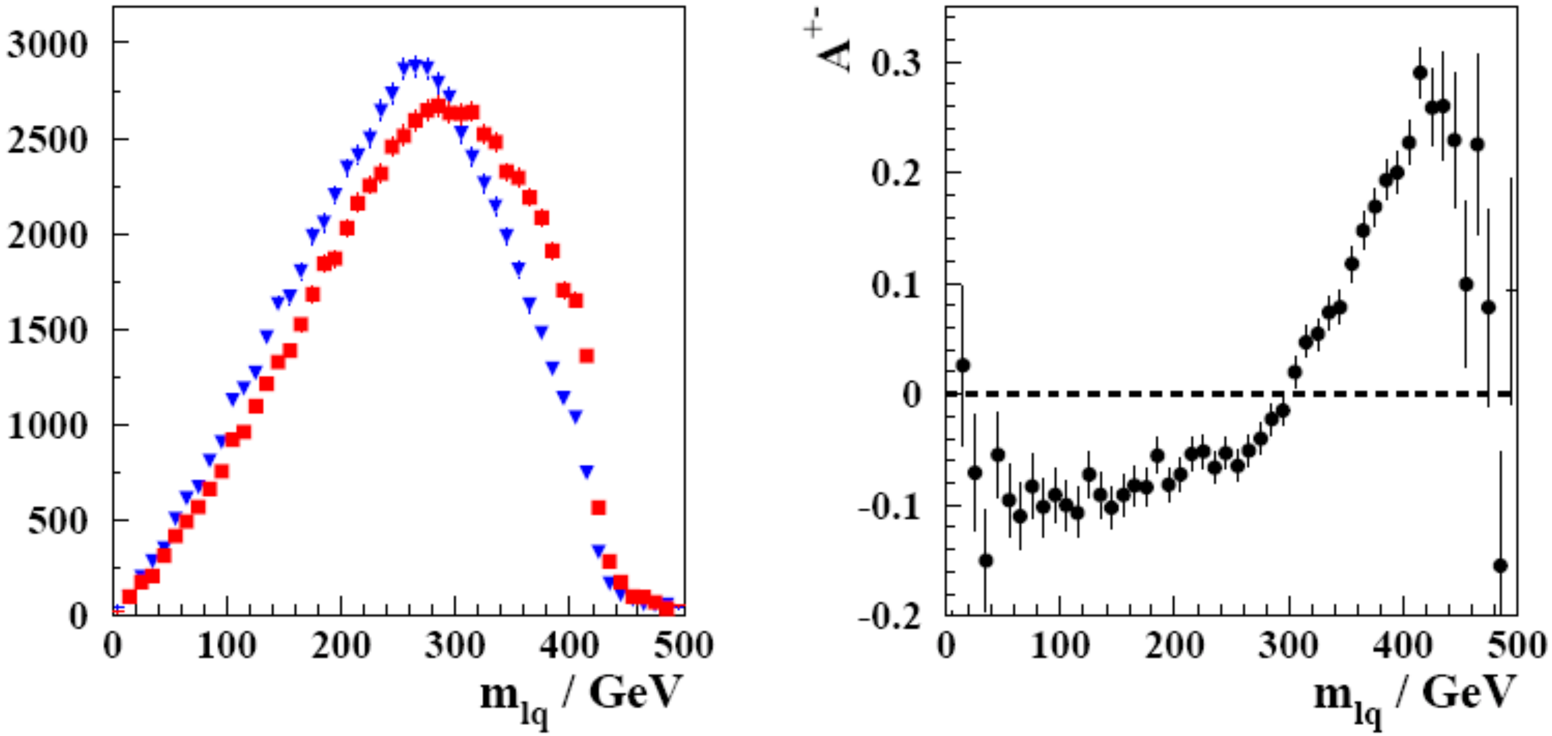
If equal numbers of squarks and anti-squarks were produced,♪
no spin information could be obtained♪

♪
♪
♪

- It will not be possible to distinguish the near lepton ♪
  from the far lepton on an event-by-event basis♪

❖ The l^-q and l^+q distributions (parton-level)

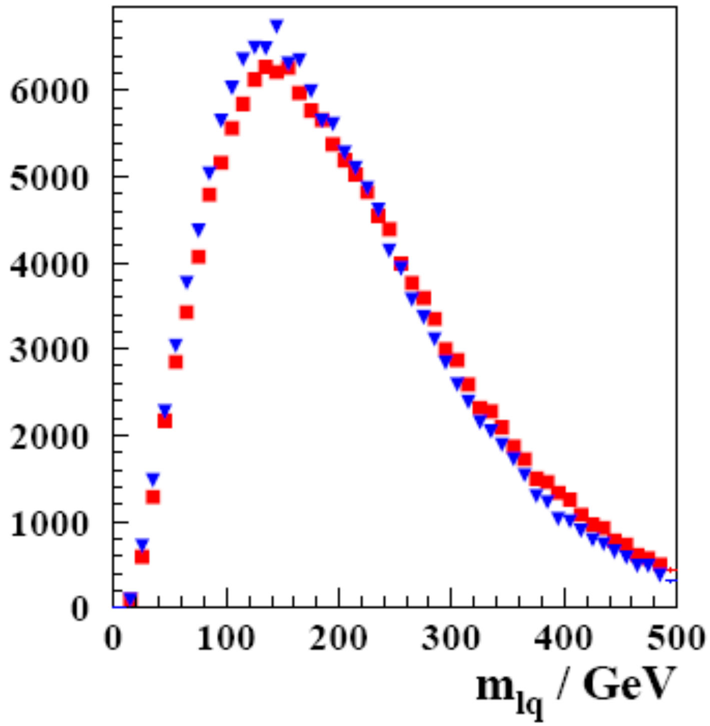
from both near and far leptons, and from squark and anti-squark

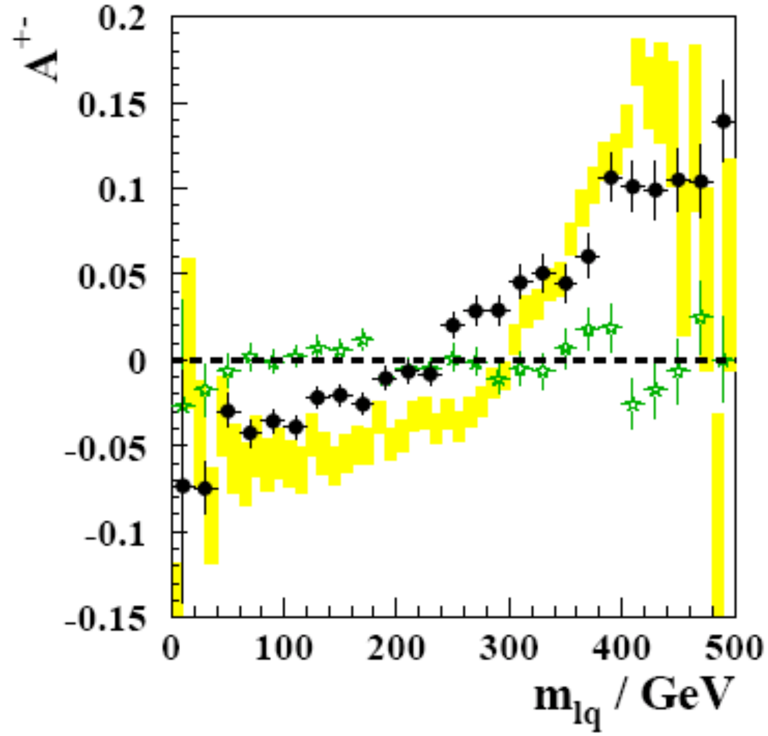


Charge asymmetry

$$A^{+-} \equiv \frac{s^+ - s^-}{s^+ + s^-}, \quad \text{where} \quad s^{\pm} = \frac{d\sigma}{d(m_{l^{\pm}q})} .$$

Including Detector Simulation and exp. cuts♪

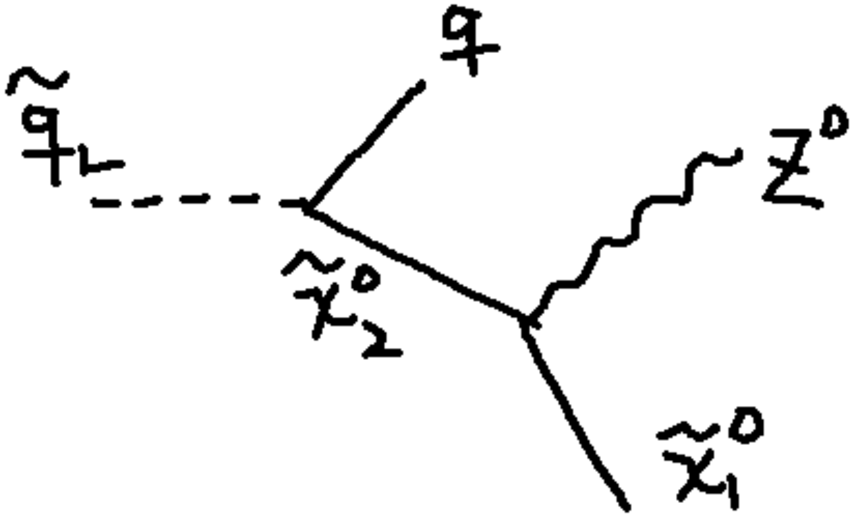




The charge asymmetry survives, and favours a spin-½ $\tilde{\chi}_2^0$

(black dots : with spin correlations,♪
green dots : switched off the spin correlations♪
yellow : parton-level asymmetry * 0.6)♪

- What if $\tilde{q}_L \to q\tilde{\chi}_2^0 \to qZ\tilde{\chi}_1^0.$?



Dominant decay mode if chi_2^0 is lighter than slepton.

Any useful spin correlation ?

Polarized neutralino decay

$$\tilde{\chi}_i^0(p, \hat{n}) \to \tilde{\chi}_j^0(q) + Z(k)$$

(YGK 2007)♪

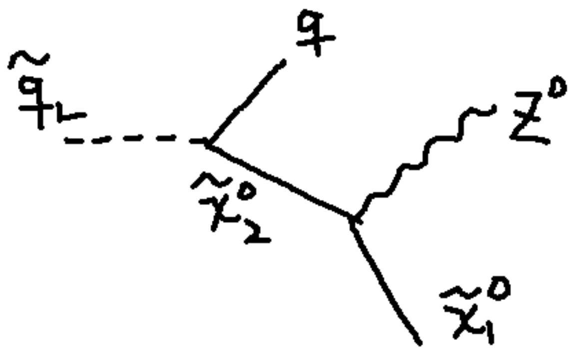
Matrix element squared♪

$$\sum_{\lambda=\pm,0} |\mathcal{M}|^2 \propto (|V|^2 + |A|^2)\left(q \cdot p + \frac{2}{m_Z^2}(k \cdot p)(k \cdot q)\right) + (|V|^2 - |A|^2)(-3m_j m_i) + 2\,\mathrm{Re}(VA^*)m_i\left((q \cdot n) + \frac{2}{m_Z^2}(k \cdot n)(k \cdot q)\right).$$

Vector coupling V is pure imaginary and axial-vector coupling A is pure real, due to Majorana nature of neutralinos.♪

$$\mathrm{Re}(VA^*) = 0$$

Flat angular distribution of Z boson w.r.t ♪
the polarization vector of neutralino♪
(Choi, Drees, Song 2006)♪



Wang and Yavin (2006)

" This could be a potentially golden channel considering the leptonic decay of the Z.

Unfortunately, there are no angular correlations since the neutralio-neutralino-Z is not even partially chiral. "

Z polarization can be reconstructed (!!) via leptonic angular distribution in Z → $l^+ l^-$

(YGK 2007)

- Differential decay widths with explicit helicity of Z boson

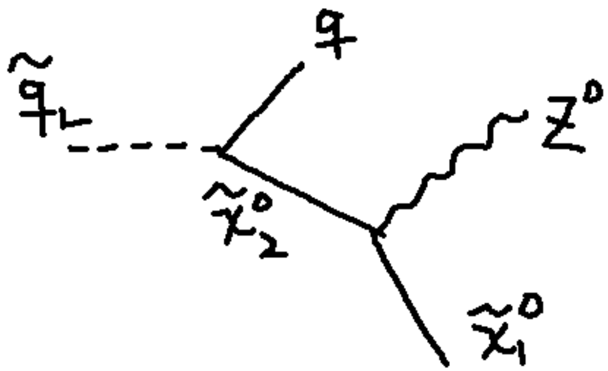
$$\frac{d\Gamma[\tilde{\chi}_i^0(\hat{n}) \to \tilde{\chi}_j^0 Z(\pm)]}{d\cos\theta} = \frac{g_Z^2 \lambda_Z^{1/2}}{64\pi m_i^3}[(|V|^2 + |A|^2) \times (m_i^2 + m_j^2 - m_Z^2) + (|V|^2 - |A|^2) \times (-2m_i m_j)](1 \pm \cos\theta), \quad (3)$$

for transverse Z

$$\cos\theta \equiv \hat{\mathrm{k}} \cdot \hat{\mathrm{n}}$$

$$\frac{d\Gamma[\tilde{\chi}_i^0(\hat{n}) \to \tilde{\chi}_j^0 Z(0)]}{d\cos\theta} = \frac{g_Z^2 \lambda_Z^{1/2}}{64\pi m_i^3}\left[(|V|^2 + |A|^2) \times \left(m_i^2 + m_j^2 - m_Z^2 + \frac{\lambda_Z}{m_Z^2}\right) + (|V|^2 - |A|^2)(-2m_i m_j)\right], \quad (4)$$

for longitudinal Z



$$\tilde{q}_L \to q\tilde{\chi}_2^0 \to qZ\tilde{\chi}_1^0.$$

(YGK 2007)♪

❖ Quark + Z boson invariant mass distributions♪

$$\frac{d\hat{\Gamma}(-)}{d\hat{m}} = \frac{4}{(1-\hat{m}_{\min}^2)^2}\hat{m}(\hat{m}^2 - \hat{m}_{\min}^2) \quad \text{for } Z(\lambda = -),$$

$$\frac{d\hat{\Gamma}(+)}{d\hat{m}} = \frac{4}{(1-\hat{m}_{\min}^2)^2}\hat{m}(1 - \hat{m}^2) \quad \text{for } Z(\lambda = +),$$

Can we see the polarization asymmetry at the LHC ?♪

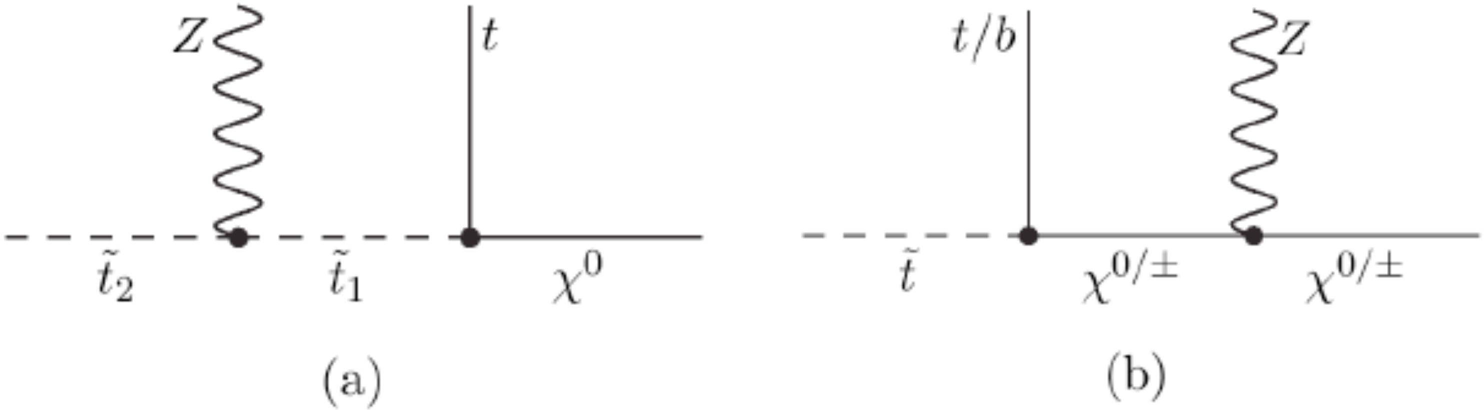
Work in progress♪

● Distinguishing decay chain with Z polarization♪
(work in progress)♪

Scalar → scalar + Z ♪ Fermion → fermion + Z♪

(only longitudinal Z possible)♪ (Both transverse and longitudianl Z)♪

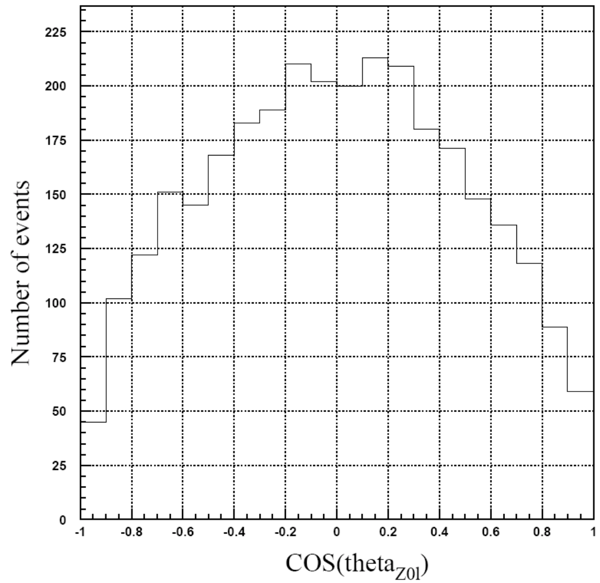
Ex)♪



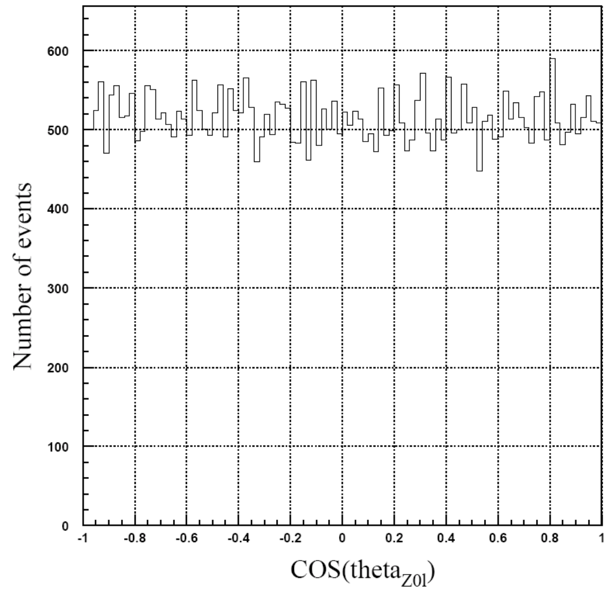
(a) Decay chain of SUSY golden region (b) An alternative chain ♪

(Perelstein and Spethmann 2007) ♪

Leptonic angular distribution of Z → $l^+ l^-$ in Z rest frame♪



for stop2 → stop1 + Z♪



for netralino2 → neutralino + Z♪

(Work in progress)♪

● Conclusions♪

- We introduced a new observable, 'gluino' m_{T2} ♪
  and showed that the maximum of the gluino m_{T2} ♪
  as a function of trial LSP mass has ♪
  a kink structure at true LSP mass from which ♪
  gluino mass and LSP mass can be determined ♪
  simultaneously. ♪

♪

- Measurement of spin at LHC is important to see ♪
  the nature of New Physics.♪
  Z polarization might be useful for the purpose♪

♪

BACKUP♪

Vector and Axial vector couplings of Ni-Nj-Z vertex in MSSM♪

$$V = -\frac{i}{2}\,\mathrm{Im}(N_{j3}N_{i3}^{*} - N_{j4}N_{i4}^{*}),$$

$$A = \frac{1}{2}\,\mathrm{Re}(N_{j3}N_{i3}^{*} - N_{j4}N_{i4}^{*}),$$

Lepton angular distribution in Z → l+ l- (in Z rest frame)♪

$$\frac{1}{\Gamma[Z \to l^+l^-]}\,\frac{d\Gamma[Z(\pm) \to l^+l^-]}{d\cos\theta_l} = \frac{3}{8}\left[1 + \cos^2\theta_l \pm 2\,\xi_l\cos\theta_l\right]$$

$$\frac{1}{\Gamma[Z \to l^+l^-]}\,\frac{d\Gamma[Z(0) \to l^+l^-]}{d\cos\theta_l} = \frac{3}{4}\,\sin^2\theta_l\ ,$$

$$\xi_l = 2v_l a_l/(v_l^2 + a_l^2) \simeq -0.147$$

The balanced m_{T2} solution

$$\left(m_{T2}^{\text{bal}}\right)^2 = m_\chi^2 + A_T + \sqrt{\left(1 + \frac{4m_\chi^2}{2A_T - \left(m_{vis}^{(1)}\right)^2 - \left(m_{vis}^{(2)}\right)^2}\right)\left(A_T^2 - \left(m_{vis}^{(1)} m_{vis}^{(2)}\right)^2\right)},$$

where

$$\begin{aligned} A_T &\equiv \alpha_1^0 \alpha_2^0 + \vec{\alpha_1} \cdot \vec{\alpha_2} \\ &= E_T^{vis(1)} E_T^{vis(2)} + \mathrm{p}_T^{vis(1)} \cdot \mathrm{p}_T^{vis(2)} \end{aligned}$$

For the m_{T2} solution, we can consider ♪
the first decay products as having total mass m_{T2} ,♪
total transverse momentum♪ $p_T^{(1)} = p_T^{q(1)} + p_T^{\chi(1)}$
and total transverse energy $E_T^{(1)} = E_T^{q(1)} + E_T^{\chi(1)}$

Similarly, for the second products, we have ♪
m_{T2}, $p_T^{(2)} = p_T^{q(2)} + p_T^{\chi(2)}$,♪ $E_T^{(2)} = E_T^{q(2)} + E_T^{\chi(2)}$

$$p_T^{(1)} = -p_T^{(2)} , \quad E_T^{(1)} = E_T^{(2)}$$ ♪

Arbitrary back-to-back transverse boost the systems♪
♪
$$p_T^{(1)\prime} = \gamma p_T^{(1)} + \gamma\beta E_T^{(1)}$$
♪
$$p_T^{(2)\prime} = \gamma p_T^{(2)} - \gamma\beta E_T^{(2)}$$
♪
Then,♪ $$p_T^{(1)\prime} + p_T^{(2)\prime} = \gamma(p_T^{(1)} + p_T^{(2)}) = 0.$$
♪
$$p_T^{\chi(1)\prime} + p_T^{\chi(2)\prime} = -(p_T^{q(1)\prime} + p_T^{q(2)\prime})$$
♪
We have valid splitting of total MET and thus m_{T2} solution. ♪

● Inclusive m_{T2} (II)

Invariant mass of visible part for gluino deca
y

Sqrt[(P_gluino – P_lsp)^2] in generator level

(Cho,Choi,YGK,Park)